Title: Analysis of the entropy vector approach to distinguish classical and quantum causal structures

Date: Sep 20, 2016 03:30 PM

URL: http://pirsa.org/16090057

Abstract: <p>Bell's theorem shows that our intuitive understanding of causation must be overturned in light of quantum correlations. Nevertheless, quantum mechanics does not permit signalling and hence a notion of cause remains. Understanding this notion is not only important at a fundamental level, but also for technological applications such as key distribution and randomness expansion. It has recently been shown that a useful way to determine which classical causal structures give rise to a given set of correlations is to use entropy vectors. We consider the question of whether such vectors can lead to useful certificates of non-classicality. We find that for a family of causal structures that include the usual bipartite Bell structure they do not, in spite of the existence of non-classical correlations. Furthermore, we find that for many causal structures non-Shannon entropic inequalities give additional constraints on the sets of possible entropy vectors in the classical case. They hence lead to tighter approximations of the set of realisable entropy vectors, which enables a sharper distinction of different causal structures. Whether these improved characterisations are also valid for the quantum case remains an open problem whose resolution would have implications for the discrimination of classical and quantum causes.</p>

Analysis of the entropy vector approach to distinguish classical and quantum causal structures

Mirjam Weilenmann

joint work with Roger Colbeck

University of York

20th September 2016

Based on arXiv:1603.02553 and arXiv:1605.02078.

Causal Structures - Classical and Quantum Cause

- Correlations between two space-like separated parties.

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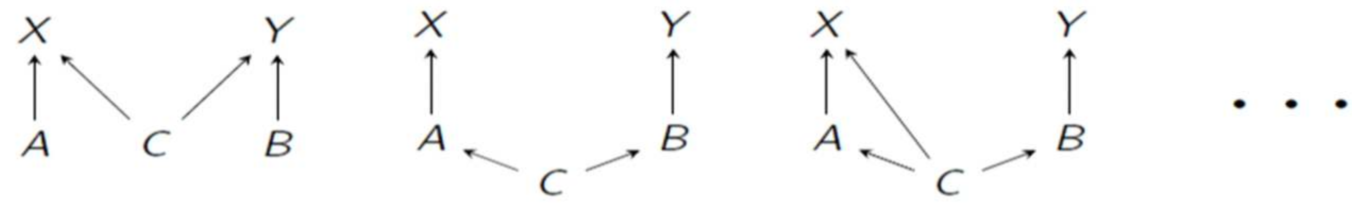

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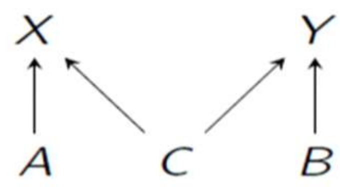

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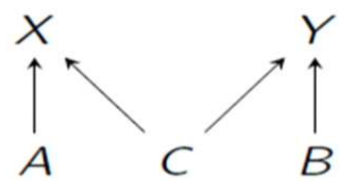

- Reconsider in light of Bell's theorem
  - free choice of settings
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  - "classical" notion of cause
- Which assumption to reject in the quantum case?

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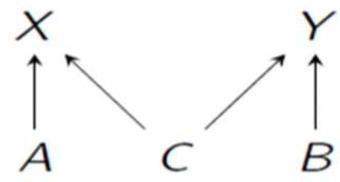

- Reconsider in light of Bell's theorem
  - free choice of settings
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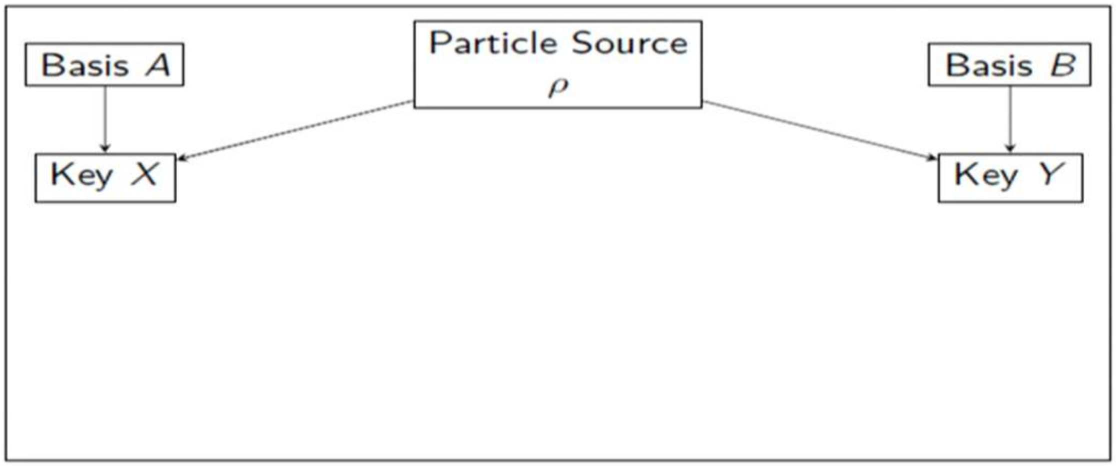
- Which assumption to reject in the quantum case?
  $\longrightarrow$ Recent result: Explanations by means of classical causal models require fine-tuning.

C. J. Wood & R. W. Spekkens, New J. Phys. 17, 2015.

Causal Structures - Classical and Quantum Cause

- Certificates of non-classicality in device-independent cryptography.
- Example: Key distribution protocol



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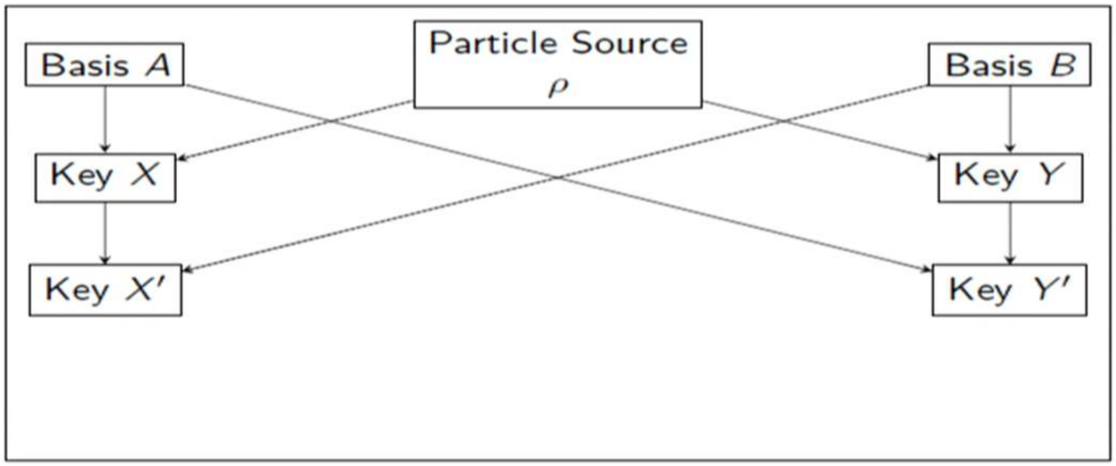

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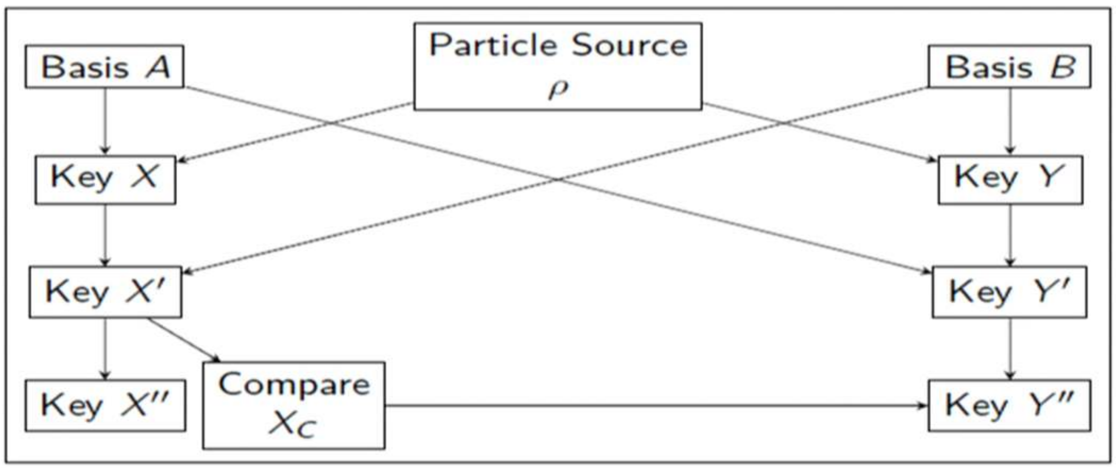

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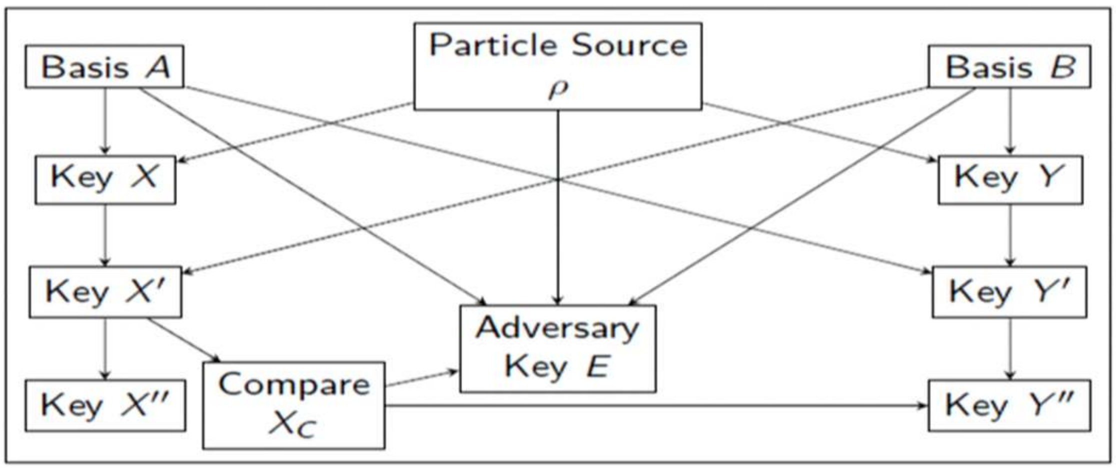

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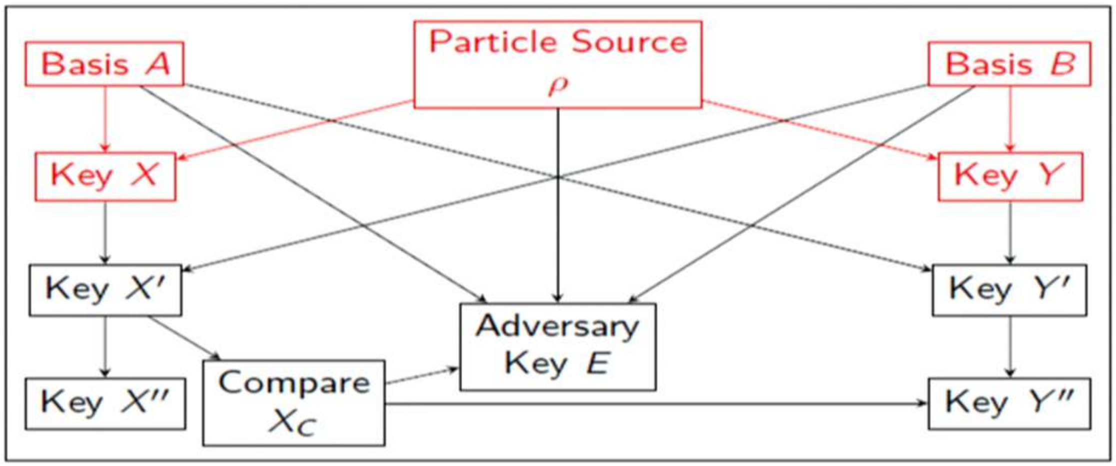

- Bell inequality violation involved: No classical causal explanation for correlations.

Causal Structures - Classical and Quantum Networks

Definition

A Causal Structure C is a set of nodes arranged in a directed acyclic graph, a subset of which is observed.

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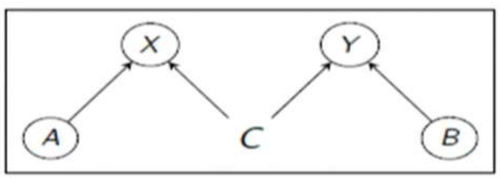
Classical causal structures C^{C}:

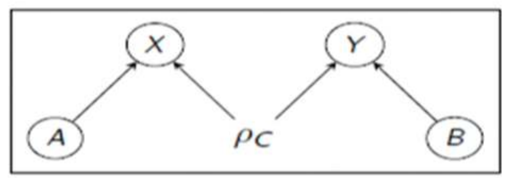
- Nodes $\longleftrightarrow$ random variables.



$$P_{ABXY} = \sum_{C} P_{X|AC} P_{Y|BC} P_A P_B P_C$$

Quantum causal structures C^{Q}:

- Observed $\longleftrightarrow$ random variables.
- Unobserved $\longleftrightarrow$ quantum states.



$$P_{ABXY} = \mathrm{tr}((E_A^X \otimes F_B^Y)\rho_C) P_A P_B$$

Classical Causal Structures - Bayesian Networks

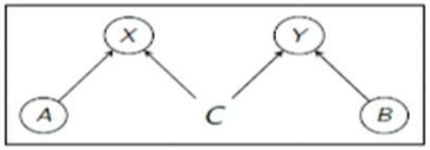
Compatibility of distribution of $\{X_1, X_2, \ldots, X_n\}$ with C^{C}:

$$P_{X_1 \ldots X_n} = \prod_i P_{X_i | \,\mathrm{pa}(X_i)}.$$

Theorem (Pearl)

A probability distribution is compatible with C^{C} iff every variable X_i is independent of its non-descendants conditioned on its parents.

Example:



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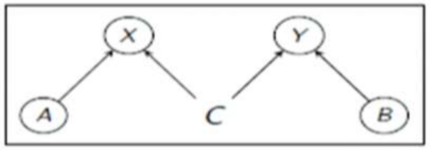

$P_{ABCXY} = P_{X|AC} P_{Y|BC} P_A P_B P_C$

- $P_{C|AB} = P_C$,
- $P_{A|BCY} = P_A$, $P_{B|ACX} = P_B$
- $P_{X|BYAC} = P_{X|AC}$, $P_{Y|AXBC} = P_{Y|BC}$.

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Entropy Vector Approach

$$H(X_S) = -\sum P_{X_S}(x_s) \log_2(P_{X_S}(x_s))$$

- Conditional independences in C^{C} as linear entropy equalities

$$I(X_i : \mathrm{nd}(X_i) \mid \mathrm{pa}(X_i)) = 0.$$

- Convex cone $\overline{\Gamma}_n^*(C^{\mathrm{C}})$ of entropy vectors compatible with C^{C}.

R. Yeung, IEEE Trans. Inf. Th., 43, 1997.
R. Chaves & T. Fritz, Phys. Rev. A, 85, 2012.

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Question

Given an arbitrary entropy vector, does there exist a corresponding probability distribution?

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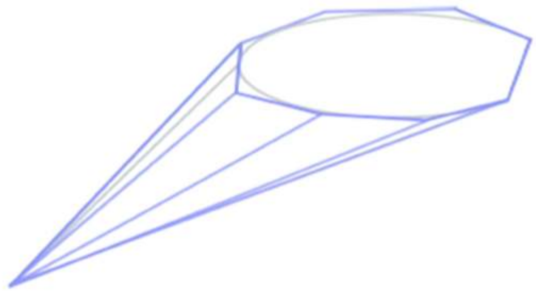
Entropy Vector Approach



Necessary conditions: Shannon inequalities

- $H(X_S) \geq 0$,
- $H(X_S|X_T) \geq 0$,
- $I(X_S : X_T|X_U) \geq 0$,

for X_S, X_T, $X_U \subseteq \{X_1, X_2, \ldots, X_n\}$ disjoint.

Shannon Cone: $\Gamma_n = \left\{ v \in \mathbb{R}_{\geq 0}^{2^n-1} \mid M_{\mathrm{SH}}.v \geq 0 \right\}$.

- Infinitely many (linear) *non–Shannon* inequalities needed to characterise $\overline{\Gamma}_n^*$ for $n \geq 4$.

Reminder: Shannon's entropy measures:

- $H(X_S|X_T) = H(X_S X_T) - H(X_T)$.
- $I(X_S : X_T|X_U) = H(X_S X_U) + H(X_T X_U) - H(X_S X_T X_U) - H(X_U)$.

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Entropy Vector Approach

- Restrictions due to causal structure C^{C}:

$$I(X_i : \mathrm{nd}(X_i) | \, \mathrm{pa}(X_i)) = 0.$$

$$\Gamma^* \left(C^{\mathrm{C}} \right) = \left\{ v \in \Gamma_n^* \mid M_{\mathrm{CI}} \left(C^{\mathrm{C}} \right) \cdot v = 0 \right\},$$
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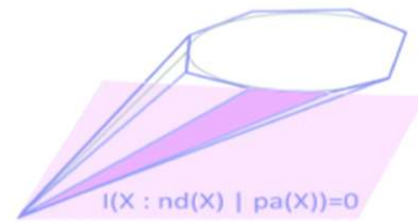

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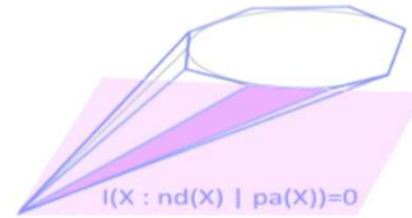
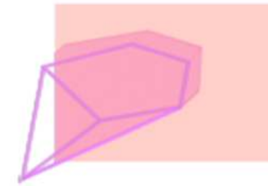


- Consider *marginal scenario* $\mathcal{M}$ of observed $\{X_1, X_2, \ldots, X_k\}$:

Projection $\pi_{\mathcal{M}} : \mathbb{R}^{2^n - 1} \longrightarrow \mathbb{R}^{2^k - 1}$.



$$\Gamma^*_{\mathcal{M}}(C^{\mathrm{C}}) = \left\{ w \in \mathbb{R}^{2^k-1}_{\geq 0} \mid \exists v \in \Gamma^*(C^{\mathrm{C}}) \text{ s.t. } w = \pi_{\mathcal{M}}(v) \right\},$$
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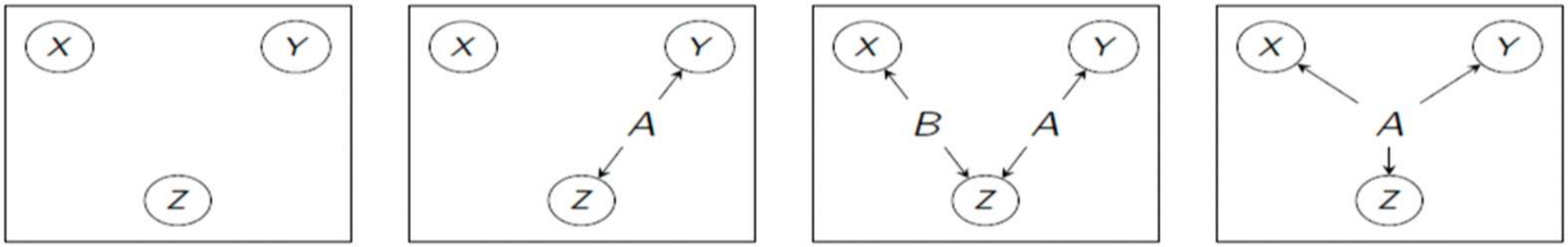

$$I(X:YZ)=0$$
$$I(Y:XZ)=0$$
$$I(Z:XY)=0$$

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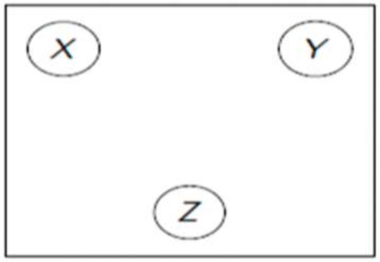

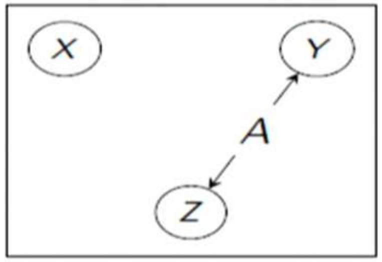

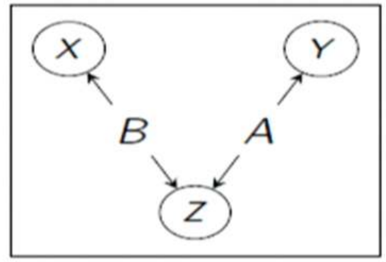

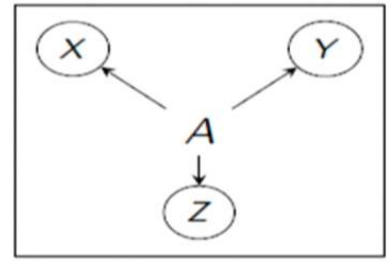

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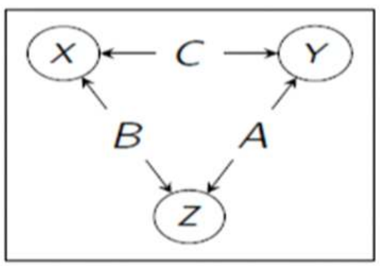

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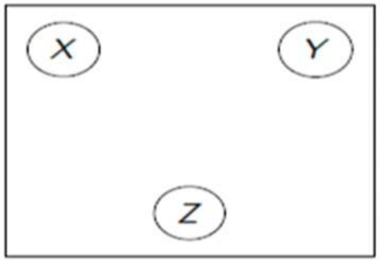

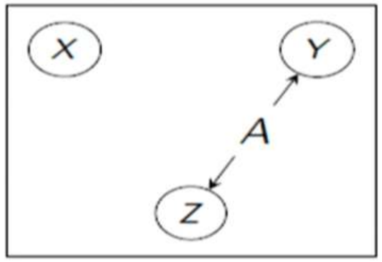

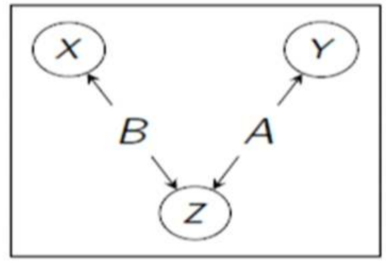

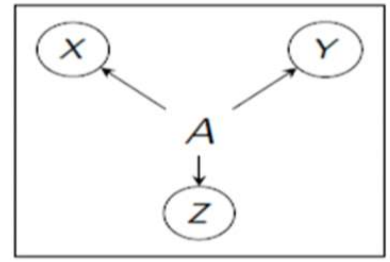

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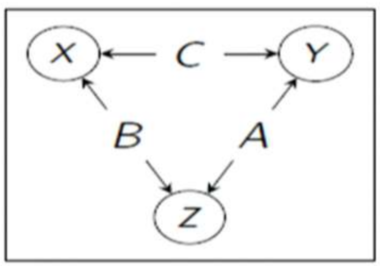

Additional inequalities, e.g.

$$H(X|Y)+H(X|Z)\geq H(X),$$

$$I(Y:Z|X)+H(Y|X)+H(Z|X)\geq 3I(Y:Z).$$

T. Fritz, New J. Phys. 14, 2014.
R. Chaves & T. Fritz, Phys. Rev. A, 85, 2012.

Entropy Vector Approach – Quantum Extension

Von Neumann entropy: $H(\rho) = -\operatorname{tr}(\rho \log \rho)$.

R. Chaves et al. Nat. Commun., 6, 2015.

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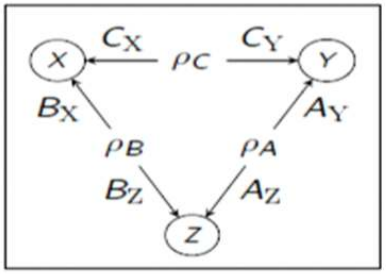
Outer approximation of marginal quantum cone:

- Inequalities derived using only observed variables from the classical case.
- Less inequalities for unobserved quantum nodes:
  - Conditional entropy may be negative.
  - No conditioning on quantum parents.
  - Data processing inequalities included instead.

$\rightarrow$ Convex cone $\Gamma_{\mathcal{M}}\left(C^{\mathrm{Q}}\right)$ approximates $\Gamma^*_{\mathcal{M}}\left(C^{\mathrm{Q}}\right)$.



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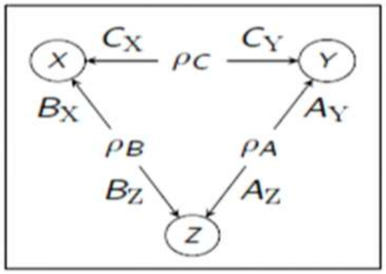

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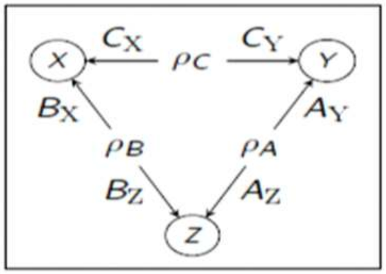

Additional inequalities, e.g.

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but not:

$$I(Y:Z|X) + H(Y|X) + H(Z|X) \geq 3I(Y:Z).$$

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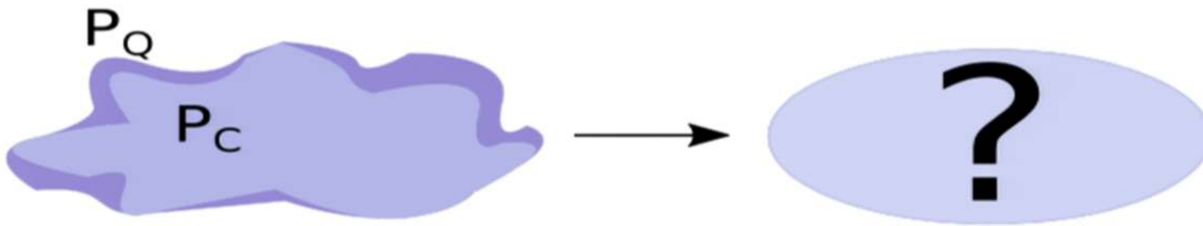
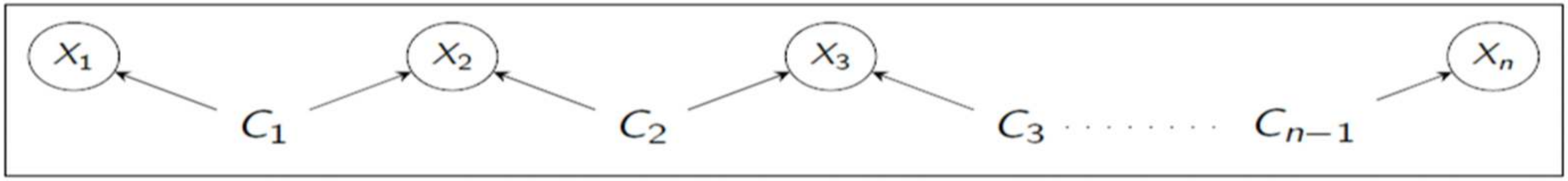
Entropy Vector Approach – Classical versus Quantum



Question

Can a quantum causal structure C^Q be distinguished from the classical C^C using the entropy vector method?

Line-like Causal Structures



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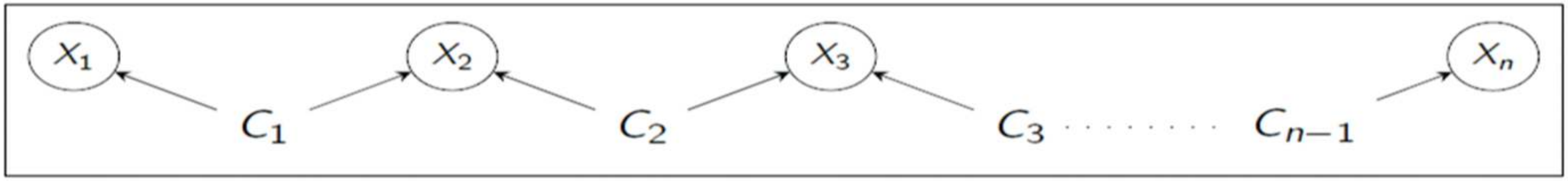


Theorem

For line-like causal structures, P_n, the classical and quantum entropic cones coincide, i.e., for any $n \in \mathbb{N}$,

$$\overline{\Gamma}^*_{\mathcal{M}}(P_n^{\mathrm{C}}) = \overline{\Gamma}^*_{\mathcal{M}}(P_n^{\mathrm{Q}}).$$

Non–Shannon Inequalities

- Complete characterisation of $\overline{\Gamma}_n^*$ for $n \geq 4$ unknown.

Non–Shannon Inequalities

- Complete characterisation of $\overline{\Gamma}_n^*$ for $n \geq 4$ unknown.
- Infinitely many independent non–Shannon inequalities known.
- First example:

Proposition (Zhang & Yeung)
For four discrete random variables T, U, V, and W,

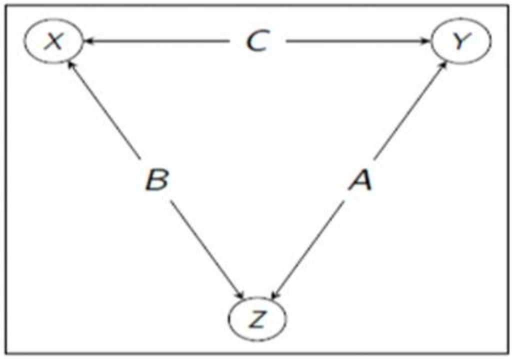
$$-H(T) - H(U) - \frac{1}{2}H(V) + \frac{3}{2}H(TU) + \frac{3}{2}H(TV) + \frac{1}{2}H(TW) + \frac{3}{2}H(UV) + \frac{1}{2}H(UW) - \frac{1}{2}H(VW) - 2H(TUV) - \frac{1}{2}H(TUW) \geq 0.$$

Z. Zhang & R. Yeung, IEEE Trans. Inf. Th., 44, 1998.

Example: Triangle Causal Structure

Necessary conditions for entropy vectors $\mathbf{H} \in \Gamma^*_{\mathcal{M}}(C_3^{\mathrm{C}})$:

- 6-variable Shannon constraints.
- Independences M_{CI} :
  - $I(A : BCX) = 0$,
  - $I(B : ACY) = 0$,
  - $I(C : ABZ) = 0$,
  - $I(X : AYZ|BC) = 0$,
  - $I(Y : BXZ|AC) = 0$,
  - $I(Z : CXY|AB) = 0$.



$$\Gamma(C_3^{\mathrm{C}}) = \left\{ v \in \Gamma_6 \mid M_{\mathrm{CI}}\left(C_3^{\mathrm{C}}\right) \cdot v = 0 \right\}.$$

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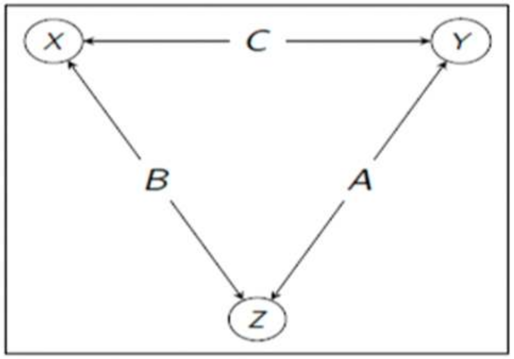


$$\Gamma(C_3^{\mathrm{C}}) = \left\{ v \in \Gamma_6 \mid M_{\mathrm{CI}}\left(C_3^{\mathrm{C}}\right) \cdot v = 0 \right\}.$$

Projection to marginal scenario $\mathcal{M}$ of observed $\{X, Y, Z\}$:

$$\Gamma_{\mathcal{M}}(C_3^{\mathrm{C}}) = \left\{ w \in \Gamma_3 \mid M_{\mathcal{M}}(C_3^{\mathrm{C}}) \cdot w \geq 0 \right\}.$$

R. Chaves & T. Fritz, Phys. Rev. A, 85, 2012.

Example: Triangle Causal Structure

Constraints on $\Gamma_{\mathcal{M}}(C_3^{\mathrm{C}}) = \left\{ w \in \Gamma_3 \mid M_{\mathcal{M}}(C_3^{\mathrm{C}}) \cdot w \geq 0 \right\}$:

- Shannon inequalities Γ_3.
- Additional inequalities (and permutations) $M_{\mathcal{M}}(C_3^{\mathrm{C}})$:

$-H(X) - H(Y) - H(Z) + H(XY) + H(XZ) \geq 0,$

$-3H(X) - 3H(Y) - 3H(Z) + 2H(XY) + 2H(XZ) + 3H(YZ) - H(XYZ) \geq 0,$

$-5H(X) - 5H(Y) - 5H(Z) + 4H(XY) + 4H(XZ) + 4H(YZ) - 2H(XYZ) \geq 0.$

Question

For the triangle causal structure, is the Shannon approximation to the entropic cone tight, i.e., is

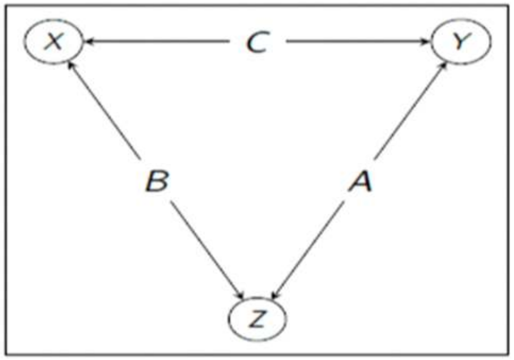
$$\overline{\Gamma}^*_{\mathcal{M}}(C_3^{\mathrm{C}}) = \Gamma_{\mathcal{M}}(C_3^{\mathrm{C}}) \ ?$$

R. Chaves & T. Fritz, Phys. Rev. A, 85, 2012.

Example: Triangle Causal Structure

Necessary conditions for entropy vectors $\mathbf{H} \in \Gamma^*_{\mathcal{M}}(C_3^{\mathrm{C}})$:

- 6-variable Shannon constraints.
- Independences M_{CI} :
  - $I(A : BCX) = 0$,
  - $I(B : ACY) = 0$,
  - $I(C : ABZ) = 0$,
  - $I(X : AYZ|BC) = 0$,
  - $I(Y : BXZ|AC) = 0$,
  - $I(Z : CXY|AB) = 0$.



- Zhang & Yeung's inequality.

$$\Gamma'(C_3^{\mathrm{C}}) = \left\{ v \in \Gamma_6 \mid M_{\mathrm{CI}}\left(C_3^{\mathrm{C}}\right) \cdot v = 0, M_{\mathrm{ZY}}\left(C_3^{\mathrm{C}}\right) \cdot v \geq 0 \right\}.$$

Projection to marginal scenario $\mathcal{M}$ of observed $\{X, Y, Z\}$:

$$\Gamma'_{\mathcal{M}}(C_3^{\mathrm{C}}) = \left\{ w \in \Gamma_3 \mid M'_{\mathcal{M}}(C_3^{\mathrm{C}}) \cdot w \geq 0 \right\}.$$

Example: Triangle Causal Structure – New Inequalities

- Inequalities derived from Zhang & Yeung's inequality (and their permutations) approximate $\Gamma^*_{\mathcal{M}}(C_3^{\mathrm{C}})$ further:

$$-4H(X) - 4H(Y) - 4H(Z) + 3H(XY) + 3H(XZ) + 4H(YZ) - 2H(XYZ) \geq 0,$$
$$-2H(X) - 2H(Y) - 2H(Z) + 3H(XY) + 3H(XZ) + 3H(YZ) - 4H(XYZ) \geq 0,$$
$$-8H(X) - 8H(Y) - 8H(Z) + 7H(XY) + 7H(XZ) + 7H(YZ) - 5H(XYZ) \geq 0.$$

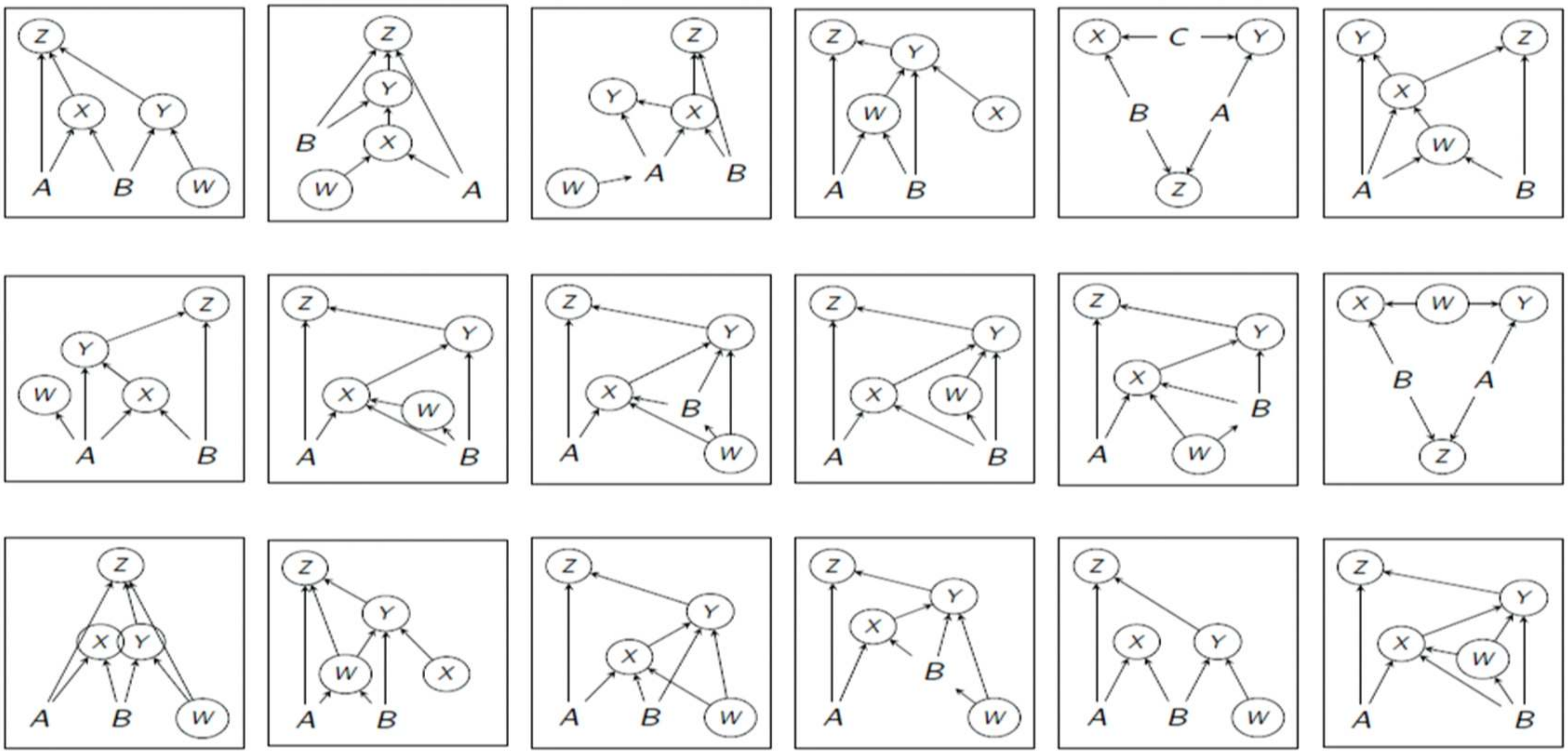
$$\overline{\Gamma}^*_{\mathcal{M}}(C_3^{\mathrm{C}}) \neq \Gamma_{\mathcal{M}}(C_3^{\mathrm{C}})$$

- Families of inequalities, for instance:

Proposition
For all $s \in \mathbb{N}$ and permutations of X, Y, and Z,

$$\left(-\frac{1}{2}s^2 - \frac{3}{2}s\right)[H(X) + H(Z)] + (-s-1)H(Y) + \left(s^2 + 2s\right)H(XZ)$$
$$+ \left(\frac{1}{2}s^2 + \frac{3}{2}s + 1\right)[H(XY) + H(YZ)] + \left(-s^2 - 2s - 1\right)H(XYZ) \geq 0.$$

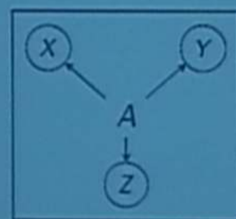
Non-Shannon Inequalities for *Interesting* Causal Structures

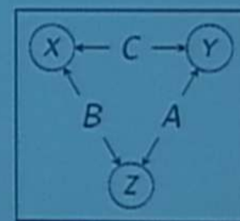


J. Henson et al., New J. Phys. 16, 2014.

Entropy Vector Approach & Non-Shannon Inequalities

- Non-Shannon inequalities tighten the entropic characterisation of causal structures.
- Allow for a better distinction of different causal structures, for instance





- Helps toward an understanding of whether there is classical quantum separation in this approach.
- No quantum violations of classical entropic constraints found in triangle scenario & provably non-classical correlations not detected.

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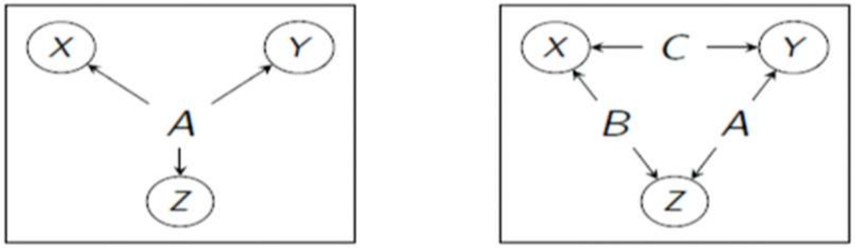

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Entropy Vector Approach

Open Problem

Is there a causal structure C, for which the classical and the quantum entropy cones differ, i.e., is

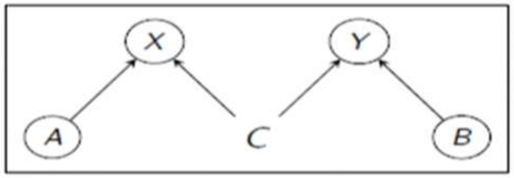
$$\overline{\Gamma}^*_{\mathcal{M}}(C^{\mathrm{C}}) \neq \overline{\Gamma}^*_{\mathcal{M}}(C^{\mathrm{Q}}) \ ?$$

Related Methods Based on Entropy

- Condition on outputs of certain variables

$$H(Y|X)_{11} + H(X|Y)_{10} + H(X|Y)_{01} - H(X|Y)_{00} \geq 0\,,$$

where $H(X|Y)_{ab}$ is the conditional entropy of the conditional distribution $P_{XY|A=a,B=b}$.



$$P_{X_0X_1Y_0Y_1} = \sum_C P_{X|A=0,C}P_{X|A=1,C}P_{Y|B=0,C}P_{Y|B=1,C}P_C\,.$$

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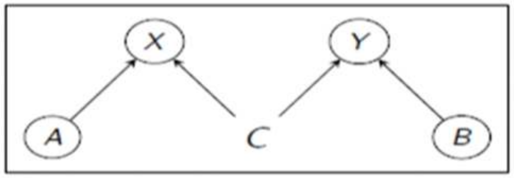

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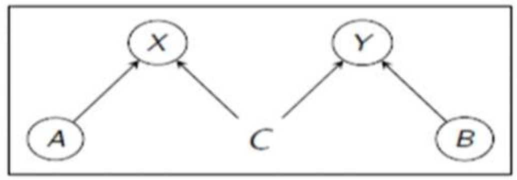

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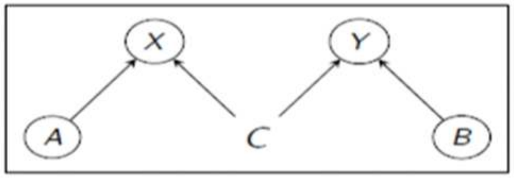

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- Technique applicable to other causal structures such as bilocality.
- For causal structures without observed inputs **not** clear how to apply this method, e.g. for the triangle causal structure.

S. Braunstein & C. Caves, Phys. Rev. Lett. 61, 1988.
R. Chaves & T. Fritz, Phys. Rev. A, 85, 2012.

Conclusions

- Entropy vector approach useful for distinguishing different causal structures.
- Important classical – quantum separation in line-like causal structures not detectable with the entropy vector approach.
- Alternative methods: e.g. consider entropies conditioned on values of outermost nodes.
- Non-Shannon inequalities tighten the entropic description of numerous causal structures & allow for them to be better distinguished.
- No quantum violations of classical entropic constraints found.
- Open problem: can the entropy vector method ever distinguish quantum from classical cause?