

Title: PSI 2016/2017 Quantum Field Theory I - Lecture 5

Date: Oct 17, 2016 09:00 AM

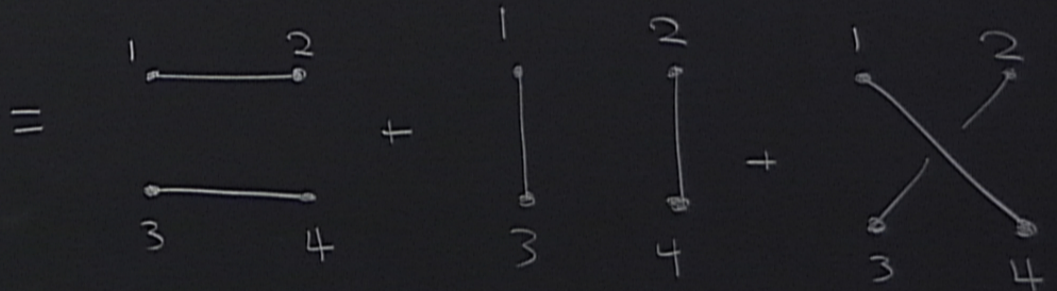
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Abstract:

Feynman Diagrams

$$\Delta_{ij} = \Delta_F(x_i - x_j)$$

$$\langle 0 | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | 0 \rangle = \Delta_{12} \Delta_{34} + \Delta_{13} \Delta_{24} + \Delta_{14} \Delta_{23}$$



$$\langle \Omega | T \varphi_1 \varphi_2 | \Omega \rangle = \frac{\langle 0 | T \varphi_1 \varphi_2 \exp[-i \int d^4x H_{\text{int}}(x)] | 0 \rangle}{\langle 0 | T \exp[-i \int d^4x H_{\text{int}}(x)] | 0 \rangle} \equiv G(x_1, x_2)$$

$$H_{\text{int}}(x) = \int d^3y \frac{\lambda}{4!} \varphi^4(y)$$

$$\text{numerator} = \langle 0 | T \varphi_1 \varphi_2 + T \varphi_1 \varphi_2 - i \frac{\lambda}{4!} \int d^4y \varphi^4(y) | 0 \rangle + \mathcal{O}(\lambda^2)$$

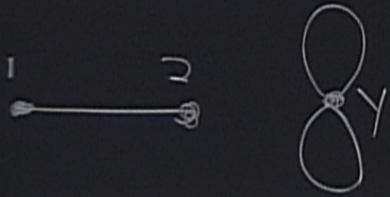
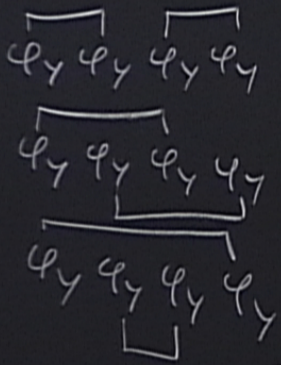
$$\langle 0 | T \varphi_1 \varphi_2 | 0 \rangle = \overset{1}{\longleftarrow} \longrightarrow \overset{2}{}$$

$$\langle 0 | T \varphi_1 \varphi_2 \frac{-i\lambda}{4!} \int d^4 y \varphi^4(y) | 0 \rangle = 3 \left(\frac{-i\lambda}{4!} \right) \Delta_{12} \int d^4 y \Delta_{yy} \Delta_{yy} +$$

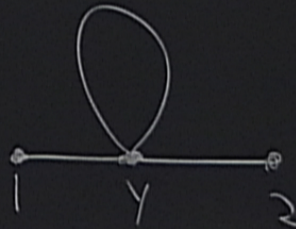
$$\begin{array}{c} \overbrace{\varphi_y \varphi_y} \quad \overbrace{\varphi_y \varphi_y} \\ \overbrace{\varphi_y \varphi_y \varphi_y \varphi_y} \\ \overbrace{\varphi_y \varphi_y \varphi_y \varphi_y} \end{array}$$

$$= \text{diagram 1} + \text{diagram 2} +$$

$$3\left(-\frac{i\lambda}{4!}\right)\Delta_{12}\int d^4y\Delta_{yy}\Delta_{yy} + 12\left(-\frac{i\lambda}{4!}\right)\int d^4y\Delta_{1y}\Delta_{2y}\Delta_{yy}$$



+



Position space Feynman rules for n-point functions

1. For each line $\overset{x}{\rule{1cm}{0.4pt}}\overset{y}{\phantom{\rule{1cm}{0.4pt}}} = \Delta_F(x-y)$

2. For each vertex $\underset{z}{\times} = -i\lambda \int d^4z$ (depends on Hint)

3. For each external point: $\bullet \rightarrow x = 1$ (will important for

4. Divide by symmetry factor.

les for n-point functions

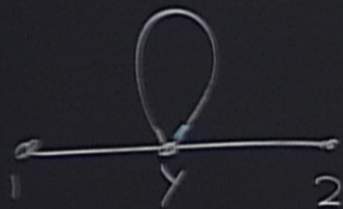
$$\text{---}\gamma = \Delta_F(x-y)$$

$$= -i\lambda \int d^4z \quad (\text{depends on Hint})$$

point: $\bullet x = 1$ (will important for fermions + gauge bosons)

factor.

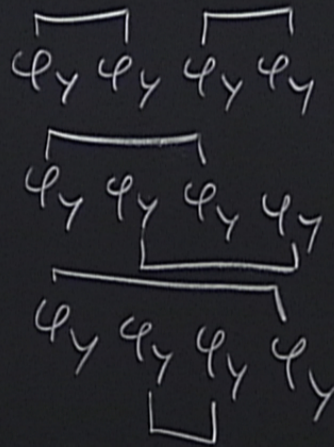
Symmetry Factor



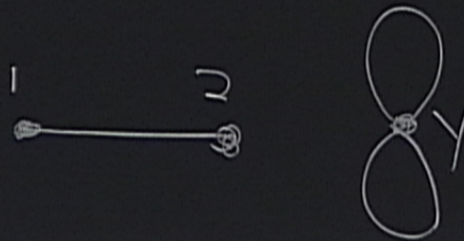
has coefficient $\frac{12}{4!} = \frac{1}{2} = \frac{1}{S}$ ← symmetry factor

$S = \#$ possible ways to exchange components of diagram
w/o changing diagram.

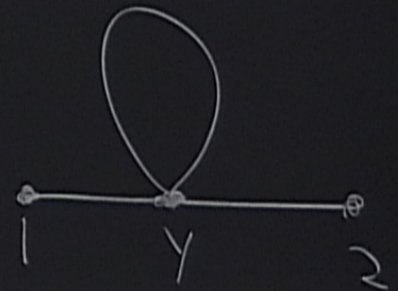
$$\langle 4 | 10 \rangle = 3 \left(\frac{4!}{4!} \Delta_{12} \right) \alpha \gamma \Delta \gamma \gamma \Delta \gamma \gamma \quad 12 \left(\frac{4!}{4!} \right) \alpha \gamma \Delta$$



$=$

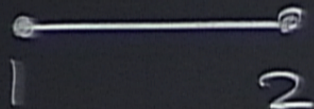


$+$



3 types of symmetries:

- exchanging two ends of an internal line
- exchanging two lines
- exchanging two vertices



$$S = 2 \cdot 2 \cdot 2 = 8$$

↑ ↑ ↑
 ends ends exchanging
 of of lines
 upper lower
 line line

$$\frac{1}{8} = \frac{3}{4!}$$

$$\text{denominator} = 1 + \underbrace{8_y}_{\theta(\lambda)} + \underbrace{8_y 8_z + \text{diagram with two loops} + \text{diagram with three loops}}_{\theta(\lambda^2)} + \dots$$

$$e^8 = 1 + 8 + \frac{1}{2}(8)^2 + \frac{1}{3!}(8)^3 + \dots$$

$$\text{denom} = \sum_{\substack{\{n_i\} \\ \uparrow \\ \text{sets of integers}}} \prod_i \frac{1}{n_i!} (V_i)^{n_i}$$

$$\{V_i\} = \{8, \infty, \Theta, \dots\}$$

$$= \left(\sum_{n_1} \frac{1}{n_1!} V_1^{n_1} \right) \left(\sum_{n_2} \frac{1}{n_2!} V_2^{n_2} \right) \dots$$

$$= \prod_i \left(\sum_{n_i} \frac{1}{n_i!} V_i^{n_i} \right)$$

$$= \prod_i \exp(V_i) = \exp\left(\sum_i V_i\right) = \exp[8 + \infty + \Theta + \dots]$$

$$\{V_i\} = \{8, \infty, \bigcirc, \dots\}$$

$$\{C_i\} = \left\{ \text{---} \bullet \text{---} \bullet \text{---}, \text{---} \bullet \text{---} \text{---}, \text{---} \bullet \text{---} \bullet \text{---} \bullet \text{---}, \dots \right\}$$

$$\text{numerator} = \sum_j \sum_{\{n_i\}} \prod_i C_j \frac{1}{n_i!} (V_i)^{n_i}$$

$$= \sum_j C_j \sum_{\{n_i\}} \prod_i \frac{1}{(n_i)!} (V_i)^{n_i}$$

$$= \sum_j C_j \exp\left(\sum_i V_i\right) \leftarrow \begin{array}{l} \text{vacuum diagrams} \\ \text{cancel} \end{array}$$

$\langle \Omega | T \varphi_1 \dots \varphi_n | \Omega \rangle = \text{sum of all diagrams with } n \text{ external points}$
+ no vacuum diagrams

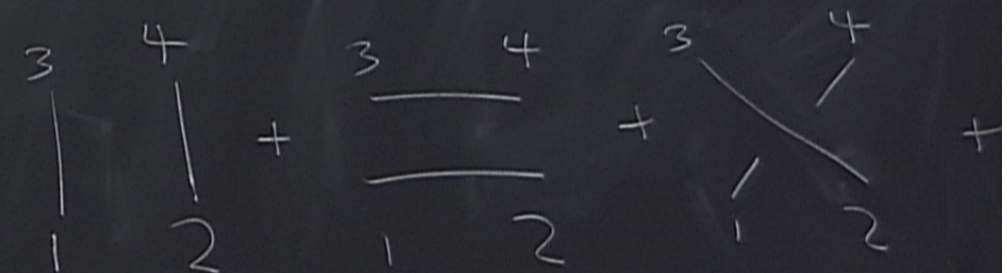
es $(2 \rightarrow 2)$

$$= i^4 \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 e^{i(p_1 \cdot x_1 + p_2 \cdot x_2 - p_3 \cdot x_3 - p_4 \cdot x_4)} \\ \cdot (\partial_1^2 + m^2)(\partial_2^2 + m^2)(\partial_3^2 + m^2)(\partial_4^2 + m^2) \langle \Omega | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | \Omega \rangle$$

Scattering Amplitudes (2→2)

$$\text{LSZ: } \langle f | S | i \rangle = i^4 \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 e^{i(p_1 \cdot x_1 - p_2 \cdot x_2 + p_3 \cdot x_3 - p_4 \cdot x_4)}$$

$$\cdot (\partial_1^2 + m^2)(\partial_2^2 + m^2)(\partial_3^2 + m^2)(\partial_4^2 + m^2)$$

$$\langle \Omega | T \phi_1 \phi_2 \phi_3 \phi_4 | \Omega \rangle =$$


$$i^4 \Delta_{yy} \Delta_{yy} + \dots$$

$$x_1 d^4 x_2 d^4 x_3 d^4 x_4 e^{i(p_1 \cdot x_1 + p_2 \cdot x_2 - p_3 \cdot x_3 - p_4 \cdot x_4)}$$

$$(\partial_2^2 + m^2)(\partial_3^2 + m^2)(\partial_4^2 + m^2) \langle \Omega | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | \Omega \rangle$$

$$\begin{array}{c} 4 \\ | \\ 2 \end{array} + \begin{array}{cc} 3 & 4 \\ \hline & \hline \\ 1 & 2 \end{array} + \begin{array}{cc} 3 & 4 \\ \diagdown & / \\ 1 & 2 \end{array} + O(\lambda) \quad \begin{array}{c} \uparrow \\ \text{time} \end{array}$$

$$\begin{array}{c} 3 \\ | \\ 1 \end{array} \begin{array}{c} 4 \\ | \\ 2 \end{array} = \Delta_{13} \Delta_{24}$$

$$(\partial_1^2 + m^2)(\partial_3^2 + m^2) \Delta_F(x_1 - x_3) = F(x_1 - x_3)$$

$$\int d^4 x_1 d^4 x_3 e^{i(p_1 \cdot x_1 - p_3 \cdot x_3)} F(x_1 - x_3) =$$

$$X_{13} = X_1 + X_3$$

$$\bar{X}_{13} = X_1 - X_3$$

$$P_{13} = \frac{P_1 + P_3}{2}$$

$$\bar{P}_{13} = \frac{P_1 - P_3}{2}$$

$$i(\bar{P}_{13} \cdot X_{13} + P_{13} \cdot \bar{X}_{13}) F(\bar{X}_{13})$$

$$X_{13} d^4 \bar{X}_{13} e$$

$$i\bar{P}_{13} \cdot X_{13} \tilde{F}(P_{13}) = (2\tilde{\pi})^4 \int^{(4)} (P_1 - P_3) \hat{\tilde{F}}\left(\frac{P_1 + P_3}{2}\right)$$

$$\langle f | S | i \rangle_{II} \propto \delta^{(4)}(p_1 - p_3) \delta^{(4)}(p_2 - p_4) \quad \text{no scattering}$$

same logic for $\times$

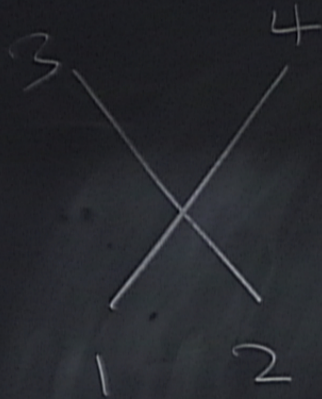
$$\langle f | S | i \rangle = \propto \delta^{(4)}(p_1 + p_2) \delta^{(4)}(p_3 + p_4) \quad \text{vanishes}$$

$$p_1^0 + p_2^0 \geq 2m > 0$$

same for

ring

Only fully connected diagrams contribute
to $n \rightarrow m$ scattering



$$= -i\lambda \int d^4 y \Delta_{1y} \Delta_{2y} \Delta_{3y} \Delta_{4y}$$

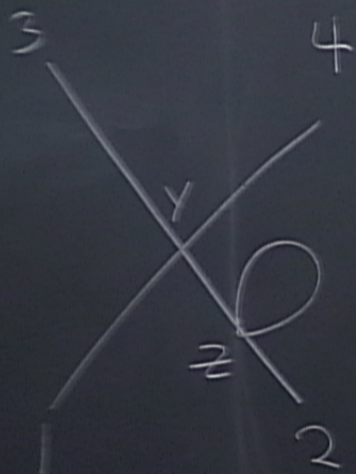
$$(\partial_1^2 + m^2) \Delta_{1y} = -i\delta^4(x_1 - y)$$

$$\langle f | S | i \rangle_X = -i\lambda \int d^4 y e^{i(p_1 + p_2 - p_3 - p_4) \cdot y}$$

$$= -i\lambda (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4)$$

$$\langle f | S | i \rangle_{cc} = i M(i \rightarrow f) (2\pi)^4 \delta(\sum p)$$

$$H_{int} = \frac{\lambda}{4!} \int d^3y \varphi^4(y)$$



$$= \frac{(-i\lambda)^2}{2} \int d^4y \int d^4z \Delta_{1y} \Delta_{3y} \Delta_{4y} \Delta_{2z} \Delta_{zz} \Delta_{yz}$$

$$d^4z \, e^{i(p_1 - p_3 - p_4) \cdot y} \, e^{i p_2 \cdot z} \, \Delta_{zz} \Delta_{yz}$$

↑ doing x_i integrals + using that Δ is Green's func

$$\langle TISII \rangle_{\phi} = \frac{(-i\lambda)^2}{2} \int d^4x \int d^4z \ e^{i(p_1 - p_3 - p_4) \cdot y} e^{i p_2 \cdot z}$$

↑ doing xi integrals + using

$$\Delta_{yz} = \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\epsilon} e^{ik(y-z)}$$

$$= \frac{(-i\lambda)^2}{2} \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\epsilon} \frac{i}{q^2 - m^2 + i\epsilon} (2\pi)^4 \delta^4(k+q)$$

$$\int d^4y \int d^4z \ e^{i(p_1 - p_3 - p_4) \cdot y} \ e^{i p_2 \cdot z} \Delta_{zz} \Delta_{yz}$$

↑ doing x_i integrals + using that Δ is Green's func

$$\Delta_{yz} = \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\epsilon} e^{ik(y-z)}$$

$$\int \frac{d^4k}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\epsilon} \frac{i}{q^2 - m^2 + i\epsilon} (2\pi)^4 \delta^4(p_1 + k - p_3 - p_4) (2\pi)^4 \delta^{(4)}(p_2 - k)$$

$$p_2 = z \quad \Delta_{zz} \Delta_{yz}$$

using that Δ is Green's func

$$\delta^{(4)}(p_1 + k - p_3 - p_4) (\hat{e}_{11})^4 \delta^{(4)}(p_2 - k)$$

$$\int d^4k \delta^{(4)}(p_2 - k) \frac{1}{k^2 - m^2 + i\epsilon}$$

$$= \frac{1}{p_2^2 - m^2} = \frac{1}{0}!$$

$$\int d^4z \ e^{i(p_1 - p_3 - p_4) \cdot y} \ e^{i p_2 \cdot z} \Delta_{zz} \Delta_{yz}$$

↑ doing x_i integrals + using that Δ is Green's func

$$\int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\epsilon} e^{ik(y-z)}$$

$$\Delta_{zz} = \int \frac{d^4q}{(2\pi)^4} \frac{i}{q^2 - m^2 + i\epsilon}$$

$$\int \frac{d^4q}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\epsilon} \frac{i}{q^2 - m^2 + i\epsilon} (2\pi)^4 \delta^{(4)}(p_1 + k - p_3 - p_4) (2\pi)^4 \delta^{(4)}(p_2 - k)$$

$$= \Delta_{yz}$$

Δ is Green's func

$$= \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 - m^2 + i\epsilon}$$

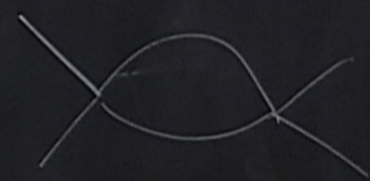
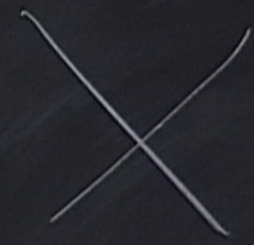
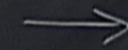
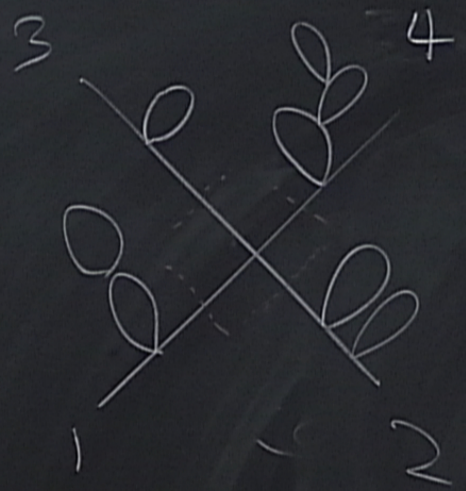
$$(p_3 - p_4) (2\pi)^4 \delta^{(4)}(p_2 - k)$$

$$\int d^4 k \delta^{(4)}(p_2 - k) \frac{1}{k^2 - m^2 + i\epsilon}$$

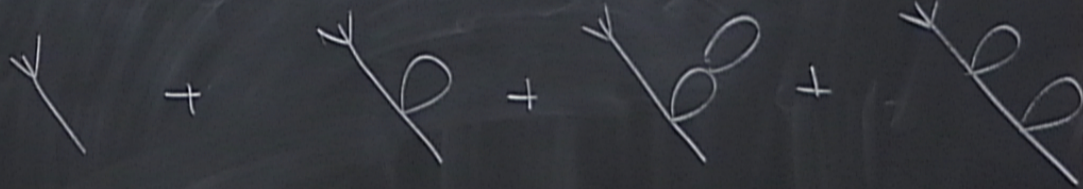
$$= \frac{1}{p_2^2 - m^2} = \frac{1}{0} !$$

Need to amputate!

Amputation:



Single particle state in interacting theory
is not the same as a single particle state
in the free theory

$$\text{one particle} = \text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4}$$
The equation shows four Feynman diagrams representing one-particle states. The first diagram is a simple line with an arrow at the top. The second diagram is a line with a self-energy loop. The third diagram is a line with two self-energy loops. The fourth diagram is a line with three self-energy loops. Each diagram is separated by a plus sign.

Momentum Space Feynman Rules for iM

iM = sum of all completely connected, amputated diagrams

1. For each internal line $\frac{1}{\vec{p}} = \frac{1}{p^2 - m^2 + i\epsilon}$

2. For each vertex: $\times = -i\lambda$ (specific to ϕ^4 theory)

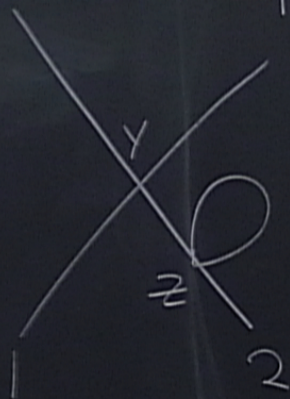
3. For each external line $\rightarrow = 1$ (will change for fermions + gauge bosons)

4. Impose momentum conservation at each vertex

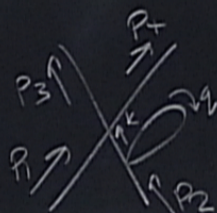
grams

heavy)

ions + gauge boson



$$= \left(\frac{-i\lambda}{2} \right)^2 \int d^4y \int d^4z \Delta_{1y} \Delta_{3y} \Delta_{4y} \Delta_{2z} \Delta_{zz} \Delta_{yz}$$



Need to amputate!

$$\frac{i}{q^2 - m^2 + i\epsilon} (2\pi)^4 \delta^{(4)}(p_1 + k - p_3 - p_4) (2\pi)^4 \delta^{(4)}(p_2 - k)$$

4. Impose momentum conservation at each vertex
5. Divide by symmetry factor.
6. Integrate over any undetermined momenta