Title: PSI 2016/2017 Quantum Field Theory I - Lecture 7

Date: Oct 19, 2016 09:00 AM

URL: http://pirsa.org/16100007

Abstract:

Introduction to Loops

$$\mathcal{L}_{naive} = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2 + \frac{1}{3!}g\varphi^3$$

LSZ: $\langle\Omega|\varphi(x)|\Omega\rangle = 0$

$\langle k|\varphi(x)|\Omega\rangle = e^{ik\cdot x}$

$$\mathcal{L} = \frac{1}{2}Z_\varphi(\partial\varphi)^2 - \frac{1}{2}Z_m m^2\varphi^2 + \frac{1}{3!}Z_g g\varphi^3 + Y\varphi$$

$$\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{int}$$

$$\mathcal{L}_{free} = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2$$

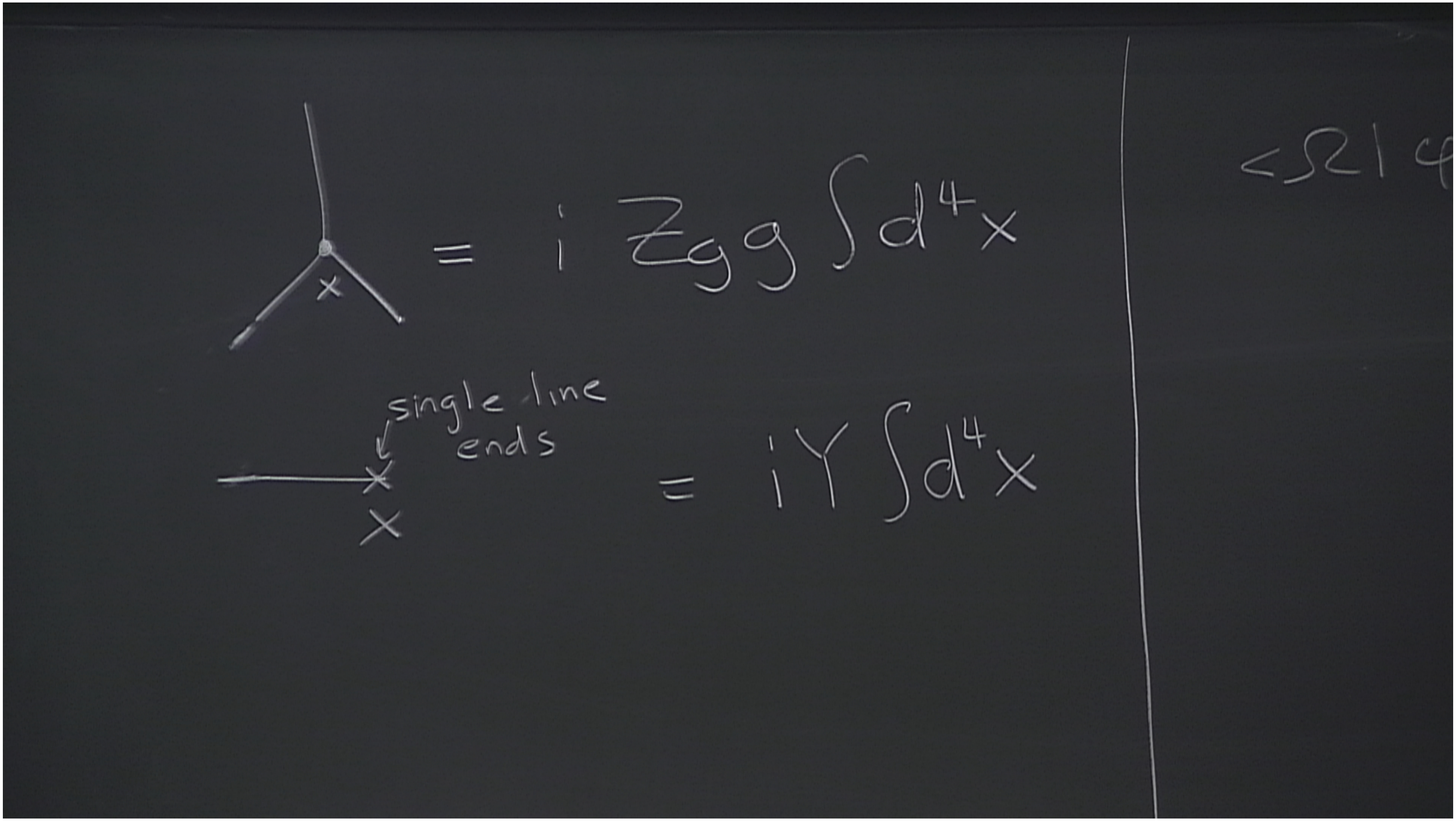
$$\mathcal{L}_{int} = \frac{1}{3!}Z_g g\varphi^3 + \mathcal{L}_{ct}$$

$$\mathcal{L}_{ct} = \frac{1}{2}(Z_\varphi - 1)(\partial\varphi)^2 - \frac{1}{2}(Z_m - 1)m^2\varphi^2 + Y\varphi$$

counterterm

$$Y = 0 + \mathcal{O}(g)$$

$$Z_i = 1 + \mathcal{O}(g^2)$$


$= i Z_g g \int d^4x$
single line ends
$= iY \int d^4x$
$\langle \Omega |$

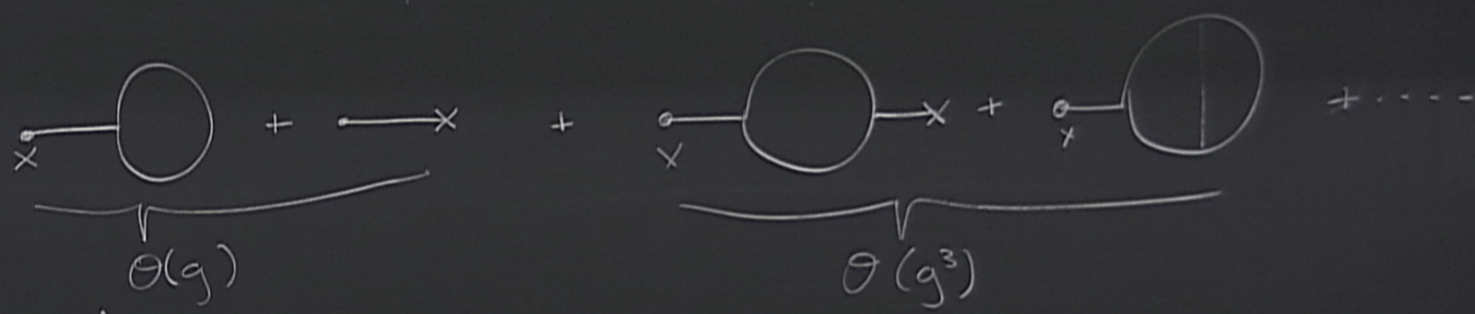
$\ldots m m^2 \varphi^2 + \frac{1}{3!} Z_g g \varphi^3 + Y\varphi$

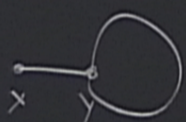
Counterterm

$Y = 0 + O(g)$

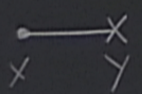
$Z_1 = 1 + O(g^2)$

$\langle \Omega | \varphi(x) | \Omega \rangle = 0$

$=$ 



$= \frac{1}{2}\left(i \cancel{Z_g}^{\,1} g\right) \int d^4y \; \Delta_F(x-y)\, \Delta_F(y-y)$



$= \; iY \int d^4y \; \Delta_F(x-y)$

$$\langle\Omega|\varphi(x)|\Omega\rangle = \left(iY + \frac{1}{2} i g\, \Delta_F(0)\right) \int d^4y\, \Delta_F(x-y) \quad + \mathcal{O}(g^3)$$

$$Y = -\frac{1}{2} g\, \Delta_F(0) + \mathcal{O}(g^3)$$

$$\Delta_F(0) = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2-m^2+i\varepsilon} \cancel{e^{-ip\cdot 0}} = \infty$$

Regularize this result by: imposing a cutoff Λ

$$\Delta_F(0) \rightarrow \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2-m^2+i\varepsilon} \left(\frac{\Lambda^2}{p^2-\Lambda^2+i\varepsilon}\right)^2$$

$$+\tfrac{1}{2}ig\,\Delta_F(0))\int d^4y\,\Delta_F(x-y) \quad + \mathcal{O}(g^3)$$

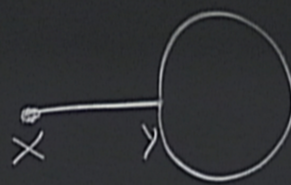
$$g\,\Delta_F(0) + \mathcal{O}(g^3)$$

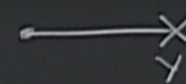
$$\frac{i}{p^2-m^2+i\epsilon}\,\cancel{e^{-ip\cdot 0}} \to 0 \quad = \infty$$

s result by: imposing a cutoff Λ

$$\Delta_F(0) \to \int \frac{d^4p}{(2\pi)^4}\,\frac{i}{p^2-m^2+i\epsilon}\left(\frac{\Lambda^2}{p^2-\Lambda^2+i\epsilon}\right)^2 = \frac{\Lambda^2}{16\pi^2}$$

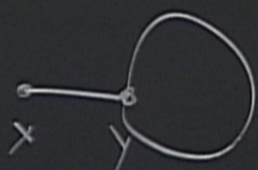
$\langle\Omega|\varphi(x)|\Omega\rangle = 0$

$=$ 

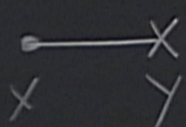
$+$ 

$+$ 

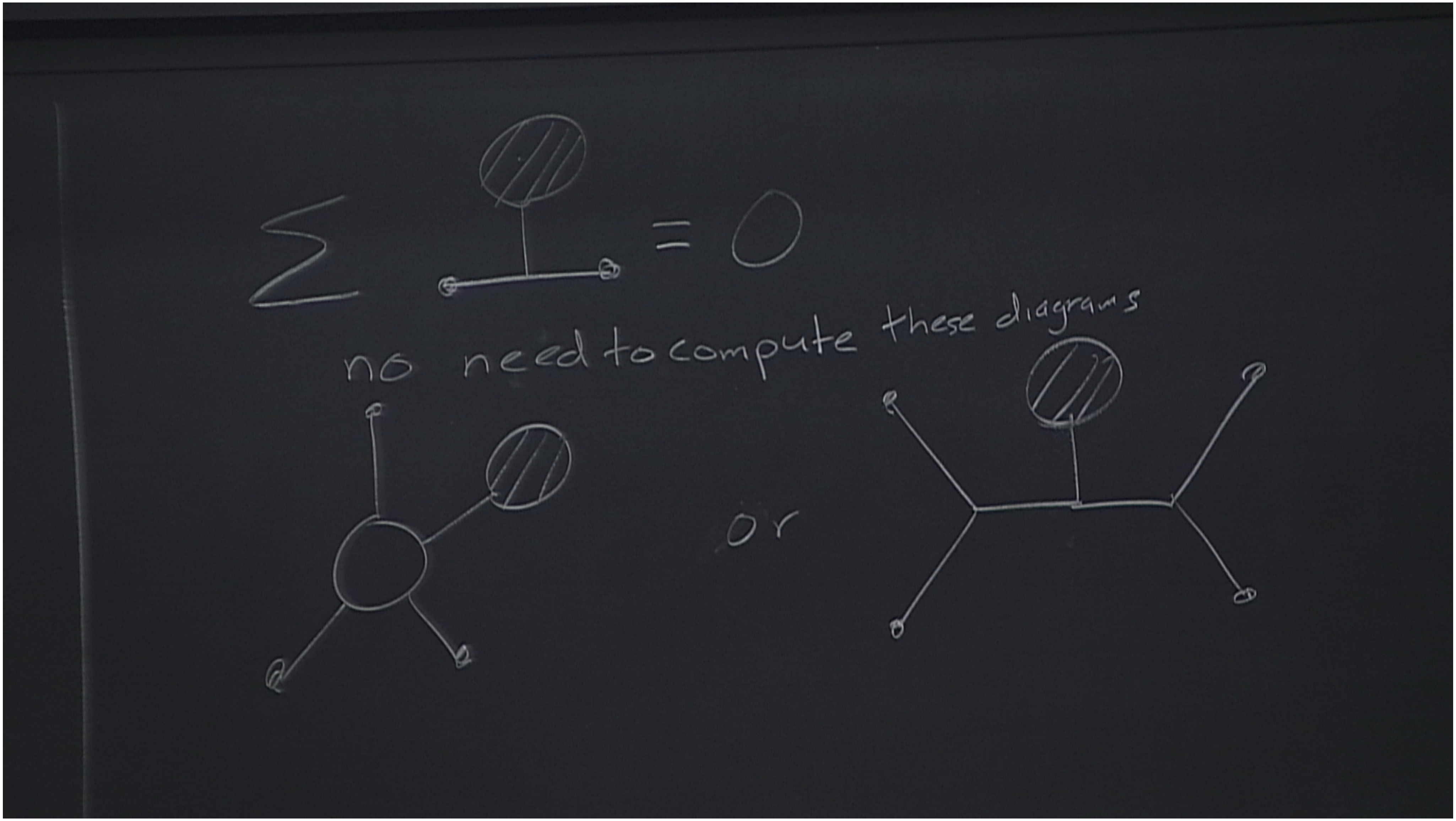
$\mathcal{O}(g)$ $\mathcal{O}(g$



$= \frac{1}{2}\left(i\, \overset{1}{\cancel{Z_g}}\, g\right)\int d^4y \;\Delta_F(x-y)\,\Delta_F(y-y)$



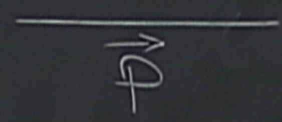
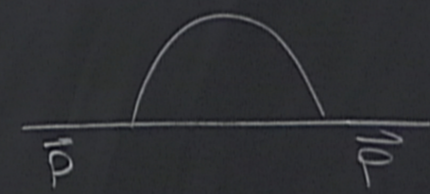
$= \; iY\int d^4y\,\Delta_F(x-y)$



= 0
no need to compute these diagrams
or

Momentum Space Propagator

$$G(p, p') = (2\pi)^4 \delta^4(p + p')\, G_2(p)$$

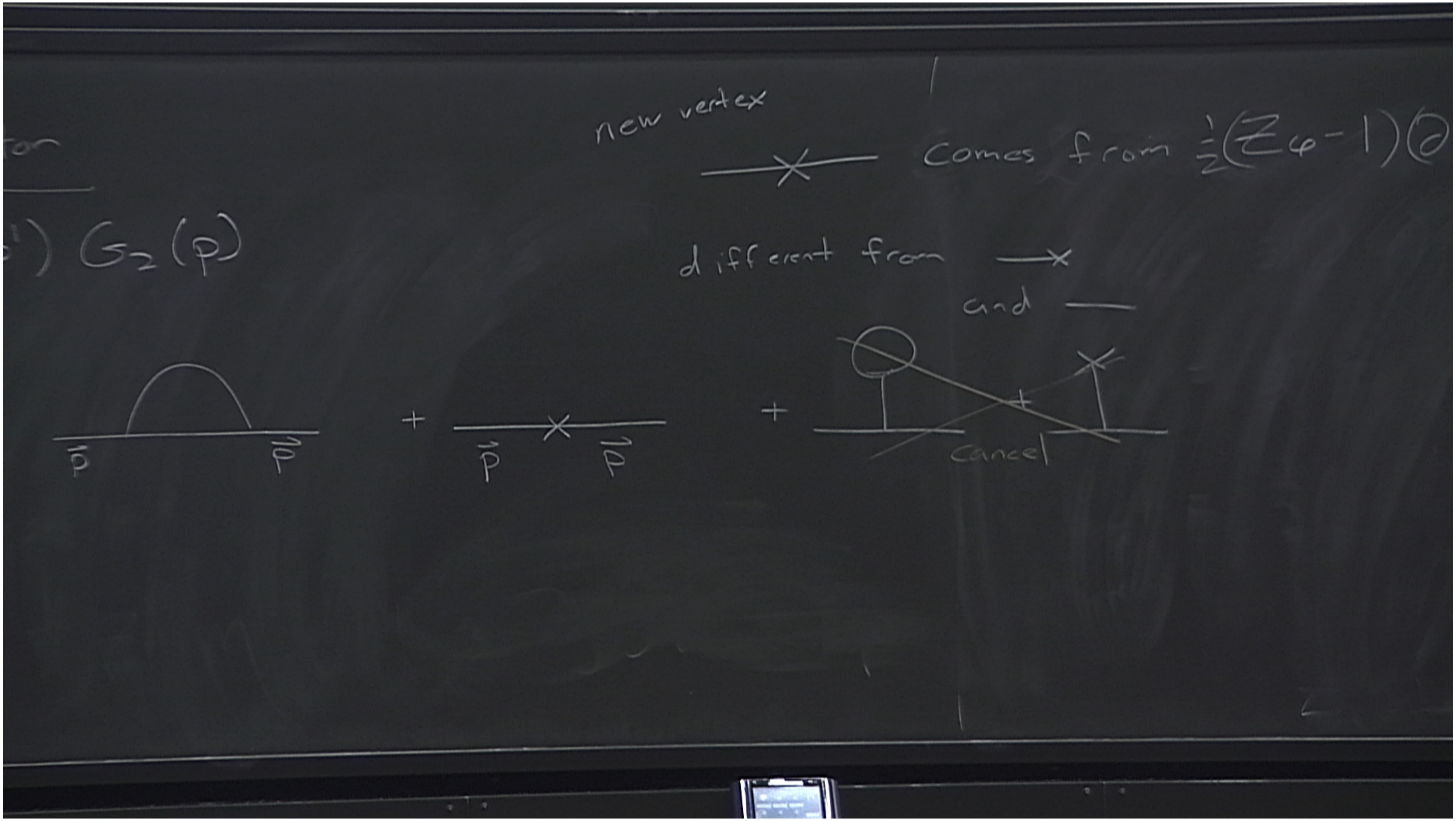
$G_2(p) =$  $+$ 

new vertex

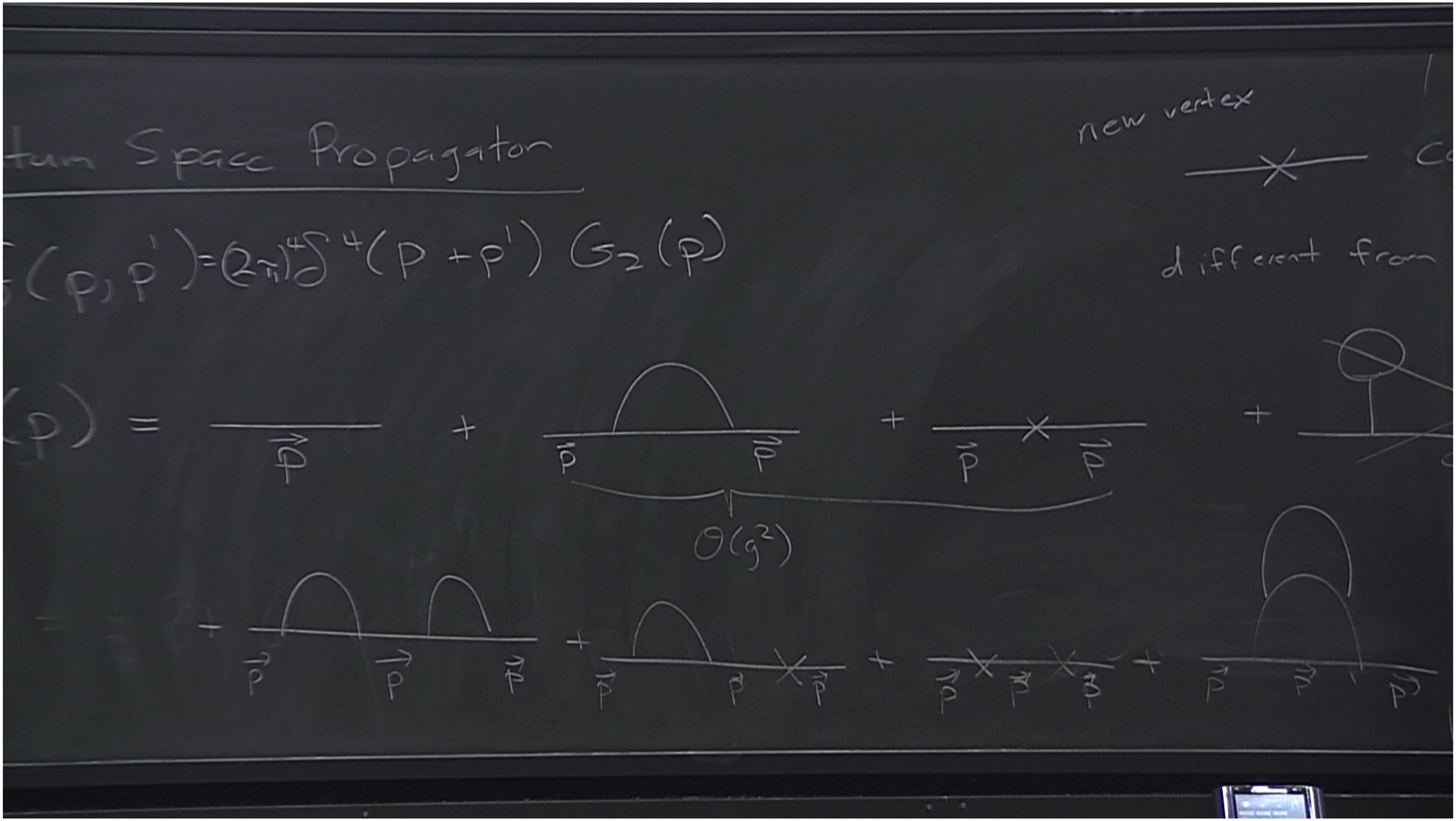
—×— comes from $\frac{1}{2}(Z_\varphi - 1)(\partial\varphi)^2$ and $\frac{1}{2}(Z_m - 1)\varphi^2$

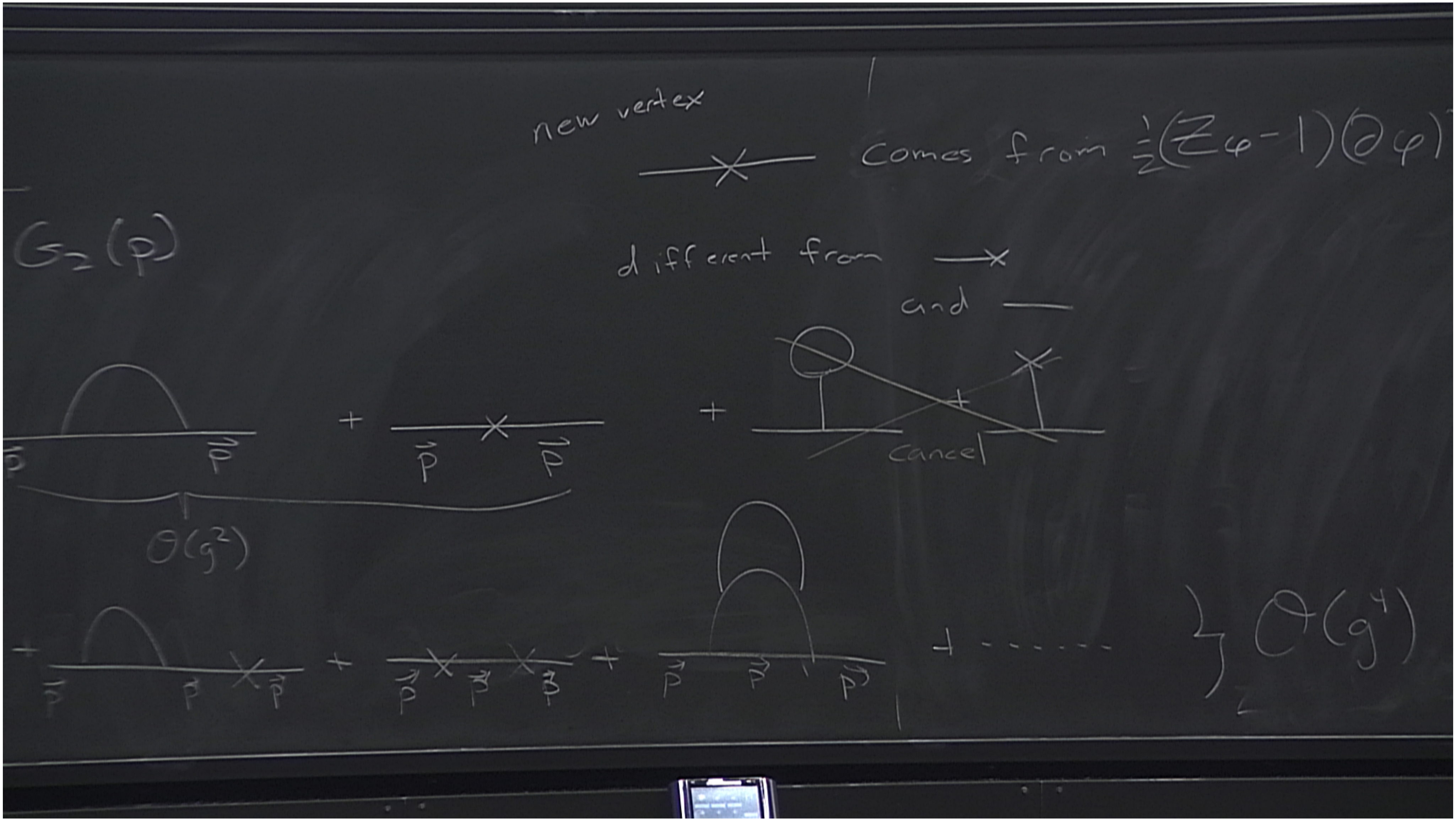
different from —×

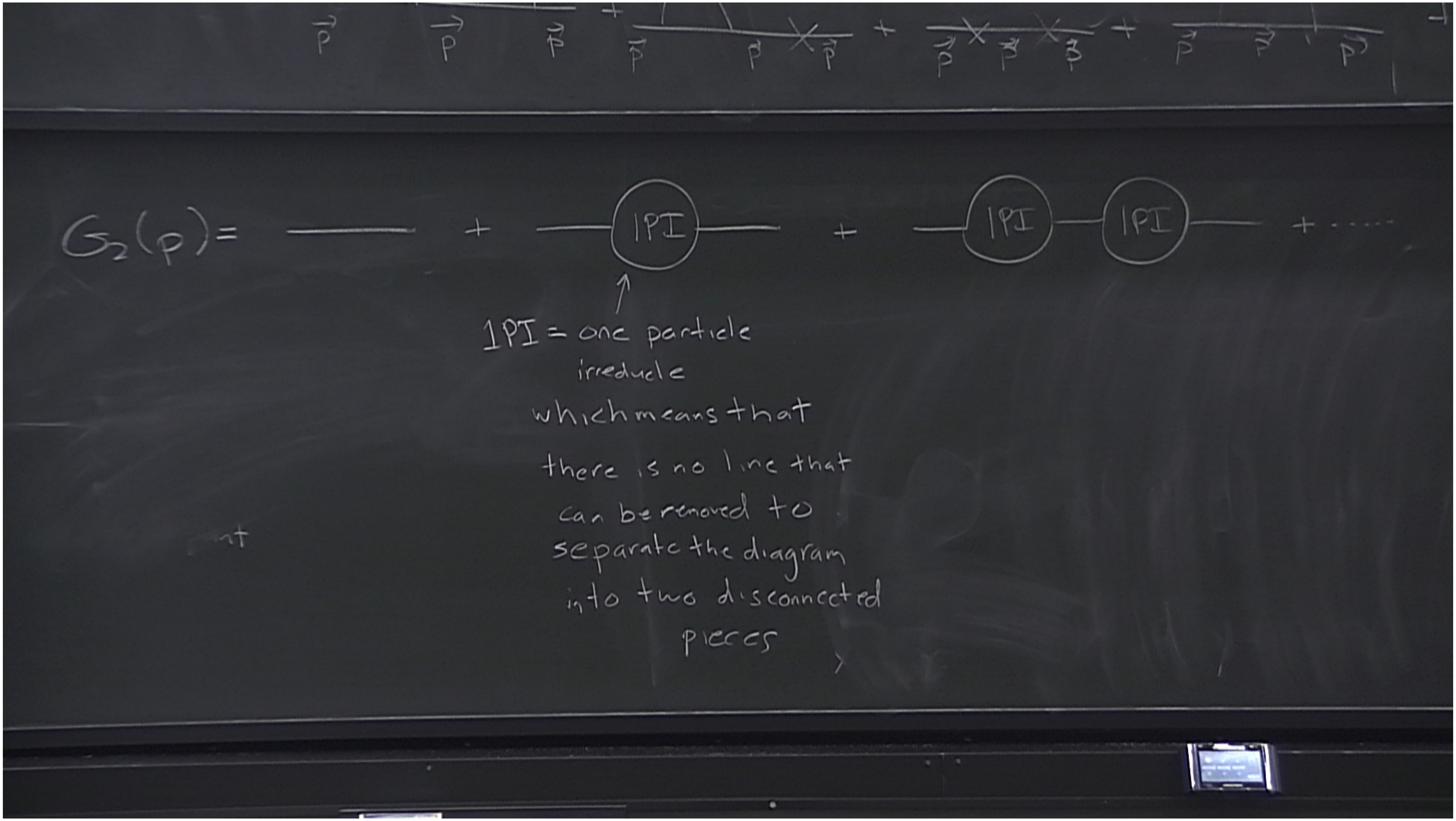
and ——



) $G_2(p)$
new vertex
comes from $\frac{1}{2}(Z_\varphi - 1)(\partial$
different from
and
$\vec{p}$
$\vec{p}$
$\vec{p}$
$\vec{p}$
cancel


tum Space Propagator
(p, p') = (2π)^4 δ^4(p + p') G_2(p)
new vertex
different from
(p) =
O(g^2)


new vertex
comes from $\frac{1}{2}(Z_\varphi - 1)(\partial\varphi)^2$
$G_2(p)$
different from
and
cancel
$O(g^2)$
$O(g^4)$


G2(p) =
1PI
1PI
1PI
1PI = one particle
irreducle
which means that
there is no line that
can be removed to
separate the diagram
into two disconnected
pieces

$$G_2(p) = \frac{i}{p^2 - m^2 + i\varepsilon} + \left(\frac{i}{p^2 - m^2 + i\varepsilon}\right)^2 \left(-i\Sigma(p^2)\right)$$

contribution from all 1PI diagrams

$$+ \left(\frac{i}{p^2 - m^2 + i\varepsilon}\right)^3 \left(-i\Sigma(p^2)\right)^2$$

$$= \frac{i}{p^2 - m^2 + i\varepsilon} \sum_{n=0}^{\infty} \left(\frac{\Sigma(p^2)}{p^2 - m^2 + i\varepsilon}\right)^n$$

$1PI$ = one par[...]

irreducle

which mea[...]

there is n[...]

can be re[...]

separate

into two

p[...]

$$G_2(p,p') \overset{FT}{\Longleftrightarrow} G(x,y) = \langle \Omega | \varphi(x)\varphi(y) | \Omega \rangle$$

$$G_2(p) = \frac{i}{p^2 - m^2 + i\varepsilon} \frac{1}{1 - \frac{\Sigma(p^2)}{p^2 - m^2 + i\varepsilon}}$$

$$= \frac{i}{p^2 - m^2 - \Sigma(p^2) + i\varepsilon}$$

Exact result

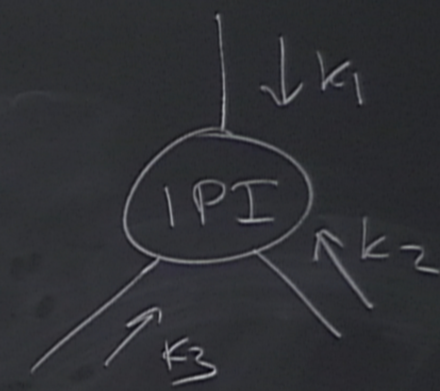
If $m^2 = m_{ph}^2$

we need $\Sigma(m^2) = 0$ so that G_2^{1PI} has a pole at $p^2 = m^2$

$\Sigma'(m^2) = 0$ so that residue of G_2^{1PI} at $p^2 = m^2$ is 1

can use these conditions to fix Z_φ, Z_m order by order in g

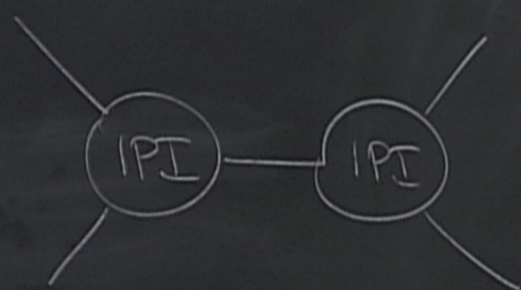
$$i V_3(k_1, k_2, k_3) =$$

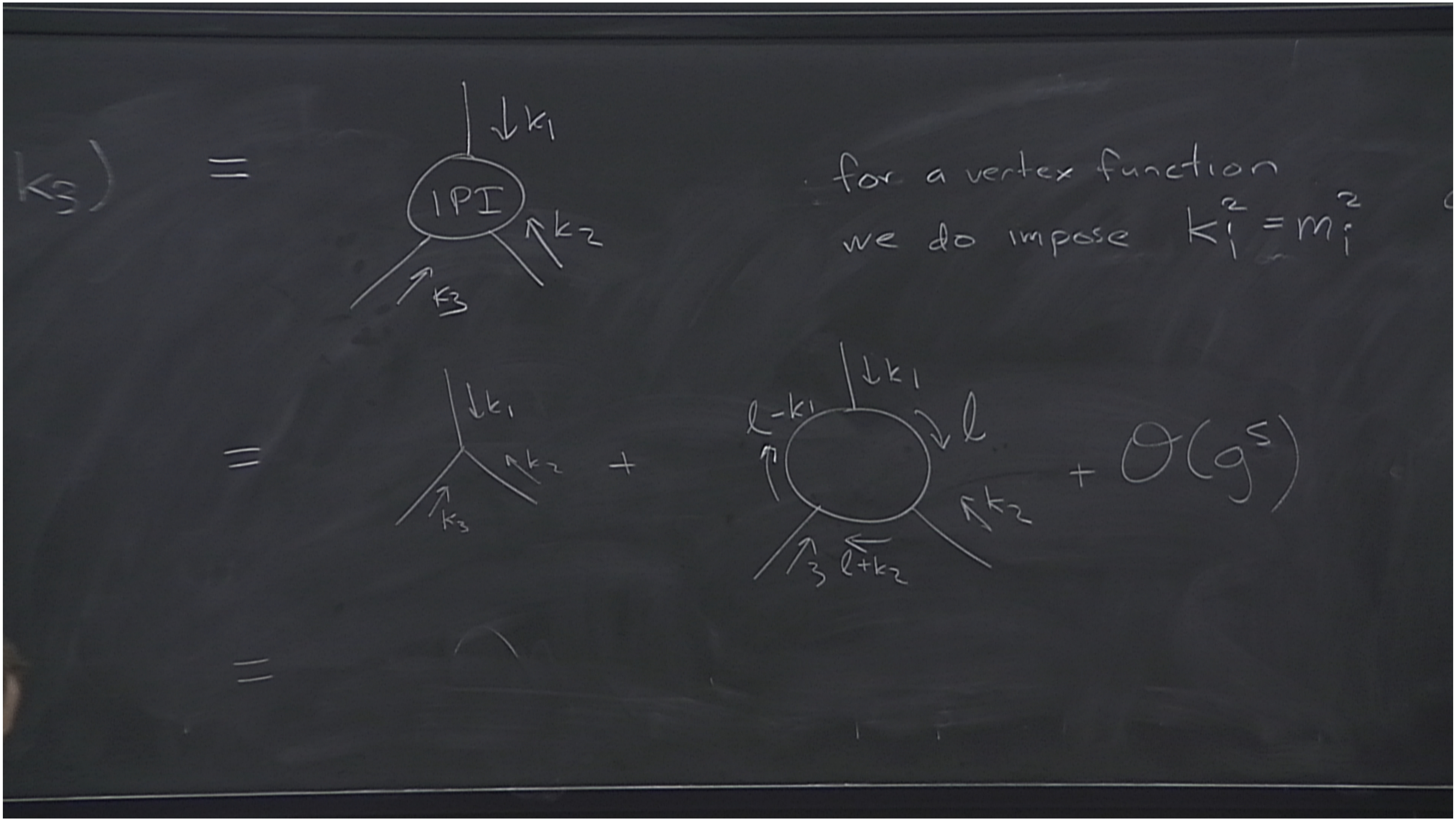


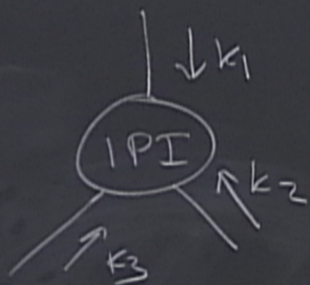
for a vertex function

we do impose $k_i^2 = m_i^2$

otherwise same as $i\mathcal{M}$




k3) =
1PI
k1
k2
k3
for a vertex function
we do impose $k_i^2 = m_i^2$
=
+
l-k1
l
l+k2
3
$+ O(g^5)$
=

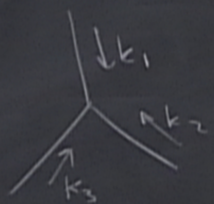
$=$ 

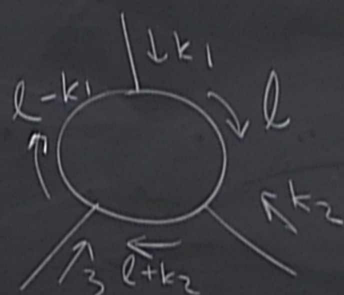
for a vertex function

we do not impose $k_i^2 = m_i^2$

for external particle

otherwise same as $i\mathcal{M}$

$=$ 

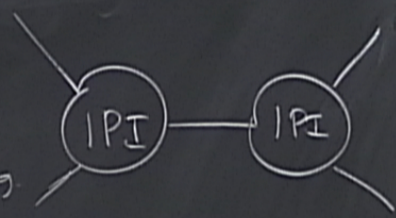
$+$ 

$+ \mathcal{O}(g^5)$

$$= iZ_g g + (i\cancelto{1}{Z_g}\, g)^3 \int \frac{d^4 l}{(2\pi)^4} \frac{i}{l^2-m^2+i\varepsilon} \frac{i}{(l+k_2)^2-m^2+i\varepsilon} \frac{i}{(l-k_1)^2-m^2+i\varepsilon}$$

Could demand some σ is related to g in a particular way

we would have to compute e.g.



$$\sigma = \# g^2$$

$$V_3(0,0,0) = g$$ can be easier

- Z_i, Y formally infinite

$\rightarrow$ need to regulate (2)

- solve renormalization conditions (1)

$$\langle\Omega|\varphi(x)|\Omega\rangle = 0$$

$$\langle k|\varphi(x)|\Omega\rangle = e^{ikx} \Longleftrightarrow \Sigma(m^2) = 0 = \Sigma'(m^2)$$

$$V_3(0,0,0) = g$$

- compute physical quantities ③

→ they will be independent of regulator

- remove regulator ④

- There are other ways to renormalize

ours is on-shell scheme

- More in QFT II