Title: Feynman Path Integrals Over Entangled States

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Abstract: <p>Entanglement is fundamental to quantum mechanics. It is central to the EPR paradox and Bellâ€™s inequality. Tensor network states constructed with explicit entanglement structures have provided powerful new insights into many body quantum mechanics. In contrast, the saddle points of conventional Feynman path integrals are not entangled, since they comprise a sequence of classical field configurations. The path integral gives a clear picture of the emergence of classical physics through the constructive interference between such sequences, and a compelling scheme for adding quantum corrections using diagrammatic expansions. We combine these two powerful and complementary perspectives by constructing Feynman path integrals over sequences of matrix product states, such that the dominant paths support a degree of entanglement. We develop a general formalism for such path integrals and give a couple of simple examples to illustrate their utility [arXiv:1607.01778]. </p>

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Feynman Path Integrals over Entangled States

Andrew G. Green

Steve Simon[1]
Chris Hooley[2]
Jonathan Keeling[2]

[arXiv:1607.01778]

[1]University of Oxford, [2]University of St Andrews

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Abstract:

Entanglement is fundamental to quantum mechanics. It is central to the EPR paradox and Bell's inequality, and gives robust criteria to compress the description of quantum states. In contrast, the Feynman path integral shows that quantum transition amplitudes can be calculated by summing sequences of states that are not entangled at all. This gives a clear picture of the emergence of classical physics through the constructive interference between such sequences. Accounting for entanglement is trickier and requires perturbative and non-perturbative expansions.

We combine these two powerful and complementary insights by constructing Feynman path integrals over sequences of states with a bounded degree of entanglement.

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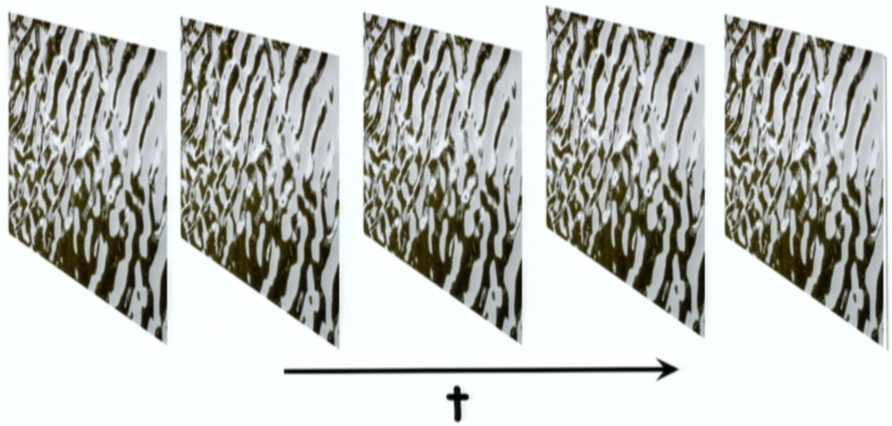
Outline:

- Goal and Central Idea
- Technical Background
- Formulating the Path Integral
- Illustrative Examples
- General Formulation
- Discussion and Conclusions

Towards a Path Integral over MPS

Goal: Import insights from tensor networks into a path integral over tensor network states



Feynman Path Integral

- Sum over classical/product state trajectories
- Sequence of classical/product state field configurations
- Can we do the same with weakly entangled states?
- Sequence of weakly entangled field configurations.

Towards a Path Integral over MPS

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Goal and Central Idea

$$\mathcal{Z} = Tr\ e^{-\beta\mathcal{H}}$$

$$\mathbb{1}\ \mathbb{1}\ \mathbb{1}\ \mathbb{1}\ \mathbb{1}\ \mathbb{1}\ \mathbb{1}\qquad \mathbb{1} = \int D\psi |\psi\rangle\langle\psi|$$

$$= \int D\psi e^{\int d\tau \left[\langle\psi|\partial_\tau \psi\rangle - \langle\psi|\hat{\mathcal{H}}|\psi\rangle\right]}$$

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- Sum over classical/product state trajectories
- Insert resolutions of identity over over-complete set
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Feynman Pat

- Sum o
- Insert … e set
- Usuall
- Can w … es?

Q. What is the Measure?

Q. Is the theory local?

Q. What is the Berry Phase?

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Insights and New Perspectives

- Saddle points equations $\delta\mathcal{S}/\delta A = 0 \Rightarrow$ TDVP
- Saddlepoints with features not present in product states
- Not always adiabatically connected to product states
- Instantons @ χ=1 => Saddle point at χ>1
- Deconfined criticality => Ginzburg-Landau in A
- Perturbative corrections to MPS?

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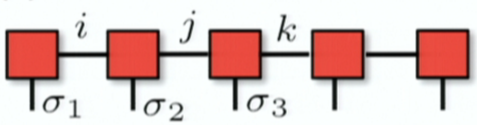
Technical Background

Tensor Networks

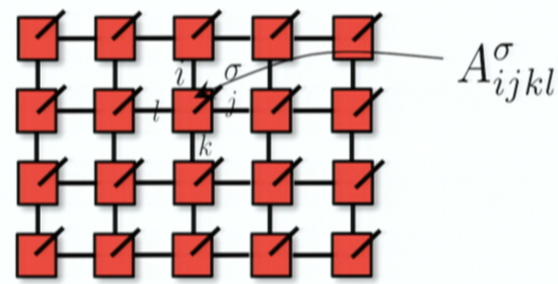
- Class of variational wavefunctions
- Embody insights about entanglement structure
- Describe groundstates of local Hamiltonians efficiently
- Exact for some model H (AKLT, Majumdar-Ghosh, etc)

Matrix Product States

$$|\phi\rangle = \sum_{\{\sigma\}} A_i^{\sigma_1} A_{ij}^{\sigma_2} A_{jk}^{\sigma_3} A_{kl}^{\sigma_4} \ldots |\sigma_1, \sigma_2, \sigma_3, \sigma_4, \ldots\rangle$$



Projected Entangled Pair States

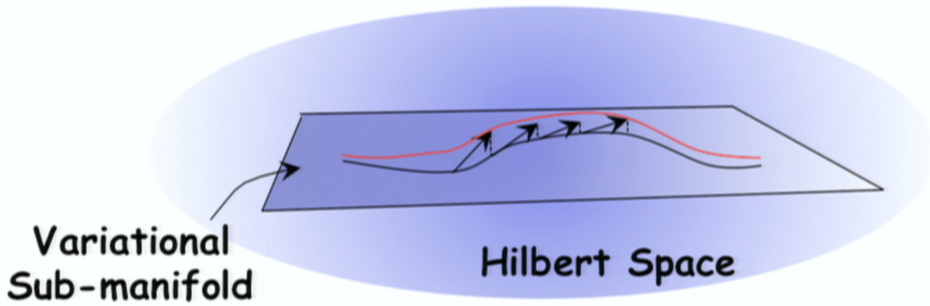


- Tensor networks are a restricted sum of product states
- Over-complete cover of Hilbert space

Technical Background

Time dependence of Tensor Network States

- Bond-order grows under Hamiltonian evolution
- TDVP: Continually Project back to fixed bond order
- Resulting equations are semi-classical
- Variational manifold forms a semi-classical phase space



$$\langle \partial_{\bar{A}_i} \psi | \partial_{A_j} \psi \rangle \dot{A}_j = i \langle \partial_{\bar{A}_i} \psi | \hat{\mathcal{H}} | \psi \rangle$$

[Haegeman et al PRL 2011]

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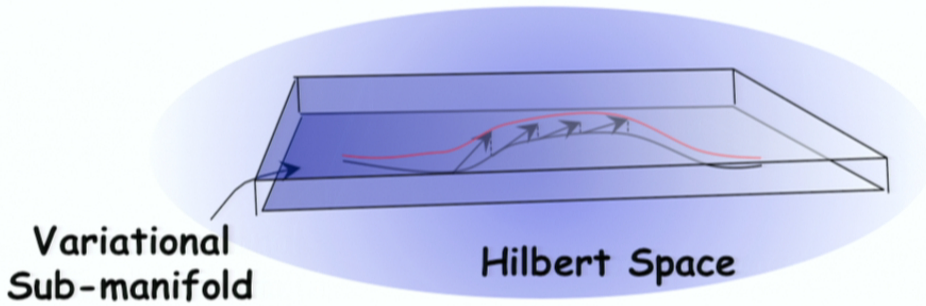


Manifold ~ semi-classical phase space
Required when tunnelling/instantons drive physics

Technical Background

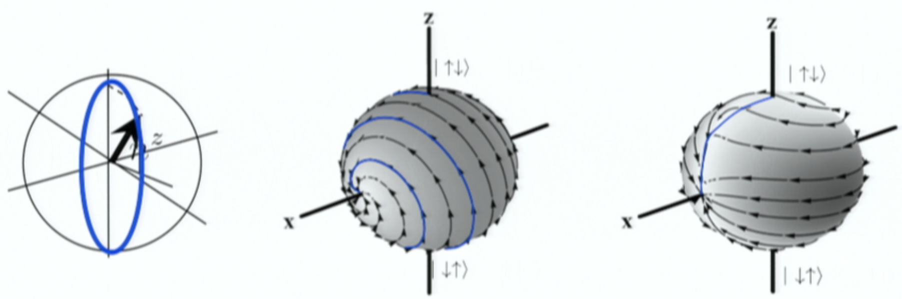
Entanglement Structure vs Tunnelling Trajectories

- Tensor networks are a restricted sum of product states
- Transfer weight => tunnelling between product states

e.g. 2 spins $\mathcal{H} = J\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$ start with $|\uparrow\downarrow\rangle$

- **MPS description:** $|\psi\rangle = n_1|\mathbf{l}_1, \mathbf{l}_2\rangle + n_2|-\mathbf{l}_1, -\mathbf{l}_2\rangle = \frac{1+e^{iJt}}{2}|\uparrow\downarrow\rangle + \frac{1-e^{iJt}}{2}|\downarrow\uparrow\rangle$

$$\mathbf{n} = (n_1^*, n_2^*)\boldsymbol{\sigma}\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$$



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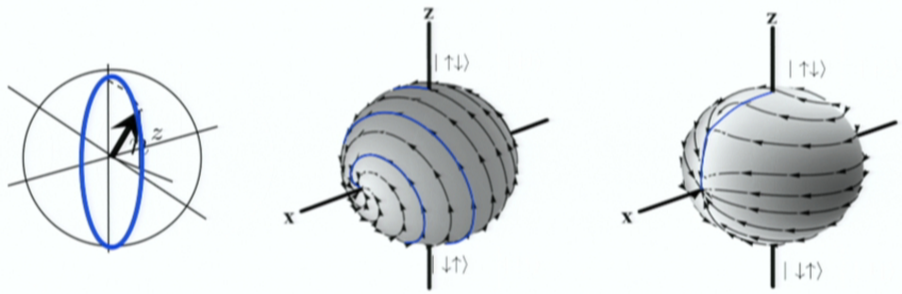


- **Product States:** Imaginary time excursions/instantons

Technical Background

Instantons in MPS Field Theory

- Disconnected configurations for product states
- Smooth field/tensor for MPS
- Instantons at χ_0 -> semi-classical configurations at $\chi > \chi_0$

Two Ways to Include Q Fluctuations in Field Theory

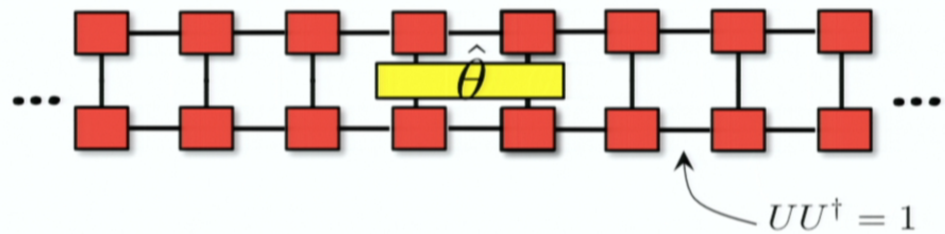
- i. Expand about semi-classical saddle point
- ii. Increase bond-order of field integral.
- Complementary - use simultaneously for different effects

Technical Background

Locality and Contractibility

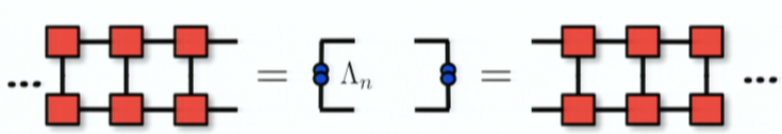
- Expectations of local operators are non-local in terms of A_n
- However, result of contracting to left or right is finite tensor Λ_n
- Expectations are local in terms of $\{A_n, \Lambda_n\}$

$\langle\psi|\hat{\theta}|\psi\rangle =$



Environment:

- Finite amount of data in 1d



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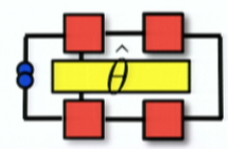

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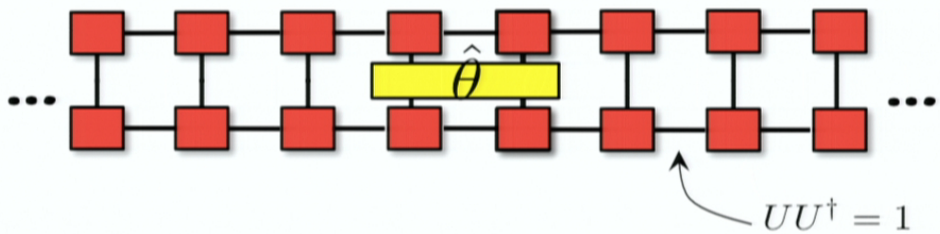
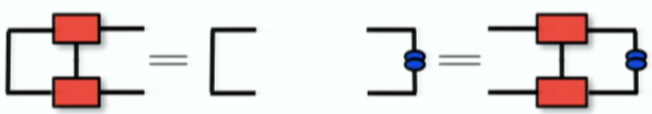


Environment:

- Difference equation



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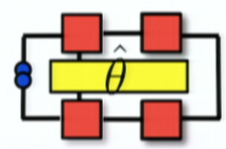


$\sum_{\sigma} A^{\sigma}$

Finite contractibility => Local Field Path Integral

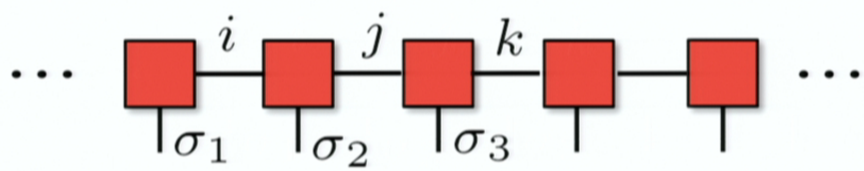
Formulating the Path Integral

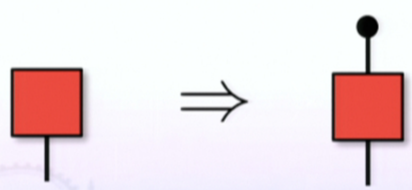
MPS data

$$\mathcal{Z} = Tr\, e^{-\beta\mathcal{H}} = \int [DA] e^{-\mathcal{S}[A]}$$

$\mathbb{1}\ \mathbb{1}\ \mathbb{1}\ \mathbb{1}\ \mathbb{1}\ \mathbb{1}\ \mathbb{1}$

The Measure:





Unitary acting on reference spin state v^{δ}

Formulating the Path Integral

UCL

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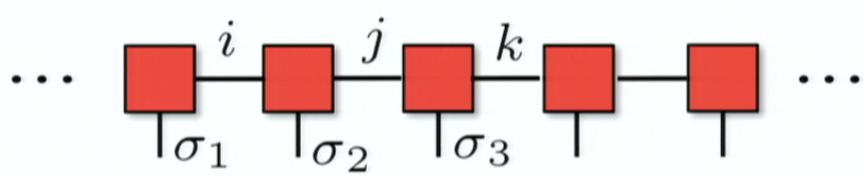

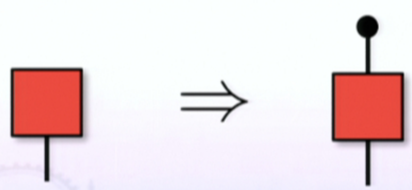

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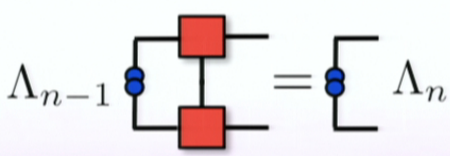
Locality:

- Field theory is not local just in terms of A
- Introduce extra fields Λ to describe the environment.
- Introduce Λ with δ-functional constraint



$$\sum_\sigma A^{\sigma\dagger}\Lambda_{n-1}A^\sigma = \Lambda_n$$

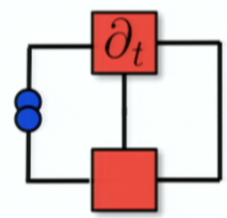
Formulating the Path Integral

$$\mathcal{Z} = Tr e^{\beta \mathcal{H}} = \int DA\, D\Lambda\, \delta\left[\Lambda_n - f(A_n, \Lambda_{n-1})\right] e^{-\mathcal{S}[A,\Lambda]}$$

Berry Phase:

- **Geometrical term in action - also local**

$$\langle \psi | \partial_t \psi \rangle = \sum_{\sigma} Tr \left[A_n^{\sigma\,\dagger} \Lambda_{n-1} \partial_t A_n^{\sigma} \right] =$$



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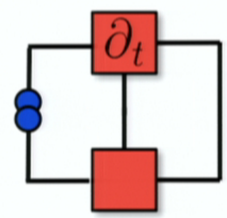

Towards a Path Integral over MPS

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Illustrative Examples

- Any entanglement in path integral => new physics
- Restricted form illustrates utility of approach
- Include local singlets and triplets
- Akin to translationally invariant bond operators

Simple Parametrization

Singlet vs triplet order

$$|A\rangle = \sum_\sigma A^\sigma|\sigma\rangle = \begin{pmatrix} 0 & n_1|\mathbf{1}\rangle & n_2|-\mathbf{1}\rangle \\ |\mathbf{1}\rangle & 0 & 0 \\ |-\mathbf{1}\rangle & 0 & 0 \end{pmatrix}$$

Spin coherent states

$$\Lambda_i = \mathrm{diag}\left(\lambda_i, (1-\lambda_i)|n_1^{i-1}|^2, (1-\lambda_i)|n_2^{i-1}|^2\right)$$

alternate $\lambda_i = \lambda$, $\lambda_i = 1-\lambda$ **from site to site**

Berry Phase

$$\mathcal{S}_B = \sum_i \int dt \left[\lambda_i\langle \mathbf{n}_i|\dot{\mathbf{n}}_i\rangle + (\lambda_{i-1}n^z_{i-1} + \lambda_i n^z_i)\langle \mathbf{l}_i|\dot{\mathbf{l}}_i\rangle\right]$$

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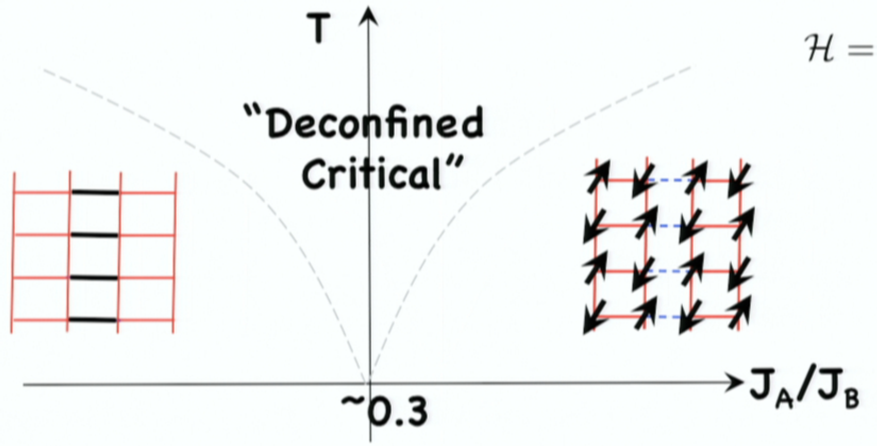
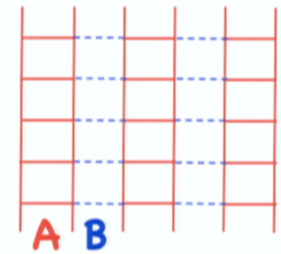
Illustrative Examples

a) Columnar VBS - Neel



$$\mathcal{H} = J_A \sum_{\langle ij \rangle} \hat{\sigma}_i.\hat{\sigma}_j + J_B \sum_{\langle ij \rangle} \hat{\sigma}_i.\hat{\sigma}_j$$



MPS ⟶ direction

- No Conventional Ginzburg-Landau theory
- Quasi 1D - entanglement mainly in horizontal direction
  - MPS description => parametrize both phases
- Entanglement is a Ginzburg-Landau order parameter
- Construct critical theory in terms of MPS tensors

[Senthil, et al Science (2004), PRB (2004)]

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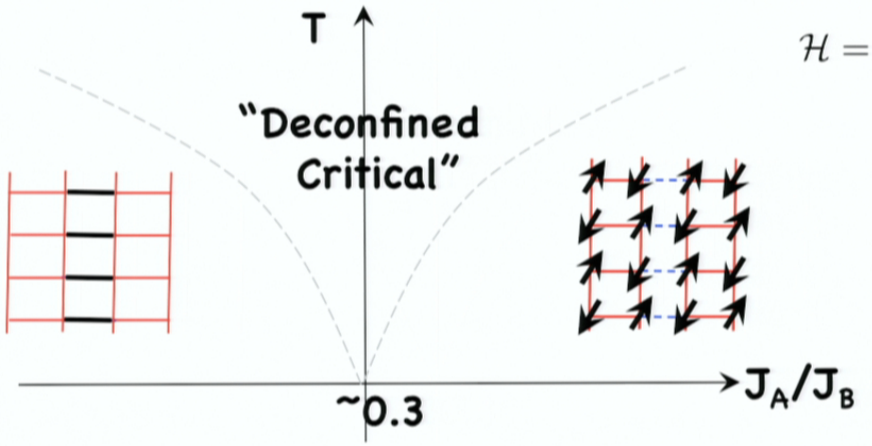

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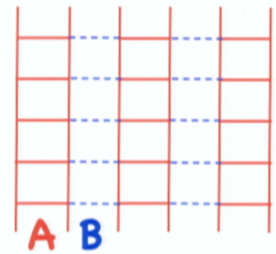

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  - MPS description => parametrize both phases
- Entanglement is a Ginzburg-Landau order parameter
- Construct critical theory in terms of MPS tensors

[Senthil, et al Science (2004), PRB (2004)]

Illustrative Examples

- Any entanglement in path integral => new physics
- Restricted form illustrates utility of approach
- Include local singlets and triplets
- Akin to translationally invariant bond operators

Simple Parametrization

Singlet vs triplet order

$$|A\rangle = \sum_\sigma A^\sigma |\sigma\rangle = \begin{pmatrix} 0 & n_1|\mathbf{1}\rangle & n_2|-\mathbf{1}\rangle \\ |\mathbf{1}\rangle & 0 & 0 \\ |-\mathbf{1}\rangle & 0 & 0 \end{pmatrix}$$

Spin coherent states

$$\Lambda_i = \text{diag}\left(\lambda_i, (1-\lambda_i)|n_1^{i-1}|^2, (1-\lambda_i)|n_2^{i-1}|^2\right)$$

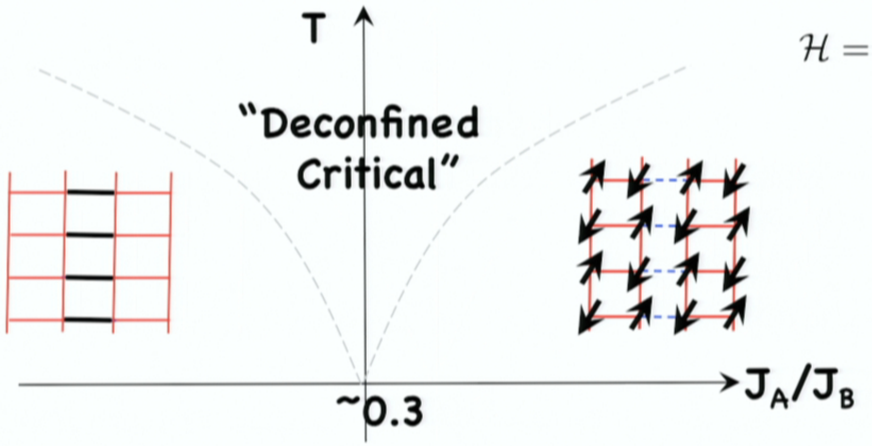
alternate $\lambda_i = \lambda$, $\lambda_i = 1-\lambda$ from site to site

Berry Phase

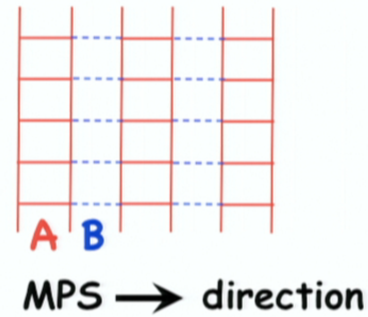
$$\mathcal{S}_B = \sum_i \int dt \left[\lambda_i \langle \mathbf{n}_i|\dot{\mathbf{n}}_i\rangle + (\lambda_{i-1} n_{i-1}^z + \lambda_i n_i^z)\langle \mathbf{l}_i|\dot{\mathbf{l}}_i\rangle\right]$$

Illustrative Examples

a) Columnar VBS - Neel



$$\mathcal{H} = J_A \sum_{\langle ij \rangle} \hat{\sigma}_i.\hat{\sigma}_j + J_B \sum_{\langle ij \rangle} \hat{\sigma}_i.\hat{\sigma}_j$$



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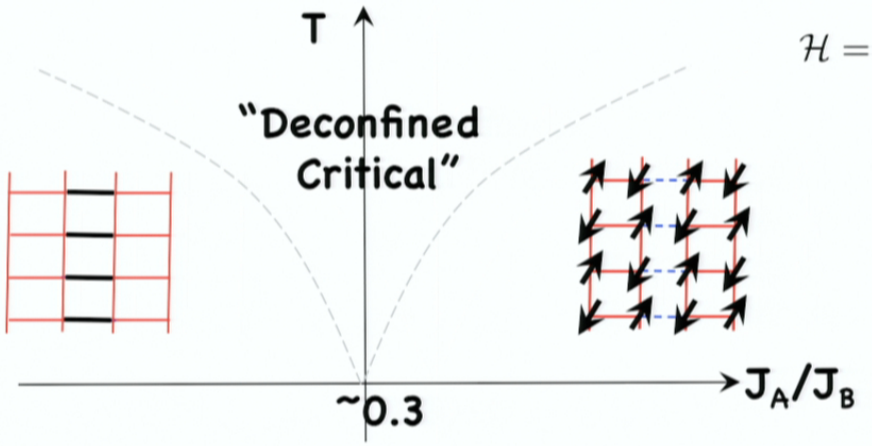

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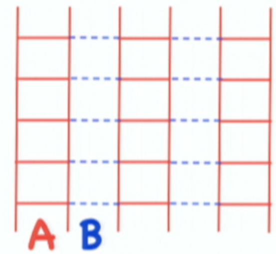

MPS ⟶ direction

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Illustrative Examples

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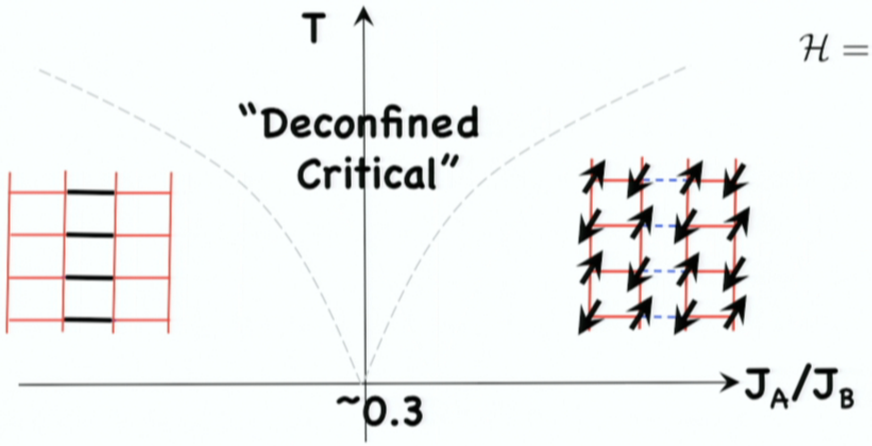

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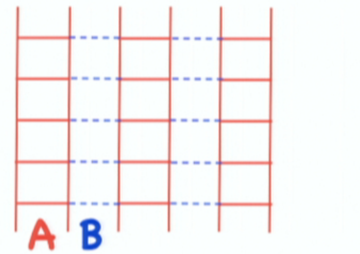


MPS → direction

Groundstate: $\lambda = 1, \mathbf{l}_i = -\mathbf{l}_{i+1}$

$$|A\rangle = \begin{pmatrix} 0 & n_1|\mathbf{l}\rangle & n_2|-\mathbf{l}\rangle \\ |\mathbf{l}\rangle & 0 & 0 \\ |-\mathbf{l}\rangle & 0 & 0 \end{pmatrix}$$

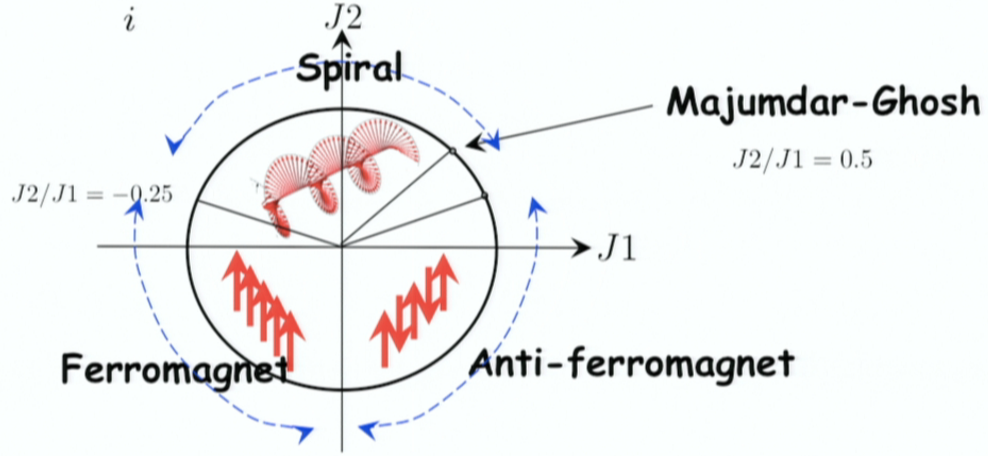
$n_z = 1$ **AFM**

$n_1 = n_2 = 1/\sqrt{2}$ **VBS**

Illustrative Examples

b) J_1–J_2 Model:

$$\mathcal{H} = \sum_i \left[J_1 \hat{\sigma}_i . \hat{\sigma}_{i+1} + J_2 \hat{\sigma}_i . \hat{\sigma}_{i+2} \right]$$



- Coherent state solution => incomensurate spiral
- Instantons proliferate and drive dimerisation when $J_1>0$
- Can capture with MPS field theory
- (would not be possible with conventional bond operators)
- Fluctuations determine entanglement via Q Order-by-Disorder

Illustrative Examples

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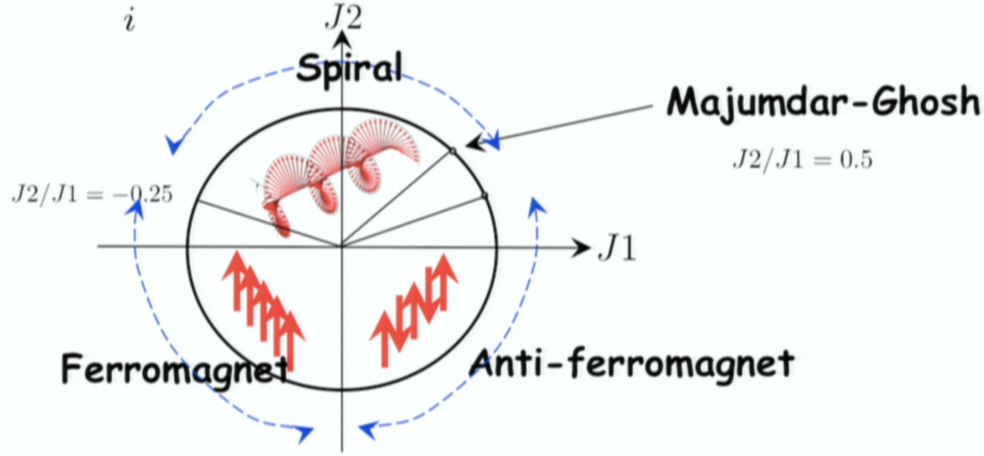

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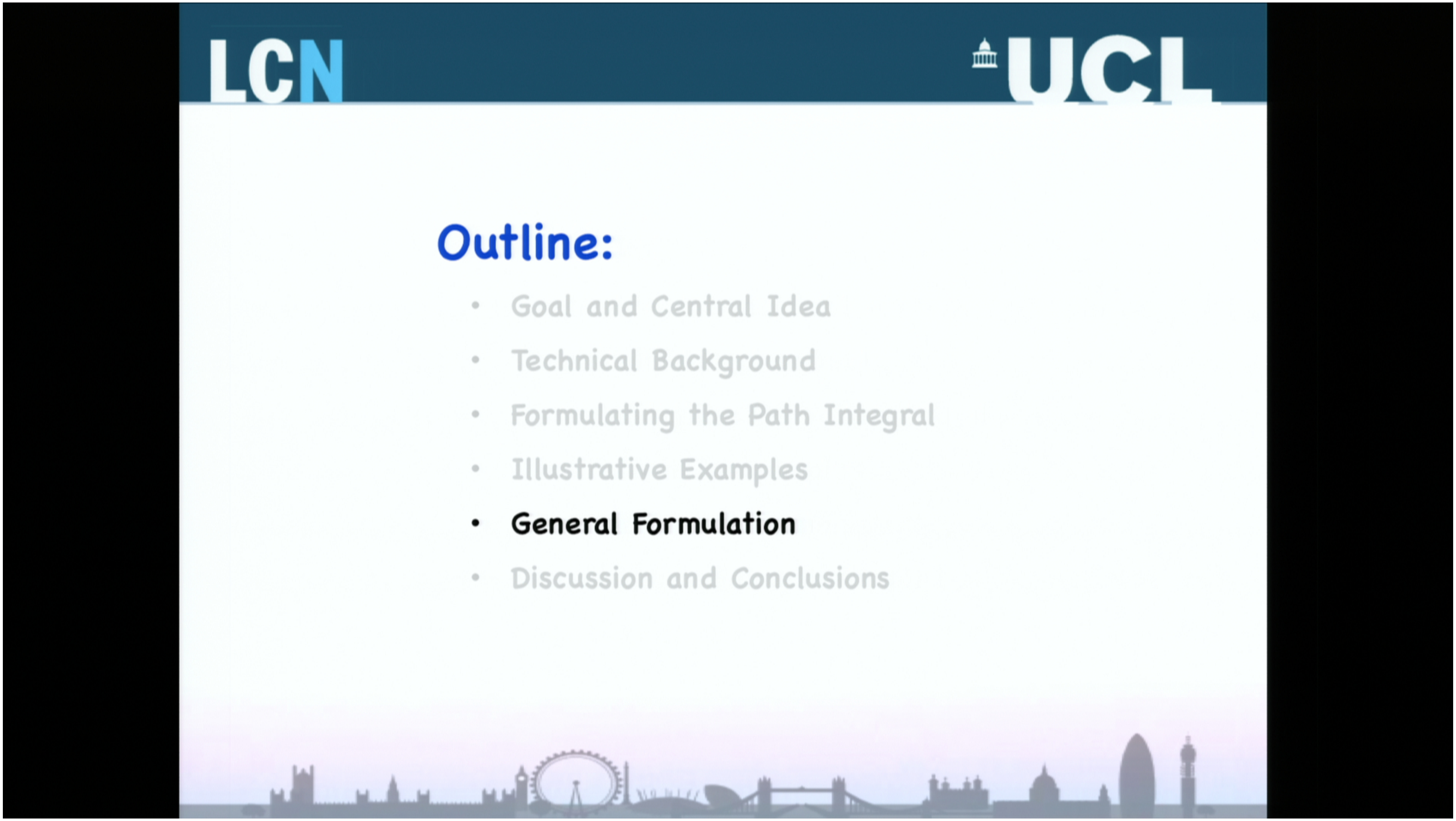

LCN
UCL
Outline:
Goal and Central Idea
Technical Background
Formulating the Path Integral
Illustrative Examples
General Formulation
Discussion and Conclusions

General Formulation

UCL

Geometry of the Berry Phase

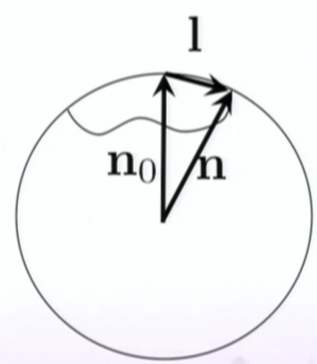
- Appealing geometrical interpretation for single spin
- Area of periodic path => quantization of spin
- Expect similar interpretation for MPS

Small Paths: Product States

- Expand in the vicinity of a reference state $\mathbf{n}_0$
- $\mathbf{n} = \mathbf{n}_0\sqrt{1-|\mathbf{l}|^2} + \mathbf{l}$, $\mathbf{n}_0 \cdot \mathbf{l} = 0$, $l = l_1 + il_2$

$$\mathcal{S}_B = \frac{S}{4\pi}\sum_n \int dt \int_0^1 d\tau \mathbf{n}_n \cdot \partial_t \mathbf{n}_n \times \partial_\tau \mathbf{n}_n$$

$$\rightarrow \sum_n \int dt l_n^* \dot{l}_n$$



General Formulation

Small Paths: MPS

- Expand in vicinity of reference state
- Reference MPS and tangent vectors

$$|A\rangle = \sum_{\{\sigma\},i,j,\ldots} \ldots A_{ij}^{\sigma_{n-1}} A_{jk}^{\sigma_n} A_{kl}^{\sigma_{n+1}} \ldots |\sigma_1, \sigma_2, \sigma_3 \ldots\rangle$$

$$|dA_n\rangle = \sum_{\{\sigma\},i,j,\ldots} \ldots A_{ij}^{\sigma_{n-1}} dA_{jk}^{\sigma_n} A_{kl}^{\sigma_{n+1}} \ldots |\sigma_1, \sigma_2, \sigma_3 \ldots\rangle$$

- Gauge fix – overlap local and orthonormal
- Parameterize by D x (d-1)D matrix $x(t)$
- Berry Phase $\mathcal{S}_B = \sum_n \int dt Tr[x_n^\dagger \dot{x}_n]$

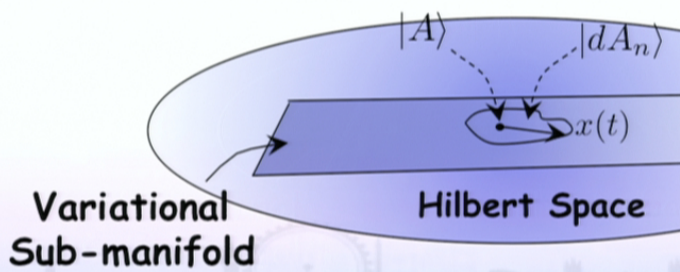
Compare with

$$\mathbf{n}_0$$

$$\mathbf{l}$$

$$\mathbf{n}_0 \cdot \mathbf{l} = 0$$

$$\mathcal{S}_B = \sum_n \int dt\ l_n^* \dot{l}_n$$



Details: [Haegeman et al PRL 2011]
Choose $dA_{il}^{\sigma}(x) = \Lambda_L^{-1/2} V^{\sigma} x \Lambda_R^{-1/2}$
where $V_{\beta}^{(\sigma\alpha)}$ is a matrix of (d-1)D, dD dimension null vectors of $(A^{\sigma\dagger}\Lambda_L^{1/2})_{\alpha,\beta}$
Reshaped according to $(A^\dagger \Lambda_L^{1/2})_{\alpha,(\beta\sigma)}$

General Formulation

General Path: Product States

- General parametrization of manifold; $\mathbf{n}$ or spinor n^{σ}
- Berry phase

$$\mathcal{S}_B = \frac{S}{4\pi}\sum_n \int dt \int_0^1 d\tau \; \mathbf{n}_n \cdot \partial_t \mathbf{n}_n \times \partial_\tau \mathbf{n}_n = \sum_n \int dt \; n_n^{\sigma\dagger} \dot{n}_n^{\sigma}$$

- Comparing with previous spinwave expansion

$$\mathcal{S}_B = \sum_n \int dt \; l_n^* \dot{l}_n \to \sum_n \int dt \; n_n^{\sigma\dagger} \dot{n}_n^{\sigma} \qquad l_n \to n_n^{\sigma}$$

General Path: MPS

- Anticipate

$$\mathcal{S}_B = \sum_n \int dt \; Tr[x_n^{\dagger} \dot{x}_n] \to \sum_n \int dt \; Tr[z_n^{\sigma\dagger} \dot{z}_n^{\sigma}]$$

Identify general parametrization $x_n \to z_n^{\sigma}$

General Formulation

General Path: MPS

- **MPO contains reference MPS and tangent vectors**

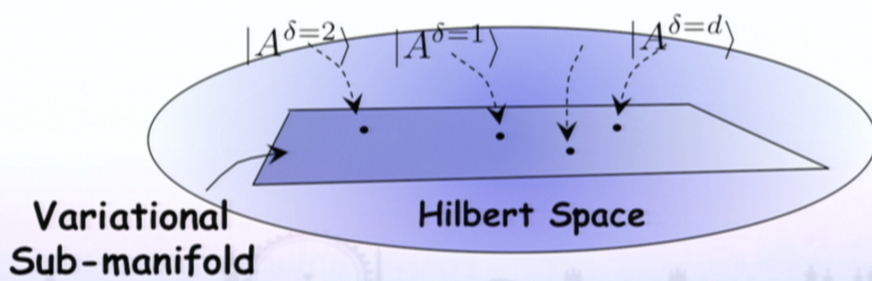
$$A^{\sigma}_{ij} = A^{\sigma,\delta=1}_{ij}$$
$$dA^{\sigma}_{il}(x) = A^{\sigma,\delta\neq1}_{ij} x^{\delta\neq1}_{jk} \Lambda^{-1/2}_{kl}$$

- **Generic point on manifold – alternative representation**

$$A^{\sigma}_{il(n)}(z) = A^{\sigma\alpha}_{ij(n)} z^{\alpha}_{jk(n)} \Lambda^{-1/2}_{kl(n)},$$

- **Constraints**

$$\Lambda_{(n)} = z^{\dagger\sigma}_{(n)} z^{\sigma}_{(n)} = A^{\sigma\mu}_{(n+1)} z^{\mu}_{(n+1)} z^{\dagger\nu}_{(n+1)} A^{\dagger\nu\sigma}_{(n+1)} \qquad \sum_{\alpha=1}^{d} Tr[z^{\alpha\dagger} z^{\alpha}] = 1.$$



Reduces to CP_1 at bond-order 1

General Formulation

General Path: MPS

- **Generic point on manifold**

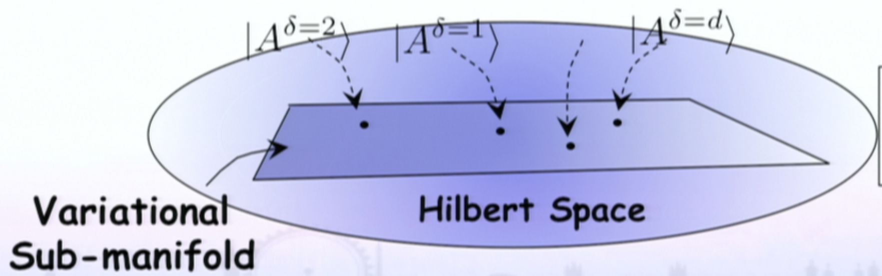
$$A^{\sigma}_{il(n)}(z) = A^{\sigma\alpha}_{ij(n)} z^{\alpha}_{jk(n)} \Lambda^{-1/2}_{kl(n)},$$

- **Berry Phase**

$$\mathcal{S}_B = \sum_n \sum_\alpha \int dt \; Tr[z^{\alpha}_{(n)}{}^{\dagger} \dot{z}^{\alpha}_{(n)}]$$

- **Partition Function**

$$\mathcal{Z} \;=\; \int Dz \, \delta\left[z^{\sigma\dagger} z^{\sigma} - A^{\sigma\mu} z^{\mu} z^{\nu\dagger} A^{\nu\sigma\dagger}\right] \delta\left[Tr[z^{\sigma\dagger} z] - 1\right] e^{-\mathcal{S}}$$



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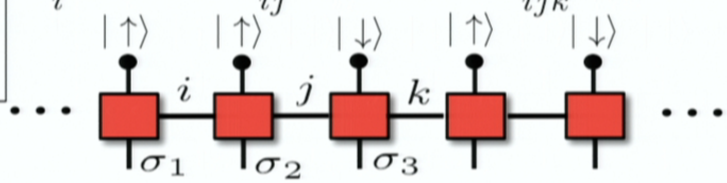
General Formulation

Non-locality of Berry Phase

- **MPO diagonalize H – generate eigenstates from L-bits**

$$\mathcal{H} = \sum_i \alpha_i \tau_i^z + \sum_{ij} \beta_{ij} \tau_i^z \tau_j^z + \sum_{ijk} \gamma_{ijk} \tau_i^z \tau_j^z \tau_k^z \ldots,$$

Highly non-local in spins



- **Restrict to coherent/product states of L-bits** $|\Phi\rangle = |\boldsymbol{\eta}_1, \boldsymbol{\eta}_2, \ldots\rangle$
- **Berry Phase:** $$\mathcal{S}_B = \sum_n \int dt \, \langle \boldsymbol{\eta}_n | \dot{\boldsymbol{\eta}}_n \rangle$$
- **Recover with the choice** $z_{ij}^\sigma = \eta^\sigma \Lambda_{ij}^{1/2}$

=> $$\mathcal{Z} = \int D\eta \delta[|\eta^\sigma|^2 - 1] e^{-\mathcal{S}[\eta]}$$

Outline:

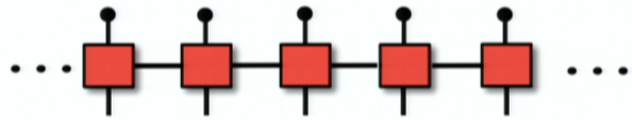
- Goal and Central Idea
- Technical Background
- Formulating the Path Integral
- Illustrative Examples
- General Formulation
- **Discussion and Conclusions**

Discussion

Extension to Higher Dimensions

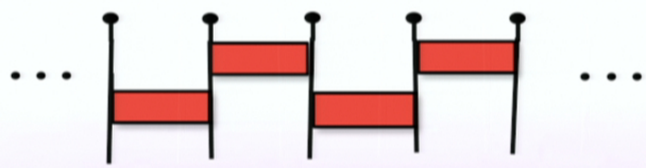
One dimension

- MPS: Environment Λ finite in one dimension
- States finitely contractible t~Poly[N]
- May always find canonical gauge
- Field theory local in $\{A_n, \Lambda_n\}$



Higher Dimensions

- Finite depth circuit is finitely contractible in any dimension
- Easy to write a field theory for these...



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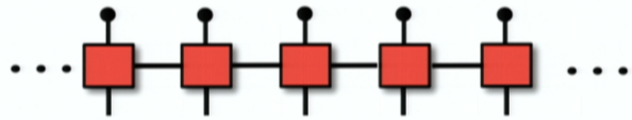

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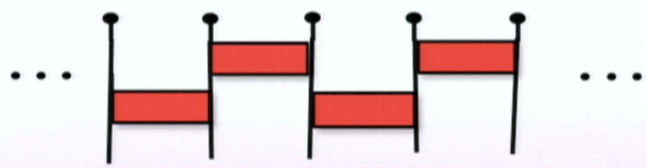
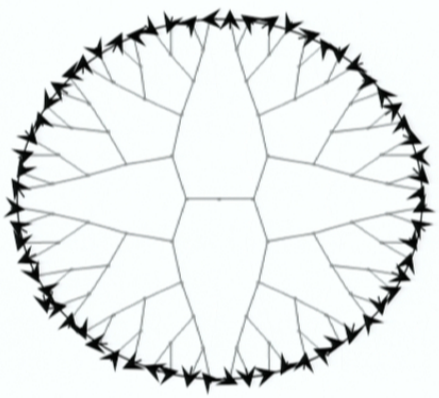


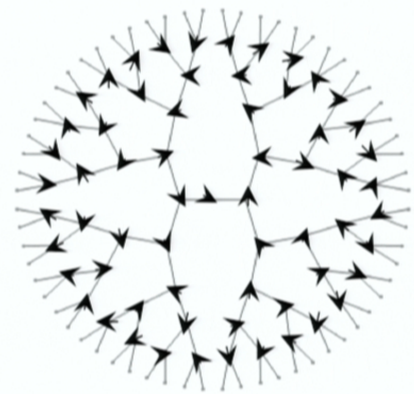
Discussion

Field Theory RG vs Tensor Network RG

- Various RG Schemes
- MERA (multiscale entanglement renormalization ansatz) [Vidal, PRL99, 220405(2007)]
- TRG (tensor RG) [Verstrate et al, Adv. Phys 57, 143 (2008)]
- SRG (second RG) [Xie et al PRL103, 16069 (2009)]
- HOTRG (higher order TRG) [Xie et al PRB86, 045139 (2012)]
- Exact Holographic Mapping [Xiao-Liang Qi[ArXiv:1309.6282]



Wavelet/RG trans of boundary $|\psi\rangle$, $\hat{\mathcal{H}}$ and $\hat{\theta}$ to bulk



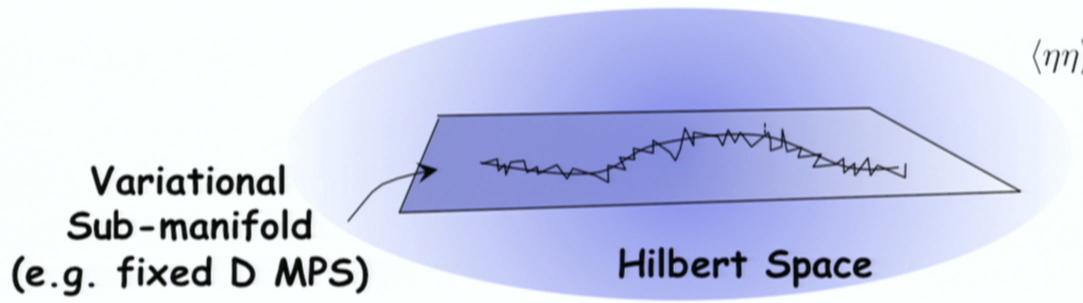
- Relation to AdS/CFT [Swingle, Phys. Rev. D 86, 065007 (2012),ArXiv:1209.3304]
- Applying RG to field theory over MPS ->[S.-S. Lee,NPB 832,56 (2010); 851,143(2011)]

Environmental Restriction of Entanglement

Quantum Langevin Equation for Entangled States

$$\gamma = \frac{1}{\omega}\langle \hat{X}\hat{X}\rangle^R$$

$$\langle \eta\eta\rangle = \frac{i}{2}\langle \hat{X}\hat{X}\rangle^K \approx 2\gamma T$$



TDVP — Fluctuation — Dissipation

$$-i\langle \partial_{A_\alpha}\psi|\partial_{A_\beta}\psi\rangle \dot{A}_\beta = \langle \partial_{A_\alpha}\psi|\hat{\mathcal{H}}|\psi\rangle + \gamma * \partial_{A_\alpha} F d_t F + \eta \partial_{A_\alpha} F$$

- **Couple to bosonic bath via** $\langle \mathcal{H}_{int}\rangle_{\text{spins}} = \hat{X}F(A)$
- **Derive from Keldysh field theory over MPS states**
- **Environmental restriction of useable entanglement**

Conclusions

- Path Integral over Tensor Network States (MPS data)

$$\mathcal{Z} = Tr\ e^{-\beta\mathcal{H}} = \int [DA] e^{-\mathcal{S}[A]}$$

- Imports insights about entanglement to field theory
- Instantons => saddles of higher bond order
- Managed for 1d - exploring applications and extensions

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