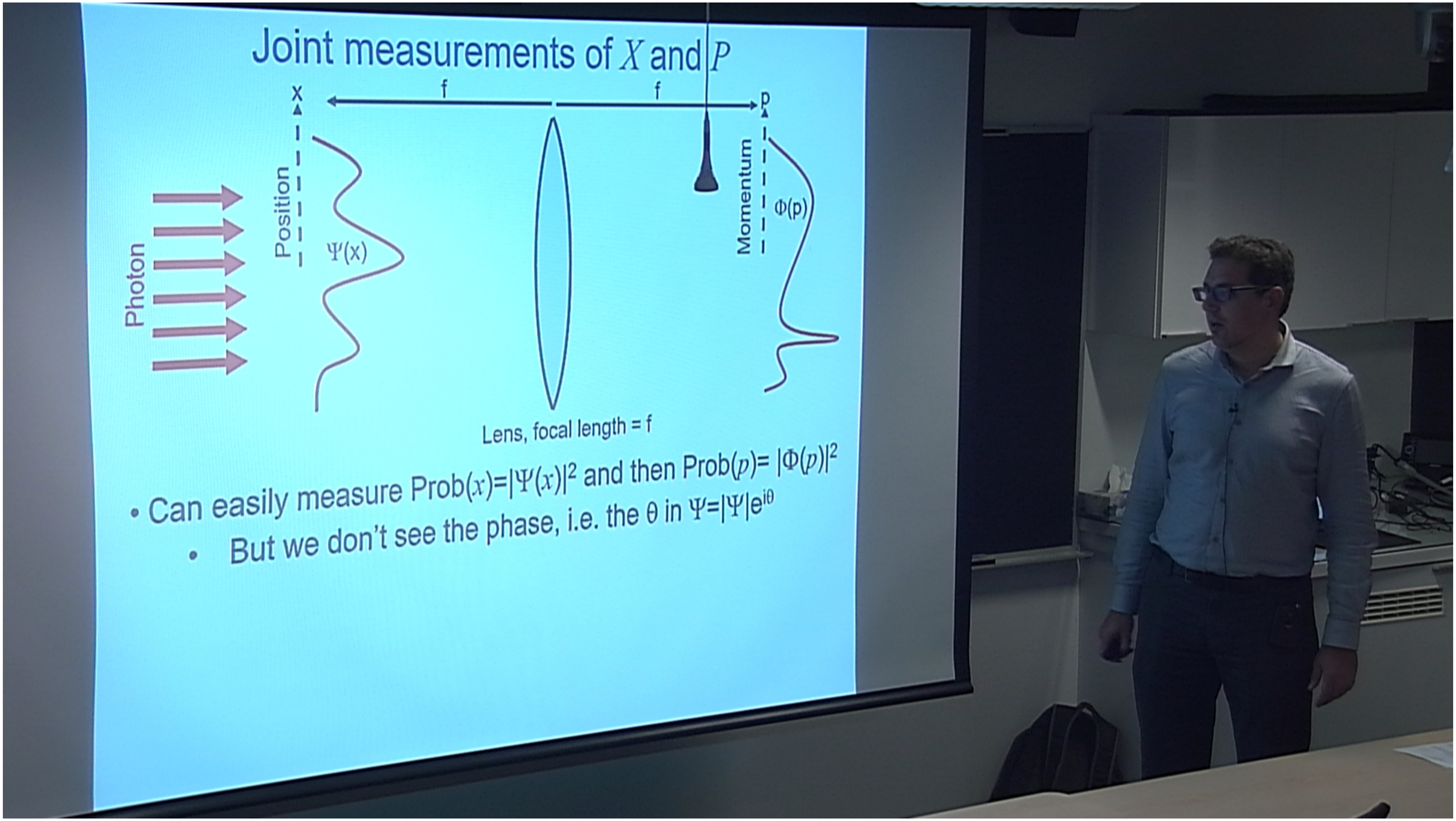
Title: NaÃ¯ve experiments for measuring incompatible observables

Date: Sep 23, 2016 02:30 PM

URL: http://pirsa.org/16090055

Abstract: Sets or pairs of incompatible observables, such as momentum and position, play a pivotal role in a wide range of distinctly quantum effects and applications, including quantum cryptography, the Heisenberg Uncertainty Principle, quantum state tomography, and Bellâ€™s inequalities. In particular, in quantum physics, we are prohibited from precisely measuring the values of incompatible observables, a fact that is at the heart of the nature of the quantum state. In this talk, I will explore an assortment of strategies that simple-mindedly attempt to circumvent this prohibition. Motivated by these naÃ¯ve strategies, we experimentally investigate the use of weak measurement and optimal quantum cloning to perform joint measurements on photons. The direct outcome of these measurements are, depending on the strategy, the wavefunction, the Dirac distribution, and the density matrix of the measured quantum system. Consequently, these naÃ¯ve strategies provide new ways to characterize quantum systems and to understand the very entities that we are measuring, such as the wavefunction.


Joint measurements of X and P
x
f
f
p
Photon
Position
Ψ(x)
Lens, focal length = f
Momentum
Φ(p)
• Can easily measure Prob(x)=|Ψ(x)|² and then Prob(p)= |Φ(p)|²
• But we don't see the phase, i.e. the θ in Ψ=|Ψ|e^iθ

# Naïve schemes to measure incompatible observables

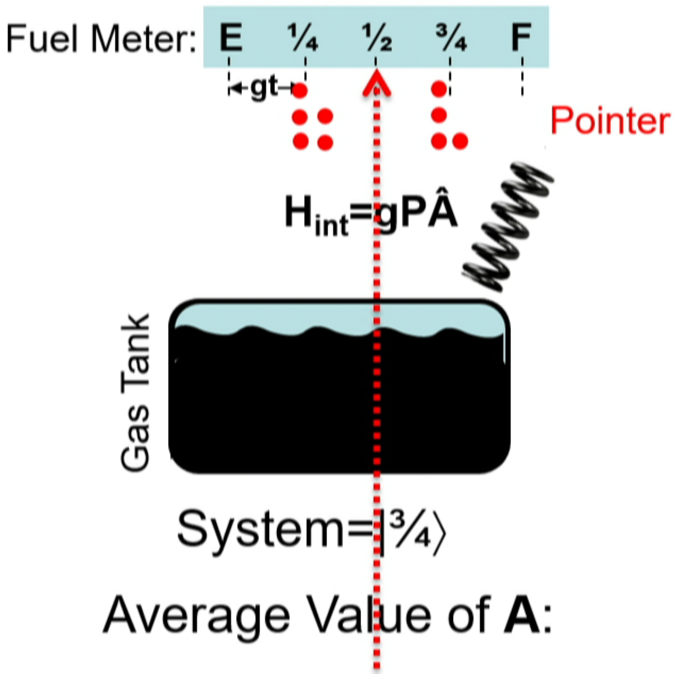
- How would a person ignorant of the laws of (quantum) physics try to measure X and P jointly on a single quantum system?

Do they


?

# Von Neumann's Quantum Measurement

## Strong Measurement




$\langle\psi|\hat{A}|\psi\rangle$

Born Rule!

Model both the measured system and the measurement apparatus as quantum systems.

e.g. The pointer needle on a fuel gauge has a wavefunction and so does the gas tank.

# Naïve Strategy #1: Learn a bit about both X and P

Balance coupling strength to two pointers for simultaneous measurement of X and P

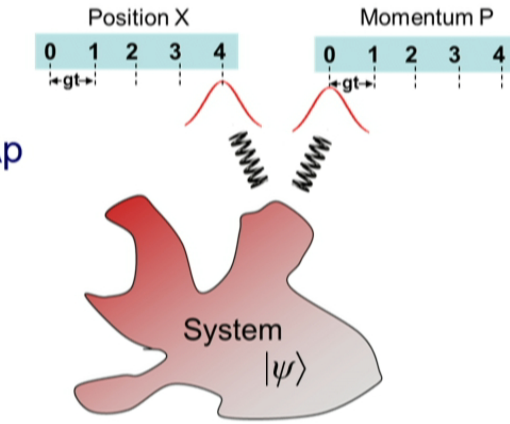
- Disturbance and Resolution are equal Δx = Δp

B.S.T.J. BRIEFS

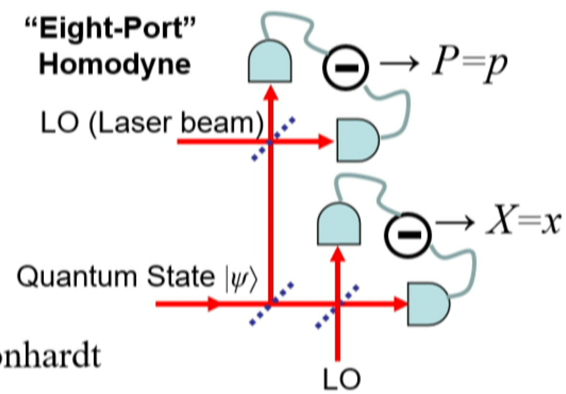
On the Simultaneous Measurement of a Pair of Conjugate Observables

By E. ARTHURS and J. L. KELLY, JR.

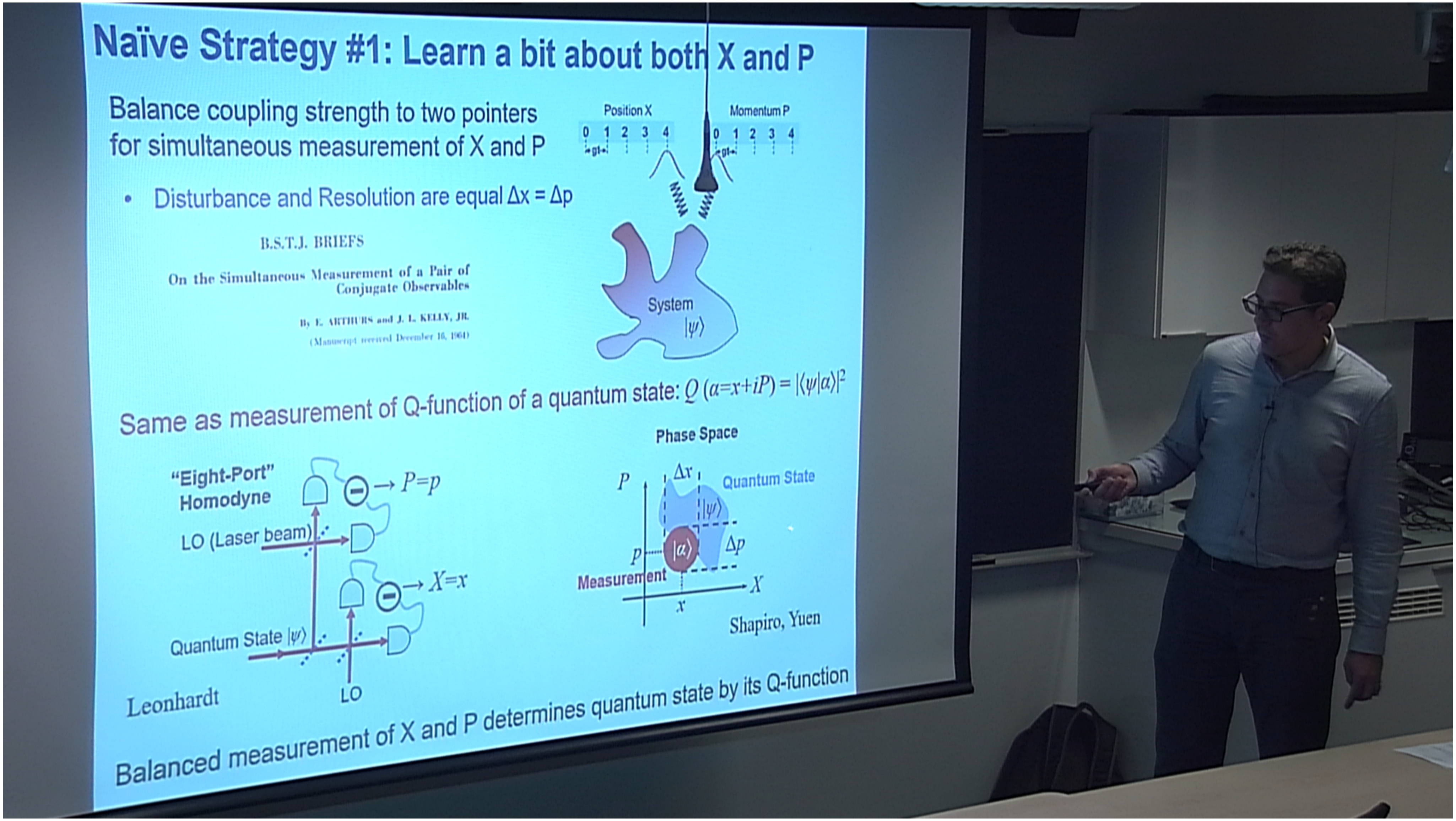
(Manuscript received December 16, 1964)




Same as measurement of Q-function of a quantum state: $Q\,(\alpha = x + iP) = |\langle\psi|\alpha\rangle|^2$




Leonhardt


Naïve Strategy #1: Learn a bit about both X and P
Balance coupling strength to two pointers
for simultaneous measurement of X and P
Disturbance and Resolution are equal Δx = Δp
Position X
Momentum P
0 1 2 3 4
0 1 2 3 4
B.S.T.J. BRIEFS
On the Simultaneous Measurement of a Pair of Conjugate Observables
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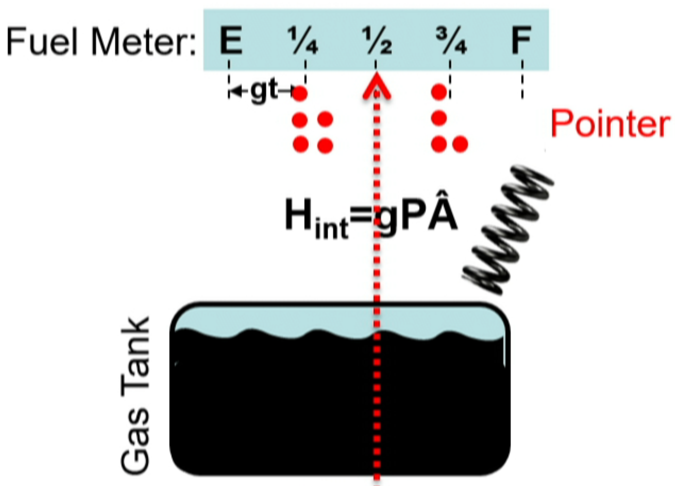
System
|ψ⟩
Same as measurement of Q-function of a quantum state: Q (α=x+iP) = |⟨ψ|α⟩|²
"Eight-Port" Homodyne
P=p
LO (Laser beam)
X=x
Quantum State |ψ⟩
LO
Leonhardt
Phase Space
P
Δx
Quantum State
|ψ⟩
Δp
p
|α⟩
Measurement
X
x
Shapiro, Yuen
Balanced measurement of X and P determines quantum state by its Q-function

# Weak Quantum Measurement

## Strong Measurement

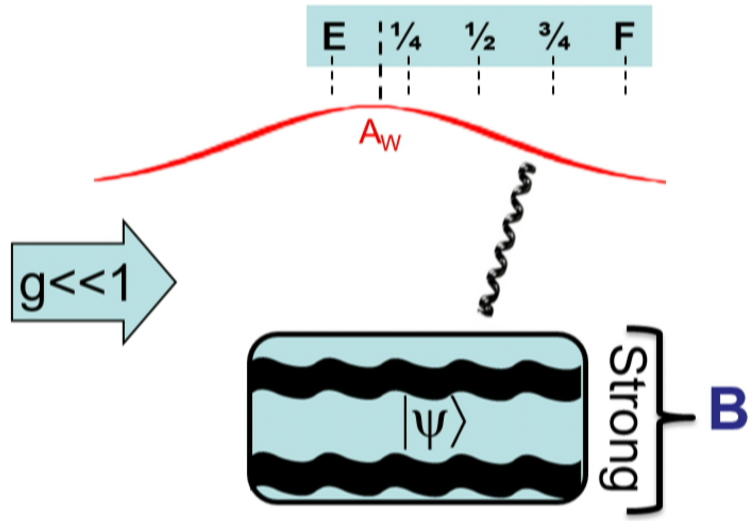



System=|¾⟩

Average Value of **A**:

$\langle\psi|\hat{A}|\psi\rangle$

## Weak Measurement



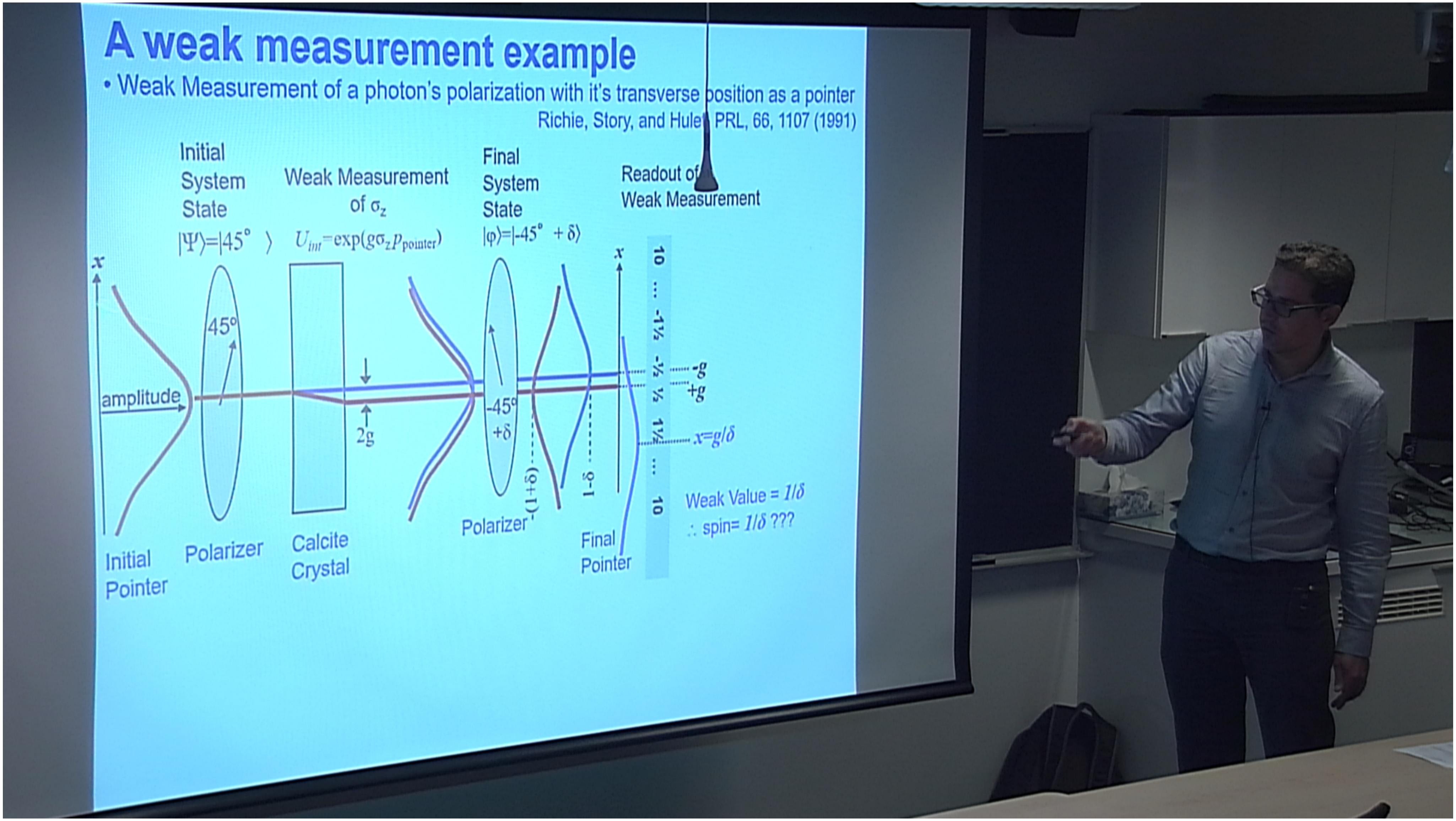

In the cases where result of **B** is b

Average Value of **A**:

$$A_W = \frac{<b|A|\psi>}{<b|\psi>}$$

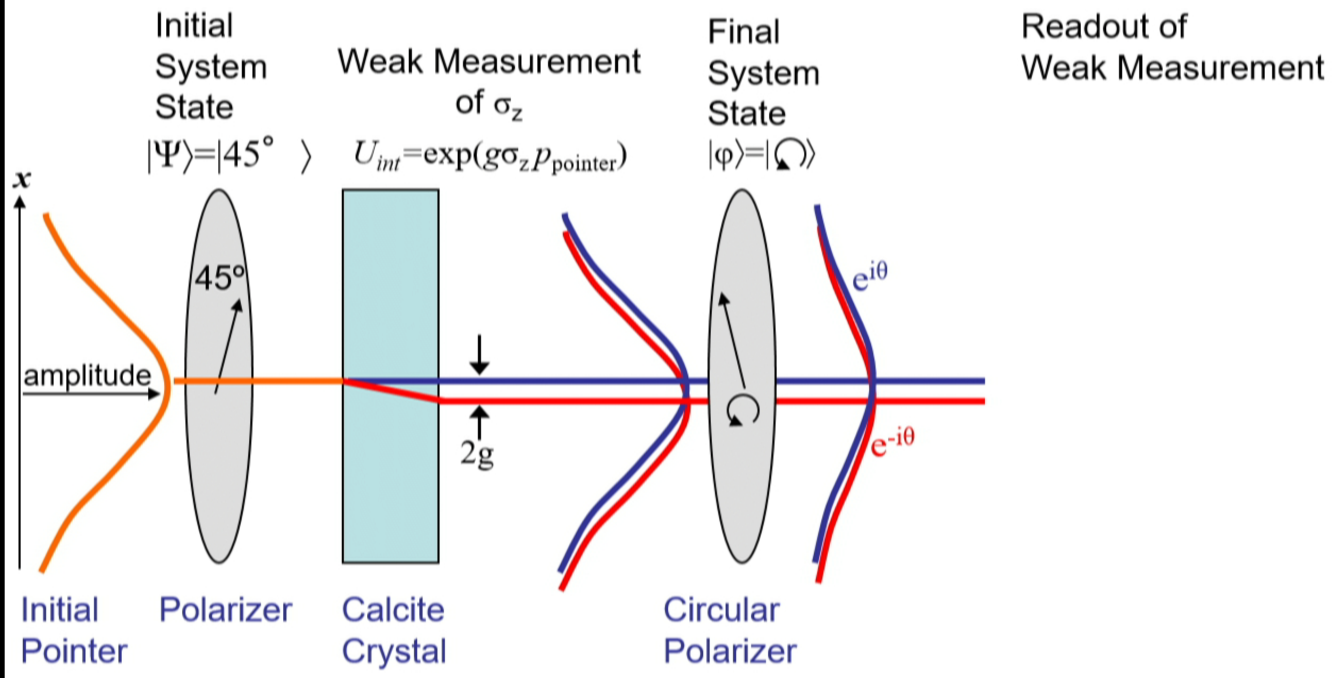
**Real** part of $A_W$ is the position shift of the pointer

**Imaginary** part of $A_w$ is the momentum shift of the pointer


A weak measurement example
• Weak Measurement of a photon's polarization with it's transverse position as a pointer
Richie, Story, and Hulet PRL, 66, 1107 (1991)
Initial System State
|Ψ⟩=|45° ⟩
Weak Measurement of σz
Uint=exp(gσz ppointer)
Final System State
|φ⟩=|-45° + δ⟩
Readout of Weak Measurement
x
amplitude
45°
2g
-45° +δ
-(g+1)
g-1
x
-g
+g
x=g/δ
10 ... -1½ -½ ½ 1½ ... 10
Weak Value = 1/δ
∴ spin= 1/δ ???
Initial Pointer
Polarizer
Calcite Crystal
Polarizer
Final Pointer

# The imaginary part of the weak value

- Weak Measurement also shifts the conjugate pointer variable - momentum



# Naïve Strategy #2:
# Gently measure X so that you don't disturb P

- What if we do a weak measurement of $X$, and then make a strong measurement of $P$?

i.e. **A** $= |x\rangle\langle x| = \pi$, Initial state= $|\psi\rangle$, Strong measurement result $P=p$

Average shift of the pointer:

$$A_W = \frac{\langle b|A|\psi\rangle}{\langle b|\psi\rangle}$$

$$\pi_w = \frac{\langle p|x\rangle\langle x|\psi\rangle}{\langle p|\psi\rangle}$$

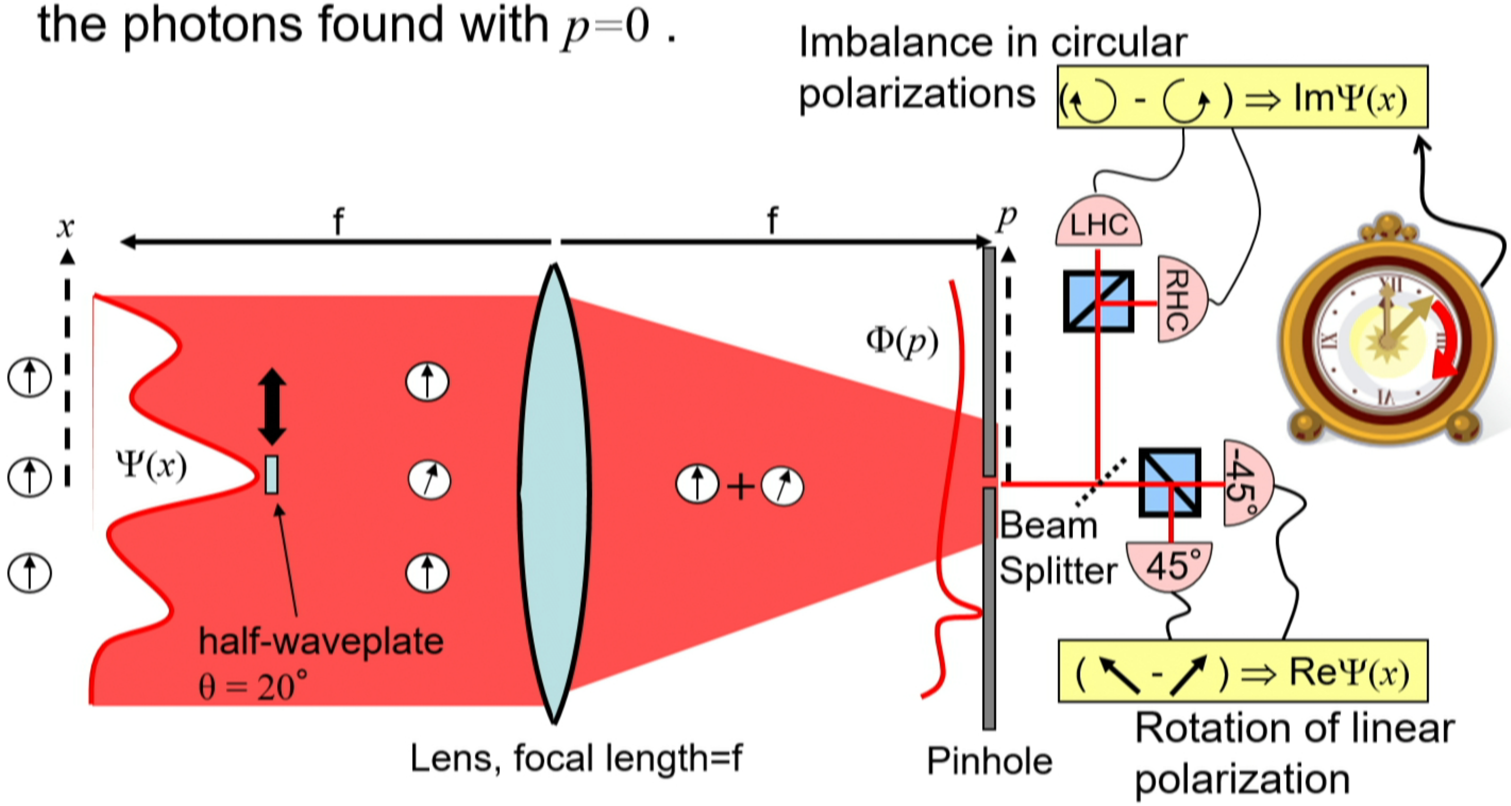
And if $p=0$,

$$\pi_w = \frac{1/\sqrt{2\pi}\cdot\langle x|\psi\rangle}{\sqrt{\mathrm{Prob}(p=0)}} = \boxed{k\cdot\psi(x)}$$

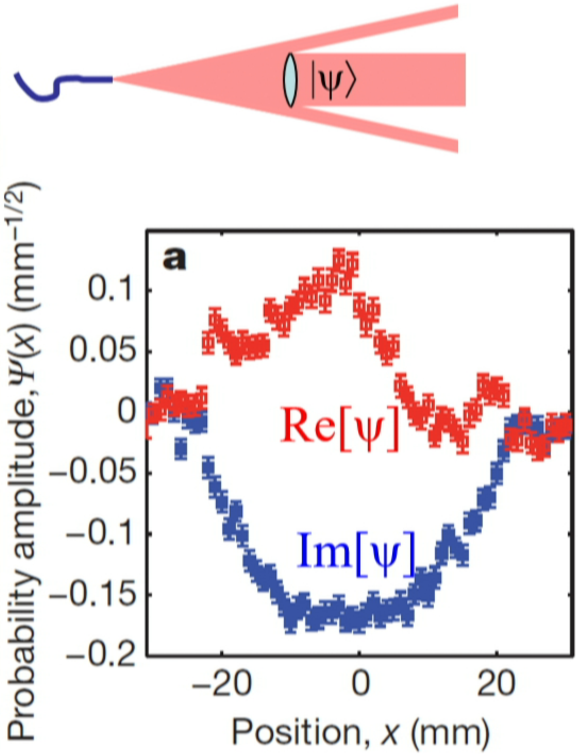
- **The average shift of the pointer (i.e. rotation of the polarization) is proportional to the wavefunction**

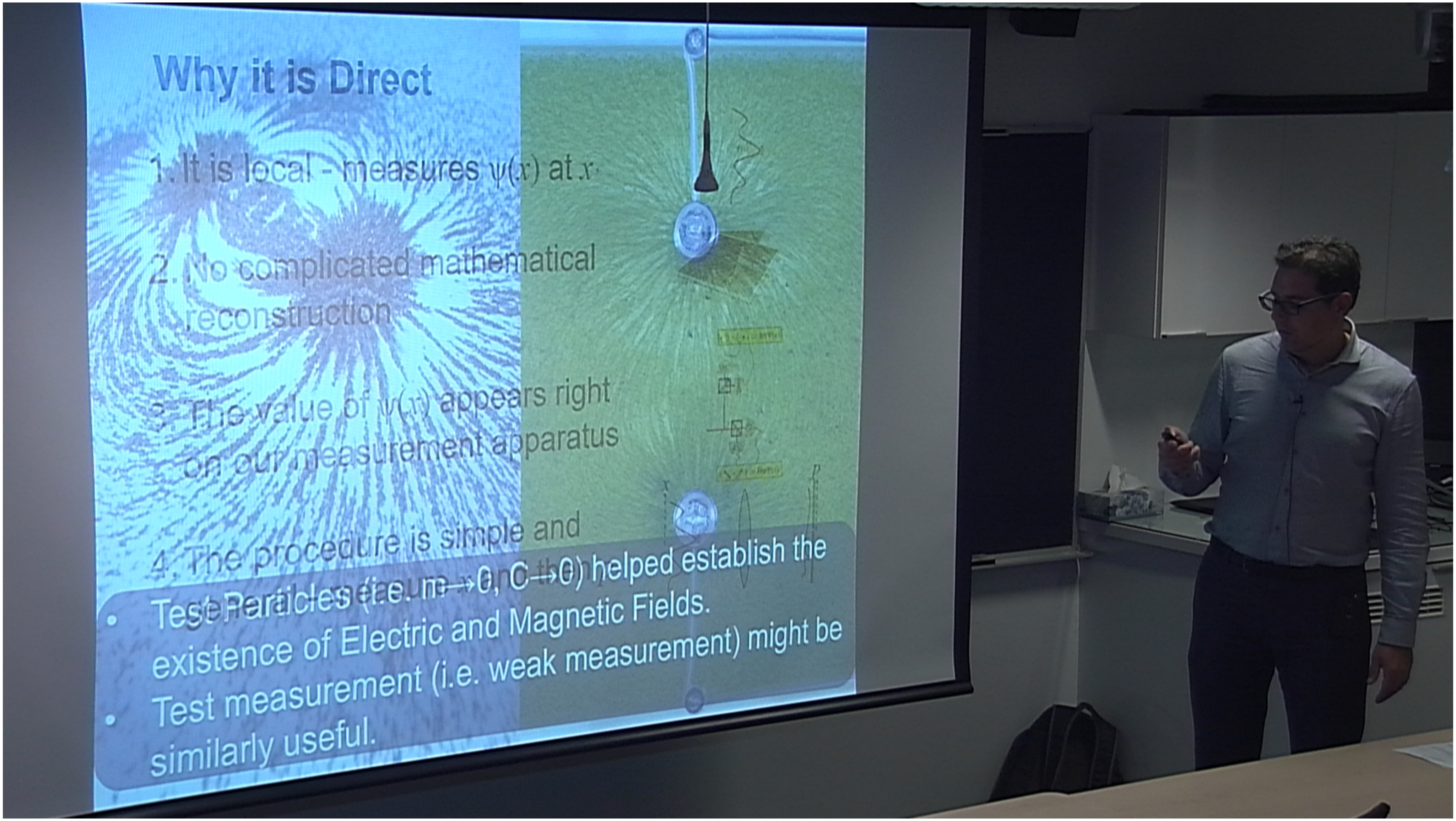
# Direct Measurement of the Wavefunction

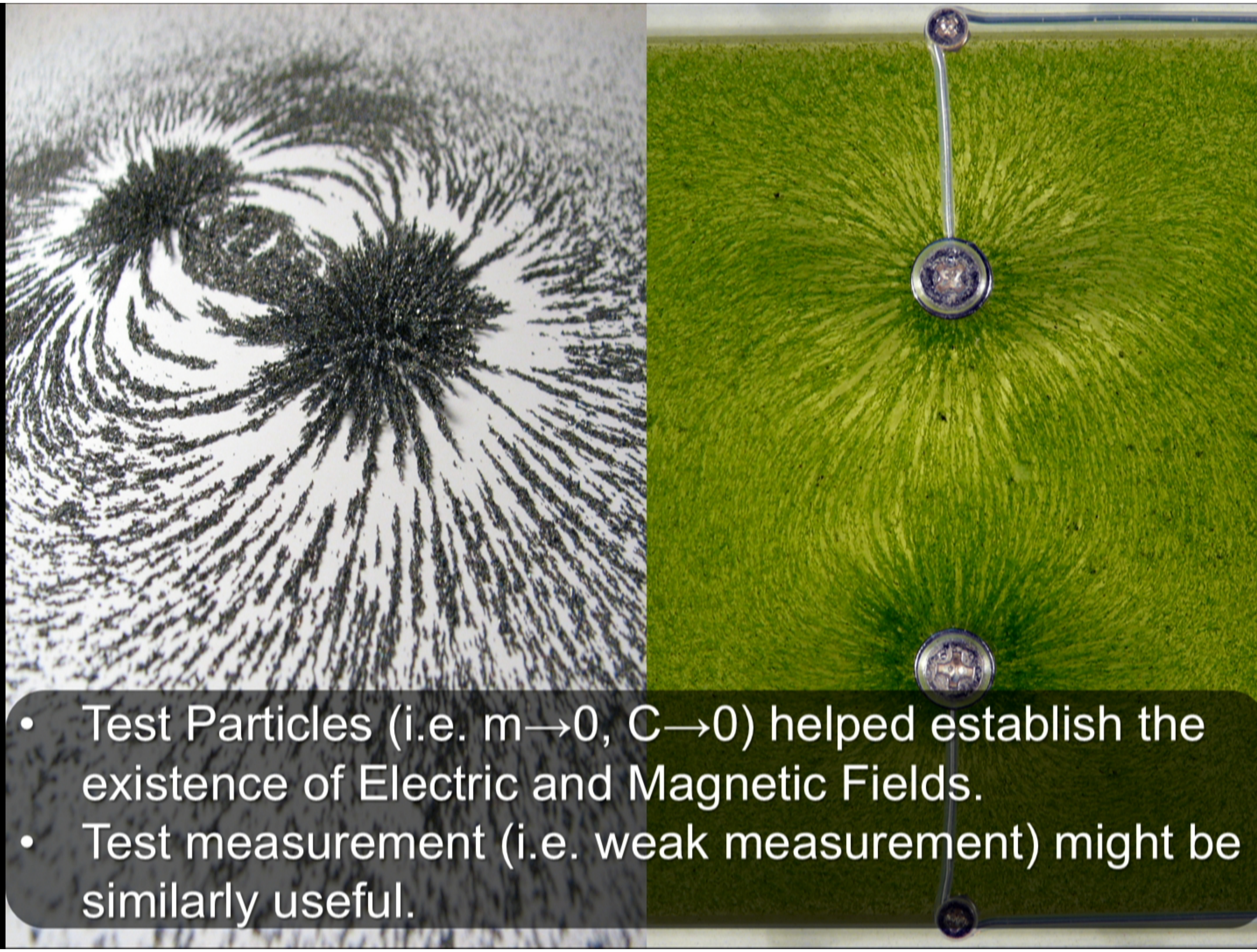
•Weakly measure $|x\rangle\langle x|$ then strongly measure $p$, and keep only the photons found with $p=0$ .



# Direct Measurement of the Wavefunction




Why it is Direct
1. It is local - measures ψ(x) at x
2. No complicated mathematical reconstruction
3. The value of ψ(x) appears right on our measurement apparatus
4. The procedure is simple and
Test Particles (i.e. m→0, C→0) helped establish the existence of Electric and Magnetic Fields.
Test measurement (i.e. weak measurement) might be similarly useful.



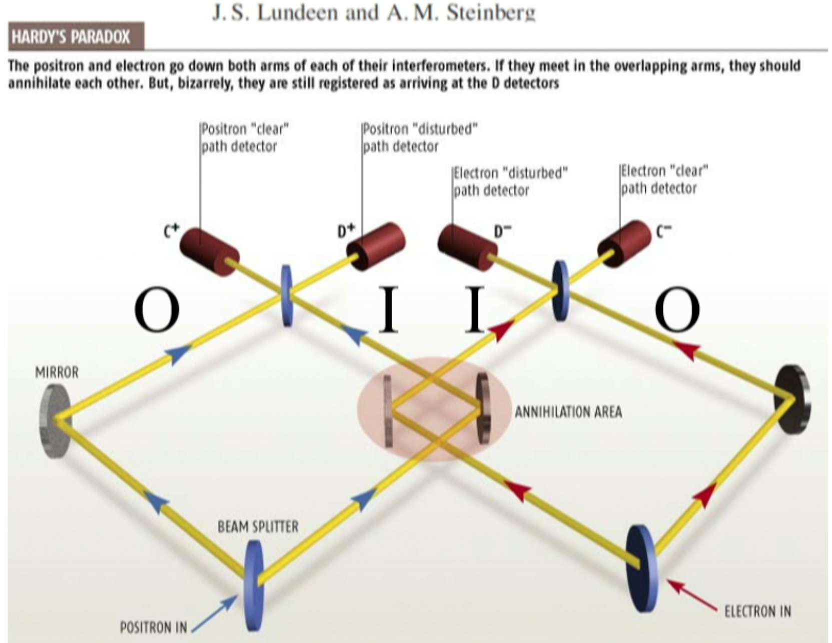
- Test Particles (i.e. m→0, C→0) helped establish the existence of Electric and Magnetic Fields.
- Test measurement (i.e. weak measurement) might be similarly useful.

# Direct Measurement of an Entangled Quantum State

PRL 102, 020404 (2009) PHYSICAL REVIEW LETTERS week ending 16 JANUARY 2009

**Experimental Joint Weak Measurement on a Photon Pair as a Probe of Hardy's Paradox**

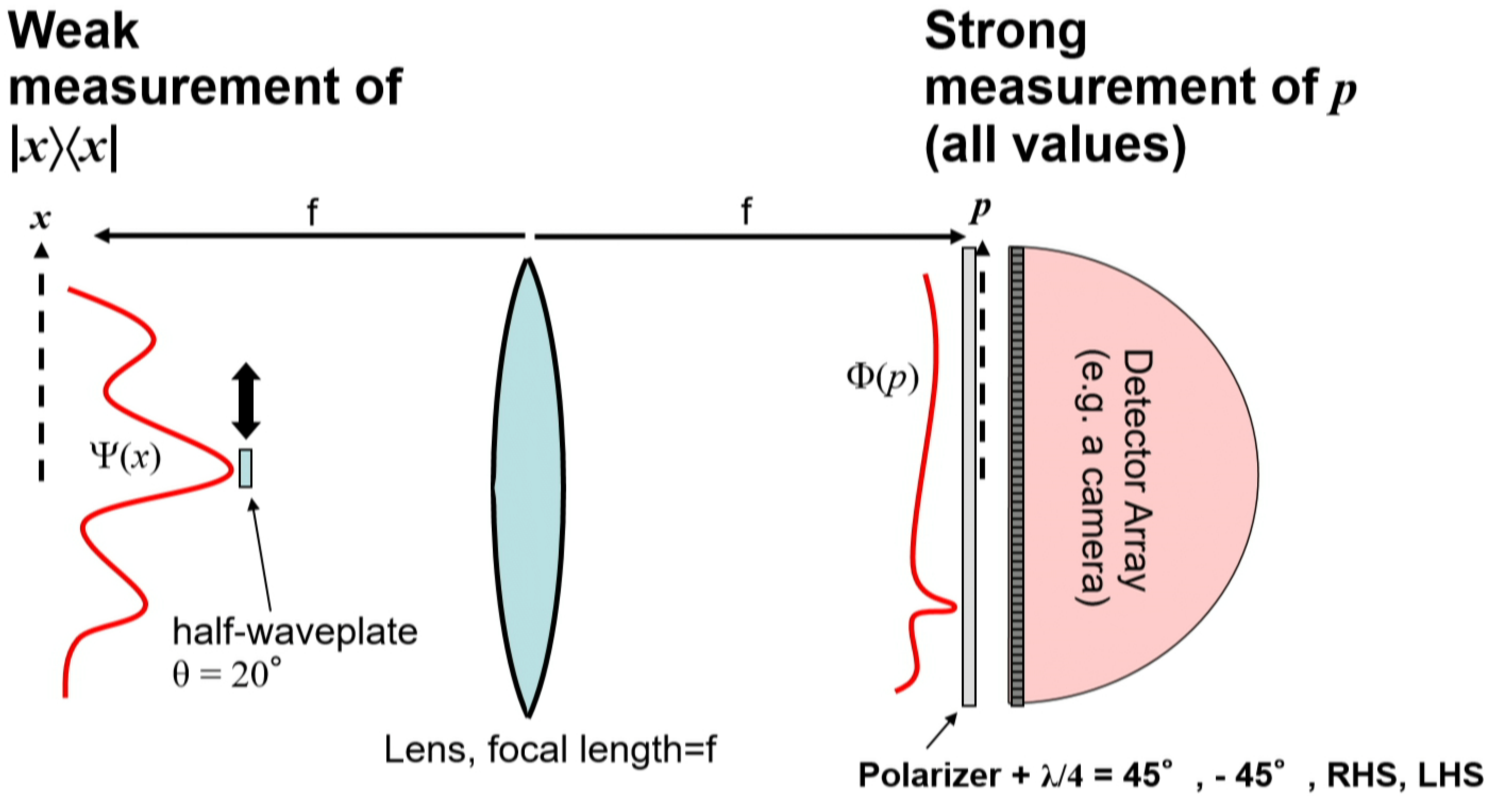
J. S. Lundeen and A. M. Steinberg




## Theoretical Quantum State:

$$|\psi\rangle = 1\,|IO\rangle + 1\,|OI\rangle - 1\,|OO\rangle + 0\,|II\rangle$$

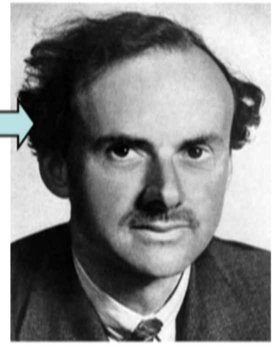
# Joint measurement of X and P



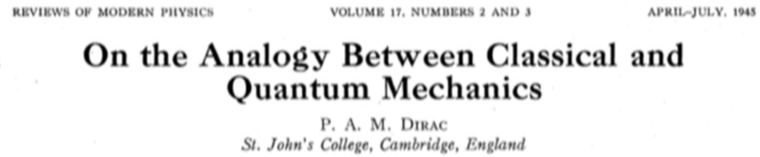

- Make a joint measurement of $|x\rangle\langle x|$ and $|p\rangle\langle p|$ to measure a mixed state $\rho$

# Dirac's Distribution

José Moyal re-invented the Wigner function



Paul Dirac thought it was a poor idea.



REVIEWS OF MODERN PHYSICS VOLUME 17, NUMBERS 2 AND 3 APRIL–JULY, 1945

**On the Analogy Between Classical and Quantum Mechanics**

P. A. M. DIRAC
*St. John's College, Cambridge, England*

$$D_\rho(x,p) = \langle p|\,|x\rangle\langle x|\, \boldsymbol{\rho}\, |p\rangle$$

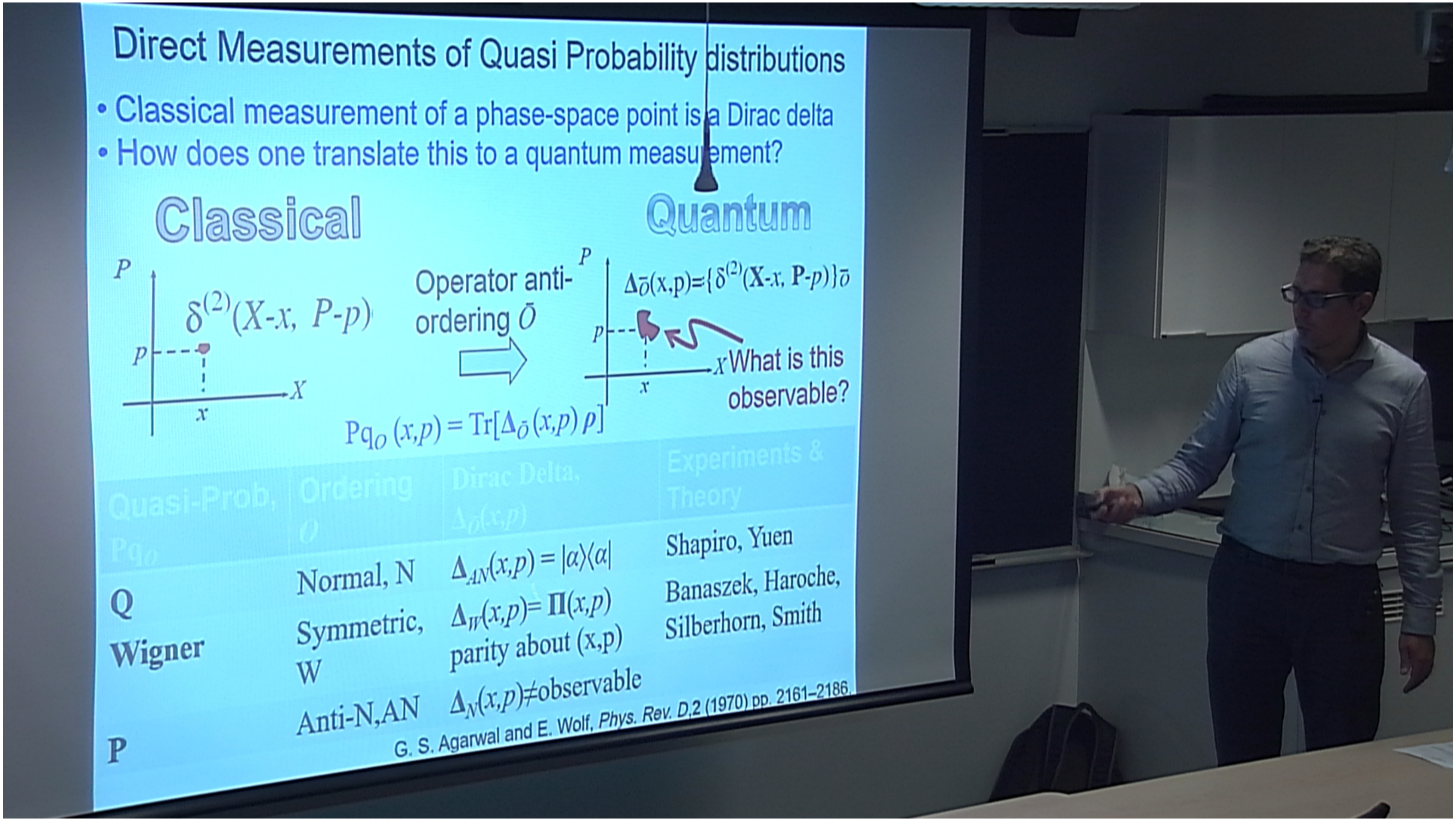
- In physics, the Dirac Distribution was forgotten as a theoretical novelty (There was no way to measure it!)

The distribution is complex!

- The Solution is *Weak Measurement*:
- We call the average result of a joint weak-strong A-B measurement the ***weak average*** = $\langle BA\rangle$

$$= \mathrm{Tr}[|p\rangle\langle p|\,|x\rangle\langle x|\, \boldsymbol{\rho}] = D_\rho(x,p)$$

Lundeen, Bamber, PRL 108, 070402 (2012)


Direct Measurements of Quasi Probability distributions
• Classical measurement of a phase-space point is a Dirac delta
• How does one translate this to a quantum measurement?
Classical
Quantum
$\delta^{(2)}(X-x,\ P-p)$
Operator anti-ordering $\bar{O}$
$\Delta_{\bar{O}}(x,p)=\{\delta^{(2)}(\mathbf{X}-x,\ \mathbf{P}-p)\}_{\bar{O}}$
What is this observable?
$\mathrm{Pq}_O(x,p) = \mathrm{Tr}[\Delta_{\bar{O}}(x,p)\,\rho]$
Quasi-Prob, $\mathrm{Pq}_O$
Ordering $O$
Dirac Delta, $\Delta_{\bar{O}}(x,p)$
Experiments & Theory
Q
Normal, N
$\Delta_{AN}(x,p) = |\alpha\rangle\langle\alpha|$
Shapiro, Yuen
Wigner
Symmetric, W
$\Delta_W(x,p)= \Pi(x,p)$ parity about (x,p)
Banaszek, Haroche, Silberhorn, Smith
P
Anti-N,AN
$\Delta_N(x,p)\neq$observable
G. S. Agarwal and E. Wolf, Phys. Rev. D,2 (1970) pp. 2161–2186.

# X-P ordered Quasi-Prob Distributions

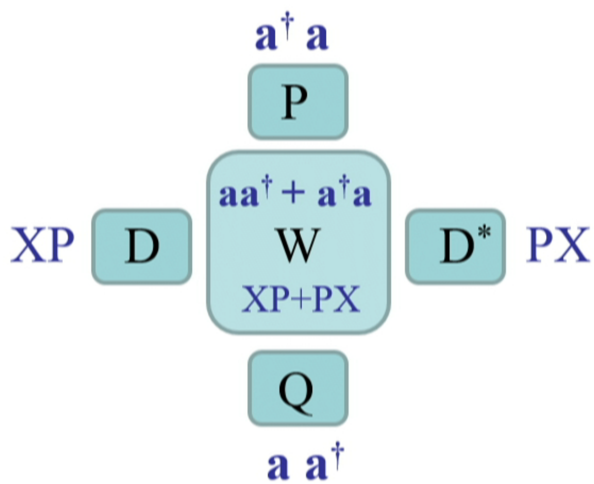
Standard S: **X** to the left of **P**

Anti-Standard AS: **P** to the left of **X**

$$\boldsymbol{\Delta}_{AS}(x,p) = \{\delta^{(2)}(\mathbf{X}-x,\ \mathbf{P}-p)\}_S$$
$$= \delta(\mathbf{P}-p)\delta(\mathbf{X}-x,)$$
$$= |p\rangle\langle p||x\rangle\langle x|$$

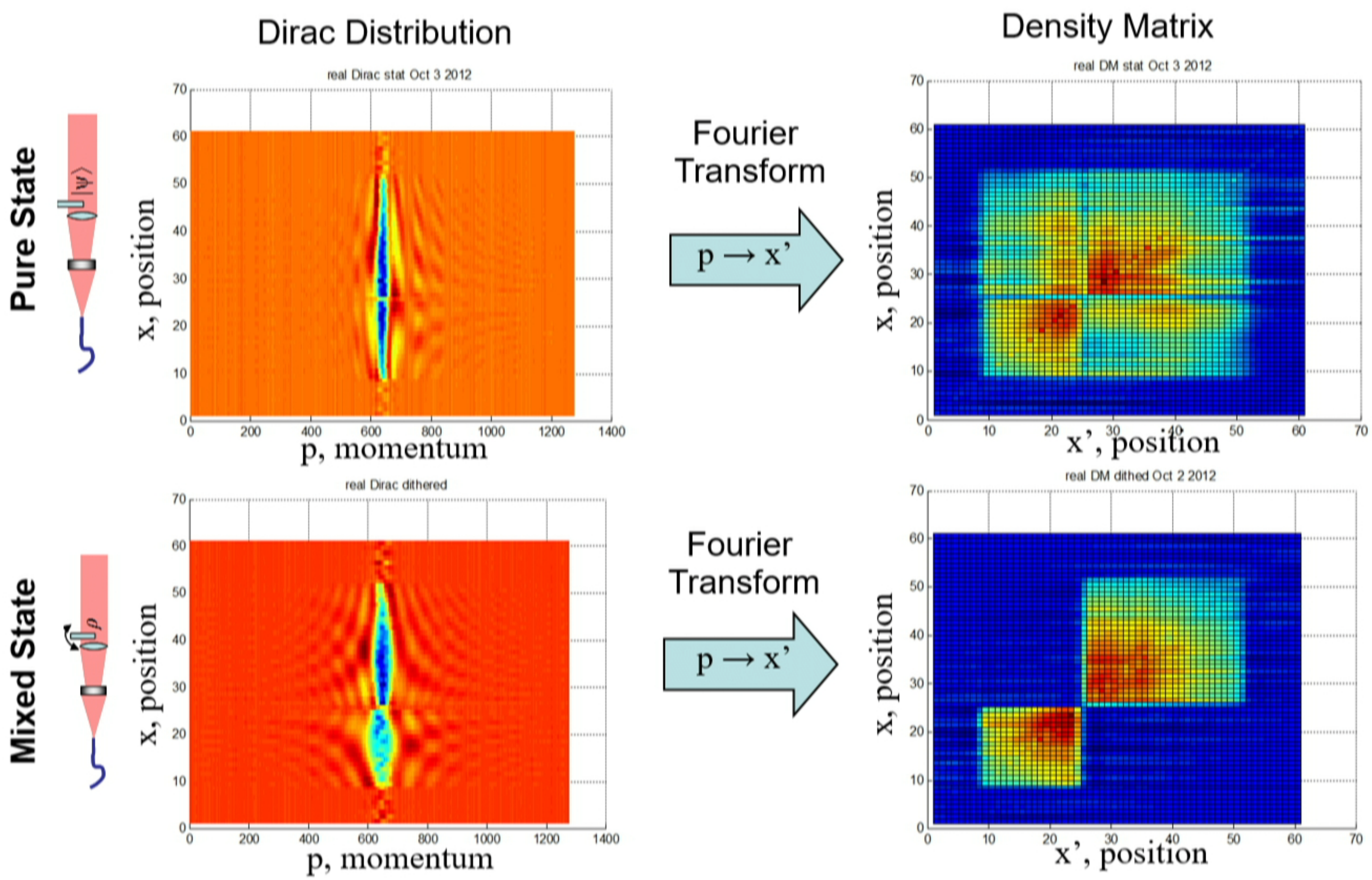
$$Pq_S\,(x,p) = \mathrm{Tr}[|p\rangle\langle p|x\rangle\langle x|\,\rho] = \langle p|x\rangle\langle x|\,\rho\,|p\rangle = D_\rho(x,p)$$

## Constellation of Quasi-Prob Distibutions




- The Dirac Distribution fits within the textbook definition of a quasi-probability

# Direct Measurement of Mixed Quantum States



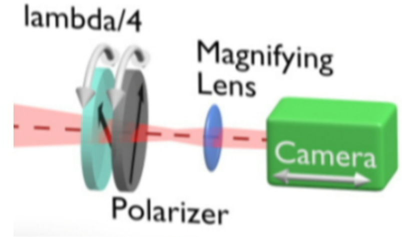

- **Simple Generalization allows us to completely measure mixed states**

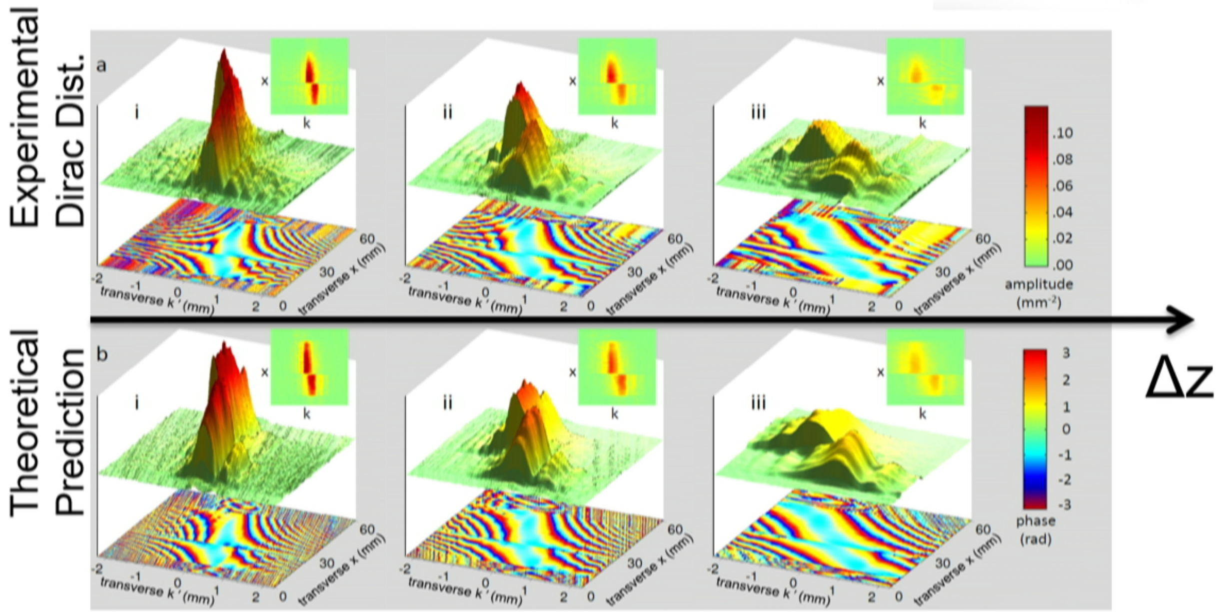
# Bayesian Propagation of the Dirac Distribution

H. F. Hofmann, New Journal of Physics, 14, 043031 (2012):

Use Baye's law to propagate the Dirac Distribution

Move camera by Δz to allow the Dirac Distribution to evolve under free propagation before the strong measurement

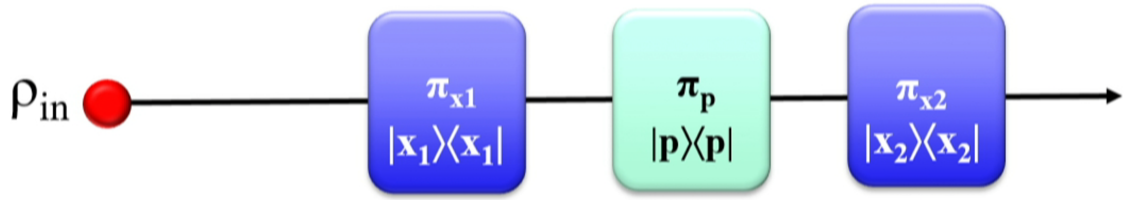



- The experiment confirms that the Dirac Distribution evolves in much the same way that a classical probability distribution evolves

Naïve strategy #3:
What if we gently measure many complementary variables in a row?

- e.g. Jointly measure X then P then X again?
- Use weak measurement!



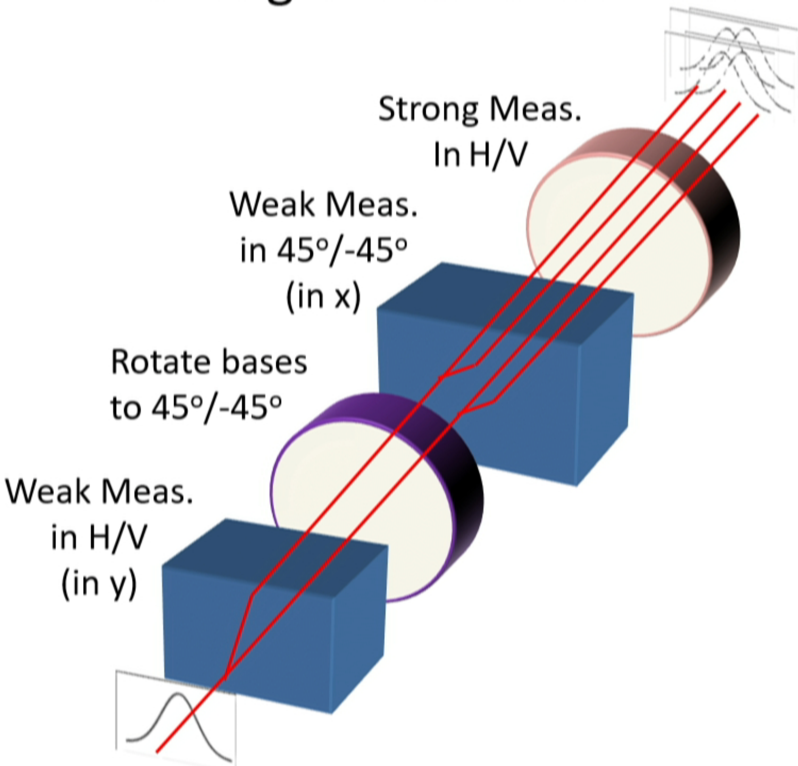

Average result is $\mathrm{Tr}[\pi_{x1}\, \pi_p\, \pi_{x2}\, \rho_{in}] = \rho_{in}(x_1,x_2)$

- We can know any chosen element $\rho_{in}(x_1,x_2)$ of the density matrix? e.g. a particular coherence, entanglement witnesses, etc.

Theory: Lundeen & Bamber PRL 108, 070402 (2012).

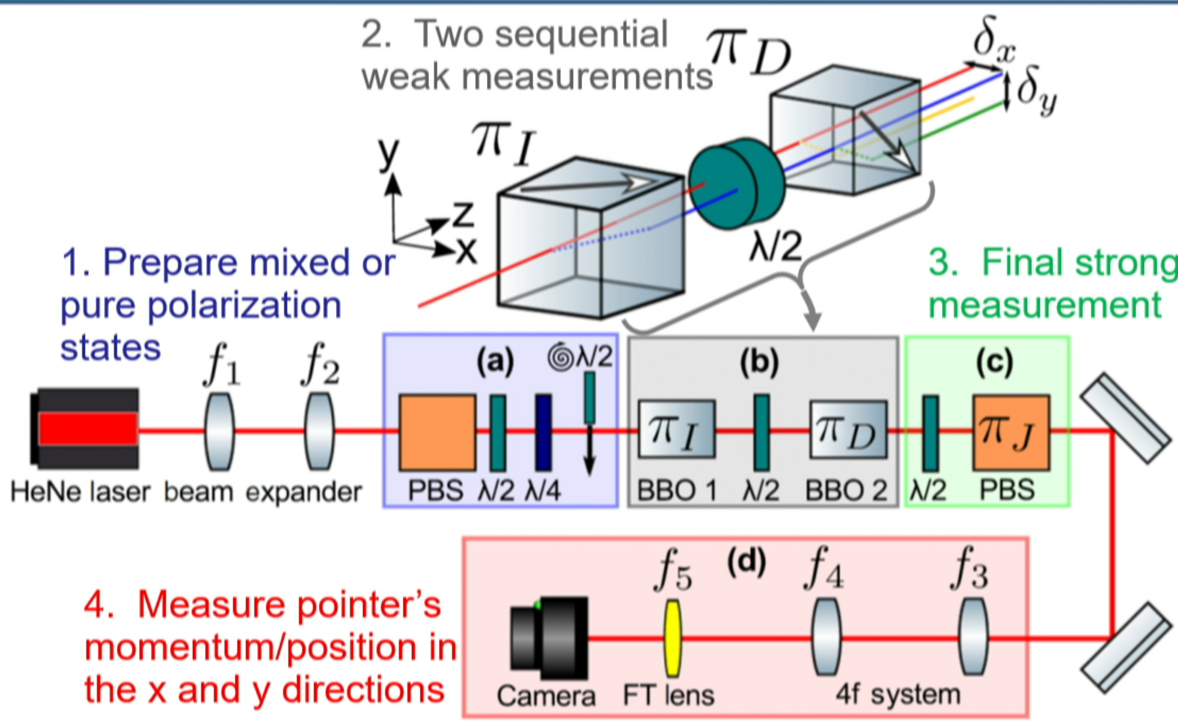
# Direct Measurement of the Density Matrix

Use two sequential weak measurements and then a strong one to obtain each density matrix element




GS Thekkadath,…, JS Lundeen, PRL 117, 120401 (2016)

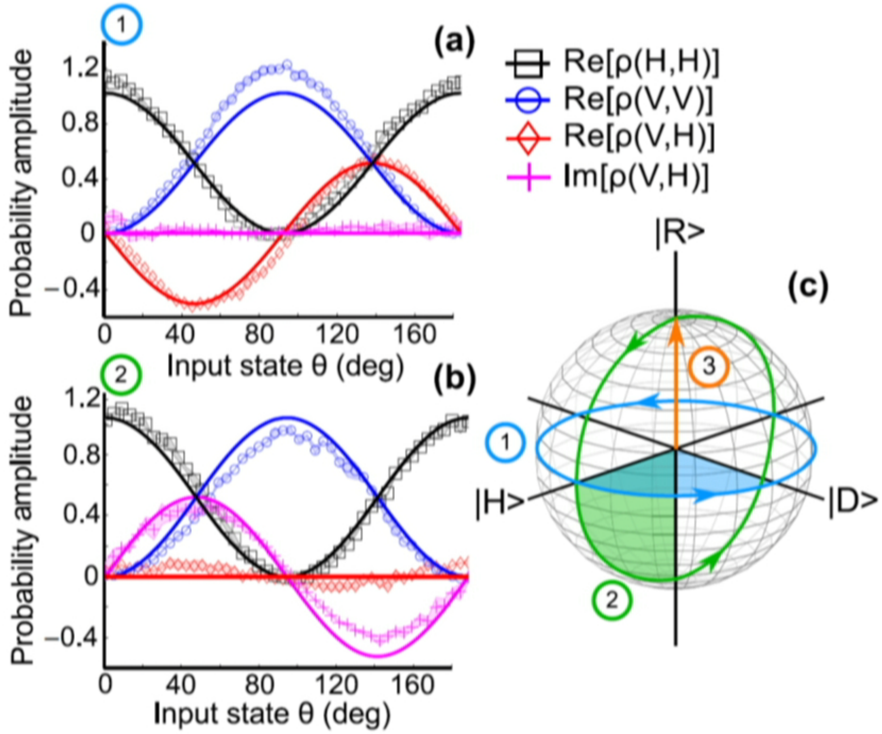
# Experimental Setup



# Direct Measurement of the Density Matrix

Max Planck - University of Ottawa Center for Extreme and Quantum Photonics

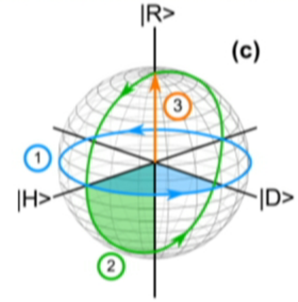
Measuring the density matrix, traversing the Poincare Sphere

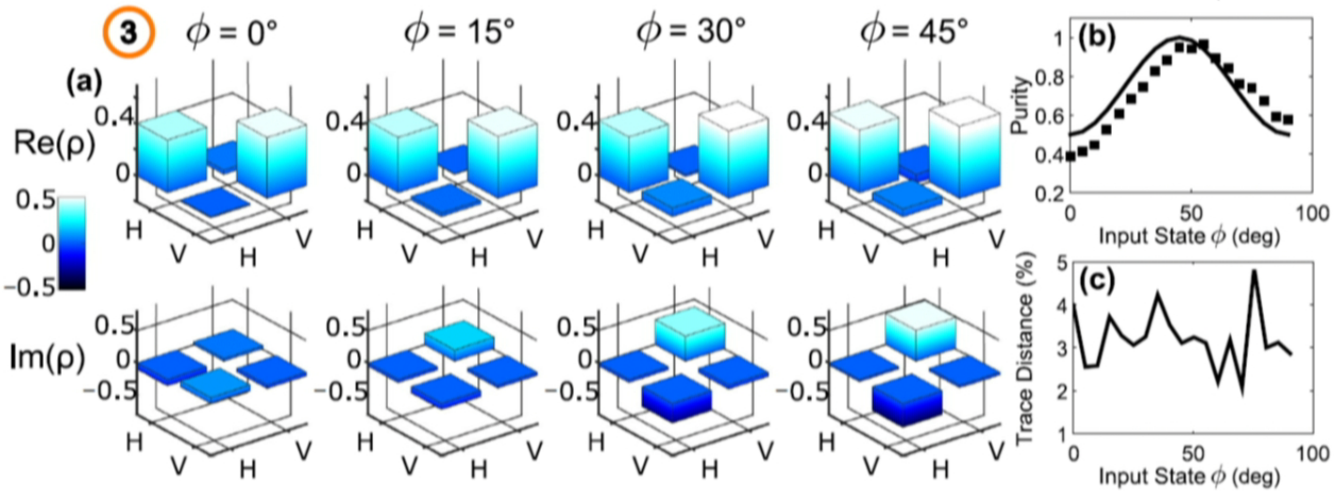



For the pure states along paths 1 and 2, the theoretical and experimental density matrix elements agree well.

# Direct Measurement of the Density Matrix

To test mixed states, we vary our input state along path 3 in the Poincare Sphere



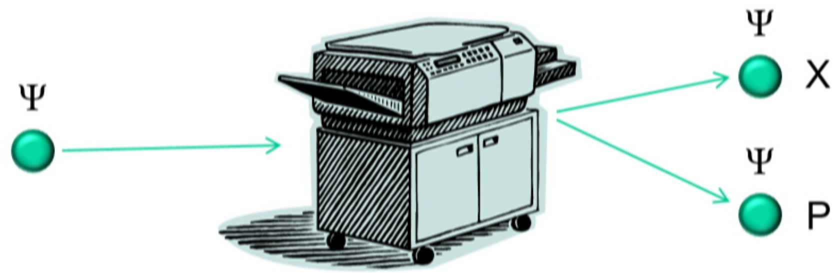



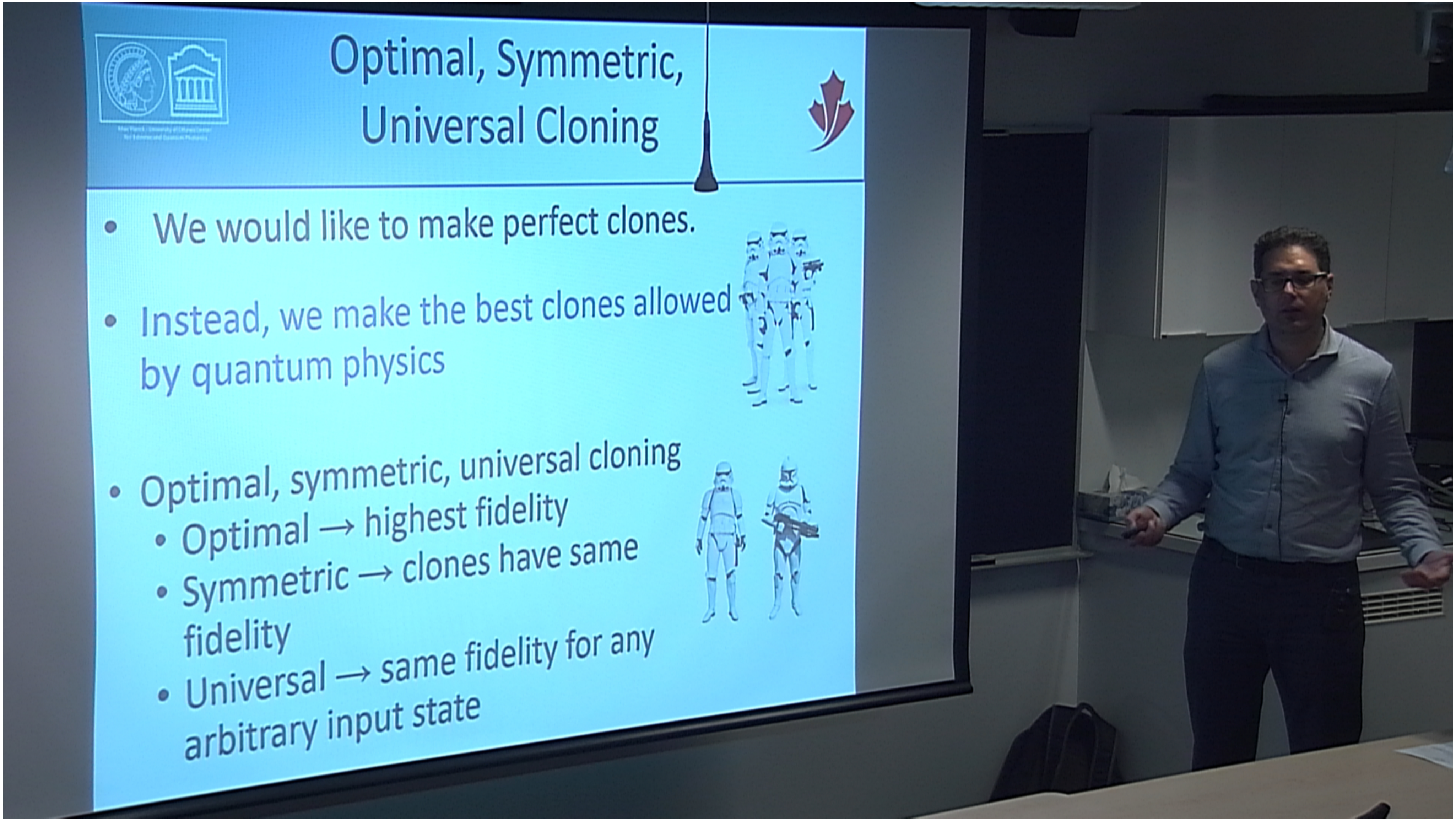

- The measured and expected purities match
- The trace distance between the expected and measured density matrices is less than 0.05

# Naïve Strategy #4: Copy then Measure

What happens when you make complementary measurements (i.e. X & P) on copies of a particle?


$$|\Psi\rangle_1 \rightarrow |\Psi\rangle_1 \, |\Psi\rangle_2$$




Optimal, Symmetric, Universal Cloning
We would like to make perfect clones.
Instead, we make the best clones allowed by quantum physics
Optimal, symmetric, universal cloning
Optimal → highest fidelity
Symmetric → clones have same fidelity
Universal → same fidelity for any arbitrary input state

# General Theory for Optimal Cloning

Two clones are created by projecting the input state and a completely mixed state onto the symmetric subspace

$$\rho_{12} = [\Pi^{+}(\rho_{in} \otimes \mathbf{I})\Pi^{+}]/\text{Norm},$$

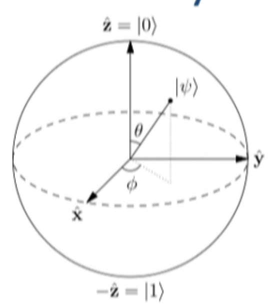
$$\text{where } \rho_{in} = |\Psi\rangle\langle\Psi| \text{ and } \mathbf{I}\text{=identity}$$

Each clone is in a mixed state,

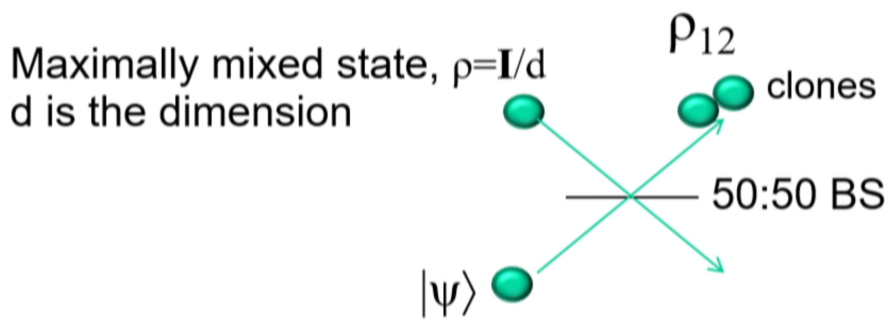
$$\rho_1 = \text{Tr}_2[\rho_{12}] = k(d)\, |\Psi\rangle\langle\Psi| + [1\text{-}k(d)]\, \mathbf{I}/d,$$

$$\text{where } k(d) = (2+d)/(2+2d)$$

For d=2, the Bloch vector shrinks by a factor = 2/3



# Optimal Cloning Device: the beamsplitter



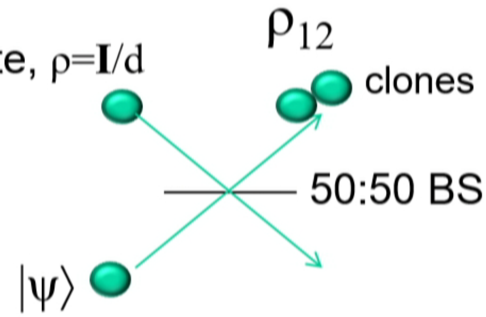

Irvine,…, 92, 047902, Bouwmeester, PRL (2004)

- Consider two photons entering opposite ports of a beamsplitter
- When alike they always bunch, exiting one port together

## Joint Measurements on Optimal Clones

# Optimal Cloning Device: the beamsplitter

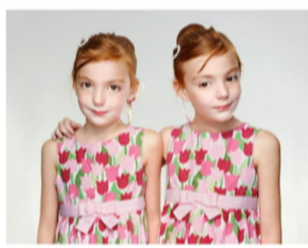
Maximally mixed state, $\rho = \mathbf{I}/d$
d is the dimension




Irvine,…, 92, 047902,
Bouwmeester, PRL (2004)

- Consider two photons entering opposite ports of a beamsplitter
- When alike they always bunch, exiting one port together

Mixed state is $|\psi\rangle$
50% of the time
(perfect cloning)



# Optimal Cloning Device: the beamsplitter

Maximally mixed state, $\rho=\mathbf{I}/d$
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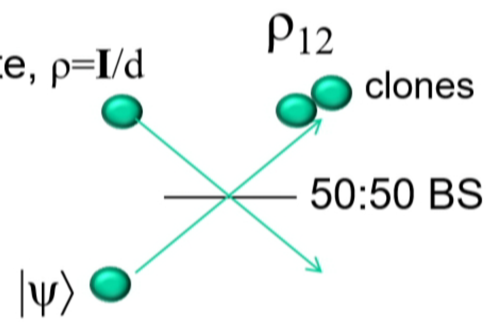


Irvine,…, 92, 047902,
Bouwmeester, PRL (2004)

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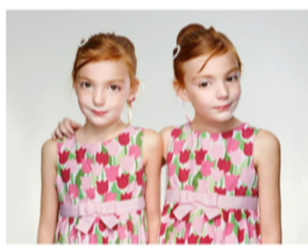
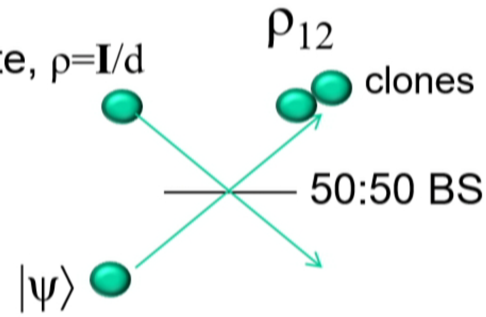


And $|\psi^{\perp}\rangle$ 50% of the time (imperfection!)

## Joint Measurements on Optimal Clones

# Optimal Cloning Device: the beamsplitter

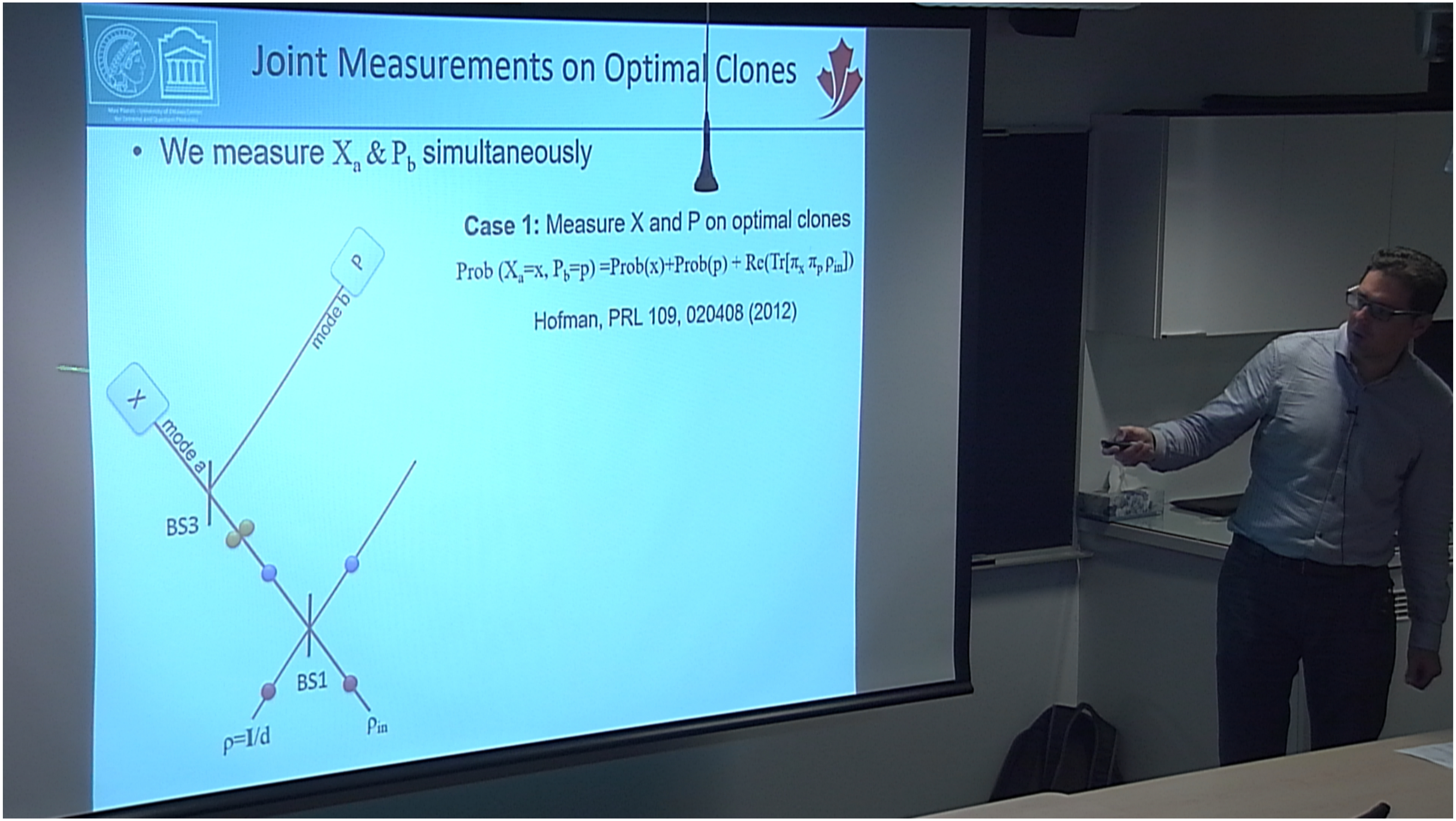
Maximally mixed state, $\rho = \mathbf{I}/d$
d is the dimension




Irvine,…, 92, 047902, Bouwmeester, PRL (2004)

- Consider two photons entering opposite ports of a beamsplitter
- When alike they always bunch, exiting one port together

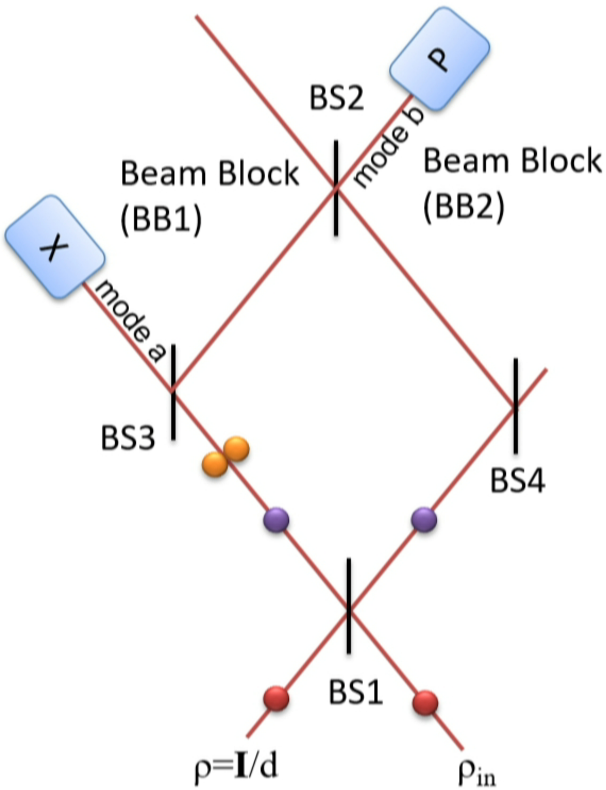
| Permutation Symmetry | States | # of photons at a output | Field Parity |
|---|---|---|---|
| Anti-symmetric ($\Pi^-$) | $\lvert HV\rangle - \lvert VH\rangle$ | 1 | odd |
| Symmetric ($\Pi^+$) | $\lvert HV\rangle + \lvert VH\rangle$<br>$\lvert HH\rangle$<br>$\lvert VV\rangle$ | 2 | even |


Joint Measurements on Optimal Clones
• We measure $X_a$ & $P_b$ simultaneously
Case 1: Measure X and P on optimal clones
$\text{Prob}(X_a=x, P_b=p) = \text{Prob}(x)+\text{Prob}(p) + \text{Re}(\text{Tr}[\pi_x \pi_p \rho_{in}])$
Hofman, PRL 109, 020408 (2012)
P
mode b
X
mode a
BS3
BS1
$\rho=\mathbb{I}/d$
$\rho_{in}$

# Joint Measurements on Optimal Clones

- We measure $X_a$ & $P_b$ simultaneously

Cernoch et al. PRL 100,180501(2008)




**Case 1:** Measure X and P on optimal clones

$$\text{Prob}\,(X_a=x, P_b=p) = \text{Prob}(x)+\text{Prob}(p) + \text{Re}(\text{Tr}[\pi_x\, \pi_p\, \rho_{in}])$$

Hofman, PRL 109, 020408 (2012)

**Case 2:** Mix cloning ($\Pi^+$ projection) with $\Pi^-$ projection: $\Pi^- + i\Pi^+ = \sqrt{}$SWAP gate.

Measure X and P on two photons:

$$\text{Prob}\,(X=x, P=p) = \text{Prob}(x)+\text{Prob}(p) + \text{Im}(\text{Tr}[\pi_x\, \pi_p\, \rho_{in}])$$

# Cloning and the SWAP Gate

- The SWAP gate exchanges the state of two particles


- SWAP, S can be written in terms of symmetric projector (our **optimal cloner**!)

$$\tfrac{1}{2}(I + S) = \Pi^{+}$$

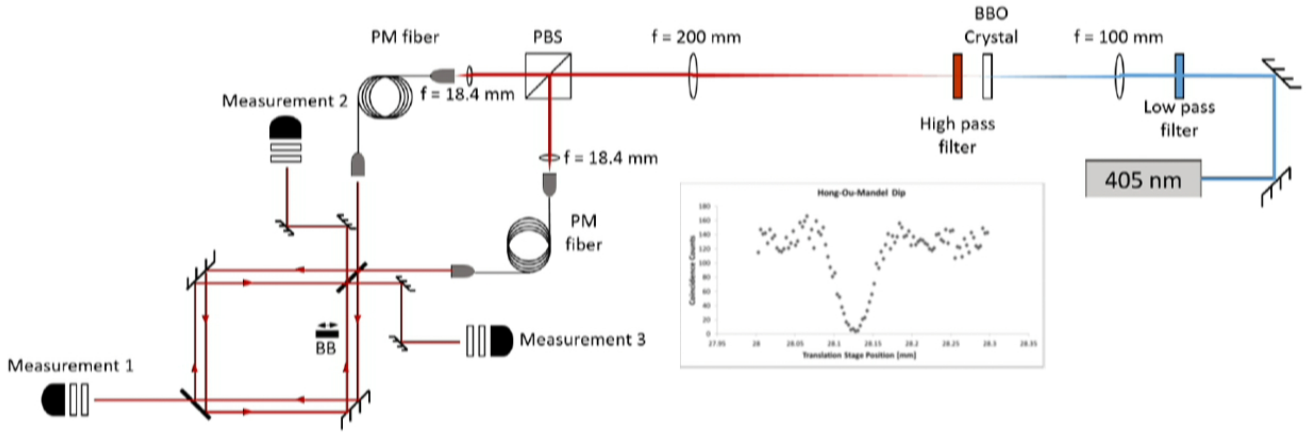
- Square root SWAP is $\sqrt{S} = 1/\sqrt{2}\,(I \pm i\,S) = \Pi^{\pm i}$

$$\text{or } \Pi^{\pm i} = \Pi^{+} \pm i\,\Pi^{-}$$

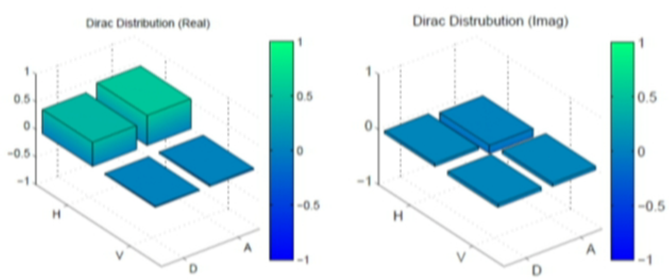
$$Re(D(x,p)) = Prob(x1, p2|\,\Pi^{+}) - Prob(x1, p2|\,\Pi^{-})$$
$$Im(D(x,p)) = Prob(x1, p2|\,\Pi^{+i}) - Prob(x1, p2|\,\Pi^{-i})$$

- The Dirac Distribution appears to be intimately related to symmetries in optimal Cloning

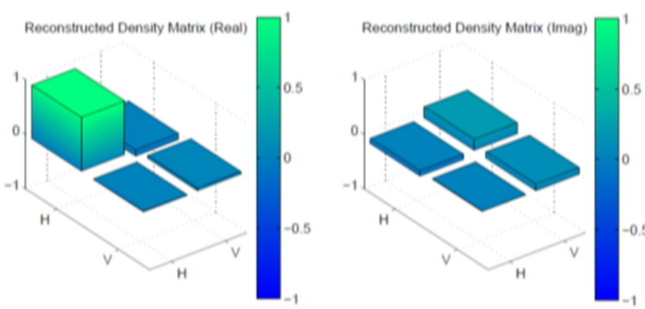
# Experimental Setup




Dirac Distribution for Target State $|H\rangle$




Reconstructed Density Matrix for Target State $|H\rangle$

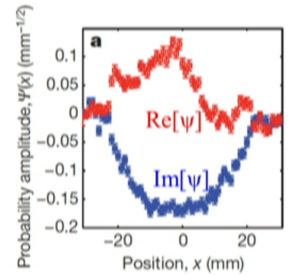



- For an H polarized input photon, we experimentally observe that joint measurements on clones give the Dirac Distribution

# Conclusions

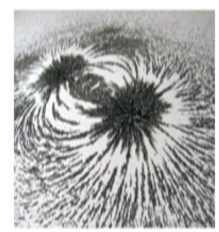
- Joint measurements complementary variables can completely characterize a state by measuring its:

1. Q-function (locally)
2. Wavefunction (locally)
3. Dirac Distribution (mixed states)
4. Density matrix (mixed states, one element at a time)




- Weak measurement and the Dirac Distribution appear to be related to symmetries in optimal cloning (Why??)


- Direct measurement gives quantum states an operational meaning: *in terms of a set of procedures in the laboratory*



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