Title: Direct experimental reconstruction of the Bloch sphere

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Abstract: The scientific journey from the first hints of quantum behaviour to the Bloch sphere in your textbook was a long and tortuous one. But using some of the technological and conceptual fruits of that journey, we show that an experiment can manifest the Bloch sphere via an analysis that doesn't require any quantum theory at all. Our technique is to fit experimental data to a generalised probabilistic theory, which allows us to infer both the dimension and shape of the state and measurement spaces of the system under study. We test our technique on an experiment measuring a variety of single-photon polarization states. As expected, the reconstructed state space closely resembles the Bloch sphere, and we are able to place small upper bounds on how much the true theory describing our experiment could possibly deviate from quantum mechanics.

Generalised probabilistic theories

PHYSICAL REVIEW A **75**, 032304 (2007)

Information processing in generalized probabilistic theories

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(Received 25 May 2006; published 5 March 2007)

I introduce a framework in which a variety of probabilistic theories can be defined, including classical and quantum theories, and many others. From two simple assumptions, a tensor product rule for combining separate systems can be derived. Certain features, usually thought of as specifically quantum, turn out to be generic in this framework, meaning that they are present in all except classical theories. These include the nonunique decomposition of a mixed state into pure states, a theorem involving disturbance of a system on measurement (suggesting that the possibility of secure key distribution is generic), and a no-cloning theorem. Two particular theories are then investigated in detail, for the sake of comparison with the classical and quantum cases. One of these includes states that can give rise to arbitrary nonsignaling correlations, including the superquantum correlations that have become known in the literature as nonlocal machines or Popescu-Rohrlich boxes. By investigating these correlations in the context of a theory with well-defined dynamics, I hope to make further progress with a question raised by Popescu and Rohrlich, which is why does quantum theory not allow these strongly nonlocal correlations? The existence of such correlations forces much of the dynamics in this theory to be, in a certain sense, classical, with consequences for teleportation, cryptography, and computation. I also investigate another theory in which all states are local. Finally, I raise the question of what further axiom(s) could be added to the framework in order to identify quantum theory uniquely, and hypothesize that quantum theory is optimal for computation.

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I. INTRODUCTION

A question periodically raised is what is responsible for the power of quantum computation (or cryptography, or information processing in general)? At a recent meeting in

broad range of different theories can be defined. The framework, described in Secs. II–IV, is based on that used by Hardy in his derivation of quantum theory from simple axioms [14]. The basic idea is that a state is represented as a vector of probabilities of measurement outcomes. Transfor-

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Quantum Theory From Five Reasonable Axioms

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Selected for a Viewpoint in *Physics*

PHYSICAL REVIEW A **84**, 012311 (2011)

Informational derivation of quantum theory

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We derive quantum theory from purely informational principles. Five elementary axioms—causality, perfect distinguishability, ideal compression, local distinguishability, and pure conditioning—define a broad class of theories of information processing that can be regarded as standard. One postulate—purification—singles out quantum theory within this class.

DOI: 10.1103/PhysRevA.84.012311 PACS number(s): 03.67.Ac, 03.65.Ta

I. INTRODUCTION

the very beginning. Von Neumann himself expressed

Generalised probabilistic theories

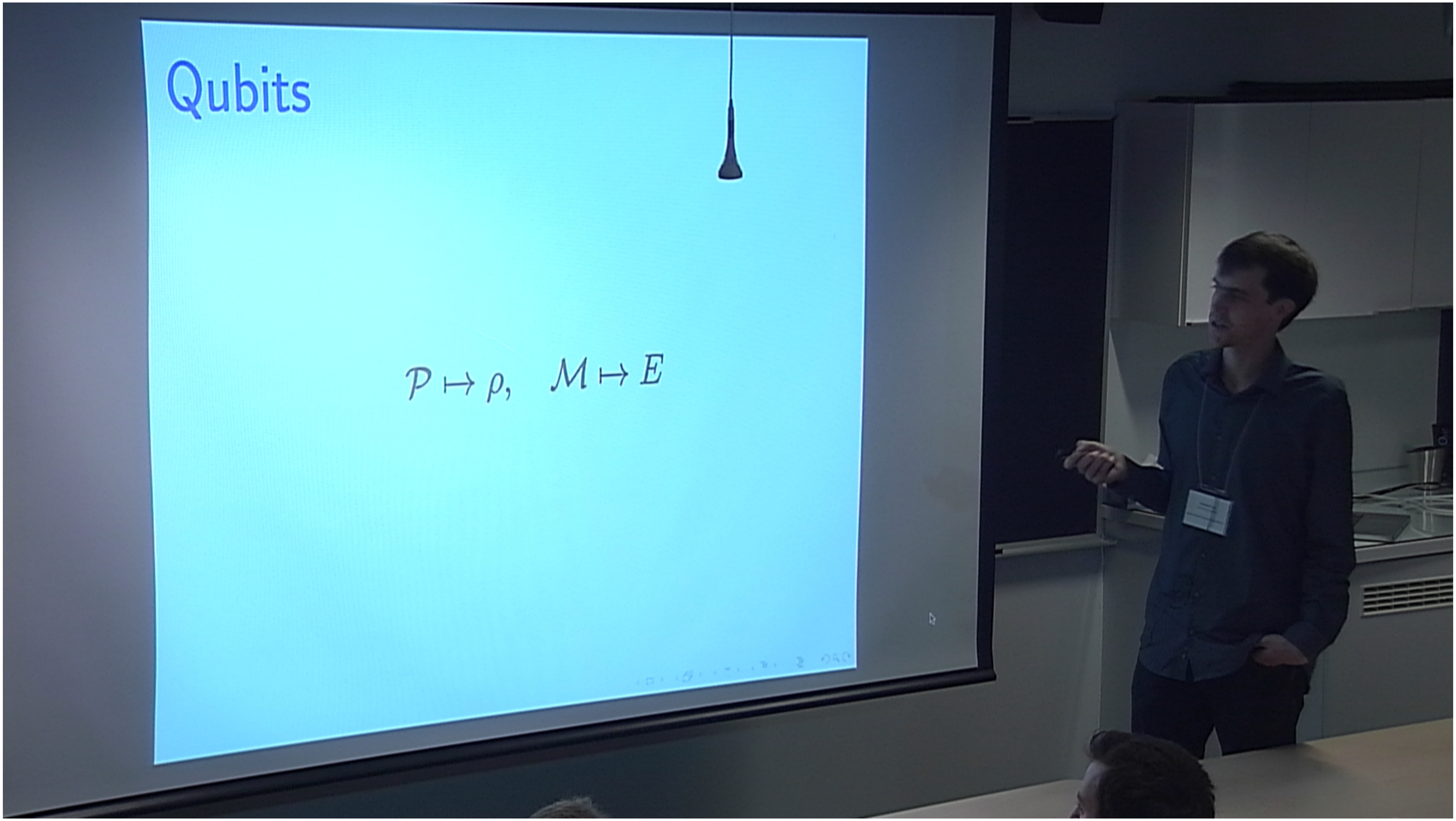
$$p(k|\mathcal{P}_i, \mathcal{M}_j)$$

Generalised probabilistic theories

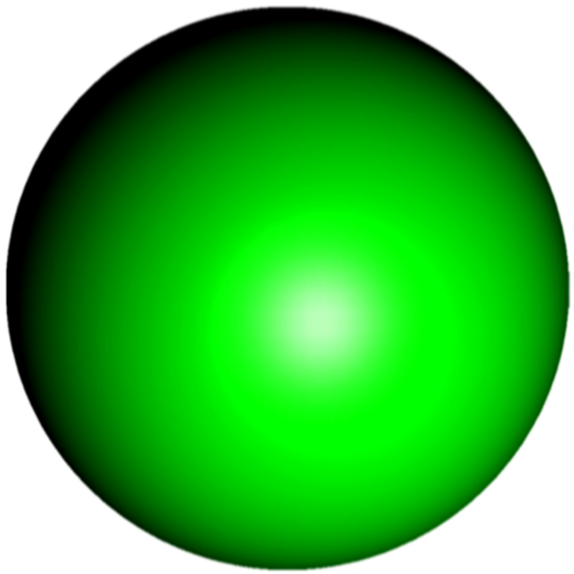
$$\begin{pmatrix} 1 & p(0|\mathcal{P}_1,\mathcal{M}_1) & p(0|\mathcal{P}_1,\mathcal{M}_2) & \dots \\ 1 & p(0|\mathcal{P}_1,\mathcal{M}_1) & p(0|\mathcal{P}_1,\mathcal{M}_2) & \dots \\ 1 & p(0|\mathcal{P}_2,\mathcal{M}_1) & p(0|\mathcal{P}_2,\mathcal{M}_2) & \dots \\ 1 & p(0|\mathcal{P}_3,\mathcal{M}_1) & p(0|\mathcal{P}_3,\mathcal{M}_2) & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

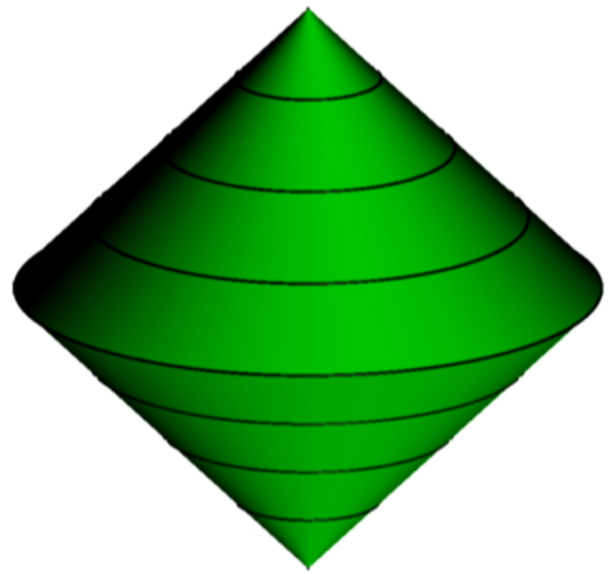
Generalised probabilistic theories

$$\begin{pmatrix} 1 & P_1^{(1)} & P_1^{(2)} & P_1^{(3)} \\ 1 & P_2^{(1)} & P_2^{(2)} & P_2^{(3)} \\ 1 & P_3^{(1)} & P_3^{(2)} & P_3^{(3)} \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix} \begin{pmatrix} 1 & M_1^{(0)} & M_2^{(0)} & \dots \\ 0 & M_1^{(1)} & M_2^{(1)} & \dots \\ 0 & M_1^{(2)} & M_2^{(2)} & \dots \\ 0 & M_1^{(3)} & M_2^{(3)} & \dots \end{pmatrix}$$


Qubits
$\mathcal{P} \mapsto \rho, \quad \mathcal{M} \mapsto E$

Qubits

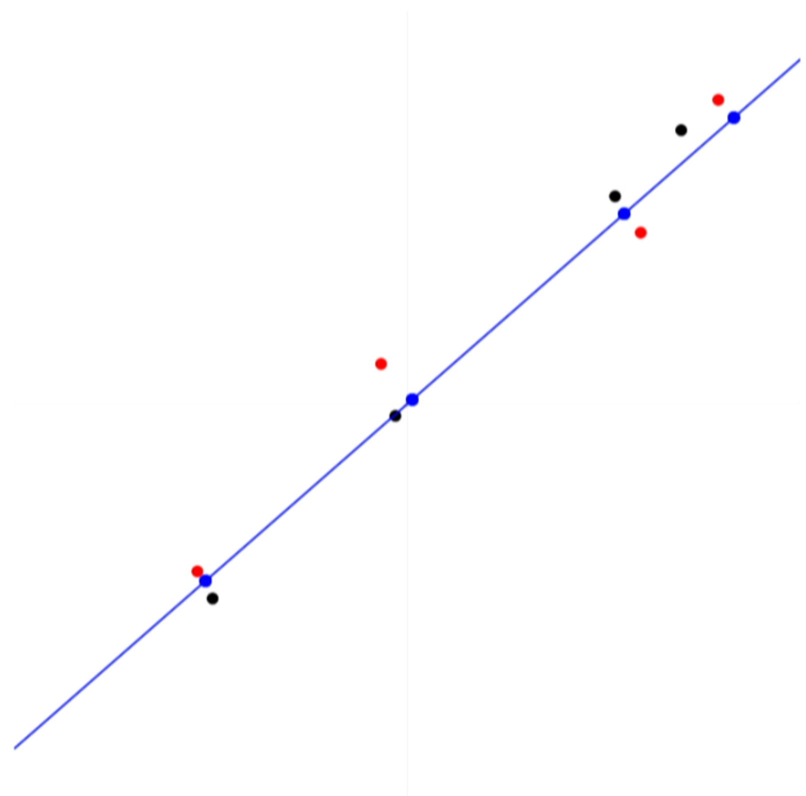




$$\left(P^{(1)}\right)^2 + \left(P^{(2)}\right)^2 + \left(P^{(3)}\right)^2 \leq 1$$

$$\left(M^{(1)}\right)^2 + \left(M^{(2)}\right)^2 + \left(M^{(3)}\right)^2 \leq \left(M^{(0)}\right)^2$$

Rank of data

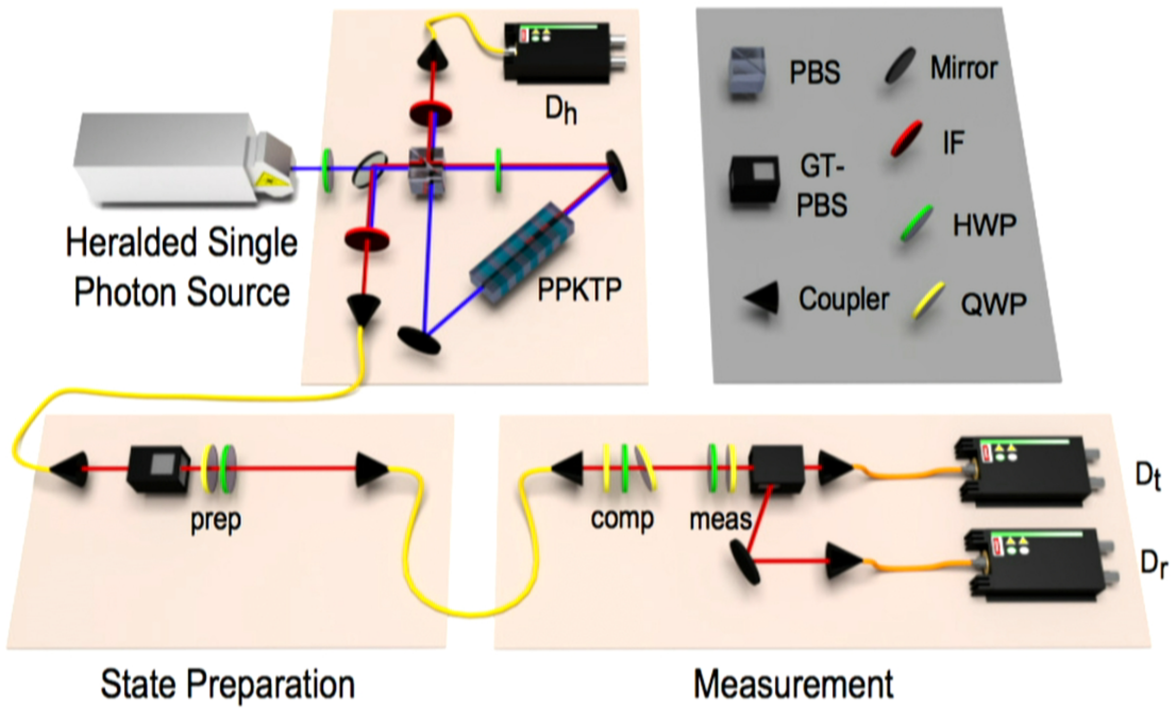


Rank of data

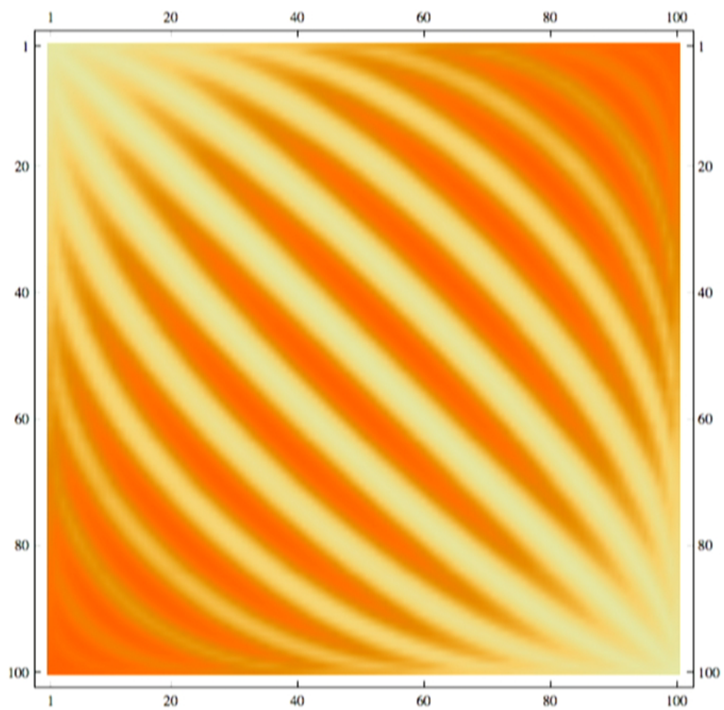
$$\text{minimize} \quad \sum_{i,j} \frac{(D_{i,j} - F_{i,j})^2}{(\sigma_{i,j})^2}$$

$$\text{subject to} \quad \mathrm{rank}(F) = r$$

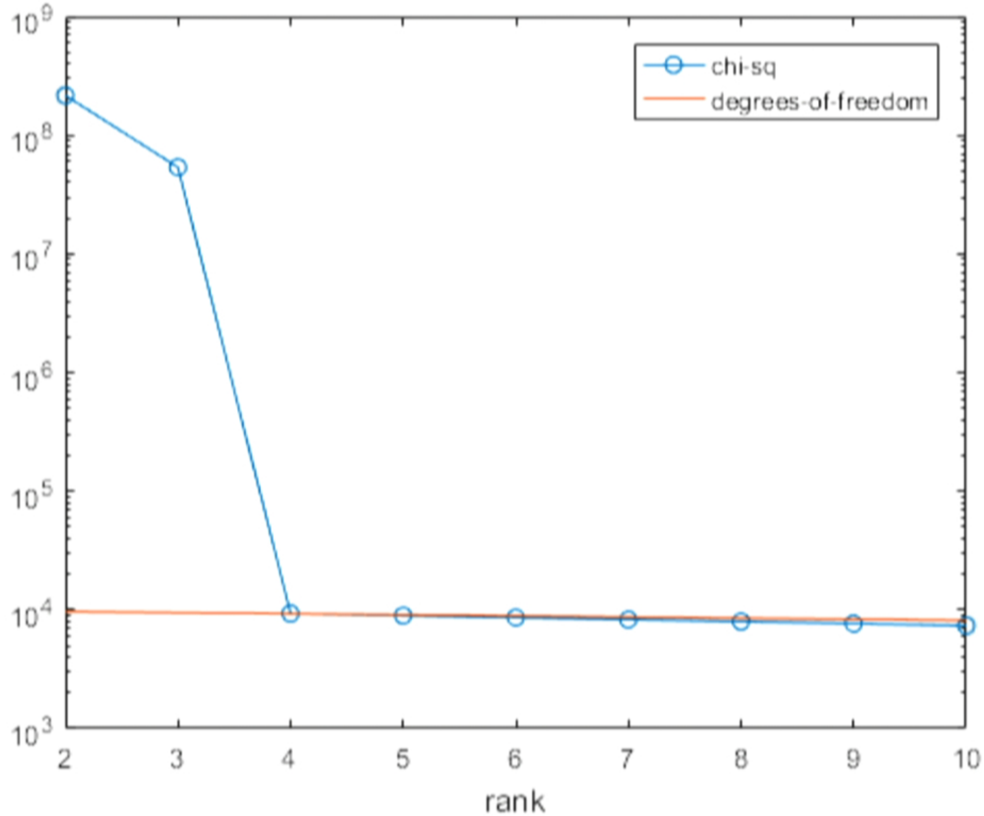
Experimental setup



Results

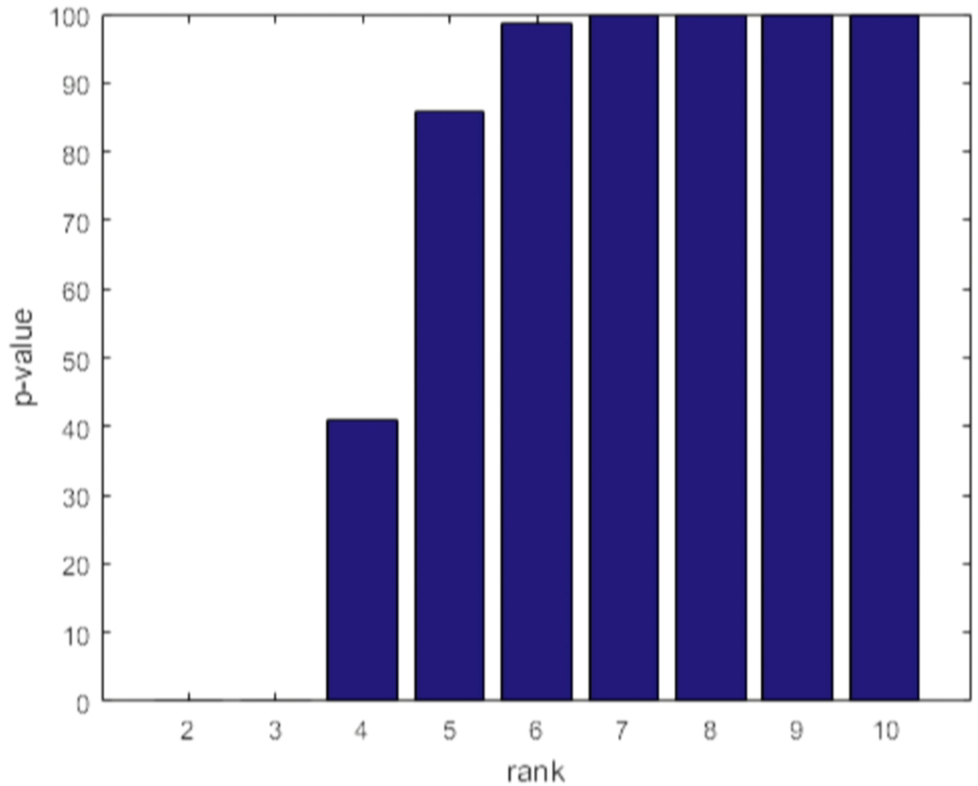


Fitting

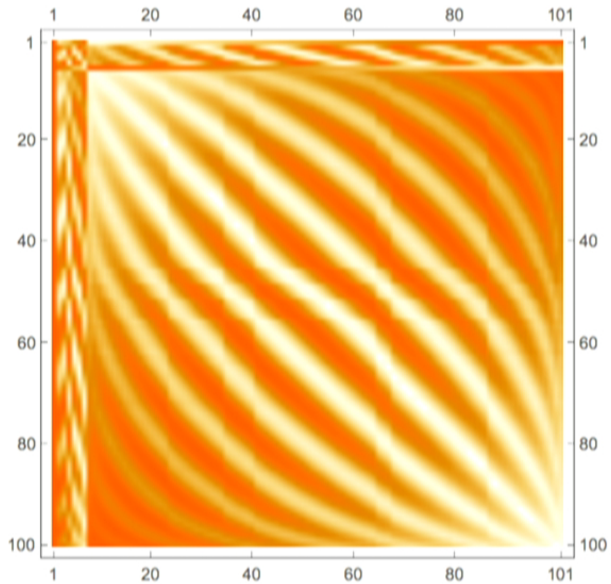


chi-sq
degrees-of-freedom
10^9
10^8
10^7
10^6
10^5
10^4
10^3
2
3
4
5
6
7
8
9
10
rank

Fitting

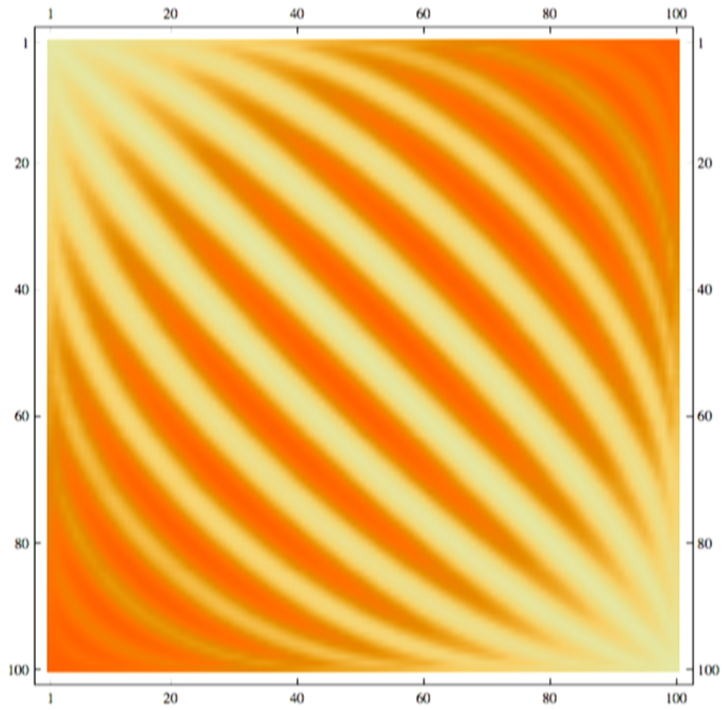


Predictive power



Model rank	RMSE
4	0.003
5	0.006
6	0.035

Fitted rank-4 GPT



$$P = 41\%$$

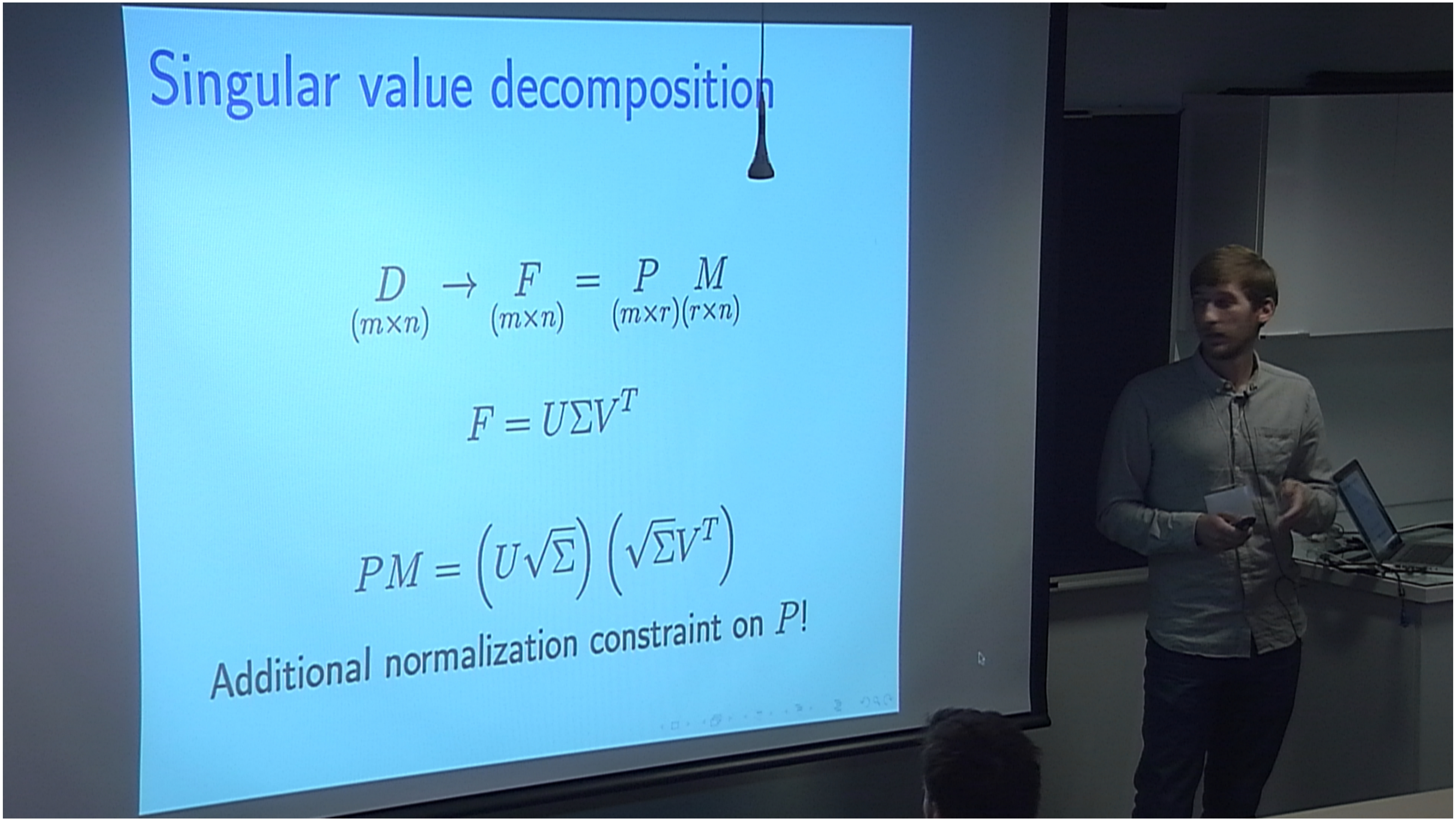
Singular value decomposition

$$\underset{(m\times n)}{D} \rightarrow \underset{(m\times n)}{F} = \underset{(m\times r)}{P}\,\underset{(r\times n)}{M}$$

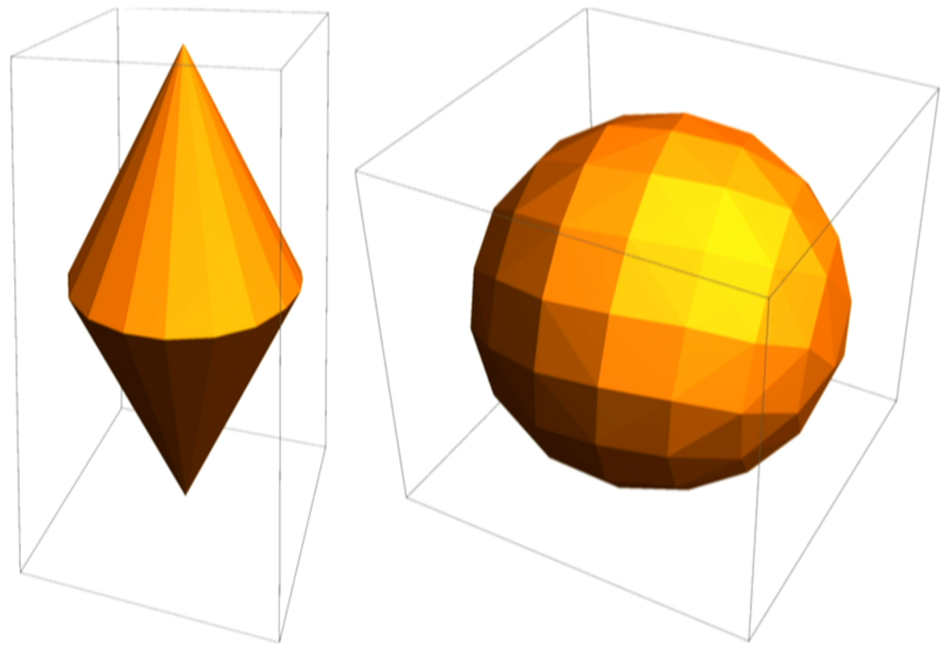
Singular value decomposition

$$\underset{(m\times n)}{D} \rightarrow \underset{(m\times n)}{F} = \underset{(m\times r)}{P}\,\underset{(r\times n)}{M}$$

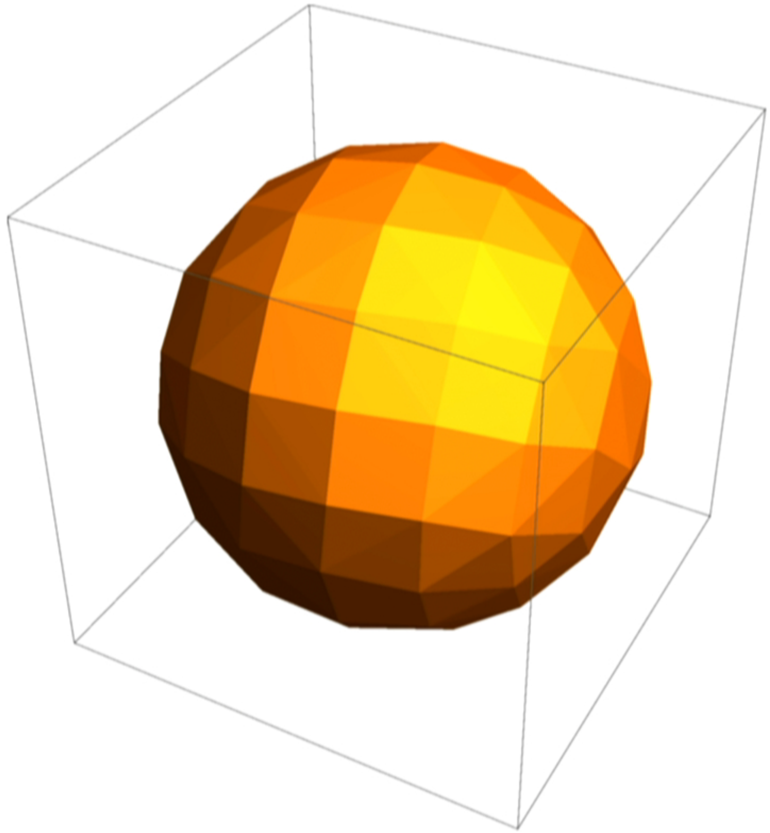
$$F = U\Sigma V^T$$


Singular value decomposition
$\underset{(m\times n)}{D} \to \underset{(m\times n)}{F} = \underset{(m\times r)}{P}\,\underset{(r\times n)}{M}$
$F = U\Sigma V^T$
$PM = \left(U\sqrt{\Sigma}\right)\left(\sqrt{\Sigma}V^T\right)$
Additional normalization constraint on P!

Effects



States



Dual of effects

- Measured state space incomplete
  - noise, finite number
- *Dual* effect space contains all *logically-valid* state vectors

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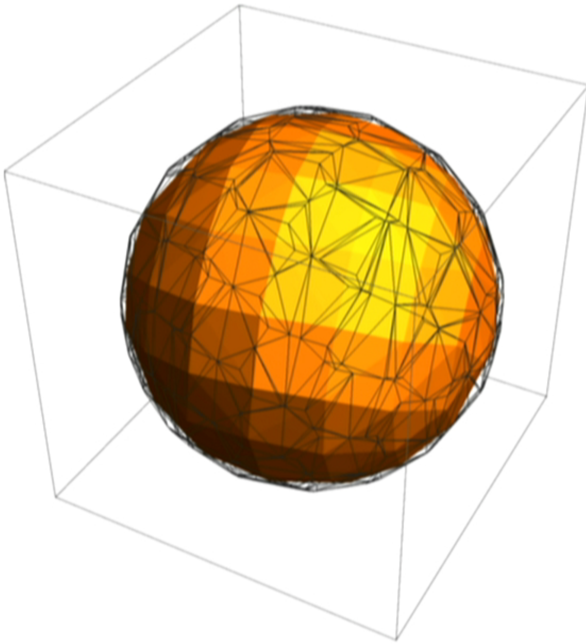
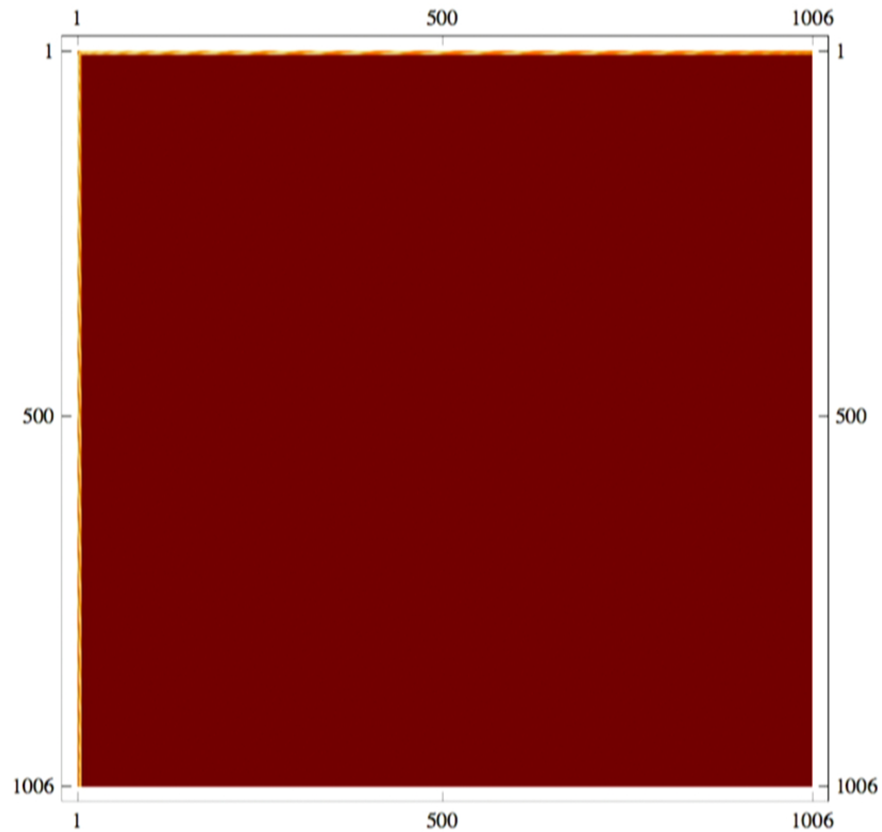
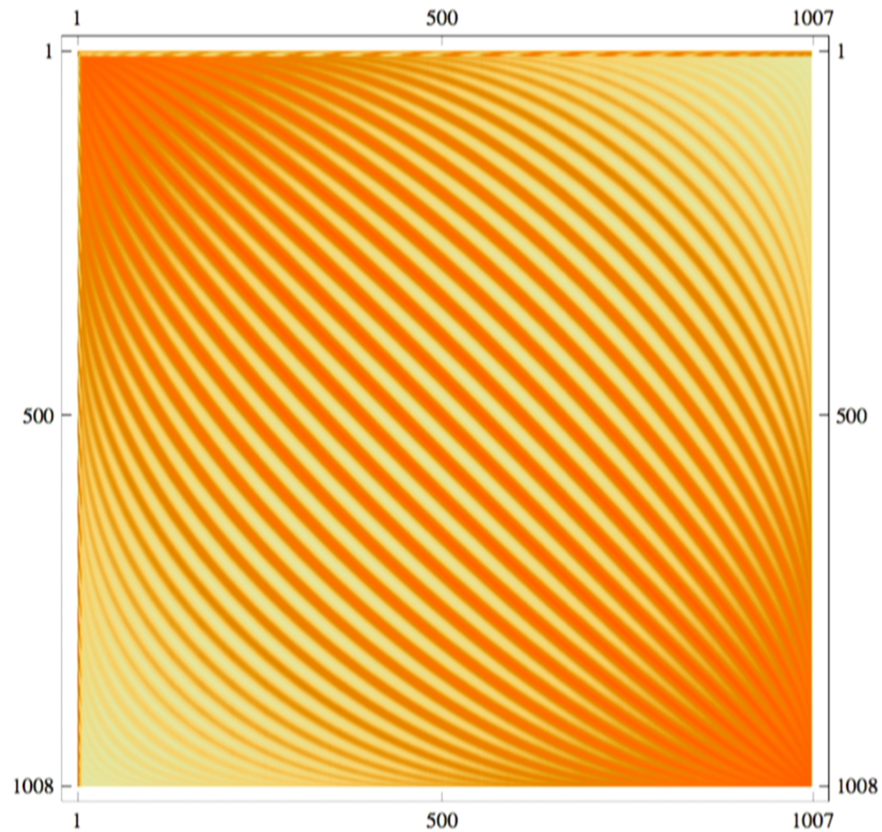
- Measured state space incomplete
  - noise, finite number
- *Dual* effect space contains all *logically-valid* state vectors
  - LV-states return valid probabilities



More data!



More data!



Duality gap

- GPT-of-best-fit not quite self-dual.
  - True spaces are somewhere in-between
- Compare (4-d) volumes of spaces
- 100 x 100 data:

$$V_P / V_{\bar{E}} = 0.91267 \pm 0.00001$$

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  - True spaces are somewhere in-between
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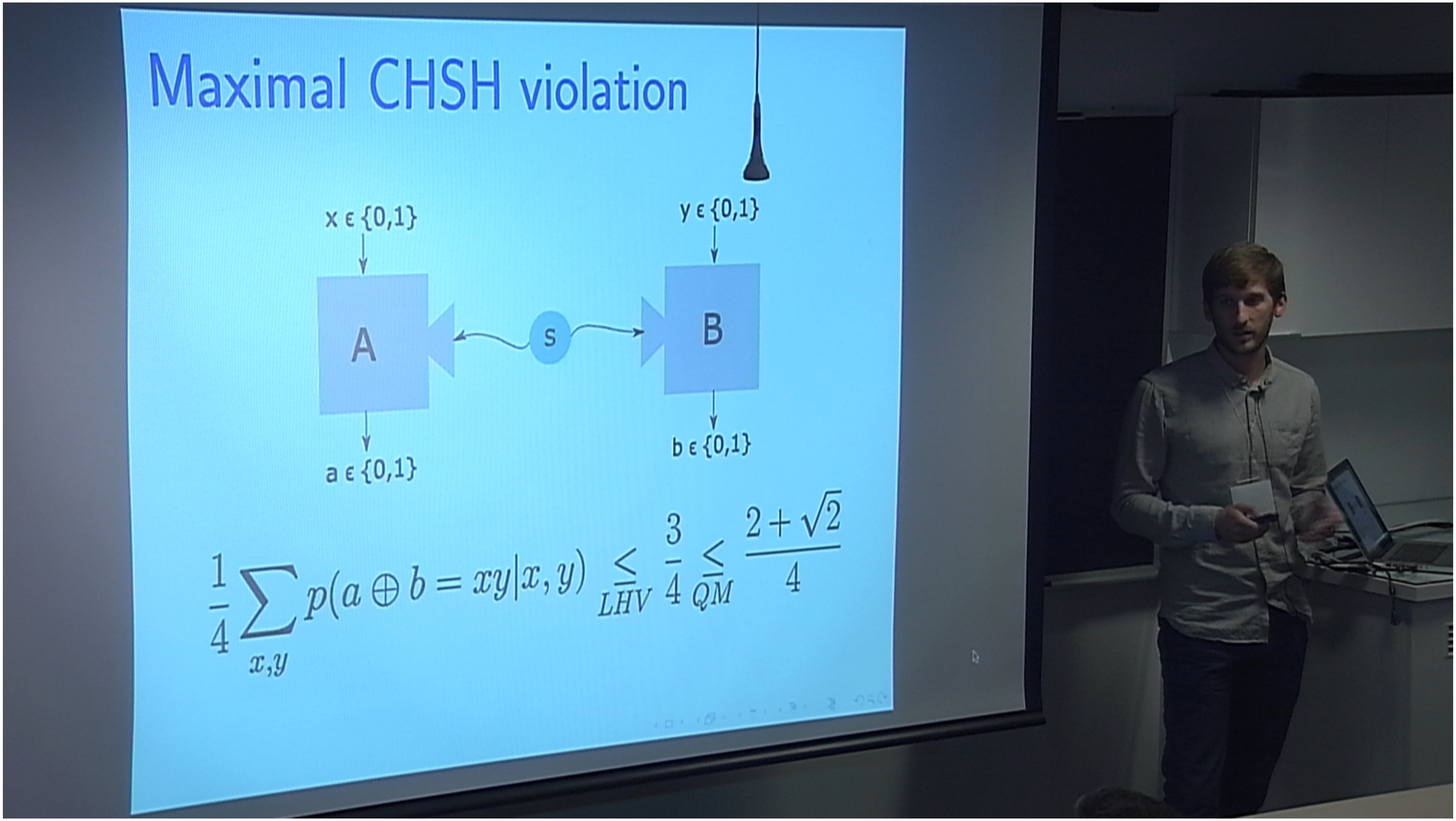
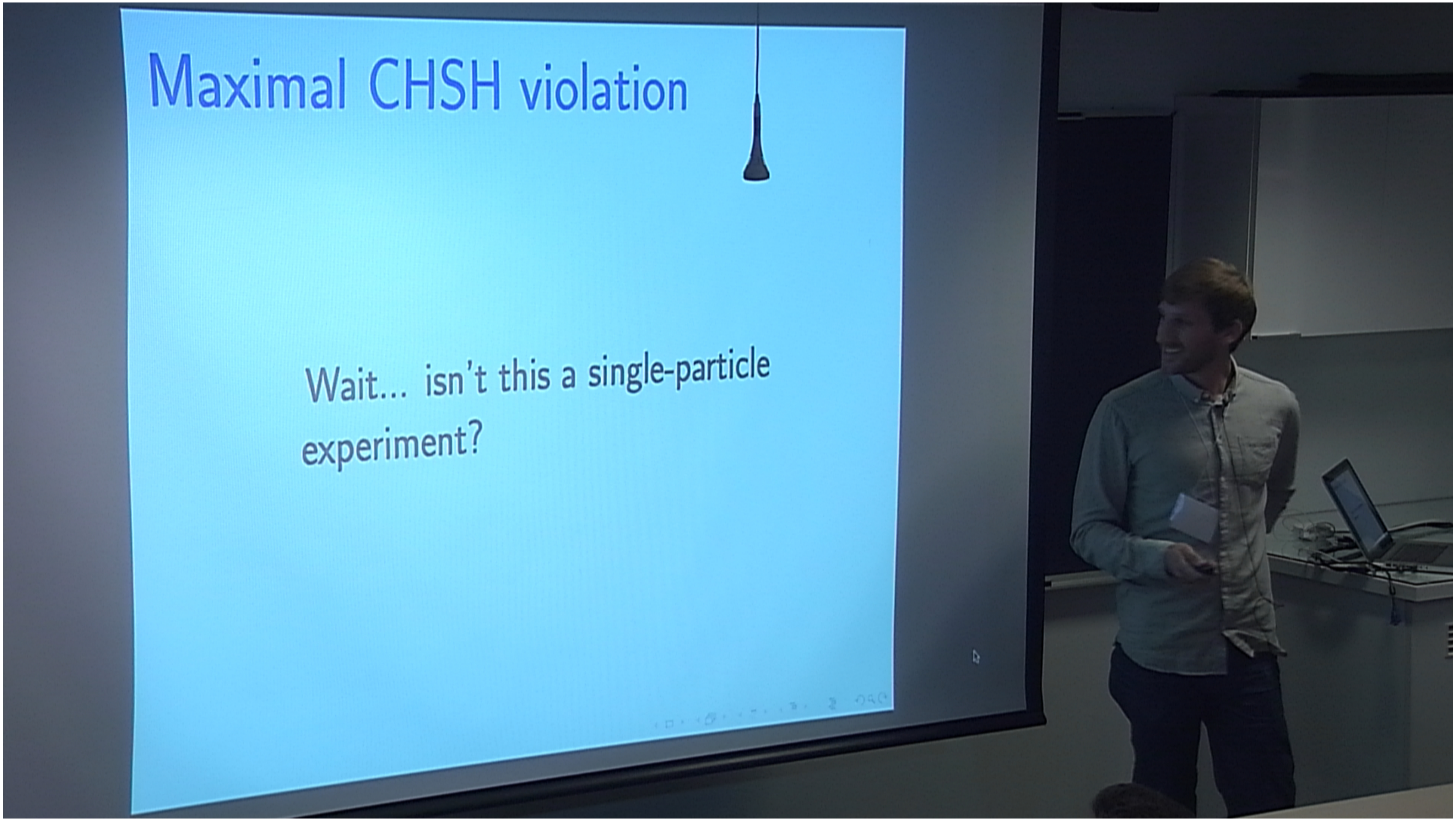
- 100 x 100 data:

$$V_P/V_{\bar{E}} = 0.91267 \pm 0.00001$$

- 1000 x 1000 data:

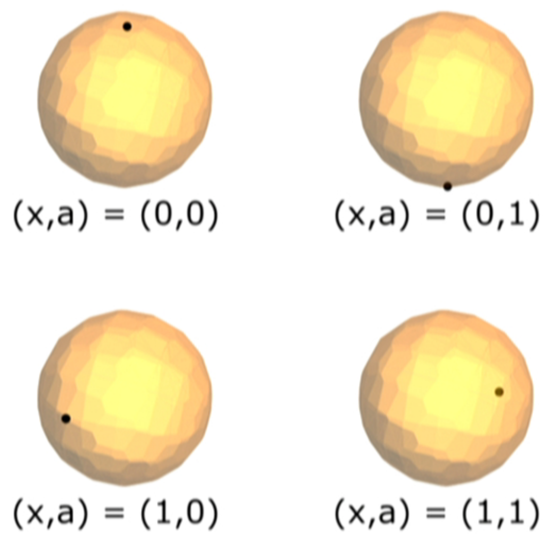
$$V_P/V_{\bar{E}} = 0.968 \pm 0.001$$

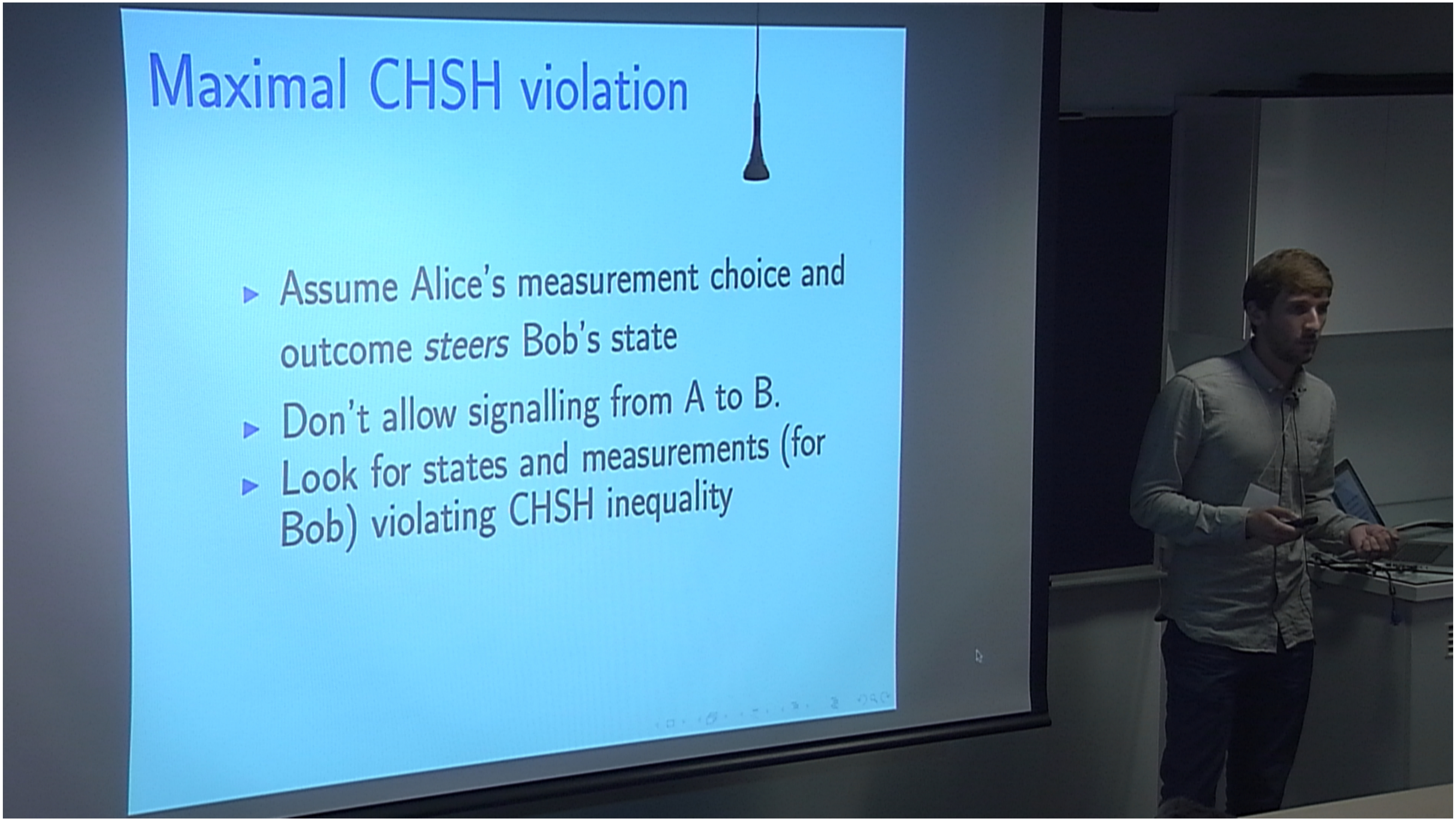

Maximal CHSH violation
x ϵ {0,1}
y ϵ {0,1}
A
s
B
a ϵ {0,1}
b ϵ {0,1}
$\frac{1}{4}\sum_{x,y} p(a \oplus b = xy|x,y) \underset{LHV}{\leq} \frac{3}{4} \underset{QM}{\leq} \frac{2+\sqrt{2}}{4}$

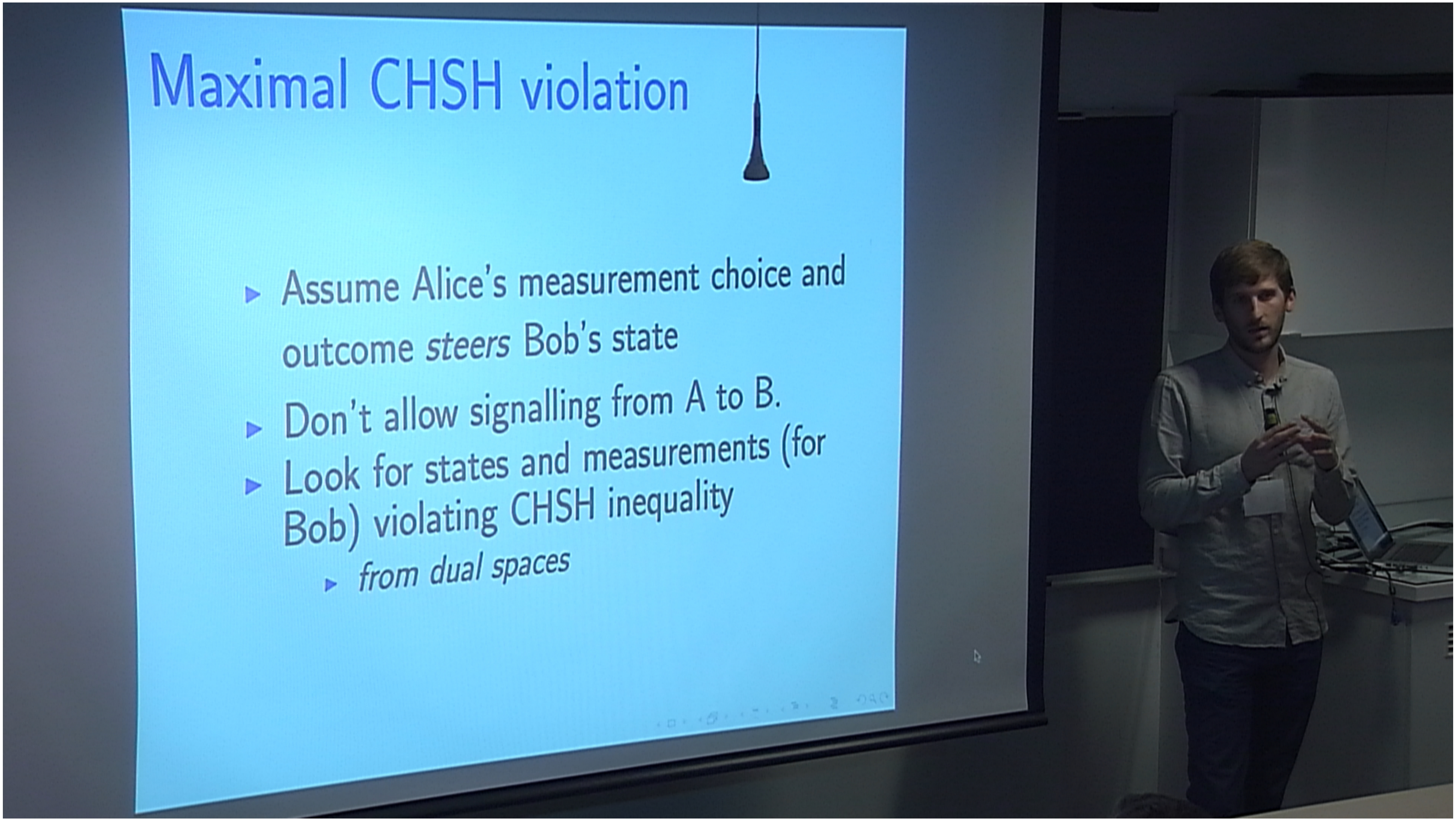

Maximal CHSH violation
Wait... isn't this a single-particle experiment?

Maximal CHSH violation

- Assume Alice's measurement choice and outcome *steers* Bob's state

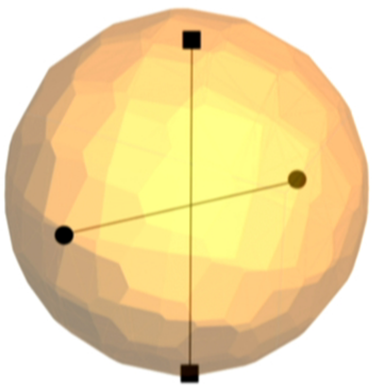



Maximal CHSH violation
Assume Alice's measurement choice and outcome steers Bob's state
Don't allow signalling from A to B.
Look for states and measurements (for Bob) violating CHSH inequality


Maximal CHSH violation
Assume Alice's measurement choice and outcome steers Bob's state
Don't allow signalling from A to B.
Look for states and measurements (for Bob) violating CHSH inequality
from dual spaces

Maximal CHSH violation

- Choose two pairs of (extremal) effects for Bob from dual state space.
- Maximize violation – without signalling – over all normalized, steerable states from dual effect space (with an LP).



Maximal CHSH violation

- Choose two pairs of (extremal) effects for Bob from dual state space.
- Maximize violation – without signalling – over all normalized, steerable states from dual effect space (with an LP).
- Repeat for all pairs of Bob's measurements, keep maximum violation

Maximal CHSH violation

For our GPT-of-best-fit we find

$$\frac{1}{4}\sum_{x,y} p(a \oplus b = xy | x, y) \leq 0.87196 \pm 0.00006$$

Maximal QM value:

$$\frac{2+\sqrt{2}}{4} \approx 0.85355$$

Future work

- POVMs
- Transformations
- Higher dimensions!
- Treatment of errors
- Other systems