Title: The relative locality of quantum spacetime

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Abstract: <p>Should we revisit the concept of space solely based on quantum mechanics?</p>

<p>Do we need a radically new physical principle to address the problem of quantum gravity?</p>

<p>In this talk I will adress these questions. I will review what are the central challenges one faces when trying to understand the theory of quantum gravity from first principles and focus on the main one which is non-locality.</p>

<p>I will present a collection of results and ideas that have been developed in the recent years that provides a radical new perspective on these issues.</p>

<p>One of the central concept I'll present is the idea that locality has to be made relative, and how this idea goes back to one of the founder of quantum mechanics: Max Born. I'll also explain how these new ideas remarkably force us to revisit the concept of space itself and propose a natural generalization that incorporate quantum mechanics in its fabric called modular space. I'll also sketch how these foundational ideas quite unexpectedly links with the most recent developments on the geometry of string theory, and generalized geometry.</p>

The Relative Locality of Quantum Space-time

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based on 1307.7080, 1405.3949, 1502.08005 and 1606.01829 and more…
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Road map

Our concepts of space and time have radically evolved over history. Our concept of matter too but these developments haven't really affected our fundamental picture of space and time yet ! e.g in the standard model of particle physics and cosmology.

Is there a notion of space and time that organically include quantum mechanics in its fabric?

- Quantum gravity: Lessons on quantum spacetime?
- Going Beyond Quantum gravity models: We need a new physical principle that reconcile the relativity principle and the presence of a fundamental scale
- Relative Locality, Born geometry and quantum geometry
- Modular space Quantum mechanics and String Theory.

Quantum + Gravity

- Should we care about putting together gravity and the quantum?

Matter is quantum and coupled to GR, GR singularities, quantum nature of large scale structure in cosmology

- If one believe that quantum gravity is only about computing small quantum correction to gravitational phenomena then we already have a theory of quantum gravity: Effective field theory (EFT)

We expect radically new phenomena to become visible, not just small correction to known phenomena, more than EFT.

Singular limits

In any breakthrough, invisible phenomena become visible:
The lower order description is a singular limit of the higher one (M-Berry). That is a mathematically consistent description which cannot reveal certain observables.

- Eulerian fluid is a singular limit of the viscous fluid, planes can't fly
- geometric optics-wave optics: No central bright spot
- Classical-Quantum: Aharonov-Bohm phases are invisible
- Non-relativivistic/relativistic Quantum Mechanics: Anti-particle
- Newton-GR: No gravity waves

  - What are the new invisibles to be revealed ?
  - In what sense is today's physics is a singular limit?

Unification

In any breakthrough, invisible phenomena become visible, but also a fundamental form of unification takes place. Seemingly opposite concepts of the original picture are unified in the more advanced one .This unification is encoded in terms of a universal conversion factor.

- h: Unification of wave and particle $\Delta E = \hbar\omega$
- c: Unification of space and time $E = \Delta M c^2$
- G: Unification of Inertial and gravitational mass
- k: Unification of Energy and information $S = k \ln W$
- h,c: Unification of quanta and fields
- G,c: Unification of matter and geometry $R_s c^2 = 2GM$

What's next? A deeper unification of quanta and geometry ?

Unification of spacetime and energy-momenta?

Quantum + Gravity

- Should we care about putting together gravity and the quantum?

We expect radically new phenomena to become visible, not just small corrections to known phenomena, more than EFT.

- Quantising gravity? : Doesn't work non renormalisable, Asymptotic Safety.
- Quantising geometry ? : Background independence and non local observables, space is fundamentally discrete, built in the Hilbert space bases. But the challenge is reconciliation with the General relativity principle outside the classical limit.
- String Theory ?: The probe is the fundamental, delocalising the probe, consistent with relativity, but it hasn't changed yet our understanding of space and time at the fundamental level.
- Emergent models ?: CDT, Causal sets, Horava Gravity, Non-commutative geometry or Holography AdS/CFT

- Why are we not done yet? what are we missing? what haven't we tried?

Lessons

- We have many different approaches to the problem. What have we learned so far? What do they all have in common?

- The only common theme between all of them is non-locality
- non-local observable in background independent approach,
- non local probes
- non locality of holography
- discreteness
- non local fixed point etc…

Why? Quantum theory is our fundamental framework yet such basic concepts as space and time have not been affected by it. Space and time as they appear in $\Phi(x)$ are still treated as classical concepts

The Challenges of non Locality

- We expect that any theory of quantum gravity will involve some non-locality. How do we deal with non-locality without opening Pandora's Box?

- Locality is built in Field Theory and General Relativity:

locality of asymptotic states,
locality of interactions,
locality of RG =separation of scales.

These are the foundations of modern physics. The organizational principle of effective field theory.

We need to specify what type of non-locality is viable, we need a new principle to tame non-locality.

Non Locality of QM

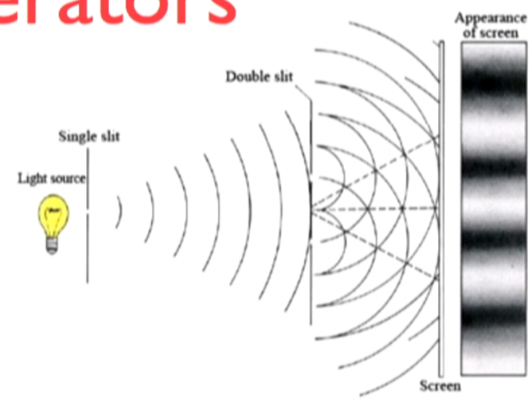
- Both QM and GR exhibits non locality:
- QM:
  - Heisenberg non locality $\Delta x \Delta p \geq \hbar/2$
  - Entanglement non locality : "Spooky action". A form of kinematical non-locality
  - Aharanov-Bohm non locality = non locality of interferences. A form of dynamical non locality

  What operator can measure the interference? These are the modular operators: A new type of observables with no classical analog

Modular operators

Suppose we have two waves $\psi_1(x), \psi_2(x)$
of non overlapping support
Consider a wave

$\langle x|\alpha\rangle = \psi_1(x) + e^{i\alpha}\psi_2(x)$



No polynomial in $\hat{x}, \hat{p}$ can detect α

$$\partial_\alpha \langle\alpha|\hat{x}^n\hat{p}^m|\alpha\rangle = 0$$

But if $L \sim x_1 - x_2$

$$\partial_\alpha \langle\alpha|e^{iL\hat{p}/\hbar}|\alpha\rangle \neq 0$$

modular operator such as

- have no classical analog
- satisfy non local equations of motions

$$i\partial_t e^{iL\hat{p}/\hbar} = (V(q) - V(q+L))e^{iL\hat{p}/\hbar}$$

dynamical non locality

However modular variables are contextual

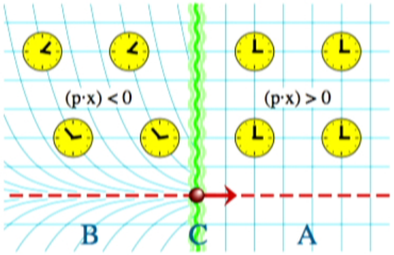
happens because wavefunctions can be non-analytic
but $e^{iL\hat{p}/\hbar}f(x) = f(x+L)$ for any func with a Fourier transform

Non Locality of GR

- Diffeomorphism invariance ⟶ gravity observables are non local (Generalised Gauss Law)= Holography.
- Due to causality there is no-screening, the gravity charges = is the energy E: it has to be positive.

Non local memory effect:

$$\Delta z \sim Gp\frac{\delta x}{x}$$



t'Hooft: One should introduce a generalisation of S-matrix in the gravitational context to account for these effects

Both gravity and quantum mechanics leads to new class of non local observables

What kind of non locality?

A new take on quantum gravity: It should emerge from a theory which is quantum and has a fundamental delocalisation scale and satisfies the relativity principle. Non locality cannot be arbitrary.

One of the fundamental challenges is to reconcile having a fundamental scale with Lorentz invariance

Relative Locality is taken as the organizational feature allowing us to tame non locality.

In relative locality processes among probes define via localization their own notion of space-time: Locality is relative. Relative locality also incorporates the ability to change polarization: Born duality is the new symmetry principle to preserve

ST is Lorentz invariant and possesses a fundamental length scale it should therefore organize itself under the relative locality principle.

What is Relative locality?

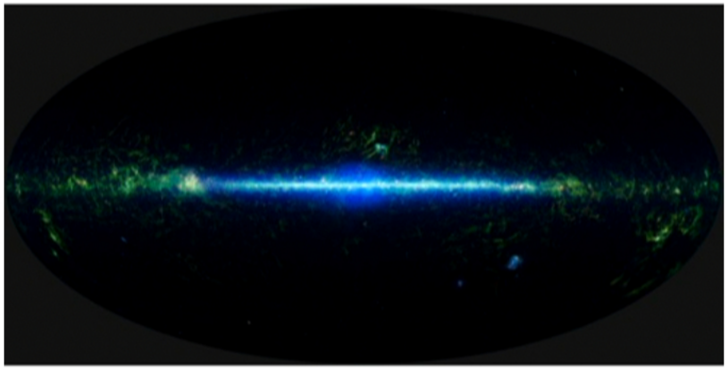
- Absolute locality is the hypothesis that the concept of spacetime is independent of the nature of probed used. It is a universal notion.
- Relative locality is on the contrary exploring the idea that spacetime is a notion which depends on the quantum nature of probe used i-e energy and quantum numbers.
- The usual spacetime notion is adapted to probes which are Point-like and classical.

What is the proper notion of quantum spacetime adapted to quantum and non-local probes ?

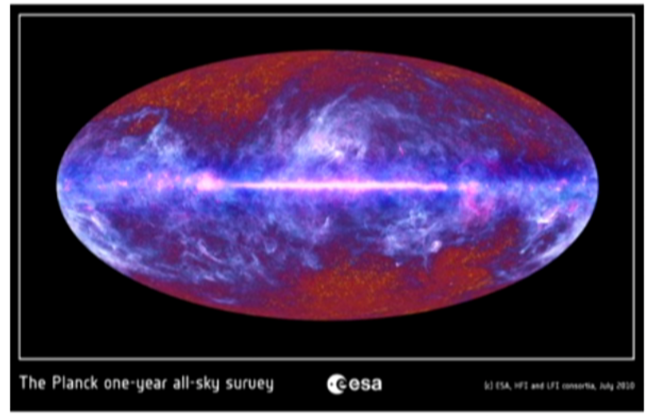
- Since spacetime appears as a choice of polarization in QM and polarizations can change, preserving this symmetry means that Spacetime is relative to energy-momentum in phase space.

Why? How to implement it? What are the elements?

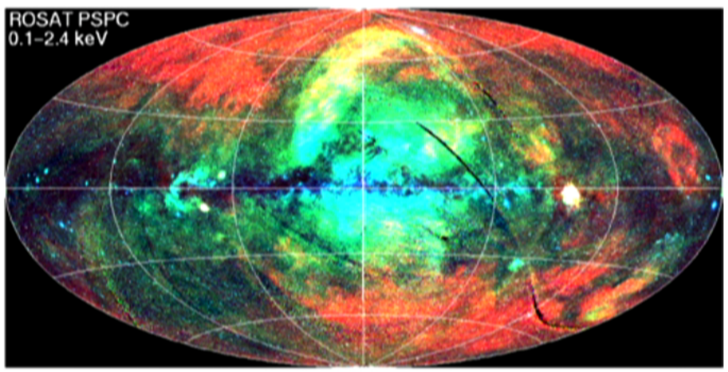
Relative Locality: Illustration full sky survey:



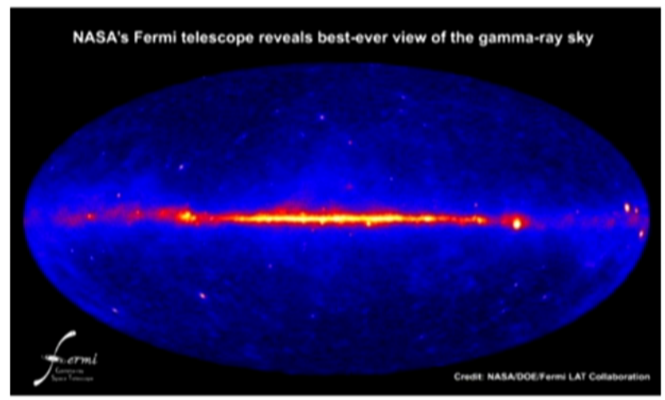
Wise infrared



Planck microwave

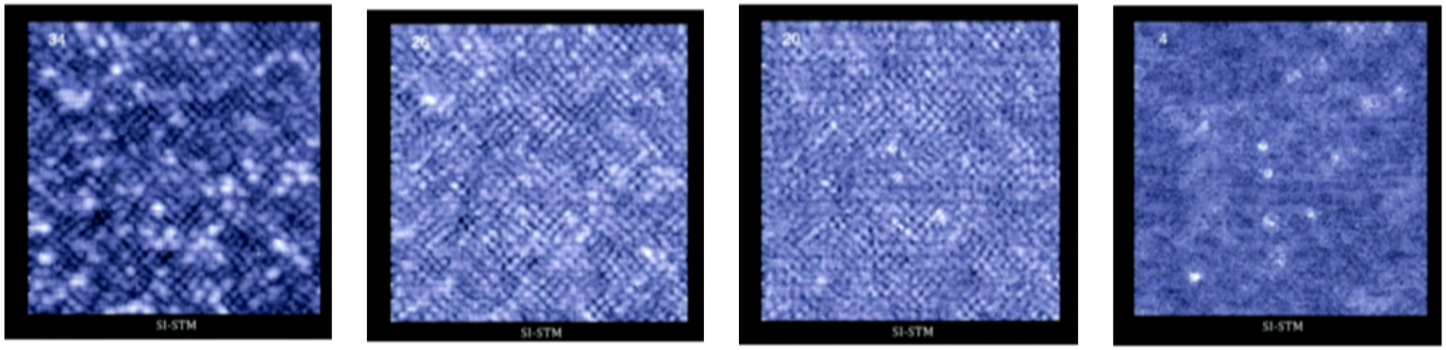


Rosat X-ray



Fermi Gamma ray

Quantum Visualization of wave function S. Davis



High E
disordered

Quasi-Particle interferences:
Friedel oscillations

ordered

Low E
translation invariant

The classical question: is it ordered or disordered? is ill-defined in QM it depends on the observation not just the system!

In the same region of space we can have different eigenstates of different energy, it is disordered at a given energy and ordered at another.

beholder's eye

Analogy: Quantum crystal = spacetime
electrons= probes.

Here locality is relative
but relativity is missing

G as conversion factor

In special relativity there is a universal speed that is also understood as a universal conversion factor between space and time. This leads to observer dependent space and time and their unification in a geometrical structure the Minkowski space-time.

G_N is usually seen as a running coupling constant.

In General Relativity due to equivalence principle G_N/c^2 can be understood as a universal inverse tension, a conversion factor between space-time and energy-momentum.
This leads to observer dependent space-times and energy-momentum and their unification in a geometrical structure the relativistic phase space of individual probes.

G as conversion factor

In General Relativity due to equivalence principle G_N can be understood as a universal inverse tension.

The running of a dimensional coupling is always fixed in terms of another dimensionful parameter. We choose Planck units as units of energy and time where G_N doesn't run. Thus we are extending the equivalence principle to quantum probes

Together with $\hbar$ this gives us a fundamental length scale and a fundamental energy scale that can be used to unify distances and momenta

We emphasize that we are talking here about the relativistic phase space of quantum probes.

What kind of geometry on phase space expresses the relative locality principle?

Relative locality and Phase space

- The geometry of spacetime is encoded in a Lorentzian metric which encodes in its causal structure the difference between space like and time like.
- Similarly the geometry of phase space is encoded into a geometric structure that we call a Born geometry which encodes the difference between space-time like and energy-momentum like.

To unify space-time with energy and momenta we need, a fundamental length scale λ and energy scale ϵ

$$\mathbb{X}^A = \begin{pmatrix} x^\mu \\ \tilde{x}_\mu \end{pmatrix} \in \boldsymbol{P} \quad \text{with} \quad \{x, \tilde{x}\} = \frac{1}{2\pi}$$

$$x = \hat{q}/\lambda \qquad \tilde{x} = \hat{p}/\epsilon$$

$$\hbar = 2\pi\lambda\epsilon \qquad G_N = \frac{\lambda}{\epsilon}$$

Born Geometry I

Phase space $\boldsymbol{P}$ possesses 3 natural structures:
A symplectic structure ω and 2 metrics.
The Quantum metric $\boldsymbol{H}$ and the locality metric η

Born Geometry: $(\boldsymbol{P}, \omega, H, \eta)$

The symplectic structure $\omega_{AB}\mathrm{d}\mathbb{X}^A\mathrm{d}\mathbb{X}^B = \frac{1}{\hbar}\mathrm{d}p_a \wedge \mathrm{d}q^a$
is encoded in the commutator

At the quantum level $[x^a, \tilde{x}_b] = \frac{i}{2\pi}\delta^a_b$

Born Geometry II

$(\boldsymbol{P}, \omega, H, \eta)$

Quantum metric H

$$\mathrm{d}s_H^2 = H_{AB}\mathrm{d}\mathbb{X}^A\mathrm{d}\mathbb{X}^B = \frac{1}{\hbar}\left(\frac{\mathrm{d}q^2}{G} + G\mathrm{d}p^2\right)$$

signature $(2, 2(d-1))$

For weakly gravitating objects $G\Delta E << \Delta L$

This metric reduces to the usual spacetime metric where spacetime is viewed as a slice of phase space

$$\mathrm{d}s_H^2 \propto \mathrm{d}q^2$$

gravitational tension is huge

$$\frac{c^2}{G} \sim 10^{17}\,\frac{\mathrm{kg}}{\mathring{A}}$$

In relative locality the spacetime metric is the leftover of the quantum metric when we ignore energy change through motion.

Born geometry III $(\boldsymbol{P}, \omega, H, \eta)$

The locality metric η

encodes the distinction between spacetime like and energy-momentum like.

$$\mathrm{d}s_\eta^2 = \eta_{AB}\mathrm{d}\mathbb{X}^A\mathrm{d}\mathbb{X}^B = \frac{2}{\hbar}\mathrm{d}p\mathrm{d}q$$

signature (d, d)

Vector tangents to spacetime are null with respect to η

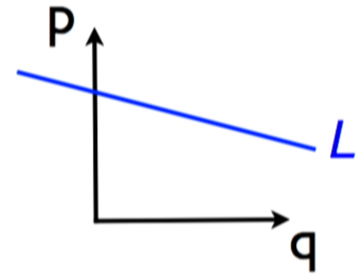
Vector tangents to momentum space are also null wrt η

This new metric captures the essence of relative locality.

Curving η: Gravitising the quantum

Geometry of Phase space

In relative locality space-time is not an absolute notion it is a Lagrangian manifold.

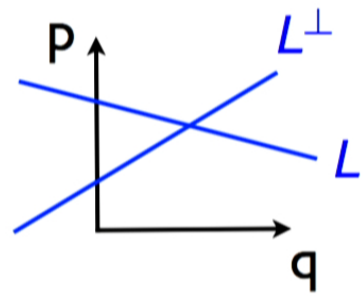


A subset of max dim of P such that $\omega|_L = 0$

Which one?

η determines which direction in phase space is a spacetime-like direction

By the same token it also determines which direction in phase space is energy-momentum like



What about quantum?

So far we have presented the classical side of relative locality. At the quantum level phase is promoted to a non-commutative Heisenberg algebra.

In QM Euclidean space appears simply as a choice of polarization: That is in the argument of the wave function. This is the quantum analog of a choice of Lagrangian

$$\Psi(x) \rightarrow \Phi(x)$$

Similarly Lorentzian space appears simply as a field label. Classical locality is built in the field definition as it is built in the Schrodinger representation.

$$(i\partial_t - H)\Psi = 0 \rightarrow \Box_g \Phi = 0$$

Can we define a notion of quantum space? quantum space-time?

Quantum spaces are?

Quantum spaces are simply choices of polarizations of the Heisenberg algebra= quantum Born geometry

$$[x^a, \tilde{x}_b] = \frac{i}{2\pi}\delta^a_b$$

The Schrodinger representation is the representation diagonalising the maximally commuting sub-algebra $F(\hat{x})$

In general a Schrodinger like polarization is associated with classical Lagrangian sub-manifold of phase space.

By definition a quantum space is a commutative *-subalgebra of the Heisenberg algebra

Is there more than Lagrangian?

Modular space

Flat Modular space are quantum space associated with abelian subgroup of the Heisenberg group. $[x^a, \tilde{x}_b] = \frac{i}{2\pi}\delta^a_b$

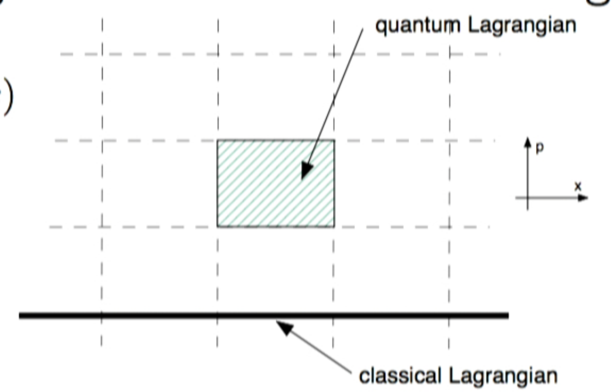
Such groups are generated by modular observables

$$[e^{2i\pi x}, e^{2i\pi \tilde{x}}] = 0$$

A quantum algebra possesses more commutative directions than a classical Poisson algebra $\{e^{2i\pi x}, e^{2i\pi \tilde{x}}\} \neq 0$

Modular space: Quantum Lagrangian with no classical analog

Modular uncertainty: can specify $(x, \tilde{x})$ within a cell but no knowledge of which cell



Schrodinger rep is a singular limit.

Modular space

Usually space determines what set of commuting measurement can be performed. We reverse the logic and define space as the maximal set of commutative operations. In this way modular spaces are fundamentally quantum. Also Modular space has a built-in length and energy scales.

Is there a physical system where modular spacetime is realized ?

Yes there is: In string theory

We can show that the target space of closed string is a relativistic version of modular space. Modularity and Born Duality are the target space realization of T-duality.

Modular space $[e^{2i\pi x}, e^{2i\pi \tilde{x}}] = 0$

A generic commutative *subalgebra is associated with a lattice $\Lambda \in \mathbf{P}$

This defines a phase space torus $T_\Lambda = P/\Lambda$

This lattice needs to be integral with respect to a neutral metric η compatible with ω (Lagrangian ω for = null for η)

The Hilbert space corresponds to sections of a U(1) bundle

$$\mathcal{H}_\Lambda = \Gamma(L_\Lambda) \qquad L_\Lambda \to T_\Lambda = \mathbf{P}/\Lambda$$

is needed in order to define the lift $T_\Lambda \to L_\Lambda$

In plain english: A modular wave function is quasi-periodic

$$\Psi(x+a, \tilde{x}) = e^{2i\pi a\tilde{x}}\Psi(x, \tilde{x}) \qquad \Psi(x, \tilde{x}+\tilde{a}) = \Psi(x, \tilde{x})$$

A vacuum determines a quantum metric H

Lorentz covariance of modular space

In order to construct $\mathcal{H}_\Lambda = \Gamma(L_\Lambda)$ we need a lift $T_\Lambda \to L_\Lambda$

η determines the polarisation, it also encodes the ambiguity of operator ordering

H determines the vaccua (pure space)

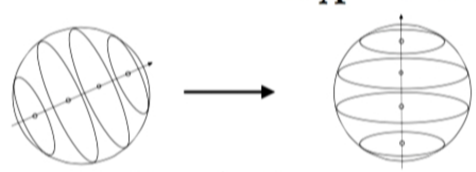
$$(\boldsymbol{P}, \omega, H, \eta)$$

This structure allows to reconcile for the organically fundamental discreetness and translational and Lorentz symmetries.

Classically $T_\Lambda \neq T_{\Lambda'}$ the symmetry is broken $\Lambda' = O\Lambda$

Since Modular space is a choice of quantum polarization that diagonalize G_Λ and since $[G_\Lambda, G_{O\Lambda}] \neq 0$ there exits an equivalence between modular states $\Psi_\Lambda \to \Psi_{O\Lambda} = U_{O\Lambda}\Psi_\Lambda$

Analog to rotation of a spin



$\sigma_x \to \sigma_z$

String theory and modular space

Closed string theory possess left and right movers. In principle both left and right movers can experience different geometries. What does that mean?

We have provided a formulation of string theory called metastring which allows the deformation in which generalized string geometry can be described geometrically. In this formulation we find that the string target is classically a Born geometry $P(\eta, \omega, H)(\mathbb{X})$

At the quantum level the string target is fundamentally non-commutative

$$[\mathbb{X}^A(\sigma), \mathbb{X}^B(\sigma')] = \omega^{AB} + \eta^{AB}\theta(\sigma, \sigma')$$

zero mode non-commutativity

oscillator non-commutativity

String theory and modular space

Off-shell the closed string lives in a non-commutative space. The on-shell conditions and mutual locality on the world sheet imposes that the string propagates in a sub-commutative algebra: A relativistic modular spacetime.

In the limit where we contract the modular spacetime to a Schrodinger spacetime and ignore non-commutativity, we recover the usual formulation of Polyakov: a string propagating in spacetime

In the limit where $\omega \to 0$ while $\eta \neq 0$ we recover the generalized geometry developed in mathematics and in double field theory in an attempt to incorporate T-duality in a geometrical setting

In general we have access to a new type of ``non geometrical'' background= modular spaces (e.g asymmetric orbifolds)

Lorentz covariance of modular space

The structure group that preserve (ω, η, H) is Lorentz

Besides the translations the Heisenberg group is invariant under

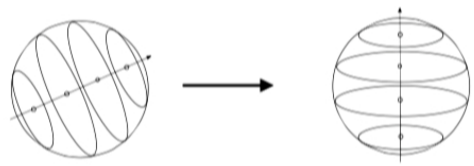
$$\mathrm{Sp}(2d)$$

The choice of modular polarisation break it down to

$$\mathrm{Sp}(2d) \cap \mathrm{O}(d,d) = GL(d)$$ a frame

The choice of vacua break it further to

$$\mathrm{Sp}(2d) \cap \mathrm{O}(d,d) \cap \mathrm{O}(2, 2(d-1)) = O(1,(d-1))$$ Lorentz



A Modular field in a boosted space is a superposition of unboosted modular fields $\Psi_\Lambda \to \Psi_{O\Lambda} = U_{O\Lambda}\Psi_\Lambda$

Conclusion

We have reviewed concepts of space and time and proposed a new notion of space which is

- fundamentally quantum
- fundamentally non-local
- respecting the principle of relative locality
- reconciling discreetness with relativity
- Appearing in the full expression of String theory

We haven't discuss how it leads to a

- generalisation of the concept of causality
- a generalisation of the concepts of fields
- or how to curve it

'It is only the beginning

- What are the new invisibles? No quantitive prediction, but qualitatively relative locality $\longrightarrow$ fundamental UV-IR mixing, no separation of scale. It goes beyond effective field theory.