Title: Yang-Mills conformal gravity

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Abstract: <p>We reconsider a gauge theory of gravity in which the gauge group is the conformal group SO(4,2), and the action is of the Yang-Mills form, quadratic in the curvature. The vacuum sector of the resulting gravitational theory exhibits local conformal symmetry. We allow for conventional matter coupled to the spacetime metric as well as matter coupled to the field that gauges special conformal transformations. When the theory is linearized about flat space, we find there is a long range gravitational force in addition to Newtonâ€™s inverse square law. Furthermore, the cosmological sector of the theory exhibits late time acceleration, an early time bounce, and a post-bounce quasi-de Sitter ``inflationary'' phase of arbitrary duration (without an inflaton).</p>

Yang-Mills conformal gravity

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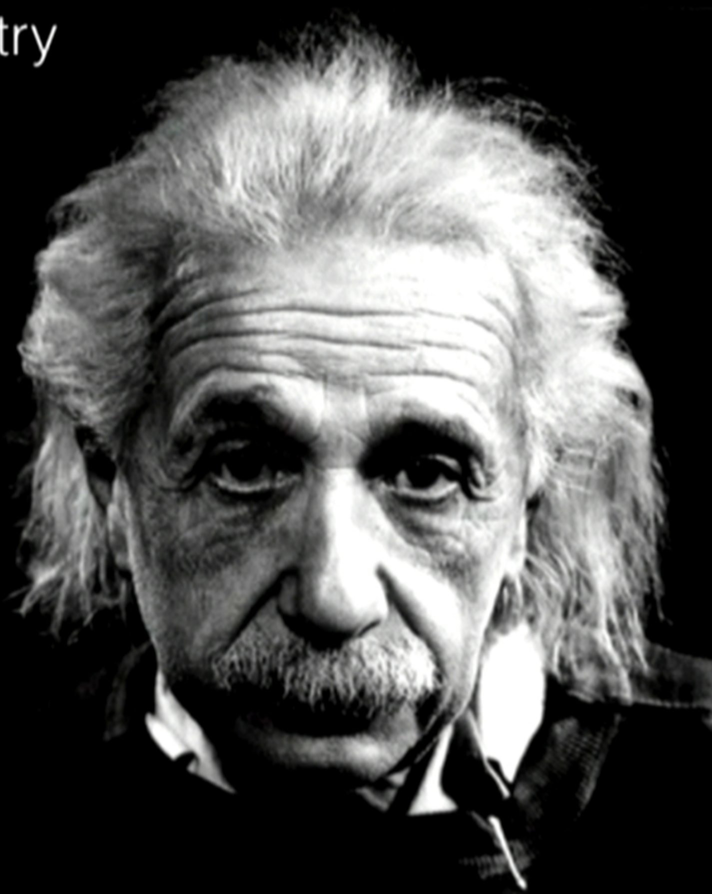
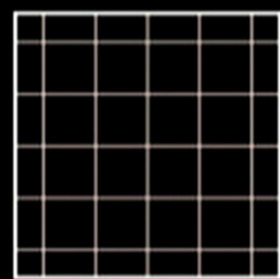
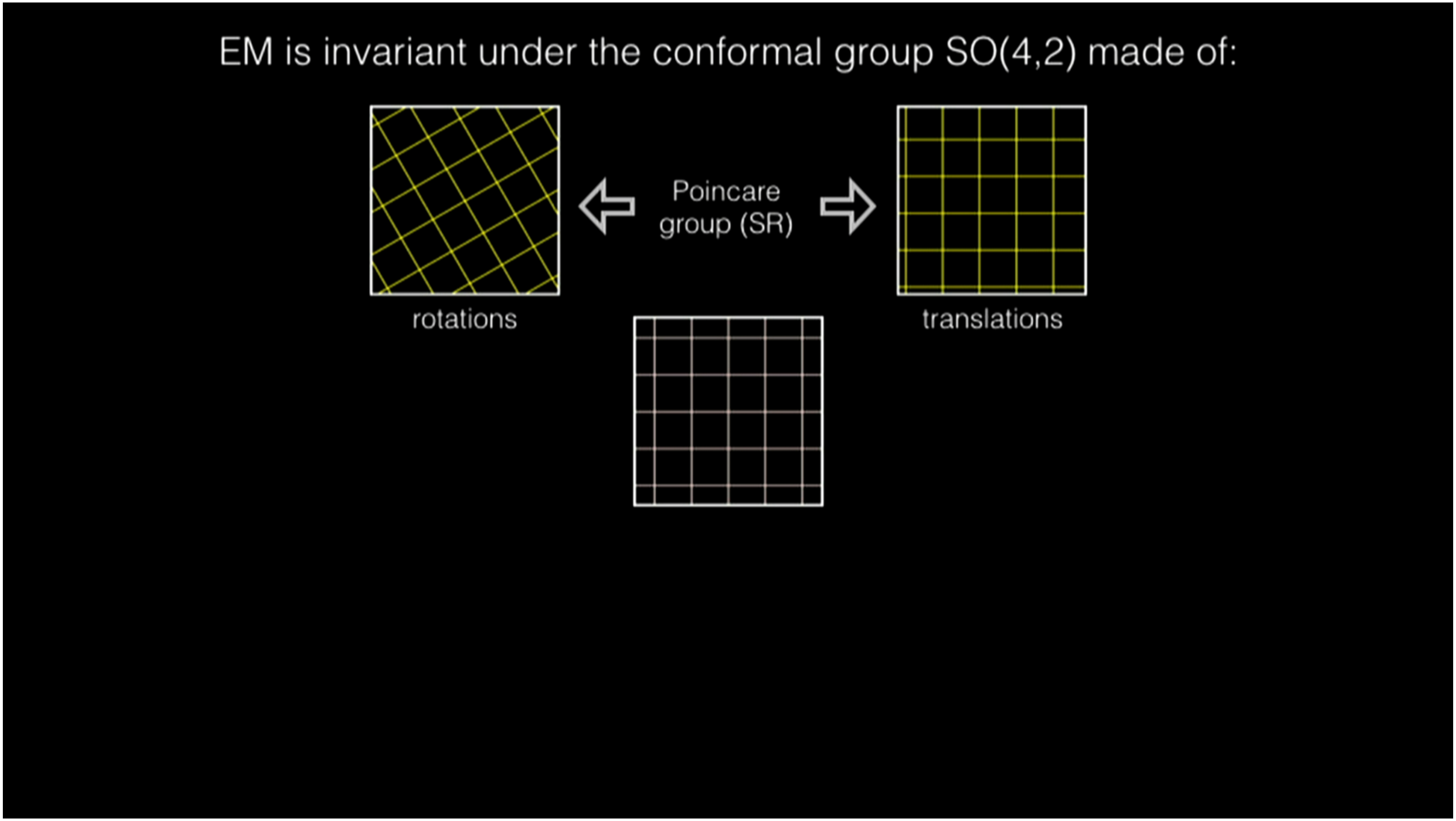
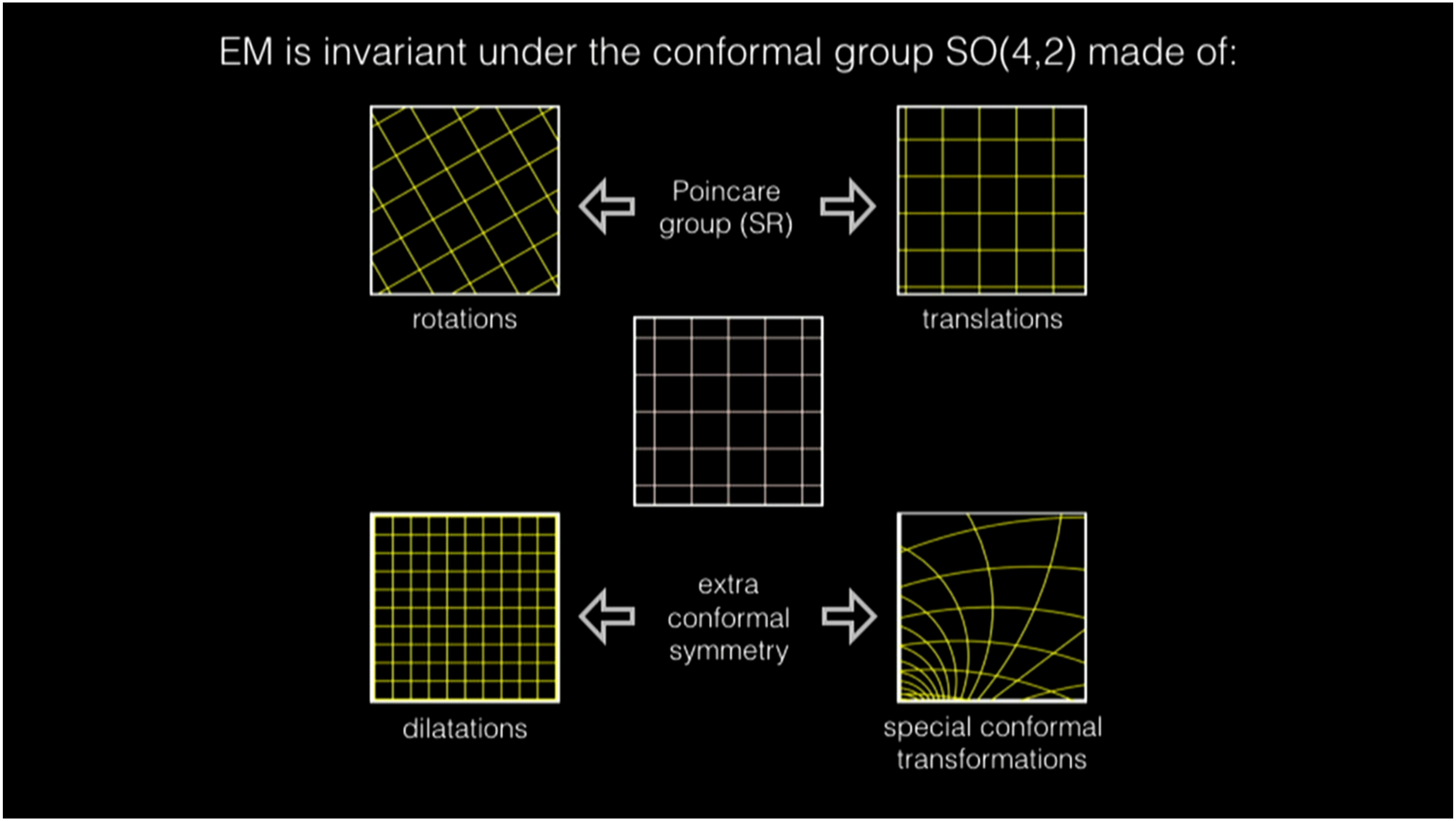
special relativity (SR): a theory of mechanics which respects the Lorentz symmetry of electromagnetism (EM)



EM is invariant under the conformal group SO(4,2) made of:




EM is invariant under the conformal group SO(4,2) made of:
Poincare
group (SR)
rotations
translations


EM is invariant under the conformal group SO(4,2) made of:
Poincare group (SR)
rotations
translations
extra conformal symmetry
dilatations
special conformal transformations

earliest conformal gravity proposal by Weyl (1918) as part of a unified gravity-EM theory:

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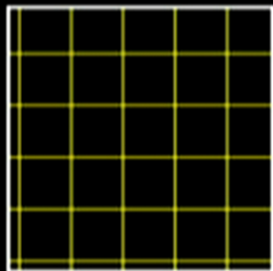
$$S = \int d^4x \sqrt{-g}\, C_{\alpha\beta\gamma\delta} C^{\alpha\beta\gamma\delta} + S_{\rm m}$$

recently studied by Mannheim et al

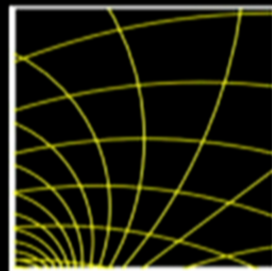
another approach: Yang Mills (YM) gravity

4 generators: $\mathbf{P}_a$



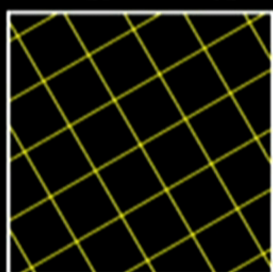
translations

4 generators: $\mathbf{K}_a$



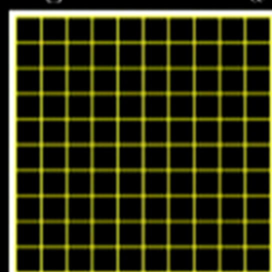
special conformal transformations

6 generators: $\mathbf{J}_{ab} = -\mathbf{J}_{ba}$



rotations

1 generator: $\mathbf{D}_a$



dilatations

SO(4,2)

use generators to form a vector potential:

$$\mathbf{A}_\alpha = A^A_\alpha \mathbf{J}_A = e^a_\alpha \mathbf{P}_a + l^a_\alpha \mathbf{K}_a + \omega^{ab}_\alpha \mathbf{J}_{ab} + q_\alpha \mathbf{D}$$

$$\mathbf{J}_A = \{\mathbf{P}_a, \mathbf{K}_a, \mathbf{J}_{ab}, \mathbf{D}\}$$

define field strength $\mathbf{F}_{\alpha\beta} = F^A_{\alpha\beta}\mathbf{J}_A$ of vector potential:

$$F^A_{\alpha\beta} = \partial_\alpha A^A_\beta - \partial_\beta A^A_\alpha + f^A{}_{BC} A^B_\alpha A^C_\beta$$

$$[\mathbf{J}_A, \mathbf{J}_B] = f^C{}_{AB}\mathbf{J}_C$$

write down YM action

$$S = -\frac{1}{2g^2_{\mathrm{YM}}}\int d^4x\sqrt{-g}g^{\alpha\mu}g^{\beta\nu}h_{AB}F^A_{\alpha\beta}F^B_{\mu\nu} + S_{\mathrm{m}}$$

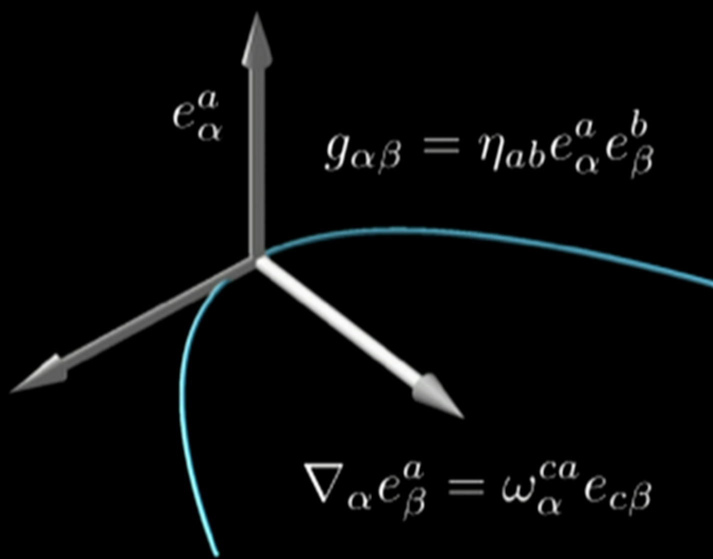
$$h_{AB} = \mathrm{Tr}(\mathbf{J}_A\mathbf{J}_B) = f^M{}_{AN}f^N{}_{BM}$$

to obtain a theory of gravity: identify components of vector potential with geometric quantities on a 4-manifold

$$\mathbf{A}_\alpha = A^A_\alpha \mathbf{J}_A = e^a_\alpha \mathbf{P}_a + l^a_\alpha \mathbf{K}_a + \omega^{ab}_\alpha \mathbf{J}_{ab} + q_\alpha \mathbf{D}$$

$e^a_\alpha \rightarrow$ orthonormal frame

$\omega^{ab}_\alpha \rightarrow$ connection 1-forms



e^a_α

$$g_{\alpha\beta} = \eta_{ab} e^a_\alpha e^b_\beta$$

$$\nabla_\alpha e^a_\beta = \omega^{ca}_\alpha e_{c\beta}$$

affine connection is metric compatible but not necessarily torsion-free couple things to keep in mind:

$$S = -\frac{1}{2g_{\mathrm{YM}}^2}\int d^4x\sqrt{-g}g^{\alpha\mu}g^{\beta\nu}h_{AB}F^A_{\alpha\beta}F^B_{\mu\nu} + S_{\mathrm{m}}$$

the metric $g = e^a \otimes e_a$ is an algebraic function of the gauge potential $\mathbf{A}$:

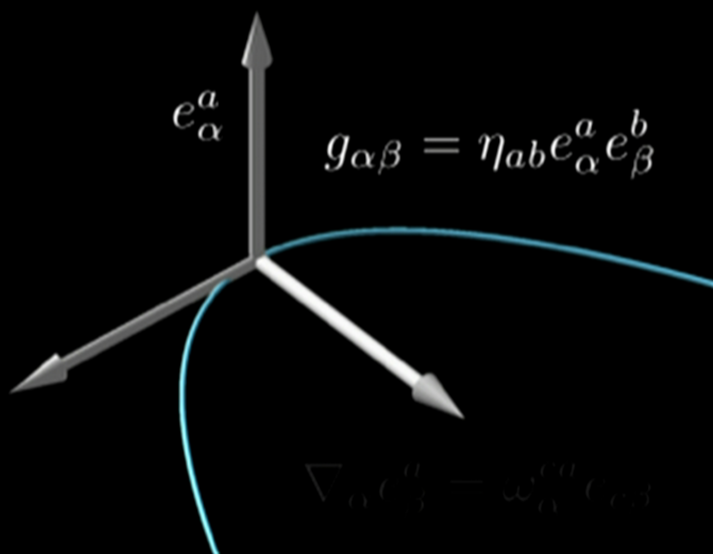
$$\mathbf{A}_\alpha = A^A_\alpha \mathbf{J}_A = e^a_\alpha \mathbf{P}_a + l^a_\alpha \mathbf{K}_a + \omega^{ab}_\alpha \mathbf{J}_{ab} + q_\alpha \mathbf{D}$$

to obtain a theory of gravity: identify components of vector potential with geometric quantities on a 4-manifold

$$\mathbf{A}_\alpha = A_\alpha^A \mathbf{J}_A = e_\alpha^a \mathbf{P}_a + l_\alpha^a \mathbf{K}_a + \omega_\alpha^{ab} \mathbf{J}_{ab} + q_\alpha \mathbf{D}$$

$e_\alpha^a \rightarrow$ orthonormal frame

$\omega_\alpha^{ab} \rightarrow$ connection 1-forms



couple things to keep in mind:

$$S = -\frac{1}{2g_{\rm YM}^2}\int d^4x\sqrt{-g}g^{\alpha\mu}g^{\beta\nu}h_{AB}F^A_{\alpha\beta}F^B_{\mu\nu} + S_{\rm m}$$

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$$\mathbf{A}_\alpha = A^A_\alpha \mathbf{J}_A = e^a_\alpha \mathbf{P}_a + l^a_\alpha \mathbf{K}_a + \omega^{ab}_\alpha \mathbf{J}_{ab} + q_\alpha \mathbf{D}$$

a number of authors have considered gravity as a gauge theory with various groups:

gauge group	authors
Poincare	Utiyama (1956) Kibble (1961) Ivanov & Niederle (1982)
de Sitter	Townsend (1977) Ivanov & Niederle (1982) Huang et al (2008; 2009a,b; 2013a,b)
conformal	Mansouri (1979) Ivanov & Niederle (1982) Wheeler et al (1991;2014a,b)
other	MacDowell & Mansouri (1977)

decompose this current in terms of SO(4,2) generators:

$$j^{B\nu}\mathbf{J}_B = a^{a\nu}\mathbf{P}_a + b^{a\nu}\mathbf{K}_a + c^{ab\nu}\mathbf{J}_{ab} + d^\nu \mathbf{D}$$

$$b_{\alpha\beta} \propto \frac{1}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_{\mathrm{m}})}{\delta g^{\alpha\beta}}$$

"ordinary" stress tensor

$$a_{\alpha\beta} \propto \frac{e^a_\beta}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_{\mathrm{m}})}{\delta l^{a\alpha}}$$

$$c_{\alpha\beta\gamma} \propto \frac{e^a_\alpha e^b_\beta}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_{\mathrm{m}})}{\delta \omega^{ab}_\gamma}$$

$$d_\alpha \propto \frac{1}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_{\mathrm{m}})}{\delta q^\alpha}$$

these represent matter fields coupled to other components of the gauge potential

$$\mathbf{A}_\alpha = A^A_\alpha \mathbf{J}_A = e^a_\alpha \mathbf{P}_a + l^a_\alpha \mathbf{K}_a + \omega^{ab}_\alpha \mathbf{J}_{ab} + q_\alpha \mathbf{D}$$

with matter current defined the equations of motion are:

$$D_\mu F^{B\mu\nu} = k^{B\nu} + j^{B\nu}$$

with matter current defined the equations of motion are:

gauge covariant derivative

$$D_\mu F^{B\mu\nu} = \hat{\nabla}_\mu F^{B\mu\nu} + f^B{}_{CD} A^C_\mu F^{D\mu\nu}$$

matter current

$$D_\mu F^{B\mu\nu} = k^{B\nu} + j^{B\nu}$$

“source” due to dependence of metric on tetrad:

$$\propto F^{A\rho\mu} F^{B\sigma}{}_\mu - \frac{1}{4} g^{\rho\sigma} F^A_{\mu\nu} F^{B\mu\nu}$$

second order field equations for gauge potential that are complicated when expanded out

ordinary YM: action invariant under
$\delta A_\alpha^A = \partial_\alpha \epsilon^A + f^A{}_{BC} A_\alpha^B \epsilon^C$ with

$$\epsilon^A \mathbf{J}_A = \epsilon^a \mathbf{P}_a + \lambda^a \mathbf{K}_a + \Lambda^{ab} \mathbf{J}_{ab} + \Omega \mathbf{D}$$

$$S = -\frac{1}{2g_{\mathrm{YM}}^2} \int d^4x \sqrt{-g}\, g^{\alpha\mu} g^{\beta\nu} h_{AB} F_{\alpha\beta}^A F_{\mu\nu}^B + S_{\mathrm{m}}$$

YM gravity: since metric depends on $\{e_\alpha^a\} \subset \{A_\alpha^A\}$,
S invariant under transformations generated by

$$\epsilon^A \mathbf{J}_A = \lambda^a \mathbf{K}_a + \Lambda^{ab} \mathbf{J}_{ab} + \Omega \mathbf{D}$$

without loss of generality we can choose a gauge where $q_\alpha = 0$

$$\mathbf{A}_\alpha = A^A_\alpha \mathbf{J}_A = e^a_\alpha \mathbf{P}_a + l^a_\alpha \mathbf{K}_a + \omega^{ab}_\alpha \mathbf{J}_{ab} + q_\alpha \mathbf{D}$$

equations of motion (after a partial gauge fixing):

$$a^{\alpha\nu} = \hat{\nabla}_\mu T^{\alpha\mu\nu} + R^{\alpha\nu} - \tfrac{1}{2}(f^{(\alpha\nu)} + \tfrac{1}{2} f g^{\alpha\nu})$$

$$b^{\alpha\nu} = \hat{\nabla}_\mu \theta^{\alpha\mu\nu} - f_{\lambda\mu}\Phi^{\alpha\lambda\mu\nu} - f^{\alpha}{}_{\mu}\mathscr{F}^{\mu\nu} - \tfrac{1}{2}\tau^{\alpha\nu}$$

$$c^{\alpha\beta\nu} = \hat{\nabla}_\mu \Phi^{\alpha\beta\mu\nu} + \tfrac{1}{2}\theta^{[\alpha\beta]\nu} - \tfrac{1}{2} f^{[\alpha|\mu|}T^{\beta]\nu}{}_{\mu}$$

$$d^{\nu} = (2\hat{\nabla}_\mu + \nabla_\mu)\mathscr{F}^{\mu\nu} + \tfrac{1}{2}\nabla_\mu (f^{(\mu\nu)} - g^{\mu\nu} f) + 2\mathscr{F}_{\mu\sigma} T^{[\sigma\mu]\nu}$$

fine print

$$R^{\mu\nu}{}_{\alpha\beta} = e^{\mu}_{a} e^{\nu}_{b} (d\omega^{ab} + \omega^{ac} \wedge \omega_{c}{}^{b})_{\alpha\beta} = \text{Riemann tensor}$$

$$T^{\alpha}{}_{\beta\gamma} = e^{\alpha}_{a}(de^{a} + \omega^{ac} \wedge e_{c})_{\beta\gamma} = 2\Gamma^{\alpha}{}_{[\beta\gamma]} = \text{torsion tensor}$$

$$\tau^{\rho\sigma} = h_{AB}\left(F^{A\rho\mu} F^{B\sigma}{}_{\mu} - \tfrac{1}{4} g^{\rho\sigma} F^{A}_{\mu\nu} F^{B\mu\nu}\right)$$

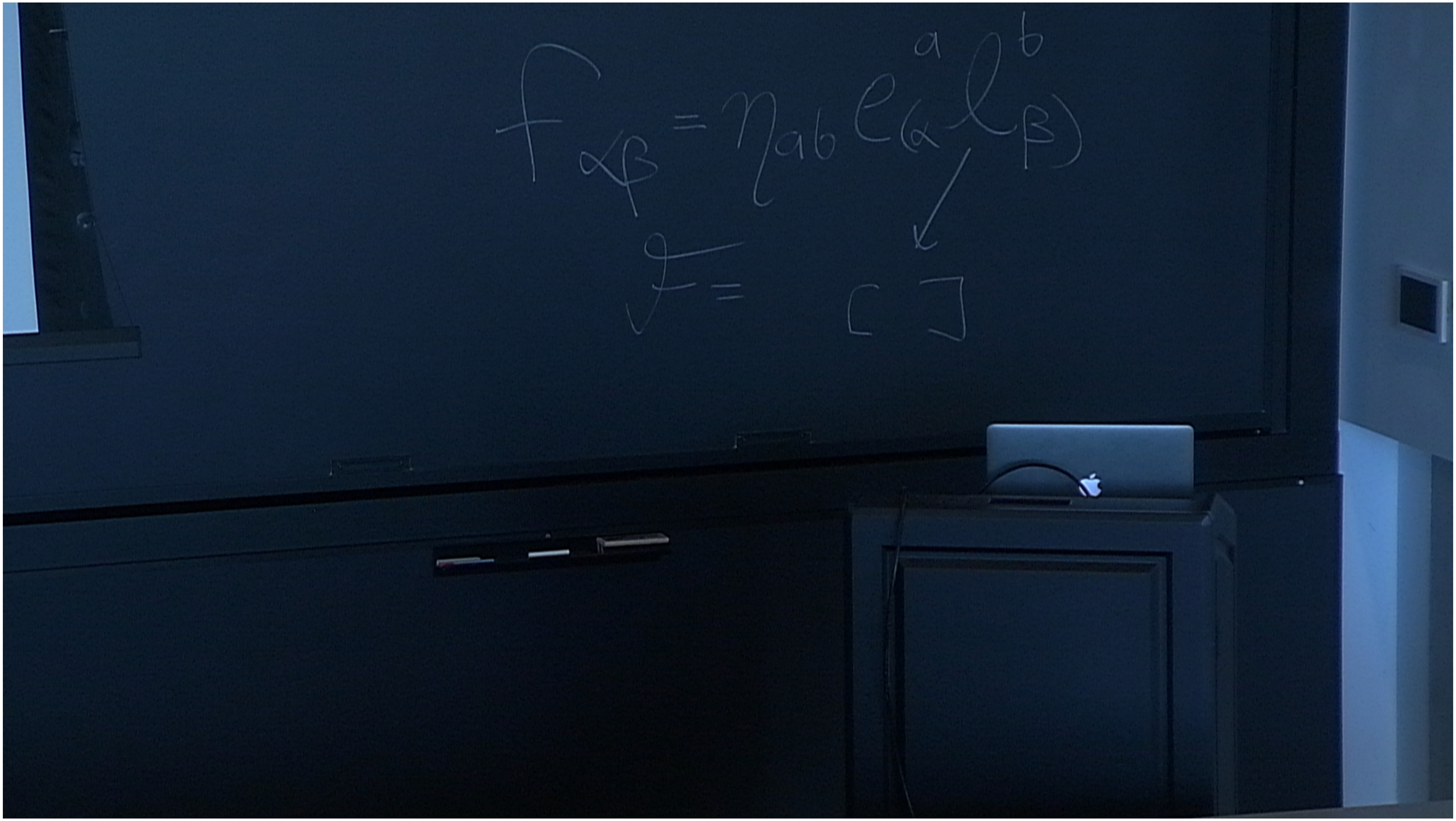
$$f_{\alpha\beta} = \eta_{ab} e^{a}_{\alpha} l^{b}_{\beta} \quad \mathscr{F}_{\alpha\beta} = \tfrac{1}{2} f_{[\alpha\beta]}$$

$$\theta^{\mu\alpha\beta} = \nabla^{\alpha} f^{\mu\beta} - \nabla^{\beta} f^{\mu\alpha} + f^{\mu\sigma} T_{\sigma}{}^{\alpha\beta}$$

$$\Phi^{\alpha\beta\gamma\delta} = R^{\alpha\beta\gamma\delta} - \tfrac{1}{2}(g^{\gamma[\alpha} f^{\beta]\delta} - g^{\delta[\alpha} f^{\beta]\gamma})$$

$$\nabla_{\alpha} = \text{covariant derivative from affine connection } \Gamma^{\alpha}{}_{\beta\gamma}$$

$$\hat{\nabla}_{\alpha} = \text{Levi-Civita covariant derivative}$$


$f_{\alpha\beta} = \eta_{ab} e^a_{(\alpha} l^b_{\beta)}$
$\mathcal{J} = [\ ]$

linearize about Minkowski space with no matter:

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta} \qquad H_{\alpha\beta} = h_{\alpha\beta} - \tfrac{1}{4}\eta_{\alpha\beta}h$$

assume torsion and some matter fields vanish at linear level:

$$c_{\alpha\beta\gamma} = 0 = d_\alpha \qquad T^{\alpha}{}_{\beta\gamma} = 0$$

obtain Maxwell-Klein-Gordon-like equations:

$$\nabla^{[\alpha}\mathscr{F}^{\beta\nu]} = 0 \qquad \nabla^{\alpha}\mathscr{F}_{\alpha\beta} = -\tfrac{1}{6}\nabla_\beta a \qquad \Box a = 0$$

plus fourth-order equation for metric perturbations:

$$-\tfrac{1}{4}\big(\tfrac{2}{3}\partial_\alpha\partial_\beta\partial^\mu\partial^\nu H_{\mu\nu} - 2\Box\partial^\nu\partial_{(\alpha}H_{\beta)\nu} + \tfrac{1}{3}\eta_{\alpha\beta}\partial^\mu\partial^\nu\Box H_{\mu\nu} + \Box^2 H_{\alpha\beta}\big) = \tfrac{1}{2}\Box a^{\mathrm{TF}}_{\alpha\beta} + \tfrac{1}{4}b_{\alpha\beta} + \tfrac{1}{6}\partial_\alpha\partial_\beta a$$

"TF" = trace free

isn't fourth-order bad news (ghosts etc)?

$$-\tfrac{1}{4}(\tfrac{2}{3}\partial_\alpha\partial_\beta\partial^\mu\partial^\nu H_{\mu\nu} - 2\Box\partial^\nu\partial_{(\alpha}H_{\beta)\nu} + \tfrac{1}{3}\eta_{\alpha\beta}\partial^\mu\partial^\nu\Box H_{\mu\nu} + \Box^2 H_{\alpha\beta}) = \tfrac{1}{2}\Box a^{\mathrm{TF}}_{\alpha\beta} + \tfrac{1}{4}b_{\alpha\beta} + \tfrac{1}{6}\partial_\alpha\partial_\beta a$$

recall field equations for gauge vector are second order:

$$D_\mu F^{B\mu\nu} = k^{B\nu} + j^{B\nu}$$

hence theory only looks fourth order
when written in terms of the metric

[similar thing happens for $f(R)$ gravity; c.f. Deser (2009)] specialize to static sources and transverse gauge:

⇩

$$h^{\mathrm{TF}}_{\alpha\beta}(\mathbf{r}) = \int d^3\mathbf{r}'\, \frac{a^{\mathrm{TF}}_{\alpha\beta}(\mathbf{r}')}{2\pi|\mathbf{r}-\mathbf{r}'|} + \int d^3\mathbf{r}'\, \frac{|\mathbf{r}-\mathbf{r}'|}{8\pi} b_{\alpha\beta}(\mathbf{r}')$$

inverse square law

long range gravitational force

action: $S = -\frac{1}{2g_{\rm YM}^2}\int d^4x\sqrt{-g}g^{\alpha\mu}g^{\beta\nu}h_{AB}F^A_{\alpha\beta}F^B_{\mu\nu} + S_{\rm m}$

DOFs: $\mathbf{A}_\alpha = A^A_\alpha \mathbf{J}_A = e^a_\alpha \mathbf{P}_a + l^a_\alpha \mathbf{K}_a + \omega^{ab}_\alpha \mathbf{J}_{ab} + q_\alpha \mathbf{D}$

full EOMs pretty complicated, let's simplify things:

gauge choice (without loss of generality):	$q_\alpha = 0$
no torsion assumption (gauge invariant):	$de^a + \omega^{ac} \wedge e_c = 0$
technical assumption (gauge invariant):	$\eta_{ab}(e^a_\alpha l^b_\beta - e^a_\beta l^b_\alpha) = 0$
matter action:	$S_{\rm m} = S_{\rm m}[g_{\alpha\beta}, l^a_\alpha, \psi]$

EOMs now less complicated:

$$0 = B^{\alpha\nu} + \frac{1}{16} g_{\rm YM}^2 T^{\alpha\nu} - \nabla_\mu \nabla^{[\nu} \bar{a}^{\mu]\alpha} - Q^{\alpha\nu}$$

$$0 = \nabla^\alpha a_{\alpha\beta}$$

$$0 = g^{\mu\nu} \nabla_\beta a_{\mu\nu} \qquad \bar{a}_{\alpha\beta} = a_{\alpha\beta} - \frac{1}{6} g_{\alpha\beta} a$$

$B^{\mu\nu}$ = Bach tensor involving fourth order metric derivatives

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_{\rm m})}{\delta g^{\mu\nu}} = \text{familiar stress energy tensor}$$

$$a^{\mu\nu} = \frac{g_{\rm YM}^2}{4\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_{\rm m})}{\delta l_\nu^b} e^{b\mu} = \text{coupling of matter to } l_\alpha^a$$

$Q^{\mu\nu}$ = tensor quadratic in Ricci, Weyl and $a_{\mu\nu}$ tensors

EOMs now less complicated:

$$0 = B^{\alpha\nu} + \frac{1}{16} g_{\rm YM}^2 T^{\alpha\nu} - \nabla_\mu \nabla^{[\nu} \bar{a}^{\mu]\alpha} - Q^{\alpha\nu}$$
$$0 = \nabla^\alpha a_{\alpha\beta}$$
$$0 = g^{\mu\nu} \nabla_\beta a_{\mu\nu}$$

residual symmetries:

local conformal transformations $(\delta g_{\alpha\beta} = \epsilon g_{\alpha\beta})$

diffeomorphisms $(\delta g_{\alpha\beta} = \pounds_\zeta g_{\alpha\beta})$

assume cosmological symmetries (isotropy and homogeneity) and FRW metric ansatz:

$$ds^2 = -dt^2 + A^2(t)\left(\frac{dr^2}{1 - kr_0^2/r^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2\right)$$

tensor characterizing exotic matter couplings must take form consistent with symmetries:

$$a^{\mu\nu} = \frac{g_{\rm YM}^2}{4\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_{\rm m})}{\delta l_\nu^b} e^{b\mu} = [\xi_1(t) + \xi_2(t)]\, u^\mu u^\nu + \xi_2(t) g^{\mu\nu}$$

$$a^{\mu\nu} = \frac{g_{\mathrm{YM}}^2}{4\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_{\mathrm{m}})}{\delta l_\nu^b} e^{b\mu} = -\frac{4}{3}\frac{\Pi}{A^4(t)} \left(u^\mu u^\nu + \frac{1}{4} g^{\mu\nu} \right) + \Lambda g_{\mu\nu}$$

assume ordinary matter is dust and radiation:

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_{\rm m})}{\delta g^{\mu\nu}} = T^{(\rm m)}_{\mu\nu} + T^{(\rm r)}_{\mu\nu}$$

remaining EOMs lead to an effective Friedmann equation:

"wrong sign" dark radiation

$$H^2 = \frac{\dot{A}^2}{A^2} = \frac{g^2_{\rm YM}}{8}\left(\frac{\rho_{\rm m} + \rho_{\rm r}}{\Lambda + \frac{\Pi}{A^4}}\right) - \frac{k}{r_0^2 A^2} + \frac{\Lambda}{3} - \frac{\Pi}{3A^4}$$

pretty nonstandard

dark energy

late time limit of Friedmann equation $A \gg (\Pi/\Lambda)^{1/4}$:

$$H^2 \approx \frac{\rho_{\rm m} + \rho_{\rm r}}{3M_{\rm Pl}^2} - \frac{k}{r_0^2 A^2} + \frac{\Lambda}{3}$$

where we made the identification $M_{\rm Pl}^2 = 8\Lambda/3g_{\rm YM}^2$

can rewrite the Friedmann equation with a time dependent Newton's constant:

$$H^2 = \frac{8\pi G_{\rm eff}(A)}{3}\left(\rho_{\rm m}+\rho_{\rm r}\right) - \frac{k}{r_0^2 A^2} + \frac{\Lambda}{3} - \frac{\Pi}{3A^4}$$

$$G_{\rm eff}(A) = \frac{3g_{\rm YM}^2}{64\pi}\left(\frac{A^4}{\Lambda A^4 + \Pi}\right) = \frac{1}{8\pi M_{\rm Pl}^2}\left(\frac{A^4}{A^4 + \Pi/\Lambda}\right)$$

in the early time limit…

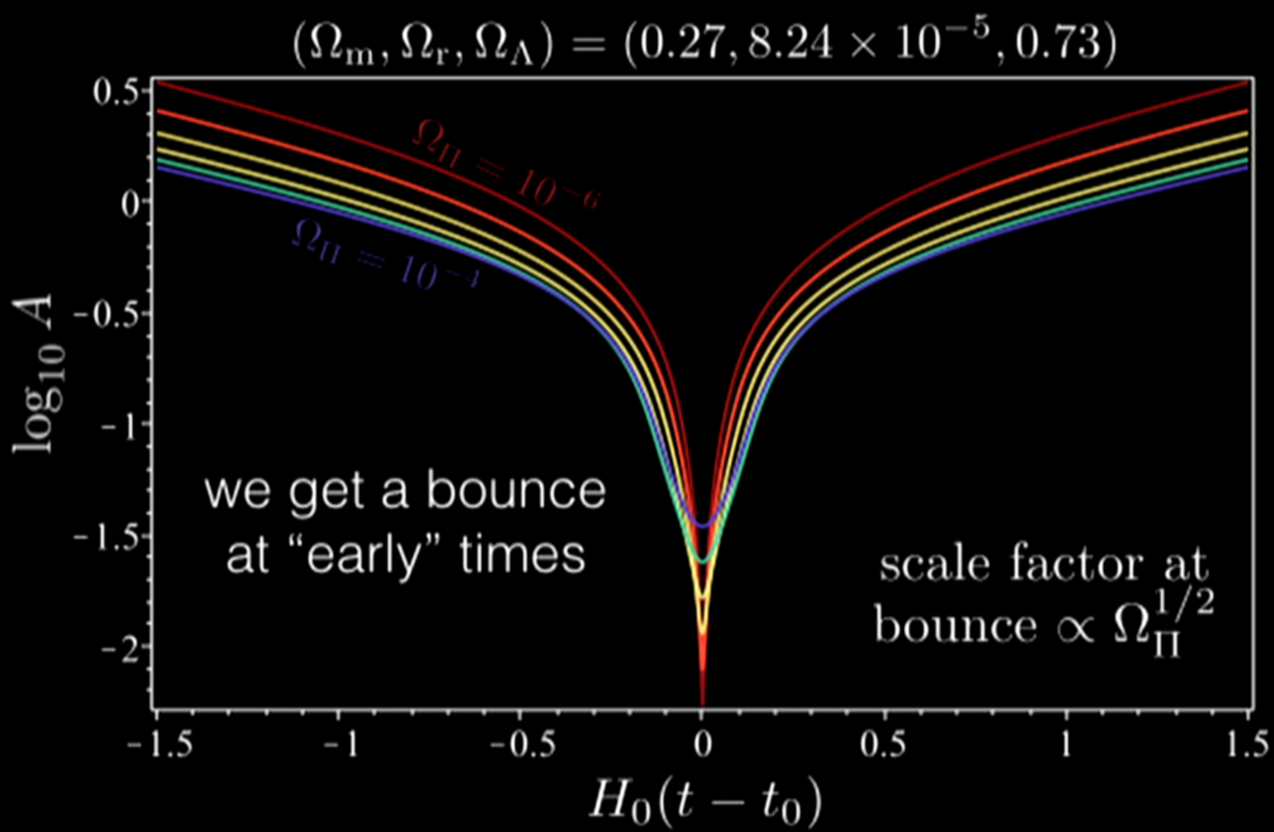
dominates and diverges to negative infinity

this must be positive

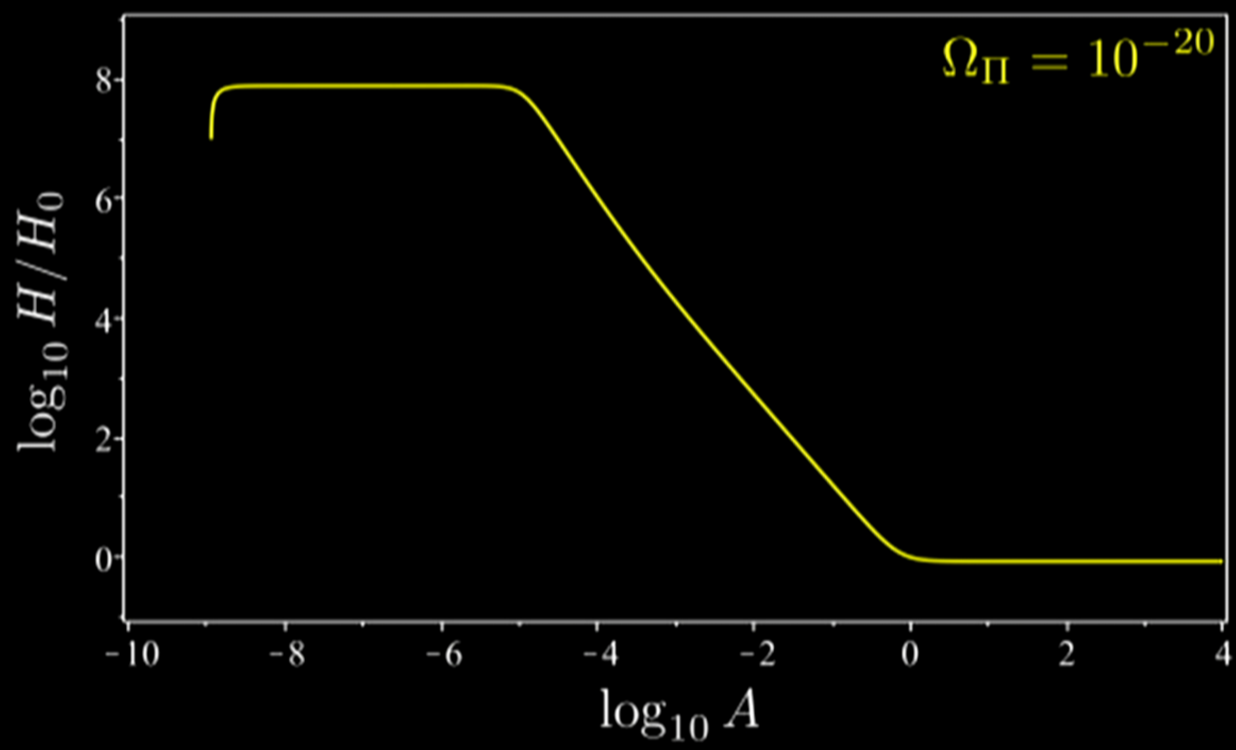
$$H^2 = \frac{8\pi G_{\rm eff}(A)}{3}(\rho_{\rm m} + \rho_{\rm r}) - \frac{k}{r_0^2 A^2} + \frac{\Lambda}{3} - \frac{\Pi}{3A^4}$$

approximately constant due to screening

range of scale factors near zero are forbidden $\Rightarrow$ there must be a cosmological bounce or Einstein-static past attractor simulation results

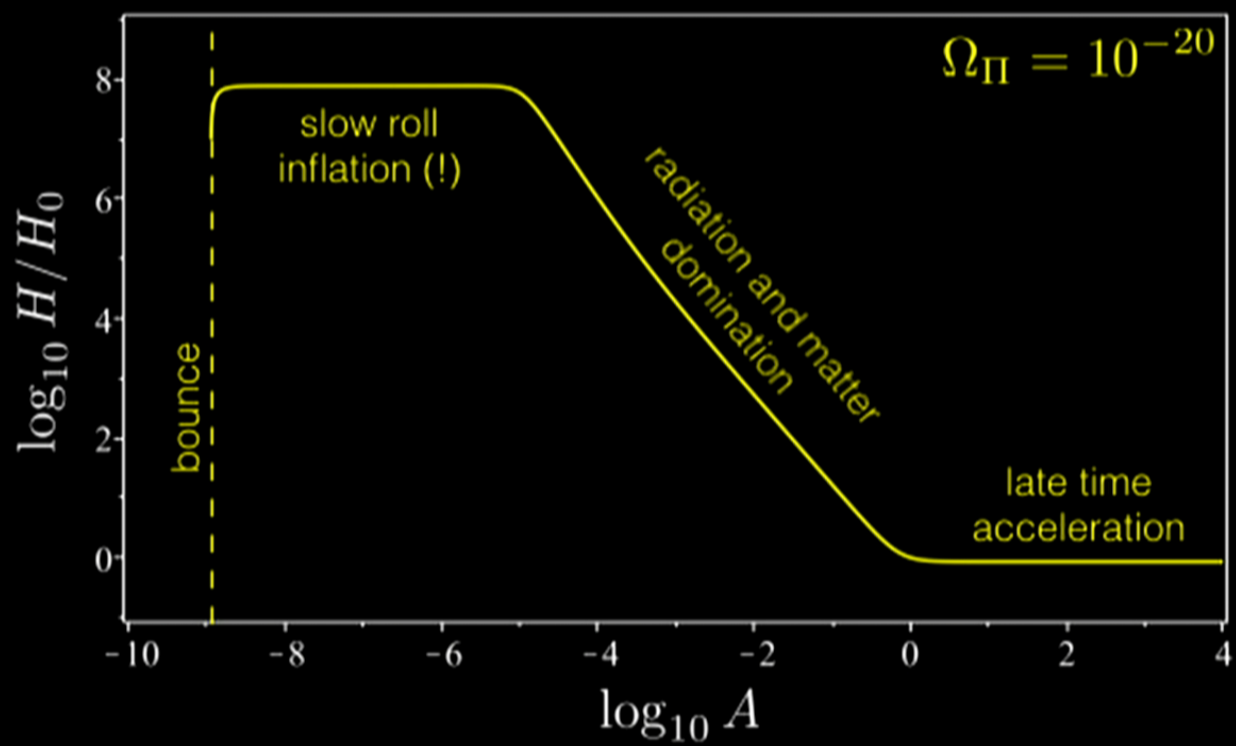


after any cosmological bounce there is an
“inflationary period” of accelerated expansion



we can see what kind of inflation is in our
model by plotting the Hubble factor

after any cosmological bounce there is an "inflationary period" of accelerated expansion



we can see what kind of inflation is in our model by plotting the Hubble factor quasi-de Sitter phase occurs when:

$$H^2 = \frac{\dot{A}^2}{A^2} = \frac{g_{\mathrm{YM}}^2}{8}\left(\frac{\rho_{\mathrm{m}} + \rho_{\mathrm{r}}}{\Lambda + \frac{\Pi}{A^4}}\right) - \frac{k}{r_0^2 A^2} + \frac{\Lambda}{3} - \frac{\Pi}{3A^4}$$

quasi-de Sitter phase occurs when:

$\rho_{\mathrm{r}} \gg \rho_{\mathrm{m}}$ and $\Pi \gg \Lambda A^4$

these are small

$$H^2 = \frac{\dot{A}^2}{A^2} = \underbrace{\frac{g_{\mathrm{YM}}^2}{8}\left(\frac{\rho_{\mathrm{m}} + \rho_{\mathrm{r}}}{\Lambda + \frac{\Pi}{A^4}}\right)}_{\sim \text{constant}} - \frac{k}{r_0^2 A^2} + \frac{\Lambda}{3} - \frac{\Pi}{3A^4}$$

condition for inflation:

$$\frac{\Pi^2}{g_{\mathrm{YM}}^2 \rho_{\mathrm{r},0}} \ll A^4 \ll \frac{\Pi}{\Lambda}$$

how much inflation?

$$e\text{-folds of inflation} = \ln \frac{A_{\rm end}}{A_{\rm start}} \sim 66 - \frac{1}{4} \ln \frac{\Omega_\Pi}{g_{\rm YM}^2}$$

what's the inflationary energy scale?

$$E_{\rm inf} \sim 5 \times 10^{17}\,{\rm GeV} \left(\frac{\Omega_\Pi}{g_{\rm YM}^2} \right)^{-1/4}$$

what's the Yang-Mills coupling?

$$g_{\rm YM}^2 = \frac{8}{3} \frac{\Lambda}{M_{\rm Pl}^2} \sim 10^{-120}$$

[using observational values for usual cosmological parameters] how much inflation?

$$e\text{-folds of inflation} = \ln \frac{A_{\rm end}}{A_{\rm start}} \sim 66 - \frac{1}{4} \ln \frac{\Omega_\Pi}{g_{\rm YM}^2}$$

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$$E_{\rm inf} \sim 5 \times 10^{17}\,{\rm GeV} \left(\frac{\Omega_\Pi}{g_{\rm YM}^2} \right)^{-1/4}$$

what's the Yang-Mills coupling?

$$g_{\rm YM}^2 = \frac{8}{3} \frac{\Lambda}{M_{\rm Pl}^2} \sim 10^{-120}$$

[using observational values for usual cosmological parameters] in cosmological case we found:

$$a^{\mu\nu} = \frac{g_{\rm YM}^2}{4\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_{\rm m})}{\delta l^b_\nu}e^{b\mu} = -\frac{4}{3}\frac{\Pi}{A^4(t)}\left(u^\mu u^\nu + \frac{1}{4}g^{\mu\nu}\right) + \Lambda g_{\mu\nu}$$

this term was crucial for recovering Newton's constant and correct late time limit in Friedmann equation add $a_{\mu\nu} = \Lambda g_{\mu\nu}$ to our previous assumptions:

gauge choice (without loss of generality):	$q_\alpha = 0$
no torsion assumption (gauge invariant):	$de^a + \omega^{ac} \wedge e_c = 0$
technical assumption (gauge invariant):	$\eta_{ab}(e^a_\alpha l^b_\beta - e^a_\beta l^b_\alpha) = 0$
matter action:	$S_{\rm m} = S_{\rm m}[g_{\alpha\beta}, l^a_\alpha, \psi]$

$$G_{\alpha\beta} + \Lambda g_{\alpha\beta} - \frac{3}{4\Lambda}\left(8B_{\alpha\beta} + g_{\alpha\beta}C^2 - 4C_{\mu\nu\rho\alpha}C^{\mu\nu\rho}{}_{\beta}\right) = \frac{3g_{\rm YM}^2}{8\Lambda}T_{\alpha\beta}$$

$$G_{\alpha\beta} = \text{Einstein tensor}$$
$$B_{\alpha\beta} = \text{Bach tensor}$$
$$C_{\alpha\beta\gamma\delta} = \text{Weyl tensor}$$

Schwarzschild de-Sitter is a solution:

$$ds^2 = -f\,dt^2 + \frac{dr^2}{f} + r^2\,d\Omega^2 \qquad f = 1 - \frac{r_0}{r} - \frac{\Lambda r^2}{3}$$

possible consistency with solar system tests, but probable deviations from GR in gravitational wave dynamics

- looked at Yang-Mills gravity based on the conformal SO(4,2) group
- linear theory about Minkowski space exhibits a long range gravitational force
- cosmological solutions exhibit a bounce, long-lived quasi-de Sitter inflation, and late time acceleration
- like Lambda-CDM, no explanation of why the observed cosmological constant is so small in Planck units
- next steps: classical and quantum cosmological perturbations, including torsion, linear theory about de Sitter backgrounds, gravitational waves…

$$G_{\alpha\beta} + \Lambda g_{\alpha\beta} - \frac{3}{4\Lambda}\left(8B_{\alpha\beta} + g_{\alpha\beta}C^2 - 4C_{\mu\nu\rho\alpha}C^{\mu\nu\rho}{}_{\beta}\right) = \frac{3g_{\mathrm{YM}}^2}{8\Lambda}T_{\alpha\beta}$$

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