Title: Supersymmetric Field Theories for Mathematicians

Date: Sep 28, 2016 02:00 PM

URL: http://pirsa.org/16090062

Abstract:

$$S^2 S_+ \to V$$
$$S^2 S_- \to V$$

(n,m) susy

$$S_+ \otimes \mathbb{C}^n \oplus S_- \otimes \mathbb{C}^m$$

R-symmetry

$$SO(n) \times SO(m)$$

$\mathrm{Spin}(5,\mathbb{C}) = \mathrm{Sp}(4,\mathbb{C})$

$S = \mathbb{C}^4$ = fun. rep of $\mathrm{Sp}(4,\mathbb{C})$

$V = \left(\Lambda^2 S\right)/\mathbb{C}\cdot\omega$ $\quad\omega$ = symplectic form

$V \subseteq \Lambda^2 S$

n extended SUSY, $S \otimes \mathbb{C}^{2n}$

$\mathbb{C}^{2n}$ is symplectic

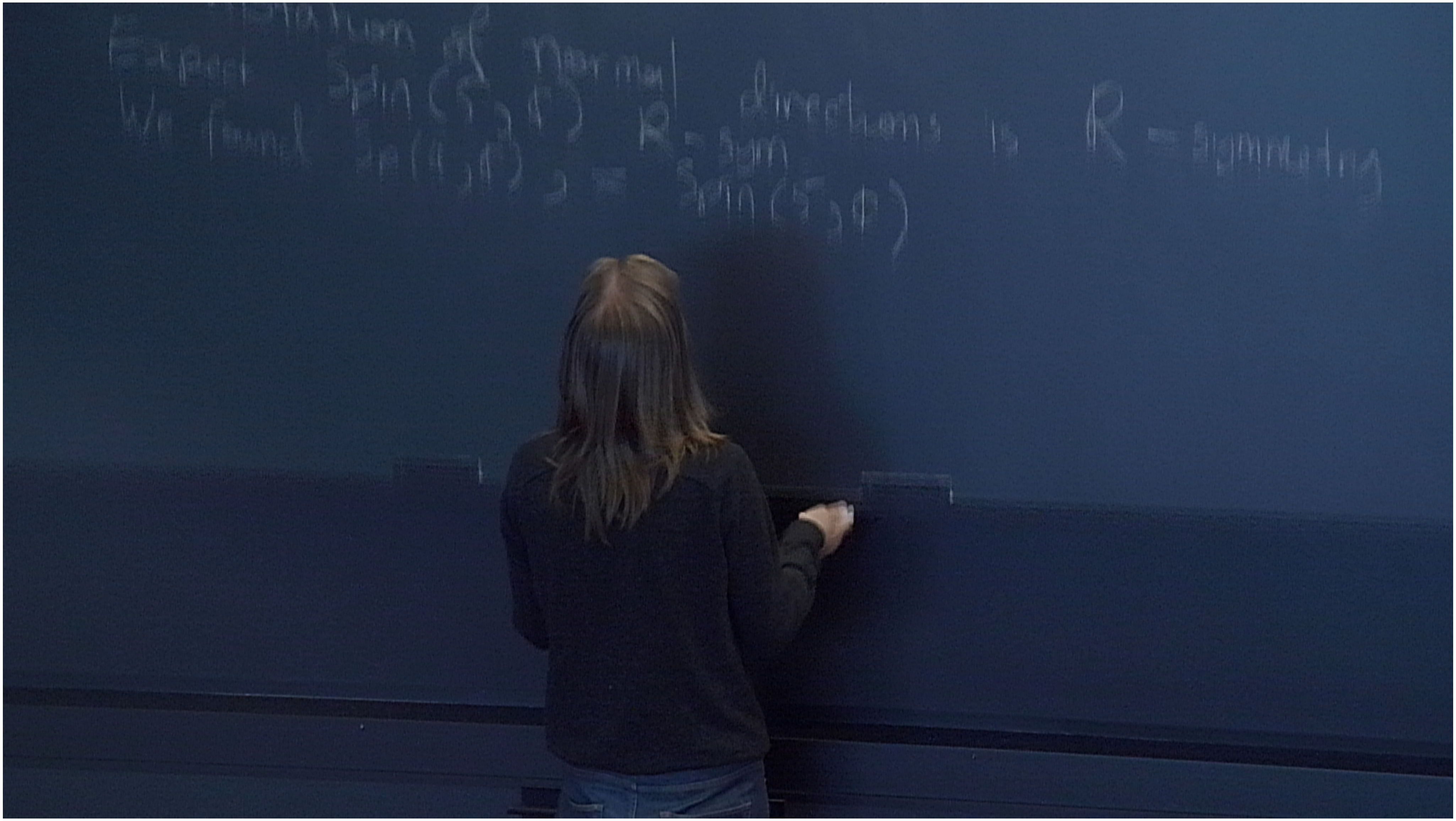
R-symmetry is $\mathrm{Sp}(2n,\mathbb{C})$

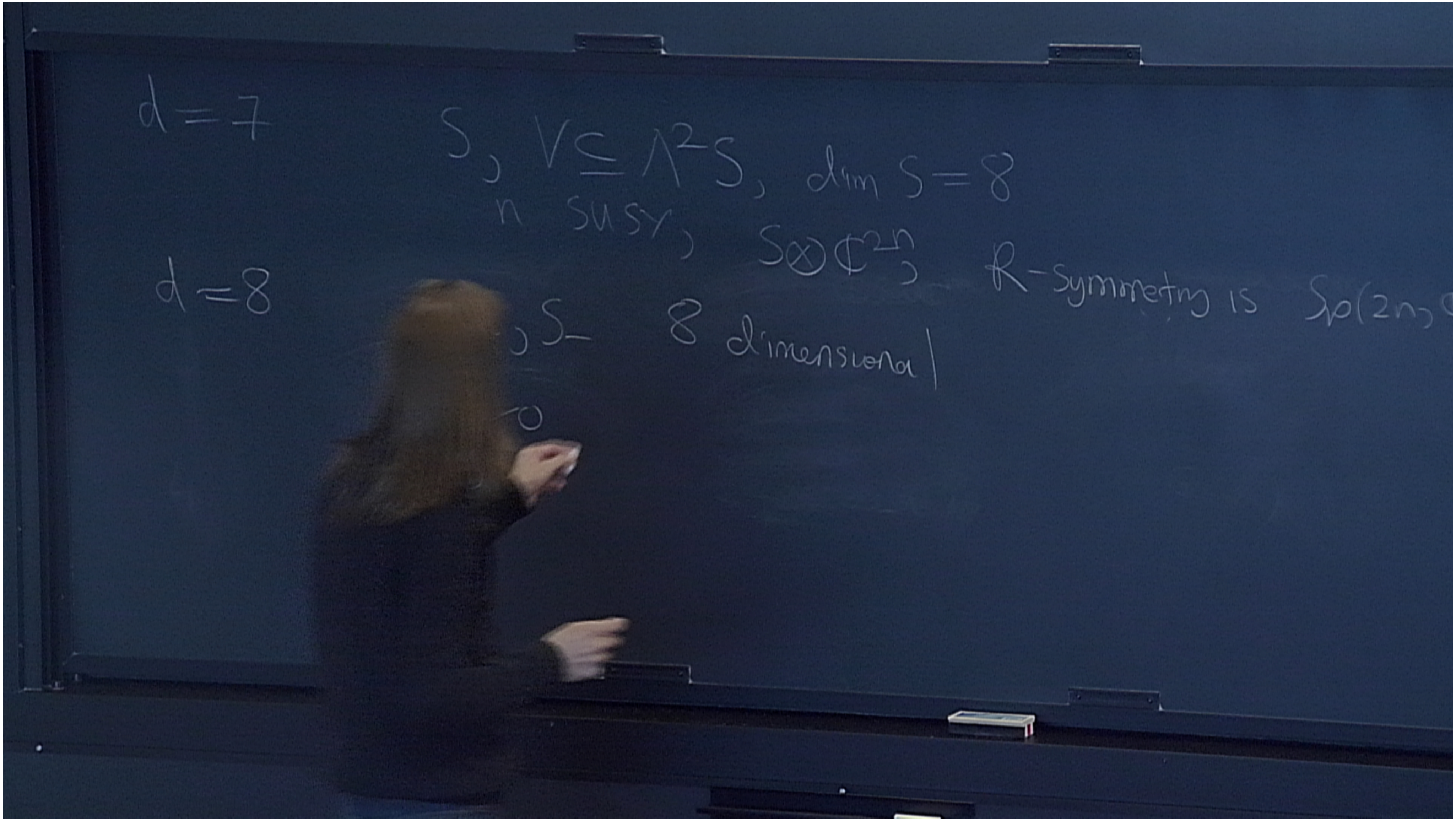
Low dimensional susy

d even $\quad S_+, S_-$ of dim^n $\quad 2^{d/2-1}$

d odd $\quad S$ of dim^n $\quad 2^{(d-1)/2}$

V vector rep.


Expect
normal
directions is
R-symmetry
We found
R-sym


d=7
S, V ⊆ Λ²S, dim S = 8
n susy, S⊗ℂ²ⁿ, R-symmetry is Sp(2n,
d=8
S₋ 8 dimensional

$d = 10$

S_+, S_- dimn 16 $\quad V \subseteq S^2 S_+ \quad V \subseteq S^2 S_-$

10

S_+, S_- dimn 16 $\quad V \subseteq S^2 S_+ \quad V \subseteq S^2 S_-$

(n, m) susy, $\quad S_+ \otimes \mathbb{C}^n \oplus S_- \otimes \mathbb{C}^m$, $\quad R$-sym is $SO(n) \times SO(m)$

10

S_+, S_- dim 16 $\quad V \subseteq S^2 S_+ \quad V \subseteq S^2 S_-$

(n,m) SUSY, $\quad (S_+ \otimes \mathbb{C}^n \oplus S_- \otimes \mathbb{C}^m)$ $\quad$ R-sym is $SO(n) \times SO(m)$

$(1,0)$ $\quad$ 10d Super YM

$(2,0)$ $\quad$ IIB string

$(1,1)$ $\quad$ IIA string

10

S_+, S_- dimn 16 $\quad V \subseteq S^2 S_+ \quad V \subseteq S^2 S_-$

(n,m) susy, $\left(S_+ \otimes \mathbb{C}^n \oplus S_- \otimes \mathbb{C}^m\right)$ $\mathbb{R}$-sym is $SO(n) \times SO(m)$

$(1,0)$ 10d Super YM

$(2,0)$ IIB string

$(1,1)$ IIA string

10

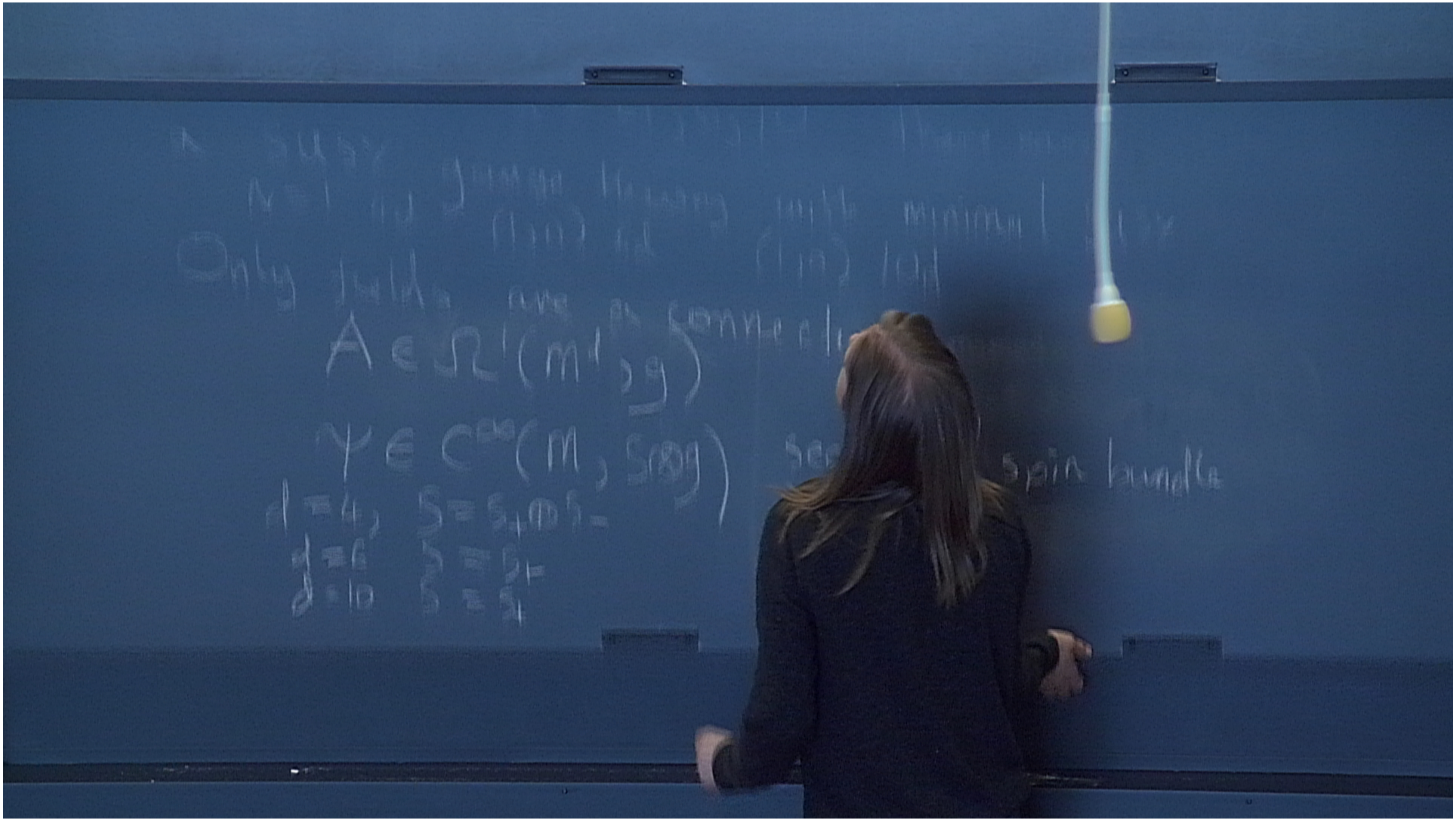
S_+ , S_- dim^n 16 $\quad V \subseteq S^2 S_+ \quad V \subseteq S^2 S_-$

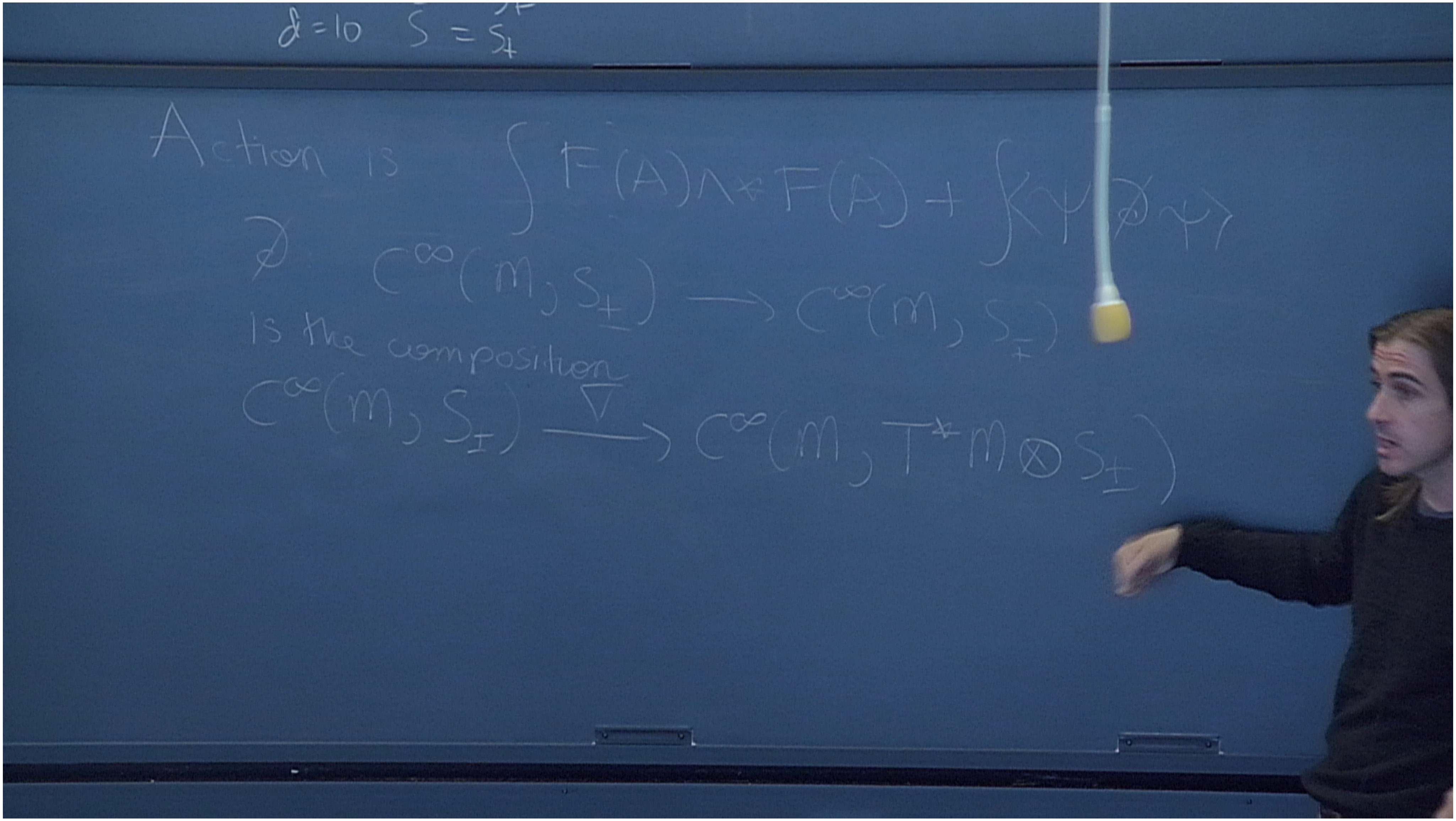
(n,m) susy, $\quad S_+ \otimes \mathbb{C}^n \oplus S_- \otimes \mathbb{C}^m)\quad$ R-sym. is $SO(n) \times SO(m)$

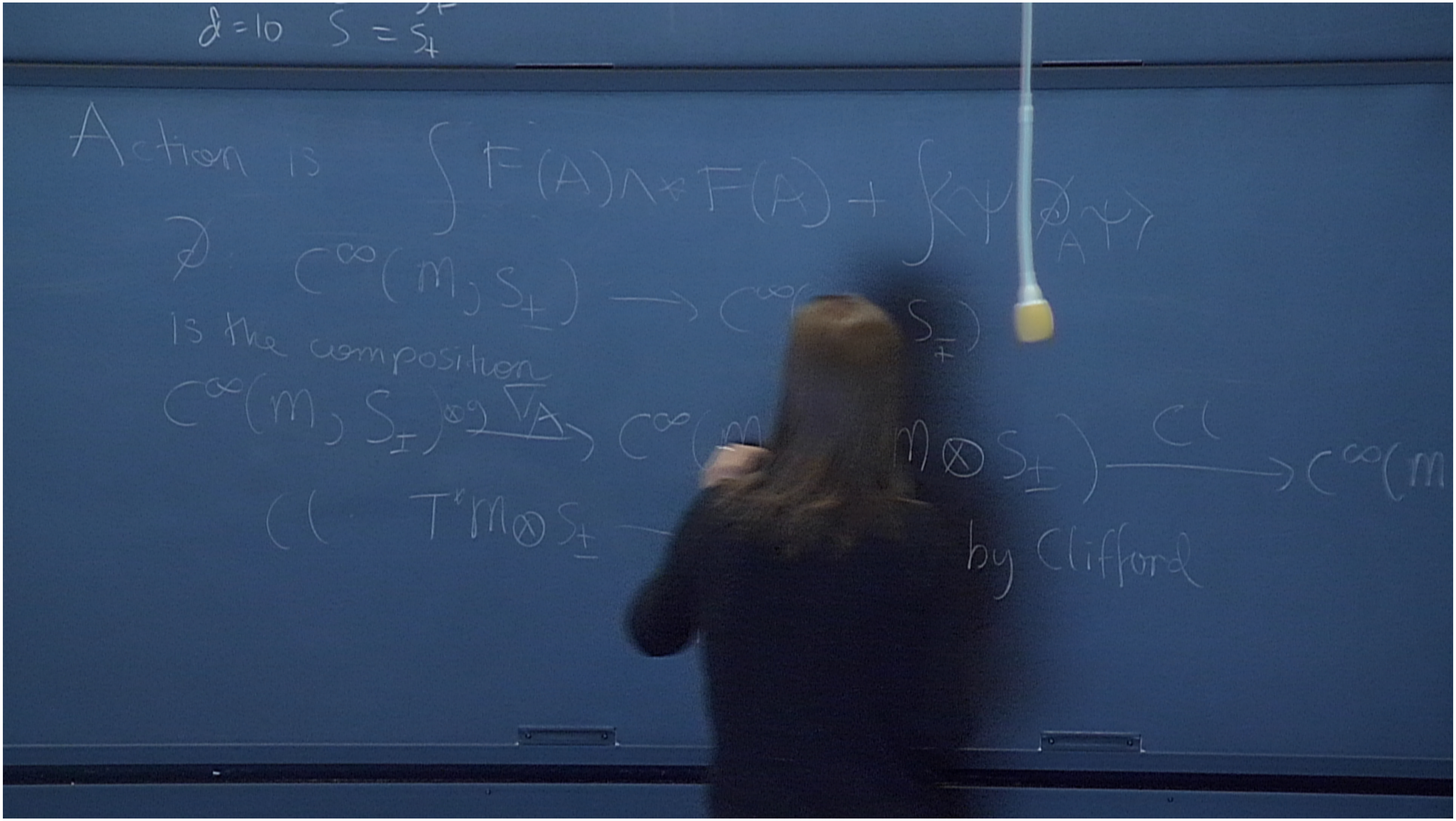
$(1,0)$ 10d Super YM

$(2,0)$ IIB string

$(1,1)$ IIA string


Only fields are connections
A ∈ Ω¹(Mᵈ, g)
ψ ∈ C^∞(M, S⊗g)
spin bundle
d=4, S=S+⊕S−
d=6 S=S+
d=10 S=S+


$\delta = 10 \quad S = S_+$
Action is $\int F(A) \wedge * F(A) + \int \langle \psi | \not{D} \psi \rangle$
$\not{D} \quad C^\infty(M, S_\pm) \longrightarrow C^\infty(M, S_\mp)$
is the composition
$C^\infty(M, S_\pm) \xrightarrow{\nabla} C^\infty(M, T^*M \otimes S_\pm)$


$d = 10$ $S = S_+$
Action is $\int F(A) \wedge * F(A) + \int \langle \psi, \not{D}_A \psi \rangle$
$\not{D} : C^\infty(M, S_\pm) \longrightarrow C^\infty(\ \ , S_\mp)$
is the composition
$C^\infty(M, S_\pm) \xrightarrow{\nabla_A} C^\infty(M \ \ M \otimes S_\pm) \xrightarrow{Cl} C^\infty(M$
$Cl \quad T^*M \otimes S_\pm \longrightarrow$
by Clifford

$d = 10 \quad S = S_+$

Action is $\int F(A) \wedge * F(A) + \int \langle \psi, \not{D}_A \psi \rangle$

$\not{D} : C^\infty(M, S_\pm) \longrightarrow C^\infty(M, S_\mp)$

is the composition

$C^\infty(M, S_\pm)^{\otimes g} \xrightarrow{\nabla_A} C^\infty(M, T^*M \otimes S_\pm^{\otimes g}) \xrightarrow{Cl} C^\infty(M$

$Cl \quad T^*M \otimes S_\pm \longrightarrow S_\mp$ given by Clifford mult.

copy of $SO(d,\mathbb{C})$

im: In dim^{n}s $d = 4, 6, 10$

1) This infinitesimal symmetry preserves the action function + commutes w. gauge

2) If V_Q = v. field on space of fields associated to Q, then

$$[V_Q, V_{Q'}] = \mathcal{L}_{\Gamma(Q \otimes Q')}$$

Lie derivative

modulo gauge symmetry + on the sol^{n}s to EOM im In dim^{n}s $d = 4, 6, 10$

1) This infinitesimal symmetry preserves the action function + commutes w. gauge

2) If $\mathcal{V}_\varphi$ = v. field on space of fields associated to φ, then

$[\mathcal{V}_\varphi, \mathcal{V}_{\varphi'}] = \mathcal{L}_{\Gamma(\varphi \otimes \varphi')}$ modulo gauge symmetry + on the sol^{n}s to EOM.

Lie derivative

... of $SO(d,\mathbb{C})$

...im

In dim^{n}s $d = 4, 6, 10$

1) This infinitesimal symmetry preserves the action function + commutes w. gauge

2) If V_Q = v. field on space of fields associated to Q, then

$$[V_Q, V_{Q'}] = \mathcal{L}_{\Gamma(Q \otimes Q')}$$

Lie derivative

modulo gauge symmetry + on the sol^{n}s to EOM.

$SO(d,\mathbb{C})$

im

In dim^{n}s $d=4,6,10$

1) This infinitesimal symmetry preserves the action function + commutes w. gauge

2) If ν_Q = v. field on space of fields associated to Q, then

$[\nu_Q, \nu_{Q'}] = \mathcal{L}_{\Gamma(Q\otimes Q')}$ modulo gauge symmetry + on the sol^{n}s to EOM

Lie derivative

aim

In dim^{n}s $d = 3, 4, 6, 10$

1) This infinitesimal symmetry preserves the action function + commutes w. gauge

2) If $\mathcal{V}_Q$ = v. field on space of fields associated to Q, then

$[\mathcal{V}_Q, \mathcal{V}_{Q'}] = \mathcal{L}_{\Gamma(Q \otimes Q')}$ modulo gauge symmetry + on the sol^{n}s to EOM

Lie derivative

M manifold

$C^\infty(M, S) \supseteq$ cov. constant spinors

$C^\infty(M, TM) \supseteq$ cov. constant vectors

these will form a smaller SUSY algebra

$d=4$ Check SUSY commutation relations

$\mathfrak{g} = \mathbb{R}$, abelian

Fields is now a linear super space, SUSY acts by linear transformations

$$\Gamma(Q_-, \psi_+) + \Gamma'(Q_+, \psi_-) \longleftarrow\!\shortmid (\psi_+, \psi_-)$$

$$\Omega^1(\mathbb{R}^4) \qquad\qquad C^\infty(M, S_+ \oplus S_-)$$

$$A \longmapsto (dA \cdot Q_+, dA \cdot Q_-)$$

$$Q = (Q_+, Q_-) \in S_+ \oplus S_-$$

Compute $[Q_+, Q_-]$ acts on $\mathcal{A}$

$$Q_-Q_+ : A \longrightarrow dA\cdot Q_+ \longrightarrow \Gamma(Q_-\otimes(dA\cdot Q_+))$$

$$Q_+Q_- \quad A \longrightarrow dA\cdot Q_- \longrightarrow \Gamma(Q_+\otimes(dA\cdot Q_-))$$

Note, $dA\,(\Gamma(Q_+\otimes Q_-)) = \Gamma((dA\,Q_+)\otimes Q_-) + \Gamma($

Compute $[Q_+, Q_-]$ acts on A

$$Q_-Q_+ : A \longrightarrow dA\cdot Q_+ \longrightarrow \Gamma(Q_- \otimes (dA\cdot Q_+))$$

$$Q_+Q_- \quad A \longrightarrow dA\cdot Q_- \longrightarrow \Gamma(Q_+ \otimes (dA\cdot Q_-))$$

Note, $dA(\Gamma(Q_+ \otimes Q_-)) = \Gamma((dA\, Q_+) \otimes Q_-) +$

$$[Q_+, Q_-] = dA \circ \Gamma(Q_+ \otimes Q_-)$$

$$= \Gamma(Q_+ \otimes Q_-) \vee dA = \mathcal{L}_{\Gamma(Q_+ \otimes Q_-)} \qquad A + d$$

$$Q_- Q_+ : A \longrightarrow dA \cdot Q_+ \longrightarrow \Gamma(Q_- \otimes (dA \cdot Q_+))$$

$$Q_+ Q_- \quad A \longrightarrow dA \cdot Q_- \longrightarrow \Gamma(Q_+ \otimes (dA \cdot Q_-))$$

Note, $dA(\Gamma(Q_+ \otimes Q_-)) = \Gamma((dA\, Q_+) \otimes Q_-) + \Gamma(Q_+ \otimes dA\, Q_-$

$$[Q_+, Q_-] = dA \cdot \Gamma(Q_+ \otimes Q_-)$$

$$= \Gamma(Q_+ \otimes Q_-) \vee dA = \mathcal{L}_{\Gamma(Q_+ \otimes Q_-)} A + d(\Gamma(Q_+ \otimes Q_-) \vee A)$$

... SUSY transformations?)

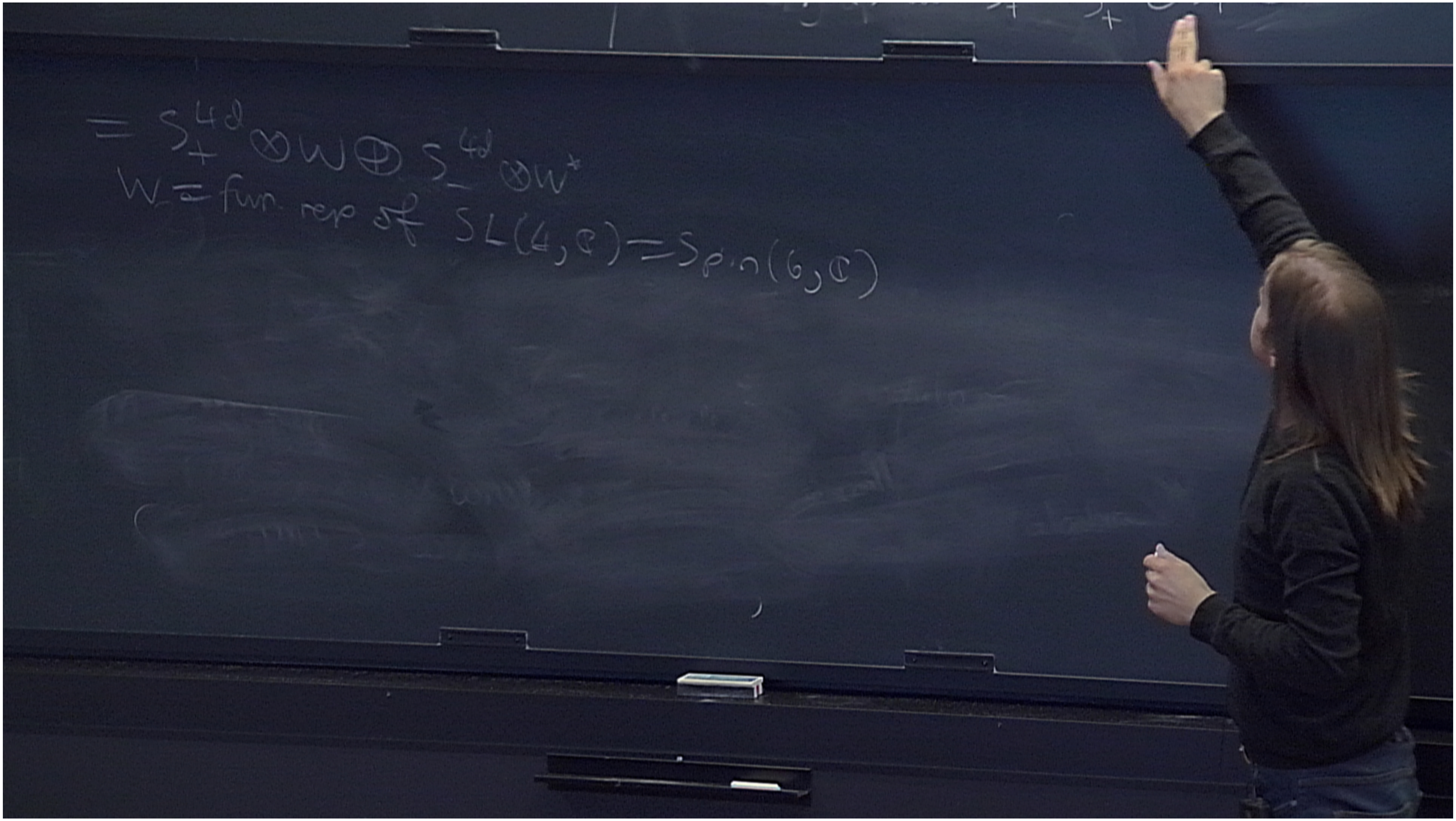
Answer $N=4, d=4$ is reduced from $N=(1,0)$ $d=10$

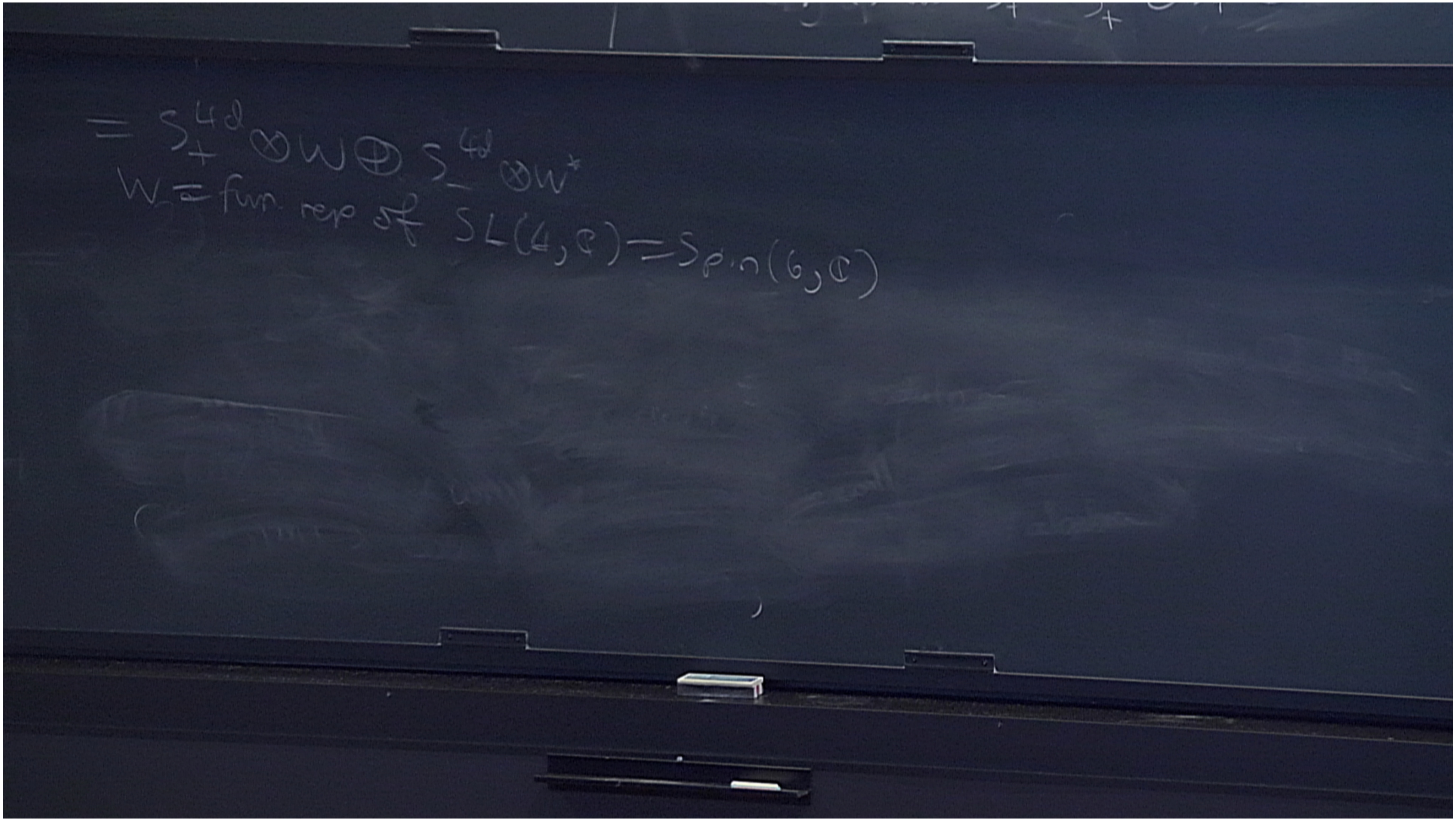
Field content $A_{4d} \in \Omega^1(\mathbb{R}^4)$, $\varphi_1 \ldots \varphi_6 \in C^\infty(\mathbb{R}^4)$

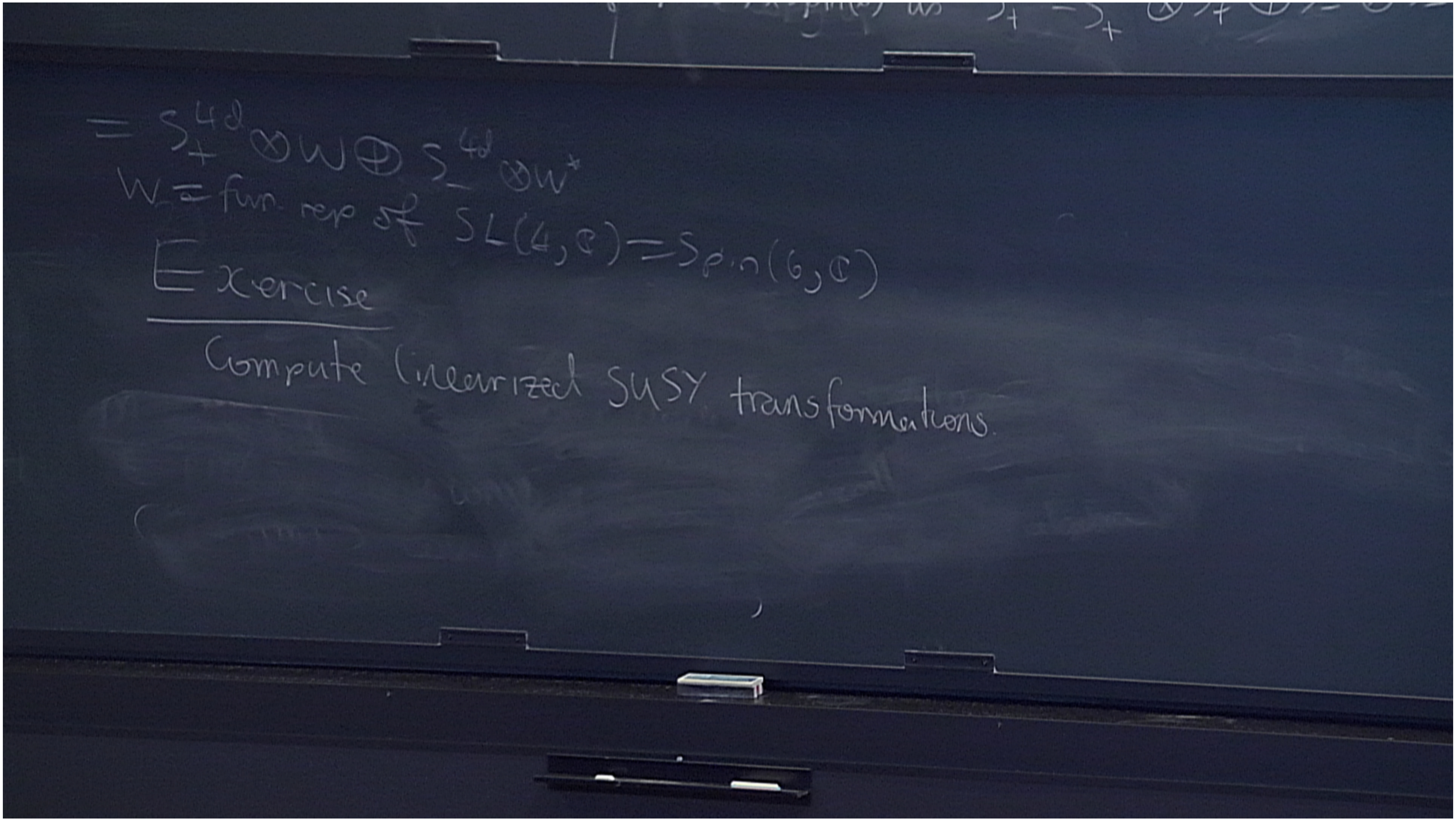
$$A_{10d} = A_{4d} + \sum_{i=1}^{6} dx_{6+i} \cdot \varphi_i$$

10d, 16 spinors in S_+^{10d}

S_+^{10d} decomposes under action of $Spin(4) \times Spin(6)$ as $S_+^{10d} = S_+^{4d}$


$= S_+^{4d} \otimes W \oplus S_-^{4d} \otimes W^*$
W = fun. rep of $SL(4,\mathbb{C}) = Spin(6,\mathbb{C})$


$= S_+^{4d} \otimes W \oplus S_-^{4d} \otimes W^*$
W = fun. rep of $SL(4,\mathbb{C}) = Spin(6,\mathbb{C})$


$= S_+^{4d} \otimes W \oplus S_-^{4d} \otimes W^*$
W = fun. rep of $SL(4,\mathbb{C}) = Spin(6,\mathbb{C})$
Exercise
Compute linearized SUSY transformations.