Title: PSI 2016/2017 Quantum Theory - Lecture 3

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Abstract:

Quantum Mechanics

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What is the space of quantum states?

$\mathbb{C}^d$

↓

$\mathbb{C}P^{d-1}$ (complex projective space)

Space of rays in $\mathbb{C}^d$ that pass through the origin

e.g. $\mathbb{C}^2$

↓

$\mathbb{C}P^1 \sim S^2$

Quantum Mechanics
Operational Quantum Mechanics

Lecture 3

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Composite quantum systems – tensor product

Two quantum systems, described by $|\psi\rangle_A$, $|\phi\rangle_B$

Joint system $|\Psi\rangle_{AB} = |\psi\rangle_A |\phi\rangle_B$ ← in what space does this object live?

Closed under superposition

If $|\psi_1\rangle_A |\phi_1\rangle_B$ and $|\psi_2\rangle_A |\phi_2\rangle_B$ then

also want $\frac{1}{\sqrt{2}} |\psi_1\rangle_A |\phi_1\rangle_B + \frac{1}{\sqrt{2}} |\psi_2\rangle_A |\phi_2\rangle_B$

Joint space is the tensor product of the space of states for systems A, B

$|\psi\rangle_A \otimes |\phi\rangle_B \in \mathbb{C}^{d_A} \otimes \mathbb{C}^{d_B}$

e.g. $|0\rangle_A \otimes |0\rangle_B$

shorthand $|0\rangle_A |0\rangle_B$

$|00\rangle_{AB}$

Also for operators $X_A \otimes Z_B$

shorthand $X_A Z_B$ not XZ

Bell states

Entangled states of two qubits

orthonormal basis

$$|\Phi_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$
$$|\Phi_{01}\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$
$$|\Phi_{10}\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$
$$|\Phi_{11}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

spin triplet

spin singlet

$$|\Phi_{ab}\rangle = (I \otimes X^a Z^b)|\Phi_{00}\rangle$$

Ensembles of quantum states

What if I have some (classical) uncertainty about the preparation?

prep.	prob.
$\|\psi_1\rangle$	p_1
$\|\psi_2\rangle$	p_2
⋮	⋮

not nec. orthogonal

Convenient description:

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

density operator (density matrix)

operator on vector (Hilbert) space of states

If $\rho = |\psi\rangle\langle\psi|$ then call it pure state

Otherwise mixed state

General properties of the density operator

An operator ρ is the density operator associated with an ensemble $\{p_i, |\psi_i\rangle\}$ iff

1) $\mathrm{Tr}[\rho] = 1$ "Trace one"

2) $\rho \geq 0$ "Positive"

Proof: Suppose $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$

Then $\mathrm{Tr}[\rho] = \sum_i p_i \mathrm{Tr}[|\psi_i\rangle\langle\psi_i|] = \sum_i p_i = 1$

and $\langle\phi|\rho|\phi\rangle = \sum_i p_i \langle\phi|\psi_i\rangle\langle\psi_i|\phi\rangle = \sum_i p_i |\langle\phi|\psi_i\rangle|^2 \geq 0 \quad \forall |\phi\rangle$

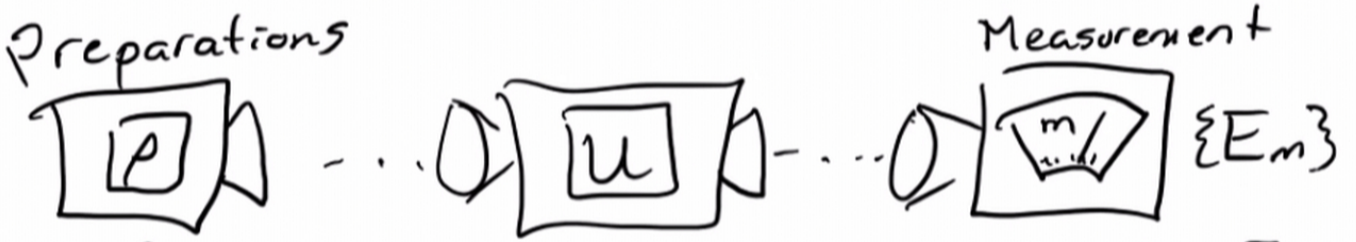
$\Rightarrow \rho \geq 0$

Conversely

$\rho \geq 0 \Rightarrow$ spectral decomposition $\rho = \sum_i \lambda_i |i\rangle\langle i| \quad \lambda_i \geq 0$

$\mathrm{Tr}[\rho] = 1 \Rightarrow \sum_i \lambda_i = 1$ i.e. prob distn

Operational quantum mechanics with density operators

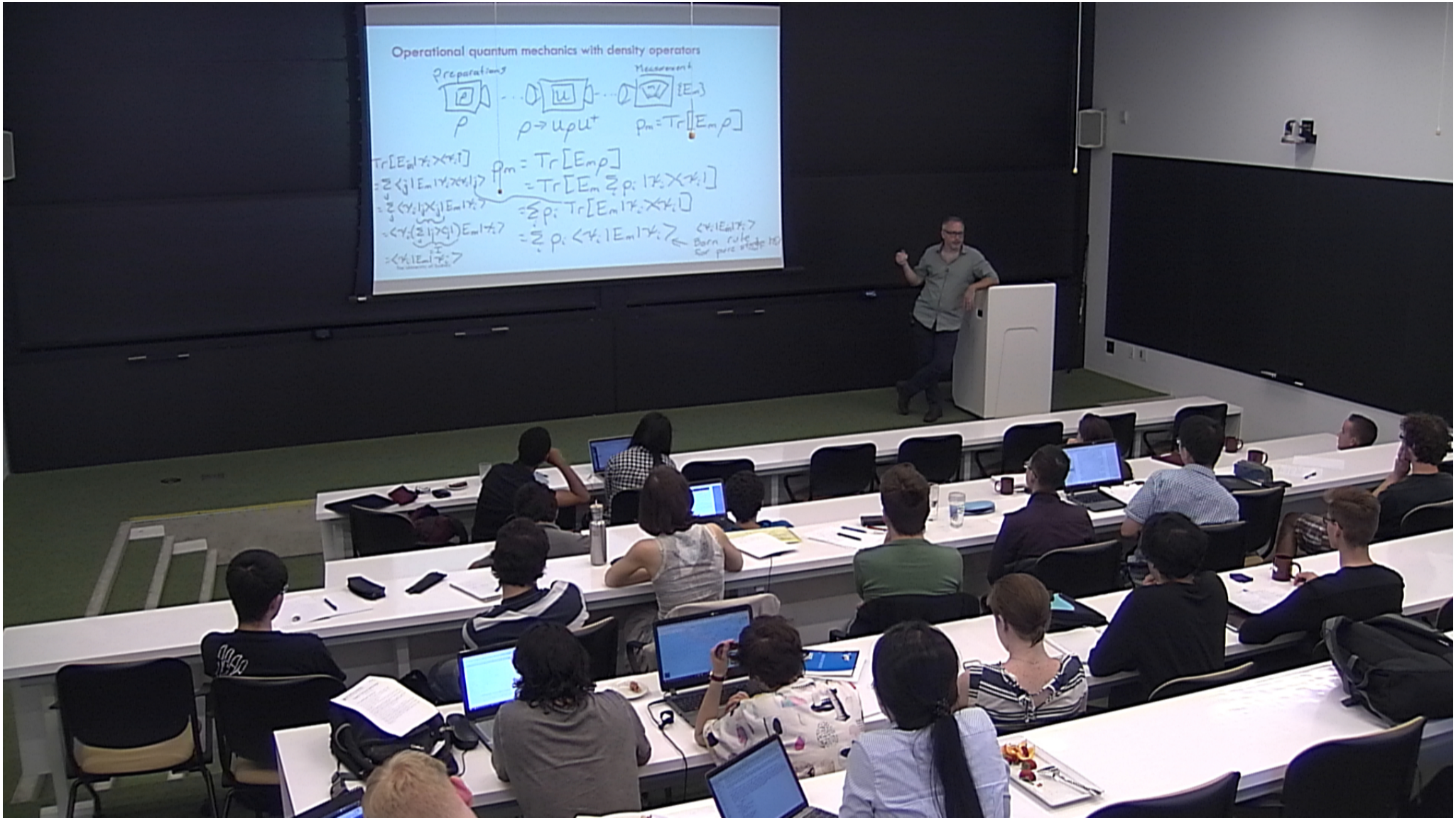


ρ $\qquad \rho \to U\rho U^\dagger \qquad p_m = \mathrm{Tr}[E_m \rho]$

$$\mathrm{Tr}[E_m |\psi_i\rangle\langle\psi_i|] = \sum_j \langle j|E_m|\psi_i\rangle\langle\psi_i|j\rangle = \sum_j \langle\psi_i|j\rangle\langle j|E_m|\psi_i\rangle = \langle\psi_i|\Big(\sum_j |j\rangle\langle j|\Big)E_m|\psi_i\rangle = \langle\psi_i|E_m|\psi_i\rangle$$

$$p_m = \mathrm{Tr}[E_m \rho] = \mathrm{Tr}\Big[E_m \sum_i p_i |\psi_i\rangle\langle\psi_i|\Big] = \sum_i p_i \,\mathrm{Tr}[E_m |\psi_i\rangle\langle\psi_i|] = \sum_i p_i \langle\psi_i|E_m|\psi_i\rangle$$

$\langle\psi_i|E_m|\psi_i\rangle$ ← Born rule for pure state $|\psi_i\rangle$


Operational quantum mechanics with density operators
Preparations
Measurement
ρ
ρ → uρu†
{Em}
pm = Tr[Em ρ]
Tr[Em|ψi><ψi|]
pm = Tr[Em ρ]
= Tr[Em Σ pi |ψi><ψi|]
= Σ pi Tr[Em|ψi><ψi|]
= Σ pi <ψi|Em|ψi>
<ψi|Em|ψi>
Born rule for pure states