

Title: Probing Cosmic Acceleration

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Abstract:

If we want a mechanism for the current cosmic expansion that is alternative to (and possibly more “natural” than) the cosmological constant, there exist intriguing proposals within the dark energy and modified gravity realm.

First, I will briefly review the status of one of the most promising ideas, massive gravity: cosmological solutions, some formal aspects and recent developments. Then, I will present recent work aimed at constraining such models with LSS probes.

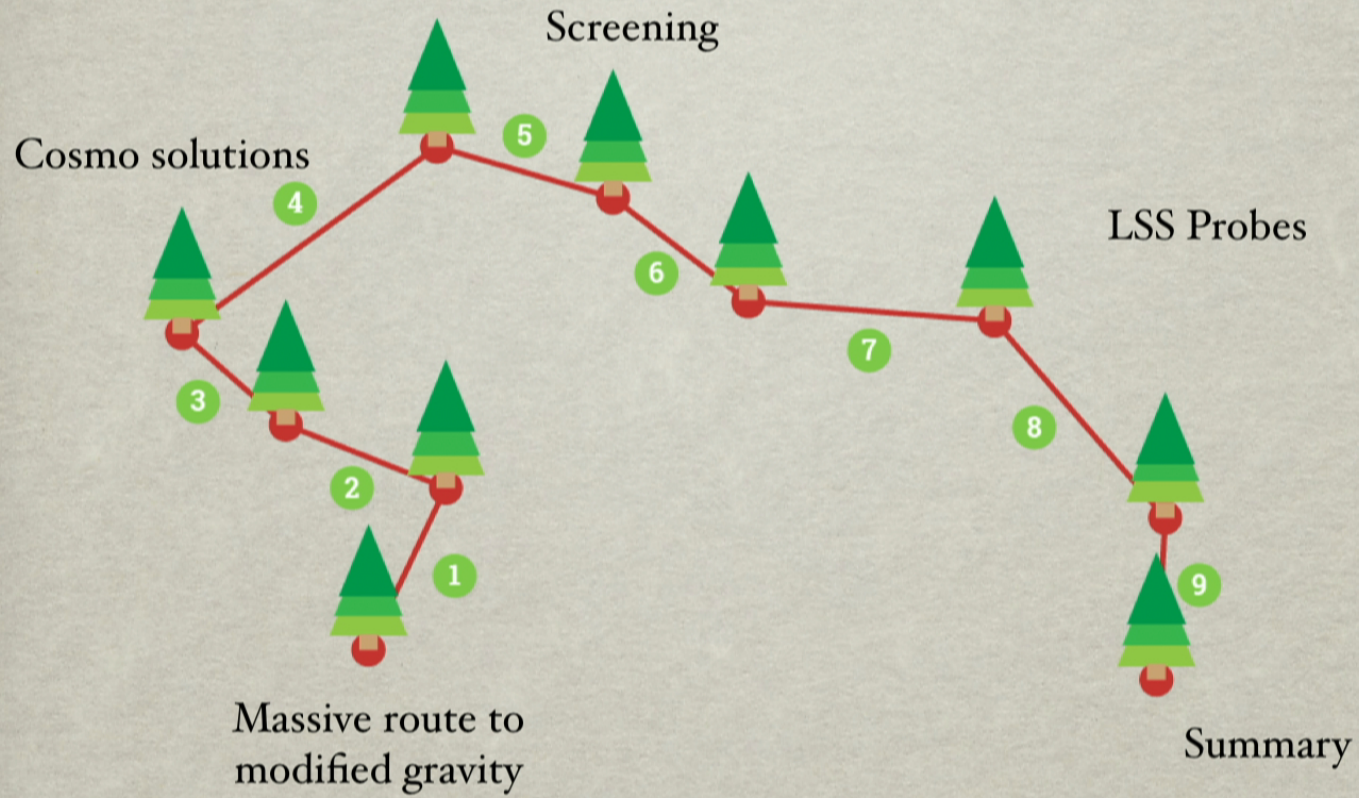
Probing Cosmic Acceleration

Matteo Fasiello



September 20th, 2016, Perimeter Institute

Outline

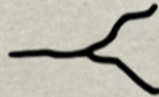


Motivations

Why modified gravity ?

Technically natural mechanism for cosmic acceleration

Why not Λ then ?



Old: “understand why the vacuum energy is so small”

Weinberg, arXiv:astro-ph/0005265



New: “why it is comparable to the present mass density”




Instrument of Choice

Non-linear massive gravity (dRGT) + extensions



Ghost-free, Lorentz-invariant 4-d theory of a ~~mass~~less massive spin 2 field

Linearly

 Fierz and Pauli (1939)

$$\underbrace{-\frac{1}{4}h^{\mu\nu}\hat{\mathcal{E}}_{\mu\nu}^{\alpha\beta}h_{\alpha\beta}}_{\text{E-H}} - \underbrace{\frac{1}{8}m^2(h_{\mu\nu}^2 - h^2)}_{\text{F-P}} ; \text{ 5 dof of healthy massive spin 2}$$

↪ breaks diff invariance: $h_{\mu\nu} \rightarrow h_{\mu\nu} + \partial_{(\mu}\xi_{\nu)}$

vDVZ discontinuity

Theory _{$m \rightarrow 0$} \neq Theory _{$m=0$}

At odds w/ observations: angle for the bending of light at impact parameter b off by 25% w.r.t. GR

Non-linearities better play a crucial role,



they do

Non Linearly

Vainshtein effect: non-linearities screen helicity-0 mode in the presence of matter

$$r_V = \left(\frac{M}{M_{\text{Pl}}^2 m^2} \right)^{1/3}$$

Most of what is verified analytically is static and spherical

screening in an area within r_V , where GR is recovered ✓ linearized theory good outside

Not easy

$$\mathcal{L}_{\text{F-P}}^{n-l} = -m^2 M_{\text{Pl}}^2 \sqrt{-g} \left([(\mathbb{I} - \mathbb{X})^2] - [\mathbb{I} - \mathbb{X}]^2 \right) \quad \text{where } \mathbb{X}_{\nu}^{\mu} = g^{\mu\alpha} \tilde{f}_{\alpha\nu}$$

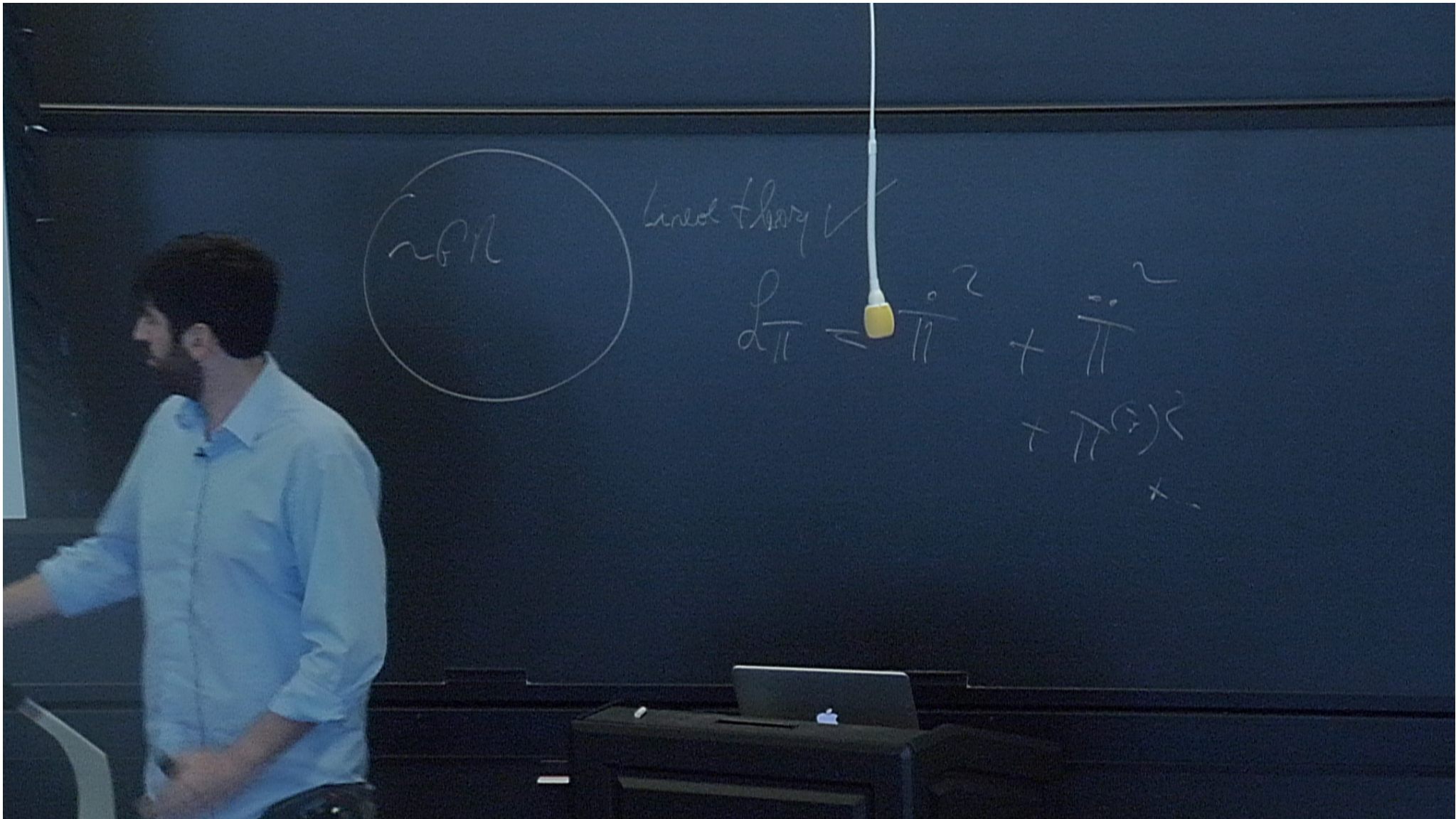
helicity-0 mode π

$$\mathbb{X}_{\nu}^{\mu} = \delta_{\nu}^{\mu} - \frac{2}{M_{\text{Pl}} m^2} \partial^{\mu} \partial_{\nu} \pi + \frac{1}{M_{\text{Pl}}^2 m^4} \partial^{\mu} \partial_{\alpha} \pi \partial^{\alpha} \partial_{\nu} \pi$$

generic non-linear interaction will carry an Ostrogradski ghost

$\sim \mathbb{R}^n$

Linear theory ✓



$$H = p_1^2 + p_2^2$$

linear theory ✓

$$L_{\pi} = \frac{1}{2} \dot{\pi}^2 + \frac{1}{2} \pi'^2 + \pi(\ddot{\pi})^2 + \dots$$

Non Linear with Special Structure, *dRGT*

de Rham, Gabadadze (2010)
de Rham, Gabadadze, Tolley (2010)

$$S_{\text{mGR}} = \frac{M_{\text{Pl}}}{2} \int d^4x \sqrt{-g} \left(R[g] + \frac{m^2}{2} \sum_{n=0}^4 \alpha_n \mathcal{L}_n[\mathcal{K}[g, f]] \right)$$

where $\mathcal{K}_\nu^\mu = \delta_\nu^\mu - \sqrt{g^{\mu\alpha} f_{\alpha\nu}}$ and

$$\mathcal{L}_0[\mathcal{K}] = 4!$$

$$\mathcal{L}_1[\mathcal{K}] = 3! [\mathcal{K}]$$

$$\mathcal{L}_2[\mathcal{K}] = 2!([\mathcal{K}]^2 - [\mathcal{K}^2])$$

$$\mathcal{L}_3[\mathcal{K}] = ([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3])$$

$$\mathcal{L}_4[\mathcal{K}] = ([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4])$$



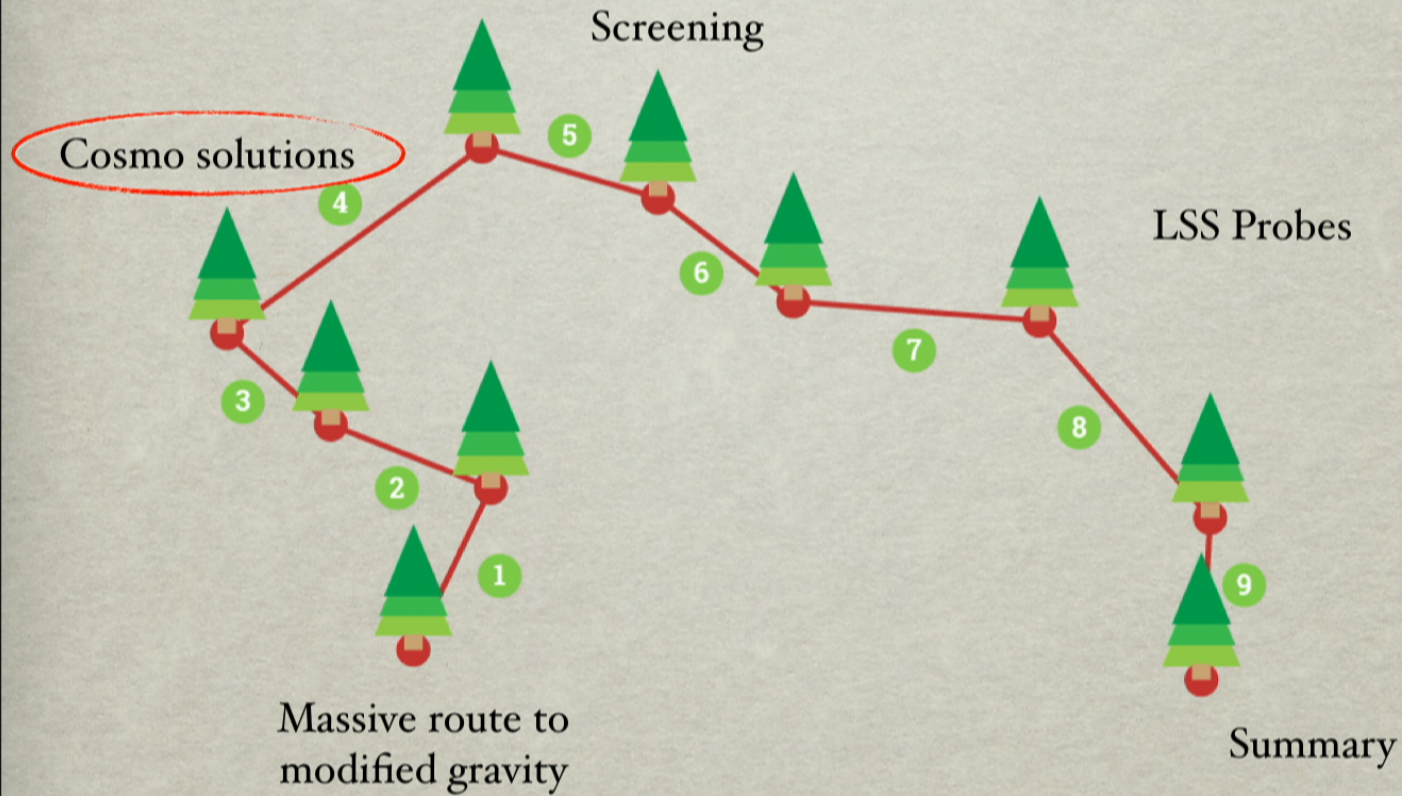
Absence of ghosts verified in countless ways at full non-linear level

Hassan, Rosen
(2011, 2011)

Extensions, e.g. bigravity = massive gravity + H-E for metric f ; 7 healthy dof

Evidence that structure of the potential is preserved under quantum corrections

Outline





Is it empty?

Is is stable?

Is is observationally viable?

No FRW solutions in dRGT if “f” Minkowski*

D'Amico, de Rham, Dubovsky,
Gabadadze, Pirtskhalava, Tolley

Yes, we can live with inhomogeneities

Vainshtein guarantees inhomogeneities unobservable before late times

Inhomogeneities only appear on scale set by inverse graviton mass



Volkov; Koyama; Gumrukcuoglu et al; Gracia, Hu, Wyman;
Kobayashi et al; DeFelice et al; Tasinato et al;
Not updated, many many more!!

*natural for interacting massive spin-2 representation of
Poincare' group

INHOMOGENEOUS SOLS

$\sim \pi$

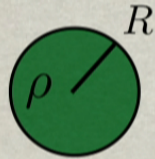
Linear theory ✓

$$L_{\pi} = \frac{\dot{\pi}^2}{\pi^2} + \frac{\ddot{\pi}^2}{\pi^2} + \pi^{(2)}$$

$$L \sim \sqrt{g} [R(g)] + F_n \left(\int \sqrt{g} F_{\mu\nu} \right)$$

Inhomogeneities carried by Stueckelbergs fields

$$\mathcal{K}_\nu^\mu = \delta_\nu^\mu - \sqrt{g^{\mu\alpha} f_{\alpha\nu}} \quad \xleftrightarrow{M_P^2 m^2 \mathcal{L}(\mathcal{K})} \quad f_{\mu\nu} = \partial_\mu \phi^a \partial_\nu \phi^b \bar{f}_{ab}$$



$$r_* = \left(\frac{\rho}{3M_P^2 m^2} \right)^{1/3} R$$

ρ_{co}

the Universe filled with pressure-less dust of density ρ

2 regimes

$$\rho > \rho_{co} ; \quad \rho < \rho_{co} ;$$

$\rho > \rho_{co}$ In a Hubble patch $1/H \sim (\rho/3M_P^2)^{1/2}$

inside the Vainshtein and therefore small corrections, $\propto \left(\frac{m}{H}\right)^k$

} matching...

$\rho < \rho_{co}$ vDVZ regime, far from GR

INHOMOGENEOUS SOLS

(Slightly) Modify Assumptions to allow for FRW

Open Universe solutions (Gumrukcuoglu et al, Vakili et al) unstable

Make reference metric dS (MF, Tolley)... 2 slides away

Generalized massive gravity, but keep the 5 d.o.f. (de Rham, MF, Tolley)... later

Reference
Bigravity: metric
 dynamical

Significant modifications (not just) to allow for FRW

- Lorentz violating massive gravity
- Quasi-dilaton massive gravity + generalizations
- Varying Mass
- Multi-vierbeins
- Extended massive gravity
- Nonlocal massive gravity

Solutions with metric “f” as dS or FRW

MF, Tolley

Add matter content to gauge model independence

$$\mathcal{L}_M \sim \int d^4x \sqrt{-g} \left[\frac{1}{2} (\partial\phi)^2 + V(\phi) \right]$$

Simple algorithm

Existence of the solution

Check stability of the theory

Early + late-time dynamics from Friedman equation

CHANGE F

Stability bound

$$H = \alpha p^2 + \beta q^2 + \gamma(\nabla q)^2 + \dots$$

coefficient of kinetic term > 0

tachyon inst.

gradient inst.

Quickest route to the Higuchi/unitarity bound in dS:

“In the the linear (massive) theory there exist a unitary spin 2 representation of the dS group iff:”

$$m^2 = 0$$

G.R.

$$m^2 = 2H^2$$

Partially massless theory

Higuchi bound
in massive gravity



$$m^2 > 2H^2$$

Massive

CHANGE F

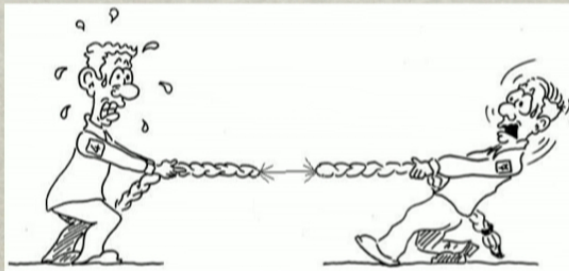
Bound from Observations

Before Dark Energy epoch sets in, G.R. good description:

$$3H^2 = \Lambda + 3m^2 \times \Theta(1) + \dots$$

$$m^2 \lesssim H^2$$

combining Stability and Observations then:



want our theory to be stable

$$m^2 > 2H^2$$

GR over many cosmo epochs

$$m^2 \lesssim H^2$$

CHANGE F

Generalized Higuchi:

$$\tilde{m}^2(H) = m^2 \frac{H}{H_f} \left((3 + 3\alpha_3 + \alpha_4) - 2(1 + 2\alpha_3 + \alpha_4) \frac{H}{H_f} + (\alpha_3 + \alpha_4) \frac{H^2}{H_f^2} \right) \geq 2H^2$$

N.B. independent on precise form of matter

Friedman Side:

$$m^2 \left(\frac{2}{3}(-6 - 4\alpha_3 - \alpha_4) + 2 \left(\frac{H}{H_f} \right) (3 + 3\alpha_3 + \alpha_4) - 2 \left(\frac{H}{H_f} \right)^2 (1 + 2\alpha_3 + \alpha_4) + \frac{2}{3} \left(\frac{H}{H_f} \right)^3 (\alpha_3 + \alpha_4) \right) \ll 2H^2$$

Combined:

$$\frac{\text{poly}_1^{(k)}(z)}{\text{poly}_2^{(k)}(z)} \gg 1$$

Hard to satisfy even using α_3, α_4

Impossible when we account $H = H(t)$

CHANGE F

What now? **Go bigravity!**

$$\mathcal{L} = \frac{1}{2} \sqrt{-g} \left[M_P^2 R(g) - m^2 \sum_{n=0}^4 \beta_n \mathcal{U}_n(g^{-1}f) \right] + \frac{1}{2} \sqrt{-f} M_f^2 R(f) + \mathcal{L}_M$$

$$g_{\mu\nu} \leftrightarrow f_{\mu\nu}, \quad M_P \leftrightarrow M_f, \quad \beta_n \leftrightarrow \beta_{4-n}^*$$

This fact must be reflected on the bound itself

Crucial for Galileon Duality

BIGRAVITY

*Matter breaks this

Stability Bound in Bigravity

MF, Tolley

Soon in a more symmetric form

Stability bound

$$\tilde{m}^2 \left[1 + \left(\frac{H_f/M_f}{H/M_P} \right)^2 \right] \geq 2H^2$$

Minisuperspace action ✓

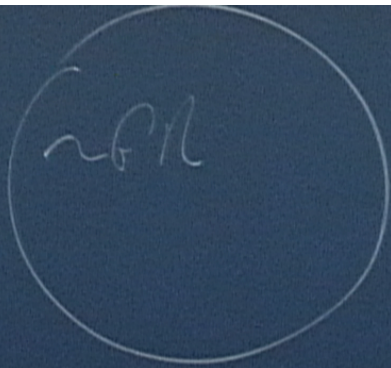
Decoupling limit ✓

PM check ✓

Symmetric ✓

Recover Massive gravity bound in the limit $M_f \rightarrow \infty$, M_P, H_f finite.

BIGRAVITY



linear theory ✓

$$L_{\pi} = \frac{\dot{\pi}^2}{\pi^2} + \frac{\ddot{\pi}^2}{\pi^2} + \pi^{(2)} \dot{\pi}^2$$

$$\frac{\dot{\pi}^2}{\pi^2} \ll \pi^2$$

$$L \sim \int g^{\mu\nu} \Pi^2 g^{\mu\nu} + \left[\frac{\alpha_m}{\beta_m} F_m \left(\int \int g^{\mu\nu} F_{\mu\nu} \right) \right]$$

Not possible before:

$$\frac{H_f}{M_f} \gg \frac{H}{M_P}$$

not directly invoking m

Friedman side

$$H^2 = \frac{1}{3M_P^2} \left[\rho(a) + \sum_{n=0}^3 \frac{3m^2 \beta_n}{(3-n)!n!} \left(\frac{H}{H_f} \right)^n \right] ; \quad H_f^2 = \frac{1}{3M_f^2} \left[\sum_{n=0}^3 \frac{3\beta_{n+1}}{(3-n)!n!} \left(\frac{H}{H_f} \right)^{(n-3)} \right]$$

$m^2 \times \Theta(1) \ll H^2$ it's the only direct requirement on m , but now:

In the $\frac{H_f}{M_f} \gg \frac{H}{M_P}$ region with $\beta_1 \neq 0$ solve for $\tilde{m}^2 H_f$, bound reads:

$$3H^2 > 2H^2 \quad \checkmark$$

The (most pressing) stability **vs** observations tension is **resolved** in bigravity!

BIGRAVITY

Stable Self-accelerating Solution

Akrami, Koivisto, Sandstad
(2012, 2013)

Set: $\beta_2 = 0 = \beta_3; \beta_1 = 2M_P^2$

$$H^2 = \frac{1}{6M_P^2} \left(\rho(a) + \sqrt{\rho(a)^2 + \frac{12 m^4 M_P^6}{M_f^2}} \right)$$

Model	B ₀	B ₁	B ₂	B ₃	B ₄	Ω _m	χ ² _{min}	p-value	log-evidence
ΛCDM	free	0	0	0	0	free	546.54	0.8709	-278.50
(B ₁ , Ω _m ⁰)	0	free	0	0	0	free	551.60	0.8355	-281.73

Observationally viable (?)! Small part of the whole table

Stability bound? It reduces to

$$\left(\frac{1}{M_P^2} + \frac{12M_f^2}{m^4\beta_1^2} H^4 \right) > 0 \quad \checkmark$$

Stable as well.

BIGRAVITY

Further work on Cosmo Solutions

Our focus has been on unitarity (Higuchi), but ought to check gradient instability

$$H = \alpha p^2 + \beta q^2 + \gamma(\nabla q)^2$$

Comelli et al (2012); Konnig et al; Comelli et al #2

Non-linearities via Vainshtein? (in progress...)



Viable (in a reduced parameter space region) bigravity model put forward by

especially useful in the “low energy regime”

De Felice et al (2014)

De Felice et al (2013)

$$h_{ij}T^{ij} = (H_{ij}^+ + C_{(r)}H_{ij}^-) T^{ij}$$

massive(massless) tensors decouple and simplify analysis of e.g. gravitational waves

BIGRAVITY

MF, Ribeiro

Generalized Massive Gravity

De Rham, MF, Tolley (2014)

Let the $\alpha_n(\beta_n)$ now depend on the Stueckelberg fields as

$$S = \frac{M_{Pl}^2}{2} \int d^4x \sqrt{-g} \left[R[g] + \frac{m^2}{2} \sum_{n=0}^4 \tilde{\alpha}_n(\tilde{\phi}^a \tilde{\phi}_a) \mathcal{U}_n[\mathcal{K}] \right] + \mathcal{L}_{\text{matter}}[g, \psi^{(i)}]$$

In pure mGR, isometry group of the reference metric is Poincare'

$$f_{\mu\nu} = \partial_\mu \phi^a \partial_\nu \phi^b \eta_{ab}$$

Now:

keep $\left\{ \begin{array}{l} \text{Lorentz} \quad \checkmark \\ \text{Translations} \quad \times \end{array} \right.$

Gen. mGR theory still has 5 d.o.f. proven ghost-free

De Rham, Keltner, Tolley (2014)

CMG

$\sim \mathbb{R}^4$

linear theory ✓

$$L_{\pi} = \frac{\dot{\pi}^2}{2} + \frac{1}{2} \pi^2$$

$\mathbb{R}^4 \hookrightarrow \mathbb{H}^2$

$$F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$$

$$L = \sqrt{-g} \mathcal{L}(g, F)$$

$$+ \frac{\alpha_m}{\beta_m} F_m \left(\int \sqrt{-g} g^{\mu\nu} F_{\mu\nu} \right)$$

$$d_m = d_m(\phi)$$

①

New Solutions

$$m^2 H \left[\frac{3}{2} \beta_1 a^2 + \beta_2 a + \frac{1}{4} \beta_3 \right] = \frac{m^2}{2a} \left[4\beta'_0 a^3 + 3\beta'_1 a^2 + \beta'_2 a + \frac{1}{6} \beta'_3 \right]$$

zero in pure massive gravity, hence
lack of solution for f Minkowski

②

Hints of self-acceleration

$$3M_{\text{PL}}^2 H^2 = \rho + \frac{m^3 M_{\text{PL}}^2}{2H} \left[\frac{(4\bar{\beta}_{0,1} + 3\bar{\beta}_{1,1} a^{-1})^2}{\bar{\beta}_{1,1}} \right] - 2m^2 M_{\text{PL}}^2 \frac{\bar{\beta}_{0,1} \bar{\beta}_2}{a \bar{\beta}_{1,1}}$$

($\bar{\beta}_{0,1}, \bar{\beta}_{1,1}, \bar{\beta}_2 \neq 0 = \forall_{\text{else}}$)

③

Stability

- Higuchi bound
- Gradient instability
- Coupling with matter
- Vector sector
- Tensor sector as usual

Full-fledged
analysis



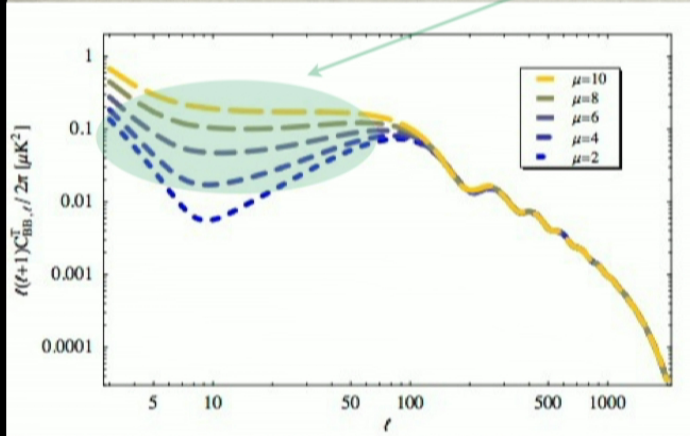
CMC

GW in L-B massive gravity and L-I bigravity

Dubovsky et al

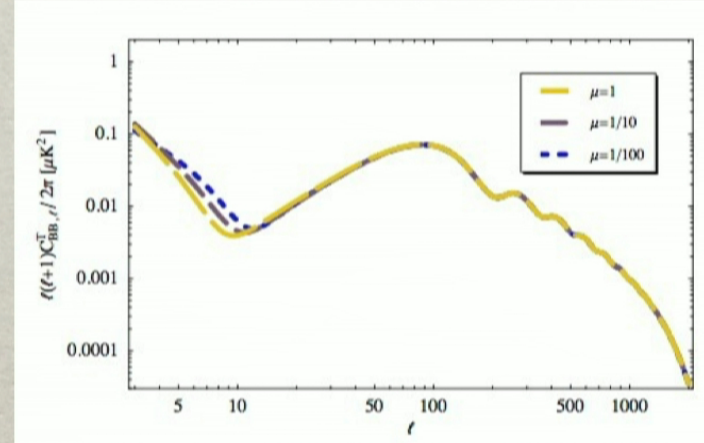
MF, Ribeiro

For effective mass smaller than H_r , low multiples would be enhanced w.r.t massless case



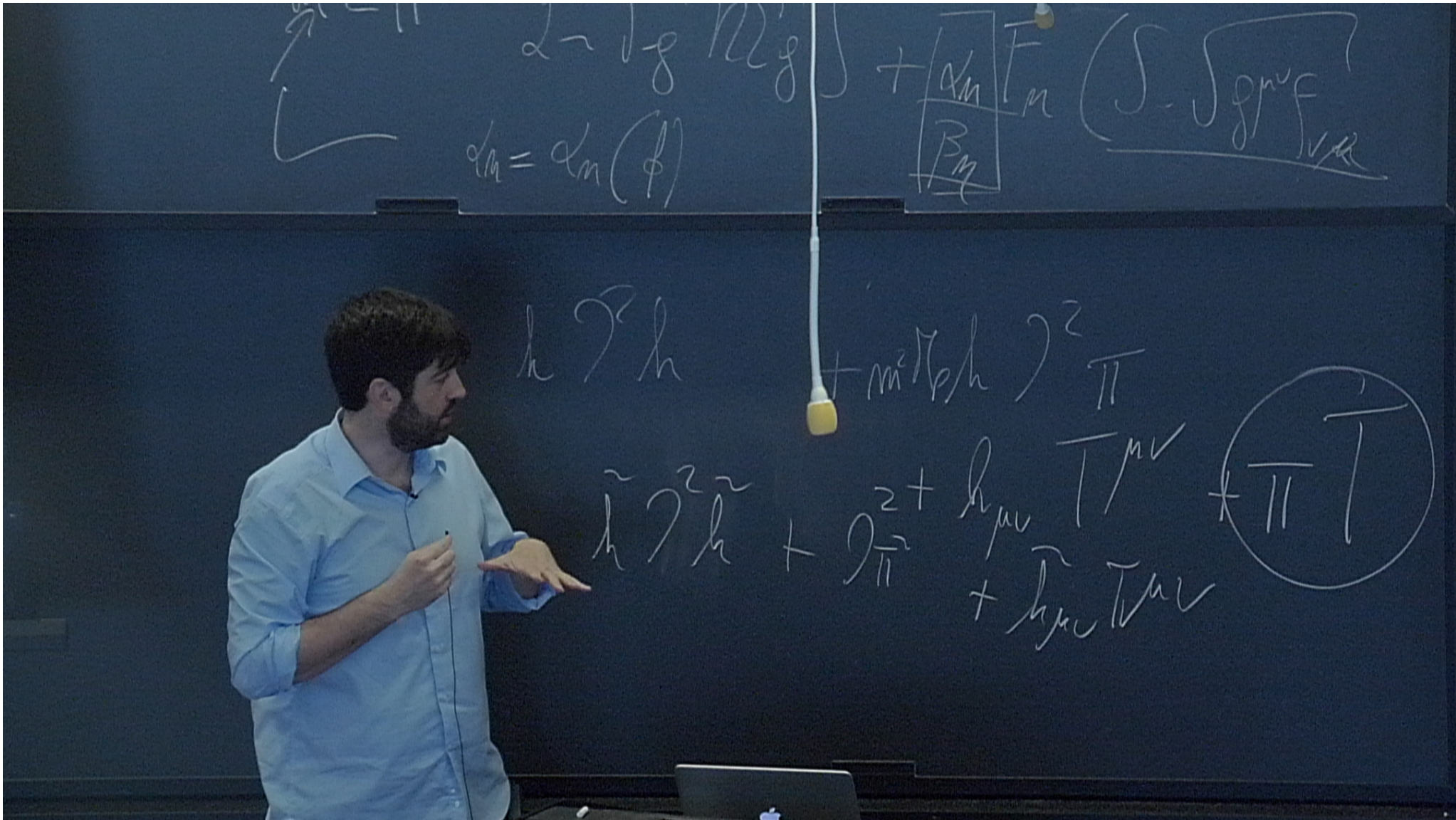
$$\frac{1}{10}H_r < m_{\text{eff}} < H_r$$

$$10^{-30}\text{eV} < m_{\text{eff}} < 10^{-29}\text{eV}$$



$$m_{\text{eff}} < \frac{1}{10}H_r$$

N.B. All plots of multipole coefficients presented here are for a massive graviton only, and were obtained through CAMB by Dubovsky et al. (2010)



$$L \sim \log M g + \frac{1}{2} \frac{d_m F_m}{B_m} \left(\int \sqrt{g^{\mu\nu}} F_{\mu\nu} \right)$$

$$d_m = d_m(\phi)$$

σDVZ $h \tilde{\mathcal{T}} h + m^2 \mathcal{P} h \tilde{\mathcal{T}}^2 \pi$

$$\tilde{h} \tilde{\mathcal{T}}^2 \tilde{h} + \tilde{\mathcal{T}}^2 + h_{\mu\nu} T^{\mu\nu} + \pi \tilde{\mathcal{T}}$$

$$+ \tilde{h}_{\mu\nu} \tilde{\mathcal{T}}^{\mu\nu}$$

$$L \sim \log M g + \frac{1}{2} \left[\frac{a_m F_m}{B_m} \right] \left(\int \sqrt{g^{\mu\nu}} F_{\mu\nu} \right)$$

$$a_m = a_m(\phi)$$

σDVZ $h \tilde{T} h + m^2 \tilde{\phi} h \tilde{T} \tilde{\phi} + \pi (\partial \pi)^2$

$$\tilde{h} \tilde{T} \tilde{h} + \tilde{T} \tilde{\pi}^2 + h_{\mu\nu} T^{\mu\nu} + \pi \tilde{T}$$

$$+ \tilde{h}_{\mu\nu} \tilde{T}^{\mu\nu} + \tilde{Z}$$



$$L \sim \int d^4x \sqrt{-g} \left(\frac{1}{2} R - \frac{1}{2} (\partial_\mu \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right)$$

$$\alpha_M = \alpha_M(\phi)$$

$\int \sqrt{-g}$

$$\Lambda_3 = (m^2 \ell_p)^{1/3}$$

$$\Lambda_2 = (m^2 \ell_p)^{1/2}$$

$$\left(\frac{1}{2} \partial_\mu \pi \partial^\mu \pi + h_{\mu\nu} T^{\mu\nu} + \pi \bar{T} \right)^2$$

$$- \frac{1}{2} \pi^2 + h_{\mu\nu} \bar{T}^{\mu\nu}$$

More...

different
backgrounds altogether
or non-trivial b. for Stueckelberg fields

$$\Phi^a \neq x^a$$

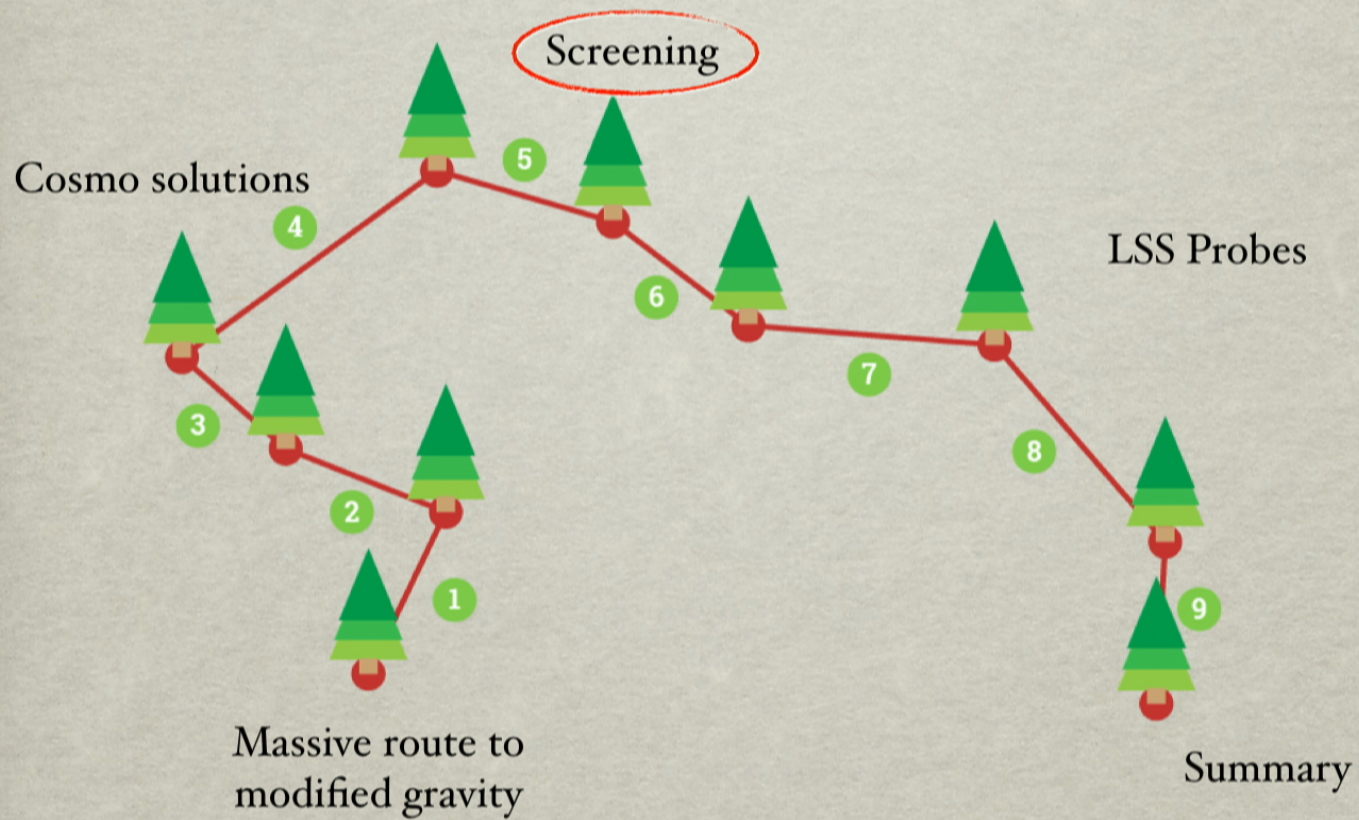


no vDVZ
Porrati (2001)
AdS

Raise strong
coupling scale
De Rham, Tolley, Zhou (2015)
Lorentz-breaking vacua but on
the Stueckelberg side

Higuchi
and
gradient ✓
Aoki, Maeda, Namba (2015)
almost FLRW but
non-linearities in
Stueckelberg

Outline



Solar system tests

Lue, Starkman (2006, i.e. DGP)
Dvali et al (2003, i.e. DGP)
...

LSS

Falck et al (2014)
Falck, Koyama, Zhao(2015)
Gronke, Mota, Winther (2015)
Ivarsen et al(2016)

Lensing
Wyman (2011)

Perturbative approach

Horndeski extensions

Koyama, Niz, Tasinato (2013)
Koyama, Sakstein (2015)
Sakstein, Koyama (2015)
Langlois (2016)
Sakstein et al(2016)

Binary pulsars

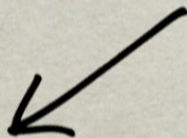
de Rham, Tolley, Wesley (2012)
de Rham, Matas, Tolley (2013)
Jimenez, Piazza, Velten (2016)

Compact obj. vs non-rel. stars
Hui, Nicolis

Tensors
not screened, possible GW
signatures in e.g. bigravity,



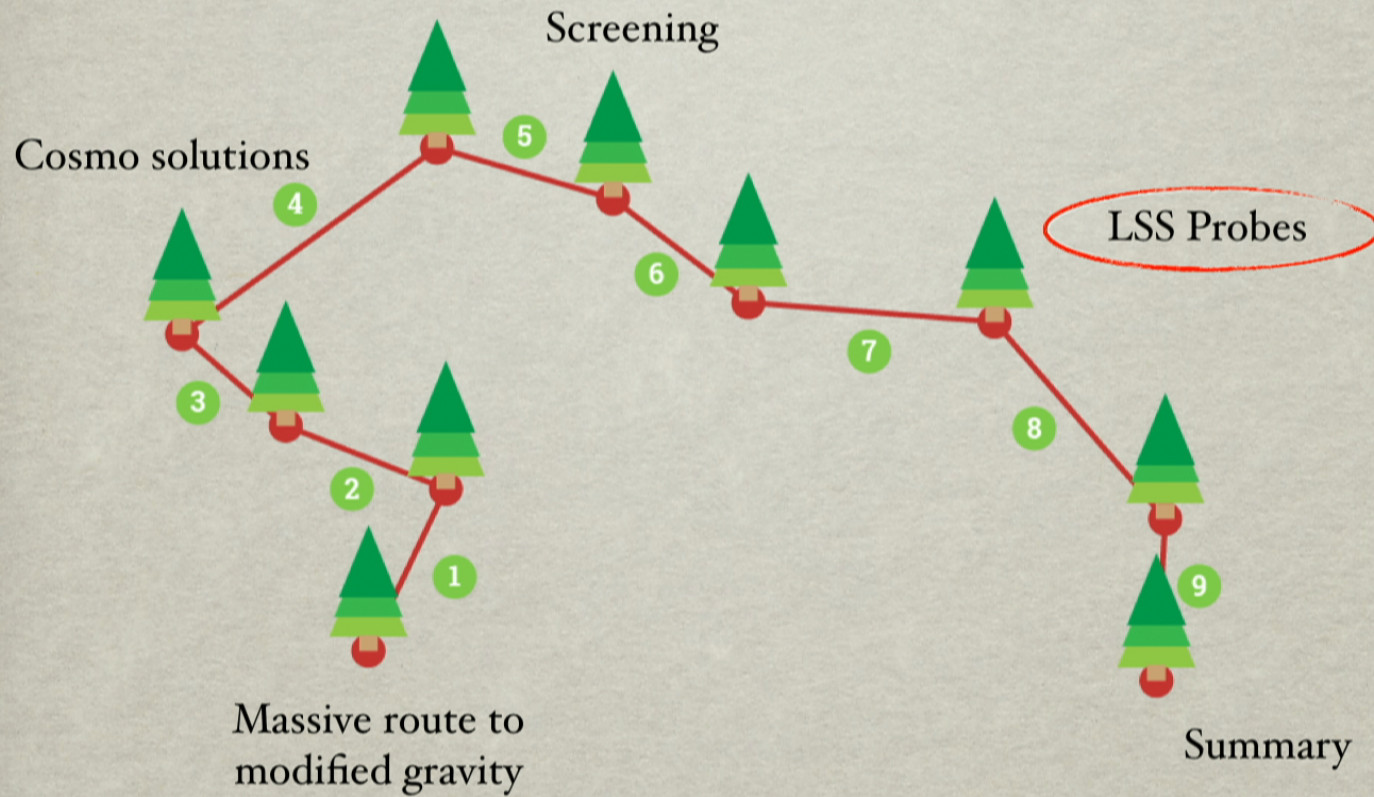
Vainshtein Screening



Numerical

Analytical handle
highly symmetric, idealized setups
(static, sph. symm etc)

Outline



Extra Scalar



$$\mathcal{L} = -\frac{1}{2}Z^{\mu\nu}(\phi, \partial\phi, \partial^2\phi, \dots)\partial_\mu\phi\partial_\nu\phi - V(\phi) + g(\phi)T$$

Screening where GR extremely well-tested, e.g. Solar system

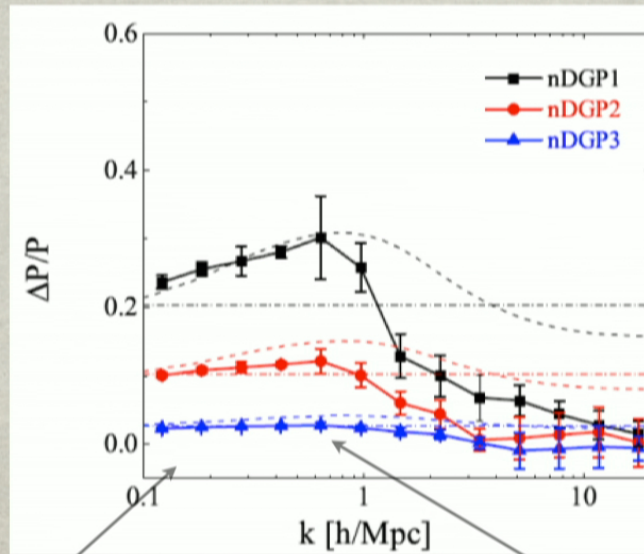
$$V(r) \sim \frac{g^2(\phi)}{Z(\phi)} \frac{e^{-\frac{m(\phi)}{\sqrt{Z(\phi)}}r}}{4\pi r} \mathcal{M}$$

Symmetron

Vainshtein

Chameleon

N-body

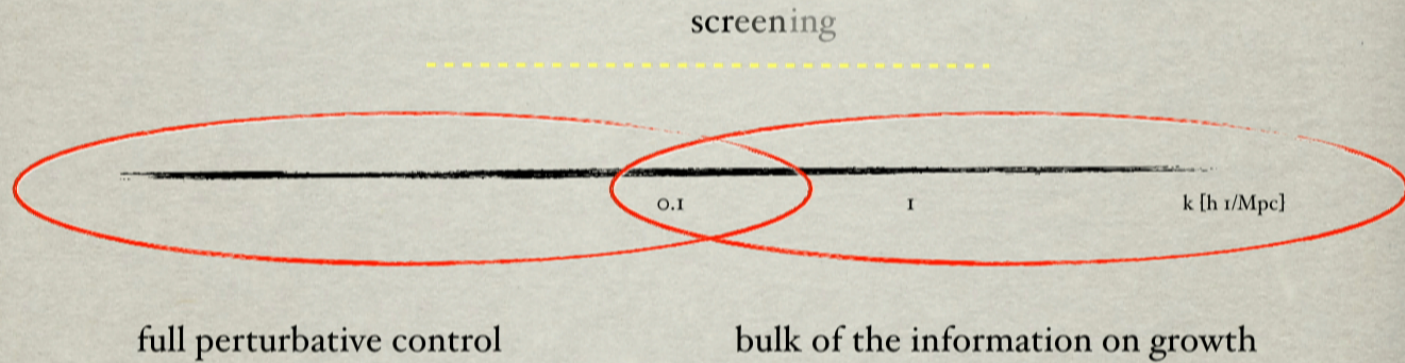


Falck, Koyama, Zhao (2015)

Linear scales

Onset Vainshtein Screening

Perturbation Theory



A lot going on to conquer the quasi-linear scales

RPT (Crocce, Scoccimarro)
TRG (Matarrese, Pietroni; Pietroni)
TSPT (Blas et al)

....

Lagrangian approach
(Matsubara; Porto et al; Vlah et al)

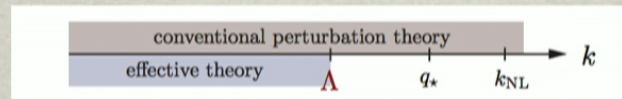
...

EFT of LSS

Perturbative approaches to LSS

conquering quasi-linear scales

EFT of LSS



Baumann et al (2010)

For dark matter (or more), use fluid description

$$\dot{\theta}_\ell + \mathcal{H}\theta_\ell + \frac{3}{2}\Omega_m\mathcal{H}^2\delta_\ell = -\frac{1}{\rho_\ell}\nabla_i\nabla_j\langle\tau_{ij}\rangle$$

the “EFTness” of the approach is in the fact one describes long-wavelength dynamic informed by a few UV inputs

$$\begin{aligned} \langle\tau_{ij}\rangle = & \rho \left[c_1 \left(\frac{\langle v_s^2 \rangle \partial^2}{\mathcal{H}^2} \right)_{ij} + c_2 \left(\frac{\langle v_s^2 \rangle \partial^2}{\mathcal{H}^2} \right)_{ij}^2 + \dots \right] \phi_\ell + \\ & + \rho \left[\left(d_1^{(n)} \left(\frac{\partial^2}{\Lambda^2} \right) + d_2^{(n)} \left(\frac{\partial^2}{\Lambda^2} \right)^2 + \dots \right) \{ v_\ell^2, \delta_\ell \phi_\ell, \dots \} \right]_{ij} \end{aligned}$$

$\alpha \sim \log M/g$
 $\alpha_m = \alpha_m(\phi)$
 $\frac{1}{\alpha_m} F_m \left(\int \sqrt{g^{\mu\nu}} F_{\mu\nu} \right)$

σ DUVZ

$\lambda_3 = (m^2/p)^{1/3}$

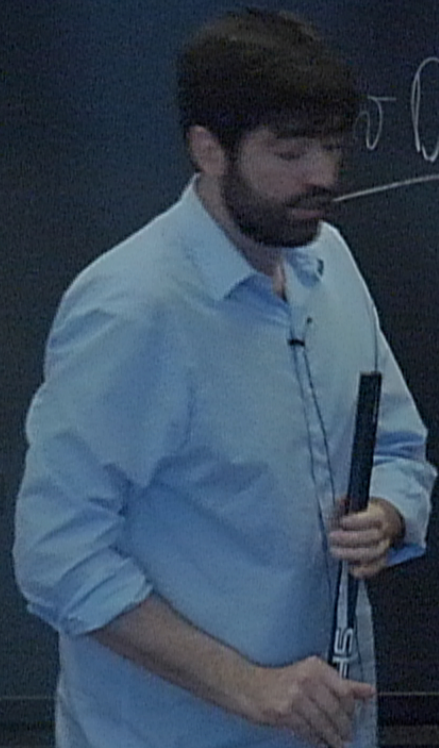
$\lambda_2 = (m^2/p)^{1/2}$

$S_m + \frac{1}{2} \left((1 + S_m) \rho_m^2 = 0 \right) (\pi)^2$
 $+ m^2 \rho_m^2$

$- \frac{1}{2} \pi^2 + h_{\mu\nu} T^{\mu\nu} + \pi \dot{\pi}$
 $+ \tilde{h}_{\mu\nu} \tilde{T}^{\mu\nu}$

$$L \sim \int d^4x \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right) + \int d^4x \sqrt{-g} \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$\alpha_m = \alpha_m(\phi)$$



$\rightarrow \text{DUAL}$

$$\Lambda_3 = (m^2 \ell_p)^{1/3}$$

$$\Lambda_2 = (m^2 \ell_p)^{1/2}$$

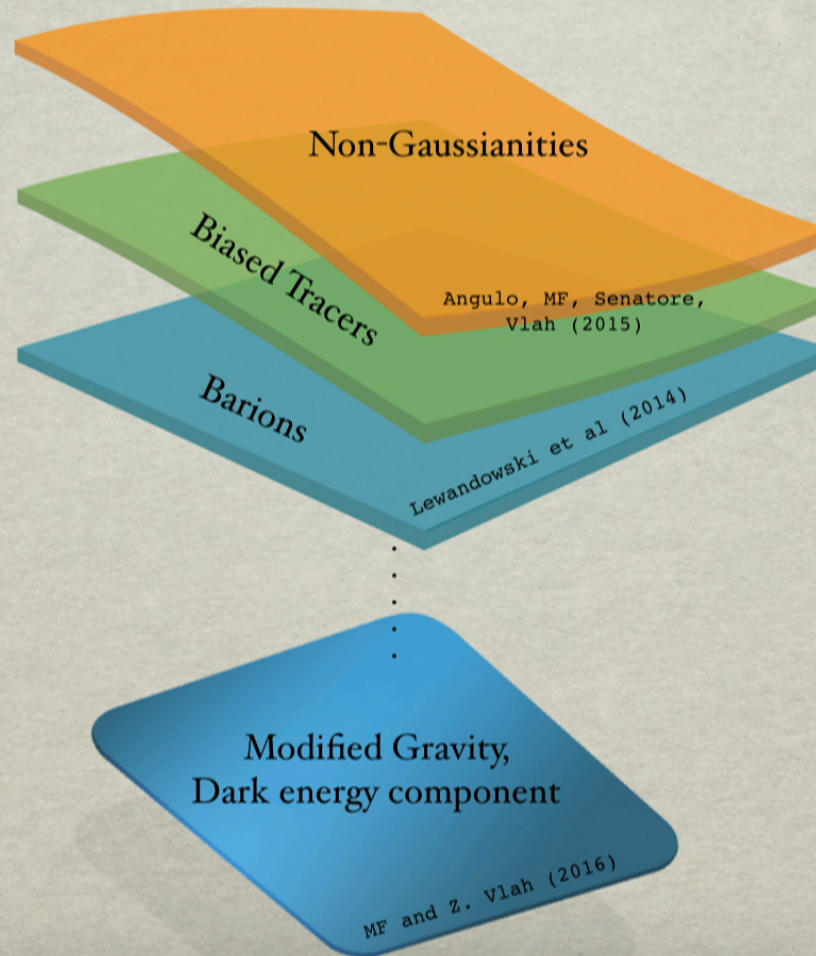
$$e_g \left(\frac{M}{\hbar c} \right)^2$$

$$S_m + \frac{1}{4} (1 + S_m) \int d^4x \sqrt{-g} \left(\partial_\mu \pi \right)^2 + m^2 \ell_p^4 \pi^2$$

$$- \frac{1}{2} \pi^2 + h_{\mu\nu} T^{\mu\nu} + \pi \dot{\pi}$$

$$+ \tilde{h}_{\mu\nu} \tilde{T}^{\mu\nu} + \pi \dot{\pi}$$

Layers of physics



Adding a MG or DDE component

Creminelli et al (2009);
Sefusatti, Vernizzi (2011);
Anselmi et al (2011);
D'Amico, Sefusatti (2011);

$$\left\{ \begin{array}{l} \frac{\partial \delta_m}{\partial \tau} + \vec{\nabla} \cdot [(1 + \delta_m) \vec{v}] = 0 \\ \frac{\partial \delta_Q}{\partial \tau} - 3\omega \mathcal{H} \delta_Q + \vec{\nabla} \cdot [(1 + \omega + \delta_Q) \vec{v}] = 0 \\ \frac{\partial v}{\partial \tau} + \mathcal{H} \vec{v} + (\vec{v} \cdot \vec{\nabla}) \vec{v} = -\nabla \Phi \end{array} \right. \quad \text{clustering quintessence, } c_s=0$$

$$\nabla^2 \Phi = \frac{3}{2} \mathcal{H}^2 \Omega_m \left(\delta_m + \delta_Q \frac{\Omega_Q}{\Omega_m} \right) \delta_T$$

known exactly only up to quadratic order

All orders, integral & differential solutions

MF, Vlah (2016)

$$\delta_{\mathbf{k}}(\eta) = \sum_{n=1}^{\infty} F_n^s(\mathbf{q}_1 \dots \mathbf{q}_n, \eta) D_+^n(\eta) \delta_{\mathbf{q}_1}^{\text{in}} \dots \delta_{\mathbf{q}_n}^{\text{in}}$$

$$\Theta_{\mathbf{k}}(\eta) = \sum_{n=1}^{\infty} G_n^s(\mathbf{q}_1 \dots \mathbf{q}_n, \eta) D_+^n(\eta) \delta_{\mathbf{q}_1}^{\text{in}} \dots \delta_{\mathbf{q}_n}^{\text{in}}$$

$$F_n(\eta) = \int_{-\infty}^{\eta} \frac{d\tilde{\eta}}{C(\tilde{\eta})} \left\{ e^{(n-1)(\tilde{\eta}-\eta)} \frac{\tilde{f}_+}{\tilde{f}_+ - \tilde{f}_-} \left[\left(\tilde{h}_{\beta}^{(n)} - \frac{\tilde{f}_-}{\tilde{f}_+} \tilde{h}_{\alpha}^{(n)} \right) + e^{\tilde{\eta}-\eta} \frac{D_-(\eta)}{\tilde{D}_-(\eta)} \left(\tilde{h}_{\alpha}^{(n)} - \tilde{h}_{\beta}^{(n)} \right) \right] \right\}$$

$$G_n(\eta) = \int_{-\infty}^{\eta} \frac{d\tilde{\eta}}{C(\tilde{\eta})} \left\{ e^{(n-1)(\tilde{\eta}-\eta)} \frac{\tilde{f}_+}{\tilde{f}_+ - \tilde{f}_-} \left[\left(\tilde{h}_{\beta}^{(n)} - \frac{\tilde{f}_-}{\tilde{f}_+} \tilde{h}_{\alpha}^{(n)} \right) + e^{\tilde{\eta}-\eta} \frac{\tilde{f}_-}{\tilde{f}_+} \frac{D_-(\eta)}{\tilde{D}_-(\eta)} \left(\tilde{h}_{\alpha}^{(n)} - \tilde{h}_{\beta}^{(n)} \right) \right] \right\}$$

$$C = 1 + (1 + \omega) \frac{\Omega_Q(\eta)}{\Omega_m(\eta)}$$

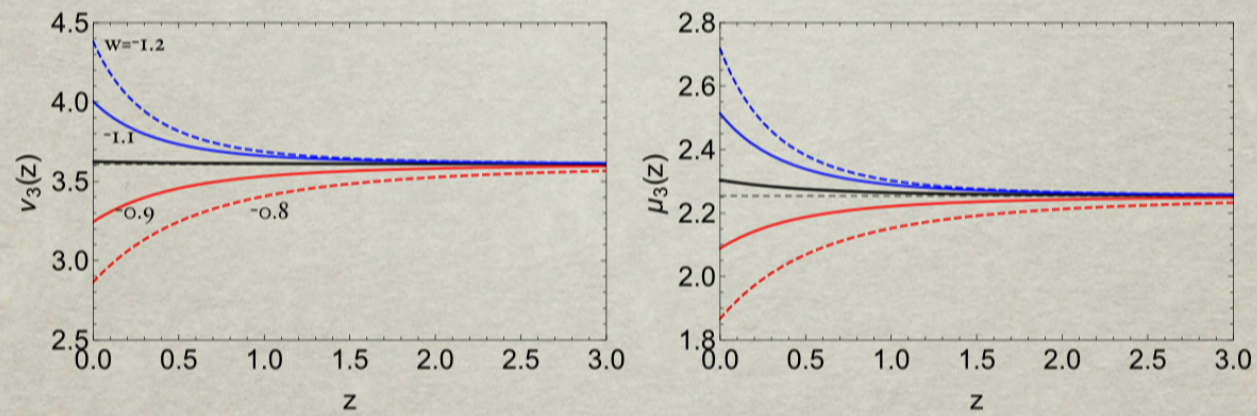
iteratively derived, first recursion are usual

$\alpha(\mathbf{q}_1, \mathbf{q}_2), \beta(\mathbf{q}_1, \mathbf{q}_2)$

related to
 \propto linear growth rate

$$F_3 = (1 - \epsilon^{(2)})\mathcal{F}_3^\epsilon + \nu_3\mathcal{F}_3^{\nu_3} + (1 - \epsilon^{(1)})\nu_2\mathcal{F}_3^{\nu_2} + \lambda_1\mathcal{F}_3^{\lambda_1} + \lambda_2\mathcal{F}_3^{\lambda_2}$$

$$G_3 = (1 - \epsilon^{(2)})\mathcal{G}_3^\epsilon + \mu_3\mathcal{G}_3^{\mu_3} + (1 - \epsilon^{(1)})\mu_2\mathcal{G}_3^{\mu_2} + \kappa_1\mathcal{G}_3^{\kappa_1} + \kappa_2\mathcal{G}_3^{\kappa_2}$$

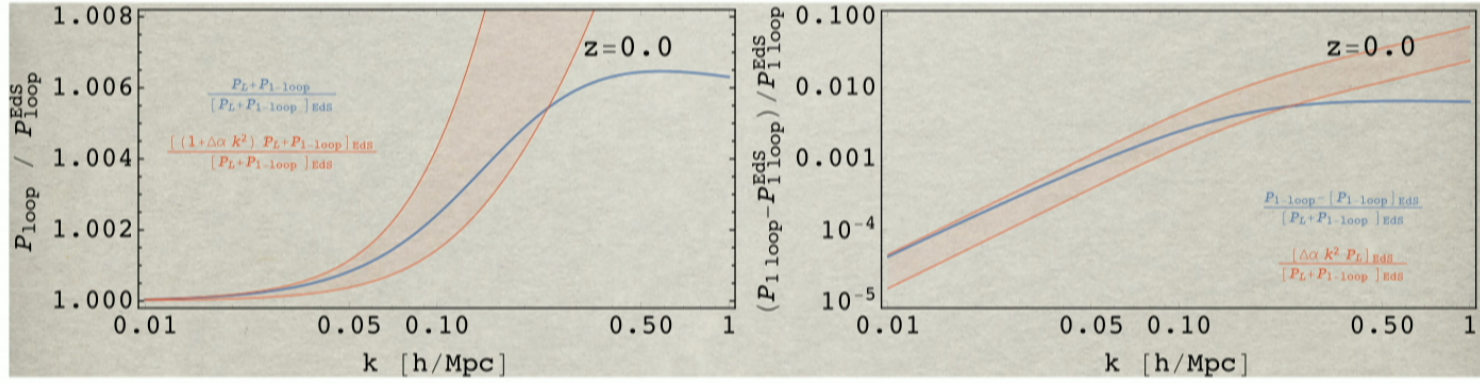


similarly for $\lambda_1, \lambda_2, \kappa_1, \kappa_2(z)$ while $\mathcal{F}_3 = \mathcal{F}_3(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3)$

Observables

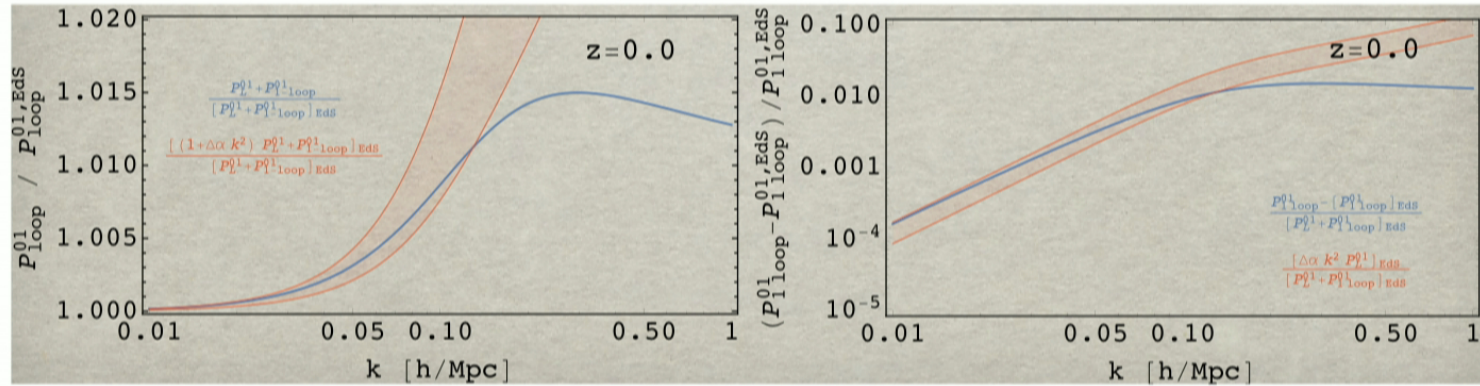
MF, Vlah (2016)

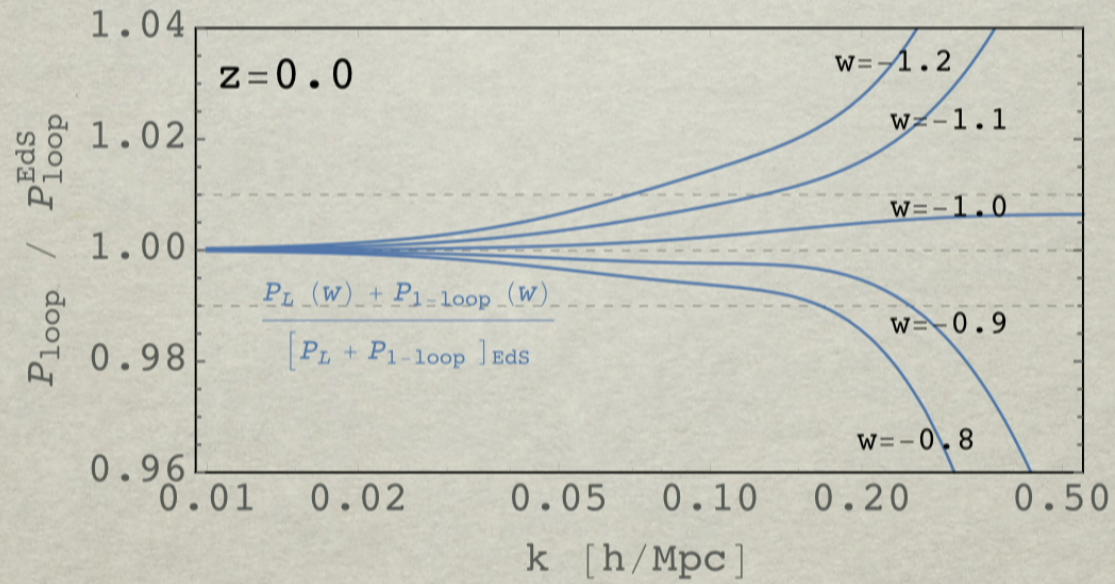
$$P_{1\text{-loop}}(k, a) = P_L(k, a) + P_{22}(k, a) + 2P_{13}(k, a) + P_{c.t.}(k, a)$$

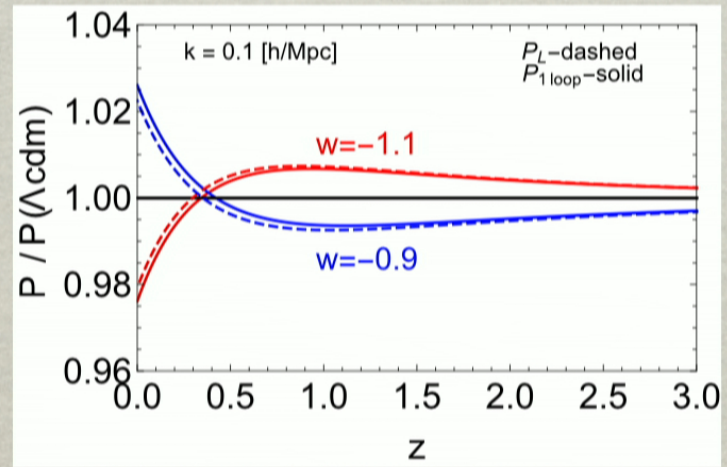


$C(\eta) = 1$

test with Λ CDM





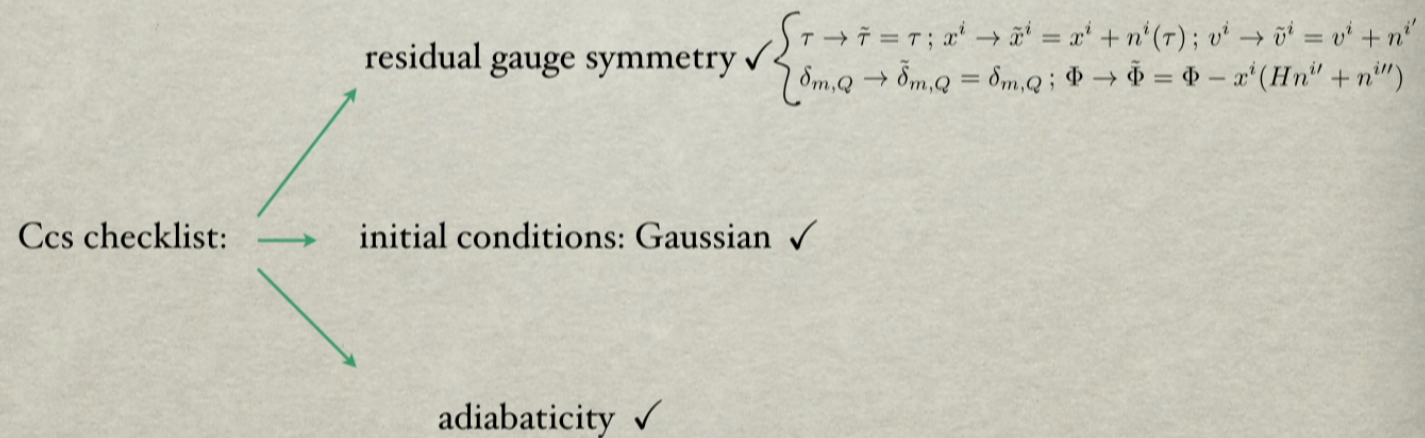


Consistency Conditions

MF, vlah, to appear

reduced system effectively 1 d.o.f.

$$\begin{cases} \delta'_T + \partial_i[(C + \delta_T)v^i] = 0, \text{ with } C = 1 + (1 + w)\frac{\Omega_Q}{\Omega_m} \\ \frac{\partial v^i}{\partial \tau} + \mathcal{H}v^i + v^j \partial_j v^i = -\nabla^i \Phi; \quad \nabla^2 \Phi = \frac{3}{2} \mathcal{H}^2 \Omega_m \delta_T, \end{cases}$$



$\mathcal{L} = \frac{1}{2} \dot{\phi}^2 - \frac{1}{2} \nabla_a \phi \nabla^a \phi - V(\phi)$
 $\delta \mathcal{L} = \delta \mathcal{L}(\phi)$
 $\delta \mathcal{L} = \delta \mathcal{L}(\phi) + \frac{\delta g_{\mu\nu}}{2} T^{\mu\nu}$

$\frac{\delta \mathcal{L}}{\delta \phi} = 0 \Rightarrow \frac{\delta \mathcal{L}}{\delta \phi} + \nabla_\mu \left(\frac{\delta \mathcal{L}}{\delta \partial_\mu \phi} \right) = 0$
 $\frac{\delta \mathcal{L}}{\delta \phi} = \frac{\delta \mathcal{L}}{\delta \phi} + \nabla_\mu \left(\frac{\delta \mathcal{L}}{\delta \partial_\mu \phi} \right) = 0 \quad (\text{EOM})$

$\delta \mathcal{L} = \frac{1}{2} \delta g_{\mu\nu} T^{\mu\nu} + \delta \pi^{\mu\nu} T_{\mu\nu}$
 $\delta \mathcal{L} = \frac{1}{2} \delta g_{\mu\nu} T^{\mu\nu} + \delta \pi^{\mu\nu} T_{\mu\nu}$

What's next

MF, Vlah



IR properties of r -loop power spectrum

Bispectrum

CQ

All ready for any $w \neq -1$, need simulations to determine “counterterm” coefficient

Expand to more complex dynamics,
which include higher derivatives (e.g. Galileons) and screening

k_{nl} vs k_V
(and its model-dependence)