

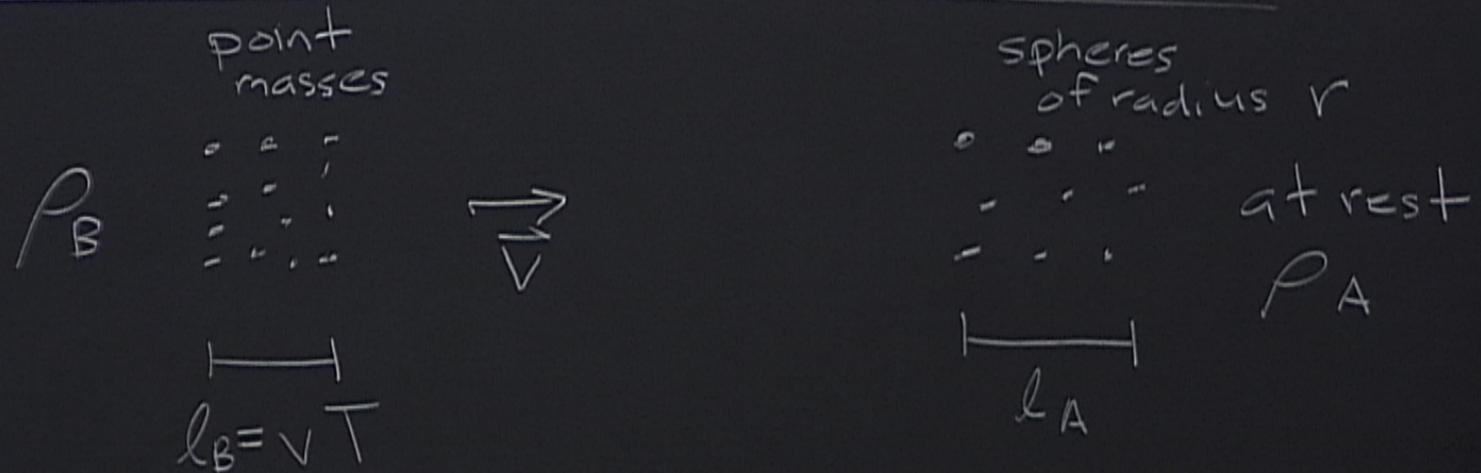
Title: PSI 2016/2017 Quantum Field Theory I - Lecture 6

Date: Oct 18, 2016 09:00 AM

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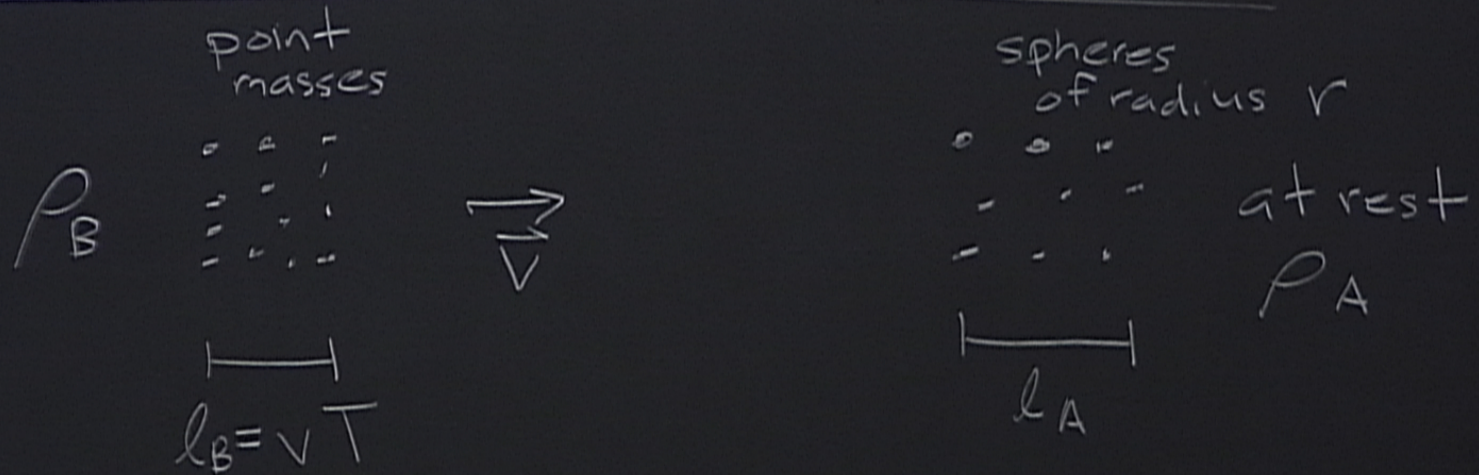
Abstract:

Cross Sections + Decay Rates

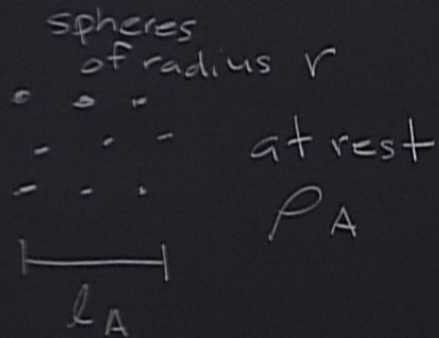


Classically: $N_{\text{collisions}} = \sigma \rho_A l_A \rho_B l_B A$

Cross Sections + Decay Rates



Classically: $N_{\text{collisions}} = \sigma P_A l_A P_B l_B A$



A = cross section
common
to two beams

$$N_{\text{collisions}} = \sigma \rho_A l_A \rho_B l_B A$$

$$N_{\text{collisions}} = \frac{N_A \sigma N_B}{A} = \frac{\sigma N_A N_B v T}{\text{vol}}$$

$$N_A = \rho_A \underbrace{l_A A}_{\text{vol}}$$

$$\frac{B}{V} = \frac{\sigma N_A N_B V T}{vol}$$

$$P = \frac{N_{collisions}}{N_A N_B} = \frac{\sigma V T}{vol}$$

↑
probability
of collisions

QFT: $2 \rightarrow n$ scattering

$$|i\rangle = |\vec{k}_1, \vec{k}_2\rangle$$

$$|f\rangle = |\vec{p}_1, \dots, \vec{p}_n\rangle$$

$$dP = \frac{|\langle f | S | i \rangle|^2}{\langle f | f \rangle \langle i | i \rangle} \prod_{j=1}^n \frac{d^3 p_j (\text{vol})}{(2\pi)^3}$$

In periodic space with $\text{vol} = L^3$

$$\vec{p}_j = \frac{2\pi}{L} \vec{n}_j$$

↑
integer components

$$\sum_{\vec{n}_j} \rightarrow \int \frac{d^3 p_j \text{vol}}{(2\pi)^3}$$

$$|\langle f | S | i \rangle|^2 = \left[(2\pi)^4 \delta^{(4)}(\sum p_j - k_1 - k_2) \right]^2 |M|^2$$

$$(\delta^{(4)})^2$$

$$[\]^2 = (2\pi)^4 \delta^{(4)}(\sum p_j - k_1 - k_2) \underbrace{(2\pi)^4 \delta^{(4)}(0)}$$

$$\int d^4x e^{i0 \cdot x} = (\text{vol}) T$$

$$\langle k|k \rangle = (2\pi)^3 2E_{\vec{k}} \delta^{(3)}(0) = 2E_{\vec{k}} \cdot \text{vol}$$

$$\langle i|i \rangle = 2E_{\vec{k}_1} 2E_{\vec{k}_2} (\text{vol})^2$$

$$\langle f|f \rangle = \prod_{j=1}^n (2E_{\vec{p}_j} \cdot \text{vol})$$

$$dP = d\Phi \frac{|M_{i \rightarrow f}|^2}{2E_k, 2E_k} \frac{T}{vol}$$

$$d\Phi = (2\pi)^4 \delta^{(4)}\left(\sum_{j=1}^n p_j - k_1 - k_2\right) \prod_{j=1}^n \frac{d^3 p_j}{(2\pi)^3 2E_{p_j}}$$

↑
phase space measure

$$dP = \frac{d\sigma v T}{vol}$$

$$d\sigma = d\Phi \cdot \frac{|M_{i \rightarrow f}|^2}{2E_{\vec{k}_1} 2E_{\vec{k}_2} \cdot V}$$

$$V = \frac{\sqrt{(k_1 \cdot k_2)^2 - m_1^2 m_2^2}}{E_{\vec{k}_1} E_{\vec{k}_2}}$$

agree if $\vec{k}_1 \parallel \vec{k}_2$

$$V = |\vec{v}_1 - \vec{v}_2|$$

Γ_{partial}

$$\Gamma_{\text{total}} = \frac{1}{\tau \leftarrow \text{lifetime}}$$

$$= \sum_{\text{decay channels}} \Gamma_{\text{partial}}$$

$$d\Gamma_{\text{partial}} \equiv \frac{dP}{T} = d\Xi \frac{|M_{1 \rightarrow f}|^2}{2E_{k_1}|_{\text{CM}}}$$

$$d\Gamma_{\text{partial}} = d\Xi \frac{|M_{1 \rightarrow f}|^2}{2m_1}$$

Identical particles in final state:

- restrict the integration

- divide by a symmetry factor

(different from Feynman diagrams)

Scalar Nucleon-Meson Theory

$$\mathcal{L} = \overset{\substack{\uparrow \text{nucleon}}}{\partial_\mu \rho^* \partial^\mu \rho} - M^2 \rho^* \rho + \frac{1}{2} \overset{\substack{\uparrow \text{meson}}}{\partial_\mu \varphi \partial^\mu \varphi} - \frac{1}{2} m^2 \varphi^2$$

$$\rho \rightarrow e^{i\theta} \rho \rightarrow \text{conserved charge}$$

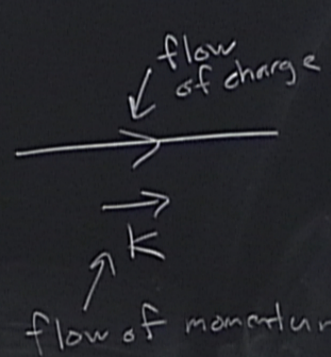
Nucleon-Meson Theory

$$\begin{aligned} \text{↙ nucleon} \quad & \star \partial_\mu \rho - M^2 \rho^* \rho + \frac{1}{2} \partial_\mu \varphi \overset{\text{↙ meson}}{\partial^\mu \varphi} - \frac{1}{2} m^2 \varphi^2 - g \rho^* \rho \varphi \end{aligned}$$

$$\rho \rightarrow e^{i\theta} \rho \rightarrow \text{conserved charge}$$

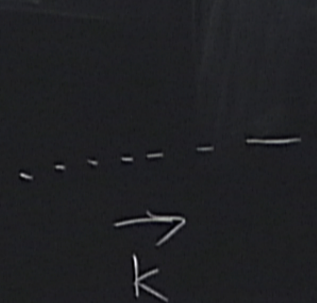
Momentum space Feynman Rules for iM

internal lines:



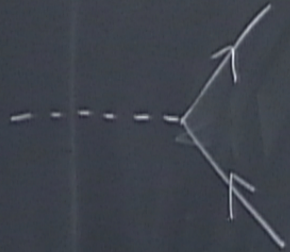
A horizontal solid line with an arrow pointing to the right. Above the line, an arrow points down and to the left with the text "flow of charge". Below the line, an arrow points up and to the left with the text "flow of momentum" and the letter k .

$$= \frac{i}{k^2 - M^2 + i\epsilon}$$



A horizontal dashed line with an arrow pointing to the right and the letter k below it.

$$= \frac{i}{k^2 - m^2 + i\epsilon}$$



$$= -ig$$

external lines = 1

momentum conservation at each vertex

divide by symmetry factor

integrate over undetermined momenta

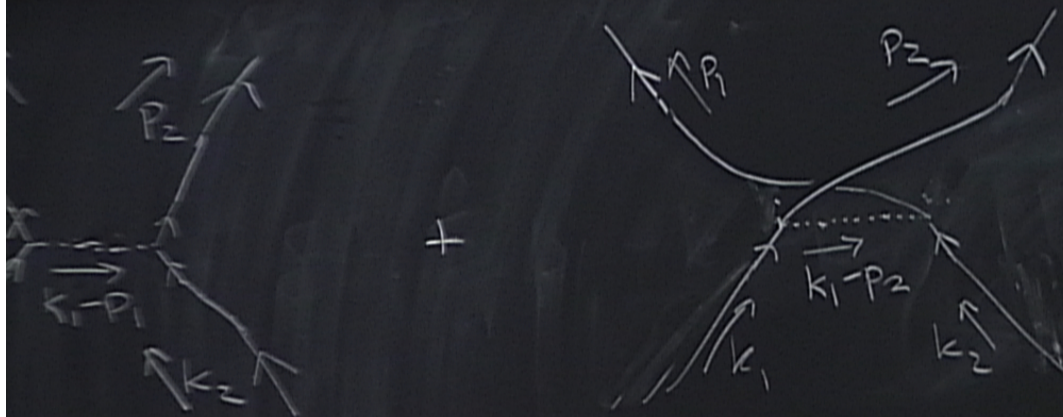
$$I \cdot |M_{i \rightarrow f}|^2$$

pp → pp Cross Section



$$iM_{pp \rightarrow pp} = (-ig)^2 \left[\frac{i}{(k_1 - p_1)^2 - m^2 + i\epsilon} + \frac{i}{(k_1 - p_2)^2 - m^2 + i\epsilon} \right]$$

pp Cross Section



$$+ \mathcal{O}(g^4)$$

$$2 \left[\frac{1}{(k_1 - p_1)^2 - m^2 + i\epsilon} + \frac{1}{(k_1 - p_2)^2 - m^2 + i\epsilon} \right] \leftarrow \text{no symmetry factor}$$

$$d\sigma = \frac{1}{2} \uparrow \text{sym factor}$$

$$d\sigma = \frac{1}{2} d\Phi_2 \frac{|M_{pp \rightarrow pp}|^2}{4\sqrt{(k_1 \cdot k_2)^2 - M^4}}$$

$\uparrow$
 symmetry
 factor

$$d\Phi_2 = (2\pi)^4 \delta^4(p_1 + p_2 - k_1 - k_2) \frac{d^3 p_1}{(2\pi)^3 2E_{\vec{p}_1}} \frac{d^3 p_2}{(2\pi)^3 2E_{\vec{p}_2}}$$

in CM frame: $\vec{k}_1 + \vec{k}_2 = 0$ $k_1^0 = k_2^0 = E$

$$\int d\Phi_2(\cdot) = \int \frac{1}{(2\pi)^2 2E_{\vec{p}_1} 2E_{\vec{p}_2}} \delta(E_{\vec{p}_1} + E_{\vec{p}_2} - 2E) d^3 p_1(\cdot) \Big|_{\vec{p}_2 = -\vec{p}_1}$$

Relativistic limit $E \gg M$

$$E_{\vec{p}_1} \approx |\vec{p}_1| = E_{\vec{p}_2}$$

3-mom cons

Vol

$$\frac{d\sigma}{d\Omega} = \frac{g^4}{128\pi^2 \sqrt{(k_1, k_2)^2}} \left[\frac{1}{(k_1 - p_1)^2} + \frac{1}{(k_1 - p_2)^2} \right]^2$$

set $M=0=m$

$$k_1 = (E, \vec{k})$$

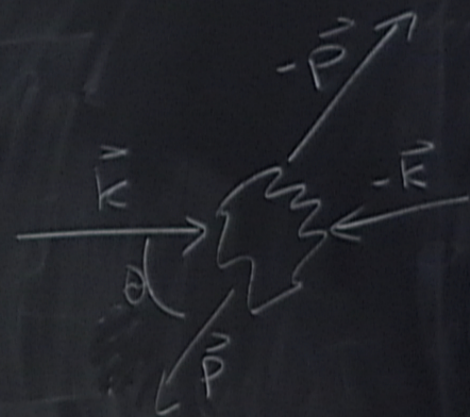
$$p_1 = (E, \vec{p})$$

$$k_2 = (E, -\vec{k})$$

$$p_2 = (E, -\vec{p})$$

$$|\vec{k}| = |\vec{p}| = E$$

$$k_1^2 = k_2^2 = p_1^2 = p_2^2 = 0$$



$$k_1 \cdot p_1 = E^2 - \vec{p} \cdot \vec{k} = E^2 - E^2 \cos \theta$$

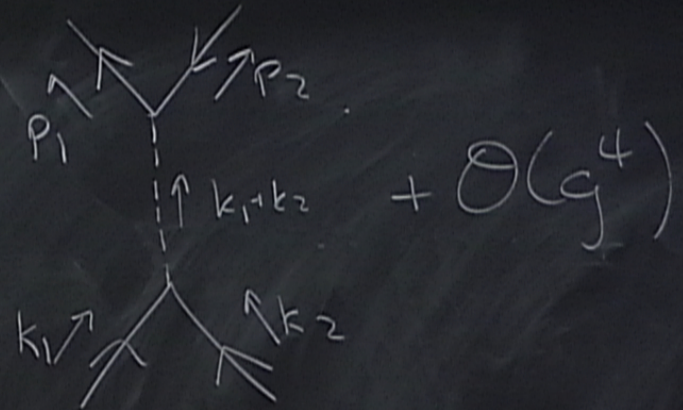
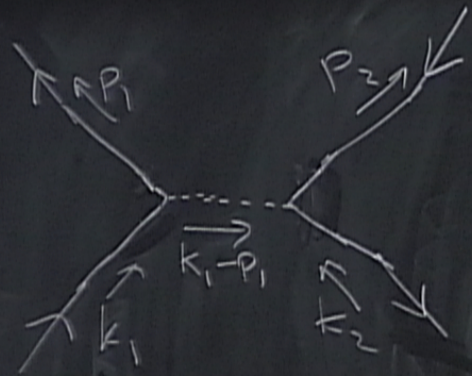
$$k_1 \cdot p_2 = E^2 + \vec{p} \cdot \vec{k} = E^2 (1 + \cos \theta)$$

$$k_1 \cdot k_2 = E^2 + \vec{k} \cdot \vec{k} = 2E^2$$

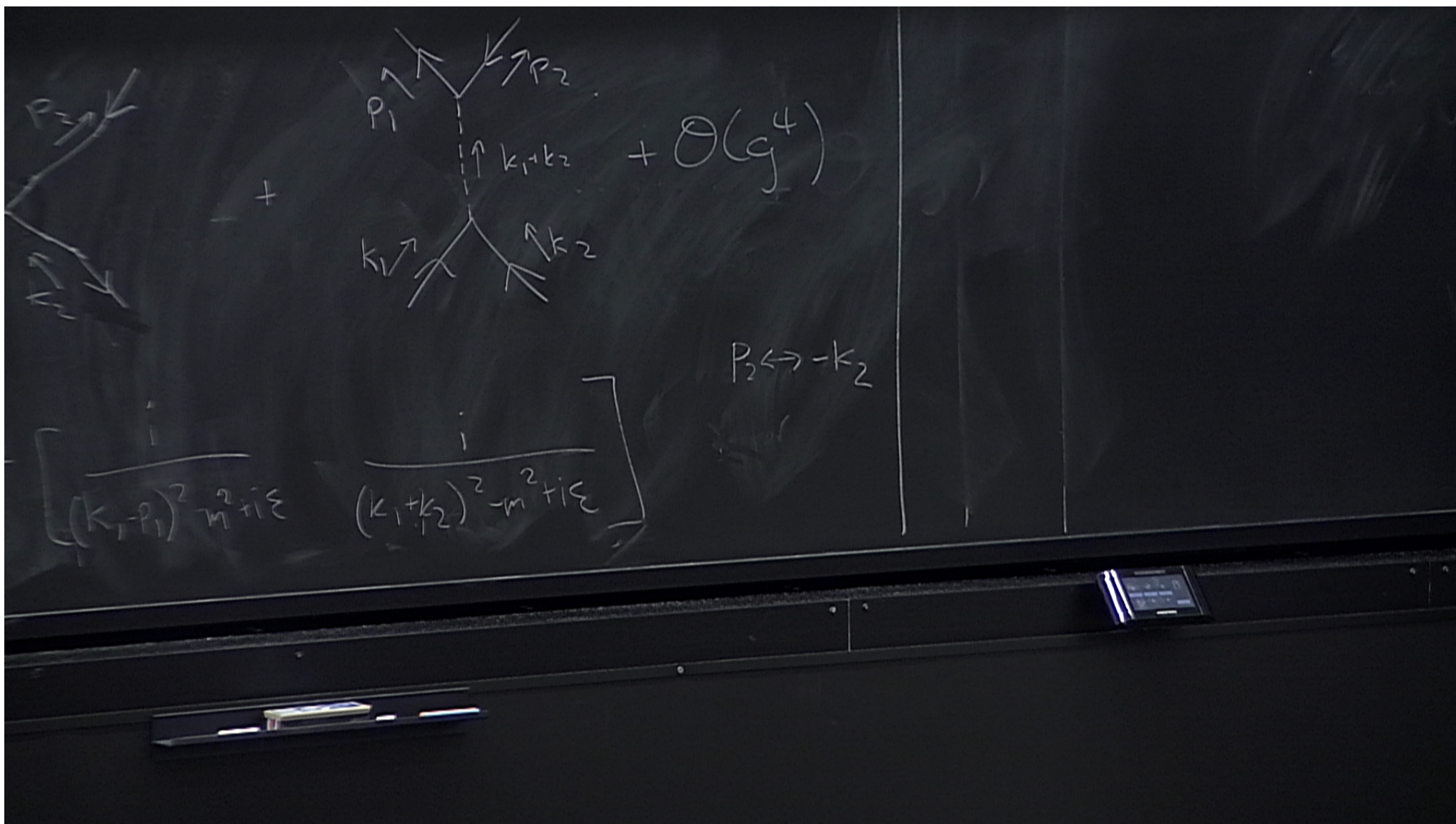
$$\frac{d\sigma}{d\Omega} = \frac{g^4}{256\pi^2 E^4} \left(\frac{1}{1 - \cos^2 \theta} \right)^2$$

↑ $\cos \theta$ int diverges

$pp^* \rightarrow pp^*$ Cross Section



$$M_{pp^* \rightarrow pp^*} = (-ig)^2 \left[\frac{i}{(k_1 - p_1)^2 - m^2 + i\epsilon} + \frac{i}{(k_1 + k_2)^2 - m^2 + i\epsilon} \right]$$

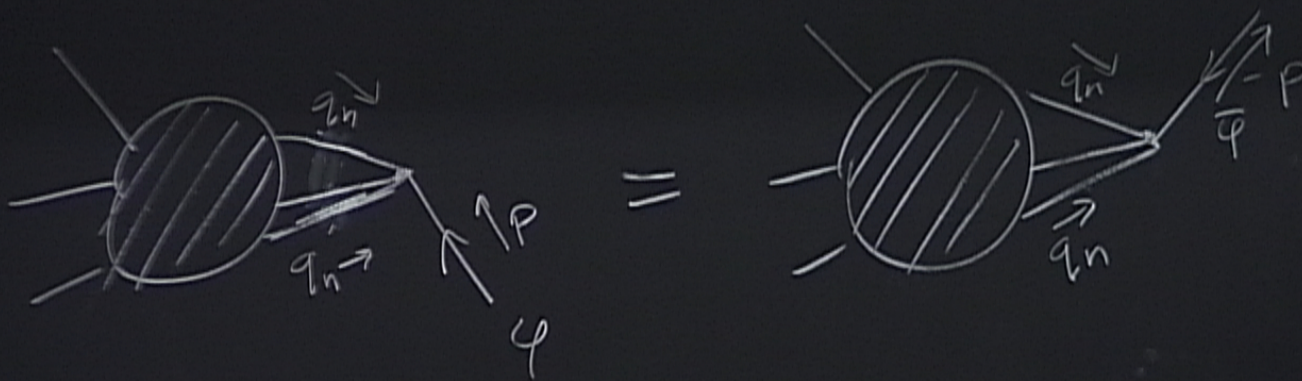


$$iM(\rho(k_1) + \rho(k_2) \rightarrow \rho(p_1) + \rho(p_2)) = iM(\rho(k_1) + \rho^*(-p_2) \rightarrow \rho(p_1) + \rho^*(-k_2))$$

Crossing Symmetry

$$iM(\varphi(p) + \dots \rightarrow \dots) = iM(\dots \rightarrow \dots \bar{\varphi}(-p))$$

↑ antiparticle of φ
 real scalar $\bar{\varphi} = \varphi$
 scalar $\bar{\varphi} = \varphi^\dagger$



Kinematics : are different