

Title: PSI 2016/2017 Quantum Field Theory I - Lecture 9

Date: Oct 21, 2016 09:00 AM

URL: <http://pirsa.org/16100009>

Abstract:

Last time

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$$

In D-dim $\gamma^\mu \rightarrow 2^{[D/2]} \times 2^{[D/2]}$ dim matrices

$\psi_a(x) \rightarrow$ Dirac spinor $2^{[D/2]}$ dim

In even dimensional

$$\psi_D = \psi_L + \psi_R$$

$\uparrow$
 left hand
 Way

In even dimensional space-time

$$\psi_D = \psi_L + \psi_R$$

left handed
Wayl

Right
handed
Wayl.

dim matrices,

dim.

Chiral Rep

$$\gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\gamma^M = \begin{pmatrix} 0 & \sigma^M \\ \bar{\sigma}^M & 0 \end{pmatrix}$$

even dimensional space-time

$$\psi = \psi_L + \psi_R$$

handed $\rightarrow$ Right handed
Wayl.

Chiral Rep

$$\gamma_5 = \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix}$$

$$\gamma^M = \begin{pmatrix} 0 & \sigma^M \\ \bar{\sigma}^M & 0 \end{pmatrix},$$

$$\sigma^M = (\mathbb{1}, \underline{\sigma})$$

$$\bar{\sigma}^M = (\mathbb{1}, -\underline{\sigma})$$

$$SO(1,3) = SU(2)_L \oplus SU(2)_R$$

Rot. $J_i = \frac{1}{2} \epsilon_{ijk} \Sigma^{ij}$

Boosts $K_i = \Sigma^{0i}$

$$N_i \equiv \frac{1}{2} (J_i - i K_i)$$

$$N_i^\dagger \equiv \frac{1}{2} (J_i + i K_i)$$

$$[J_i, J_j] = i \epsilon_{ijk} J_k$$

$$[K_i, K_j] = -i \epsilon_{ijk} J_k$$

$$[J_i, K_j] = i \epsilon_{ijk} K_k$$

$$[N_i, N_j] = i \epsilon_{ijk} N_k$$

$$\mathfrak{su}(2)_L \oplus \mathfrak{su}(2)_R$$

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$$\mathfrak{g} = \mathrm{SU}(2)_L \oplus \mathrm{SU}(2)_R$$

$$\epsilon_{ijk} \sum_j j^k$$

$$\sum^{\alpha_i}$$

$$(J_i - iK_i)$$

$$\frac{1}{2} (J_i + iK_i)$$

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$$(m, n)$$

$$3) = SU(2)_L \oplus SU(2)_R$$

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$$(\frac{1}{2}, 0) \quad (0, \frac{1}{2})$$

$$[N_i, N_j] = i \epsilon_{ijk} N_k$$

$$[N_i^\dagger, N_j^\dagger] = i \epsilon_{ijk} N_k^\dagger$$

$$(m, n)$$

$$(\frac{1}{2}, 0) \quad (0, \frac{1}{2})$$

$$\gamma^{\mu\dagger} = \gamma^0 \gamma^\mu \gamma^0$$

$$\gamma^{0\dagger} = \gamma^0 = (\gamma^0)^2 \gamma^0 = \gamma^0 \gamma^0 \gamma^0$$

$$\gamma^{i\dagger} = -\gamma^i = -(\gamma^0)^2 \gamma^i = +\gamma^0 \gamma^i \gamma^0$$

$$[L_i, N_j] = i \epsilon_{ijk} N_k$$

$$[L_i, N_j^\dagger] = i \epsilon_{ijk} N_k^\dagger$$

$$(0, \frac{1}{2})$$

$$\gamma^{\mu\dagger} = \gamma^0 \gamma^\mu \gamma^0$$

$$\gamma^{0\dagger} = \gamma^0 = (\gamma^0)^2 \gamma^0 = \gamma^0 \gamma^0 \gamma^0$$

$$\gamma^{i\dagger} = -\gamma^i = -(\gamma^0)^2 \gamma^i = +\gamma^0 \gamma^i \gamma^0$$

$$\begin{aligned} \Sigma^{\mu\nu\dagger} &= -\frac{i}{4} [\gamma^\mu, \gamma^\nu]^\dagger \\ &= -\frac{i}{4} [\gamma^{\nu\dagger}, \gamma^{\mu\dagger}] \\ &= \frac{i}{4} [\gamma^{\mu\dagger}, \gamma^{\nu\dagger}] \end{aligned}$$

$$= \gamma^0 \Sigma^{\mu\nu} \gamma^0$$

$$\psi(x) \rightarrow \exp \left[-\frac{i}{2} \omega_{mn} \Sigma^{mn} \right] \psi(x)$$

$$\psi^\dagger(x) \rightarrow \psi^\dagger(x) \exp \left[+\frac{i}{2} \omega_{mn} \Sigma^{mn\dagger} \right]$$

$$\psi^\dagger \psi$$

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$$\psi^\dagger \psi$$

$$\bar{\psi} = \psi^\dagger \gamma^0$$

$$\bar{\psi}(x) \longrightarrow$$

$$\begin{aligned}
 \bar{\psi}(x) &\longrightarrow \psi^\dagger \exp\left[\frac{i}{2} \omega_{mn} \Sigma^{mn\dagger}\right] \gamma^0 \\
 &= \psi^\dagger (\gamma^0)^2 \dots\dots\dots \\
 &= \bar{\psi} \gamma^0 \exp(\dots) \gamma^0
 \end{aligned}$$

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 \bar{\psi}(x) &\longrightarrow \psi^\dagger \exp\left[\frac{i}{2} \omega_{\mu\nu} \Sigma^{\mu\nu\dagger}\right] \gamma^0 \\
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$$\bar{\psi}(x) \longrightarrow \psi^\dagger \exp \left[\frac{i}{2} \omega_{\mu\nu} \Sigma^{\mu\nu \dagger} \right] \gamma^0$$

$$= \psi^\dagger (\gamma^0)^2 \dots \dots \dots$$

$$= \bar{\psi} \gamma^0 \exp () \gamma^0$$

$$= \bar{\psi} \exp \left[\frac{i}{2} \omega_{\mu\nu} \Sigma^{\mu\nu} \right]$$

$$\bar{\psi}(x) \psi(x) \longrightarrow \text{Lorentz scalar.}$$

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$O(1,3)$

$$\rightarrow \psi^\dagger \exp \left[\frac{i}{2} \omega_{\mu\nu} \Sigma^{\mu\nu \dagger} \right] \gamma^0$$

$$= \psi^\dagger(\gamma^0) \dots \dots \dots$$

$$= \bar{\psi} \gamma^0 \exp(\dots) \gamma^0$$

$$= \bar{\psi} \exp \left[\frac{i}{2} \omega_{\mu\nu} \Sigma^{\mu\nu} \right]$$

$\psi(x) \rightarrow$ Lorentz scalar.

$$O(1,3)$$

$$\Lambda^T \eta \Lambda = \eta$$

$$P^\mu_{\ \nu} \equiv \Lambda^\mu_{\ \nu} = \begin{pmatrix} 1 & & \\ & -1 & \\ & & -1 & \\ & & & -1 \end{pmatrix} \sim \text{Parity}$$

$$(t, \underline{x}) \longrightarrow (t, -\underline{x})$$

$$\exp\left[\frac{i}{2}\omega_{\mu\nu}\Sigma^{\mu\nu}\right]\gamma^0$$

$$(\gamma^0)^2 \dots \dots \dots$$

$$\gamma^0 \exp(\dots) \gamma^0$$

$$\exp\left[\frac{i}{2}\omega_{\mu\nu}\Sigma^{\mu\nu}\right]$$

→ Lorentz scalar.

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$$\det P = -1$$

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$$T^\mu_\nu \equiv \Lambda^\mu_\nu = \begin{pmatrix} -1 & & & \\ & +1 & & \\ & & +1 & \\ & & & +1 \end{pmatrix}$$

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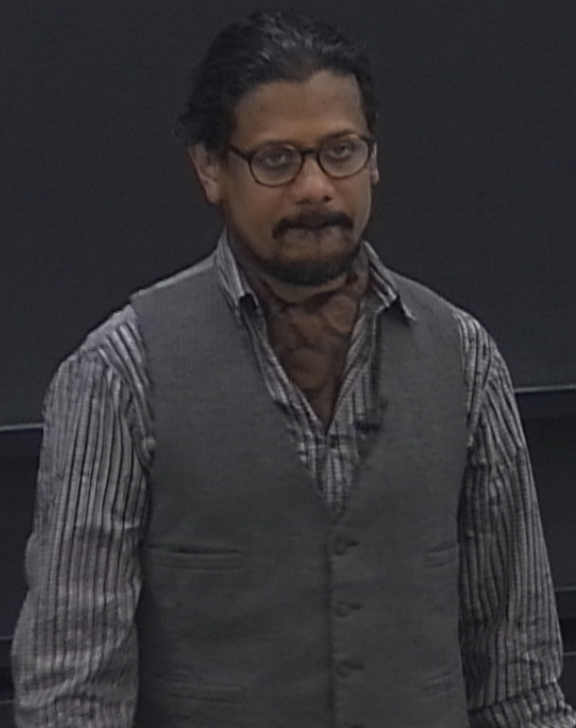
$$(t, \underline{x}) \rightarrow (t, -\underline{x})$$

$$T^\mu_\nu \equiv \Lambda^\mu_\nu = \begin{pmatrix} -1 & & & \\ & +1 & & \\ & & +1 & \\ & & & +1 \end{pmatrix} \sim \text{Time inversion}$$

$$(t, \underline{x}) \rightarrow (-t, \underline{x})$$

CPT Theorem: Relativistic QFTs need to be
invariant only under the combined transformation
of charge conjugation, parity $\neq$ time inversion

Helicity $\mathbf{J} \cdot \mathbf{P}$

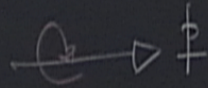


Helicity $\mathbf{J} \cdot \mathbf{P} \sim \mathbf{S} \cdot \mathbf{P}$

$$\frac{\mathbf{S} \cdot \mathbf{P}}{|\mathbf{P}|}$$

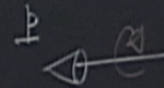
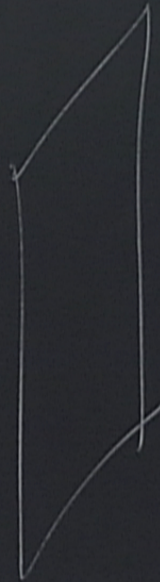
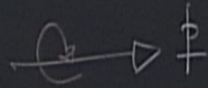
Helicity $\mathbf{J} \cdot \hat{\mathbf{p}} \sim \boldsymbol{\sigma} \cdot \hat{\mathbf{p}}$

$$\frac{\boldsymbol{\sigma} \cdot \hat{\mathbf{p}}}{|\hat{\mathbf{p}}|}$$



Helicity $\underline{J} \cdot \underline{P} \sim \underline{\sigma} \cdot \underline{P}$

$$\frac{\underline{\sigma} \cdot \underline{P}}{|\underline{P}|}$$



$$\psi_{L,R} \xrightarrow{P} \psi_{R,L}$$

$$\psi_D(x,t) \xrightarrow{P} \gamma^0 \psi_D(-x,t)$$

$$\psi^t(x,t) \xrightarrow{P} \psi^t(-x,t) \gamma^0$$

1. Scalar $\bar{\psi}(x,t) \psi(x,t) \rightarrow \bar{\psi}(x,t) \psi(x,t)$

$\psi^\dagger \gamma^0 \gamma^0 \gamma^0 \psi$

1. Scalar $\bar{\psi}(x,t)\psi(x,t) \rightarrow \bar{\psi}(x,t)\psi(x,t)$

2. Pseudo-scalar

$\psi(-x,t)$

γ^0

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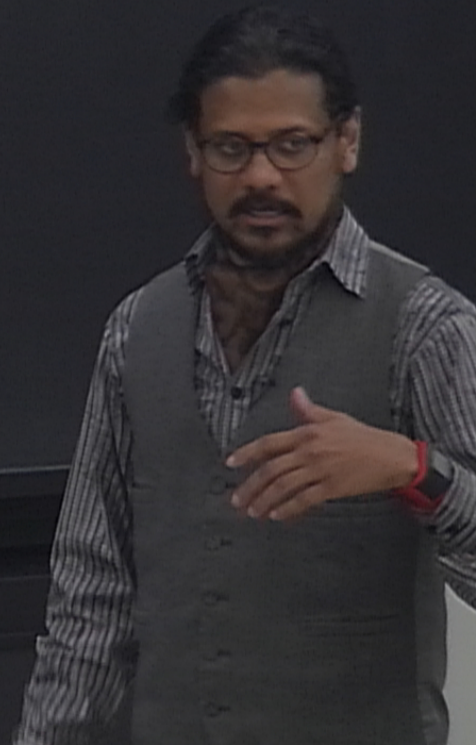
2. Pseudo-scalar : $\bar{\psi}(x,t)\gamma_5\psi(x,t)$

$\psi(-x,t)$

$$1. \text{ Scalar : } \bar{\psi}(x,t)\psi(x,t) \xrightarrow{P} \bar{\psi}(x,t)\psi(x,t)$$

$$2. \text{ Pseudo-scalar : } \bar{\psi}(x,t)\gamma_5\psi(x,t) \rightarrow -\bar{\psi}(-x,t)\gamma_5\psi(-x,t)$$

$$3. \text{ Vector : } \bar{\psi}\gamma^\mu\psi \xrightarrow{P} \begin{cases} \bar{\psi}\gamma^\mu\psi \\ -\bar{\psi}\gamma^\mu\psi \end{cases}$$



$$1. \text{ Scalar : } \bar{\psi}(x,t)\psi(x,t) \xrightarrow{P} \bar{\psi}(x,t)\psi(x,t)$$

$$2. \text{ Pseudo-scalar : } \bar{\psi}(x,t)\gamma_5\psi(x,t) \rightarrow -\bar{\psi}(-x,t)\gamma_5\psi(-x,t)$$

$$3. \text{ Vector : } \bar{\psi}\gamma^\mu\psi \xrightarrow{P} \begin{cases} \bar{\psi}\gamma^\mu\psi \\ -\bar{\psi}\gamma^\mu\psi \end{cases}$$

$$4. \text{ Pseudo or Axial Vector : } \bar{\psi}\gamma_5\gamma^\mu\psi \xrightarrow{P} \begin{cases} \bar{\psi}\gamma_5\gamma^\mu\psi \\ +\bar{\psi}\gamma_5\gamma^\mu\psi \end{cases}$$

$$1. \text{ Scalar : } \bar{\psi}(x,t)\psi(x,t) \xrightarrow{P} \bar{\psi}(x,t)\psi(x,t)$$

$$2. \text{ Pseudo-scalar : } \bar{\psi}(x,t)\gamma_5\psi(x,t) \rightarrow -\bar{\psi}(-x,t)\gamma_5\psi(-x,t)$$

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$$\bar{\psi} \stackrel{m}{\sim} \psi$$

$\mathcal{L}$ for spinors

$$\partial_m \equiv \frac{\partial}{\partial x^m}$$

$$\mathcal{L}_{\text{kin}} = i \bar{\psi}(x) \gamma^m \partial_m \psi(x)$$

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$$\partial_m \bar{\psi} \partial^m \psi$$

$\mathcal{L}$ for spinors

$$\partial_m \equiv \frac{\partial}{\partial x^m}$$

$$\mathcal{L}_{\text{kin}} = i \bar{\psi}(x) \gamma^m \partial_m \psi(x)$$

$$S_{\text{kin}} = \int \mathcal{L}_{\text{kin}} d^4x \sqrt{-g}$$

$$\psi \gamma_5 \gamma^\mu \psi \rightarrow \gamma^\mu + \psi \gamma_5 \gamma^\mu \psi$$

$$\underline{C=1, \hbar=1}$$

$$[S'_{fin}] = 0$$

(x)

$$\psi \gamma_5 \gamma^\mu \psi \rightarrow \dots + \psi \gamma_5 \gamma^\mu \psi$$

$$C=1, \hbar=1$$

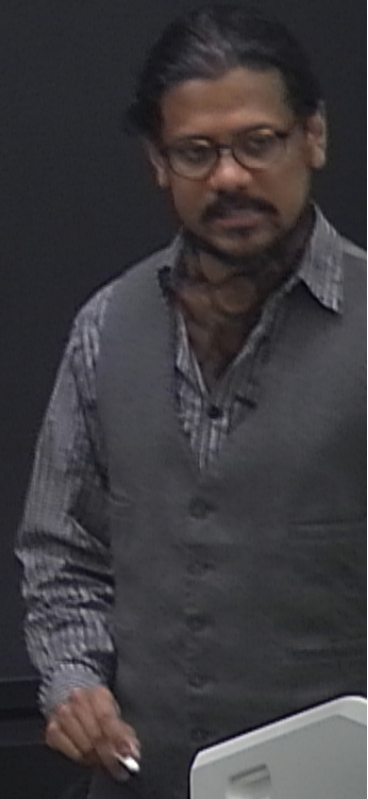
$$[S_{\text{kin}}] = 0$$

$$[d_x^4] = -4$$

$$[\partial_m] = +1$$

$$[\gamma^m] = 0$$

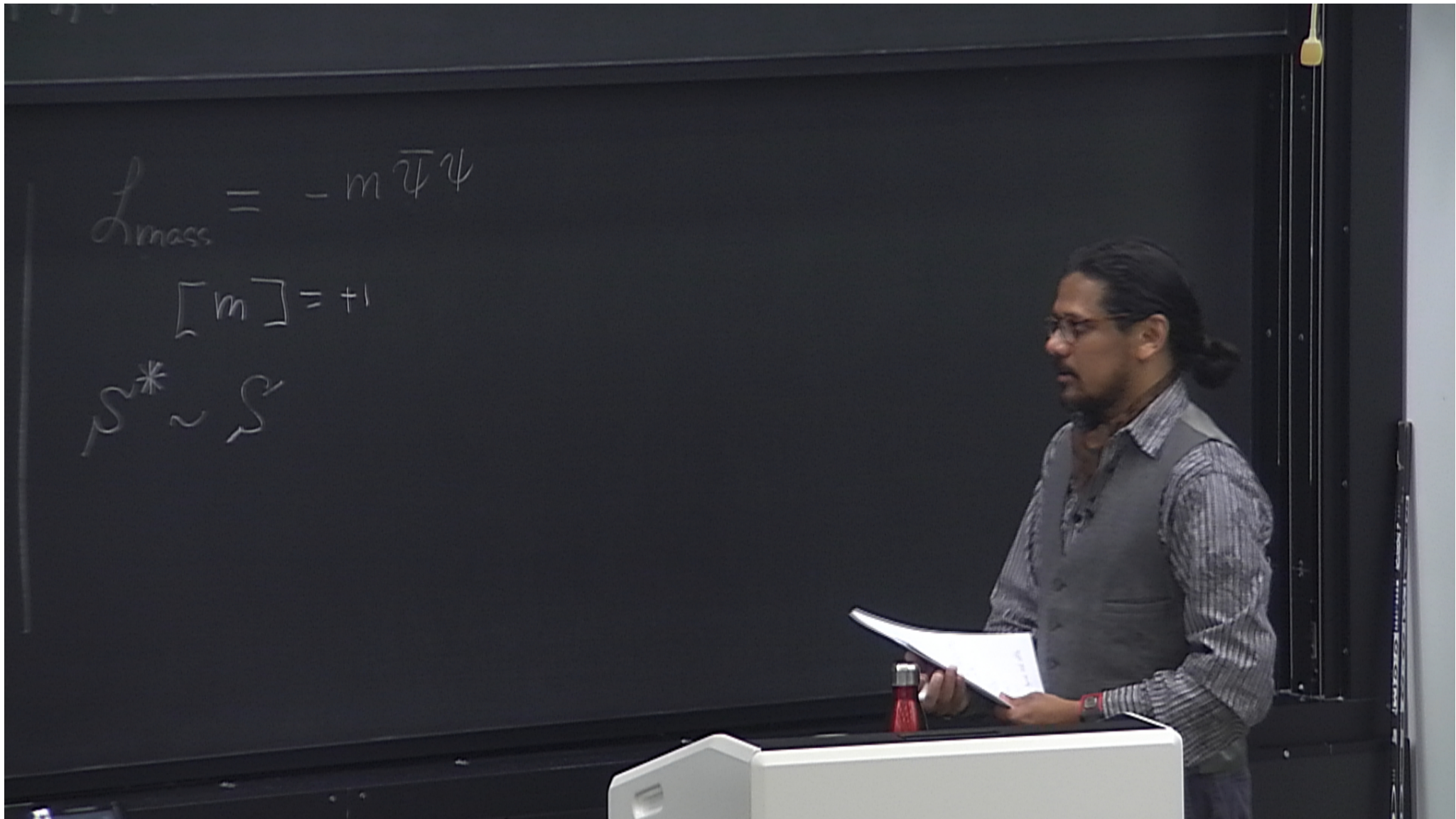
$$[\psi] = [\bar{\psi}] = \frac{3}{2}$$



$$\mathcal{L}_{\text{mass}} = -m \bar{\psi} \psi$$

$$[m] = +1$$

$$= \frac{3}{2}$$



$$(i\not{\partial} - m)\psi = 0$$

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_m$$

$$\phi = \gamma^\mu \chi_\mu$$

$$\begin{aligned} (i\not{\partial} - m)\psi &= 0 \\ \mathcal{L} &= \mathcal{L}_{\text{kin}} + \mathcal{L}_m \end{aligned}$$

$$\not{\partial} \equiv \gamma^\mu \partial_\mu$$

$$[\Sigma^{\mu\nu}, \gamma^\rho]$$

$$[\Sigma^{\mu\nu}, \gamma^\rho] = i(\eta^{\rho\nu}\gamma^\mu - \gamma^{\rho\mu}\gamma^\nu)$$

$$[M^{\alpha\beta}]^\mu{}_\nu = i(\delta^\alpha{}_\nu\eta^{\mu\beta} - \delta^\beta{}_\nu\eta^{\mu\alpha})$$

$$\frac{1}{4} [\gamma^\mu, \gamma^\nu]$$

$$[\Sigma^{\mu\nu}, \gamma^\rho] = [\mathcal{M}^{\mu\nu}]^\rho{}_\sigma \gamma^\sigma$$

$$S^{-1}(\Lambda) \gamma^\rho S(\Lambda) = \Lambda^\rho{}_\nu \gamma^\nu$$

$$\frac{1}{4} [\gamma^\mu, \gamma^\nu]$$

$$[\Sigma^{\mu\nu}, \gamma^\rho] = [\mathcal{M}^{\mu\nu}]^\rho{}_\sigma \gamma^\sigma$$

$$S^{-1}(\Lambda) \gamma^\rho S(\Lambda) = \Lambda^\rho{}_\nu \gamma^\nu$$

$$S = \begin{pmatrix} S_L & 0 \\ 0 & S_R \end{pmatrix}$$

$$[\Sigma^{\mu\nu}, \gamma^\rho] = [\mathcal{M}^{\mu\nu}]^\rho{}_\sigma \gamma^\sigma$$

$$S^{-1}(\Lambda) \gamma^\rho S(\Lambda) = \Lambda^\rho{}_\nu \gamma^\nu$$

$$S = \begin{pmatrix} S_L & 0 \\ 0 & S_R \end{pmatrix}$$

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$$

$$S_L^{-1} \sigma^\mu S_R$$

$$[\gamma^\mu, \gamma^\nu] = [\gamma^\mu]^\rho \gamma^\rho \gamma^\nu$$

$$S^{-1}(\Lambda) \gamma^\rho S(\Lambda) = \Lambda^\rho{}_\nu \gamma^\nu$$

$$S = \begin{pmatrix} S_L & 0 \\ 0 & S_R \end{pmatrix} \quad \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$$

$$S_L^{-1} \sigma^\mu S_R = \Lambda^\mu{}_\nu \sigma^\nu$$

$$S_R^{-1} \bar{\sigma}^\mu S_L = \Lambda^\mu{}_\nu \bar{\sigma}^\nu$$

$$\psi = \begin{pmatrix} \chi_L \\ \varphi_R \end{pmatrix} \quad \psi^\dagger = (\chi_L^\dagger, \varphi_R^\dagger)$$

$$\bar{\psi} = (\varphi_R^\dagger, \chi_L^\dagger)$$

$$\psi \rightarrow S \psi$$

$$\chi_L \rightarrow S_L \chi_L$$

$$\varphi_R \rightarrow S_R \varphi_R$$

$$\chi_L^\dagger \rightarrow \chi_L^\dagger S_R^{-1}$$

$$\varphi_R^\dagger \rightarrow \varphi_R^\dagger S_L^{-1}$$

$$\mathcal{L}_L = i \chi_L^\dagger \bar{\sigma}^\mu \partial_\mu \chi_L$$

$$\rightarrow i \chi_L^\dagger S_R^{-1} \bar{\sigma}^\mu \Lambda_\mu^\nu \partial_\nu S_L \chi_L$$

$$\mathcal{L}_L = i \chi_L^\dagger \bar{\sigma}^\mu \partial_\mu \chi_L$$

$$\rightarrow i \chi_L^\dagger S_R^{-1} \bar{\sigma}^\mu \Lambda_\mu^\nu \partial_\nu S_L \chi_L$$

$$= i \chi_L^\dagger \underbrace{\Lambda_\sigma^\mu \bar{\sigma}^\sigma \Lambda_\mu^\nu}_{\delta_\sigma^\nu} \partial_\nu \chi_L$$

$$= i \chi_L^\dagger \bar{\sigma}^\nu \partial_\nu \chi_L$$

$$\psi^\dagger \exp\left[\frac{i}{2} \omega_{\mu\nu} \Sigma^{\mu\nu} t\right] \gamma^0$$

$$\psi^\dagger(\gamma^0) \dots$$

$$\bar{\psi} \gamma^0 \exp(\dots) \gamma^0$$

$$\bar{\psi} \exp\left[\frac{i}{2} \omega_{\mu\nu} \Sigma^{\mu\nu}\right]$$

→ Lorentz scalar.

$O(1,3)$

$$\Lambda^T \eta \Lambda = \eta$$

$$P^\mu_\nu \equiv \Lambda^\mu_\nu = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \sim \text{Parity}$$

$$\det P = -1$$

$$(t, \underline{x}) \rightarrow (t, -\underline{x})$$

$$T^\mu_\nu \equiv \Lambda^\mu_\nu = \begin{pmatrix} -1 & & & \\ & +1 & & \\ & & +1 & \\ & & & +1 \end{pmatrix} \sim \text{Time inversion.}$$

$$(t, \underline{x}) \rightarrow (-t, \underline{x})$$

$$\psi_R' \rightarrow \psi_R^T S_L^{-1} \quad = i \chi_L^T \bar{\sigma}^0$$

$$S_L, S_L^{-1}, S_P, S_P^{-1} \in SL(2, \mathbb{C})$$

$$\mathcal{L}_R = i \psi_P^T \sigma^\mu \partial_\mu \psi_R$$

$$(\chi_L^T)^\dagger \chi_L \rightarrow \chi^T S_P^{-1} S_L \chi$$

$$\mathcal{L}_{\text{kin}}^{\text{Dirac}} = i \chi_L^T$$

$$\psi_R' \rightarrow \psi_R^T S_L^{-1} \quad = i \chi_L^T \bar{\sigma}^0$$

$$S_L, S_L^{-1}, S_P, S_P^{-1} \in SL(2, \mathbb{C})$$

$$\text{Weyl} \\ \mathcal{L}_R = i \psi_P^T \sigma^\mu \partial_\mu \psi_R$$

$$(\chi_L^T)^\dagger \chi_L \rightarrow \chi^T S_P^{-1} S_L \chi$$

$$\mathcal{L}_{\text{kin}}^{\text{Dirac}} = i \chi_L^T \bar{\sigma}^\mu \partial_\mu \chi$$

$$\mathcal{L} =$$

$$= i \chi_L^\dagger \bar{\sigma}^\nu \partial_\nu \chi_L$$

$$\mathcal{L}_{\text{kin}}^{\text{Dirac}} = i \chi_L^\dagger \bar{\sigma}^\mu \partial_\mu \chi_L + i \varphi_R^\dagger \sigma^\mu \partial_\mu \varphi_R$$

$$\mathcal{L}_{\text{mass}}^{\text{Dirac}} = -m \bar{\Psi} \Psi = -m (\varphi_R^\dagger \chi_L + \chi_L^\dagger \varphi_R)$$

$$= i \chi_L^\dagger \bar{\sigma}^\nu \partial_\nu \chi_L$$

$$\mathcal{L}_{\text{kin}}^{\text{Dirac}} = i \chi_L^\dagger \bar{\sigma}^\mu \partial_\mu \chi_L + i \varphi_R^\dagger \sigma^\mu \partial_\mu \varphi_R$$

$$\mathcal{L}_{\text{mass}}^{\text{Dirac}} = -m \bar{\Psi} \Psi = -m (\varphi_R^\dagger \chi_L + \chi_L^\dagger \varphi_R)$$

$$\psi_D = \begin{pmatrix} \chi_L \\ \phi_R \end{pmatrix}$$

$$\chi_L, \quad \sigma^2 \phi_R^*$$

$$\psi = \begin{pmatrix} \chi \\ i\sigma^2 \chi^* \end{pmatrix}$$

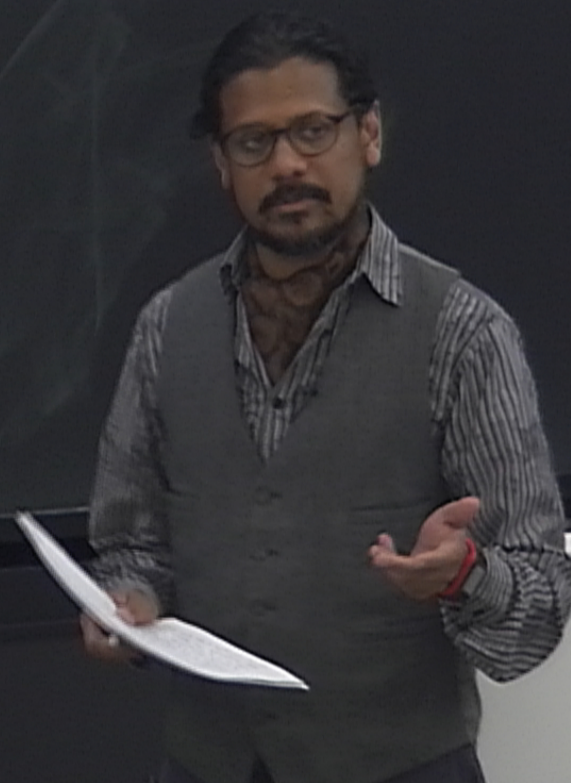
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$$\chi_L, \quad \sigma^2 \psi_R^*$$

$$\psi = \begin{pmatrix} \chi \\ i\sigma^2 \chi^* \end{pmatrix} \sim \text{Majorana spinor.}$$

$$\mathcal{C} : \psi \rightarrow \psi^{(c)} \equiv -i\gamma_2 \psi^*$$

$$\boxed{\psi^{(c)} = \psi}$$



$$\psi_p(x) = u(p) e^{-ipx}$$

$\hat{p}_4$ momentum

$$u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$i \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \begin{pmatrix} u_1(p) \\ u_2(p) \end{pmatrix} (-ip_\mu) - m \begin{pmatrix} u_1(p) \\ u_2(p) \end{pmatrix} = 0$$

$$\psi_p(x) = u(p) e^{-ipx}$$

ξ_4 momentum

$$u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

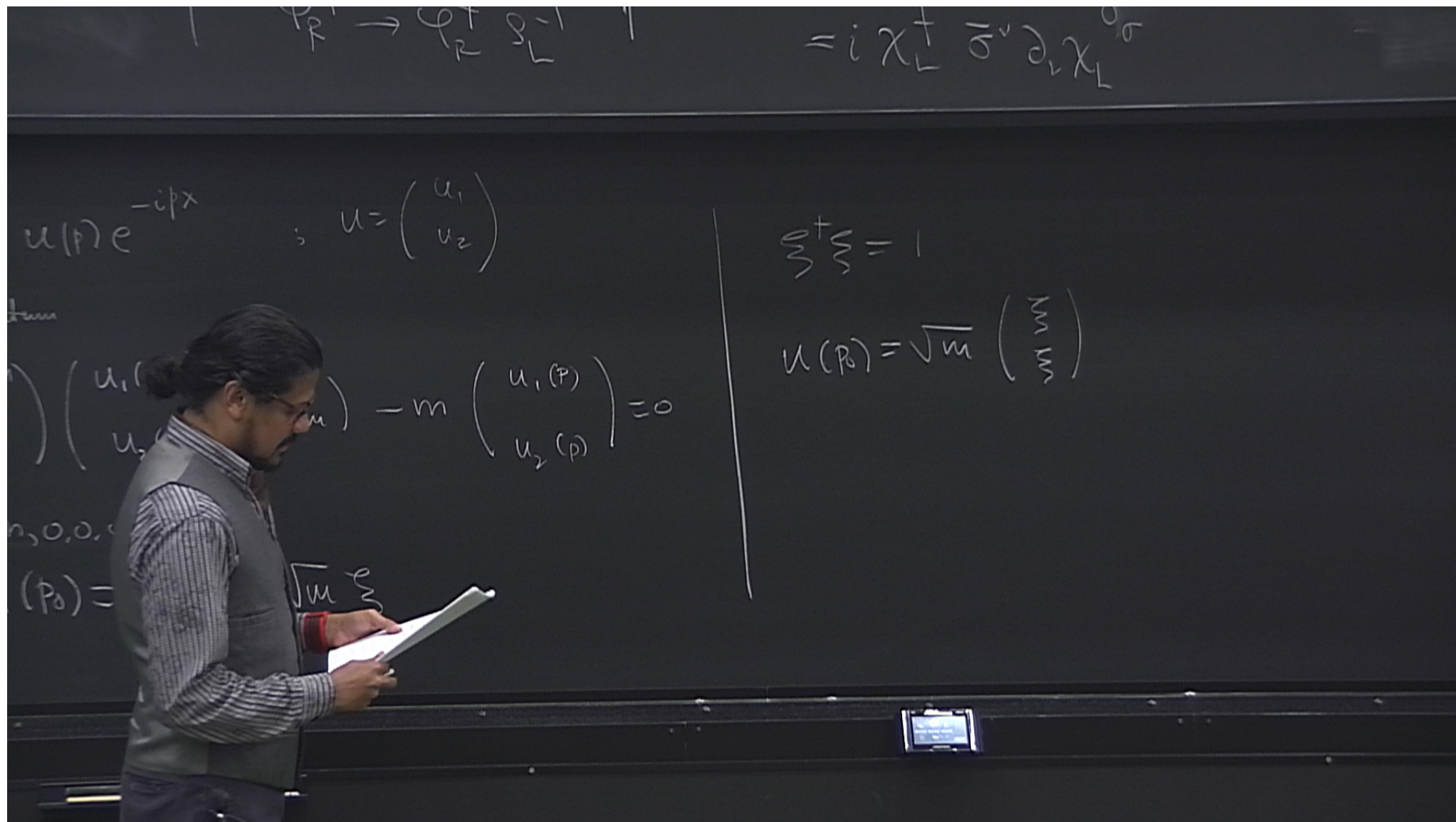
$$i \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \begin{pmatrix} u_1(p) \\ u_2(p) \end{pmatrix} (-ip_\mu) - m \begin{pmatrix} u_1(p) \\ u_2(p) \end{pmatrix} = 0$$

$$p^\mu = (m, 0, 0, 0)$$

$$u_1(p_0) = u_2(p_0) = \sqrt{m} \xi$$

$$\xi^\dagger \xi = 1$$

$$u = \sqrt{m} \begin{pmatrix} \xi \\ \xi \end{pmatrix}$$



$$\psi_R' \rightarrow \psi_R^T S_L^{-1}$$

$$= i \chi_L^\dagger \bar{\sigma}^\nu \partial_\nu \chi_L^{\sigma}$$

$$u(p) e^{-i p x} \quad ; \quad u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

then

$$\begin{pmatrix} u_1(p) \\ u_2(p) \end{pmatrix} (-i p_\mu) - m \begin{pmatrix} u_1(p) \\ u_2(p) \end{pmatrix} = 0$$

$(p_0, 0, 0, 0)$

$$u_1(p_0) = u_2(p_0) \equiv \sqrt{m} \xi$$

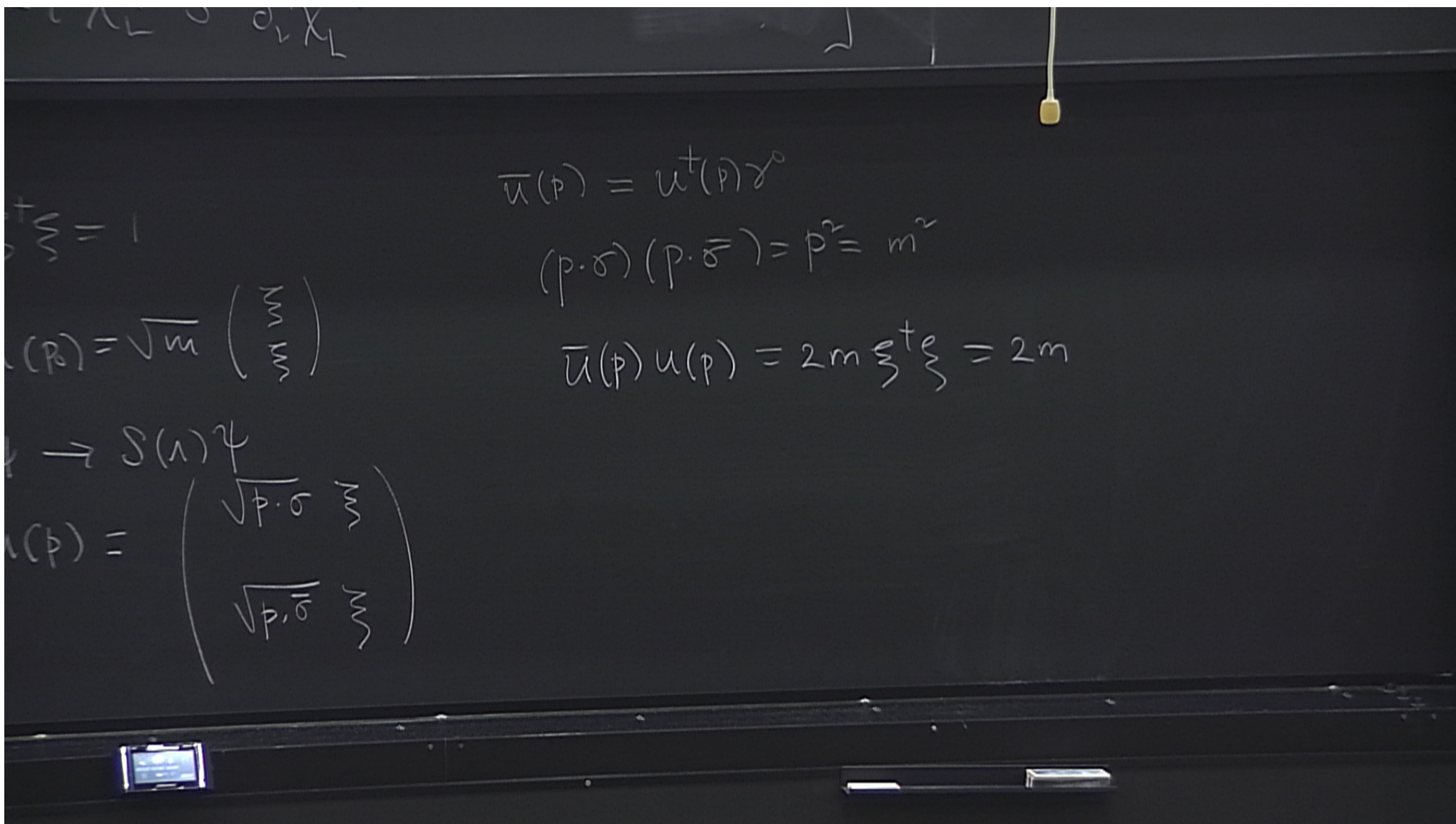
$$\xi^\dagger \xi = 1$$

$$u(p_0) = \sqrt{m} \begin{pmatrix} \xi \\ \xi \end{pmatrix}$$

$$\psi \rightarrow S(\Lambda) \psi$$

$$u(p) = \begin{pmatrix} \sqrt{p \cdot \sigma} \xi \\ \sqrt{p \cdot \bar{\sigma}} \xi \end{pmatrix}$$

$$\bar{u}(p) = u^\dagger(p)$$



$$p^0 > 0$$

$$\psi_p(x) = u(p) e^{-i p x}$$

ξ_4 momentum

$$u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$i \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \begin{pmatrix} u_1(p) \\ u_2(p) \end{pmatrix} (-i p_\mu) - m \begin{pmatrix} u_1(p) \\ u_2(p) \end{pmatrix} = 0$$

$$p^\mu = (m, 0, 0, 0)$$

$$u_1(p_0) = u_2(p_0) = \sqrt{m} \xi$$

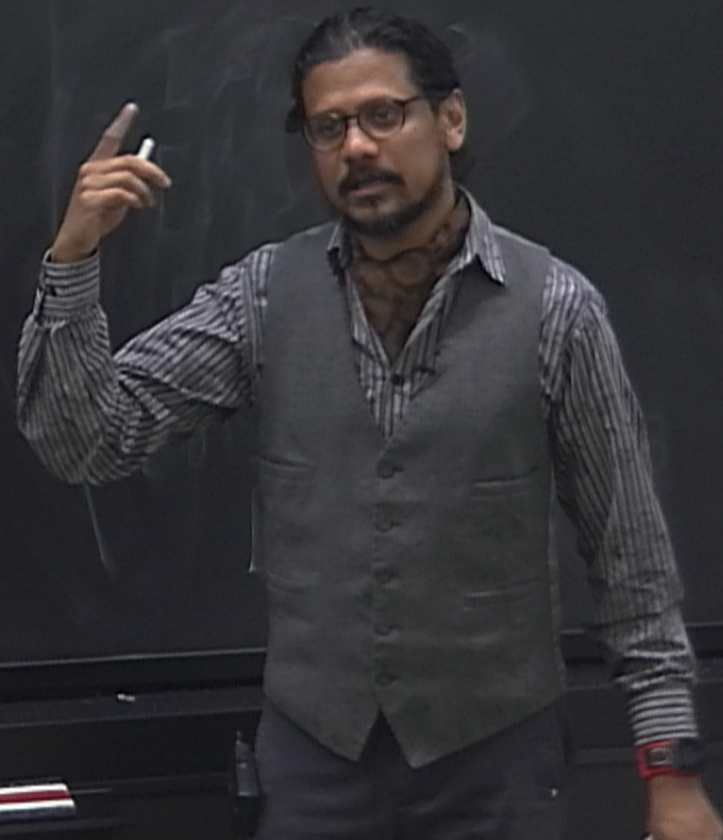
$$\xi^\dagger \xi = 1$$

$$u(p) = \sqrt{m} \begin{pmatrix} \xi \\ \xi \end{pmatrix}$$

ψ

$u(p)$

$$\psi_p(x) = U(p) e^{+ipx}, \quad p^0 > 0$$



$$\psi_p(x) = \mathcal{U}(p) e^{+ipx}, \quad p^0 > 0$$

$$\mathcal{U}(p) = \begin{pmatrix} \sqrt{p \cdot \sigma} & \eta \\ -\sqrt{p \cdot \bar{\sigma}} & \eta \end{pmatrix}$$

$$i \bar{\chi}_L \sigma^\mu \partial_\mu \chi_L$$

$$\xi = 1$$

$$u(p) = \sqrt{m} \begin{pmatrix} \xi \\ \xi \end{pmatrix}$$

$$\rightarrow S(\Lambda) \psi$$

$$u(p) = \begin{pmatrix} \sqrt{p \cdot \sigma} \xi \\ \sqrt{p \cdot \bar{\sigma}} \xi \end{pmatrix}$$

$$\bar{u}(p) = u^\dagger(p) \gamma^0$$

$$(p \cdot \sigma)(p \cdot \bar{\sigma}) = p^2 = m^2$$

$$\bar{u}(p) u(p) = 2m \xi^\dagger \xi = 2m$$

$$\xi^{(s)\dagger} \xi^{(r)} = \delta^{sr}$$

$$\psi_p(x) = U(p) e^{+ipx}, \quad p^0 > 0$$

$$U(p) = \begin{pmatrix} \sqrt{p \cdot \sigma} & \eta^s \\ -\sqrt{p \cdot \bar{\sigma}} & \eta^s \end{pmatrix}$$

$$\eta^{(s)\dagger} \eta^{(r)} = \delta^{sr}$$

$$\psi_p(x) = U(p) e^{+ipx}, \quad p^0 > 0$$

$$U^A(p) = \begin{pmatrix} \sqrt{p \cdot \sigma} & \eta^s \\ -\sqrt{p \cdot \bar{\sigma}} & \eta^s \end{pmatrix}$$

$$\eta^{(s)\dagger} \eta^{(r)} = \delta^{sr}$$

$$\bar{u}^{(s)}(p) u^{(r)}(p) = 2m \delta^{sr}$$

$$u^{(s)\dagger}(p) u^{(r)}(p) = 2E_p \delta^{sr}$$

$$i\bar{\chi}_L \sigma^\mu \partial_\mu \chi_L$$

$$\xi = 1$$

$$u(p) = \sqrt{m} \begin{pmatrix} \xi \\ \xi \end{pmatrix}$$

$$\rightarrow S(\lambda) \psi$$

$$u(p) = \begin{pmatrix} \sqrt{p \cdot \sigma} \xi \\ \sqrt{p \cdot \bar{\sigma}} \xi \end{pmatrix}$$

$$\bar{u}(p) = u^\dagger(p) \gamma^0$$

$$(p \cdot \sigma)(p \cdot \bar{\sigma}) = p^2 = m^2$$

$$\bar{u}(p) u(p) = 2m \xi^\dagger \xi = 2m$$

$$\xi^{(2)\dagger} \xi^{(1)} = \delta^{2r}$$

$$\xi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \xi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\psi_p(x) = u(p) e^{+ipx}, \quad p^0 > 0$$

$$u^s(p) = \begin{pmatrix} \sqrt{p \cdot \sigma} & \eta^s \\ -\sqrt{p \cdot \bar{\sigma}} & \eta^s \end{pmatrix}$$

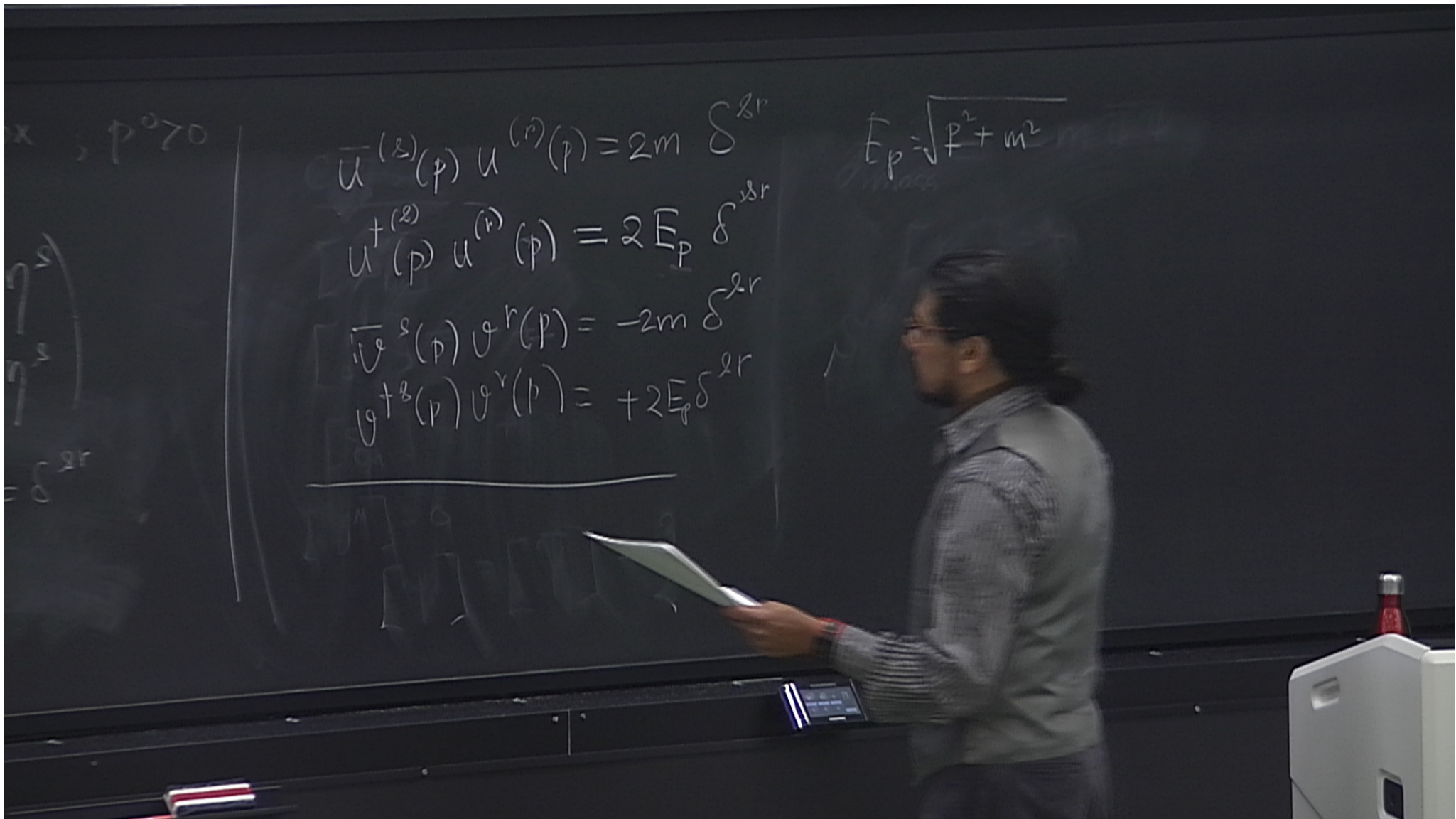
$$\eta^{(s)\dagger} \eta^{(r)} = \delta^{sr}$$

$$\bar{u}^{(s)}(p) u^{(r)}(p) = 2m \delta^{sr}$$

$$u^{(s)\dagger}(p) u^{(r)}(p) = 2E_p \delta^{sr}$$

$$\bar{u}^s(p) u^r(p) = -2m \delta^{sr}$$

$$u^{(s)\dagger}(p) u^{(r)}(p) = +2E_p \delta^{sr}$$



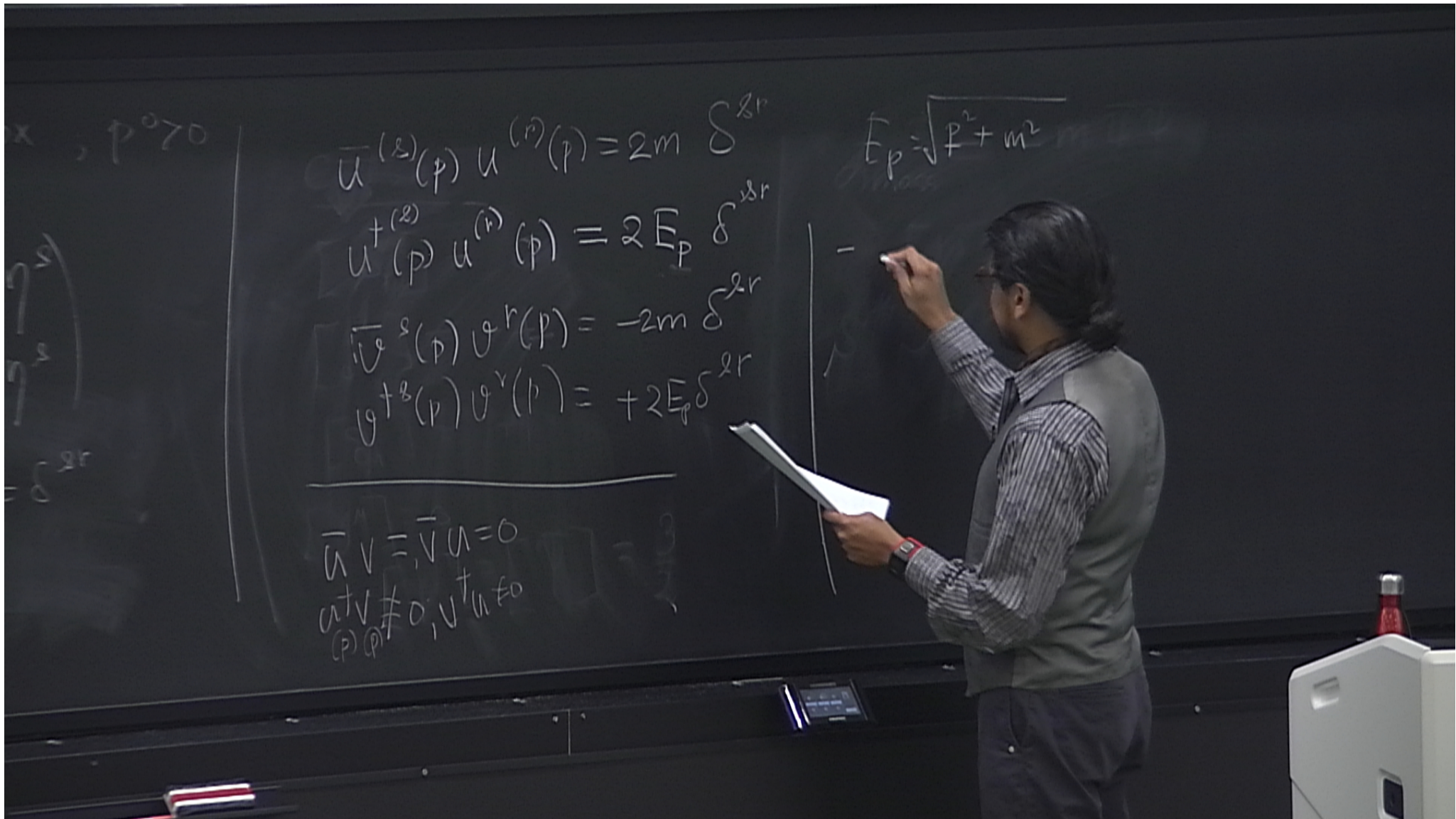
$$\bar{u}^{(s)}(p) u^{(r)}(p) = 2m \delta^{sr}$$

$$u^{(s)\dagger}(p) u^{(r)}(p) = 2E_p \delta^{sr}$$

$$\bar{v}^{(s)}(p) v^{(r)}(p) = -2m \delta^{sr}$$

$$v^{(s)\dagger}(p) v^{(r)}(p) = +2E_p \delta^{sr}$$

$$E_p = \sqrt{p^2 + m^2}$$



$$\bar{u}^{(s)}(p) u^{(r)}(p) = 2m \delta^{sr}$$

$$u^{(s)\dagger}(p) u^{(r)}(p) = 2E_p \delta^{sr}$$

$$\bar{v}^{(s)}(p) v^{(r)}(p) = -2m \delta^{sr}$$

$$v^{(s)\dagger}(p) v^{(r)}(p) = +2E_p \delta^{sr}$$

$$\bar{u} v = \bar{v} u = 0$$
$$u^\dagger v \neq 0, v^\dagger u \neq 0$$

$$E_p = \sqrt{p^2 + m^2}$$

$$x, p^0 > 0$$

$$\bar{u}^{(2)}(p) u^{(1)}(p) = 2m \delta^{2r}$$

$$u^{(2)}(p) u^{(1)}(p) = 2E_p \delta^{2r}$$

$$\bar{v}^s(p) v^r(p) = -2m \delta^{2r}$$

$$v^{+s}(p) v^r(p) = +2E_p \delta^{2r}$$

$$\bar{u} v = \bar{v} u = 0$$

$$u^{+s}(p) v^r(p) \neq 0, v^{+s}(p) u^r(p) \neq 0$$

$$E_p = \sqrt{p^2 + m^2}$$

$$-u^{+s}(-\underline{p}) v^s(\underline{p}) = 0 = v^{+s}(-\underline{p}) u^r(\underline{p})$$

$$x, p^0 > 0$$

$$\bar{u}^{(2)}(p) u^{(1)}(p) = 2m \delta^{2r}$$

$$u^{(2)}(p) u^{(1)}(p) = 2E_p \delta^{2r}$$

$$\bar{v}^s(p) v^r(p) = -2m \delta^{2r}$$

$$v^{(2)}(p) v^{(1)}(p) = +2E_p \delta^{2r}$$

$$\bar{u} v = \bar{v} u = 0$$

$$u^\dagger v \neq 0, v^\dagger u \neq 0$$

$$E_p = \sqrt{p^2 + m^2}$$

$$-u^\dagger(-\underline{p}) v^r(\underline{p}) = 0 = v^{(2)}(-\underline{p}) u^r(\underline{p})$$