Title: PSI 2016/2017 Quantum Theory - Lecture 14

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Abstract:

Quantum Mechanics
Wigner functions

Lecture 14

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Quasiprobabilities

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Quasiprobability representation:

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Quasiprobability representation: an alternative framework for quantum mechanics.

Classical hidden variables on a phase space Λ - continuous / - discrete

States: $\rho \rightarrow W_\rho(\lambda)$ like prob. distⁿ

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Quantum operations: $\mathcal{E} \rightarrow W$

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States: $\rho \Rightarrow W_\rho(\lambda)$ like prob. dist^{n} $\sum_{\lambda\in\Lambda} W_\rho(\lambda) = 1$

Quantum operations: $\mathcal{E} \Rightarrow W_\mathcal{E}(\lambda|\lambda')$ like a conditional prob dist^{n}

Measurements: $E_m \Rightarrow W(m|\lambda)$ like a conditional prob dist^{n}

Distributions are real valued, normalized as expected.

Born rule: $p_m = \mathrm{Tr}[E_m \mathcal{E}(\rho)] = W(m| \, W_\mathcal{E}(\lambda|\lambda') W_\rho(\lambda')$

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Born rule: $p_m = \mathrm{Tr}[E_m \mathcal{E}(\rho)] = \sum_{\lambda,\lambda'\in\Lambda} W(m|\lambda) W_\mathcal{E}(\lambda|\lambda') W_\rho(\lambda')$

But can take on negative values!

The Wigner Function

Weyl - Heisenberg $\quad \underbrace{[\hat{X}, \hat{P}] = i\hbar}_{\text{algebra and group}} \Rightarrow D(p,x) = e^{ip\hat{X}} e^{ix\hat{P}}$

algebra and group

Heisenberg-Weyl representation

acts irreducibly on $\mathcal{L}^2(\mathbb{R})$

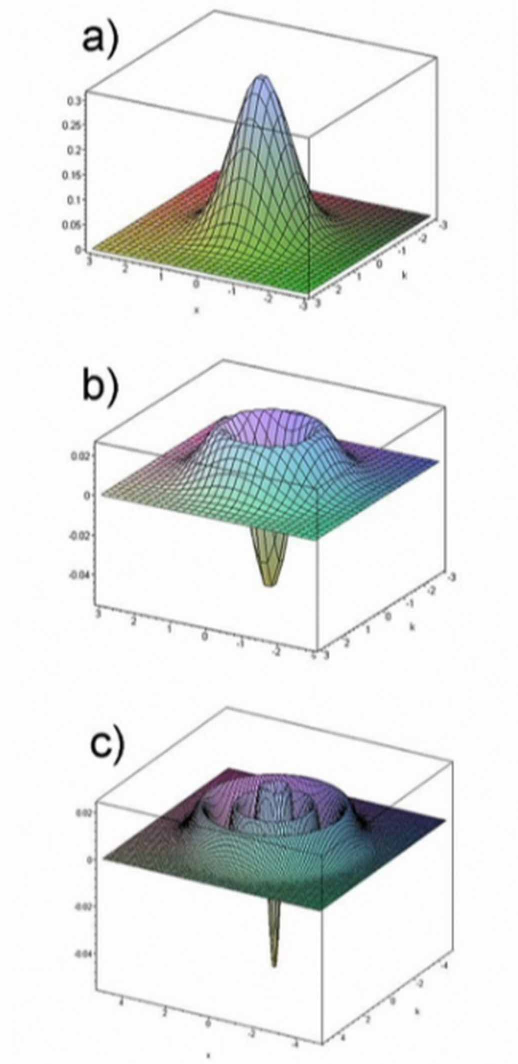
$$D(p_1, x_1) D(p_2, x_2) = e^{i\pi(p_1 x_2 - p_2 x_1)} D(p_1 + p_2, \cancel{p}\, x_1 + x_2)$$

symplectic inner product on $\mathbb{R}^2$

Characteristic function

$$\chi_\rho(p,x) = \mathrm{Tr}[\rho D(p,x)]$$

The Wigner Function



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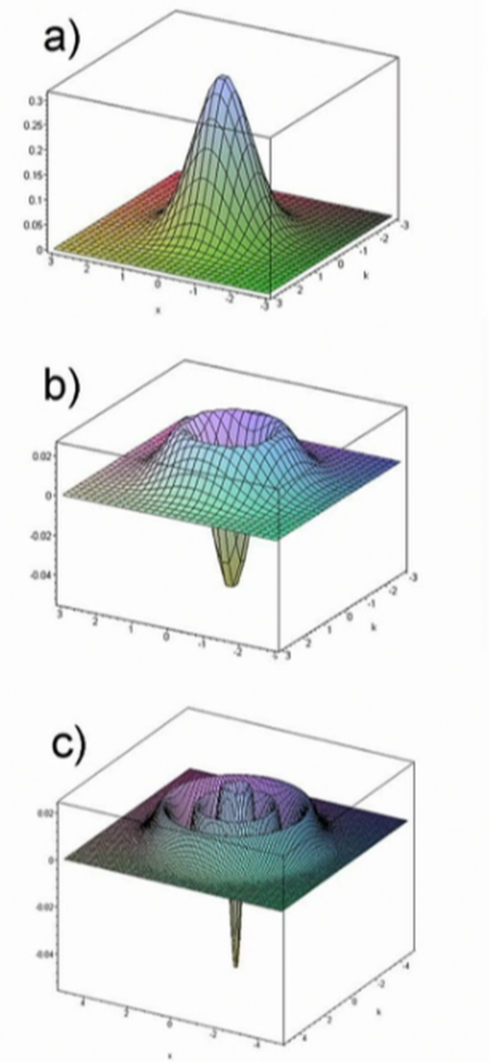
The Wigner Function

Take the usual FT over $\mathbb{R}^2$

$$W_\rho(p,x) = \mathcal{F}_{(p',x') \to (p,x)} \chi_\rho(p',x')$$

Take some pure state $\psi(x)$

$$W_\psi(p,x) = \frac{1}{\pi\hbar}\int_{-\infty}^{\infty} \psi^*(x -$$



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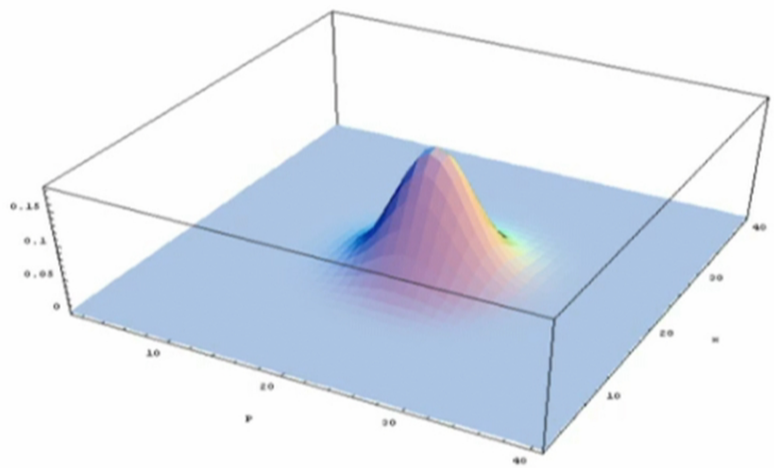
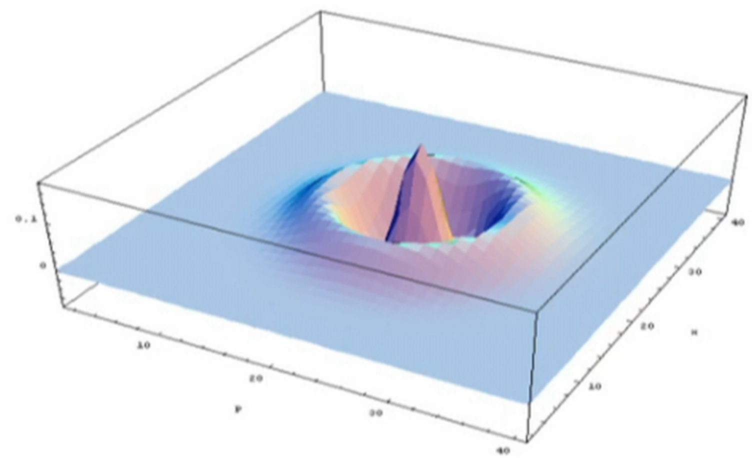
Negativity and nonclassicality



Classical



Quantum

Properties of the Wigner function

① $W(p,x)$ ~~is~~ real

② $\int_{-\infty}^{\infty} dp\, W(p,x) = \langle x|\rho|x\rangle = |\psi(x)^*\psi(x)|^{2} \geq 0$

$\int_{-\infty}^{\infty} dx\, W(p,x) = \langle p|\rho|p\rangle \geq 0$

$\iint dx\, dp\, W(p,x) = 1$

③ $|\langle\psi|\phi\rangle|^2 = 2\pi\hbar \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dp\, W_{\psi}(p,x)\, W_{\phi}(p,x)$

④ $\mathrm{Tr}(\hat{O}\rho) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dp\, W_{\hat{O}}(p,x)\, W_{\rho}(p,x)$

Frames

Dual frames formalism

Frame: operator basis on $\mathcal{H}$

Pick 2 frames: $F(\lambda) \quad \lambda \in \Lambda$

$G(\lambda) \quad \lambda \in \Lambda$

For

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For any operator A

$$A = \sum_{\lambda \in \Lambda} G(\lambda) \mathrm{Tr}[A F(\lambda)]$$

$$W_\rho(\lambda) = \mathrm{Tr}[F(\lambda)\rho]$$

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$$W_\mathcal{E}(\lambda|\lambda') = \mathrm{Tr}[G(\lambda)\mathcal{E} F(\lambda')] \leftarrow$$

ambiguous if frame is ov.

$$W_{E_m}(m|\lambda) = \mathrm{Tr}[E_m G(\lambda)]$$

Frames Dual frames formalism

Frame: operator basis on $\mathcal{H}$

Pick 2 frames: $F(\lambda)$ $\lambda \in \Lambda$

$G(\lambda)$ $\lambda \in \Lambda$

Frame is self-dual if $\{F(\lambda)\} = \{G(\lambda)\}$

For any operator A $$A = \sum_{\lambda \in \Lambda} G(\lambda) \mathrm{Tr}[A F(\lambda)]$$

Reproduces Born rule

$$W_\rho(\lambda) = \mathrm{Tr}[F(\lambda)\rho]$$

$$W_\mathcal{E}(\lambda|\lambda') = \mathrm{Tr}[G(\lambda)\mathcal{E} F(\lambda')]$$ ← ambiguous if frame is overcomplete

$$W_{E_m}(m|\lambda) = \mathrm{Tr}[E_m G(\lambda)]$$

Example: discrete Wigner function of a qubit

HW group: $D_{x,p} = i^{xp} Z^p X^x \qquad x, p \in \mathbb{Z}_2$

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$$A_{0,0} = \tfrac{1}{2}(I + X + Z + Y)$$

$$A_{x,p} = D_{x,p} A_{0,0} D_{x,p}^{-1}$$

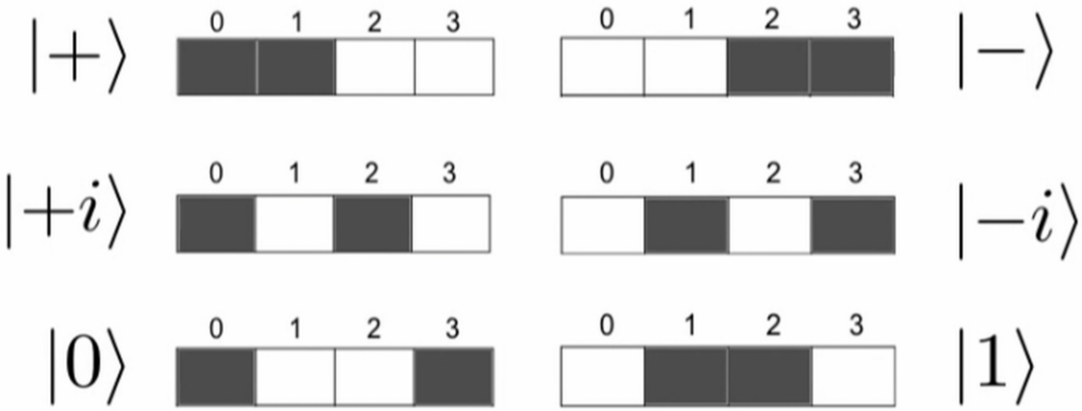
$$= \tfrac{1}{2}\left(I + (-1)^p X + (-1)^x Z + (-1)^{x+p} Y\right)$$

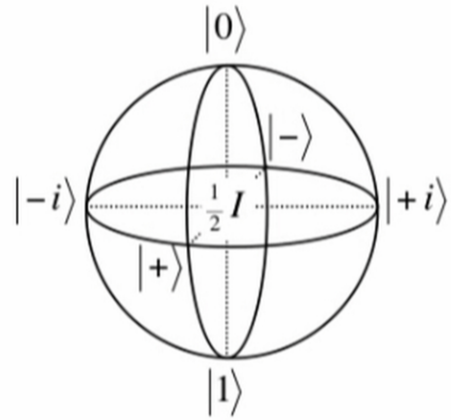
$\{A_{x,p},\ x,p \in \mathbb{Z}^2\}$ frame (self dual)

$$W_{x,p} = \mathrm{Tr}[\rho A_{x,p}]$$

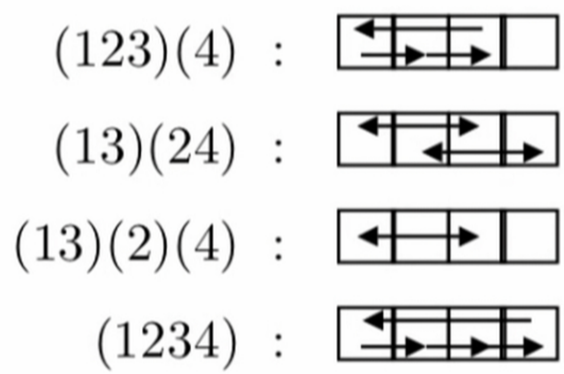
Single-qubit stabilizers with the DWF

- Single-qubit stabilizer states are all represented nonnegatively in the DWF



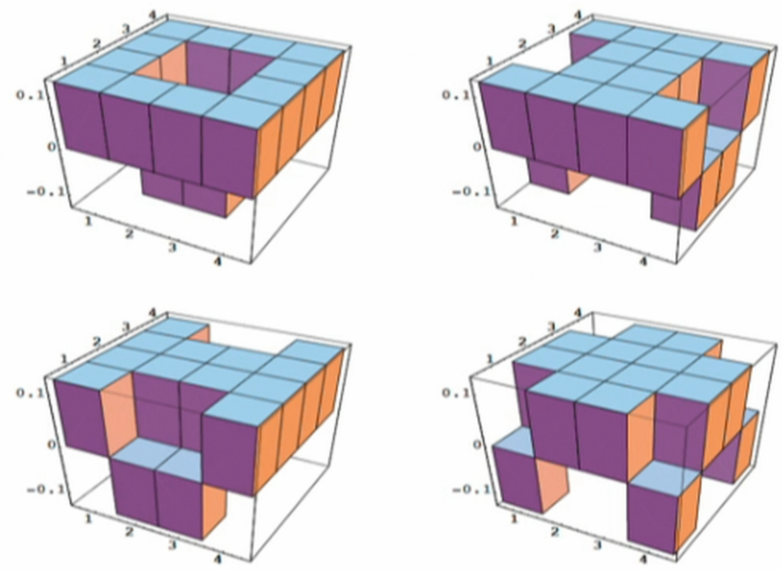


- Single-qubit 'Clifford' operations map stabilizer states to stabilizer states
- Some Clifford gates are represented non-negatively in the DWF and act as permutations on the phase space



Operationalizing nonclassicality

Negativity in a quasiprobability can be related to notions of nonclassicality



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