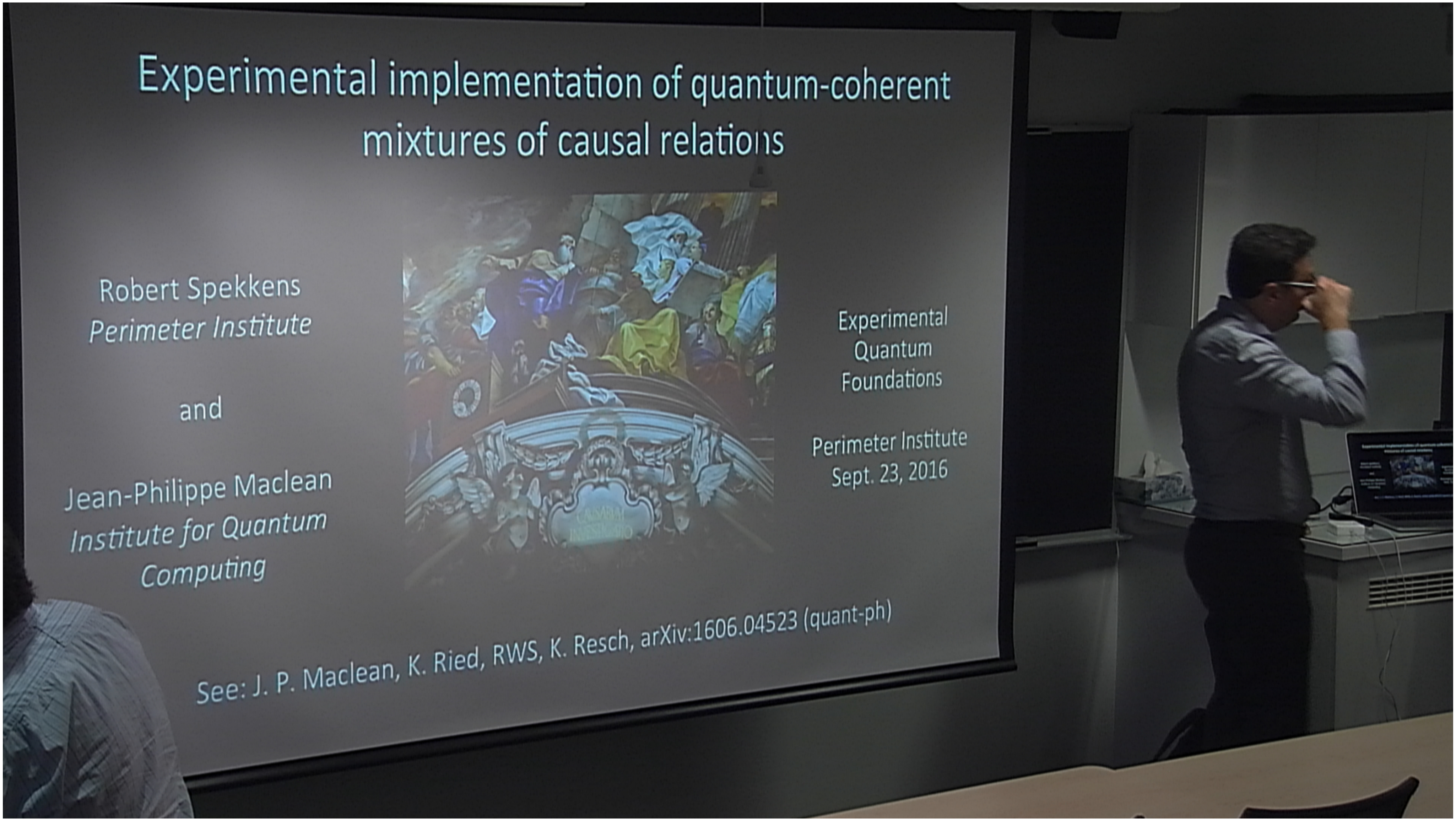
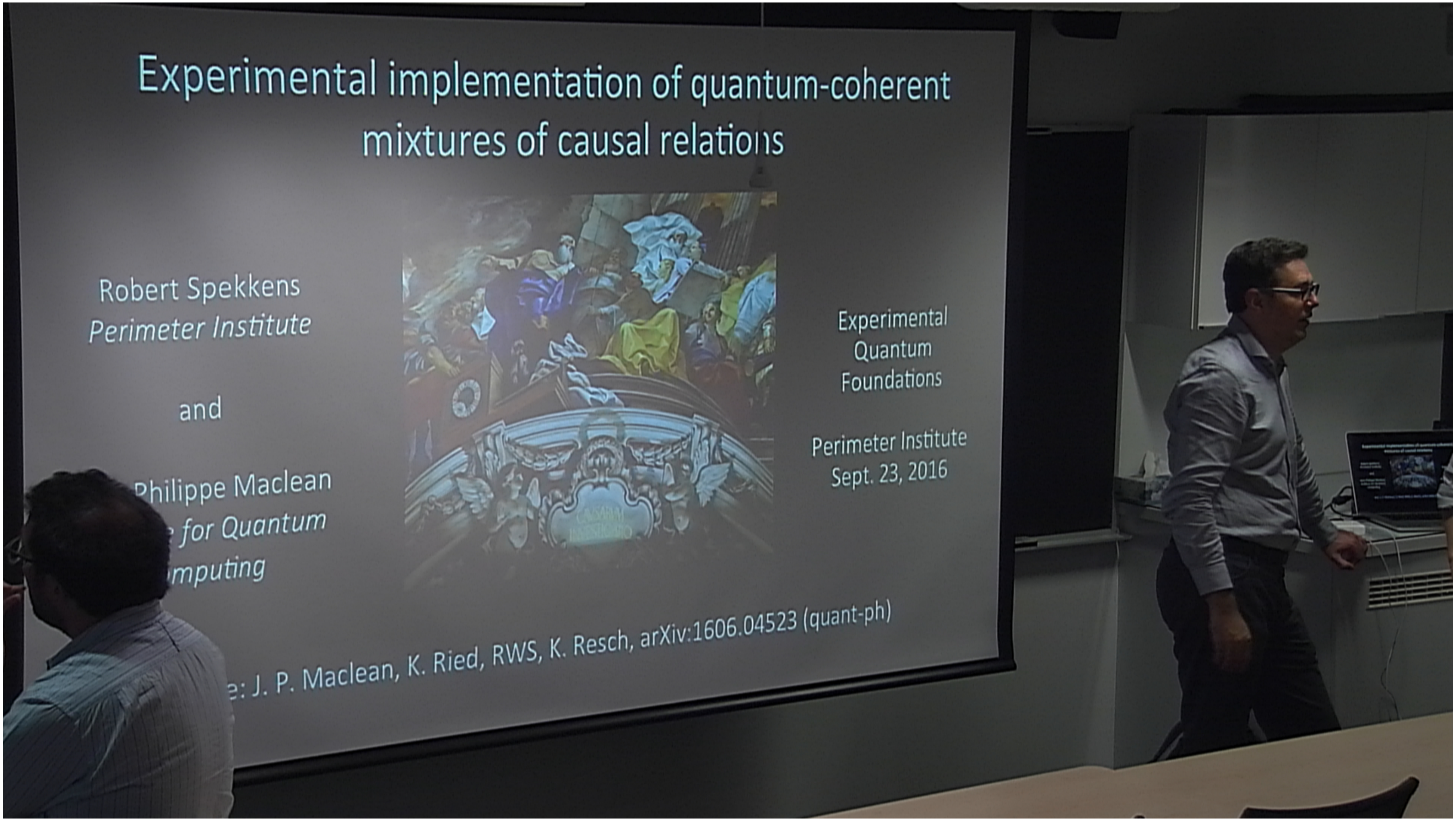
Title: Experimental implementation of quantum-coherent mixtures of causal relations

Date: Sep 23, 2016 04:00 PM

URL: http://pirsa.org/16090056

Abstract: Understanding the causal influences that hold among the parts of a system is critical both to explaining that system's natural behaviour and to controlling it through targeted interventions. In a quantum world, understanding causal relations is equally important, but the set of possibilities is far richer. The two basic ways in which a pair of time-ordered quantum systems may be causally related are by a cause-effect mechanism or by a common cause acting on both. Here, we show that it is possible to have a coherent mixture of these two possibilities. We realize such a nonclassical causal relation in a quantum optics experiment and derive a set of criteria for witnessing the coherence based on a quantum version of Berkson's paradox. (Joint work with Katja Ried and Kevin Resch)


Experimental implementation of quantum-coherent mixtures of causal relations
Robert Spekkens
Perimeter Institute
and
Jean-Philippe Maclean
Institute for Quantum Computing
Experimental Quantum Foundations
Perimeter Institute
Sept. 23, 2016
See: J. P. Maclean, K. Ried, RWS, K. Resch, arXiv:1606.04523 (quant-ph)


Experimental implementation of quantum-coherent mixtures of causal relations
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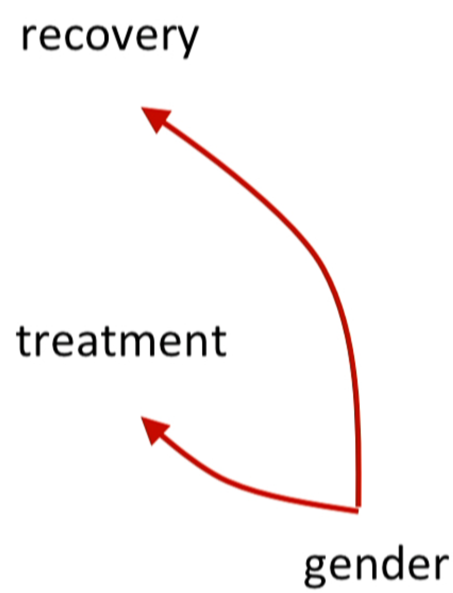
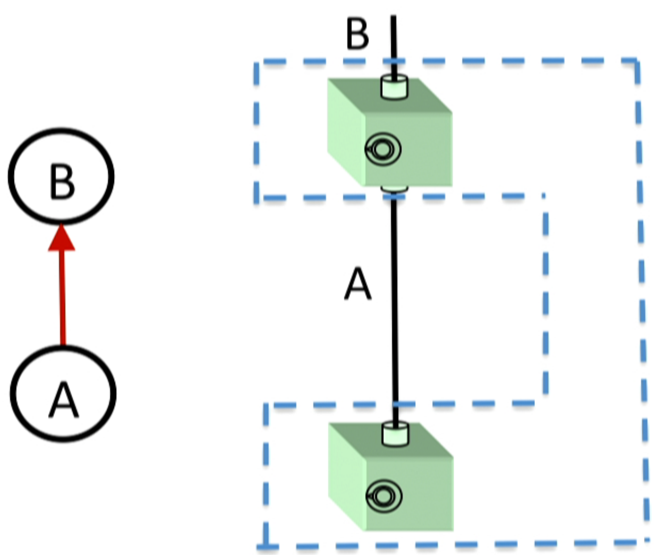
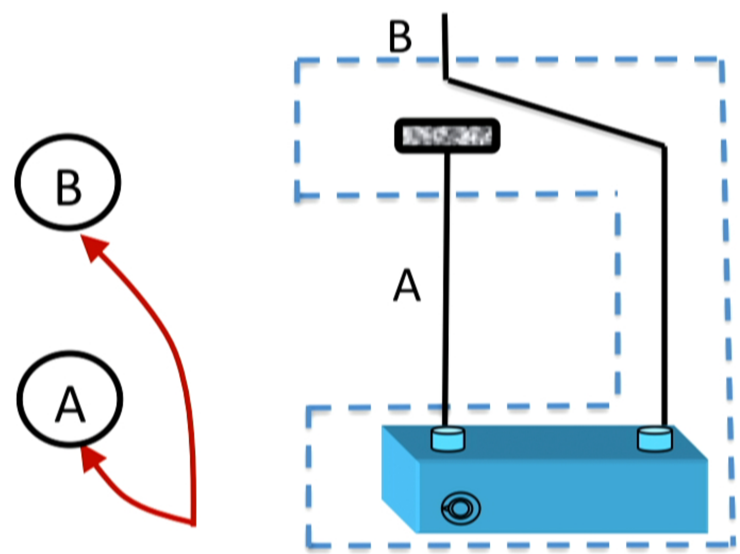
	drug	no drug
recovery	60%	48%
no recovery	40%	52%

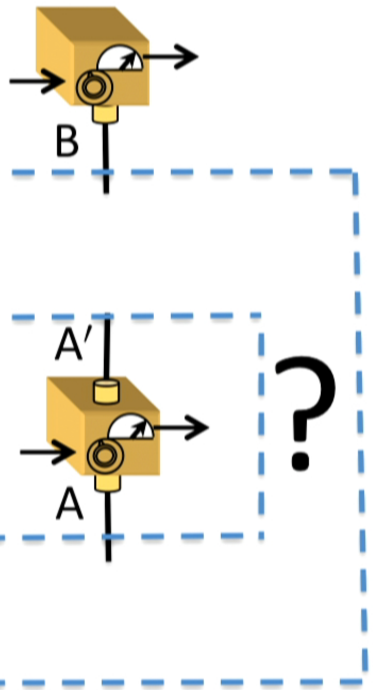


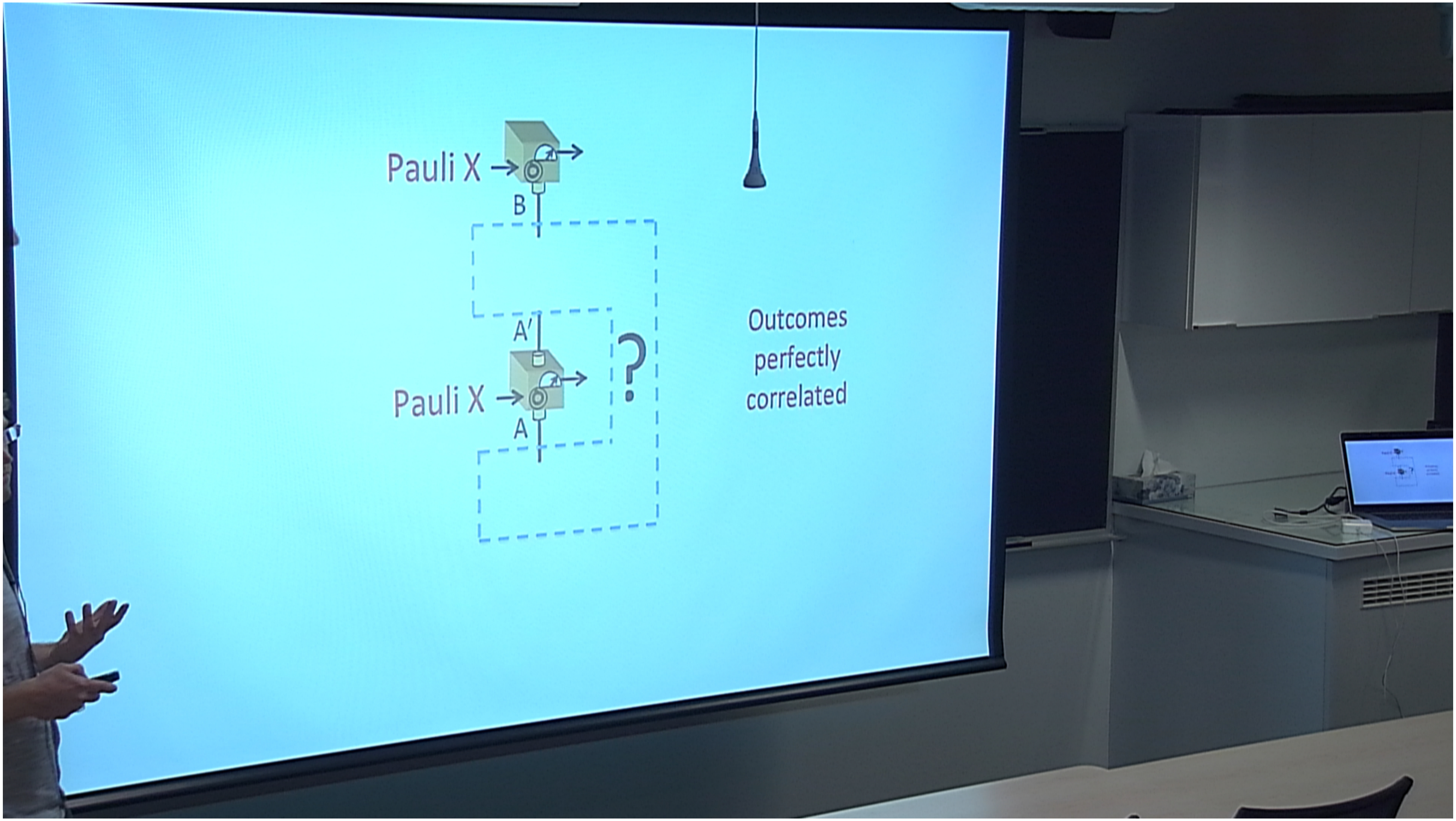
Cause-effect



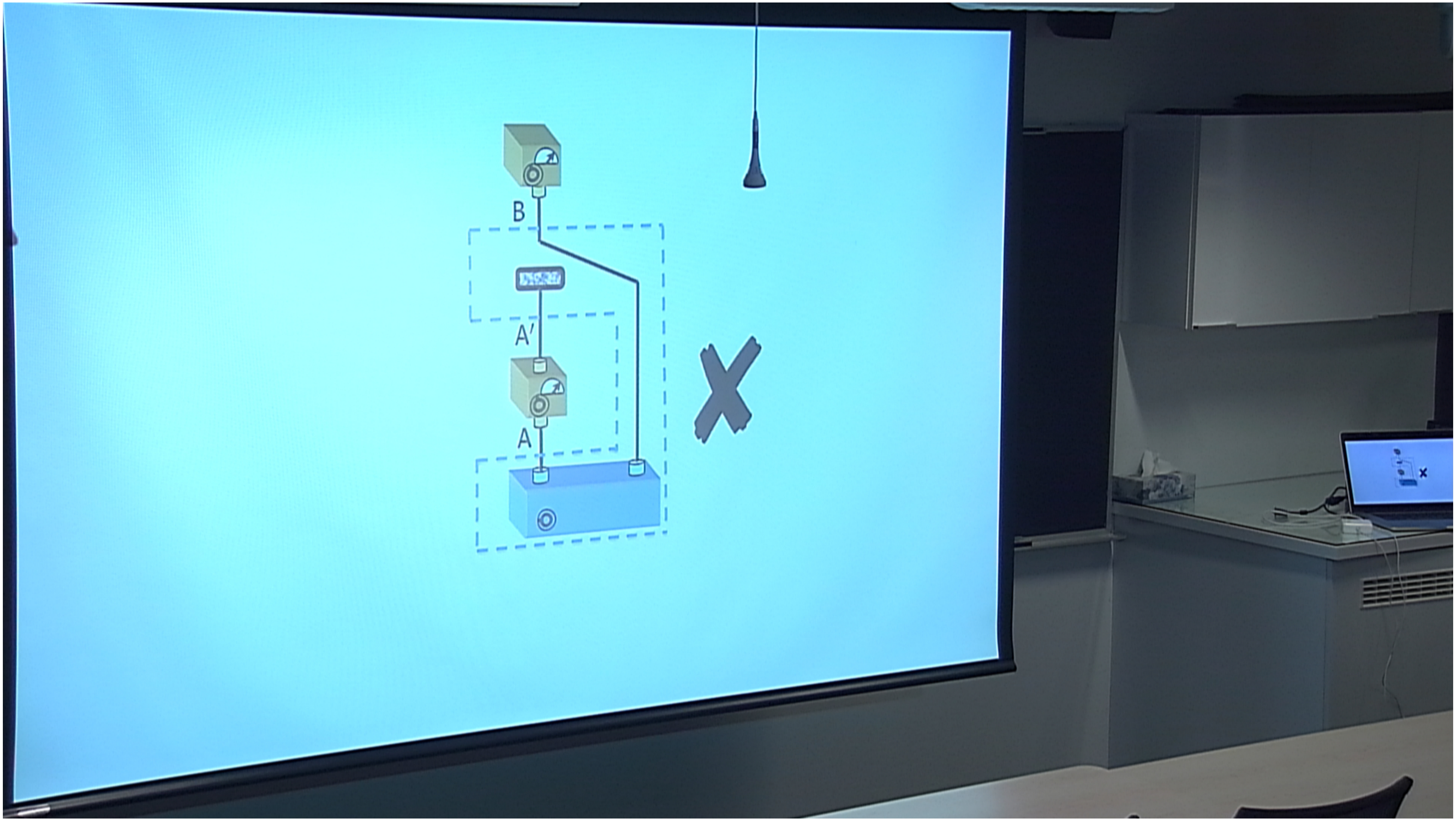
Common cause




B
A′
?
A


Pauli X
B
A′
?
Pauli X
A
Outcomes
perfectly
correlated

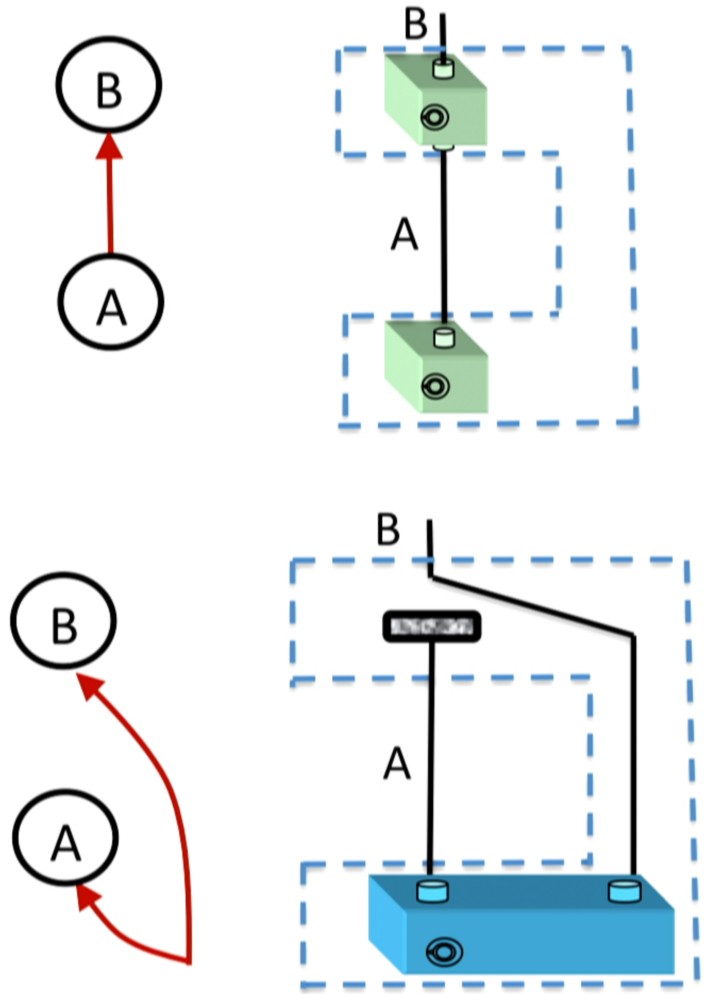

Identity map
B
A′
A

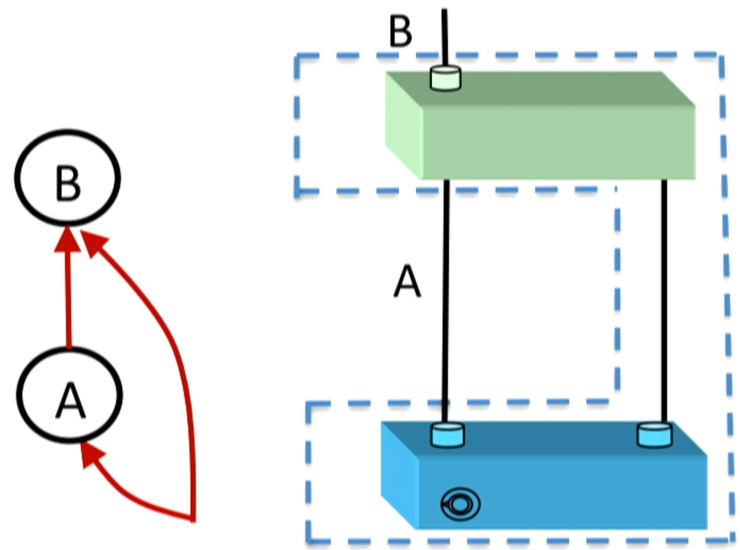

B
A′
A

A quantum-coherent mixture of causal relations

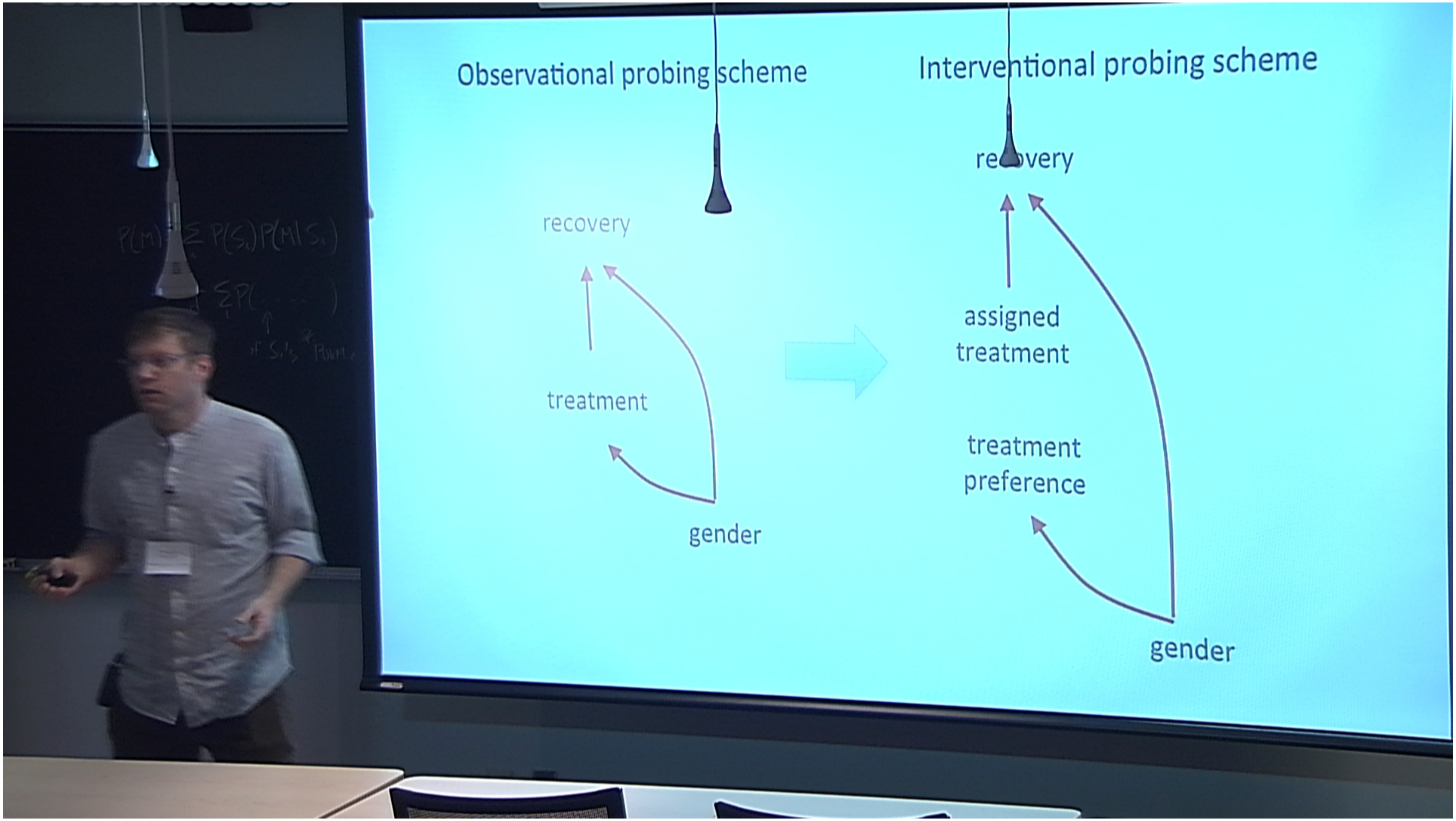
J. P. Maclean, K. Ried, RWS, K. Resch

arXiv:1606.04523 (quant-ph)

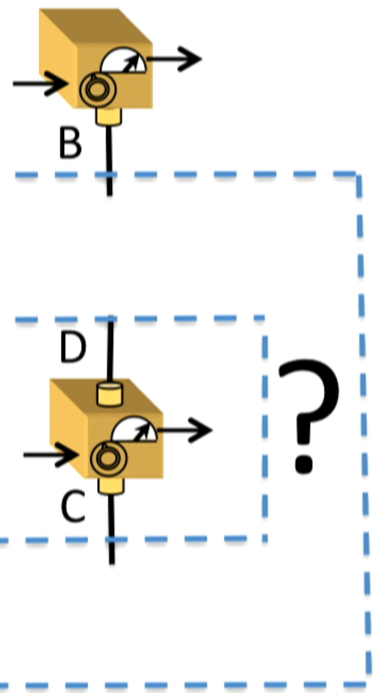




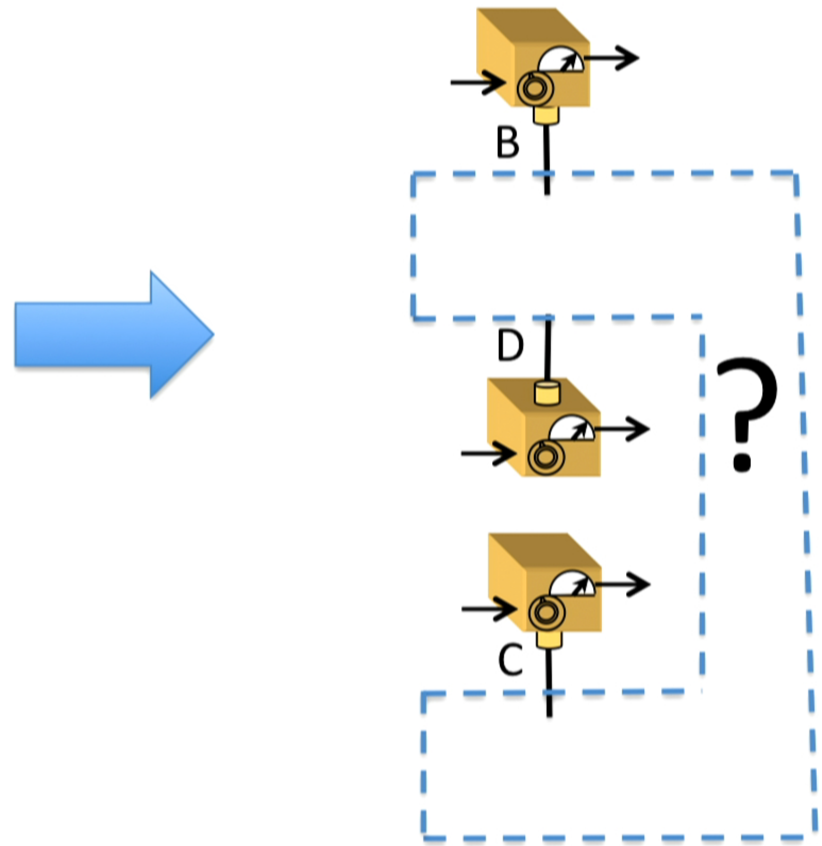
Can the two causal relations be mixed coherently?


Observational probing scheme
recovery
treatment
gender
Interventional probing scheme
assigned treatment
treatment preference
gender

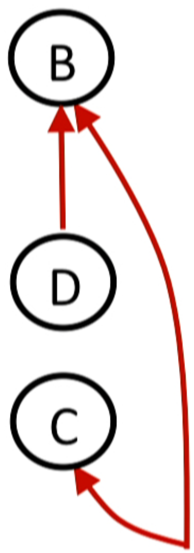
Observational probing scheme

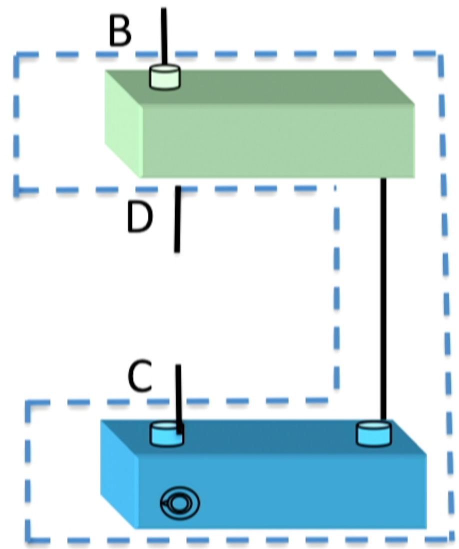


Interventional probing scheme

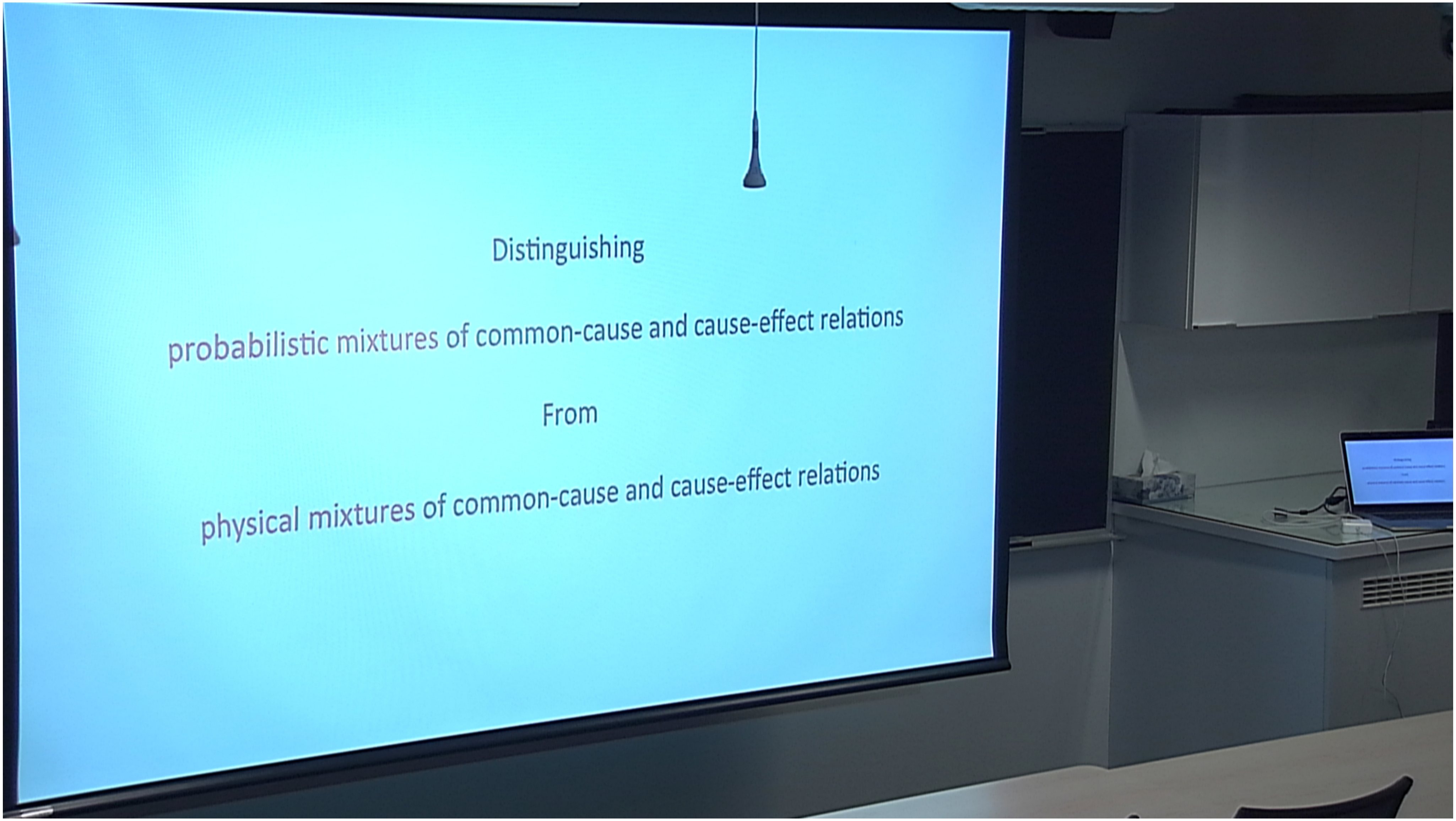


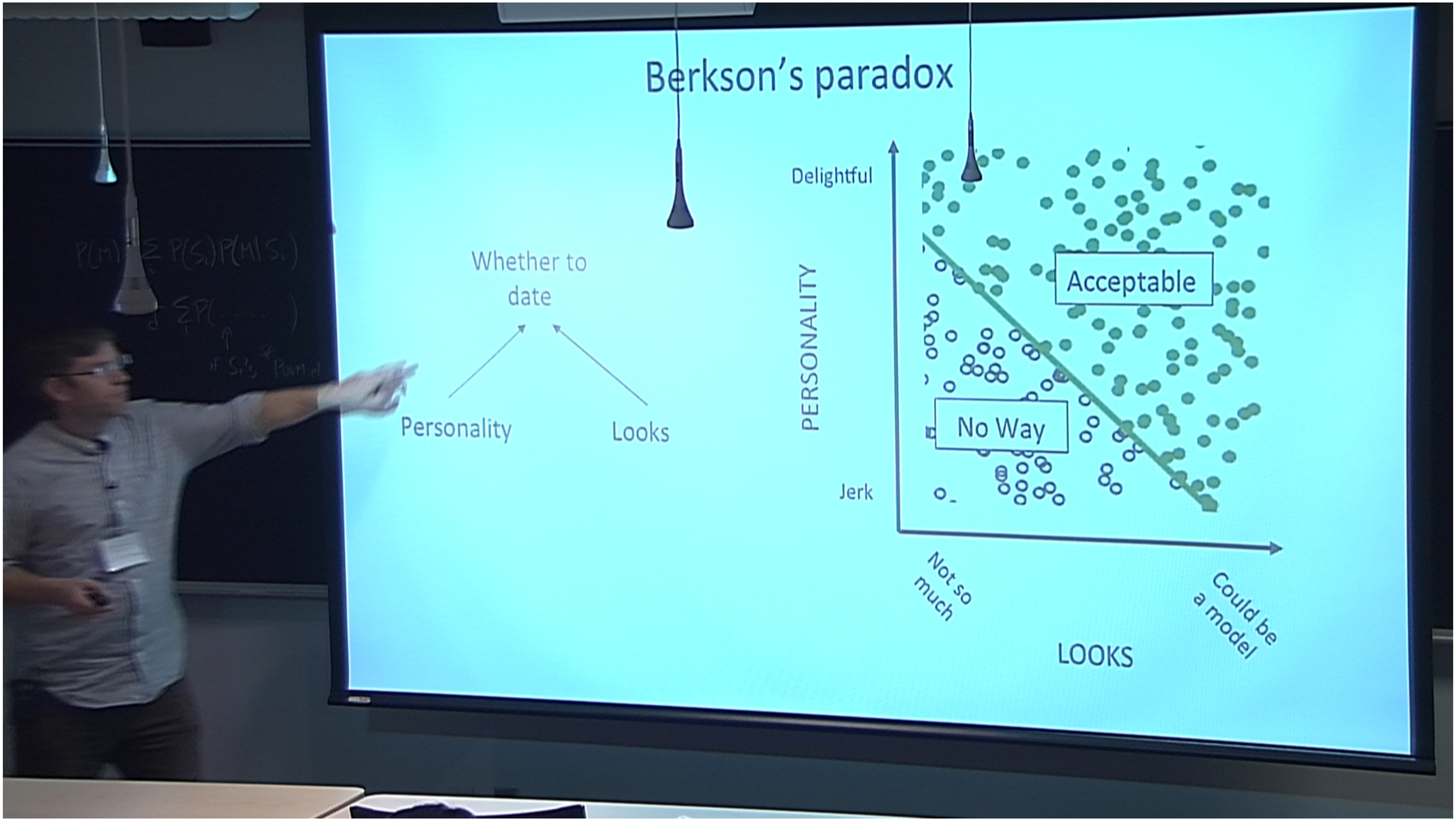
CLASSICAL



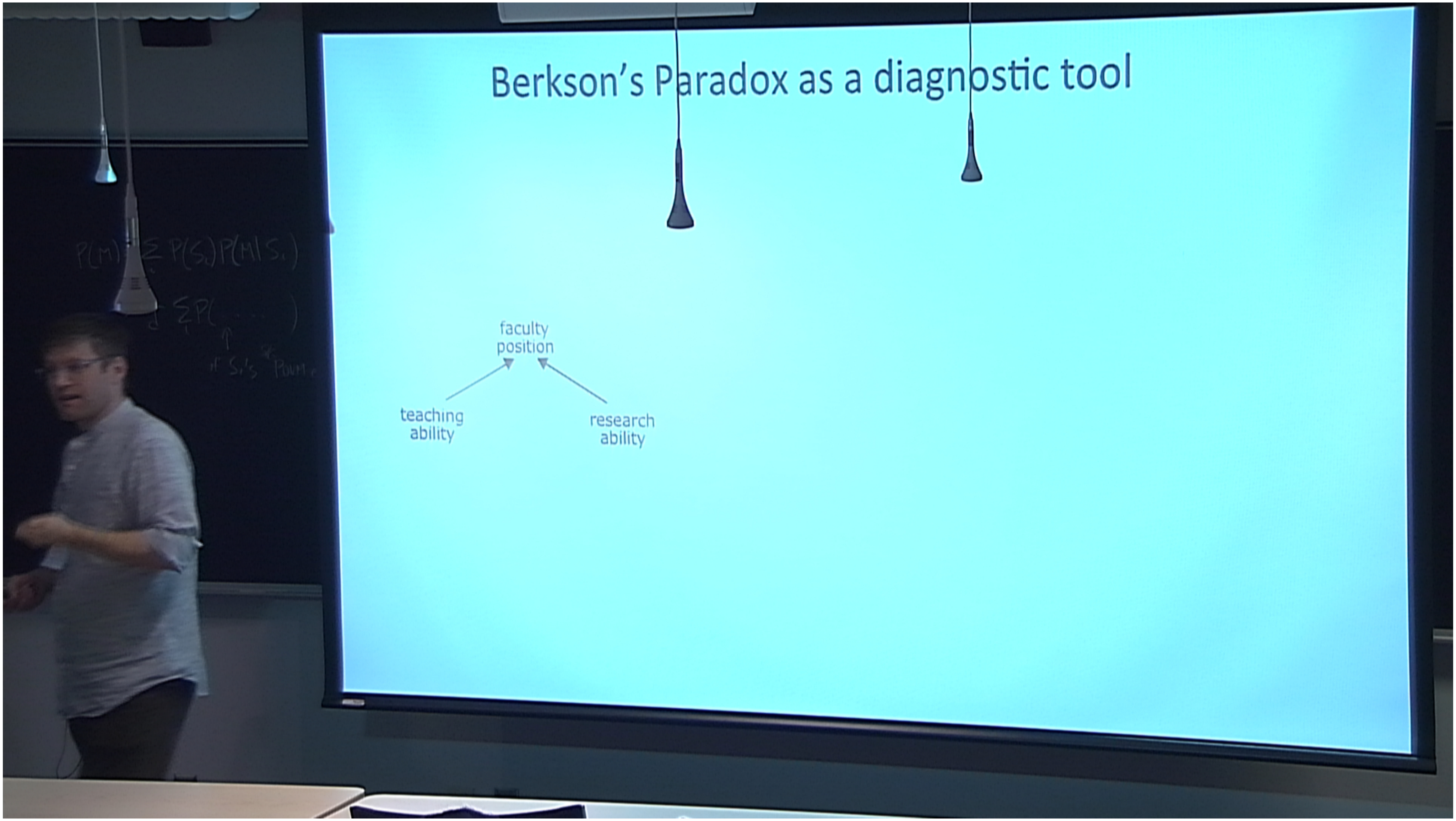


$$P(CB|D)$$

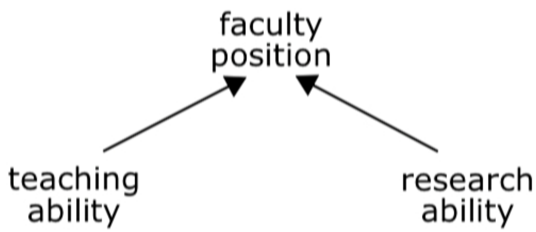

Distinguishing
probabilistic mixtures of common-cause and cause-effect relations
From
physical mixtures of common-cause and cause-effect relations

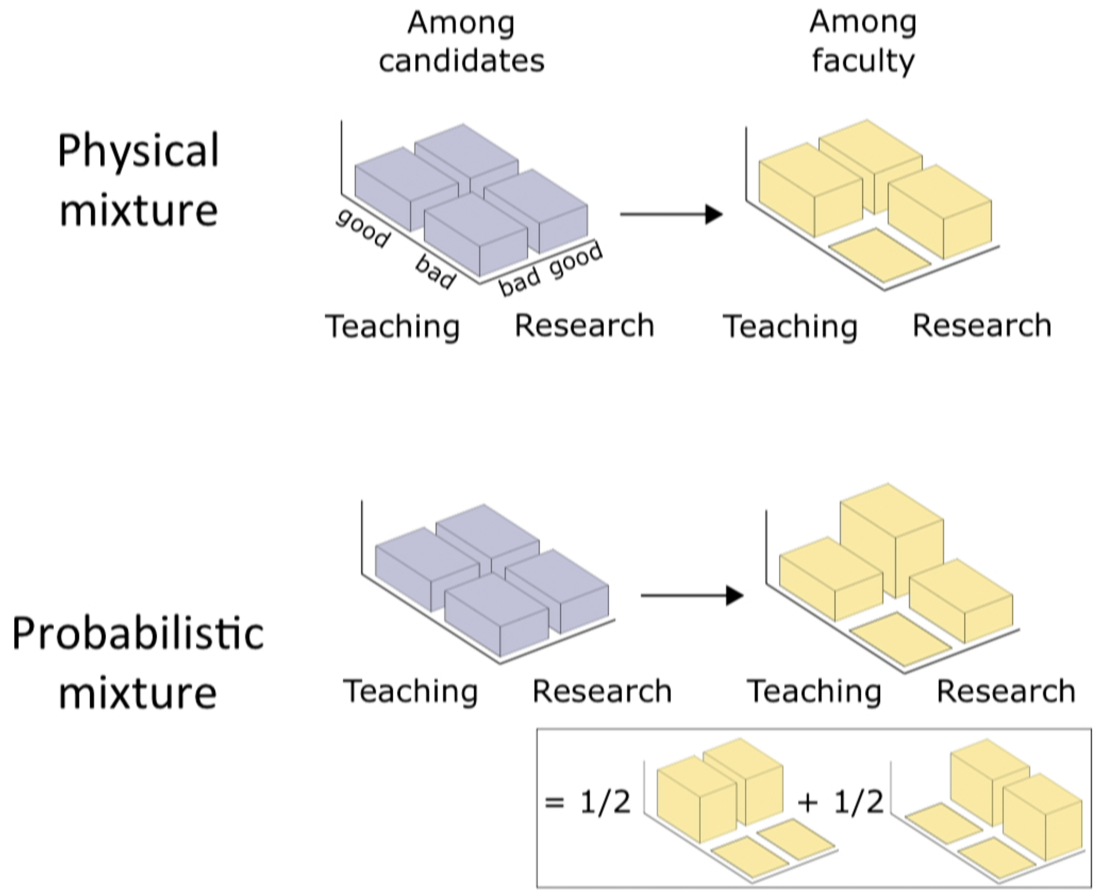


Berkson's paradox
Whether to date
Personality
Looks
Delightful
Jerk
PERSONALITY
Acceptable
No Way
Not so much
Could be a model
LOOKS

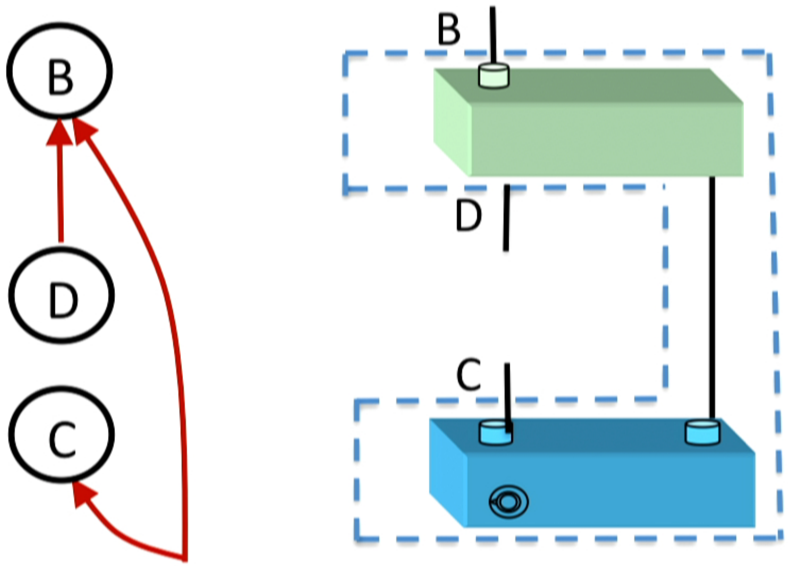

Berkson's Paradox as a diagnostic tool
faculty position
teaching ability
research ability

Berkson’s Paradox as a diagnostic tool



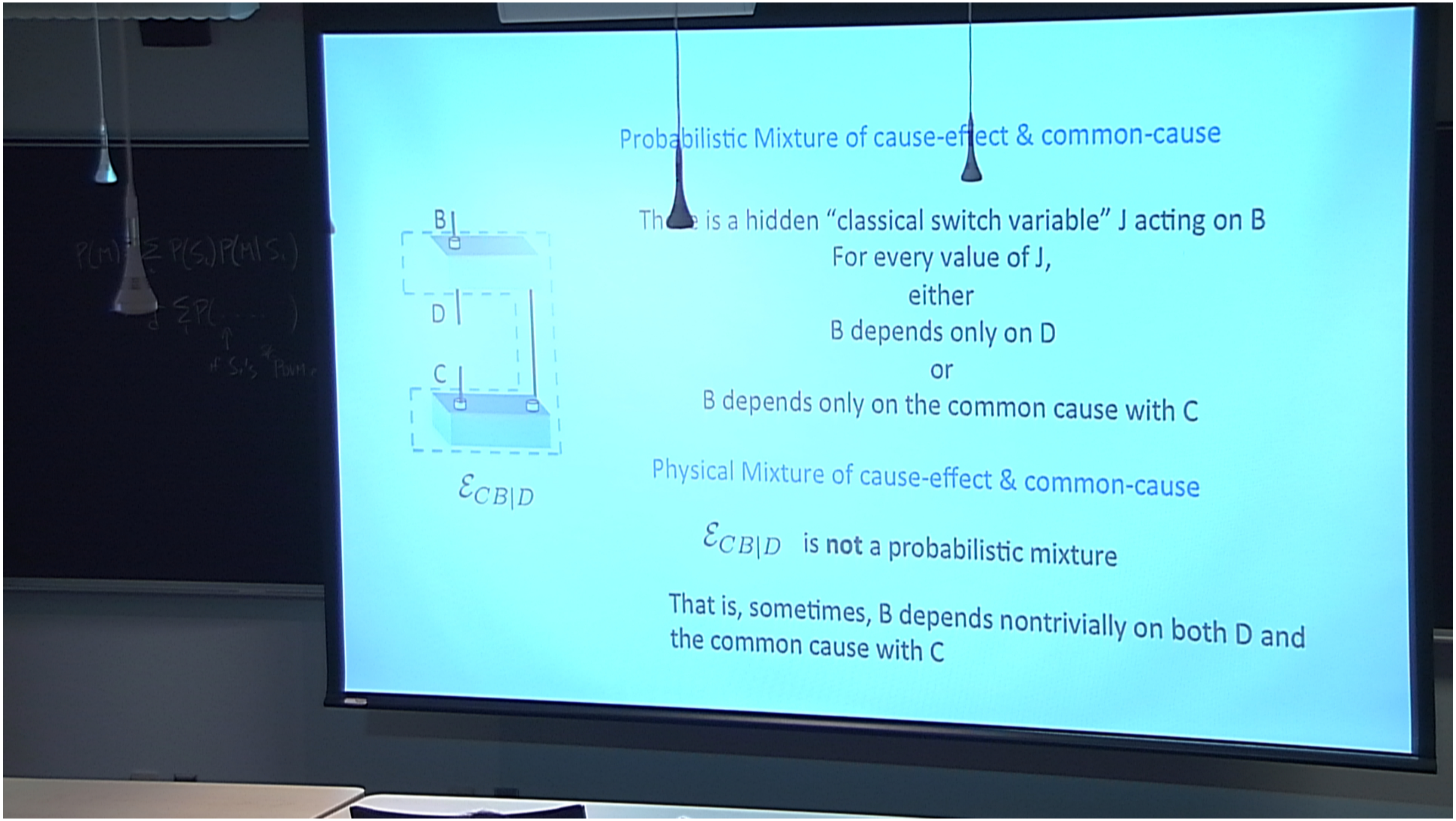


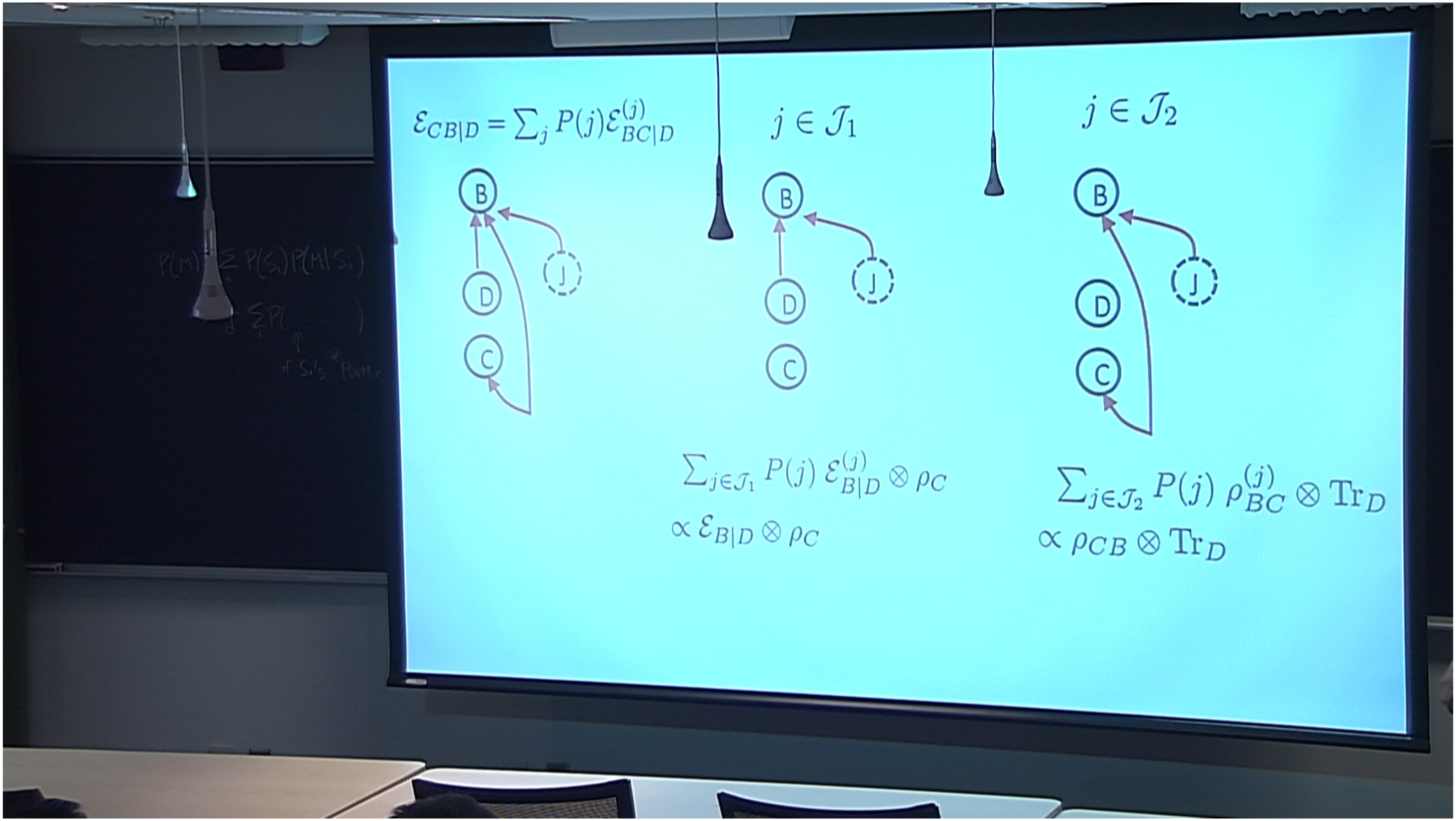
CLASSICAL



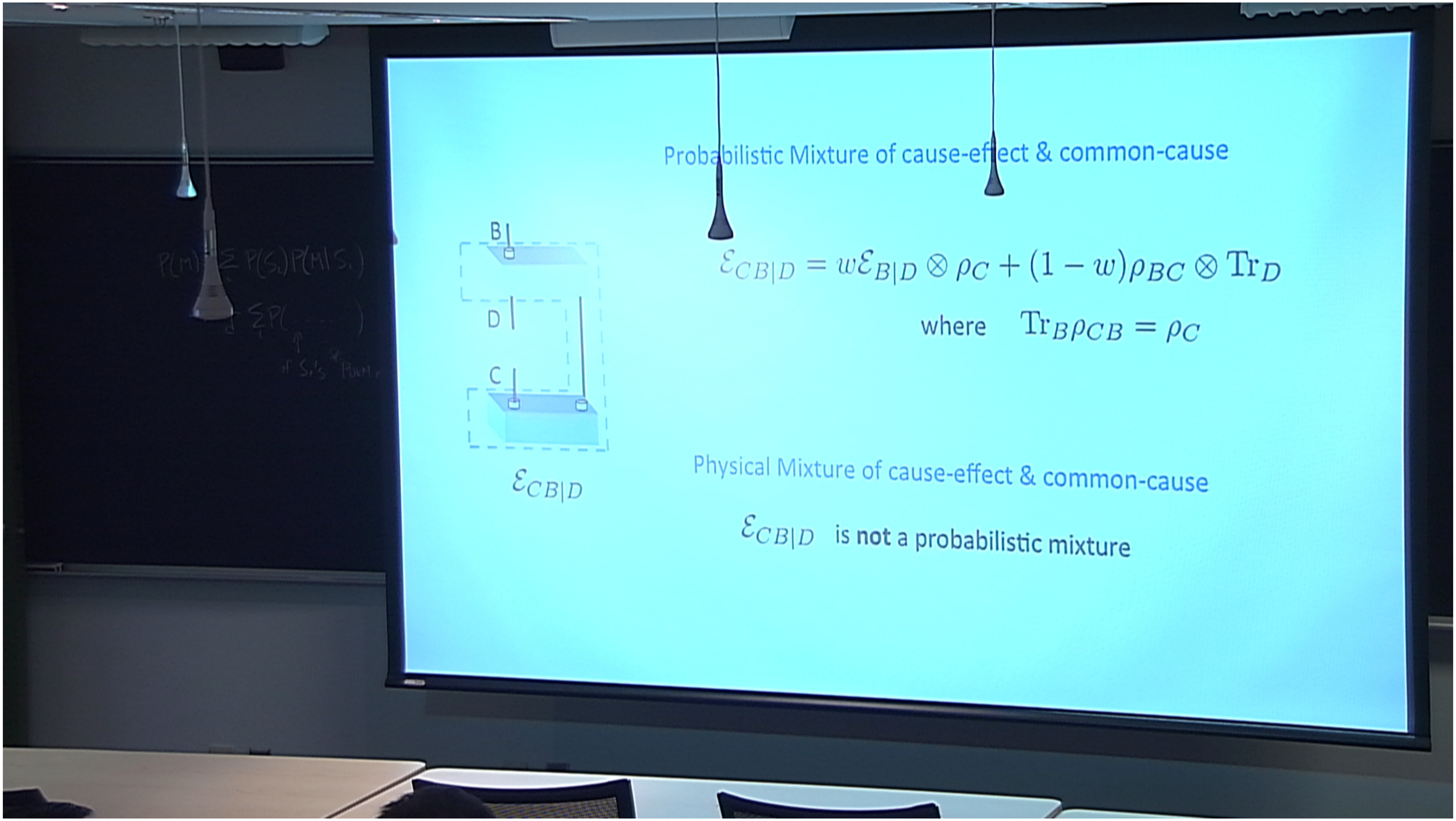
Interventionist probing → C and D are marginally independent

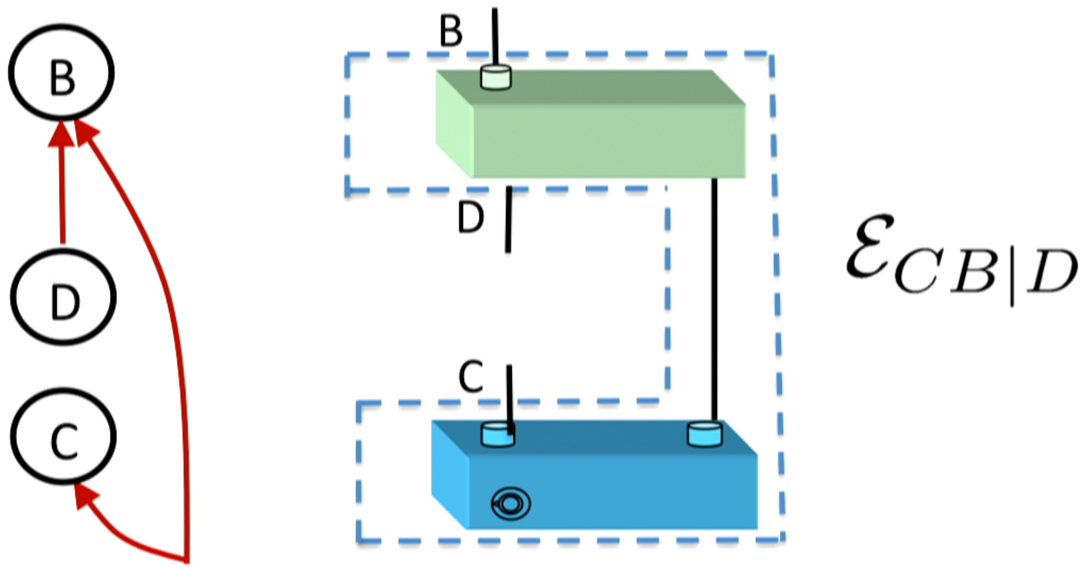
But they can *become* dependent after conditioning on B


Probabilistic Mixture of cause-effect & common-cause
B
D
C
$\mathcal{E}_{CB|D}$
Th is a hidden “classical switch variable” J acting on B
For every value of J,
either
B depends only on D
or
B depends only on the common cause with C
Physical Mixture of cause-effect & common-cause
$\mathcal{E}_{CB|D}$ is **not** a probabilistic mixture
That is, sometimes, B depends nontrivially on both D and the common cause with C

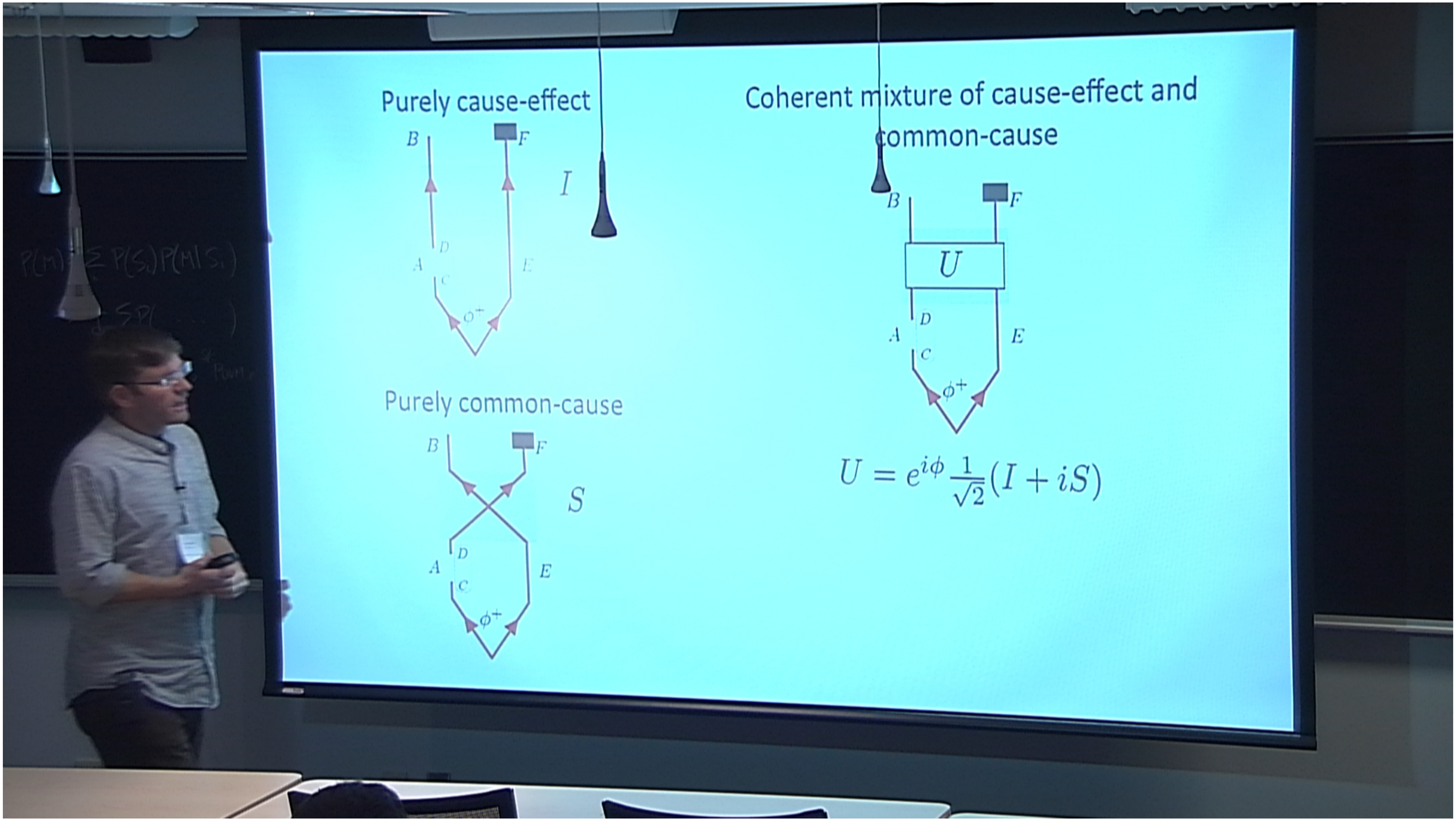


$\mathcal{E}_{CB|D} = \sum_j P(j)\mathcal{E}^{(j)}_{BC|D}$
$j \in \mathcal{J}_1$
$j \in \mathcal{J}_2$
B
J
D
C
$\sum_{j\in\mathcal{J}_1} P(j)\,\mathcal{E}^{(j)}_{B|D}\otimes\rho_C$
$\propto \mathcal{E}_{B|D}\otimes\rho_C$
$\sum_{j\in\mathcal{J}_2} P(j)\,\rho^{(j)}_{BC}\otimes \mathrm{Tr}_D$
$\propto \rho_{CB}\otimes\mathrm{Tr}_D$

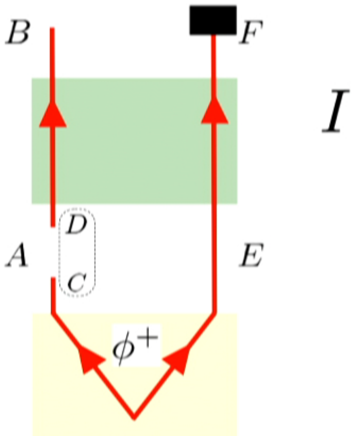

Probabilistic Mixture of cause-effect & common-cause
B
D
C
$\mathcal{E}_{CB|D} = w\mathcal{E}_{B|D} \otimes \rho_C + (1-w)\rho_{BC} \otimes \mathrm{Tr}_D$
where $\mathrm{Tr}_B \rho_{CB} = \rho_C$
$\mathcal{E}_{CB|D}$
Physical Mixture of cause-effect & common-cause
$\mathcal{E}_{CB|D}$ is **not** a probabilistic mixture



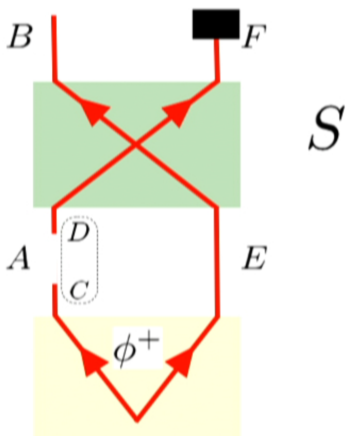
Each causal mechanism on its own should be quantum


Purely cause-effect
B
F
I
D
A
C
E
ϕ^+
Purely common-cause
B
F
S
D
A
C
E
ϕ^+
Coherent mixture of cause-effect and common-cause
B
F
U
D
A
C
E
ϕ^+
$U = e^{i\phi}\frac{1}{\sqrt{2}}(I + iS)$

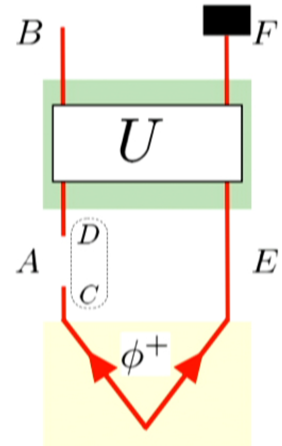
Purely cause-effect



Purely common-cause

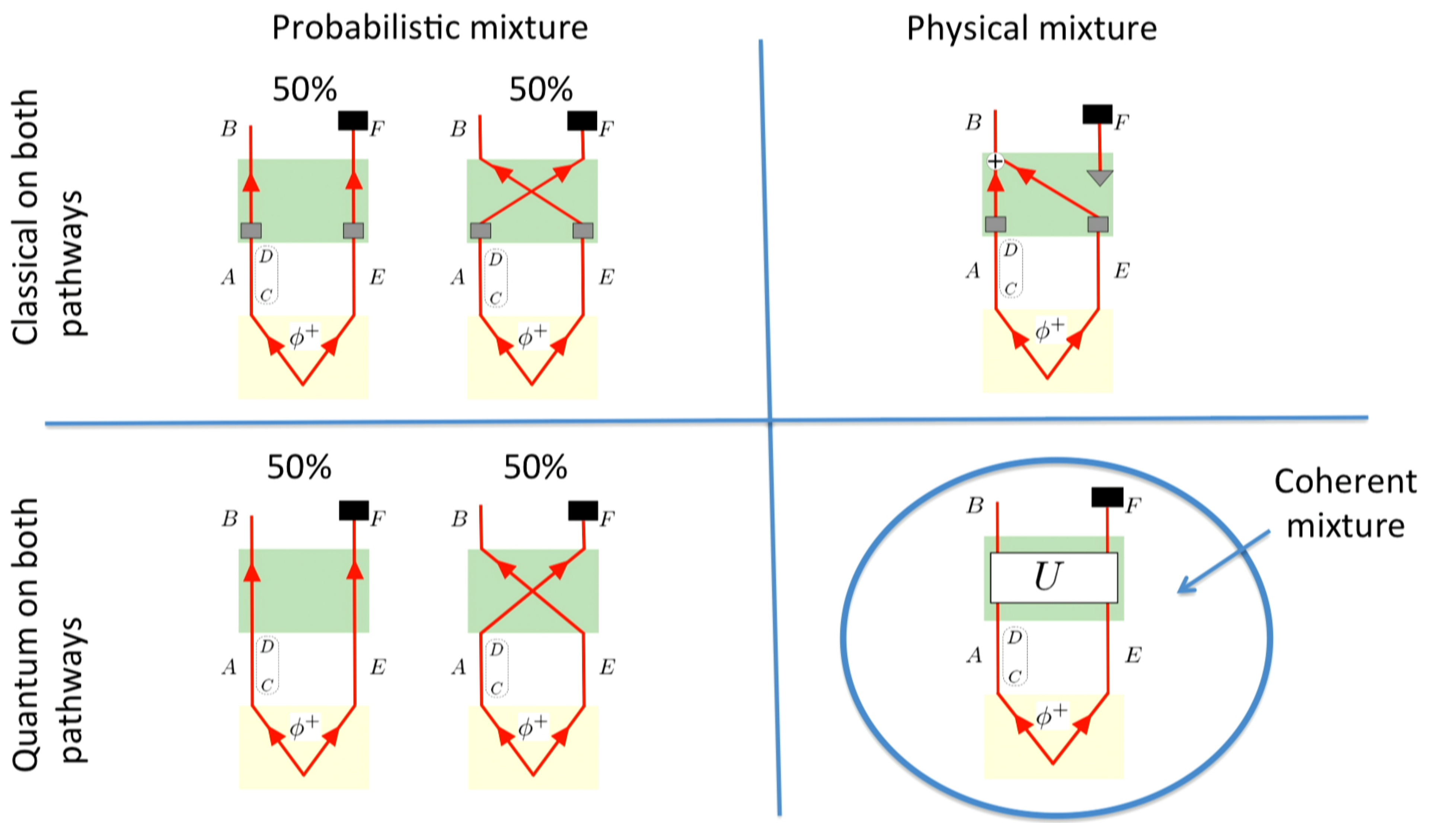


Coherent mixture of cause-effect and common-cause



$$U = e^{i\phi}\frac{1}{\sqrt{2}}(I + iS)$$

$$\begin{aligned}\tau_{CBD} = &\frac{1}{2}(\tfrac{1}{2}I_C) \otimes |\phi^+\rangle\langle\phi^+|_{B|D} + \frac{1}{2}|\phi^+\rangle\langle\phi^+|_{CB} \otimes I_D \\ &+ i\Big\{[\tfrac{1}{2}I_C \otimes |\phi^+\rangle\langle\phi^+|_{B|D}]|\phi^+\rangle\langle\phi^+|_{CB} \\ &- |\phi^+\rangle\langle\phi^+|_{CB}[\tfrac{1}{2}I_C \otimes |\phi^+\rangle\langle\phi^+|_{B|D}]\Big\},\end{aligned}$$

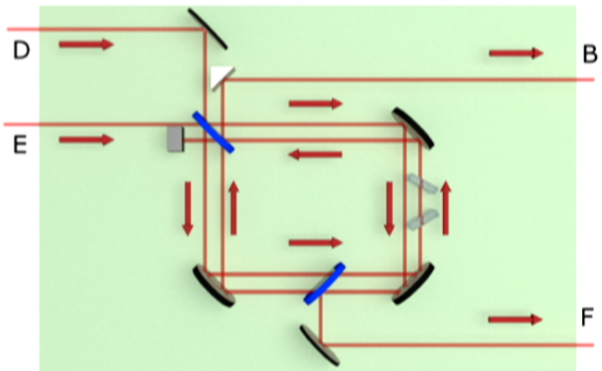


Probabilistic mixture
Physical mixture
Classical on both pathways
Quantum on both pathways
50%
50%
50%
50%
Coherent mixture
U

EXPERIMENTAL IMPLEMENTATION OF A QUANTUM COHERENT MIXTURES OF CAUSAL RELATIONS (PART II)

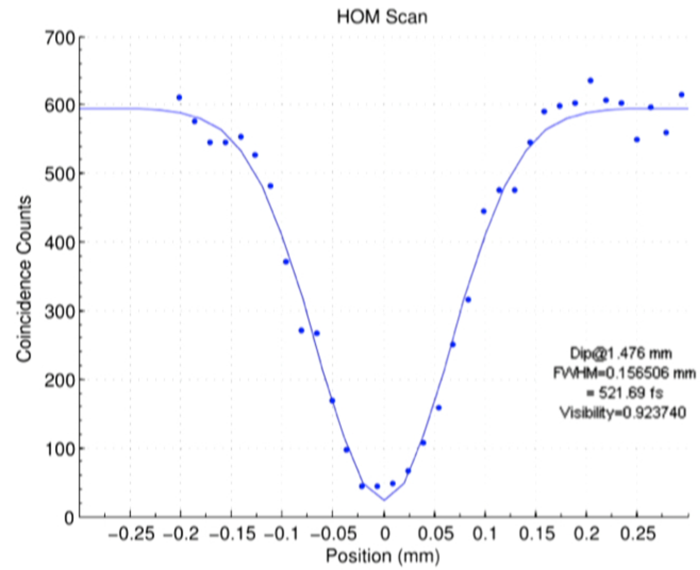
Jean-Philippe MacLean
Rob Spekkens
Katja Ried
Kevin Resch

UNIVERSITY OF WATERLOO
IQC Institute for Quantum Computing
PERIMETER INSTITUTE FOR THEORETICAL PHYSICS



$$U(\theta) = \cos(\theta/2)I + i\sin(\theta/2)S$$

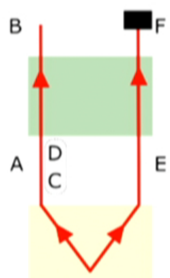
$$U(\theta) = \Pi^{+} + e^{i\theta}\Pi^{-}$$



92.3%

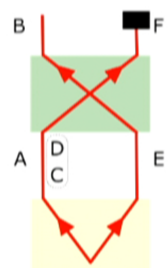
Cause-effect:

$\theta=0$



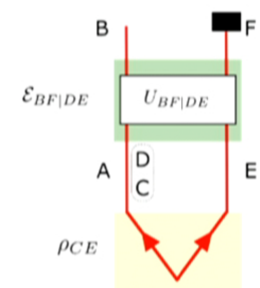
Common-cause:

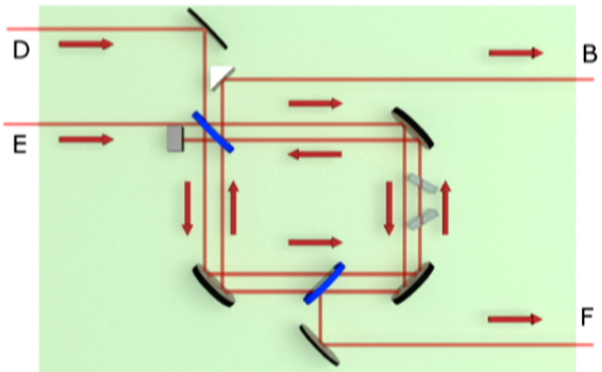
$\theta=\pi$

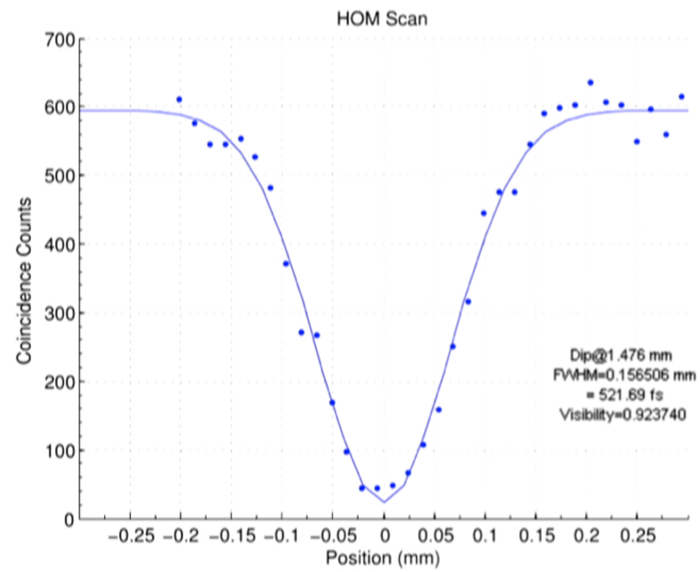


Coherent mixture:

$\theta=\pi/2$







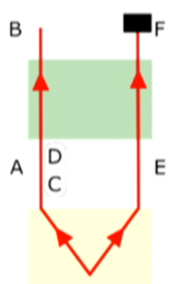
92.3%

$$U(\theta) = \cos(\theta/2)I + i\sin(\theta/2)S$$

$$U(\theta) = \Pi^{+} + e^{i\theta}\Pi^{-}$$

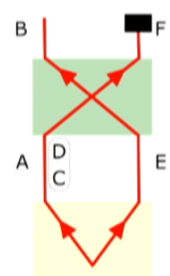
Cause-effect:

$\theta=0$



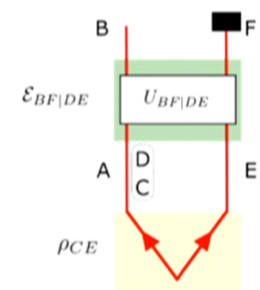
Common-cause:

$\theta=\pi$

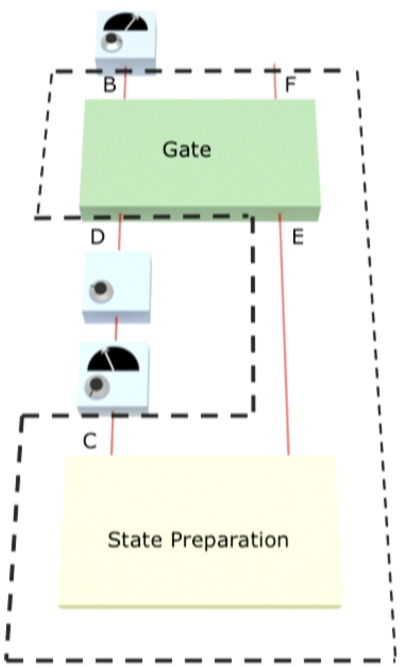


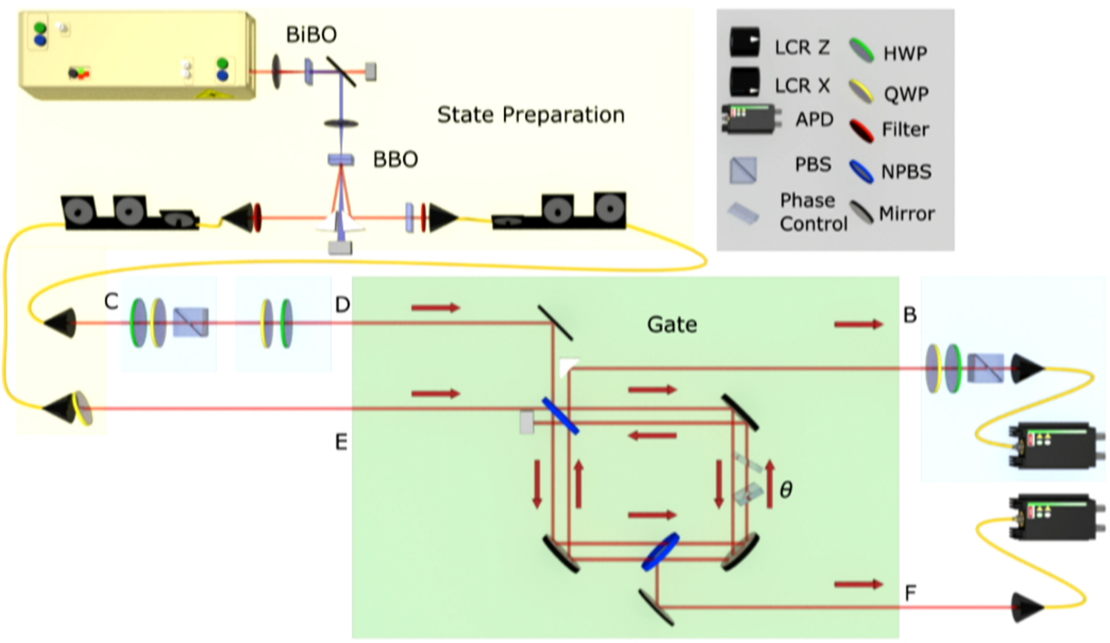
Coherent mixture:

$\theta=\pi/2$

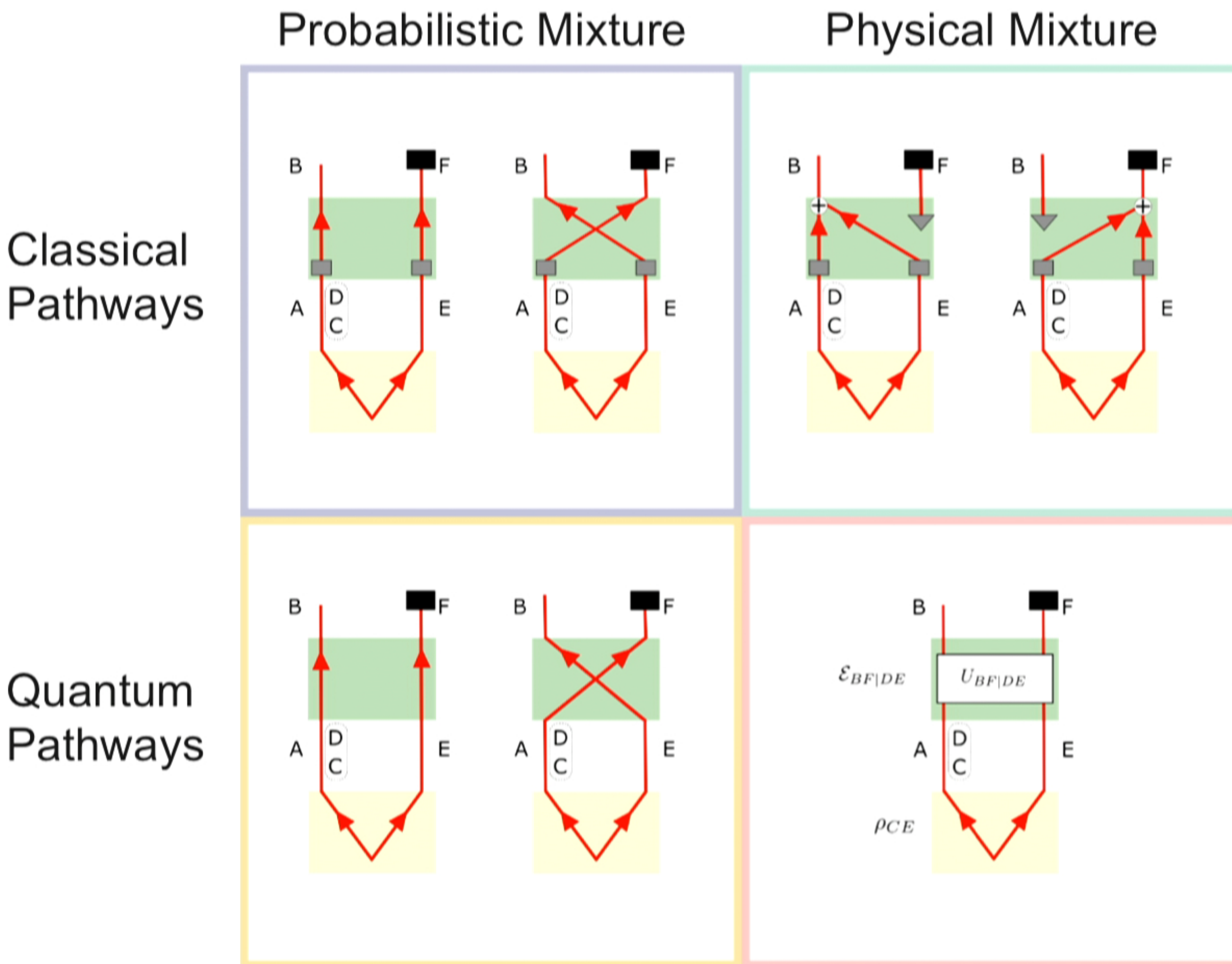


EXPERIMENTAL SETUP

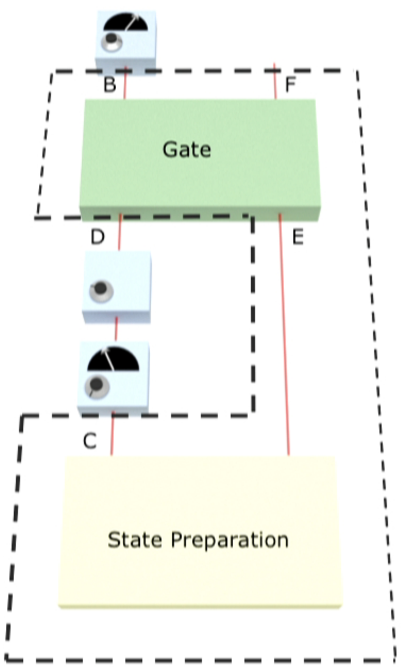


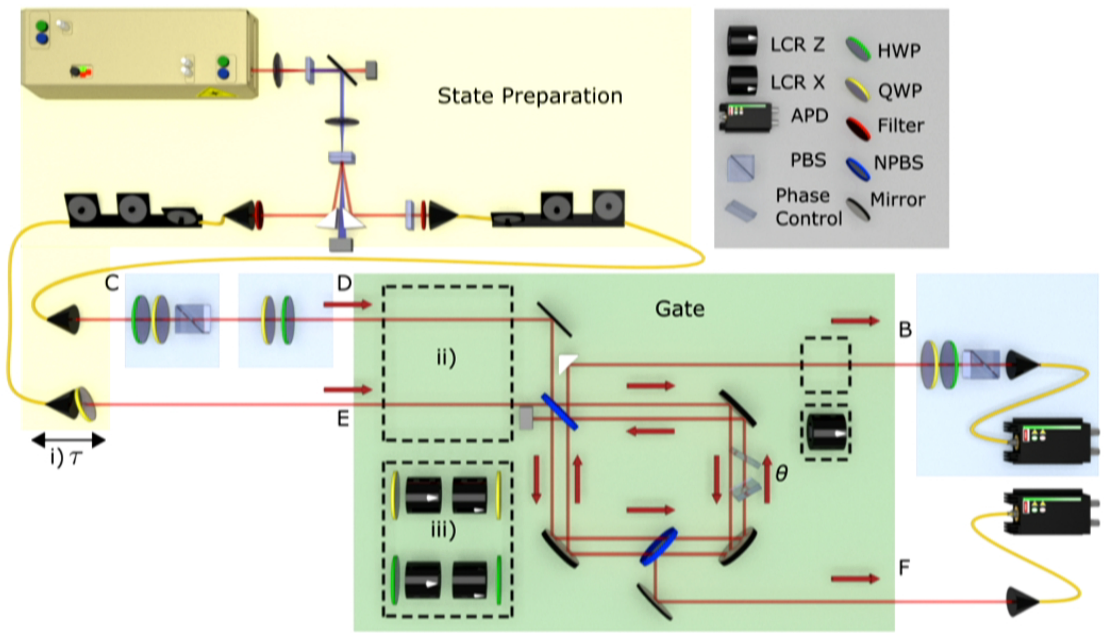


MacLean et al. “Quantum-Coherent Mixtures of Causal Relations.” *arXiv:1606.04523*

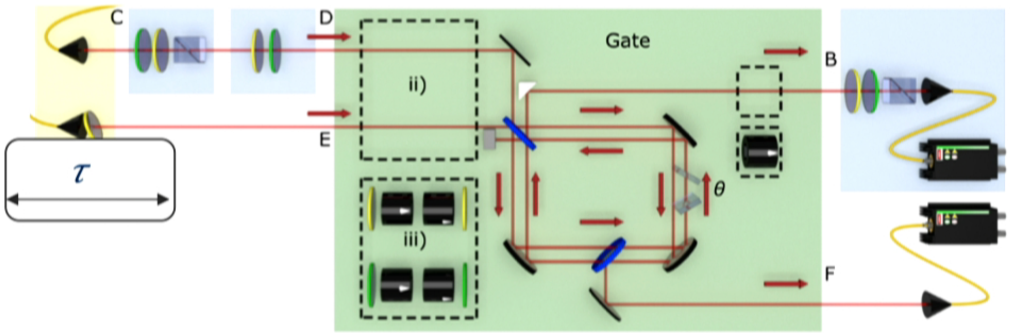

Probabilistic Mixture
Physical Mixture
Classical Pathways
Quantum Pathways
B
F
A
D
C
E
$\mathcal{E}_{BF|DE}$
$U_{BF|DE}$
ρ_{CE}

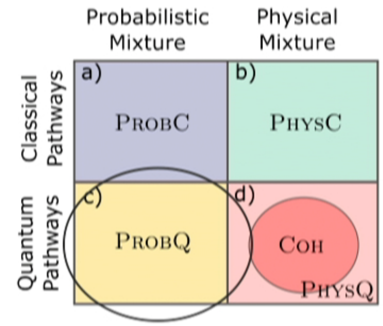
REALIZING DIFFERENT CAUSAL STRUCTURES

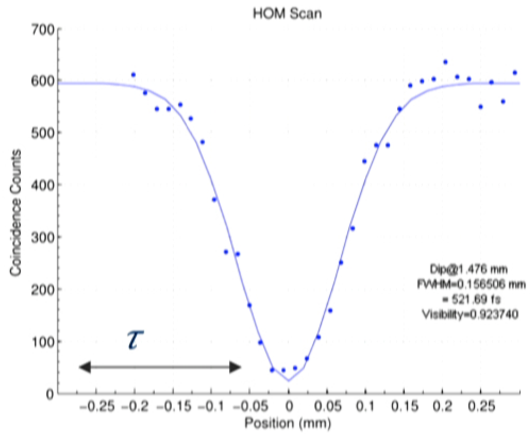


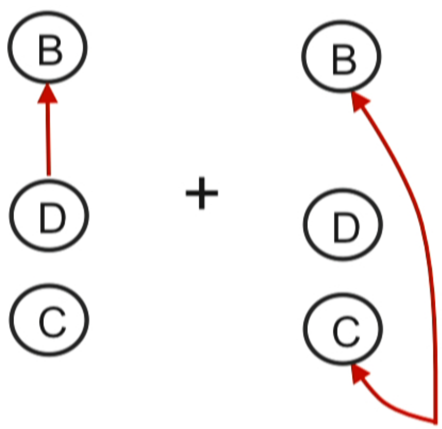


MacLean et al. "Quantum-Coherent Mixtures of Causal Relations." *arXiv:1606.04523*



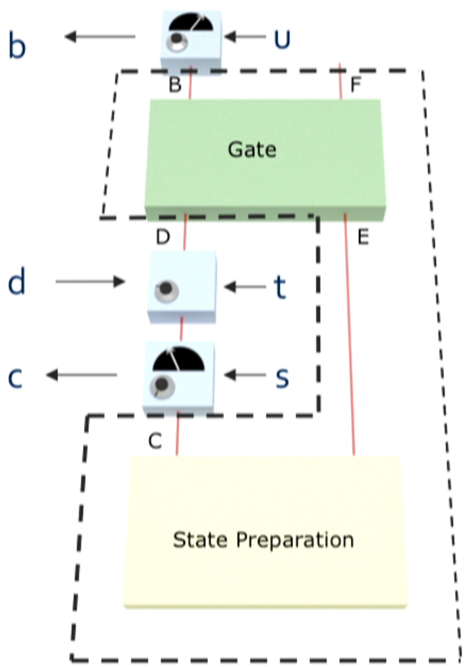






$$\mathcal{E}_{BF|DE}(\rho_{DE}) = \frac{1}{2}\Pi^+\rho_{DE}\Pi^+ + \frac{1}{2}\Pi^-\rho_{DE}\Pi^-$$

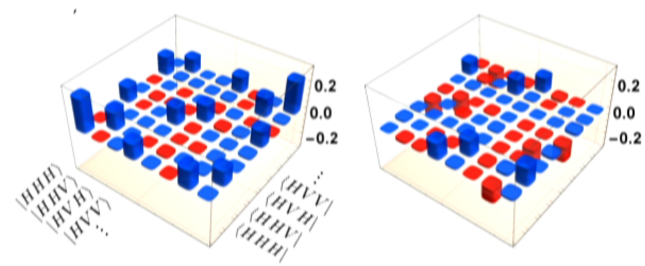
CAUSAL TOMOGRAPHY

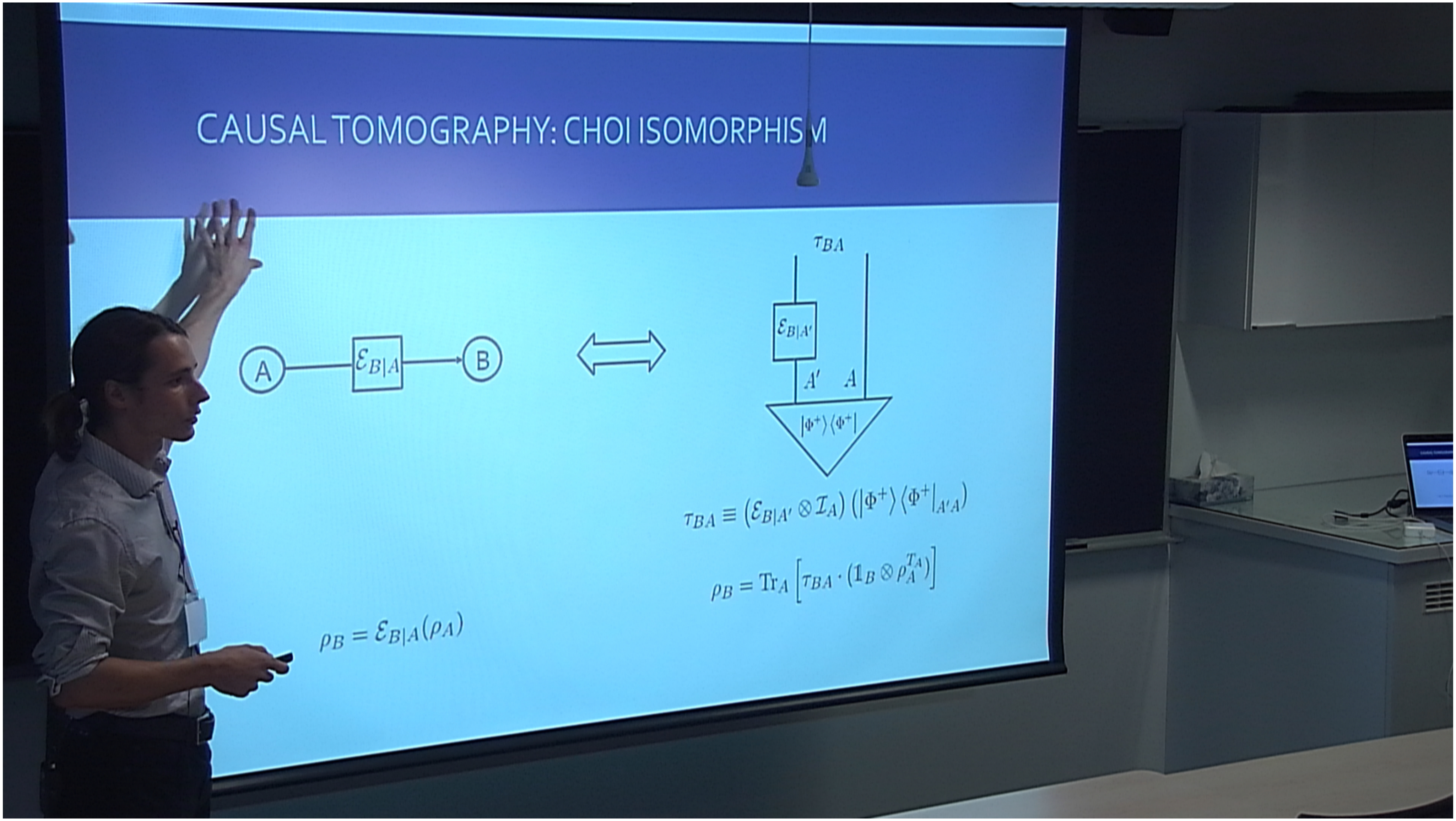


Ried et *al*. "A Quantum Advantage for Inferring Causal Structure."
Nature Physics 11, no. 5 (2015): 414–20. doi:10.1038/nphys3266.

$$P(cbd|stu) = \mathrm{Tr}\left[\tau_{CBD} \cdot \Pi_C^{s,c} \otimes \Pi_B^{u,b} \otimes T_D\left(\Pi_D^{t,d}\right)\right]$$

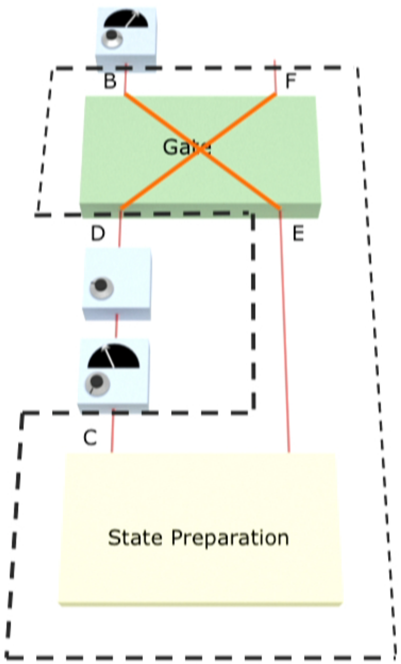
$\tau\downarrow CBD$

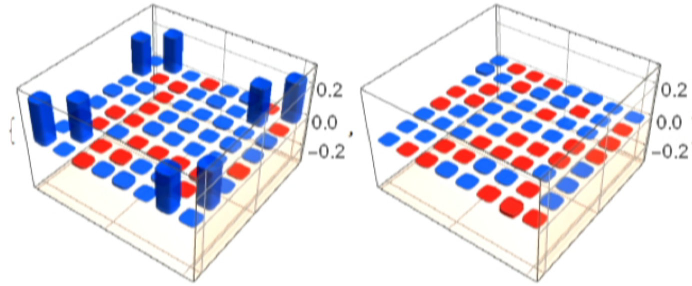



CAUSAL TOMOGRAPHY: CHOI ISOMORPHISM
A
$\mathcal{E}_{B|A}$
B
τ_{BA}
$\mathcal{E}_{B|A'}$
A'
A
$|\Phi^+\rangle\langle\Phi^+|$
$\tau_{BA} \equiv \left(\mathcal{E}_{B|A'} \otimes \mathcal{I}_A\right)\left(|\Phi^+\rangle\langle\Phi^+|_{A'A}\right)$
$\rho_B = \mathrm{Tr}_A\left[\tau_{BA} \cdot (\mathbb{1}_B \otimes \rho_A^{T_A})\right]$
$\rho_B = \mathcal{E}_{B|A}(\rho_A)$

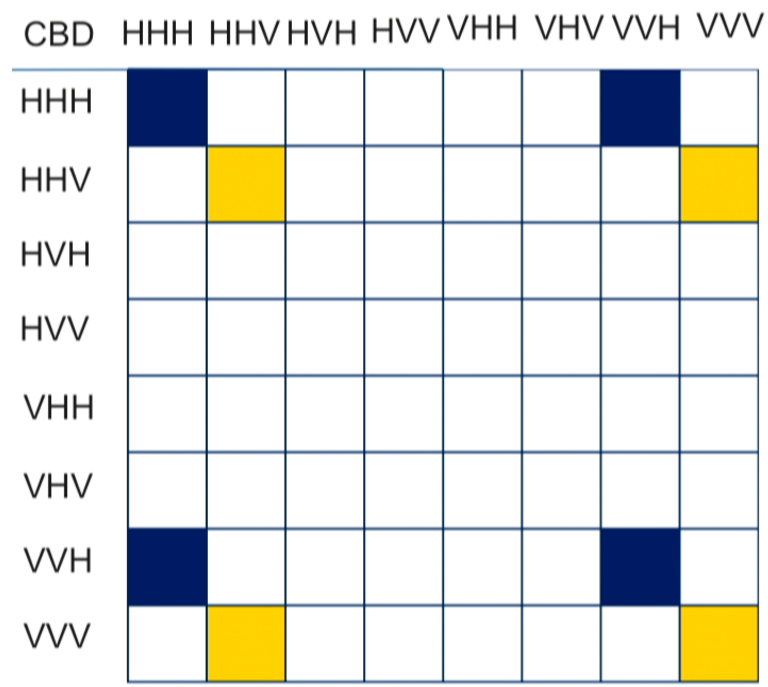
Common-cause process:

$U = S$



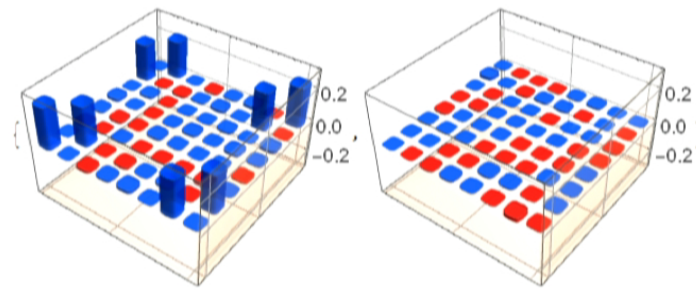


Fidelity: 92.6%

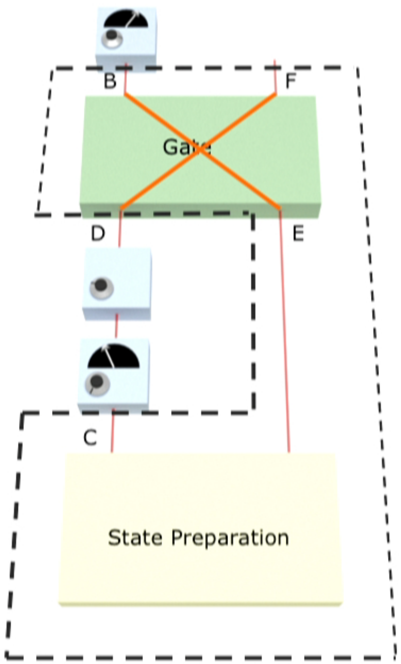


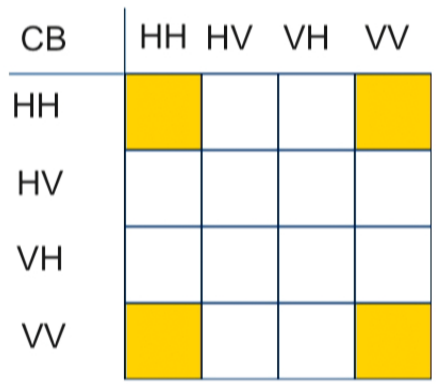
MacLean et al. "Quantum-Coherent Mixtures of Causal Relations." *arXiv:1606.04523*

Common-cause process: $U = S$



Fidelity: 92.6%





$$\otimes \left| V \right\rangle \left\langle V \right|_D$$

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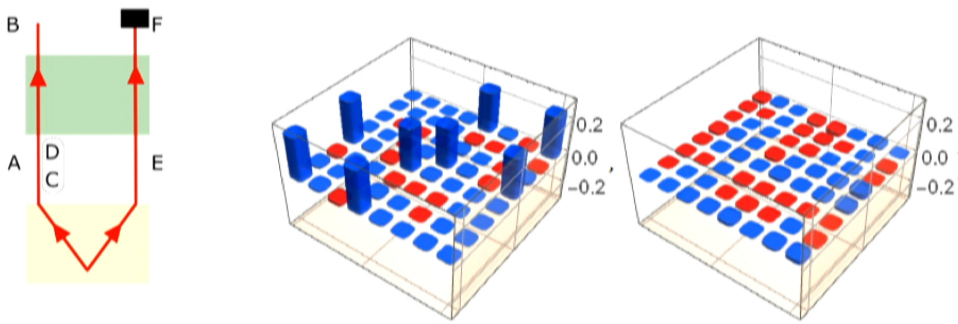
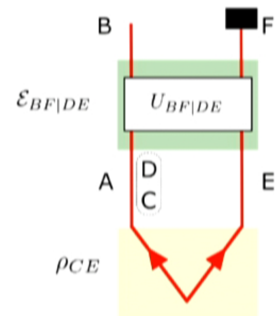
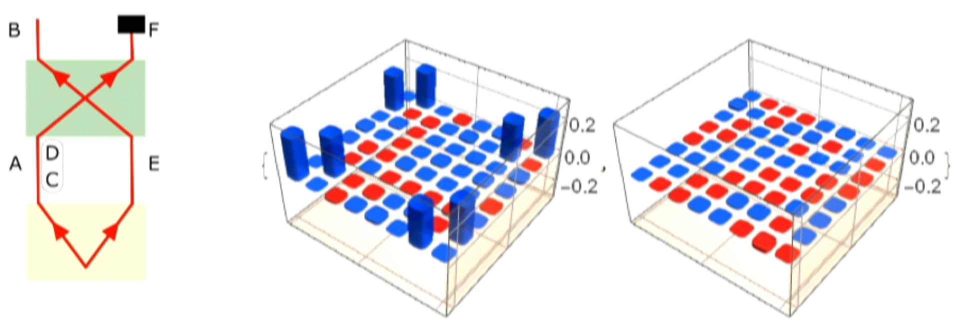
Cause-effect: $U = I$



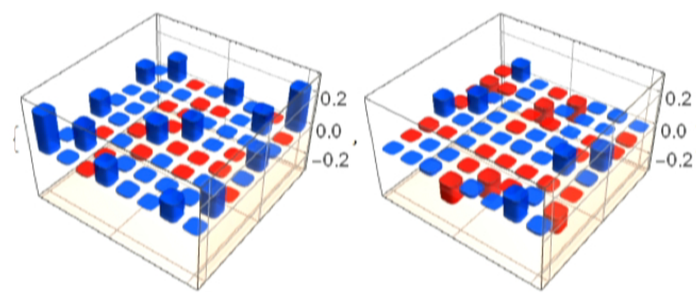
Coherent mixture: $U = \frac{1}{\sqrt{2}} I + i \frac{1}{\sqrt{2}} S$



Common-cause: $U = S$



Fidelity: 93.7%



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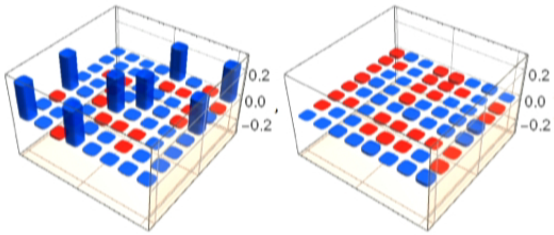
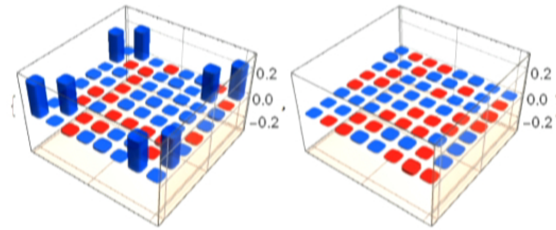
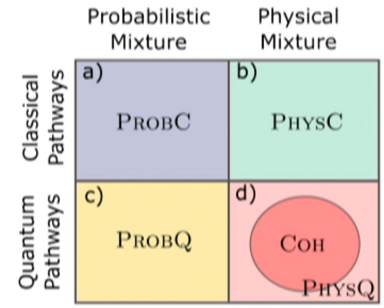
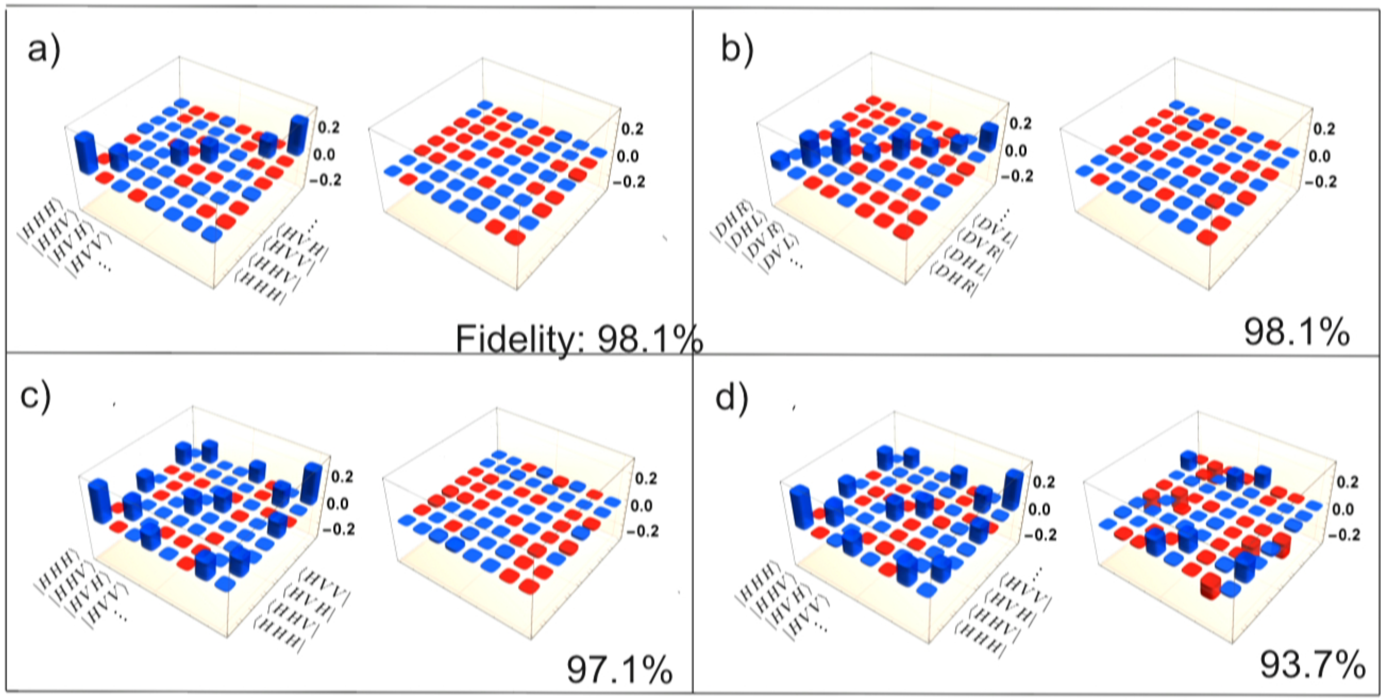
Cause-effect



Common-cause







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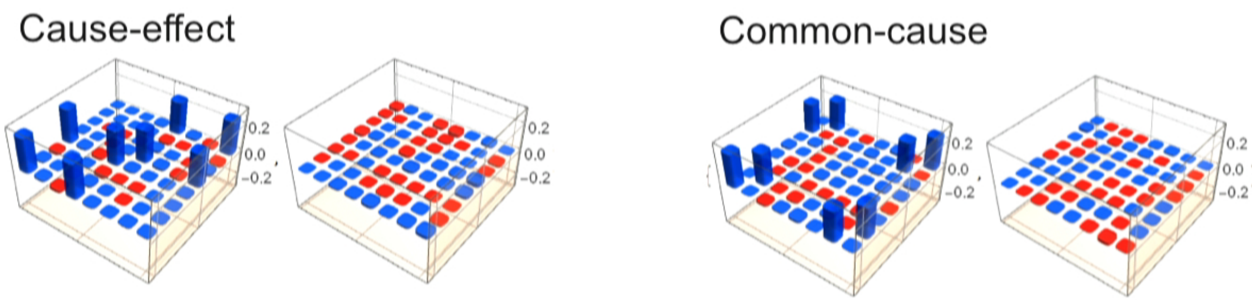

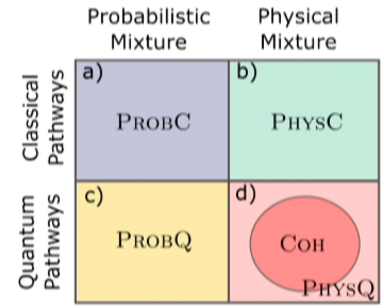

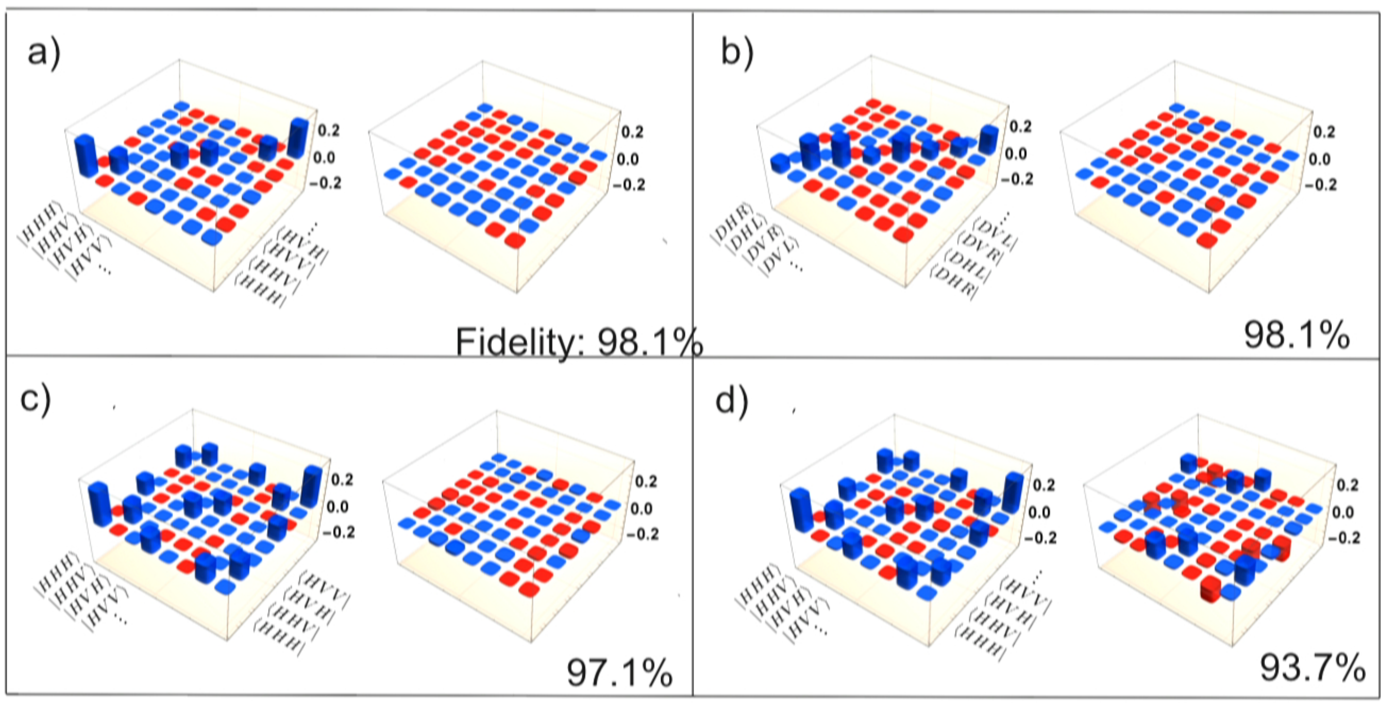

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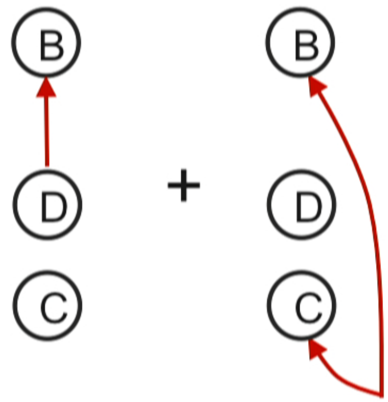
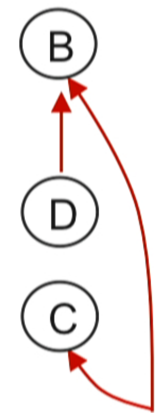
DIFFERENTIATING PROBABILISTIC FROM PHYSICAL MIXTURES

Covariance $\mathrm{Cov}(\mathrm{c},\mathrm{b}) \equiv \langle \mathrm{cb} \rangle - \langle \mathrm{c} \rangle \langle \mathrm{b} \rangle$

Witness $\mathcal{C}_{CD} = \sum_b b P(b)^2 \mathrm{Cov}(cd|b) = \mathrm{Tr}\left[\tau_{CBD} \cdot \sigma_C^s \otimes \sigma_D^t \otimes \sigma_B^u\right]$



$$\Rightarrow \mathcal{C}_{CD} = 0$$



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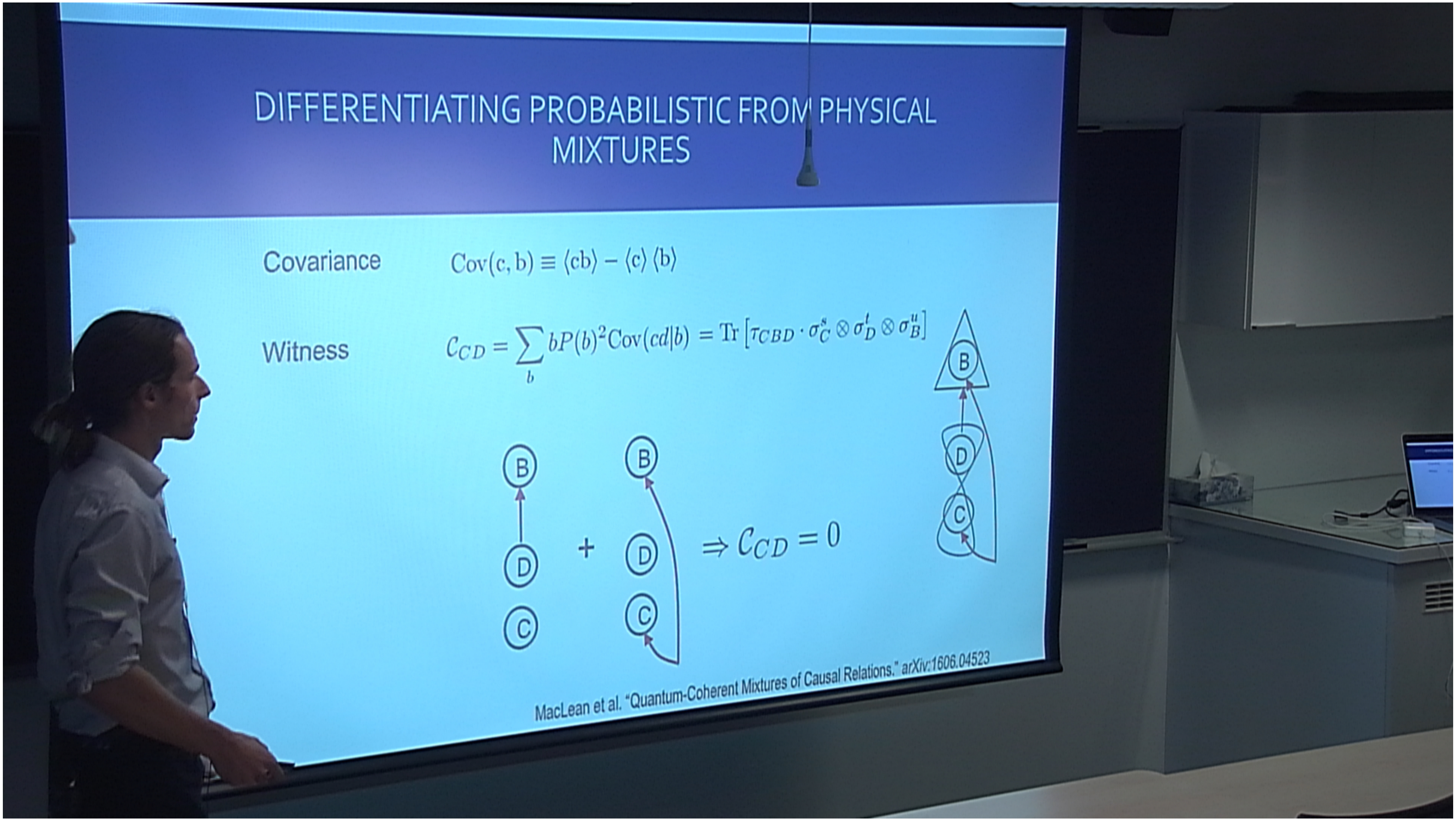
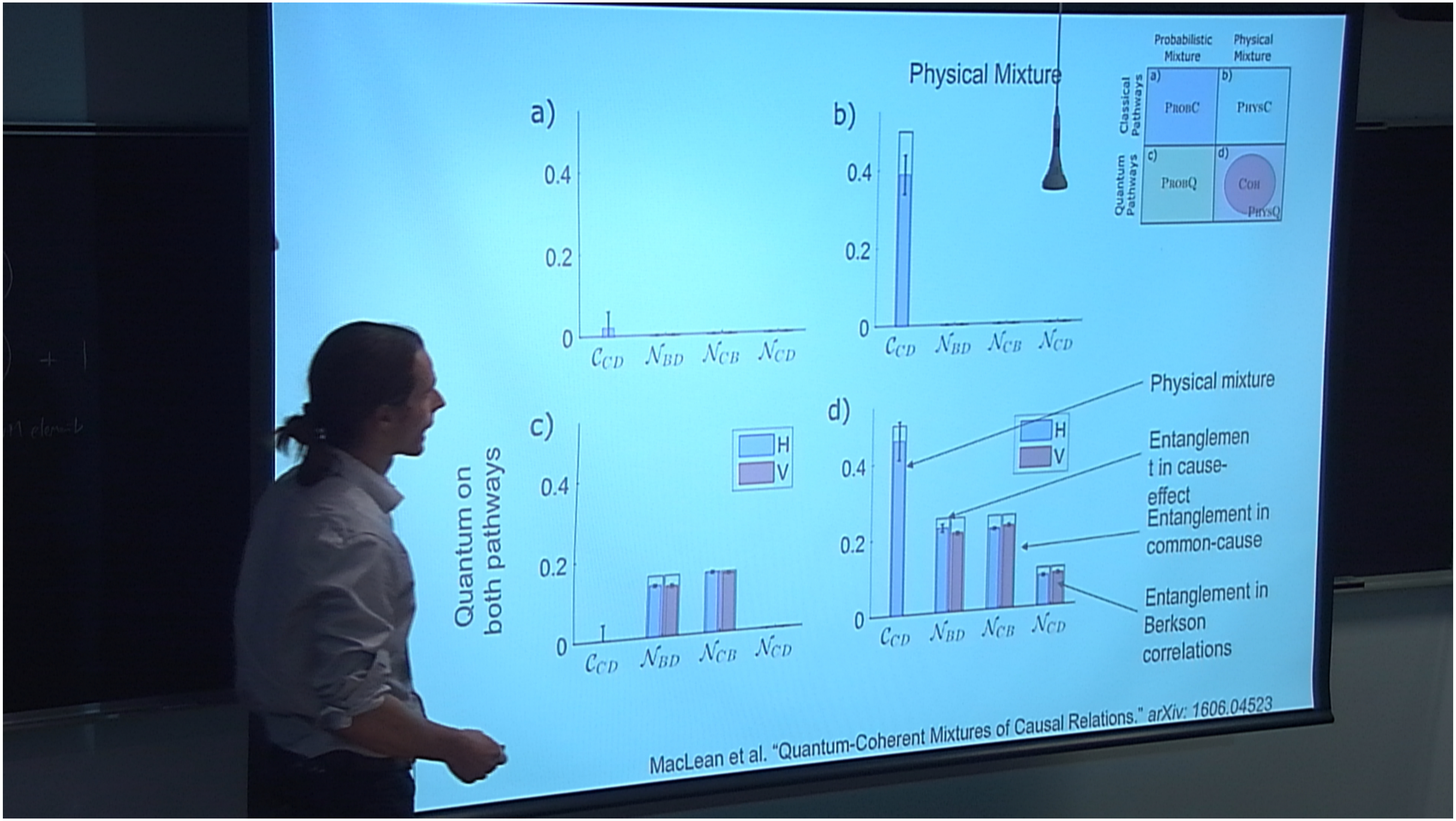
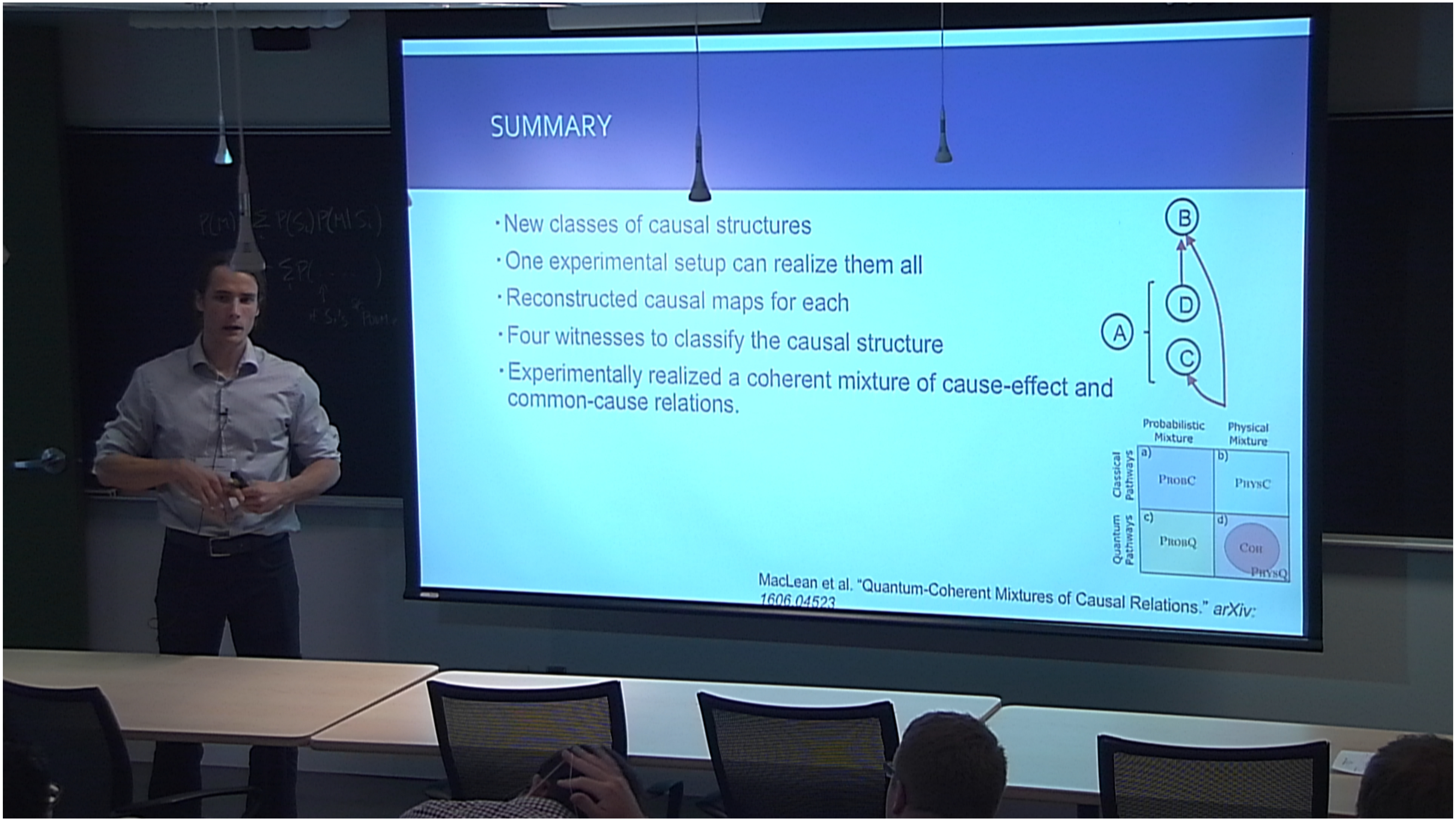
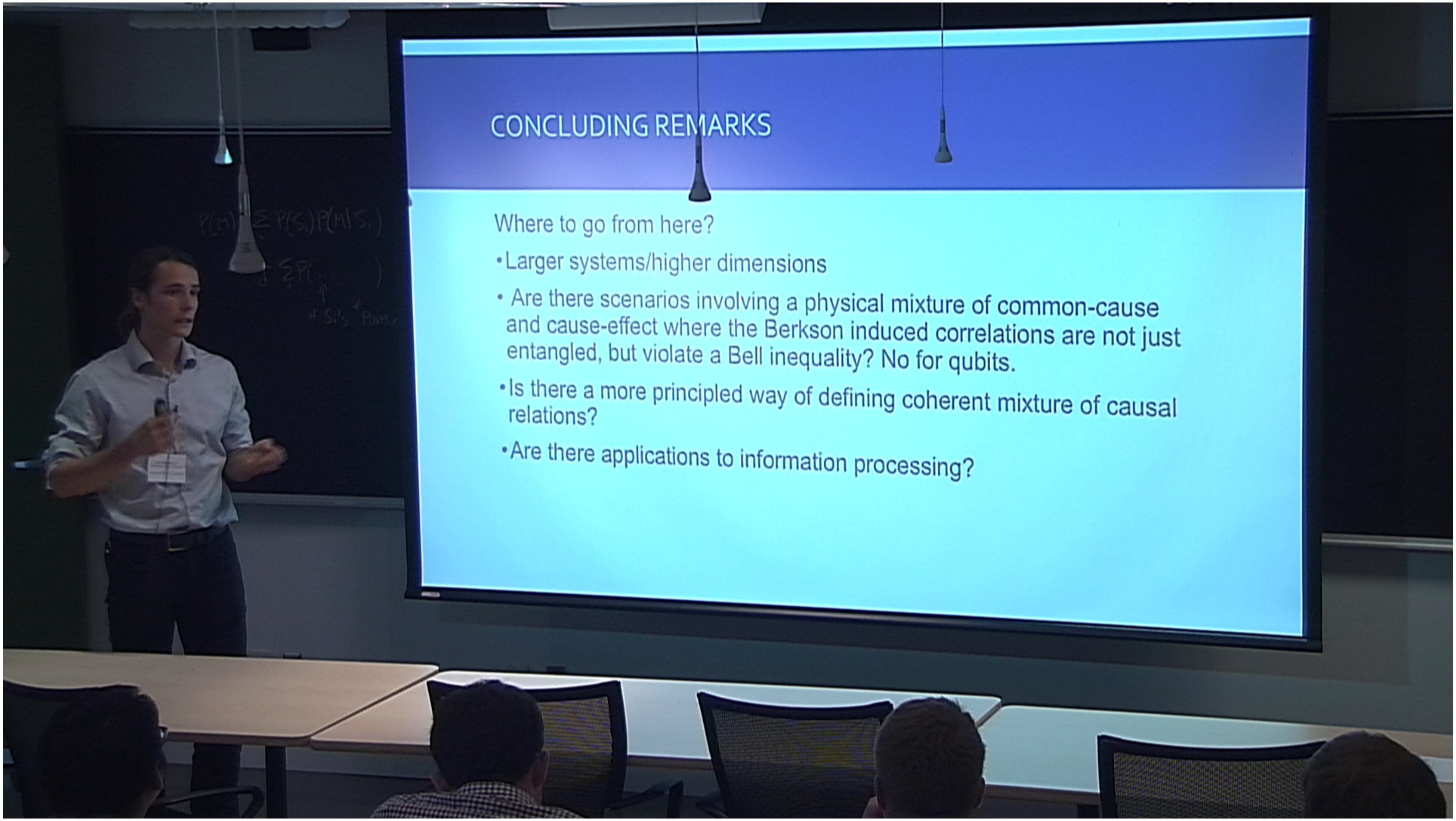

DIFFERENTIATING PROBABILISTIC FROM PHYSICAL MIXTURES
Covariance
$\mathrm{Cov}(c,b) \equiv \langle cb \rangle - \langle c \rangle \langle b \rangle$
Witness
$\mathcal{C}_{CD} = \sum_b bP(b)^2 \mathrm{Cov}(cd|b) = \mathrm{Tr}\left[\tau_{CBD} \cdot \sigma_C^s \otimes \sigma_D^t \otimes \sigma_B^u\right]$
B
D
C
+
$\Rightarrow \mathcal{C}_{CD} = 0$
MacLean et al. "Quantum-Coherent Mixtures of Causal Relations." arXiv:1606.04523


Physical Mixture
Probabilistic Mixture
Physical Mixture
Classical Pathways
Quantum Pathways
PROBC
PHYSC
PROBQ
COH
PHYSQ
a)
b)
c)
d)
0.4
0.2
0
$\mathcal{C}_{CD}$ $\mathcal{N}_{BD}$ $\mathcal{N}_{CB}$ $\mathcal{N}_{CD}$
Quantum on both pathways
H
V
Physical mixture
Entanglemen t in cause-effect
Entanglement in common-cause
Entanglement in Berkson correlations
MacLean et al. "Quantum-Coherent Mixtures of Causal Relations." arXiv: 1606.04523



SUMMARY
New classes of causal structures
One experimental setup can realize them all
Reconstructed causal maps for each
Four witnesses to classify the causal structure
Experimentally realized a coherent mixture of cause-effect and common-cause relations.
B
D
A
C
Probabilistic Mixture
Physical Mixture
Classical Pathways
Quantum Pathways
a) ProbC
b) PhysC
c) ProbQ
d) Coh
PhysQ
MacLean et al. "Quantum-Coherent Mixtures of Causal Relations." arXiv: 1606.04523


CONCLUDING REMARKS
Where to go from here?
•Larger systems/higher dimensions
• Are there scenarios involving a physical mixture of common-cause and cause-effect where the Berkson induced correlations are not just entangled, but violate a Bell inequality? No for qubits.
•Is there a more principled way of defining coherent mixture of causal relations?
•Are there applications to information processing?