

Title: PSI 2016/2017 Quantum Field Theory I - Lecture 7

Date: Oct 19, 2016 09:00 AM

URL: <http://pirsa.org/16100007>

Abstract:

Introduction to Loops

$$\mathcal{L}_{\text{naive}} = \frac{1}{2} (\partial\varphi)^2 - \frac{1}{2} m^2 \varphi^2 + \frac{1}{3!} g \varphi^3$$

$$\text{LSZ: } \langle \Omega | \varphi(x) | \Omega \rangle = 0$$

$$\langle k | \varphi(x) | \Omega \rangle = e^{ik \cdot x}$$

$$\mathcal{L} = \frac{1}{2} Z_\varphi (\partial\varphi)^2 - \frac{1}{2} Z_m m^2 \varphi^2 + \frac{1}{3!} Z_g g \varphi^3 + Y_\varphi$$

$$\mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}}$$

$$\mathcal{L}_{\text{free}} = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2$$

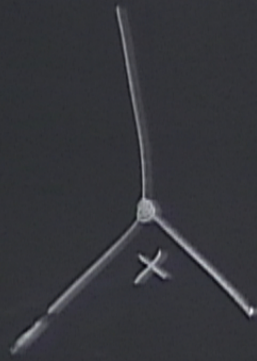
$$\mathcal{L}_{\text{int}} = \frac{1}{3!}Z_g g \varphi^3 + \mathcal{L}_{\text{ct}}$$

$$\mathcal{L}_{\text{ct}} = \frac{1}{2}(Z_\varphi - 1)(\partial\varphi)^2 - \frac{1}{2}(Z_m - 1)m^2\varphi^2 + Y\varphi$$

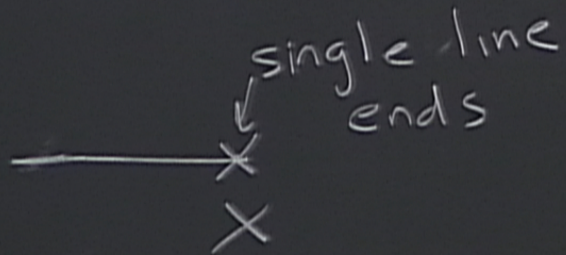
↑
counterterm

$$Y = 0 + \mathcal{O}(g)$$

$$Z_i = 1 + \mathcal{O}(g^2)$$



$$= i Z_{gg} \int d^4 x$$



$$= i Y \int d^4 x$$

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$$m^2 \varphi^2 + \frac{1}{3!} Z_g g \varphi^3 + Y \varphi$$

Counterterm

$$Y = 0 + \mathcal{O}(g)$$

$$Z_1 = 1 + \mathcal{O}(g^2)$$

$$\langle \Omega | \varphi(x) | \Omega \rangle = 0$$

$$= \underbrace{\text{diagram}_1 + \text{diagram}_2}_{\mathcal{O}(g)} + \underbrace{\text{diagram}_3 + \text{diagram}_4}_{\mathcal{O}(g^3)} + \dots$$

The diagrams are:
 1. A circle with an external line on the left.
 2. A horizontal line with an external line on the left.
 3. A circle with an external line on the left and another on the right.
 4. A circle with an external line on the left and a vertical line through its center.

$$\text{diagram}_1 = \frac{1}{2} (i Z_g g) \int d^4 y \Delta_F(x-y) \Delta_F(y-y)$$

$$\text{diagram}_2 = i Y \int d^4 y \Delta_F(x-y)$$

$$\langle \Omega | \varphi(x) | \Omega \rangle = (iY + \frac{1}{2}ig \Delta_F(0)) \int d^4y \Delta_F(x-y) + \mathcal{O}(g^3)$$

$$Y = -\frac{1}{2}g \Delta_F(0) + \mathcal{O}(g^3)$$

$$\Delta_F(0) = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - m^2 + i\epsilon} \xrightarrow{-i \cdot p_0} = \infty$$

Regularize this result by: imposing a cutoff Λ

$$\Delta_F(0) \rightarrow \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - m^2 + i\epsilon} \left(\frac{\Lambda^2}{p^2 + \Lambda^2 + i\epsilon} \right)^2$$

$$+ \frac{1}{2} i g \Delta_F(0)) \int d^4 y \Delta_F(x-y) + \mathcal{O}(g^3)$$

$$g \Delta_F(0) + \mathcal{O}(g^3)$$

$$\frac{i}{p^2 - m^2 + i\epsilon} \xrightarrow{-i \cdot p_0 \rightarrow 0} = \infty$$

result by: imposing a cutoff Λ

$$\Delta_F(0) \rightarrow \int \frac{d^4 p}{(2\pi)^4} \frac{i}{p^2 - m^2 + i\epsilon} \left(\frac{\Lambda^2}{p^2 \Lambda^2 + i\epsilon} \right)^2 = \frac{\Lambda^2}{16\pi^2}$$

$$\langle \Omega | \varphi(x) | \Omega \rangle = 0$$

$$= \underbrace{\text{diagram 1} + \text{diagram 2}}_{\theta(g)} + \underbrace{\text{diagram 3}}_{\theta(g)}$$

The diagrams represent Feynman diagrams for the vacuum expectation value of a scalar field. Diagram 1 is a tadpole with a loop on the left and a line extending to the right ending in a cross. Diagram 2 is a tadpole with a loop on the right and a line extending to the left ending in a cross. Diagram 3 is a tadpole with a loop on the left and a line extending to the right ending in a cross. A bracket under the first two diagrams is labeled $\theta(g)$, and a bracket under the third diagram is also labeled $\theta(g)$.

$$\text{diagram 1} = \frac{1}{2} (i \cancel{g} g) \int d^4 y \Delta_F(x-y) \Delta_F(y-y)$$

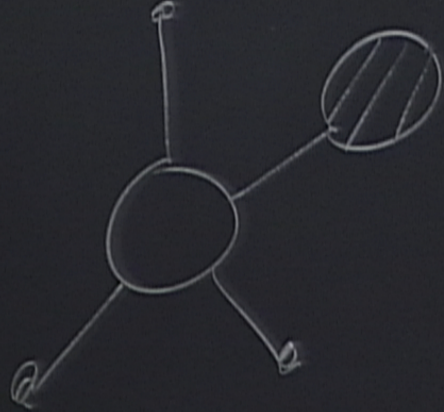
The diagram 1 is a tadpole with a loop on the left and a line extending to the right ending in a cross. The equation shows it is equal to $\frac{1}{2} (i \cancel{g} g) \int d^4 y \Delta_F(x-y) \Delta_F(y-y)$. An arrow points from the \cancel{g} term to the diagram.

$$\text{diagram 2} = iY \int d^4 y \Delta_F(x-y)$$

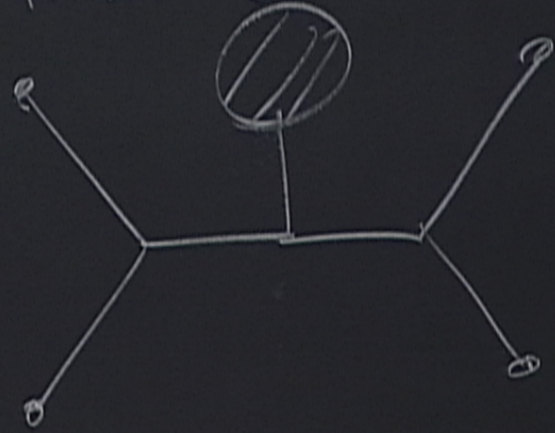
The diagram 2 is a tadpole with a loop on the right and a line extending to the left ending in a cross. The equation shows it is equal to $iY \int d^4 y \Delta_F(x-y)$.

$$\sum \text{diagram} = 0$$

no need to compute these diagrams



or



Momentum Space Propagator

$$G(p, p') = (2\pi)^4 \delta^4(p + p') G_2(p)$$

$$G_2(p) = \text{---} \underset{\substack{\uparrow \\ p}}{\text{---}} \text{---} + \text{---} \underset{\substack{\uparrow \\ p}}{\text{---}} \text{---} \text{---} \underset{\substack{\uparrow \\ p}}{\text{---}}$$

new vertex



comes from $\frac{1}{2}(Z_{\varphi}-1)(\partial\varphi)^2$ and $\frac{1}{2}(Z_m-1)\varphi^2$

different from $\rightarrow x$



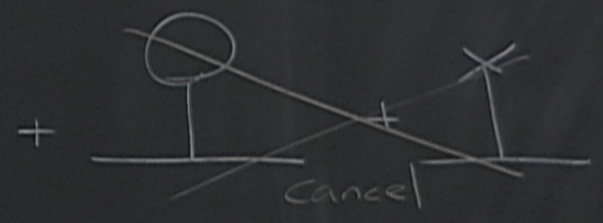
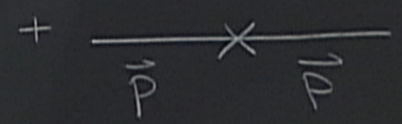
for

1) $G_2(p)$

new vertex

~~—*—~~ comes from $\frac{1}{2}(Z_4 - 1) \odot$

different from —*—
and —



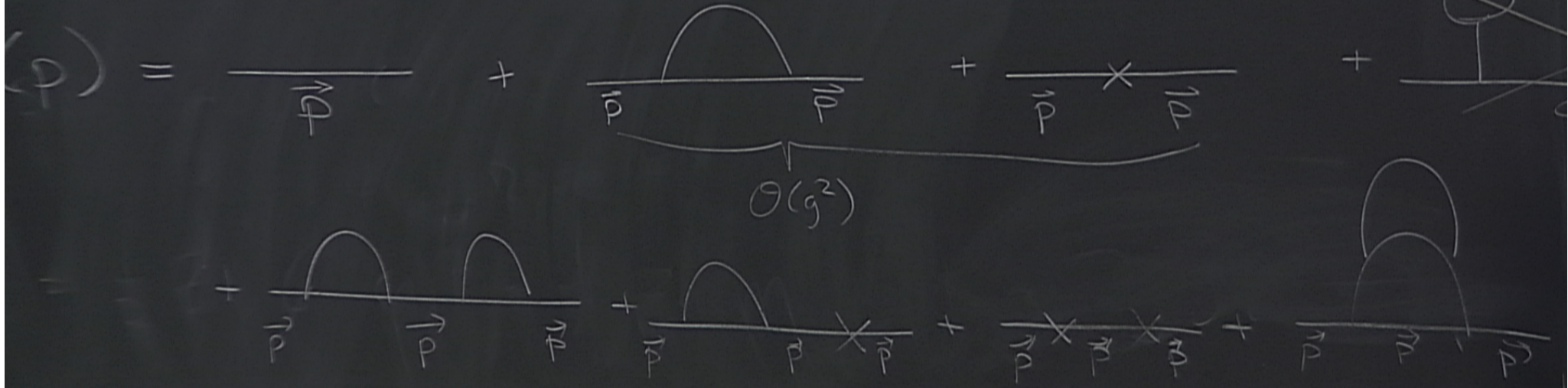
Time Space Propagator

$$G(p, p') = (2\pi)^4 \delta^4(p + p') G_2(p)$$

new vertex



different from



new vertex



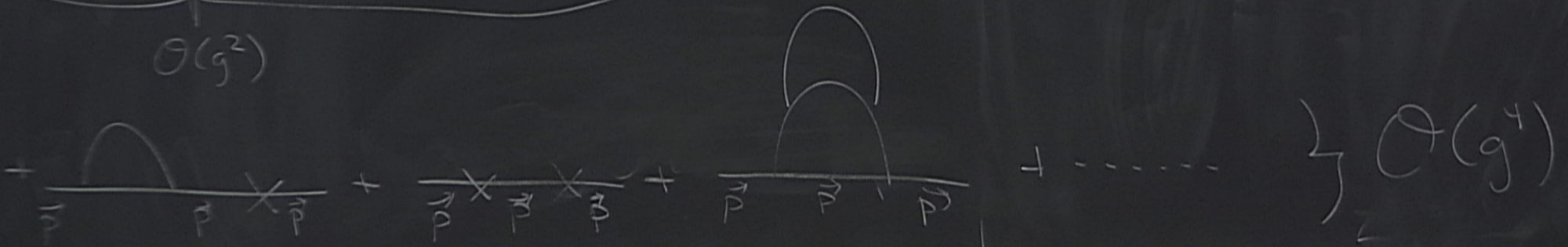
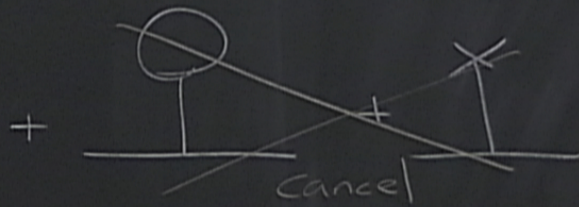
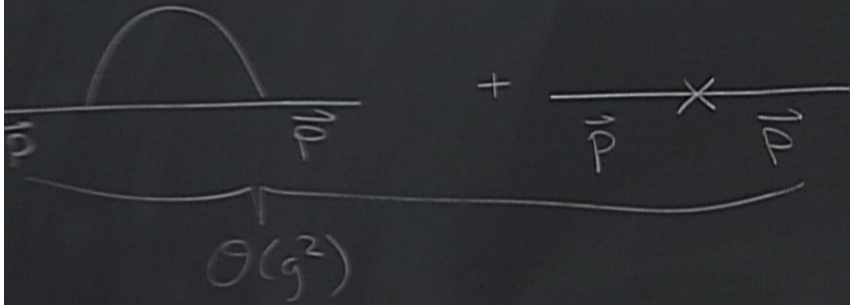
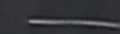
comes from $\frac{1}{2}(\mathbb{Z}_4 - 1)(\partial\varphi)$

$G_2(p)$

different from



and





$$G_2(p) = \text{---} + \text{---} \text{---} \text{IPI} \text{---} + \text{---} \text{---} \text{IPI} \text{---} \text{---} \text{IPI} \text{---} + \dots$$

↑
 IPI = one particle
 irreducible
 which means that
 there is no line that
 can be removed to
 separate the diagram
 into two disconnected
 pieces

$$G_2(p) = \frac{i}{p^2 - m^2 + i\epsilon} + \left(\frac{i}{p^2 - m^2 + i\epsilon} \right)^2 \left(-i \Sigma(p^2) \right)$$

contribution
from all 1PI
diagrams

$$+ \left(\frac{i}{p^2 - m^2 + i\epsilon} \right)^3 \left(-i \Sigma(p^2) \right)^2$$

$$= \frac{i}{p^2 - m^2 + i\epsilon} \sum_{n=0}^{\infty} \left(\frac{\Sigma(p^2)}{p^2 - m^2 + i\epsilon} \right)^n$$

$$G_2(P, P') \stackrel{FT}{\leftrightarrow} G(x, y) = \langle \Omega | \varphi(x) \varphi(y) | \Omega \rangle$$

↑
 1PI = one particle
 irreducible
 which means
 there is no
 can be renormalized
 separately
 into two
 p

$$G_2(p) = \frac{i}{p^2 - m^2 + i\epsilon} \left| - \frac{\Sigma(p^2)}{p^2 - m^2 + i\epsilon} \right.$$

$$= \frac{i}{p^2 - m^2 - \Sigma(p^2) + i\epsilon}$$

Exact result

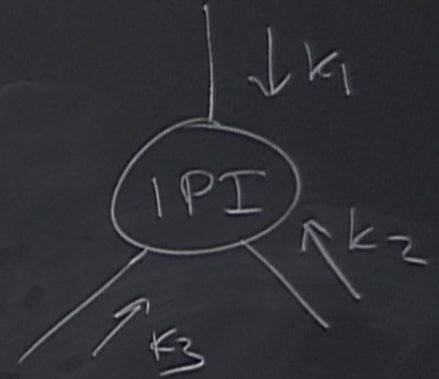
If $m^2 = m_{\text{ph}}^2$

we need $\sum(m^2) = 0$ so that $G_2^{|\text{PI}|}$ has a pole at $p^2 = m^2$

$\sum'(m^2) = 0$ so that residue of $G_2^{|\text{PI}|}$ at $p^2 = m^2$ is 1

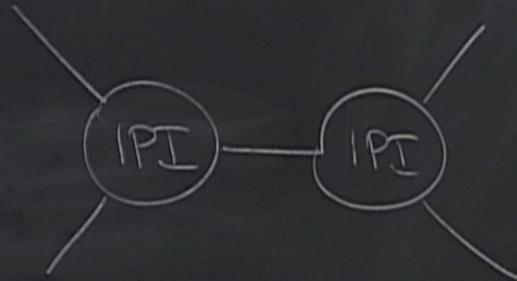
can use these conditions to fix Z_φ, Z_m order by order in g

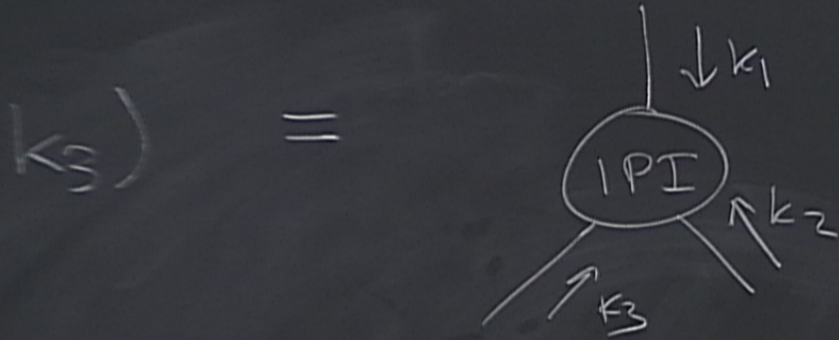
$$i V_3(k_1, k_2, k_3) =$$



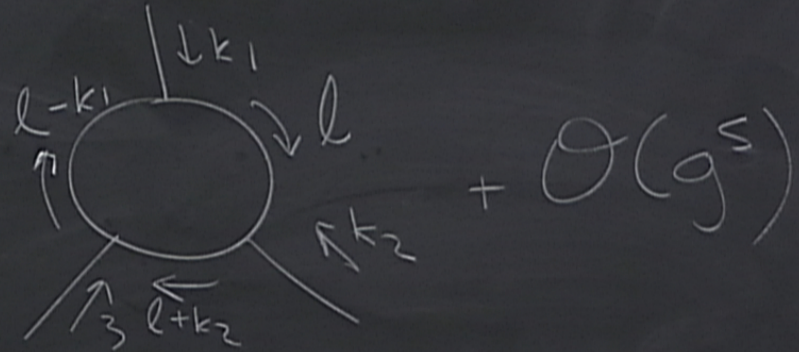
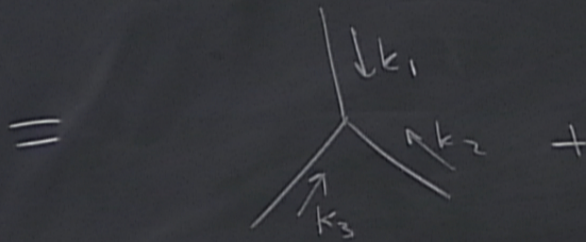
for a vertex function
we do impose $k_i^2 = m_i^2$

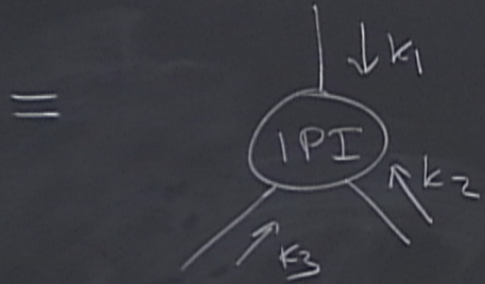
otherwise same as iM





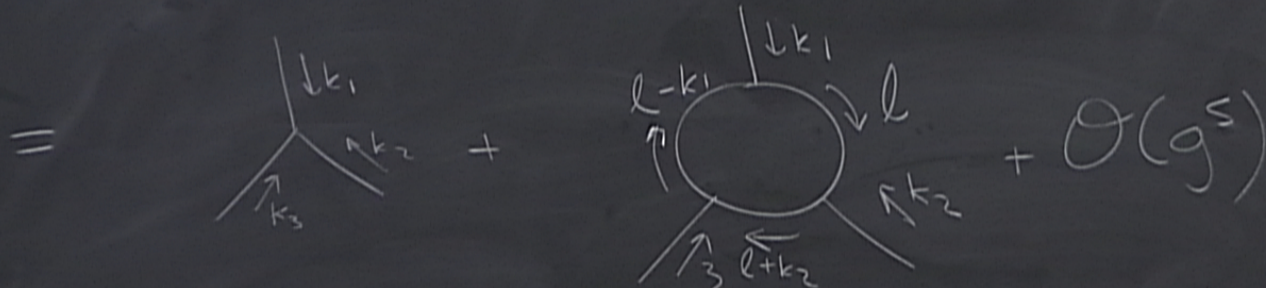
for a vertex function
we do impose $k_i^2 = m_i^2$





for a vertex function
we do not impose $k_i^2 = m_i^2$
for external particle

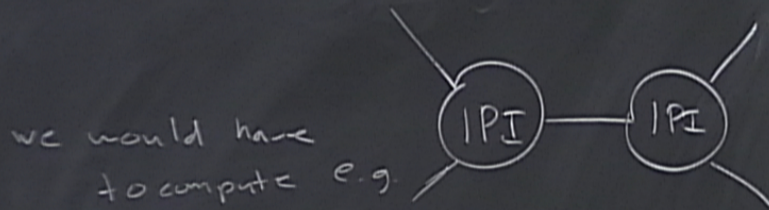
otherwise same as iM



=

$$iZg^3 + (iZg^3)^3 \int \frac{d^4 l}{(2\pi)^4} \frac{i}{l^2 - m^2 + i\epsilon} \frac{i}{(l+k_2)^2 - m^2 + i\epsilon} \frac{i}{(l-k_1)^2 - m^2 + i\epsilon}$$

Could demand some σ is related to g in a particular way



$$\sigma = \# g^2$$

$$V_3(0,0,0) = g \quad \text{can be easier}$$

- Z_1, Y formally infinite

→ need to regulate ②

- solve renormalization conditions ①

$$\langle \Omega | \varphi(x) | \Omega \rangle = 0$$

$$\langle k | \varphi(x) | \Omega \rangle = e^{ikx} \leftrightarrow \Sigma(m^2) = 0 = \Sigma'(m^2)$$

$$V_3(0, 0, 0) = g$$

- compute physical quantities ③
 - they will be independent of regulator
- remove regulator ④

- There are other ways to renormalize
ours is on-shell scheme

- More in QFT II