

Title: PSI 2016/2017 Quantum Field Theory I - Lecture 11

Date: Oct 25, 2016 09:00 AM

URL: <http://pirsa.org/16100011>

Abstract:

$$T(\psi_a(x)\chi_b(y)) = \begin{cases} \psi_a(x)\chi_b(y) & \text{if } x^0 > y^0 \\ -\chi_b(y)\psi_a(x) & \text{if } y^0 > x^0 \end{cases}$$

$$\begin{aligned} S_F(x-y)_{ab} &= \langle 0 | T \psi_a(x) \bar{\psi}_b(y) | 0 \rangle \\ &= \begin{cases} \langle 0 | \psi_a(x) \bar{\psi}_b(y) | 0 \rangle, & x^0 > y^0 \\ -\langle 0 | \bar{\psi}_b(y) \psi_a(x) | 0 \rangle, & y^0 > x^0 \end{cases} \end{aligned}$$

$$(x-y)^2 < 0$$



$$S_F = \langle 0 | T \psi(x) \bar{\psi}(y) | 0 \rangle$$

$$= i \int \frac{d^4 p}{(2\pi)^4} \frac{\not{p} + m}{p^2 - m^2 + i\epsilon} e^{-i p \cdot (x-y)}$$

$$(i \not{\partial}_x - m) S_F(x-y) = (2\pi)^4 \delta^4(x-y)$$

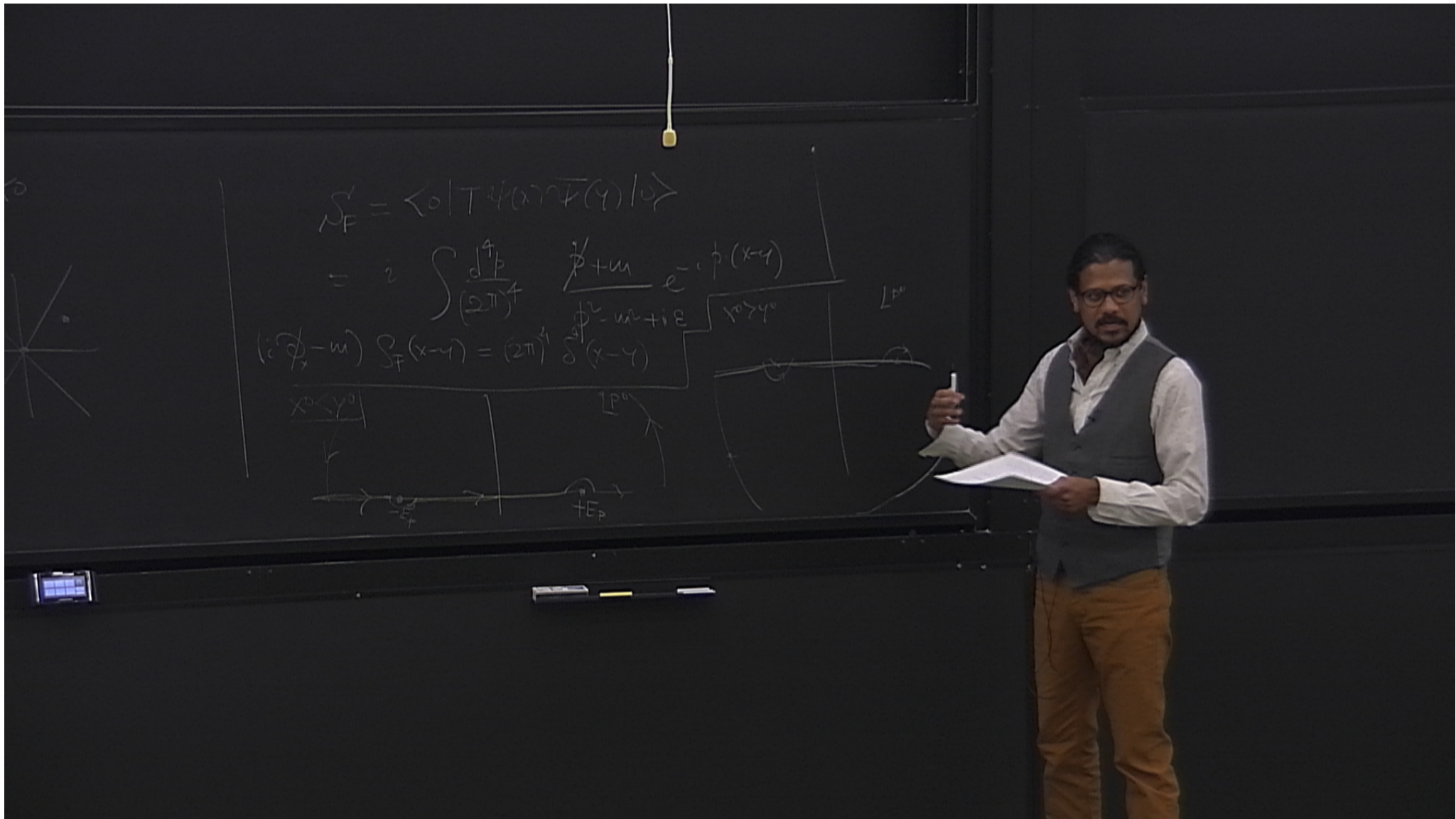
$$\begin{aligned}
 S_F &= \langle 0 | T \psi(x) \bar{\psi}(y) | 0 \rangle \\
 &= i \int \frac{d^4 p}{(2\pi)^4} \frac{\not{p} + m}{p^2 - m^2 + i\epsilon} e^{-i p \cdot (x-y)}
 \end{aligned}$$

$(i \not{\partial}_x - m) S_F(x-y) = (2\pi)^4 \delta(x-y)$

$x^0 < y^0$ $x^0 > y^0$

$-E_p$ $+E_p$

p^0



Yukawa Theory

1934 — Interacting nucleons by exchanging

pseudoscalar meson

Scattering $2 \rightarrow 2$

Heisenberg picture $|K_A, K_B\rangle_{in}$ in the asymptotic past
 $|P_1, P_2\rangle_{out}$ " " " future.

$$\begin{aligned} & \langle P_1, P_2 | K_A, K_B \rangle_{out} \\ &= \langle P_1, P_2 | S | K_A, K_B \rangle_{in} \end{aligned}$$

exchanging pseudoscalar meson

$$\begin{aligned} & \text{out} \langle p_1, p_2 | K_A, K_B \rangle_{\text{in}} \\ &= \langle \underline{p_1}, \underline{p_2} | S | \underline{K_A}, \underline{K_B} \rangle \end{aligned}$$

the part
future

$$\begin{aligned} S &= \mathbb{1} + iT \\ \langle \underline{p_1}, \underline{p_2} | iT | K_A, K_B \rangle &= (2\pi)^4 \delta^4(K_A + K_B - p_1 - p_2) \\ &\quad \times i \underbrace{\mathcal{M}(K_A K_B \rightarrow p_1 p_2)} \end{aligned}$$

Particles in $|in\rangle$ are for separated & non-interacting } by integrating with wave packets
 " " (out) " " " " " " " " " " " "

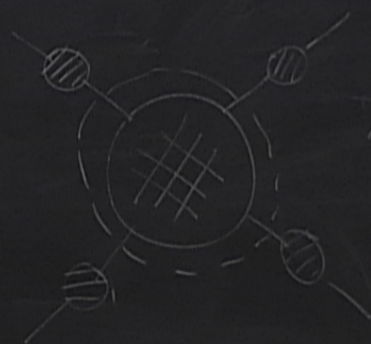
$$\# |in\rangle = a_{\underline{1}}^\dagger(\underline{k}_A) a_{\underline{1}}^\dagger(\underline{k}_B) |0\rangle$$

$$\langle \underline{p}_1, \underline{p}_2 | iT | \underline{k}_A, \underline{k}_B \rangle = \lim_{T \rightarrow \infty (1-i\epsilon)} \left\{ \langle \underline{p}_1, \underline{p}_2 | T \exp \left[-i \int_{-T}^{+T} dt H_I^{int}(t) \right] | \underline{k}_A, \underline{k}_B \rangle_0 \right\}$$

$\left. \begin{array}{l} \text{contracting} \\ \dots \end{array} \right\} \text{ by integrating with wave packets}$

$$\left. \left[\exp \left[-i \int_{-T}^{+T} dt H_I^{\text{int}}(t) \right] |K_A, E_B\rangle_0 \right\} \right.$$

LSZ: $\langle \Omega | T(\phi(x_1) \dots \phi(x_n)) | \Omega \rangle$



$\equiv \langle 0 | T \left[\exp(-i \int dt H_I(t)) \right] | 0 \rangle$
 $\phi_1 \phi_2 \phi_3 \phi_4$

$$\langle 0 | T(\phi_1) | 0 \rangle$$

$$\mathcal{L}_{\text{total}} = \mathcal{L}_f + \mathcal{L}_\phi + \mathcal{L}_{\text{Yuk}}$$

$$\mathcal{L}_f = \bar{\psi} (i \not{\partial} - m) \psi$$

$$\mathcal{L}_\phi = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} M^2 \phi^2$$

$$\mathcal{L}_{\text{Yuk}} = -\lambda \phi \bar{\psi} \psi$$

$$[\lambda] = 0$$

$$\psi(x) \rightarrow e^{i\alpha} \psi(x)$$

$$\bar{\psi}(x) \rightarrow e^{-i\alpha} \bar{\psi}(x)$$

Conserved charge

→ Nucleon #

$$\psi(x) \rightarrow e^{i\alpha} \psi(x)$$

$$\bar{\psi}(x) \rightarrow e^{-i\alpha} \bar{\psi}(x)$$

Conserved charge
 $\rightarrow$ Nucleon #

$$\langle f | T \exp \left[-i \int d^4x \mathcal{H}_{int}(x) \right] | i \rangle$$

$$T[\psi_a(x) \bar{\psi}_b(y)] = \langle \psi_a(x) \bar{\psi}_b(y) \rangle + \overline{\psi_a(x) \bar{\psi}_b(y)}$$

$$\overline{\psi_a(x) \bar{\psi}_b(y)} = \begin{cases} \{ \psi_a^{(+)}(x), \bar{\psi}_b^{(-)}(y) \} & \text{if } x^0 > y^0 \\ -\{ \bar{\psi}_b^{(+)}(y), \psi_a^{(-)}(x) \} & \text{if } y^0 > x^0 \end{cases}$$

$$= \langle 0 | T(\psi_a(x) \bar{\psi}_b(y)) | 0 \rangle$$

$$\overline{\psi \psi} = 0$$

$$\overline{\bar{\psi} \bar{\psi}} = 0$$

$$T \exp \left[-i \int d^4x \mathcal{H}_{int}(x) \right] |i\rangle$$

$$[\psi_a(x) \bar{\psi}_b(y)] = :\psi_a(x) \bar{\psi}_b(y): + \overbrace{\psi_a(x) \bar{\psi}_b(y)}$$

$$\overbrace{\psi_a(x) \bar{\psi}_b(y)} = \begin{cases} \{\psi_a^{(+)}(x), \bar{\psi}_b^{(-)}(y)\} & \text{if } x^0 > y^0 \\ -\{\bar{\psi}_b^{(+)}(y), \psi_a^{(-)}(x)\} & \text{if } y^0 > x^0 \end{cases}$$

$$= \langle 0 | T (\psi_a(x) \bar{\psi}_b(y)) | 0 \rangle$$

$$\overbrace{\psi \psi} = 0$$

$$\overbrace{\bar{\psi} \bar{\psi}} = 0$$

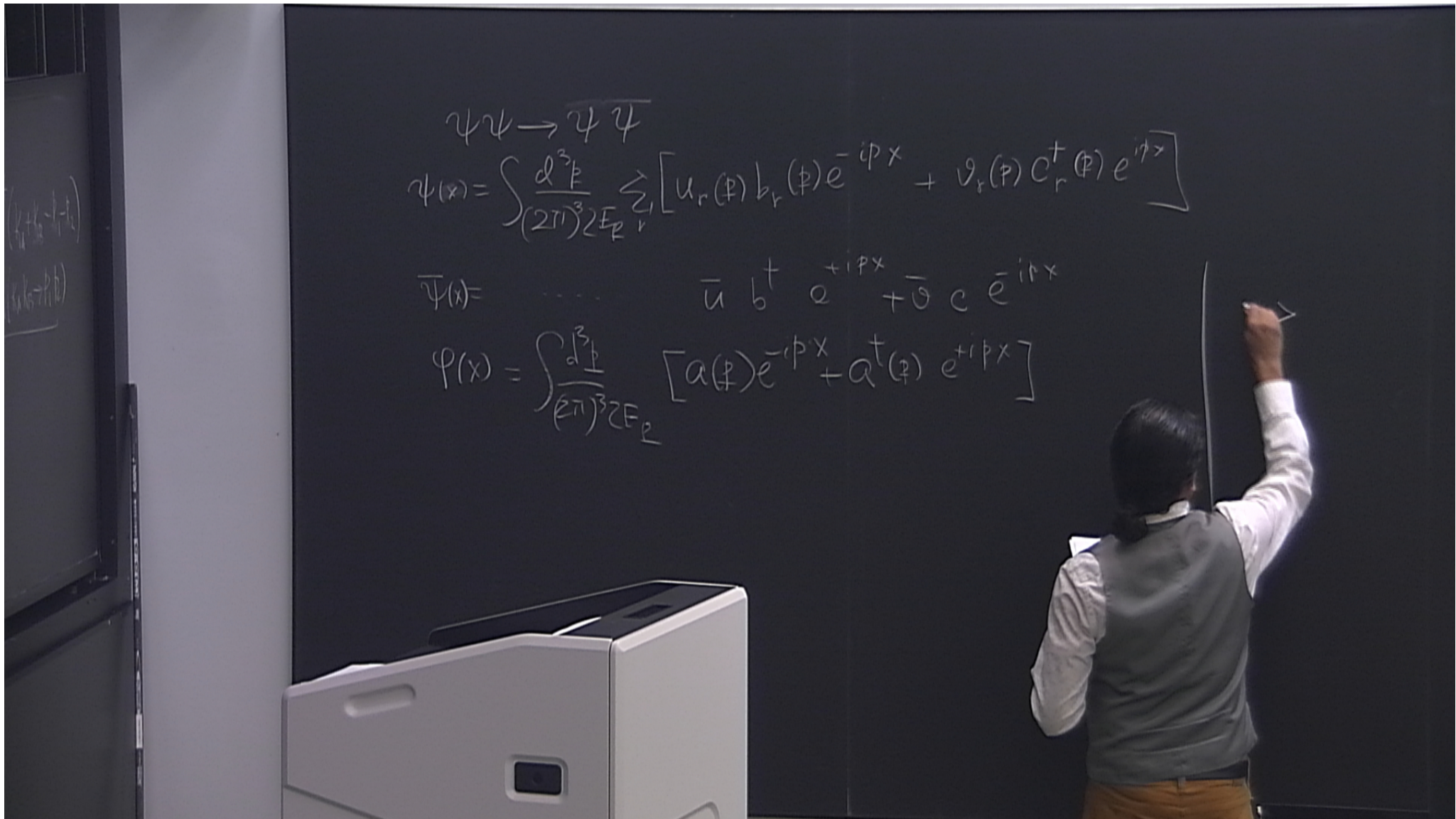
$$T [\psi_1 \bar{\psi}_2 \psi_3 \dots]$$

$$= :\psi_1 \bar{\psi}_2 \psi_3 \dots:$$

+ all possible contractions :

+ ...

$$\begin{aligned} & \overbrace{\psi_1 \bar{\psi}_2 \psi_3 \dots} \\ &= - \overbrace{\psi_1 \bar{\psi}_3 \psi_2 \dots} \\ &= - \overbrace{\psi_1 \bar{\psi}_4 \psi_2 \psi_3 \dots} \end{aligned}$$



$$\psi\psi \rightarrow \psi\psi$$

$$\psi(x) = \int \frac{d^3p}{(2\pi)^3 2E_p} \sum_r \left[u_r(p) b_r(p) e^{-ipx} + v_r(p) c_r^\dagger(p) e^{ipx} \right]$$

$$\bar{\psi}(x) = \dots \bar{u} b^\dagger e^{+ipx} + \bar{v} c e^{-ipx}$$

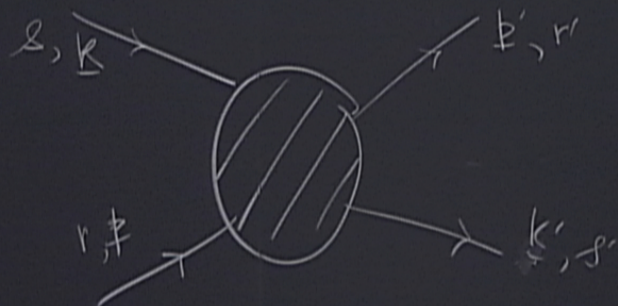
$$\varphi(x) = \int \frac{d^3p}{(2\pi)^3 2E_p} \left[a(p) e^{-ipx} + a^\dagger(p) e^{ipx} \right]$$

$$e^{i\gamma}$$

$$|in\rangle = b_s^\dagger(k) b_r^\dagger(p) |0\rangle$$

$$|out\rangle = b_{s'}^\dagger(k') b_{r'}^\dagger(p') |0\rangle$$

$$\langle out| = \langle 0| b_{r'}(p') b_{s'}(k')$$



$$H_{int}^I = \lambda \int d^3x \varphi(x) \bar{\psi}(x) \psi(x)$$

$$\varphi = \varphi^{(+)} + \varphi^{(-)}$$

$$\varphi^{(+)}(x) |in\rangle = 0$$

$$\langle out| \varphi^{(-)}(x) = 0$$

$$\mathcal{O}(\lambda) = 0$$

$$\underline{\mathcal{O}(\lambda^2)}$$

$$\begin{array}{l}
 \int d^3x \, \varphi(x) \bar{\psi}(x) \psi(x) \\
 \varphi^{(+)} + \varphi^{(-)} \\
 |in\rangle = 0 \\
 \psi(x) = 0
 \end{array}
 \quad
 \begin{array}{l}
 \mathcal{O}(\lambda) = 0 \\
 \mathcal{O}(\lambda^2) \\
 iT = \frac{(-i\lambda)^2}{2!} \overbrace{\varphi(x_1) \varphi(x_2)}^{\varphi(x_1) \varphi(x_2)} \langle f | T \int d^4x_1 d^4x_2 (\bar{\psi}\psi)_1 (\bar{\psi}\psi)_2 | in \rangle \\
 = \frac{(-i\lambda)^2}{2!} \langle f | \int d^4x_1 d^4x_2 :(\bar{\psi}\psi)_1 (\bar{\psi}\psi)_2: | in \rangle \overbrace{\varphi(x_1) \varphi(x_2)}^{\varphi(x_1) \varphi(x_2)}
 \end{array}$$

$$\begin{aligned}
 & \int d^3x \, \varphi(x) \bar{\psi}(x) \psi(x) \\
 & \varphi^{(+)} + \varphi^{(-)} \\
 & |in\rangle = 0 \\
 & \psi(x) = 0
 \end{aligned}
 \quad \left| \begin{aligned}
 & \mathcal{O}(\lambda) = 0 \\
 & \mathcal{O}(\lambda^2) \\
 & iT = \frac{(-i\lambda)^2}{2!} \overbrace{\varphi(x_1) \varphi(x_2)}^{\varphi(x_1) \varphi(x_2)} \langle f | T \int d^4x_1 d^4x_2 (\bar{\psi}\psi)_1 (\bar{\psi}\psi)_2 | in \rangle \\
 & = \frac{(-i\lambda)^2}{2!} \langle f | \int d^4x_1 d^4x_2 :(\bar{\psi}\psi)_1 (\bar{\psi}\psi)_2: | in \rangle \overbrace{\varphi(x_1) \varphi(x_2)}^{\varphi(x_1) \varphi(x_2)}
 \end{aligned} \right.$$

$$H_{int} = \lambda \int d^3x \, \psi(x) \bar{\psi}(x) \psi(x)$$

$$\psi = \psi^{(+)} + \psi^{(-)}$$

$$\psi^{(+)}(x) |in\rangle = 0$$

$$\langle out | \psi^{(-)}(x) = 0$$

$$\mathcal{O}(\lambda) = 0$$

$$\mathcal{O}(\lambda^2)$$

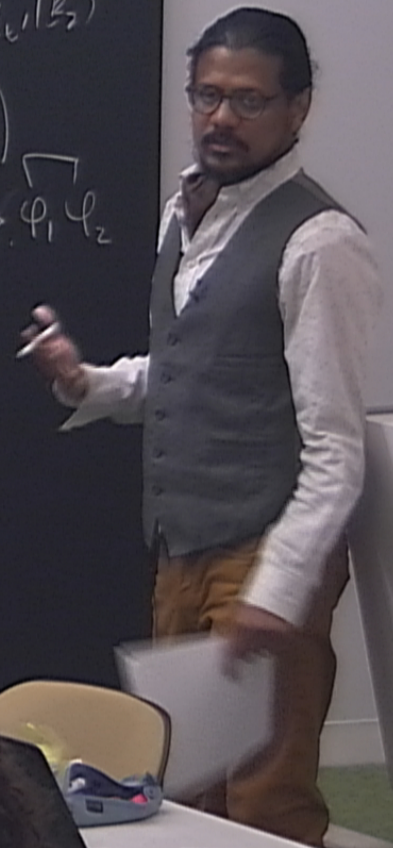
$$\overline{\psi(x_1) \psi(x_2)}$$

$$iT = \frac{(-i\lambda)^2}{2!} \langle f | T \int d^4x_1 d^4x_2 (\bar{\psi}\psi)_1 (\bar{\psi}\psi)_2 | in \rangle$$

$$= \frac{(-i\lambda)^2}{2!} \langle f | \int d^4x_1 d^4x_2 :(\bar{\psi}\psi)_1 (\bar{\psi}\psi)_2: | in \rangle \overline{\psi(x_1) \psi(x_2)}$$

$$\begin{aligned}
 x) \quad \mathcal{O}(\lambda) &= 0 \\
 \frac{\mathcal{O}(\lambda^2)}{i\Gamma} &= \frac{(-i\lambda)^2}{2!} \overbrace{\varphi(x_1) \varphi(x_2)}^{\text{}} \langle f | T \int d^4x_1 \int d^4x_2 (\bar{\psi}\psi)_1 (\bar{\psi}\psi)_2 | in \rangle \\
 &= \frac{(-i\lambda)^2}{2!} \langle f | \int d^4x_1 \int d^4x_2 :(\bar{\psi}\psi)_1 (\bar{\psi}\psi)_2: | in \rangle \overbrace{\varphi(x_1) \varphi(x_2)}^{\text{}}
 \end{aligned}$$

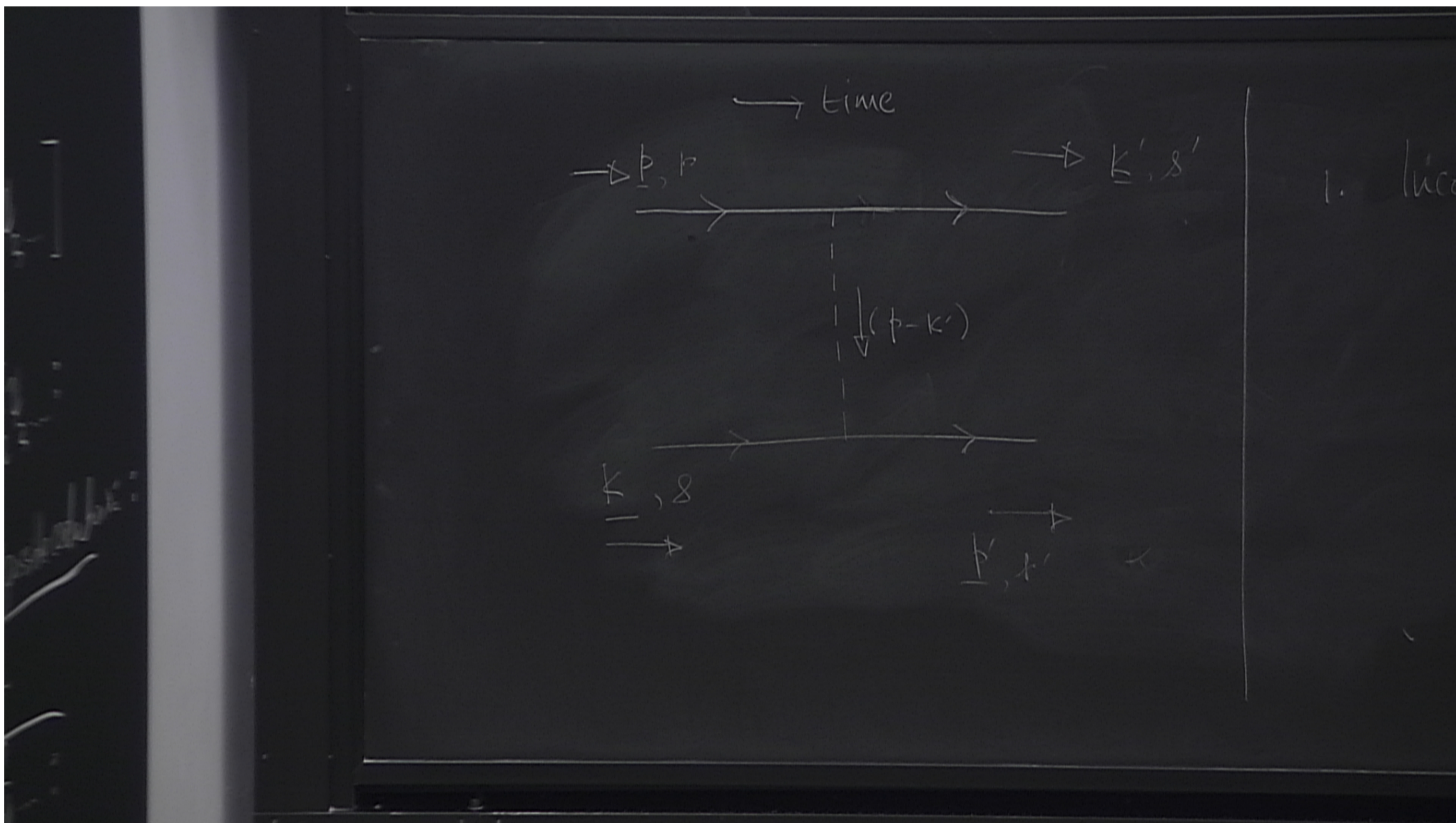
$$i T = - \frac{(-i\lambda)^2}{2!} \int d^4x_1 d^4x_2 \int \frac{d^3k_1 d^3k_2}{(2\pi)^6 4E_{k_1} E_{k_2}} \times \langle f | \sum_{l,l'} \left(\psi_1^{(-)} \cdot u_l(k_1) \bar{\psi}_2^{(-)} \cdot u_{l'}(k_2) \right. \\ \left. e^{-i(k_1 \cdot x_1 - k_2 \cdot x_2)} b_l(k_1) b_{l'}(k_2) \right) \left[b_s^\dagger(k) b_r^\dagger(k) |0\rangle \cdot \psi_1 \psi_2 \right]$$



$$i T = - \frac{(-i\lambda)^2}{2!} \int d^4x_1 d^4x_2 \int \frac{d^3k_1 d^3k_2}{(2\pi)^6 4E_{k_1} E_{k_2}} \times \langle f | \sum_{l,l'} \left(\psi_1^{(-)} \cdot u_l(k_1) \bar{\psi}_2^{(-)} \cdot u_{l'}(k_2) \right. \\ \left. e^{-i(k_1 \cdot x_1 - k_2 \cdot x_2)} b_l(k_1) b_{l'}(k_2) \right) \overbrace{b_s^\dagger(k) b_r^\dagger(k') | 0 \rangle}^{\psi_1 \psi_2}$$

$$i T = - \frac{(i\lambda)^2}{2!} \int d^4x_1 d^4x_2 \langle 0 | b_{r'}(k') b_{s'}(k) \left\{ - \bar{\psi}_1^{(-)} \cdot u_{s'}(k) \psi_2^{(-)} \cdot u_{r'}(k') e^{-i(k \cdot x_1 - k' \cdot x_2)} \right. \\ \left. - \bar{\psi}_1^{(-)} \cdot u_{r'}(k') \psi_2^{(-)} \cdot u_{s'}(k) e^{-i(k' \cdot x_1 - k \cdot x_2)} \right\} | 0 \rangle$$

$$\begin{aligned}
 iT &= -\frac{(i\lambda)^2}{2!} \int d^4x_1 d^4x_2 \langle 0 | b_{r'}(E') b_{s'}(E') \left\{ -\bar{\psi}_1^{(-)} u_{s'}(E) \bar{\psi}_2^{(-)} u_{r'}(E) e^{-i(p_1 x_1 - E' x_2)} \right\} | 0 \rangle \bar{\psi}_1 \psi_2 \\
 &= (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \left\{ (-i\lambda)^2 \left[\frac{(\bar{u}_{r'}(p_1) u_{s'}(E)) (\bar{u}_{s'}(p_2) u_{r'}(E))}{(p_1 - p_2)^2 - M^2 + i\epsilon} - \frac{(\bar{u}_{s'}(p_1) u_{r'}(E)) (\bar{u}_{r'}(p_2) u_{s'}(E))}{(p_1 - p_2)^2 - M^2 + i\epsilon} \right] \right\}
 \end{aligned}$$



3. We label fermion with an arrow
 that flows in the same direction as time
 for the external states. You write
 down the spinor factor from left to
 right as you follow the fermion
 arrow in the ^{opposite direction} direction.

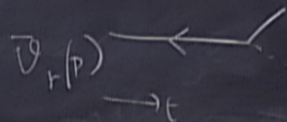
$$\psi(x) \rightarrow e$$

$$\bar{\psi}(x) \rightarrow e$$

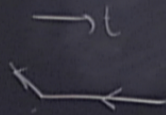
Conserved

→

4. Incoming anti-fermion $\bar{\psi}_r(p)$



5. Out anti-fermion



6.

