Title: PSI 2016/2017 Quantum Theory - Lecture 13

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Abstract:

Quantum Mechanics
Introduction to Tensor Networks

Lecture 13

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Quantum Mechanics

Introduction to Tensor Networks

Lecture 13

Bridgeman + Chubb

1603.03039

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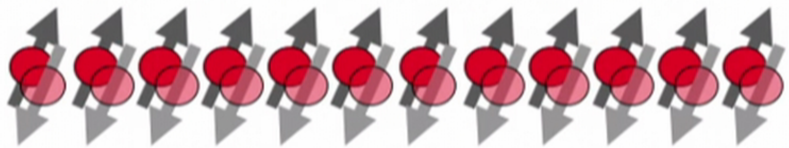
Quantum many-body physics

Describing quantum states of many particles

– Quantum spin chain – consider N quantum spin-1/2 particles in a line



Quantum many-body physics

Describing quantum states of many particles

– Quantum spin chain – consider N quantum spin-1/2 particles in a line

State of 1 spin

$\psi = \begin{pmatrix} c_+ \\ c_- \end{pmatrix}$ 2-vector

State of N spins

$$\psi_N = \begin{pmatrix} c_{+++++\dots+} \\ c_{+++++\dots-} \\ \vdots \end{pmatrix}$$

2^N vector

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$$|\psi\rangle_N = \sum_{i_1, i_2, \ldots} c_{i_1, i_2, \ldots i_N} |i_1\rangle \otimes |i_2\rangle \otimes \ldots \otimes |i_N\rangle$$

Product states / no entanglement / mean field approx.

Approximation: each spin-1/2 particle in a single-particle quantum state

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Approximation: each spin-1/2 particle in a single-particle quantum state

If i^{th} particle has state $|\psi_i\rangle = b_i^0 |\uparrow\rangle + b_i^1 |\downarrow\rangle$

No entanglement $\Rightarrow$ efficient description of N spins

Case 1: all spins in the same state $\vec{b}_i = \vec{b}$
need 2 numbers

Case 2: all in diff states
need 2N numbers

Tensor descriptions

Use tensors to represent the state coefficients (e.g., in a computer)

- General quantum state of N spins

- Unentangled quantum state of N spins

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a list of 2^N numbers

$C_{i_1, i_2, i_3, \dots i_N}$

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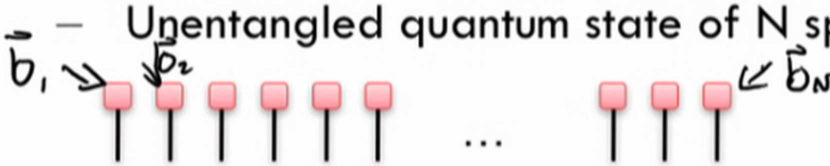

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$\Updownarrow$ anything in between?

– Unentangled quantum state of N spins

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Matrix Product State (MPS)

Constructing big tensors by contracting smaller ones

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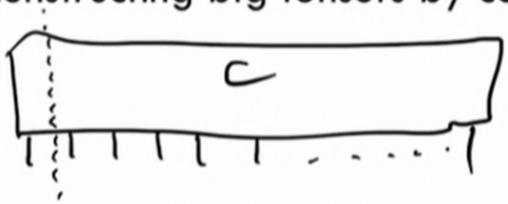

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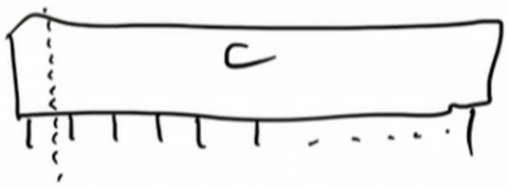

Schmidt decomposition

$$|\psi\rangle = \sum_{\nu} \sqrt{\lambda_{\nu}}\, |L_{\nu}\rangle \otimes |R_{\nu}\rangle$$

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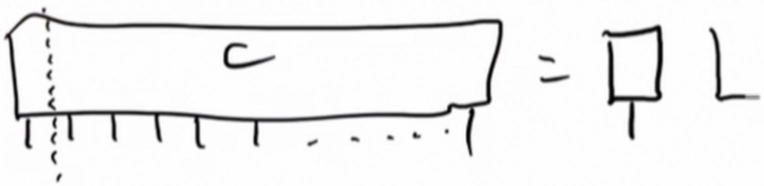

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- bounded by lesser of $\dim L$, $\dim R$

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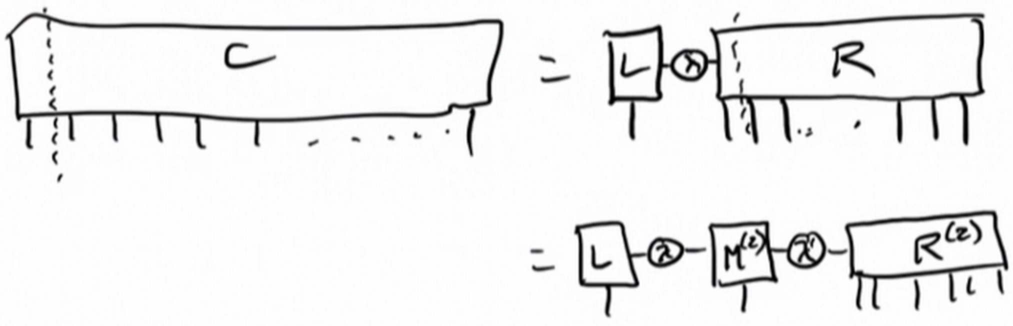

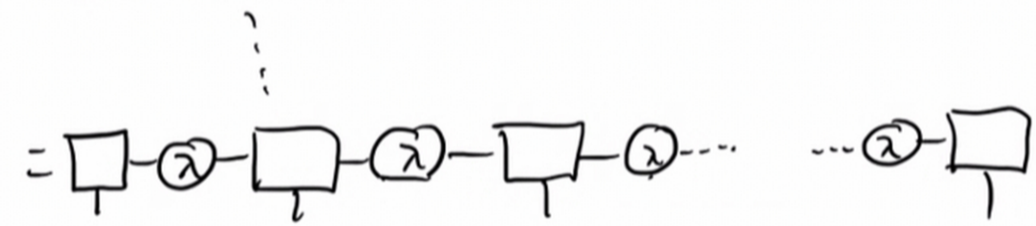

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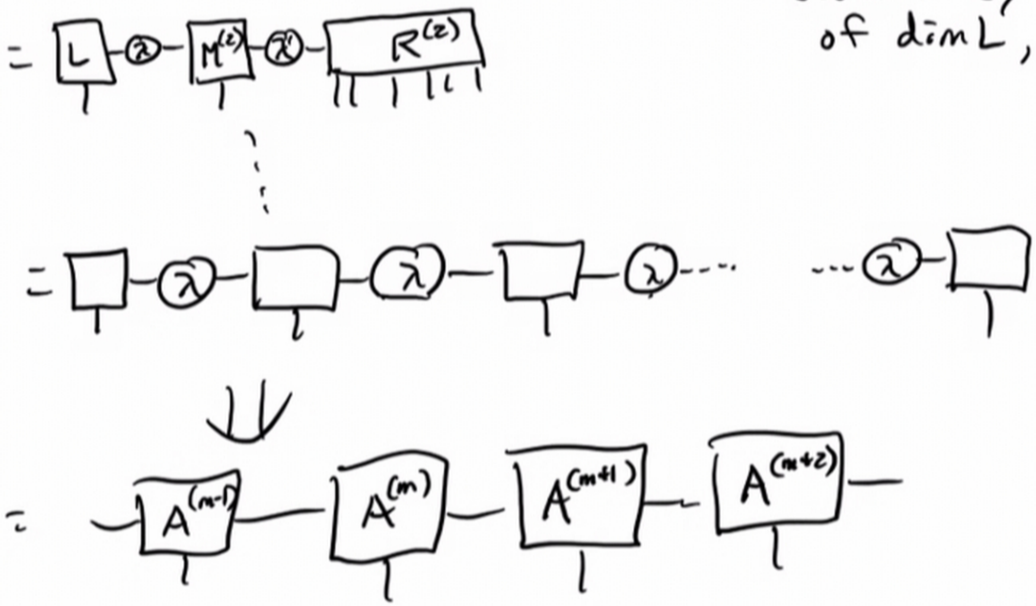

Matrix Product State (MPS)

Constructing big tensors by contracting smaller ones

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Matrix Product State (MPS)

Constructing big tensors by contracting smaller ones

$$\text{MPS}: \quad C_{i_1, i_2, \dots i_N} = A^{(1)}_{i_1} A^{(2)}_{i_2} A^{(3)}_{i_3} \cdots A^{(N)}_{i_N}$$

Matrix Product State (MPS)

Constructing big tensors by contracting smaller ones

MPS: $C_{i_1, i_2, \ldots i_N} = \mathrm{Tr}\left(A^{(1)}_{i_1} A^{(2)}_{i_2} A^{(3)}_{i_3} \cdots A^{(N)}_{i_N}\right)$

$A^{(n)}_{i_1}$ is a $D \times D$ matrix

anything
- represents # of nonzero Schmidt coeff.

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Matrix Product State (MPS)

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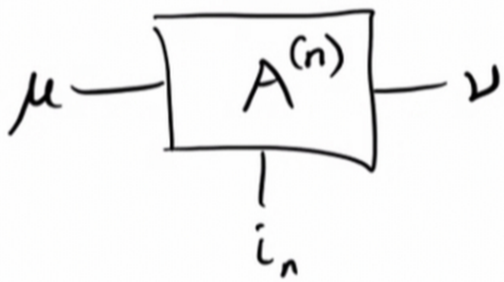

$\left(A^{(n)}_{i_n}\right)_{\mu,\nu}$

μ row

ν column

$\mu, \nu = 1, \dots D$

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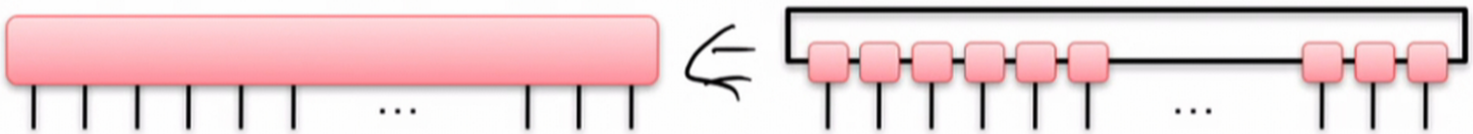

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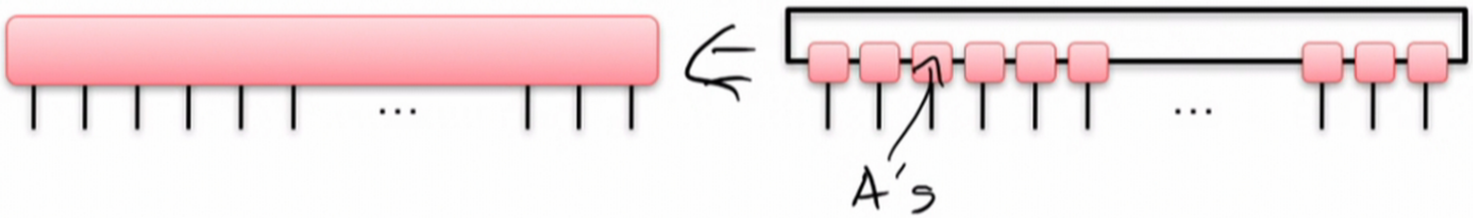
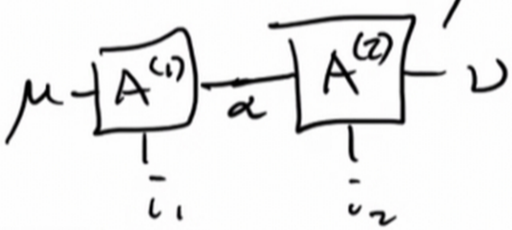


$$\left(A^{(1)}_{i_1} A^{(2)}_{i_2}\right)_{\mu\nu} = \left(A^{(1)}_{i_1}\right)_{\mu\alpha} \left(A^{(2)}_{i_2}\right)_{\alpha\nu}$$



MPS example

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$$|\psi^{(N)}\rangle = \frac{1}{\sqrt{2}}\left(|0000\cdots\cdots 0\rangle + |1111\cdots\cdots 1\rangle\right)$$

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$$A_i = A^{(n)}_{i_n} \qquad A_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad A_1 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathrm{Tr}\left(A_{i_1} A_{i_2} A_{i_3} \cdots\cdots A_{i_N}\right)$$

MPS example

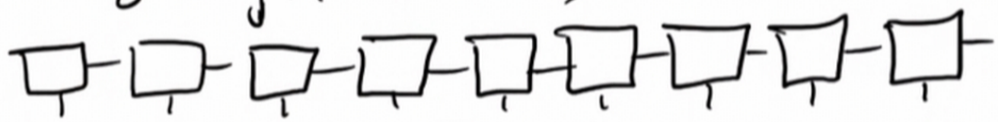
$$|\psi^{(N)}\rangle = \frac{1}{\sqrt{2}}\left(|10000\cdots\cdots 0\rangle + |11111\cdots\cdots 1\rangle\right)$$

$$A_i = A^{(n)}_{i_n} \qquad A_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad A_1 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\langle i_1,\ldots i_N|\psi^{(N)}\rangle = \frac{1}{\sqrt{2}}\mathrm{Tr}\left(A_{i_1} A_{i_2} A_{i_3} \cdots\cdots A_{i_N}\right)$$

MPS and correlation functions

$$\langle \Psi[A] | O_0 \, O_{j+1} | \Psi[A] \rangle$$

$=$ 

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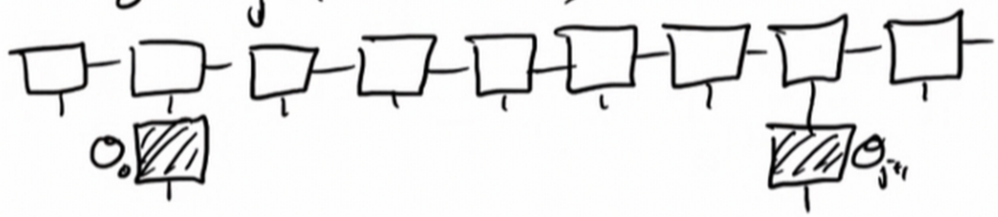
$=$ 

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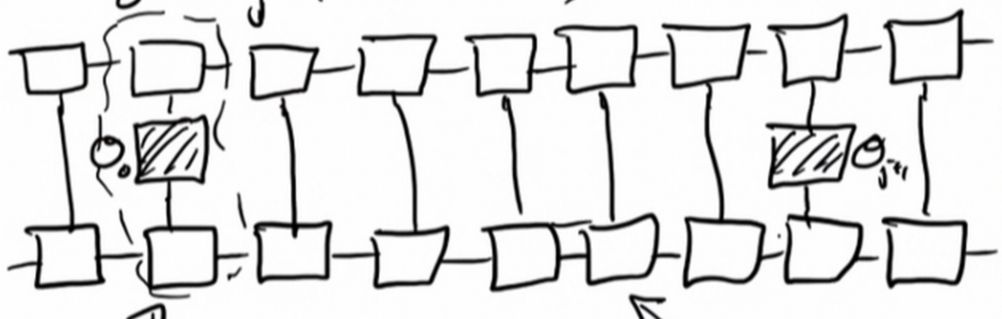
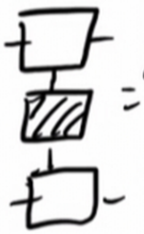
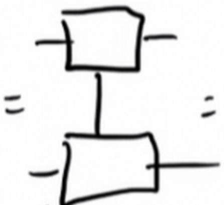


$E_O =$  $= \sum_{i,j=0}^{d-1} O_{ij} (A_i \otimes A_j^T)$

$E_I =$  $=$

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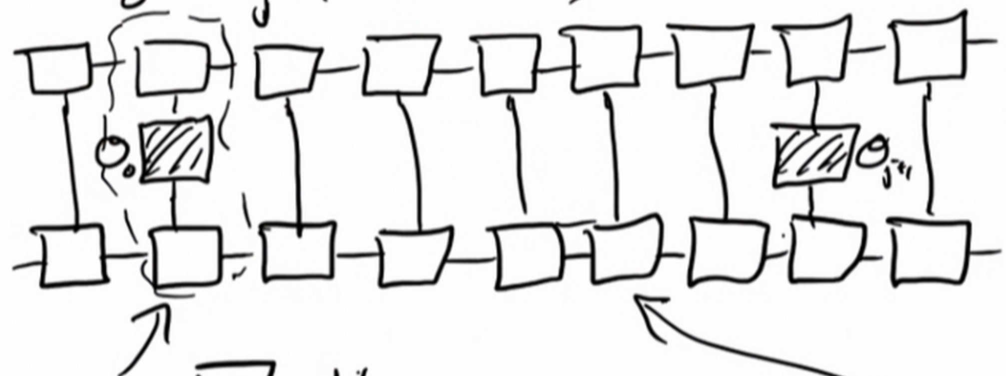
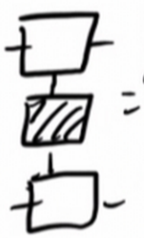
$=$ 

$E_O =$  $= \sum_{i,j=0}^{d-1} O_{ij} (A_i \otimes A_j^T)$

$E_I =$  $= \sum_{i=0}^{d-1} A_i \otimes A_i^T$

$\langle \Psi[A] | E^{(\infty)}$

MPS and correlation functions

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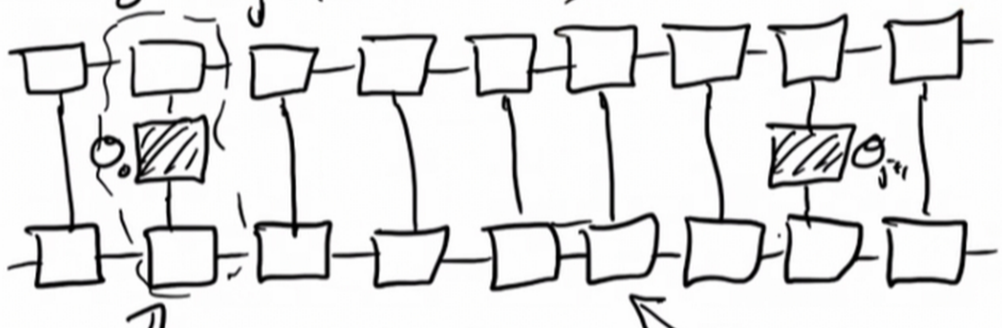
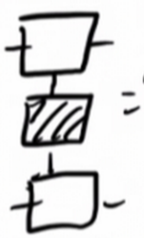
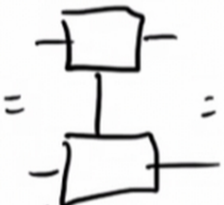
$=$ 

$E_O =$  $= \sum_{i,j=0}^{d-1} O_{ij} (A_i \otimes A_j^T)$

$E_I =$  $= \sum_{i=0}^{d-1} A_i \otimes A_i^T$

$$= \mathrm{Tr}\left(E^{(\infty)} E_{O_0} E_I^{j} E_{O_{j+1}} E^{(\infty)}\right)$$

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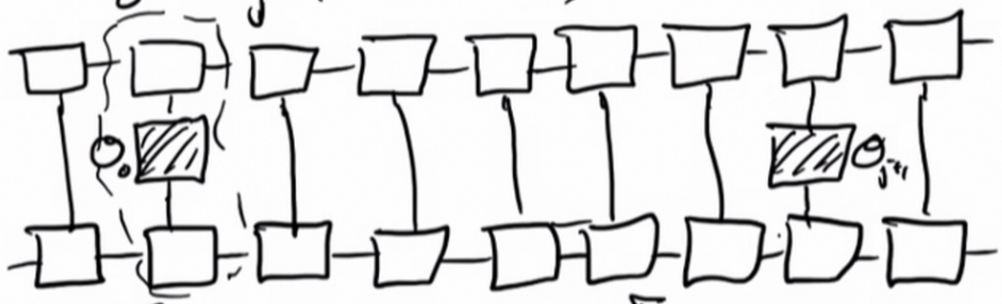
$$\langle \psi[A] | O_0 \, O_{j+1} | \psi[A] \rangle$$

$=$



$$E_O = $$

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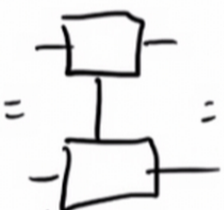

$$= \sum_{i=0}^{d-1} A_i \otimes A_i^T$$

$$= \mathrm{Tr}\left(E^{(\infty)} E_{O_0} E_I^{j} E_{O_{j+1}} E^{(\infty)}\right) = \nu^{j} \Leftarrow$$ largest eigenvalue of E_I

Spin chains, MPS, and area law

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MPS are a good description for ground states of gapped 1D spin chains

Hasting: all

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MPS are a good description for ground states of gapped 1D spin chains

Hasting: all gapped 1D spin chains have a g.s. satifying an area law (bounded entanglement between any region + its complement)

$\Rightarrow$

Spin chains, MPS, and area law

MPS are a good description for ground states of gapped 1D spin chains

Hastings: all gapped 1D spin chains have a g.s. satifying an area law (bounded entanglement between any region + its complement)

$\Rightarrow$ MPS with fixed dim. D can approx. g.s. with arbitrary accuracy