

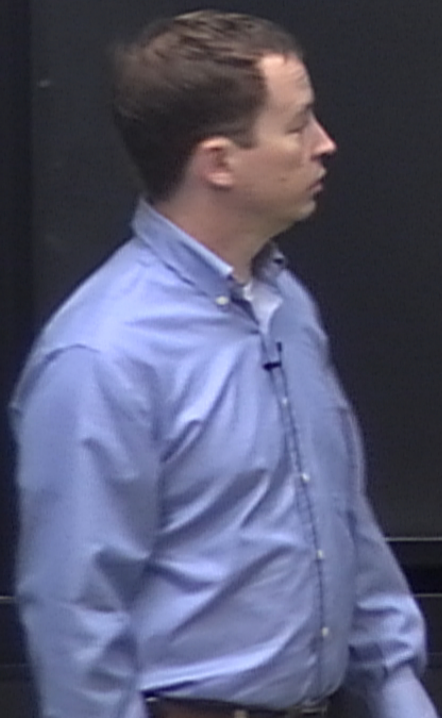
Title: PSI 2016/2017 Quantum Field Theory I - Lecture 1

Date: Oct 11, 2016 09:00 AM

URL: <http://pirsa.org/16100001>

Abstract:

Quantum Field Theory I



Quantum Field Theory I

Why QFT?

- QM + SR imply particle # not conserved

$$\Delta E \geq \frac{\hbar c}{L}$$

↖ box size

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$$\Delta E \geq \frac{\hbar c}{L} \quad \text{if } \Delta E = 2mc^2$$

$\nwarrow$ box size

then we can create particle-antiparticle pair

Quantum Field Theory I

Why QFT?

- QM + SR imply particle # not conserved

$$\Delta E \geq \frac{\hbar c}{L} \quad \text{if } \Delta E = 2mc^2$$

$\nwarrow$ box size

then we can create particle-antiparticle pair

- identical particles

- locality

Outline: Spin-0 } Dan
Spin- $\frac{1}{2}$ } Tibra
Spin-1 }

Outline: Spin-0 } Dan
Spin- $\frac{1}{2}$ } Tibra
Spin-1 }

Spin-0

Classical Field Theory

Canonical Quantization

LSZ Reduction

Interaction Picture + Wick's Theorem

Feynman Diagrams

Cross Sections + Decay Rates

Introduction to Loops

$$\hbar = c = 1$$

theory

on

Wick's Theorem

Classical Field Theory

$$\varphi(t, \vec{x}) = \varphi(x) \quad \text{generalizes}$$

generalized coordinates
in class. mech.
 $q_a(t)$

∞ many degrees of freedom

$$S = \int dt L(q_a, \dot{q}_a) \rightarrow S = \int dt \int d^3x \mathcal{L}(\varphi(x), \partial_\mu \varphi(x))$$

Classical Field Theory

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$\rightarrow$

$$S = \int dt \underbrace{\int d^3x \mathcal{L}(\varphi(x), \partial_\mu \varphi(x))}_{L(t)}$$

$$\varphi(x) \rightarrow \varphi'(x) = \varphi(\Lambda^{-1}x)$$

under $x^\mu \rightarrow (x')^\mu = \Lambda^\mu{}_\nu x^\nu$

where $\Lambda^\mu{}_\sigma \eta^{\sigma\rho} \Lambda^\nu{}_\rho = \eta^{\mu\nu}$

$$\eta^{\mu\nu} = \eta_{\mu\nu} = \begin{pmatrix} +1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$(\varphi(x), \partial_\mu \varphi(x))$

active transformations

$$\varphi(x) \rightarrow \varphi'(x) = \varphi(\overset{\substack{\uparrow \text{active transformations}}}{\Lambda^{-1}} x)$$

$$\text{under } x^\mu \rightarrow (x')^\mu = \Lambda^\mu{}_\nu x^\nu$$

$$\text{where } \Lambda^\mu{}_\sigma \eta^{\sigma\rho} \Lambda^\nu{}_\rho = \eta^{\mu\nu}$$

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Simplest theory: Klein-Gordon

$$\mathcal{L}_{KG} = \frac{1}{2} \partial^\mu \varphi \partial_\mu \varphi - \frac{1}{2} m^2 \varphi^2$$

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EOM: In class. mech.

$$\frac{\delta S}{\delta q_a(t)} = 0$$

gives

$$\frac{\partial \mathcal{L}}{\partial q_a} - \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}_a} = 0$$

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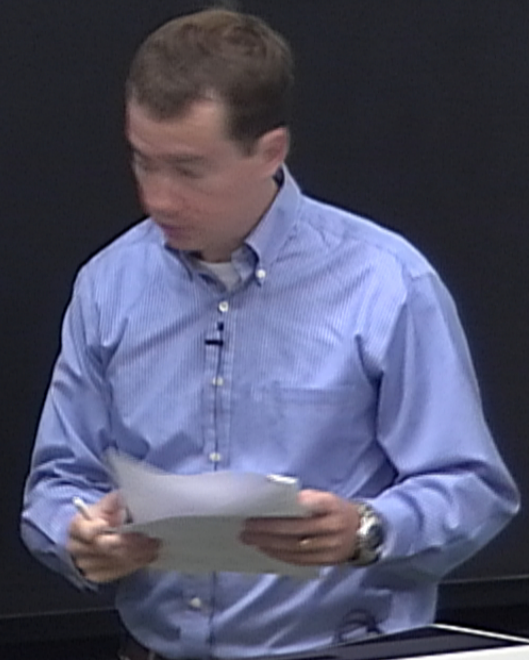
$$\frac{\partial \mathcal{L}}{\partial q_a} - \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}_a} = 0$$

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gives

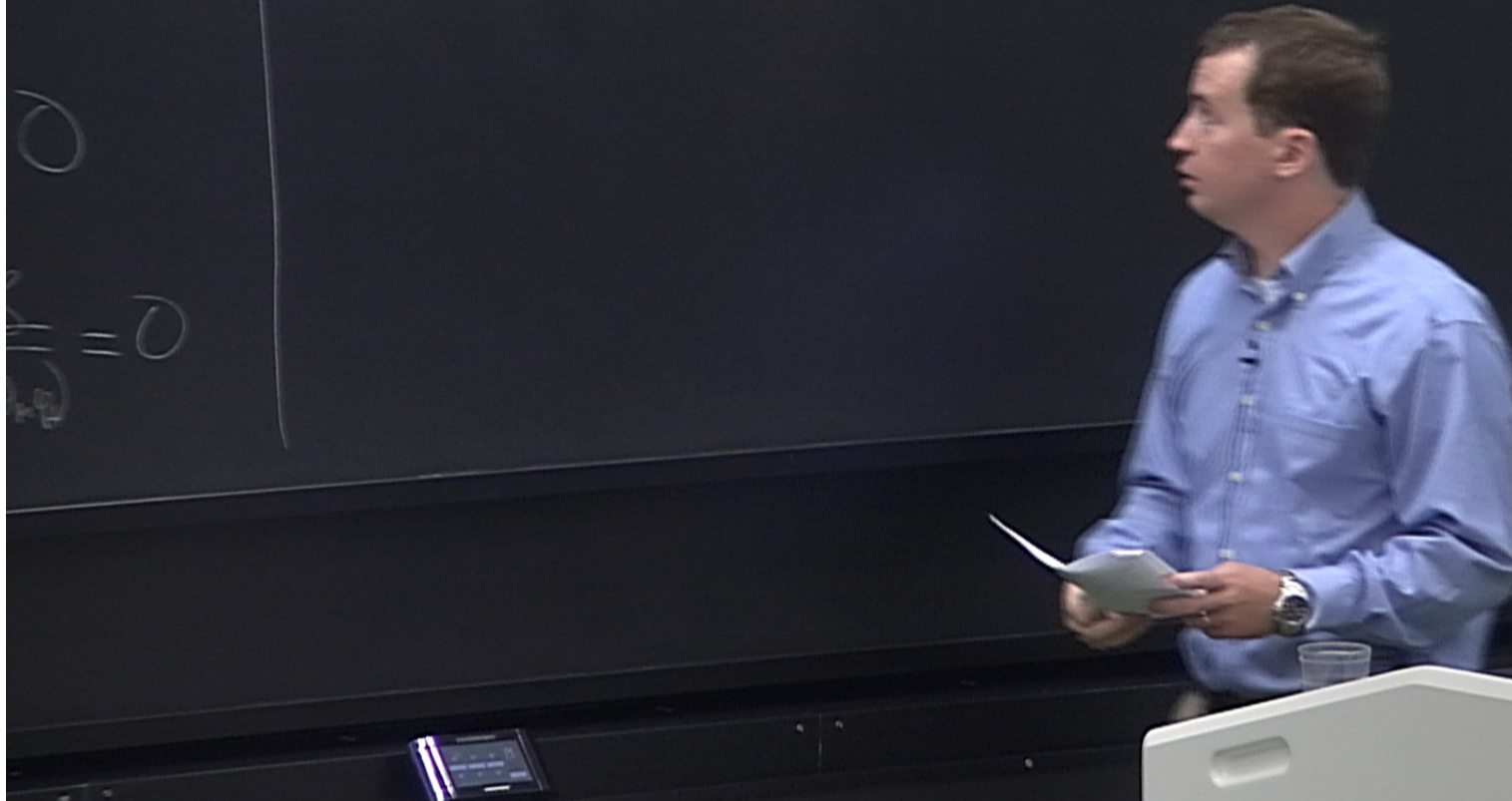
$$\frac{\partial \mathcal{L}}{\partial \varphi_A} - \partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi_A)} = 0$$

$$\text{KG EOM: } +m^2\varphi + \partial_\mu\partial^\mu\varphi = 0$$



$$KG \text{ EOM: } +m^2\varphi + \partial_\mu\partial^\mu\varphi = 0$$

Noether's Theorem



Noether's Theorem

Every continuous symmetry of the action gives rise
to a conserved current $j^\mu(x)$

↑
means $\partial_\mu j^\mu = 0$

conserved current $\Rightarrow$ conserved charge

$$Q = \int_{\mathbb{R}^3} d^3x j^0$$

$$\frac{dQ}{dt} = \int_{\mathbb{R}^3} d^3x \frac{dj^0}{dt} = - \int_{\mathbb{R}^3} d^3x \vec{\nabla} \cdot \vec{j} = 0$$

$\uparrow$ assuming $j \rightarrow 0$
as $|\vec{x}| \rightarrow \infty$

$$\vec{\nabla} \cdot \vec{j} = 0$$

assuming $j \rightarrow 0$
as $|\vec{x}| \rightarrow \infty$

conserved current $\Rightarrow$ conserved charge

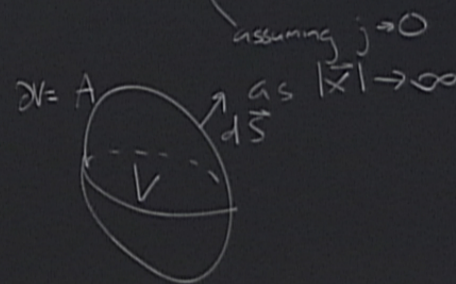
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+ charge is conserved locally

$$Q_V = \int_V d^3x j^0$$

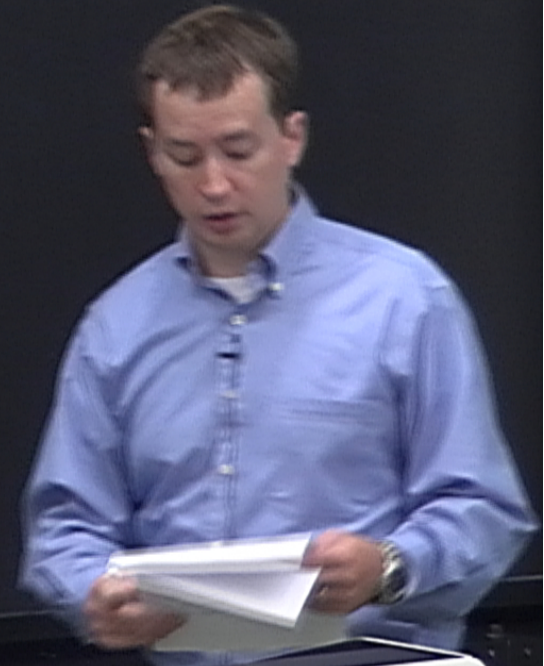
$$\frac{dQ_V}{dt} = - \int_A \vec{j} \cdot d\vec{S}$$



Proof: If we have a continuous symmetry
we can consider an infinitesimal transformation

$$\delta \varphi_a(x) = Y_a(\varphi)$$

$\rightarrow 0$
 $\rightarrow \infty$



Proof: If we have a continuous symmetry
we can consider an infinitesimal transformation

$$\delta \varphi_a(x) = \epsilon \delta \varphi_a(\varphi)$$

This is a symmetry transformation if

$$\delta \mathcal{L} = 0$$

EOM unchanged

Proof If we have a continuous symmetry
we can consider an infinitesimal transformation

$$\delta \varphi_a(x) = Y_a(\varphi)$$

This is a symmetry transformation if

$$\delta \mathcal{L} = \partial_\mu F^\mu(\varphi)$$

leaves EOM unchanged

$$\frac{dQ_v}{dt} = - \int_A \vec{j} \cdot d\vec{S}$$

Consider an arbitrary transformation $\delta\varphi_a$

$$\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\varphi_a} \delta\varphi_a + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi_a)} \partial_\mu(\delta\varphi_a)$$

$$= \left(\frac{\partial\mathcal{L}}{\partial\varphi_a} - \partial_\mu \frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi_a)} \right) \delta\varphi_a + \partial_\mu \left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi_a)} \delta\varphi_a \right)$$

$$\frac{dQ_v}{dt} = - \int_A \vec{j} \cdot d\vec{S}$$

Consider an arbitrary transformation $\delta\varphi_a$

$$\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\varphi_a} \delta\varphi_a + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi_a)} \partial_\mu(\delta\varphi_a)$$

$$= \underbrace{\left(\frac{\partial\mathcal{L}}{\partial\varphi_a} - \partial_\mu \frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi_a)} \right)}_{=0 \text{ if EOM satisfied}} \delta\varphi_a + \partial_\mu \left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi_a)} \delta\varphi_a \right)$$

If $\delta\varphi_a$ is a symmetry:

$$\delta\mathcal{L} = \partial_\mu \left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi_a)} \delta\varphi_a \right) = \partial_\mu F^\mu(\varphi)$$

Then $\partial_\mu j^\mu = 0$

$$\text{if } j^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi_a)} \delta\varphi_a - F^\mu(\varphi)$$

$$\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi_a)} \delta\varphi_a \right)$$

Example: Translations

Under $x^\nu \rightarrow x^\nu - \epsilon^\nu$
↑ infinitesimal

$$\varphi_a(x) \rightarrow \varphi_a(x) + \epsilon^\nu \partial_\nu \varphi_a(x) + \mathcal{O}(\epsilon^2)$$

↑ active transformations

Example: Translations

Under $x^\nu \rightarrow x^\nu - \epsilon^\nu$
↑ infinitesimal

$$\varphi_a(x) \rightarrow \varphi_a(x) + \epsilon^\nu \partial_\nu \varphi_a(x) + \cancel{O(\epsilon^2)}^0$$

↑ active transformations

$$\mathcal{L}(x) \rightarrow \mathcal{L}(x) + \epsilon^\nu \partial_\nu \mathcal{L}(x)$$

Example: Translations

4 trans. $\rightarrow$ 4 currents

$$\epsilon^\nu (\cdot)^M{}_\nu = \epsilon^\nu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi_a)} \partial_\nu \varphi_a - \epsilon^\nu \delta^M_\nu \mathcal{L}$$

$$\epsilon^\nu (j^M)_\nu = \epsilon^\nu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi_a)} \partial_\nu \phi_a - \epsilon^\nu \delta^\mu_\nu \mathcal{L}$$

$$j^M_\nu \equiv T^M_\nu$$

↑ energy momentum tensor

charges are 4-momentum $P^M = \int d^3x T^{0M}$

Example: $U(1)$ Internal (Global) Symmetry

Complex field $\Phi(x) = \frac{1}{\sqrt{2}} (\varphi_1(x) + i\varphi_2(x))$

$$\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi - V(|\Phi|^2)$$

Example: $U(1)$ Internal (Global) Symmetry

Complex field $\Phi(x) = \frac{1}{\sqrt{2}} (\varphi_1(x) + i\varphi_2(x))$

$$\mathcal{L} = \partial_\mu \Phi^\dagger \partial^\mu \Phi - V(|\Phi|^2)$$

$$\Phi \rightarrow e^{i\theta} \Phi \quad \Phi^\dagger \rightarrow e^{-i\theta} \Phi^\dagger$$

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Complex field $\Phi(x) = \frac{1}{\sqrt{2}} (\varphi_1(x) + i\varphi_2(x))$

$$\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi - V(|\Phi|^2)$$

$$\Phi \rightarrow e^{i\theta} \Phi \quad \Phi^* \rightarrow e^{-i\theta} \Phi^*$$

$$\delta \Phi = i\theta \Phi$$

$$\delta \Phi^* = -i\theta \Phi^*$$

$$\delta \mathcal{L} = 0$$

$$\Theta j^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Phi)} \delta \Phi$$

$$\Theta j^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Phi)} \delta \Phi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Phi^*)} \delta \Phi^*$$

$$\cancel{\Theta} j^\mu = \partial^\mu \Phi^* i \delta \Phi - i \delta \Phi^* \partial^\mu \Phi$$

$$j^\mu = \Phi^* \partial^\mu \Phi - \Phi \partial^\mu \Phi^*$$

$$\frac{d\vec{a}}{dt} = - \int_A \vec{j} \cdot d\vec{S}$$

Hamiltonian Formalism

$$\pi^a(x) = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}_a}$$

momentum conjugate to $\varphi_a(x)$

$$\frac{d\vec{a}}{dt} = - \int_A \vec{j} \cdot d\vec{S}$$

Hamiltonian Formalism

$$\tilde{\pi}^a(x) = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}^a}$$

momentum conjugate to $\varphi^a(x)$

$$\mathcal{H} = \tilde{\pi}^a(x) \dot{\varphi}^a(x) - \mathcal{L}(x)$$

↑
Hamiltonian density

$$H = \int d^3x \mathcal{H}$$

$$\mathcal{L}_{KG} = \frac{1}{2} \dot{\varphi}^2 - \frac{1}{2} (\vec{\nabla} \varphi)^2 - \frac{1}{2} m^2 \varphi^2$$

$$\pi = \dot{\varphi}$$

$$H = \int d^3x \left(\frac{1}{2} \pi^2 + \frac{1}{2} (\vec{\nabla} \varphi)^2 + \frac{1}{2} m^2 \varphi^2 \right)$$

to $\varphi_a(x)$

$$\tilde{\pi} = \dot{\varphi}$$

$$H = \int d^3x \left(\frac{1}{2} \tilde{\pi}^2 + \frac{1}{2} (\vec{\nabla} \varphi)^2 + \frac{1}{2} m^2 \varphi^2 \right)$$

not manifestly Lorentz invariant

Solution of the KG Equation (Classical)

$$\partial^2 \varphi + m^2 \varphi = 0$$

$$\varphi(t, \vec{x}) = \int \frac{d^3 k}{(2\pi)^3} e^{i \vec{k} \cdot \vec{x}} \tilde{\varphi}(t, \vec{k})$$

spatial Fourier trans

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spatial Fourier trans

solution: $\tilde{\varphi}(t, \vec{k}) = A(\vec{k}) e^{iE_{\vec{k}} t} + B(\vec{k}) e^{-iE_{\vec{k}} t}$

$$E_{\vec{k}} = \sqrt{\vec{k}^2 + m^2}$$

Solution of the KG Equation (Classical)

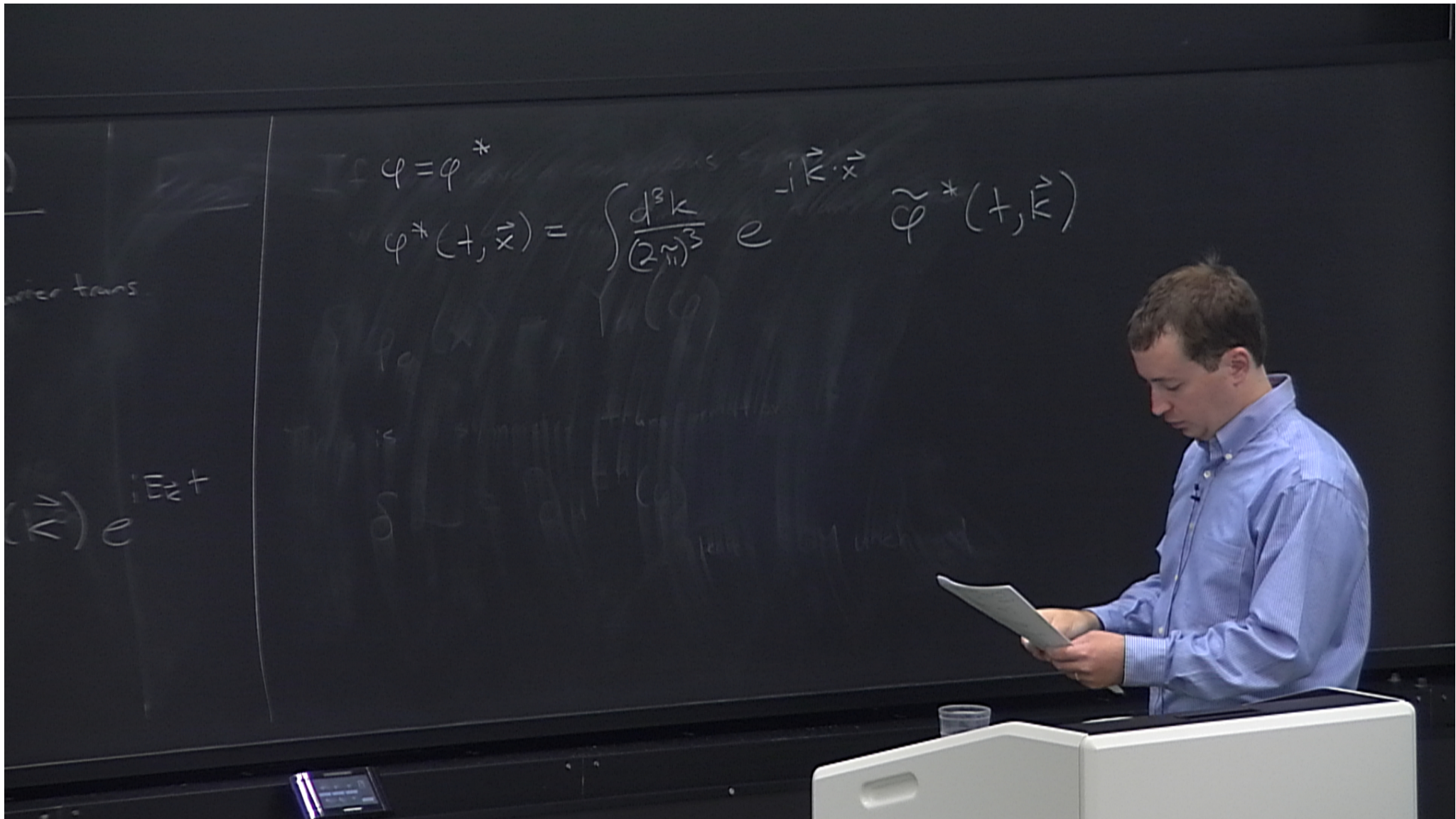
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$$\varphi = \varphi^*$$

$$\pi \rightarrow -\pi$$

$$= \int \frac{d^3 K}{(2\pi)^3}$$

$$(\lambda \nu) e^{i E_k +}$$

$$\varphi = \varphi^*$$

$$\varphi^*(t, \vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{-i\vec{k} \cdot \vec{x}} \tilde{\varphi}^*(t, \vec{k})$$

$$\vec{k} \rightarrow -\vec{k}$$

$$= \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{x}} \tilde{\varphi}^*(t, -\vec{k})$$

$$\tilde{\varphi}(t, \vec{k}) = \tilde{\varphi}^*(t, -\vec{k})$$

If $\delta\varphi_a$ is a symmetry:

$$\delta\mathcal{L} = \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi_a)} \delta\varphi_a \right) = \partial_\mu F^\mu(\varphi)$$

$$B(\vec{k}) = A^\dagger(-\vec{k})$$

$$\varphi(t, \vec{x}) = \int \frac{d^3k}{(2\pi)^3} \left[A(\vec{k}) e^{-iE_k t + i\vec{k} \cdot \vec{x}} + A^\dagger(-\vec{k}) e^{iE_k t + i\vec{k} \cdot \vec{x}} \right]$$

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$e^{-i\vec{k} \cdot \vec{x}}$ where $k^0 = E_k$

$$E_{\vec{k}} = \sqrt{k^2 + m^2}$$

$$B(\vec{k}) = A^*(-\vec{k})$$

$$\psi(t, \vec{x}) = \int \frac{d^3k}{(2\pi)^3} \left[A(\vec{k}) \underbrace{e^{-iE_{\vec{k}}t + i\vec{k} \cdot \vec{x}}}_{e^{-ik \cdot x}} + \underbrace{A^*(-\vec{k}) e^{iE_{\vec{k}}t + i\vec{k} \cdot \vec{x}}}_{\vec{k} \rightarrow -\vec{k} \text{ this term only}} \right]$$

where $k^0 = E_{\vec{k}}$

$$\psi(t, \vec{x}) = \int \frac{d^3k}{(2\pi)^3} \left[A(\vec{k}) e^{-ik \cdot x} + A^*(-\vec{k}) e^{ik \cdot x} \right]$$

$$\int \frac{d^4 k}{(2\pi)^4} 2\pi \delta(k^2)$$

$$\int \frac{d^4 k}{(2\pi)^4} \ 2\pi \delta(\underbrace{k^2 - m^2}_{k^0^2 - \vec{k}^2}) \ \Theta(k^0)$$



$$\int \frac{d^4 k}{(2\pi)^4} 2\pi \delta(\underbrace{k^2 - m^2}_{k^0^2 - \vec{k}^2}) \Theta(k^0) = \frac{1}{2E_k} \int \frac{d^3 k}{(2\pi)^3}$$

invariant
 under Lor. trans.
 that preserve arrow
 of time

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↑
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$$A(\vec{k}) = \frac{a(\vec{k})}{2E_{\vec{k}}}$$

$$\varphi(t, \vec{x}) = \int \frac{d^3k}{(2\pi)^3 2E_k} \left[a(\vec{k}) e^{-ik \cdot x} + a^\dagger(-\vec{k}) e^{ik \cdot x} \right]$$



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