Title: PSI 2016/2017 Quantum Theory - Lecture 1

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Abstract:

Quantum Mechanics

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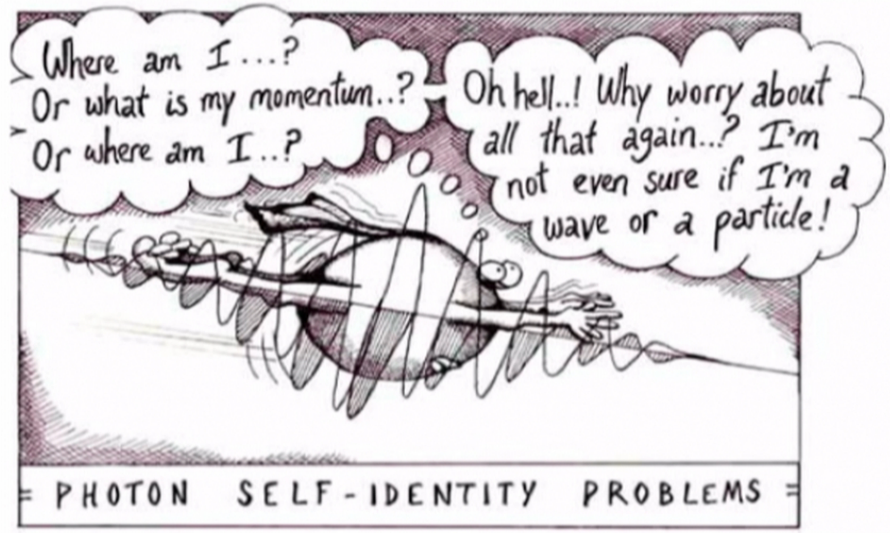
Quantum weirdness

QUANTUM PHYSICS 101

- Quantum physics is weird!
- You cannot talk about “the position” or “the momentum” of a particle
- Complementarity: Particles behave like waves, and waves behave like particles
- Just looking at a particle affects its motion
- Schrödinger’s cat: both alive and dead
- Tunnelling: a particle trapped in a box can suddenly ‘appear’ outside of the box
- Entanglement: pairs of particles can be in weird states, such that a measurement of one instantaneously changes the other
- But follow the rules and it all works!



THE FAR SIDE By GARY LARSON

“Ohhhhhhh . . . Look at that, Schuster . . . Dogs are so cute when they try to comprehend quantum mechanics.”

So what?

Quantum physics represents the most accurate scientific theory we have, but…

- … do we really *understand* it, or just ‘Shut up and calculate’?
- … why can’t we quantize gravity?
- … why do measurements obey different laws to everything else?
- … what was Einstein going on about?

If quantum mechanics is so strange and counter-intuitive, how can we hope to develop new technologies such as quantum computers which are based on this ‘weirdness’?

Let’s take up the challenge!

Quantum Mechanics
Operational Quantum Mechanics

Lecture 1

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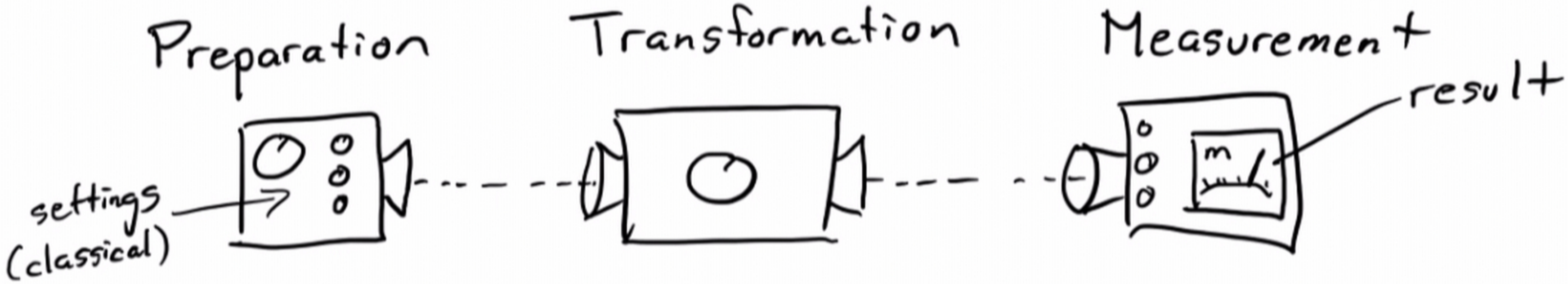
Operational quantum mechanics



Quantum states

↓

Density operators

Unitary transformation

↓

General quantum operations (CPTP maps)

Projections in a basis/ observable

↓

POVMs

Preparations – quantum states

Quantum system

described by a normalised vector (length 1)

in a complex Hilbert (vector) space of dimension $\underline{\underline{d}}$

Is d the only defining parameter?

Basically yes.

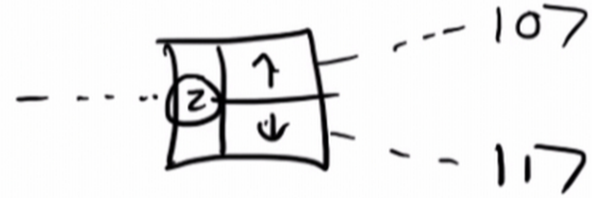
Like space of probability dist$^{\underline{n}}$s

Normalisation condition leads to (almost) all of the defining properties of transformations + measurements

* + linearity

Kets – Dirac notation

Stern Gerlach experiment



– Kets are used to label everything we know about a quantum system

$|0\rangle$ will give outcome $+1$ with certainty when SG gradient is in $+z$ direction

Could have called it $|\uparrow\rangle$, $|+z\rangle$, $|☺\rangle$

$|1\rangle$ will give outcome -1 with certainty

$\rightarrow$ Leads you to pick $d = 2$

General state:

$$|\psi\rangle = a|0\rangle + b|1\rangle$$

Normalisation

$$|a|^2 + |b|^2 = 1$$

$d=2$

Qubits and the Bloch sphere

$|\psi\rangle = a|0\rangle + b|1\rangle$ with $|a|^2 + |b|^2 = 1$

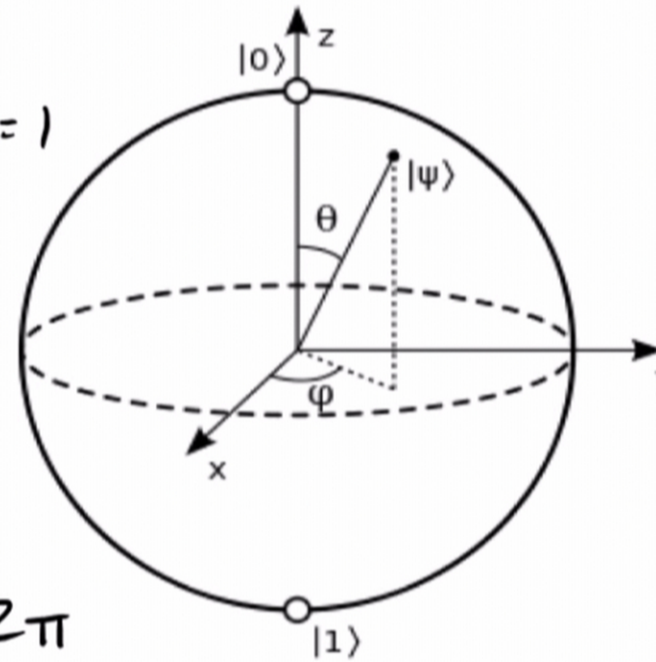
Overall phase of $|\psi\rangle$ is unobservable

$$|\psi\rangle \simeq e^{i\chi}|\psi\rangle$$

wlog choose a real, positive

$$a = \cos(\theta/2) \qquad 0 \leq \theta \leq \pi$$

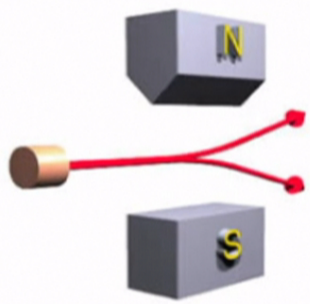
$$b = e^{i\phi}\sin(\theta/2) \qquad 0 \leq \phi < 2\pi$$



ϕ is a relative phase between $|0\rangle, |1\rangle$

$$|\psi\rangle \simeq \vec{\psi} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

Quantum Measurement – the Born rule



Quantum systems: described by normalised state vectors $|\psi\rangle$

Measurements: described by a basis of vectors, e.g.,

$|0\rangle, |1\rangle$

We can only predict the **probability** of each **outcome**.

Consider meas. in $|0\rangle, |1\rangle$ basis

$|\psi\rangle = a|0\rangle + b|1\rangle$

$$P_0 = |\langle 0|\psi\rangle|^2 = |a|^2$$

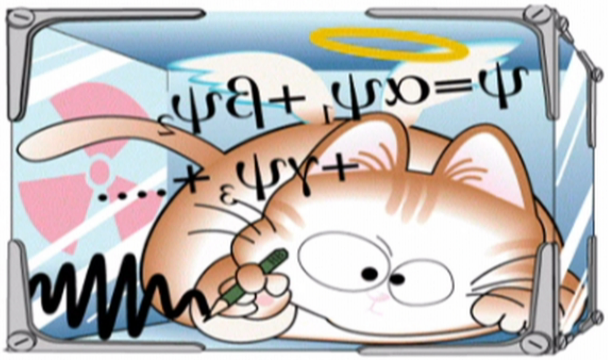
$$P_1 = |\langle 1|\psi\rangle|^2 = |b|^2$$

$$P_0 + P_1 = |a|^2 + |b|^2 = 1$$

Born rule just a way to assign probabilities to distinct outcomes

Schrödinger's Cat coherence

Schrödinger's cat: a *coherent* superposition of alive and dead



$$|\text{cat}\rangle = \frac{1}{\sqrt{2}}|\text{alive}\rangle + \frac{1}{\sqrt{2}}|\text{dead}\rangle$$

The interesting point about this state is not that it's a combination of the cat being alive and being dead.

It's that there exists a *coherence measurement* with 2 outcomes such that this state, when measured, gives '+' with certainty

Indistinguishability of non-orthogonal quantum states

Take two non-orthogonal quantum states $|\psi_0\rangle, |\psi_1\rangle$

i.e. $\langle\psi_0|\psi_1\rangle \neq 0$

Can they be distinguished with certainty with a single measurement?

Meas. basis $|\phi_0\rangle, |\phi_1\rangle$

we demand $P(0|0) = P(1|1) = 1$ $\quad P(0|1) = P(1|0) = 0$

$= |\langle\phi_0|\psi_0\rangle|^2 \quad = |\langle\phi_1|\psi_1\rangle|^2$

What's the best we can do?