Title: PSI 2016/2017 Quantum Field Theory I - Lecture 10

Date: Oct 24, 2016 09:00 AM

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Abstract:

Applications:

$$\mathcal{L}_{\text{Dirac}} = \bar{\psi}(i\not{\partial} - m)\psi$$

1. Internal global phase rotation: $\psi(x) \longrightarrow e^{i\alpha}\psi(x)$, $\alpha \rightarrow$ constant

$$\bar{\psi}(x) \longrightarrow \bar{\psi}(x)\, e^{-i\alpha}$$

$$\delta\psi = i\alpha\psi$$

$$\delta\bar{\psi} = -i\alpha\bar{\psi}$$

$$\delta\mathcal{L} = 0$$

$$J^\mu = \bar{\psi}\gamma^\mu\psi$$

$$\partial_\mu J^\mu = 0 \sim \text{e.o.m}$$

2. $m \to 0$ Chiral symmetry

$$\psi(x) \longrightarrow e^{i\alpha\gamma_5}\psi(x)$$

$$\bar{\psi}(x) = \psi^\dagger(x)\gamma^0 \longrightarrow \psi^\dagger(x)\, e^{-i\alpha\gamma_5}\gamma^0$$

$$= \bar{\psi}(x)\, e^{+i\alpha\gamma_5}$$

$$\bar{\psi}\gamma^\mu\partial_\mu\psi$$

$$J_5^\mu = \bar{\psi}\gamma^\mu\gamma_5\psi \qquad \partial_\mu J^\mu_5 = 0$$

$$J^{\mu} = a^{\nu}\, i\,\bar{\Psi}\gamma^{\mu}\partial_{\nu}\psi$$

$$T^{\mu}{}_{\nu} = i\,\bar{\Psi}\gamma^{\mu}\partial_{\nu}\psi$$

$$P_{\nu} = \int d^3x\, i\,\bar{\Psi}\gamma^{0}\partial_{\nu}\psi$$ ~ Energy-momentum four vector.

4. Lorentz transformation.

Quantization

Spin statistics theorem. 3+1 dim ⇒

half-integer spin particles satisfy Fermi-Dirac stat.

Integer spin " " Bose-Einstein "

$\partial_\mu J^\mu = 0$

$$\psi(\underline{x}) = \int \frac{d^3p}{(2\pi)^3\, 2E_{\underline{p}}} \sum_{s=1}^{2} \left\{ u_s(p)\, b_s(p)\, e^{ip\cdot \underline{x}} + v_s(p)\, c_s^\dagger(p)\, e^{-ip\cdot \underline{x}} \right\}$$

$$\psi^\dagger(x) = \qquad u^\dagger \quad b^\dagger \quad e^{-ip\cdot x} \qquad v^\dagger \quad c \quad e^{+ip\cdot \underline{x}}$$

$$\Pi_a(x) = \frac{\partial \mathcal{L}}{\partial \dot{\psi}_a} = i\psi^\dagger_a(x)$$

$$\{\psi_a(\underline{x}), \Pi_b(\underline{y})\} = i\hbar\, \delta_{ab}\, \delta^3(\underline{x}-\underline{y})$$

$$\{\psi_a(\underline{x}), \psi^\dagger_b(\underline{y})\} = \hbar\, \delta_{ab}\, \delta^3(\underline{x}-\underline{y})$$

$$\{\psi, \psi\} = 0$$

$$\{\bar{\psi}, \bar{\psi}\} = 0$$

$$\{ b_s(\mathbf{p}), b_r^\dagger(\mathbf{p}') \} = (2\pi)^3 2E_{\mathbf{p}} \, \delta_{sr} \, \delta^3(\mathbf{p} - \mathbf{p}')$$

$$\{ c_s(\mathbf{p}), c^\dagger_{s'}(\mathbf{p}') \} = (2\pi)^3 2E_{\mathbf{p}} \, \delta_{ss'} \, \delta^3(\mathbf{p} - \mathbf{p}')$$

$$\{b, b\} = \{b^\dagger, b^\dagger\} = \{c, c\} = \{c^\dagger, c^\dagger\} = \cdots = 0$$

$$\{b, c\} = \{b^\dagger, c\} = \cdots = 0$$

$$\left\{\psi_a(x), \psi^\dagger_b(y)\right\} = \int \frac{d^3p_1\, d^3p_2}{(2\pi)^6\, 2E_{p_1} 2E_{p_2}} \sum_{s,r} \Big[u_s(p_1)_a\, u_r^\dagger(p_2)_b \left\{ b_s(p_1), b_r^\dagger(p_2) \right\} e^{i p_1\cdot \underline{x} - i p_2 \cdot \underline{y}}$$

$$+\, v_s(p_1)_a\, v_r^\dagger(p_2)_b \left\{ c_s(p_1), c_r^\dagger(p_2) \right\} e^{-i p_1 \cdot \underline{x} + i p_2 \cdot \underline{y}} \Big]$$

$$\sum_s u_s \bar{u}_s = (\not{p} + m)$$

$$\sum_s v_s \bar{v}_s = (\not{p} - m)$$

$$\int \frac{d^3p}{(2\pi)^3\, 2E_{\underline{p}}} \Big\{ \left[\left(\gamma^0 p^0 - \cancel{\gamma^i p^i} + \cancel{m}\right)\gamma^0\right]_{ab} e^{i \underline{p}\cdot(\underline{x}-\underline{y})}$$

$$+ \left[\left(\gamma^0 p^0 + \cancel{\gamma^i p^i} - \cancel{m}\right)\gamma^0\right]_{ab} \quad ''$$

$$= \delta_{ab}\, \delta^3(\underline{x} - \underline{y})$$

$$p_2$$

$$= -\,|(\underline{p}_2, s_2), (\underline{p}_1, s_1)\rangle$$

$$b^\dagger_{s_1}(\underline{p}_1)\, b^\dagger_{s_2}(\underline{p}_2)|0\rangle = |(\underline{p}_1, s_1), (\underline{p}_2, s_2)\rangle$$

$$\underline{p}_1, s_1 \longleftrightarrow \underline{p}_2, s_2$$

$$H=\int d^3x\,\{\pi_a\dot{\psi}_a-\mathcal{L}\}$$

$$=\int d^3x\,\{-i\bar{\psi}\gamma^i\partial_i\psi+m\bar{\psi}\psi\}$$

$$(i\not{\partial}-m)u_s e^{-ip\cdot x}=0$$

$$(i\not{\partial}-m)v_s e^{+ip\cdot x}=0$$

$$H=\int\frac{d^3p}{(2\pi)^3}\frac{1}{2}\sum_{r=1}^{2}\{b_r^\dagger(p)b_r(p)-c_r(p)c_r^\dagger(p)\}$$

$$=\int\frac{d^3p}{(2\pi)^3}\frac{1}{2}\sum_r\{b_r^\dagger(p)b_r(p)+c_r^\dagger(p)c_r(p)-(2\pi)^3\,2E_p\,\delta_{rr}\,\delta(0)\}$$

$$:Q: = \int d^3x \, :J^0:$$

$$= \int \frac{d^3p}{(2\pi)^3 2E_p} \sum_r \left\{ b_r^\dagger(p)\, b_r(p) - c_r^\dagger(p)\, c_r(p) \right\}$$

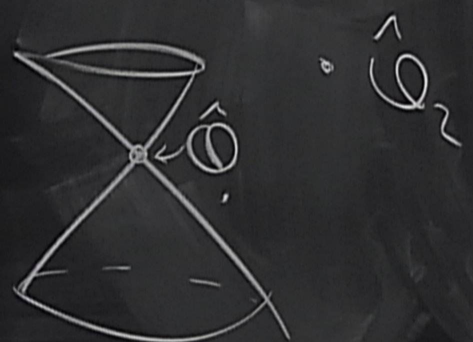
$$\psi(x) = \int \frac{d^3p}{(2\pi)^3 2E_p} \sum_{r=1}^{2} \left\{ b_r(p)\, u_r(p)\, e^{-ip\cdot x} + c_r^\dagger(p)\, v_r(p)\, e^{ip\cdot x} \right\}$$

$$= \psi^{(+)}(x) + \psi^{(-)}(x)$$

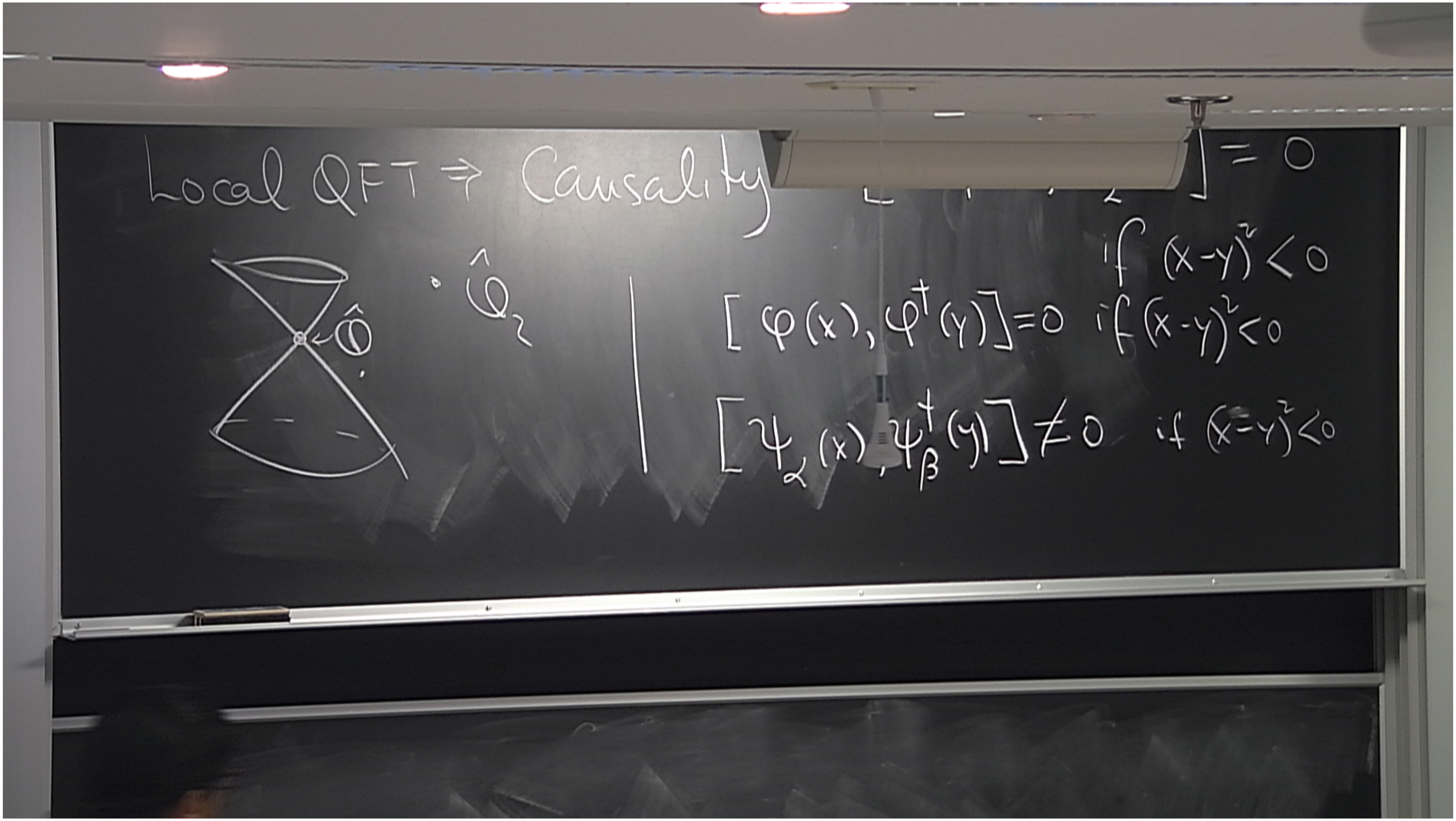
Causality & the Feynman propagator.

Local QFT ⇒ Causality

$$[\hat{O}_1(x), \hat{O}_2(y)] = 0 \quad \text{if } (x-y)^2 < 0$$



$$[\varphi(x), \varphi^\dagger(y)] = 0 \quad \text{if } (x-y)^2 < 0$$


Local QFT ⇒ Causality
$] = 0$
if $(x-y)^2 < 0$
$\hat{\mathcal{O}}$
$\hat{\mathcal{O}}_2$
$[\varphi(x), \varphi^\dagger(y)] = 0$ if $(x-y)^2 < 0$
$[\psi_\alpha(x), \psi^\dagger_\beta(y)] \neq 0$ if $(x-y)^2 < 0$

Observables of fermionic fields $\sim \bar{\Psi}\,\Gamma\,\Psi \sim \hat{\mathcal{O}}(x)$

$\hookrightarrow$ gamma matrix

$$\{\psi_\alpha(x), \bar{\psi}_\beta(y)\} = 0 \quad \text{for } (x-y)^2 < 0$$

$N = a^\dagger a$

$$[N, a^\dagger] = a^\dagger$$

$$\{\psi_\alpha(x), \bar{\psi}_\beta(y)\} = (i\not{\partial}_x + m)\left[D(x-y) - D(y-x)\right]$$

$$D(x-y) = \int \frac{d^3p}{(2\pi)^3 2E_p}\, e^{-ip\cdot(x-y)}$$