

Title: PSI 2016/2017 Quantum Field Theory I - Lecture 3

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Abstract:









# LSZ Reduction Formula

$$\langle f | \underset{\substack{\uparrow \\ \text{S-matrix}}}{S} | i \rangle_{\text{Heisenberg}} = \langle f, t=\infty | i, t=-\infty \rangle_{\text{Schrödinger}}$$

$$S = \mathbb{1} \quad \text{in a free theory}$$

$$S = \mathbb{1} + iT$$

$\nwarrow$  transfer matrix



$$T = (2\pi)^4 \delta^4(\Sigma p) M$$

$$\langle f | S - 1 | i \rangle = (2\pi)^4 \delta^4(\Sigma p) \underbrace{i \langle f | M | i \rangle}_{= i M(i \rightarrow f)}$$

↑  
matrix element

In free theory  $a_i^+(t) = \int d^3k f_i(\vec{k}) a_{\vec{k}}^+(t)$

$$f_i(\vec{k}) \propto \exp[-(\vec{k} - \vec{k}_i)^2 / 4\sigma^2]$$

want is  $a_i^+(t) | 0 \rangle$  to be localized around  $\vec{k} = \vec{k}_i$  and  $\vec{x} = 0$  at  $t = 0$

$$a_1^{\dagger}(t)a_2^{\dagger}(t)|0\rangle$$

$$\vec{k}_1 \neq \vec{k}_2$$

## Interacting Theory

vacuum changes  $|0\rangle \rightarrow |\Omega\rangle$

Assume  $a_i^{\dagger}(\pm\infty)|\Omega\rangle$  are still one particle states



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vacuum changes  $|0\rangle \rightarrow |\Omega\rangle$

Assume  $a_i^\pm(\pm\infty)|\Omega\rangle$  are still one particle states

For  $2 \rightarrow 2$  scattering

$$|i\rangle = a_1^\pm(-\infty) a_2^\pm(-\infty) |\Omega\rangle$$

$$|f\rangle = a_3^\pm(+\infty) a_4^\pm(+\infty) |\Omega\rangle$$

$a_1(t)|0\rangle$  to be localized around  $\vec{k}=\vec{k}_1$  and  $\vec{x}=0$  at

Goal:  $\langle f | S^{-1} | i \rangle$

We will use:

states

$$a_1^+(+\infty) - a_1^+(-\infty) = -i \int d^3k f_1(\vec{k}) \int d^4x \left[ e^{i\vec{k}\cdot\vec{x}} (\partial^2 + m^2) \varphi(x) \right]$$

Assumption:

$$a_k^+(t) = -i \int d^3x e^{-i\vec{k}\cdot\vec{x}} (\partial_0 \varphi(x) + i E_{\vec{k}} \varphi(x))$$



$$\left[ \partial_0 \varphi(x) + i E_k \varphi(x) \right]$$

$$\varphi(x) + E_k^2 \varphi(x)$$

$$(-\vec{\nabla}^2 + m^2) \varphi(x)$$

$$(m^2) \varphi(x)$$

$$E_k^2 = \vec{k}^2 + m^2$$

Similarly for

$$a_1(+\infty) - a_1(-\infty)$$

with  $i \rightarrow -i$  on

RHS

$$\begin{aligned}
 \langle f | S^{-1} | i \rangle &= \langle \Omega | a_4(+\infty) a_3(+\infty) a_1^+(-\infty) a_2^+(-\infty) | \Omega \rangle \\
 &= \langle \Omega | T a_4(+\infty) a_3(+\infty) a_1^+(-\infty) a_2^+(-\infty) | \Omega \rangle
 \end{aligned}$$



$$\varphi(x) \equiv I_1^+$$

$$E \varphi(x)$$

$$\langle f | S^{-1} | i \rangle = \langle \Omega | a_4(+\infty) a_3(+\infty) a_1^+(-\infty) a_2^+(-\infty) | \Omega \rangle$$

$$= \langle \Omega | T a_4(+\infty) a_3(+\infty) a_1^+(-\infty) a_2^+(-\infty) | \Omega \rangle$$

$$\text{use } a_1^+(-\infty) = a_1^+(+\infty) + I_1^+$$

$$a_4(+\infty) = a_4(-\infty) + I_4$$

$$\langle f | S^{-1} | i \rangle = \langle \Omega | T a_4(-\infty) a_3(-\infty) a_1^+(+\infty) a_2^+(+\infty) | \Omega \rangle + \dots + \langle \Omega | T I_4 I_3 I_1^+ I_2^+ | \Omega \rangle$$

$$\langle f | S^{-1} | i \rangle = \langle \Omega | a_4(+\infty) a_3(+\infty) a_1^+(-\infty) a_2^+(-\infty) | \Omega \rangle$$

$$= \langle \Omega | T a_4(+\infty) a_3(+\infty) a_1^+(-\infty) a_2^+(-\infty) | \Omega \rangle$$

$$\text{use } a_1^+(-\infty) = a_1^+(+\infty) + \boxed{I_1^+}$$

$$a_4(+\infty) = a_4(-\infty) + \boxed{I_4} \leftarrow \text{series of integrals}$$

$$\langle f | S^{-1} | i \rangle = \underbrace{\langle \Omega | T a_4(-\infty) a_3(-\infty) a_1^+(+\infty) a_2^+(+\infty) | \Omega \rangle}_{0} + \dots + \langle \Omega | T I_4 I_3 I_1^+ I_2^+ | \Omega \rangle$$

$$= \langle \Omega | a_1^+(+\infty) a_2^+(+\infty) a_4(-\infty) a_3(-\infty) | \Omega \rangle$$



LSZ Reduction Formula:

$$\langle f | S | i \rangle = i^{2+2} \left( \prod_{j=1}^{2+2} \int d^4 x_j e^{-i \lambda_j k_j \cdot x_j} (\partial_j^2 + m^2) \right) \langle \Omega | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | \Omega \rangle$$

$\uparrow$   
 $\lambda_j =$  -1 final states  
 $+1$  initial states

$\uparrow$   
 $\varphi_i \equiv \varphi(x_i)$

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# Assumptions

$$a_i^\pm(\pm\infty)|\Omega\rangle$$

$$\varphi(x)|\Omega\rangle$$

are one particle states

Goal:  $\langle f | S^{-1} | i \rangle$

We will use:

particle states

$$a_i^+(+\infty) - a_i^+(-\infty) = -i \int d^3k f_i(\vec{k}) \int d^4x \left[ e^{ik \cdot x} (\partial^2 + m^2) \varphi(x) \right] \equiv -I_i^+$$

Assumption:

$$a_k^+(+) = -i \int d^3x e^{-ik \cdot x} (\partial_0 \varphi(x) + i E_{\vec{k}} \varphi(x))$$



## Multiparticle states

Assume no bound states

$$\begin{aligned} \langle \vec{P}, \sigma | \varphi(x) | \Omega \rangle &= \langle \vec{P}, \sigma | e^{-i\hat{P} \cdot x} \varphi(0) e^{-i\hat{P} \cdot x} | \Omega \rangle \\ &= e^{iP \cdot x} \langle \vec{P}, \sigma | \varphi(0) | \Omega \rangle \\ &= e^{iP \cdot x} A_{\sigma}(\vec{P}) \neq 0 \text{ in general} \end{aligned}$$

total  
3-momentum

all other  
parameters

$$\langle \vec{p}, \sigma | a_i^\pm(\pm\infty) | \Omega \rangle = 0$$

$$\langle \Psi | a_i^\pm(\pm\infty) | \Omega \rangle = 0$$

↑ normalizable multiparticle state

$$|\Psi\rangle = \sum_{\sigma} \int d^3p \, \psi_{\sigma}(\vec{p}) |\vec{p}, \sigma\rangle$$

↑  
wavepacket  $\psi$



$$\int d^3x e^{i(\vec{k}-\vec{p})\cdot\vec{x}} = (2\pi)^3 \delta^{(3)}(\vec{k}-\vec{p})$$

$$\langle \Psi | a^\dagger | \Omega \rangle = \int \frac{d^3p}{(2\pi)^3} (2\pi)^3 (p^0 + k^0) \psi_\sigma^\dagger(\vec{p}) f_1(\vec{p}) A_\sigma(\vec{p}) e^{-i(p^0 - k^0)t}$$

$$p^0 = (\vec{p}^2 + M^2)^{1/2}$$

↑  
invariant mass of multiparticle state

$$k^0 = (\vec{p}^2 + m^2)^{1/2}$$

no bound states  $\rightarrow M \geq 2m > m$

$$p^0 > k^0$$

$$[ ] = i(p^0 + k^0) e^{i(\vec{p}-\vec{k}) \cdot \vec{x}} A_\sigma(\vec{p})$$

$$\int d^3x e^{i(\vec{E}-\vec{p}) \cdot \vec{x}} = (2\pi)^3 \delta^3(\vec{k}-\vec{p})$$

$$\langle \Psi | a_i^\dagger | \Omega \rangle = \sum_{\sigma} \int d^3p (2\pi)^3 (p^0 + k^0) \Psi_{\sigma}^*(\vec{p}) f_i(\vec{p}) A_{\sigma}(\vec{p}) e^{-i(p^0 - k^0)t}$$

$$p^0 = (\vec{p}^2 + M^2)^{1/2}$$

^  
invariant mass of multiparticle state

$$k^0 = (\vec{p}^2 + m^2)^{1/2}$$

$$\text{no bound states} \rightarrow M \geq 2m > m$$

$$p^0 > k^0$$

$\vec{p}$  integral vanishes as  $t \rightarrow \pm \infty$

$$\langle \Psi | a_i^\dagger(\pm \infty) | \Omega \rangle = 0$$



Drop  $\sim f_{\text{co}}$