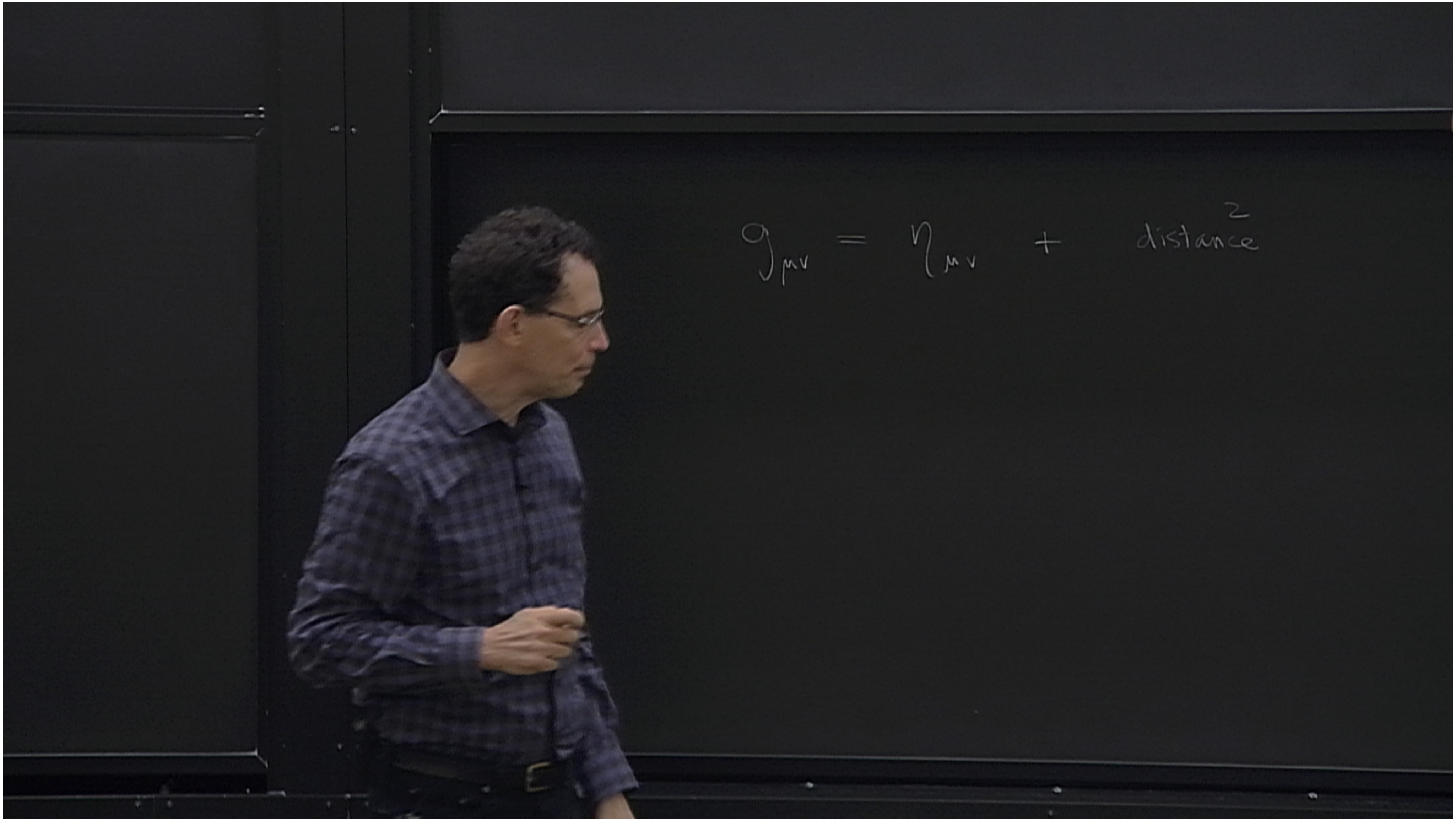
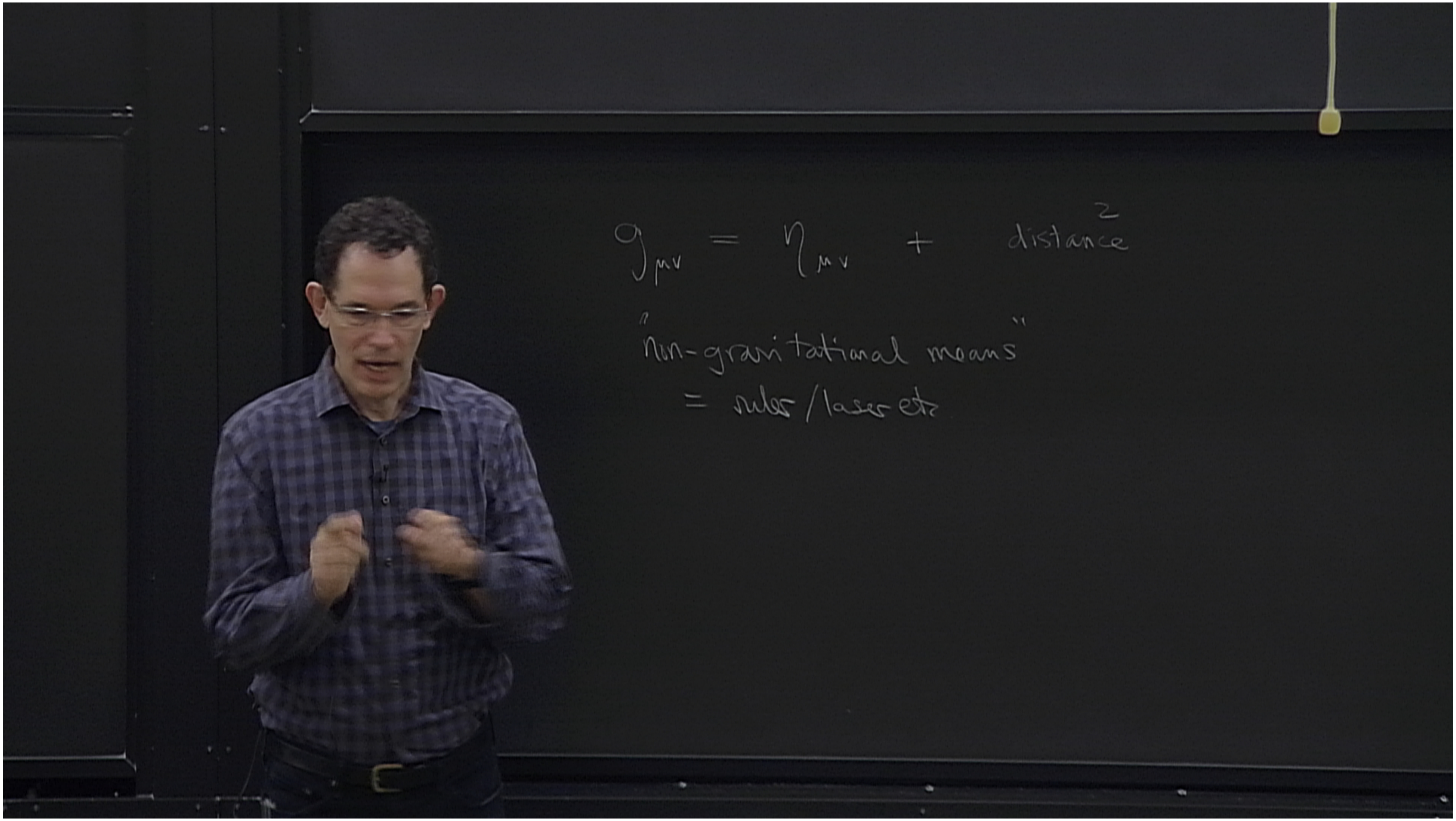
Title: PSI 2016/2017 Relativity - Lecture 12

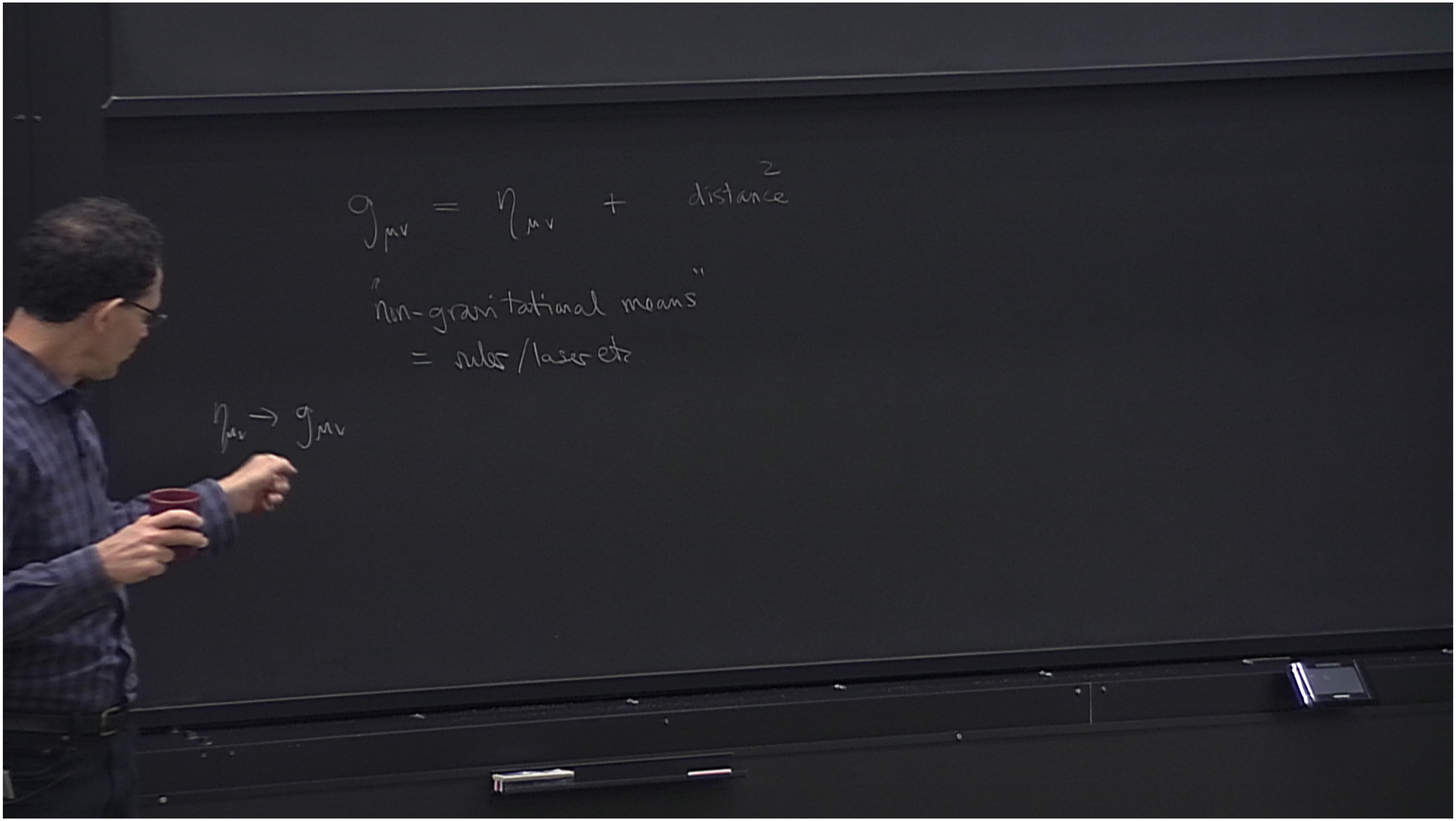
Date: Sep 21, 2016 09:00 AM

URL: http://pirsa.org/16090012

Abstract:


$g_{\mu\nu} = \eta_{\mu\nu} + \text{distance}^2$


$g_{\mu\nu} = \eta_{\mu\nu} + \text{distance}^2$
"non-gravitational means"
= ruler / laser etc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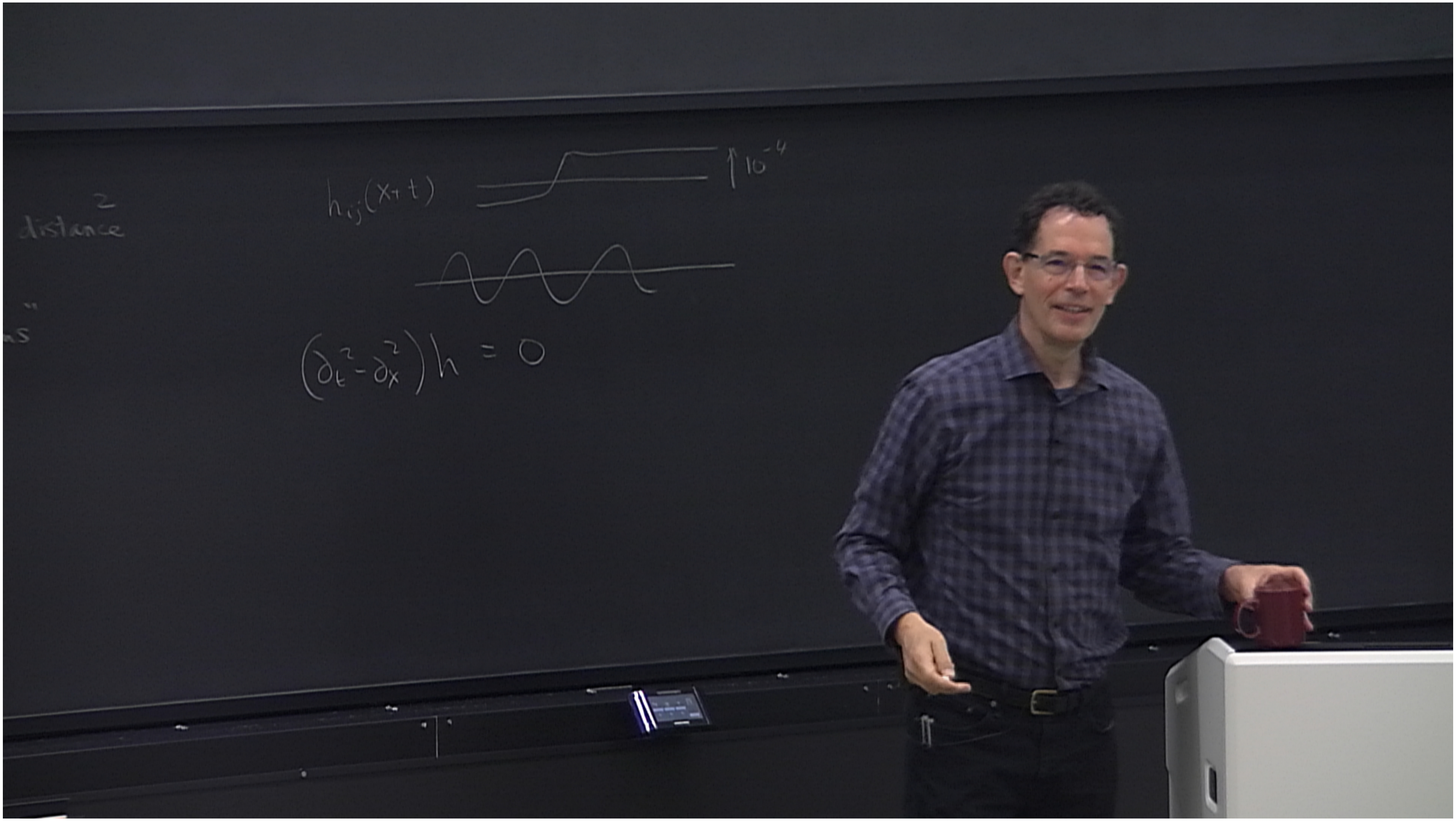
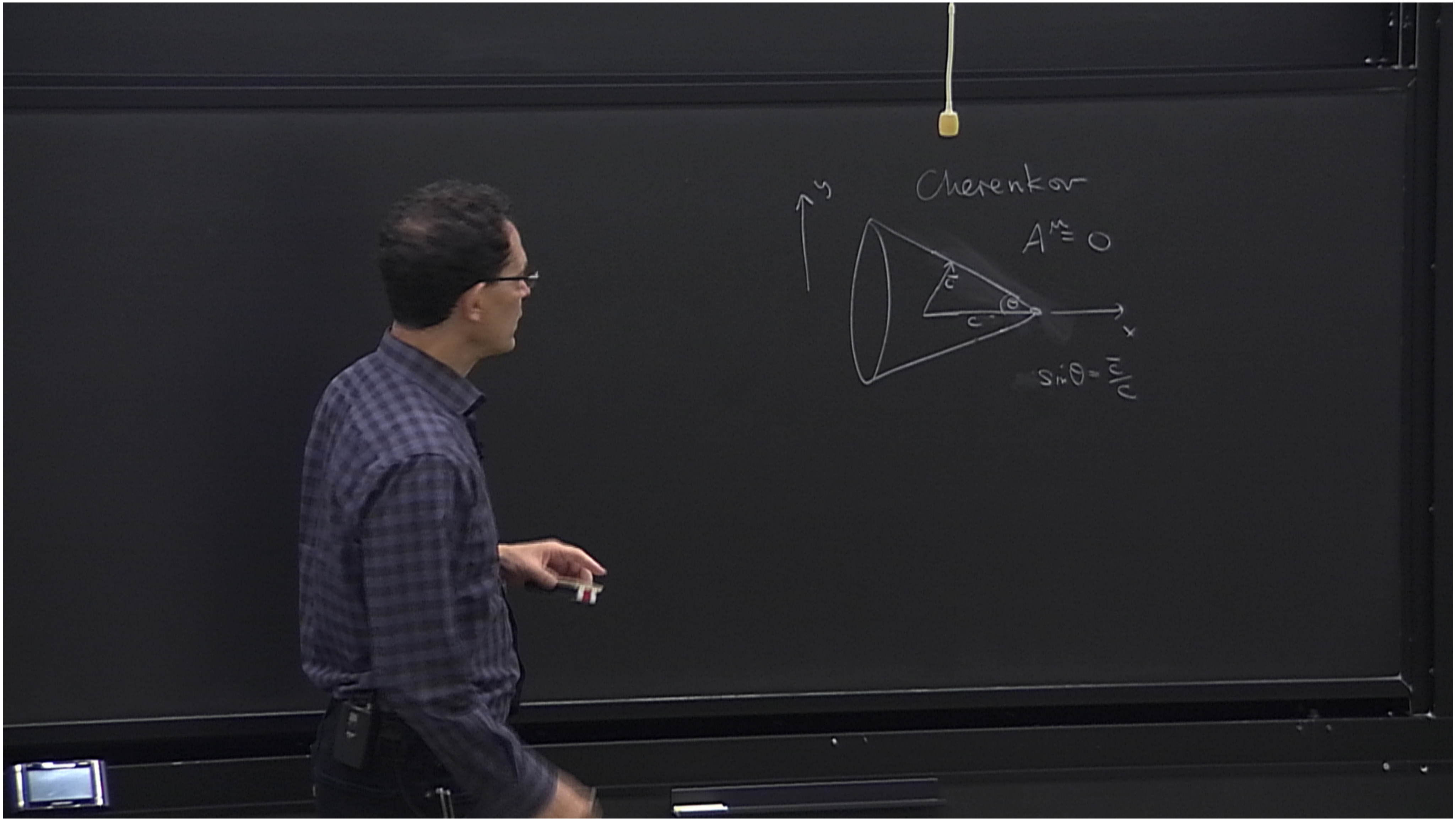
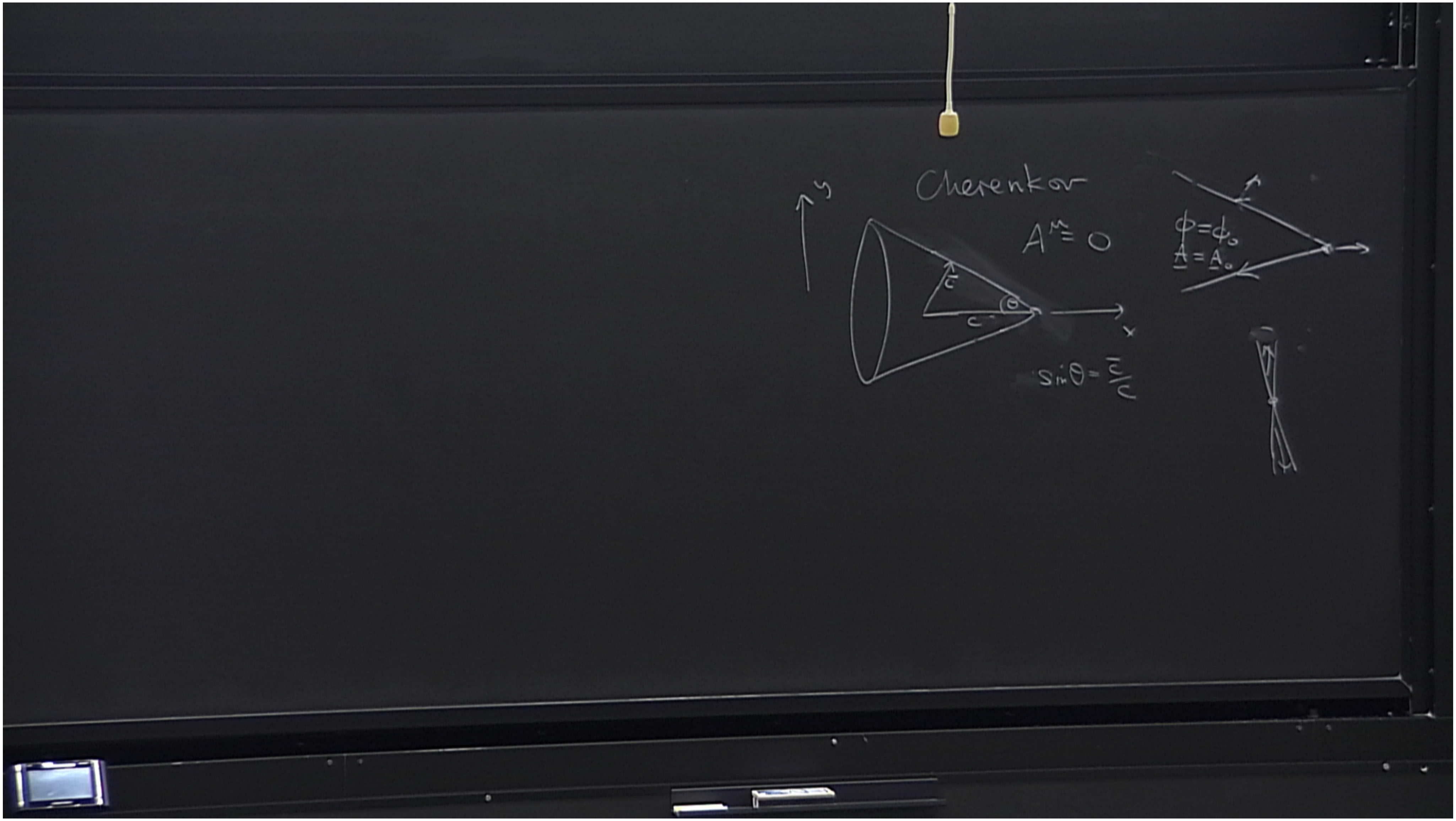
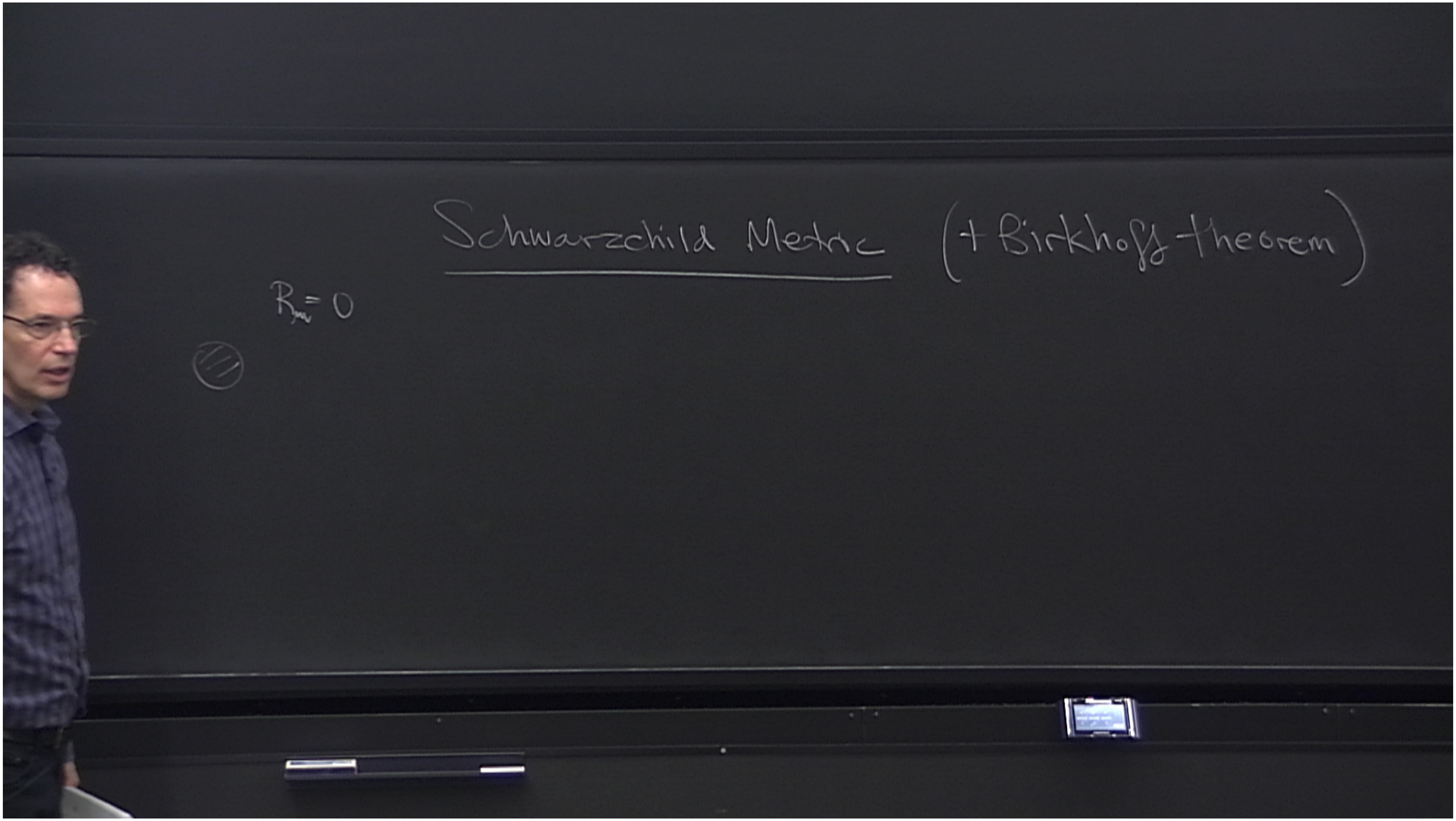
"non-gravitational means"
= rules / laser etc
$\eta_{\mu\nu} \to g_{\mu\nu}$


distance
$h_{ij}(x,t)$
10^{-4}
$(\partial_t^2 - \partial_x^2)h = 0$


Cherenkov
$A^\mu \equiv 0$
$\sin\theta = \frac{\tilde{c}}{c}$
y
x

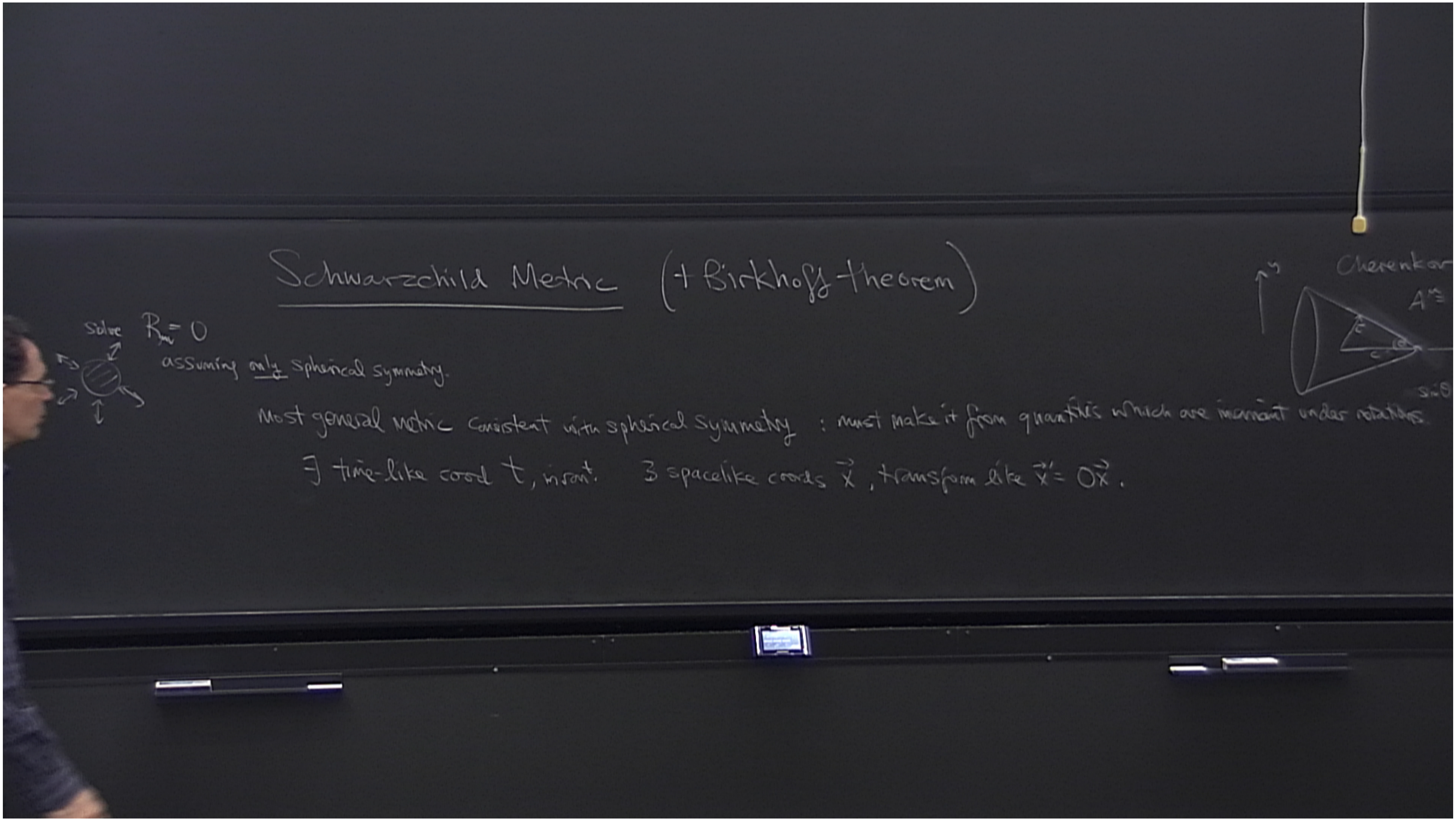

Cherenkov
y
$A^\mu = 0$
$\bar{c}$
θ
c
x
$\sin\theta = \bar{c}/c$
$\phi = \phi_0$
$\underline{A} = \underline{A}_0$

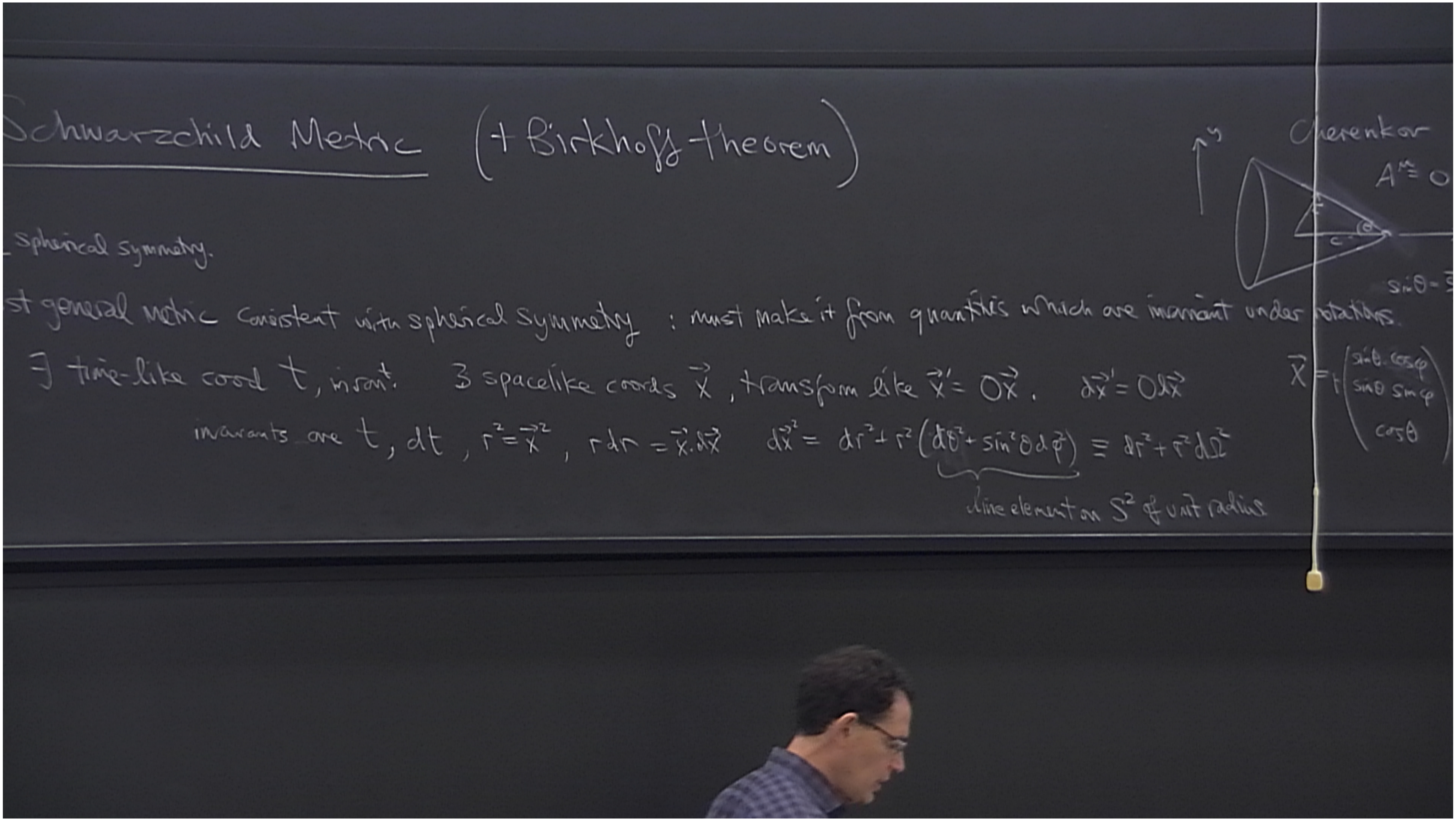

Schwarzschild Metric (+ Birkhoffs theorem)
$R_{\mu\nu} = 0$

Schwarzchild Metric (+ Birkhoff's theorem)

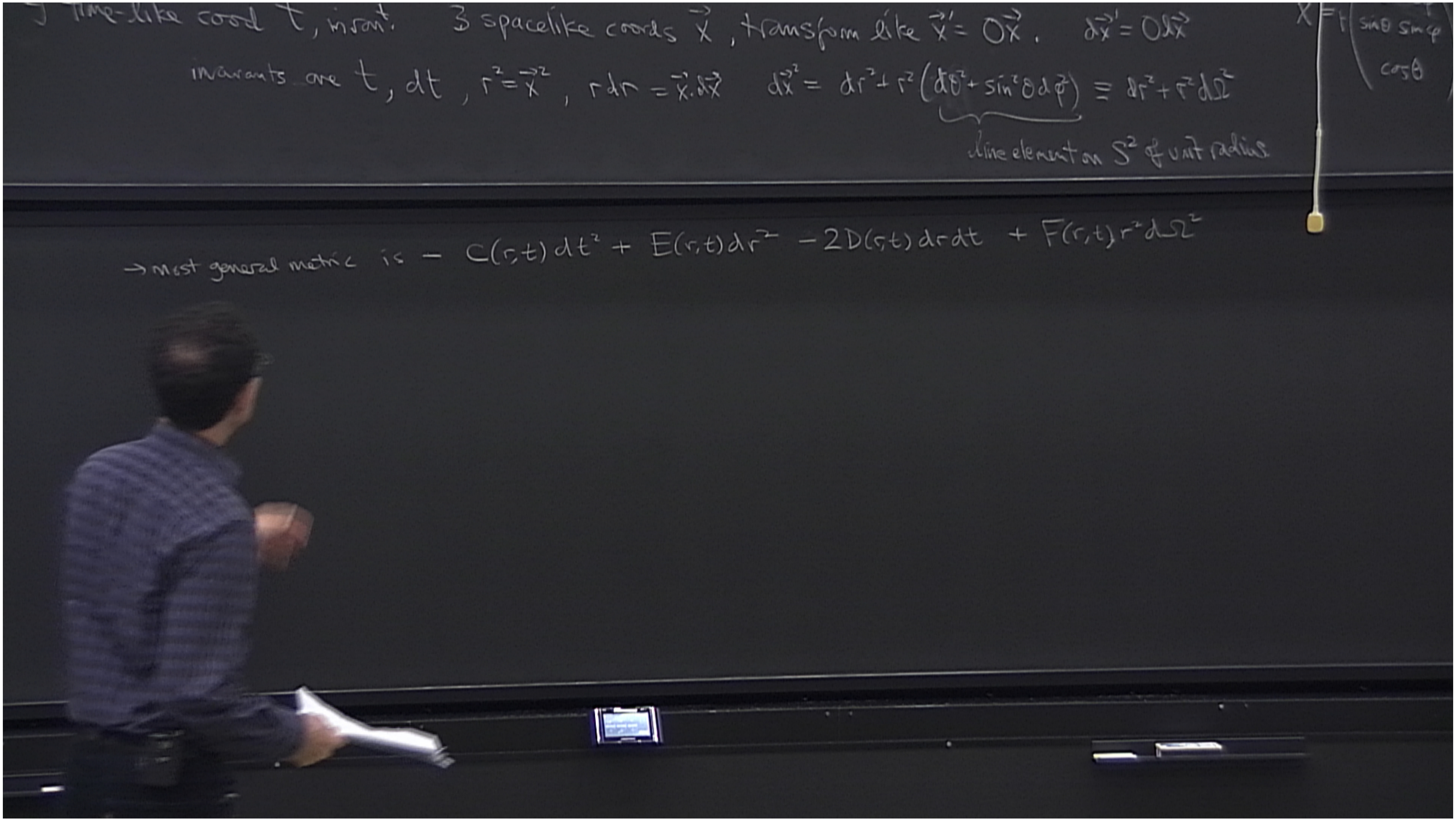
solve $R_{\mu\nu} = 0$

assuming <u>only</u> spherical symmetry.


Schwarzchild Metric (+ Birkhoff's theorem)
Solve $R_{\mu\nu} = 0$
assuming only spherical symmetry.
Most general metric consistent with spherical symmetry : must make it from quantities which are invariant under rotations.
$\exists$ time-like coord t, invariant. 3 spacelike coords $\vec{x}$, transform like $\vec{x}' = O\vec{x}$.
Cherenkov



Schwarzchild Metric (+ Birkhoff's theorem)
spherical symmetry.
st general metric consistent with spherical symmetry : must make it from quantities which are invariant under rotations.
∃ time-like coord t, invariant. 3 spacelike coords $\vec{x}$, transform like $\vec{x}' = O\vec{x}$. $d\vec{x}' = O d\vec{x}$
invariants are t, dt, $r^2 = \vec{x}^2$, $r\,dr = \vec{x}\cdot d\vec{x}$ $d\vec{x}^2 = dr^2 + r^2(\underbrace{d\theta^2 + \sin^2\theta\, d\varphi^2}_{\text{line element on } S^2 \text{ of unit radius}}) \equiv dr^2 + r^2 d\Omega^2$
Cherenkov
$A^\mu \equiv 0$
$\vec{X} = r\begin{pmatrix}\sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta\end{pmatrix}$


time-like coord t, invariant. 3 spacelike coords $\vec{x}$, transform like $\vec{x}' = O\vec{x}$. $d\vec{x}' = O\,d\vec{x}$
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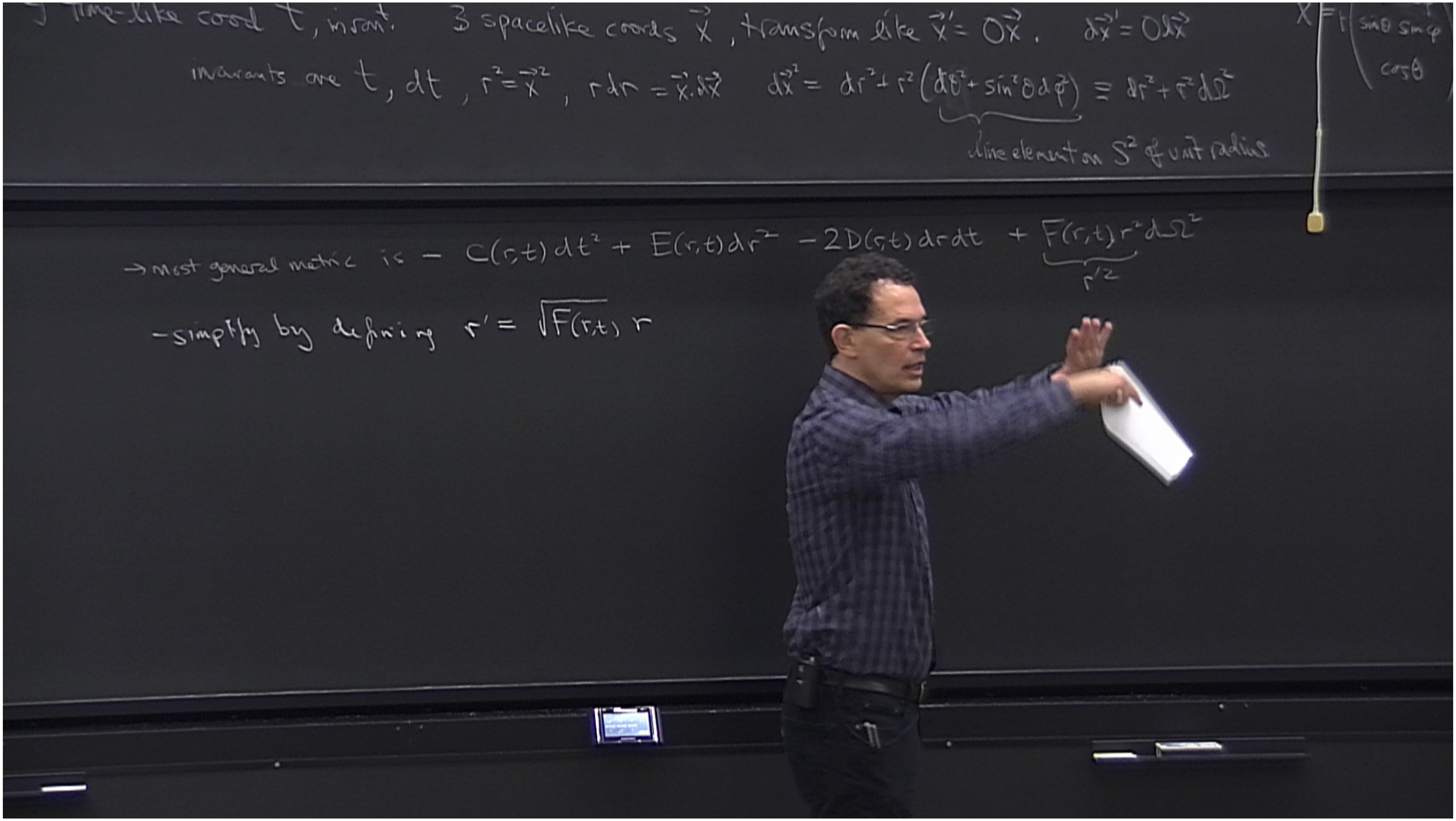
$\sin\theta \sin\varphi$
$\cos\theta$
→ most general metric is $-C(r,t)\,dt^2 + E(r,t)\,dr^2 - 2D(r,t)\,dr\,dt + F(r,t)\,r^2 d\Omega^2$



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line element on S^2 of unit radius
$\vec{x} = r\begin{pmatrix} \sin\theta \sin\varphi \\ \cos\theta \end{pmatrix}$
→ most general metric is $-C(r,t)\,dt^2 + E(r,t)\,dr^2 - 2D(r,t)\,dr\,dt + \underbrace{F(r,t)\,r^2}_{r'^2} d\Omega^2$
– simplify by defining $r' = \sqrt{F(r,t)}\, r$

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$$dr = \frac{dr}{dr'} dr'$$

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now remove cross term $\quad dt' = f(r,t)\left(C(r,t)\,dt + D(r,t)\,dr\right)$

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...op primes

$-C(r,t)\,dt^2 + E(r,t)\,dr^2 - 2D(r,t)\,dr\,dt + r^2 d\Omega^2 \quad = \quad -\frac{dt'^2}{Cf^2}$

...emove cross term

$dt' = f(r,t)\left(C(r,t)\,dt + D(r,t)\,dr\right)$

$dt'^2 = Cf^2\left(C\,dt^2 + 2D\,dr\,dt + \frac{D^2}{C} dr^2\right)$

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...general metric is $-C(r,t)\,dt^2 + E(r,t)\,dr^2 - 2D(r,t)\,dr\,dt + \underbrace{F(r,t)\,r^2}_{r'^2}d\Omega^2$

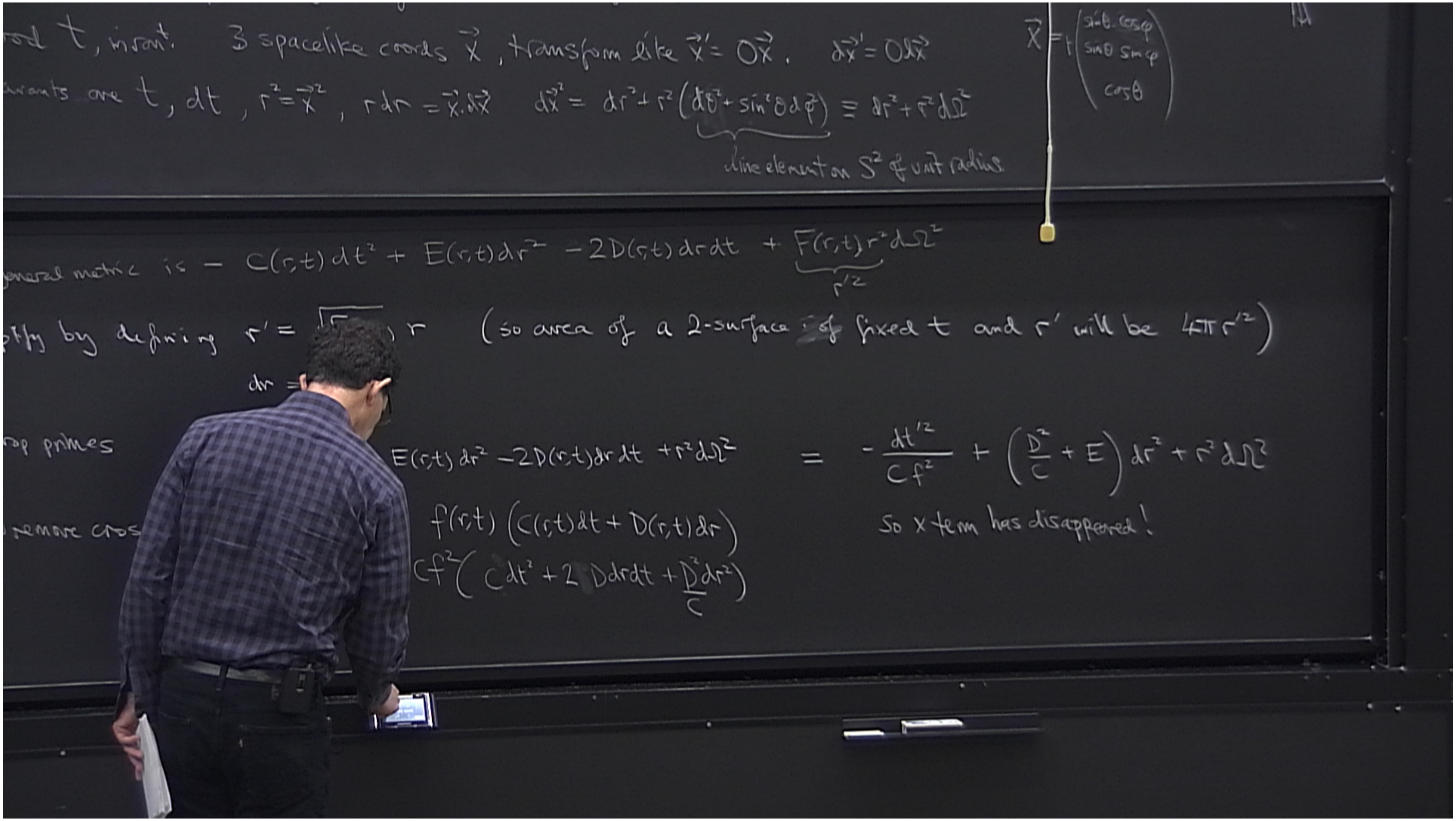
...plify by defining $r' = \sqrt{F(r,t)}\,r$ (so area of a 2-surface of fixed t and r' will be $4\pi r'^2$)

$$dr = \frac{dr}{dr'}dr'$$

...drop primes $\quad -C(r,t)\,dt^2 + E(r,t)\,dr^2 - 2D(r,t)\,dr\,dt + r^2 d\Omega^2 = -\frac{dt'^2}{Cf^2} + \left(\frac{D^2}{C} + E\right)dr^2 + r^2 d\Omega^2$

...remove cross term $\quad dt' = f(r,t)\left(C(r,t)\,dt + D(r,t)\,dr\right)$

$$dt'^2 = Cf^2\left(C\,dt^2 + 2D\,dr\,dt + \frac{D^2}{C}dr^2\right)$$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$\vec{x} = r\begin{pmatrix}\sin\theta\cos\varphi\\ \sin\theta\sin\varphi\\ \cos\theta\end{pmatrix}$
general metric is $-C(r,t)\,dt^2 + E(r,t)\,dr^2 - 2D(r,t)\,dr\,dt + \underbrace{F(r,t)\,r^2}_{r'^2} d\Omega^2$
simplify by defining $r' = \sqrt{F}\, r$ (so area of a 2-surface of fixed t and r' will be $4\pi r'^2$)
drop primes
$E(r,t)\,dr^2 - 2D(r,t)\,dr\,dt + r^2 d\Omega^2 = -\frac{dt'^2}{Cf^2} + \left(\frac{D^2}{C} + E\right)dr^2 + r^2 d\Omega^2$
remove cross
$f(r,t)\,(C(r,t)\,dt + D(r,t)\,dr)$
$Cf^2(C\,dt^2 + 2D\,dr\,dt + \frac{D^2}{C}dr^2)$
So x term has disappeared!

$$-C(r,t)\,dt^2 + E(r,t)\,dr^2 - 2D(r,t)\,dr\,dt + r^2 d\Omega^2 \quad = \quad -\frac{dt'^2}{Cf^2} + \left(\frac{D^2}{C} + E\right) dr^2 + r^2 d\Omega^2$$

now remove cross term

$$dt' = f(r,t)\left(C(r,t)\,dt + D(r,t)\,dr\right) \quad (*)$$

$$dt'^2 = Cf^2\left(C\,dt^2 + 2D\,dr\,dt + \frac{D^2}{C}dr^2\right)$$

So x term has disappeared!

BUT: must show $\exists$ a fn $t'(t,r)$ s.t. $(*)$ is true.

primes

$$-C(r,t)\,dt^2 + E(r,t)\,dr^2 - 2D(r,t)\,dr\,dt + r^2 d\Omega^2 = -\frac{dt'^2}{Cf^2} + \left(\frac{D}{C} + E\right) dr^2 + r^2 \ldots$$

now remove cross term

$$dt' = f(r,t)\left(C(r,t)\,dt + D(r,t)\,dr\right) \quad (*)$$

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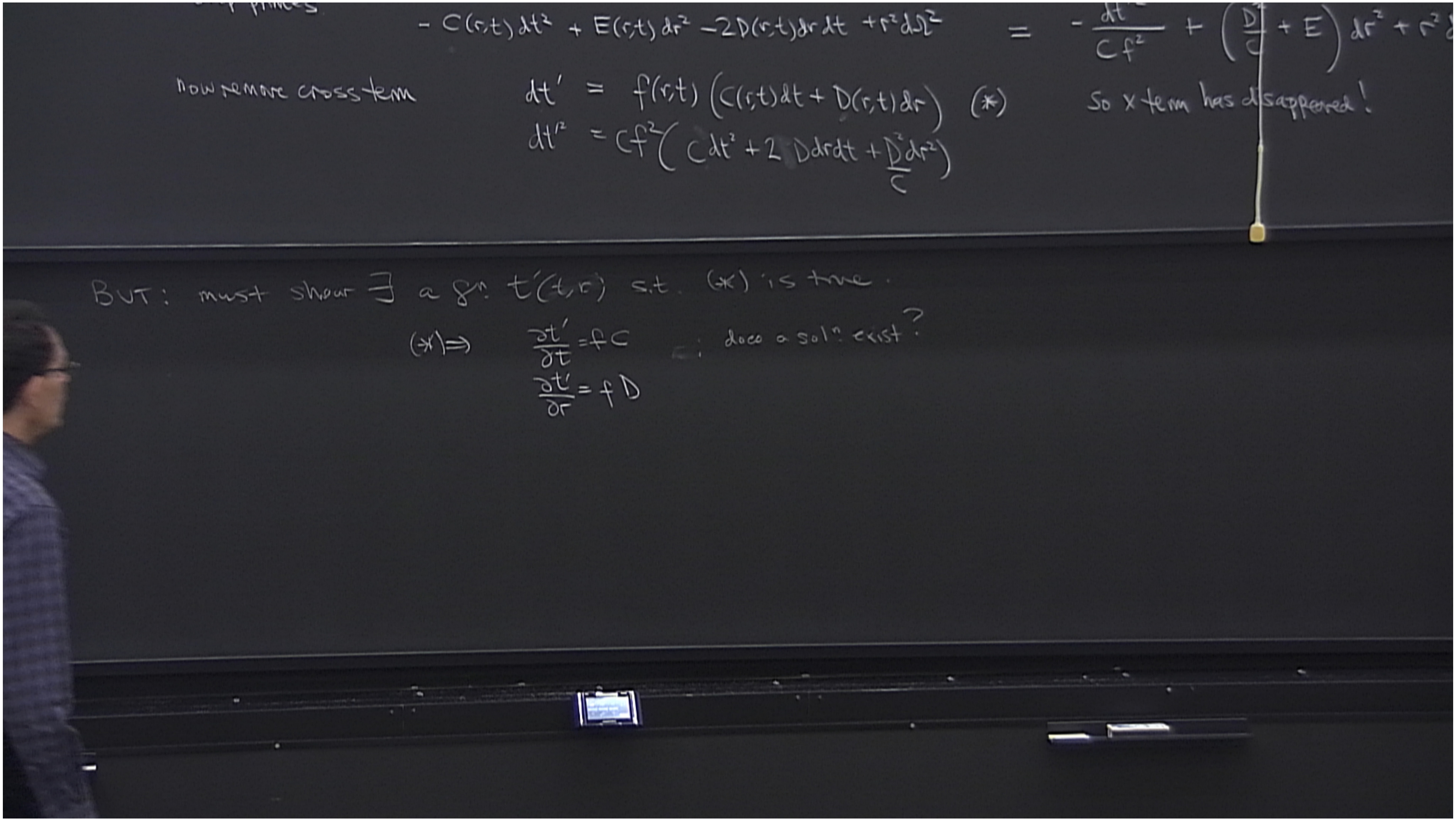
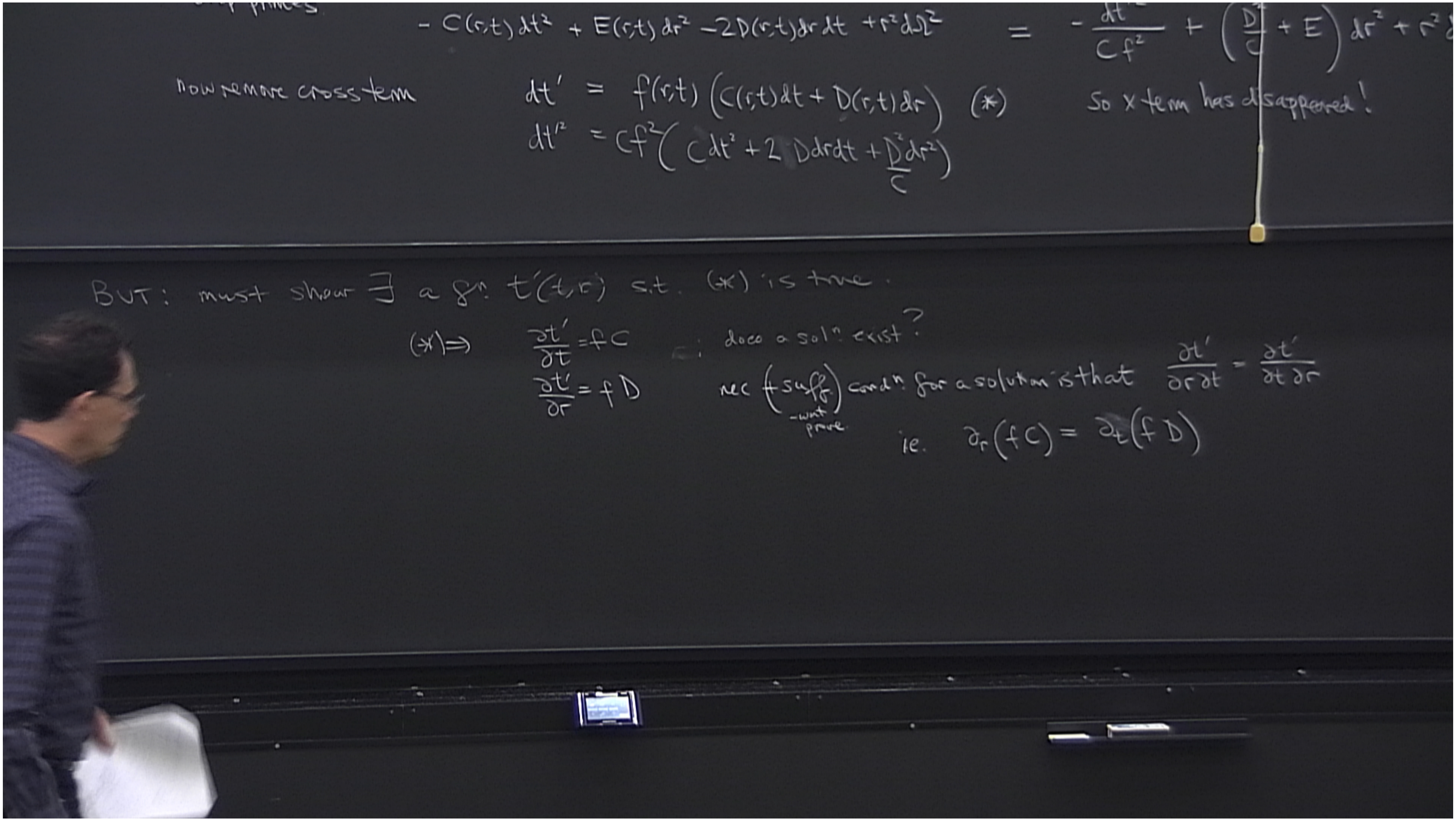
So x term has disappeared!

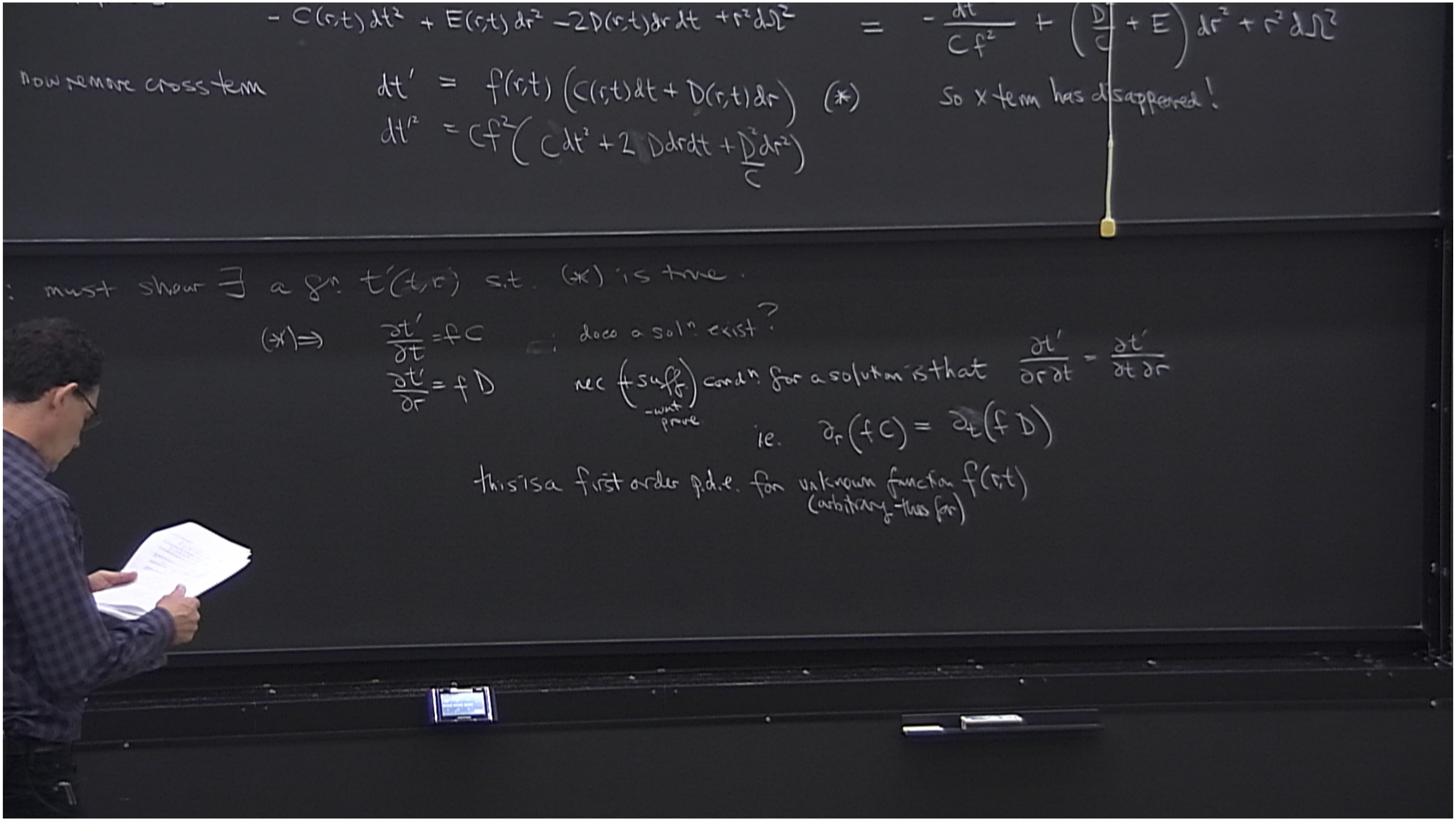
BUT: must show $\exists$ a fn $t'(t,r)$ s.t. $(*)$ is true.

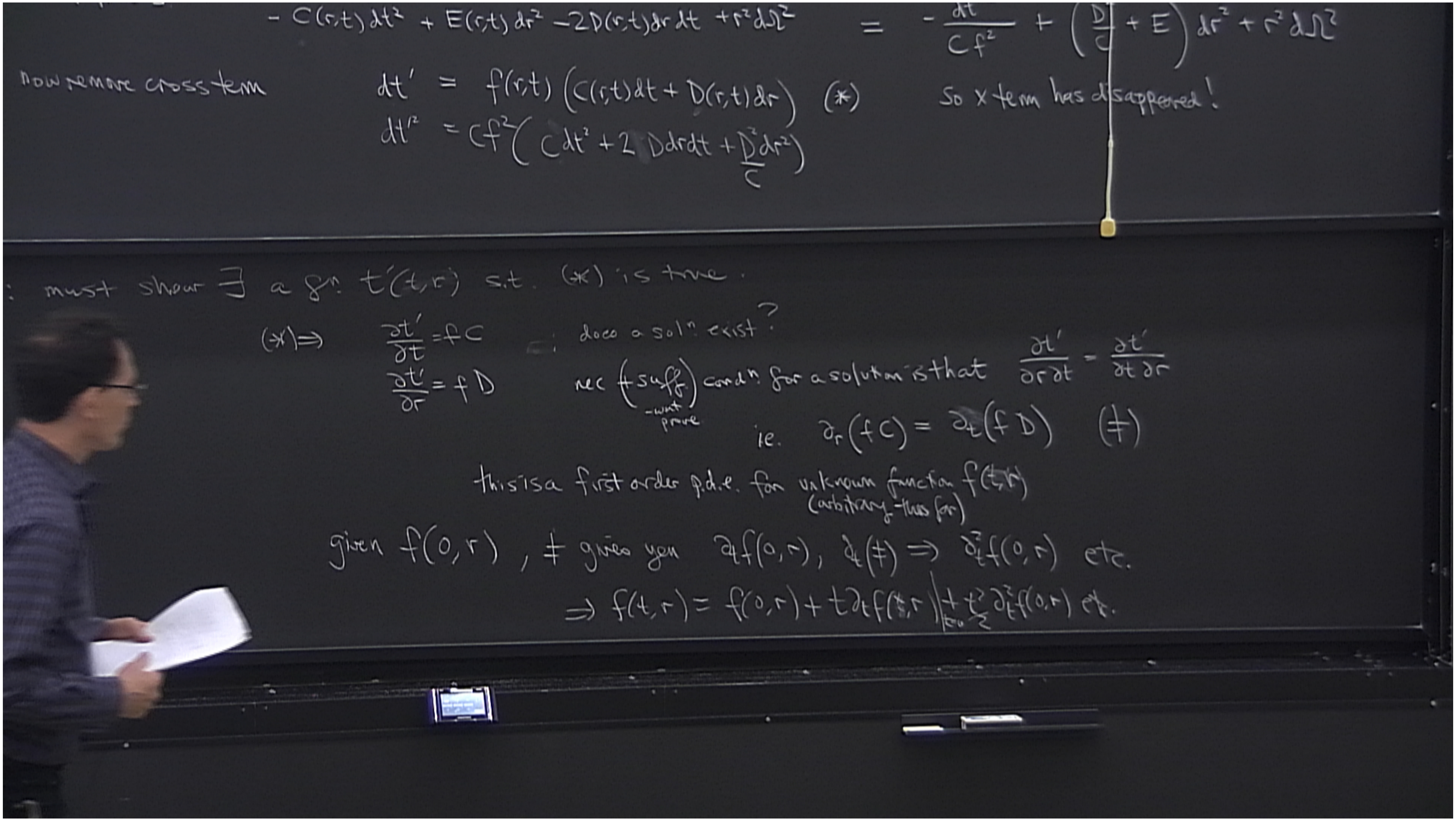
$$(*) \Rightarrow \quad \frac{\partial t'}{\partial t} = fC$$

$$\frac{\partial t'}{\partial r} = fD$$


$-C(r,t)dt^2 + E(r,t)dr^2 - 2D(r,t)dr\,dt + r^2 d\Omega^2 = -\frac{dt'^2}{Cf^2} + \left(\frac{D^2}{C} + E\right)dr^2 + r^2 d\Omega^2$
now remove cross term
$dt' = f(r,t)\left(C(r,t)dt + D(r,t)dr\right)$ (*)
$dt'^2 = Cf^2\left(C dt^2 + 2D\,dr\,dt + \frac{D^2}{C}dr^2\right)$
So x term has disappeared!
BUT: must show ∃ a fn $t'(t,r)$ s.t. (*) is true.
(*) ⇒ $\frac{\partial t'}{\partial t} = fC$
$\frac{\partial t'}{\partial r} = fD$
does a soln exist?


$-C(r,t)\,dt^2 + E(r,t)\,dr^2 - 2D(r,t)\,dr\,dt + r^2 d\Omega^2 = -\frac{dt'^2}{Cf^2} + \left(\frac{D^2}{C} + E\right)dr^2 + r^2 \ldots$
now remove cross term
$dt' = f(r,t)\left(C(r,t)\,dt + D(r,t)\,dr\right) \quad (*)$
So x term has disappeared!
$dt'^2 = Cf^2\left(C\,dt^2 + 2D\,dr\,dt + \frac{D^2}{C}dr^2\right)$
BUT: must show ∃ a fn $t'(t,r)$ s.t. (*) is true.
$(*) \Rightarrow \frac{\partial t'}{\partial t} = fC$
$\frac{\partial t'}{\partial r} = fD$
does a soln exist?
nec (+ suff) condn for a solution is that $\frac{\partial t'}{\partial r \partial t} = \frac{\partial t'}{\partial t \partial r}$
- won't prove
ie. $\partial_r(fC) = \partial_t(fD)$


$-C(r,t)dt^2 + E(r,t)dr^2 - 2D(r,t)dr\,dt + r^2 d\Omega^2 = -\frac{dt'^2}{Cf^2} + \left(\frac{D^2}{C} + E\right)dr^2 + r^2 d\Omega^2$
now remove cross term
$dt' = f(r,t)\left(C(r,t)dt + D(r,t)dr\right)$ (*)
So x term has disappeared!
$dt'^2 = Cf^2\left(C\,dt^2 + 2D\,dr\,dt + \frac{D^2}{C}dr^2\right)$
: must show ∃ a fn. $t'(t,r)$ s.t. (*) is true.
(*) ⇒ $\frac{\partial t'}{\partial t} = fC$
$\frac{\partial t'}{\partial r} = fD$
does a soln exist?
nec (+ suff) condn for a solution is that $\frac{\partial^2 t'}{\partial r \partial t} = \frac{\partial^2 t'}{\partial t \partial r}$
- won't prove
ie. $\partial_r(fC) = \partial_t(fD)$
this is a first order p.d.e. for unknown function $f(r,t)$
(arbitrary thus far)


$-C(r,t)dt^2 + E(r,t)dr^2 - 2D(r,t)dr\,dt + r^2 d\Omega^2 = -\frac{dt'^2}{Cf^2} + \left(\frac{D^2}{C} + E\right)dr^2 + r^2 d\Omega^2$
now remove cross term
$dt' = f(r,t)\left(C(r,t)dt + D(r,t)dr\right)$ (*)
$dt'^2 = Cf^2\left(C dt^2 + 2 D dr dt + \frac{D^2}{C}dr^2\right)$
So x term has disappeared!
: must show ∃ a fn $t'(t,r)$ s.t. (*) is true.
$(*) \Rightarrow \frac{\partial t'}{\partial t} = fC$, $\frac{\partial t'}{\partial r} = fD$
does a soln exist?
nec (+ suff - won't prove) condn for a solution is that $\frac{\partial t'}{\partial r \partial t} = \frac{\partial t'}{\partial t \partial r}$
ie. $\partial_r(fC) = \partial_t(fD)$ (‡)
this is a first order p.d.e. for unknown function $f(t,r)$ (arbitrary thus far)
given $f(0,r)$, ‡ gives you $\partial_t f(0,r)$, $\partial_t(‡) \Rightarrow \partial_t^2 f(0,r)$ etc.
$\Rightarrow f(t,r) = f(0,r) + t\,\partial_t f(t,r)\big|_{t=0} + \frac{t^2}{2}\partial_t^2 f(0,r)$ etc.

$$R_{\theta\theta} = 1 - \frac{1}{B} + \frac{rB'}{2B^2} - \frac{rA'}{2AB}$$

$$R_{\varphi\varphi} = \sin^2\theta \, R_{\theta\theta}$$

$\Rightarrow$ drop prime, replace arb funct^{n}s by new arb^{y} fnc

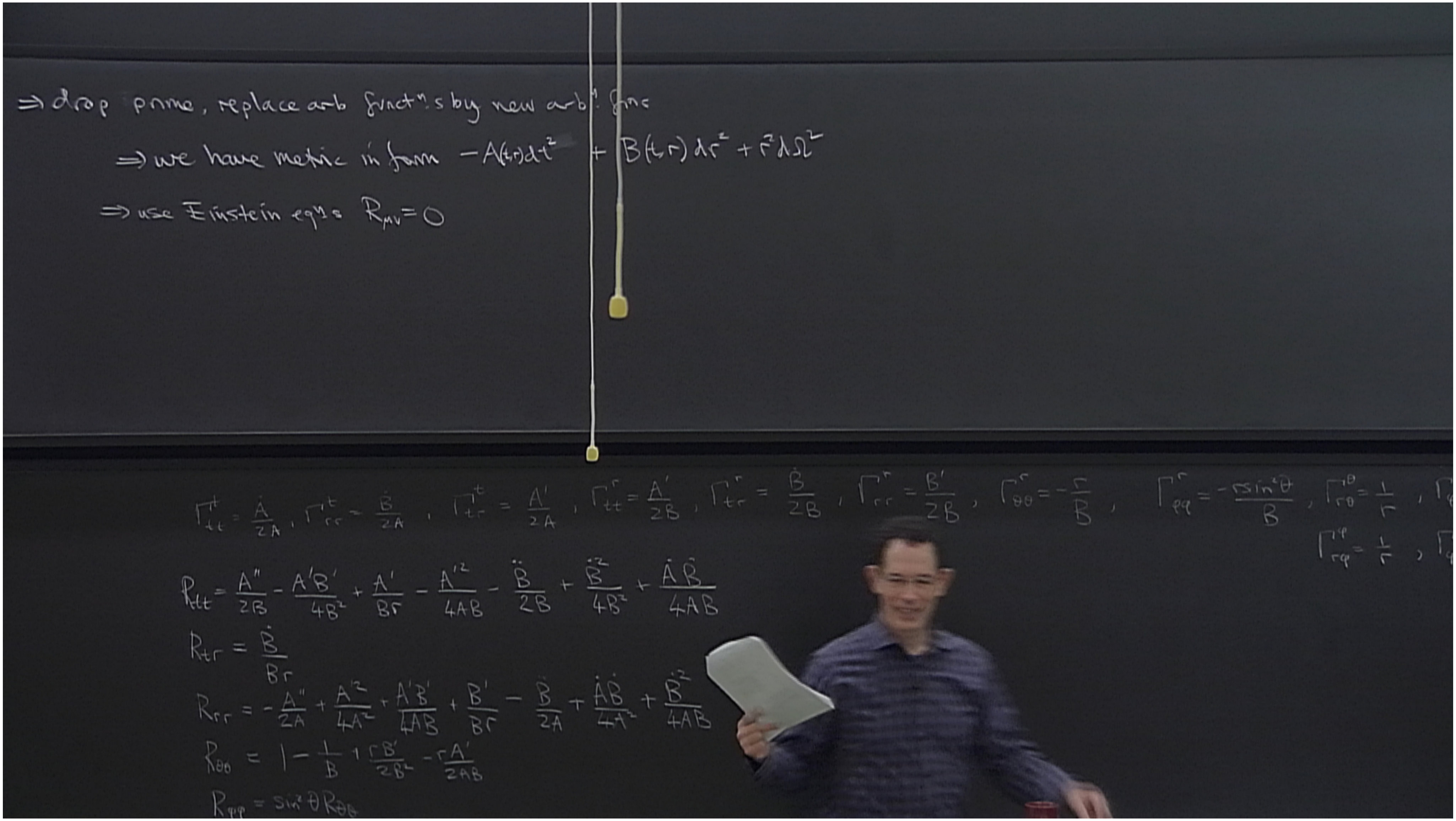
$\Rightarrow$ we have metric in form $-A(t,r)dt'^2 + B(t,r)dr^2 + r^2 d\Omega^2$

$$R_{\theta\theta} = 1 - \frac{1}{B} + \frac{rB'}{2B^2} - \frac{rA'}{2AB}$$

$$R_{\varphi\varphi} = \sin^2\theta \, R_{\theta\theta}$$

$\Rightarrow$ drop prime, replace arb functn's by new arb^{y} fnc

$\Rightarrow$ we have metric in form $-A(t,r)dt^2 + B(t,r)dr^2 + r^2 d\Omega^2$


⇒ drop prime, replace arb functⁿs by new arbⁿ func
⇒ we have metric in form $-A(t,r)dt^2 + B(t,r)dr^2 + r^2 d\Omega^2$
⇒ use Einstein eqⁿs $R_{\mu\nu} = 0$
$\Gamma^t_{tt} = \frac{\dot{A}}{2A}$, $\Gamma^t_{rr} = \frac{\dot{B}}{2A}$, $\Gamma^t_{tr} = \frac{A'}{2A}$, $\Gamma^r_{tt} = \frac{A'}{2B}$, $\Gamma^r_{tr} = \frac{\dot{B}}{2B}$, $\Gamma^r_{rr} = \frac{B'}{2B}$, $\Gamma^r_{\theta\theta} = -\frac{r}{B}$, $\Gamma^r_{\varphi\varphi} = -\frac{r\sin^2\theta}{B}$, $\Gamma^\theta_{r\theta} = \frac{1}{r}$,
$\Gamma^\varphi_{r\varphi} = \frac{1}{r}$,
$R_{tt} = \frac{A''}{2B} - \frac{A'B'}{4B^2} + \frac{A'}{Br} - \frac{A'^2}{4AB} - \frac{\ddot{B}}{2B} + \frac{\dot{B}^2}{4B^2} + \frac{\dot{A}\dot{B}}{4AB}$
$R_{tr} = \frac{\dot{B}}{Br}$
$R_{rr} = -\frac{A''}{2A} + \frac{A'^2}{4A^2} + \frac{A'B'}{4AB} + \frac{B'}{Br} - \frac{\ddot{B}}{2A} + \frac{\dot{A}\dot{B}}{4A^2} + \frac{\dot{B}^2}{4AB}$
$R_{\theta\theta} = 1 - \frac{1}{B} + \frac{rB'}{2B^2} - \frac{rA'}{2AB}$
$R_{\varphi\varphi} = \sin^2\theta R_{\theta\theta}$

$$\Gamma^t_{tt} = \frac{\dot{A}}{2A}, \quad \Gamma^t_{rr} = \frac{\dot{B}}{2A}, \quad \Gamma^t_{tr} = \frac{A'}{2A}, \quad \Gamma^r_{tt} = \frac{A'}{2B}, \quad \Gamma^r_{tr} = \frac{\dot{B}}{2B}, \quad \Gamma^r_{rr} = \frac{B'}{2B}, \quad \Gamma^r_{\theta\theta} = -\frac{r}{B}, \quad \Gamma^r_{\varphi\varphi} = -\frac{r\sin^2\theta}{B}, \quad \Gamma^\theta_{r\theta} = \frac{1}{r}, \quad \Gamma^\theta_{\varphi\varphi} = -\sin\theta\cos\theta,$$

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$$R_{tt} = \frac{A''}{2B} - \frac{A'B'}{4B^2} + \frac{A'}{Br} - \frac{A'^2}{4AB} - \frac{\ddot{B}}{2B} + \frac{\dot{B}^2}{4B^2} + \frac{\dot{A}\dot{B}}{4AB}$$

$$R_{tr} = \frac{\dot{B}}{Br}$$

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drop prime, replace arb functⁿs by new arbⁿ func

$\Rightarrow$ we have metric in form $-A(t,r)\,dt^2 + B(t,r)\,dr^2 + r^2 d\Omega^2$

$\Rightarrow$ use Einstein eqⁿs $R_{\mu\nu} = 0$

$$\Gamma^t_{tt} = \frac{\dot{A}}{2A}, \quad \Gamma^t_{rr} = \frac{\dot{B}}{2A}, \quad \Gamma^t_{tr} = \frac{A'}{2A}, \quad \Gamma^r_{tt} = \frac{A'}{2B}, \quad \Gamma^r_{tr} = \frac{\dot{B}}{2B}, \quad \Gamma^r_{rr} = \frac{B'}{2B}, \quad \Gamma^r_{\theta\theta} = -\frac{r}{B}, \quad \Gamma^r_{\varphi\varphi} = -\frac{r\sin^2\theta}{B}, \quad \Gamma^\theta_{r\theta} = \frac{1}{r}, \quad \Gamma^\theta_{\varphi\varphi} = -\sin\theta\cos\theta,$$

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$$\dot{} \equiv \frac{\partial}{\partial t}, \qquad {}' \equiv \frac{\partial}{\partial r}$$

$$\Gamma^t_{tt} = \frac{\dot{A}}{2A}, \quad \Gamma^t_{rr} = \frac{\dot{B}}{2A}, \quad \Gamma^t_{tr} = \frac{A'}{2A}, \quad \Gamma^r_{tt} = \frac{A'}{2B}, \quad \Gamma^r_{tr} = \frac{\dot{B}}{2B}, \quad \Gamma^r_{rr} =$$

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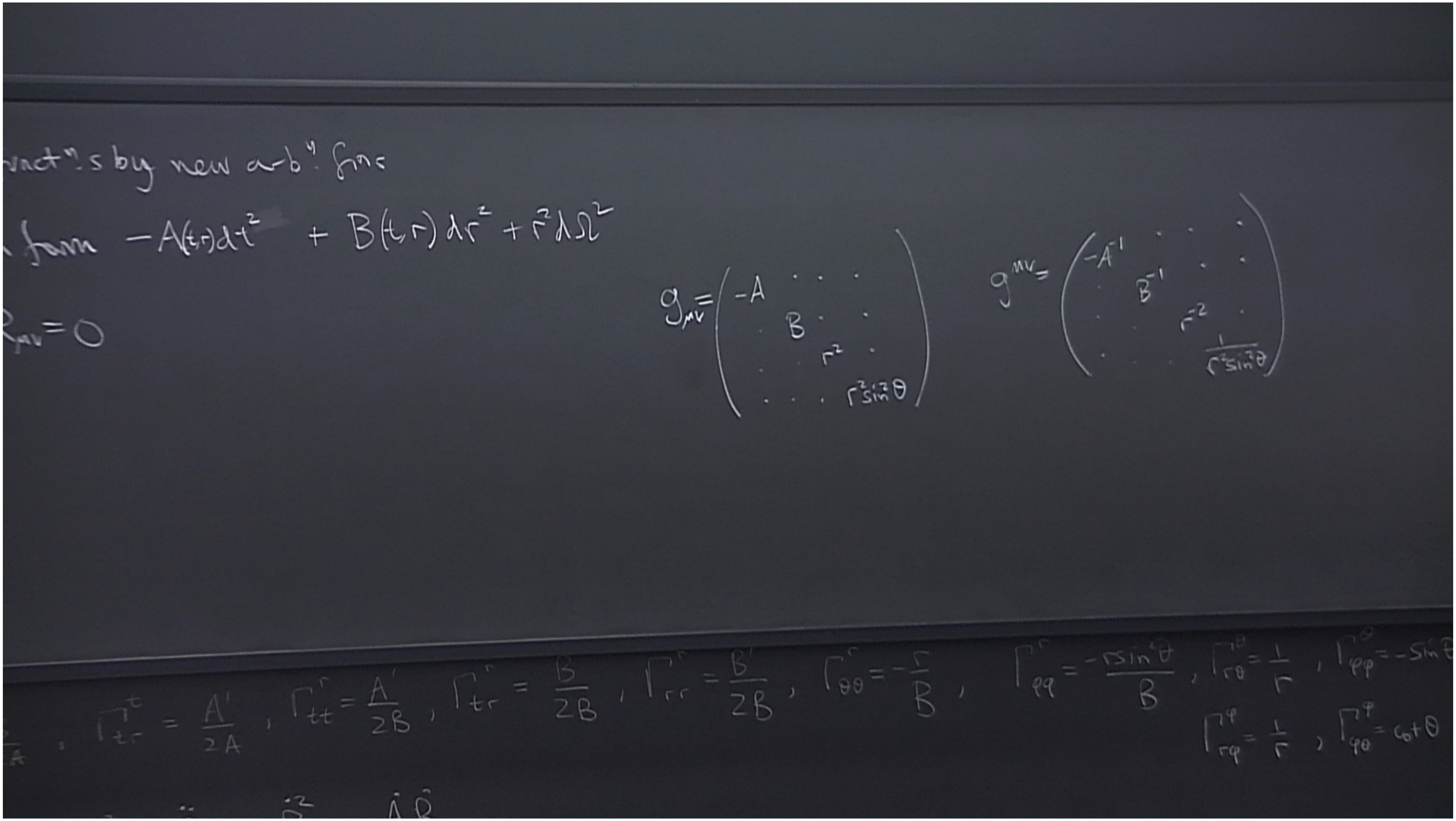
ract^{n}s by new arb^y f^{n}s
form $-A(t,r)dt^2 + B(t,r)dr^2 + r^2 d\Omega^2$
$R_{\mu\nu} = 0$
$g_{\mu\nu} = \begin{pmatrix} -A & \cdot & \cdot & \cdot \\ \cdot & B & \cdot & \cdot \\ \cdot & \cdot & r^2 & \cdot \\ \cdot & \cdot & \cdot & r^2\sin^2\theta \end{pmatrix}$
$g^{\mu\nu} = \begin{pmatrix} -A^{-1} & \cdot & \cdot & \cdot \\ \cdot & B^{-1} & \cdot & \cdot \\ \cdot & \cdot & r^{-2} & \cdot \\ \cdot & \cdot & \cdot & \frac{1}{r^2\sin^2\theta} \end{pmatrix}$
$\Gamma^t_{tr} = \frac{A'}{2A}$, $\Gamma^r_{tt} = \frac{A'}{2B}$, $\Gamma^r_{tr} = \frac{\dot{B}}{2B}$, $\Gamma^r_{rr} = \frac{B'}{2B}$, $\Gamma^r_{\theta\theta} = -\frac{r}{B}$, $\Gamma^r_{\varphi\varphi} = -\frac{r\sin^2\theta}{B}$, $\Gamma^\theta_{r\theta} = \frac{1}{r}$, $\Gamma^\theta_{\varphi\varphi} = -\sin\theta$
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Only nonzero Γ's and $R_{\mu\nu}$'s are

$$\Gamma^t_{tt} = \frac{\dot{A}}{2A}, \quad \Gamma^t_{rr} = \frac{\dot{B}}{2A}, \quad \Gamma^t_{tr} = \frac{A'}{2A}, \quad \Gamma^r_{tt} = \frac{A'}{2B}, \quad \Gamma^r_{tr} = \frac{\dot{B}}{2B}, \quad \Gamma^r_{rr} = \frac{B'}{2B}, \quad \Gamma^r_{\theta\theta} = -\frac{r}{B}$$

$$\dot{} \equiv \frac{\partial}{\partial t}, \qquad {}' \equiv \frac{\partial}{\partial r}$$

$$R_{tt} = \frac{A''}{2B} - \frac{A'B'}{4B^2} + \frac{A'}{Br} - \frac{A'^2}{4AB} - \frac{\ddot{B}}{2B} + \frac{\dot{B}^2}{4B^2} + \frac{\dot{A}\dot{B}}{4AB}$$

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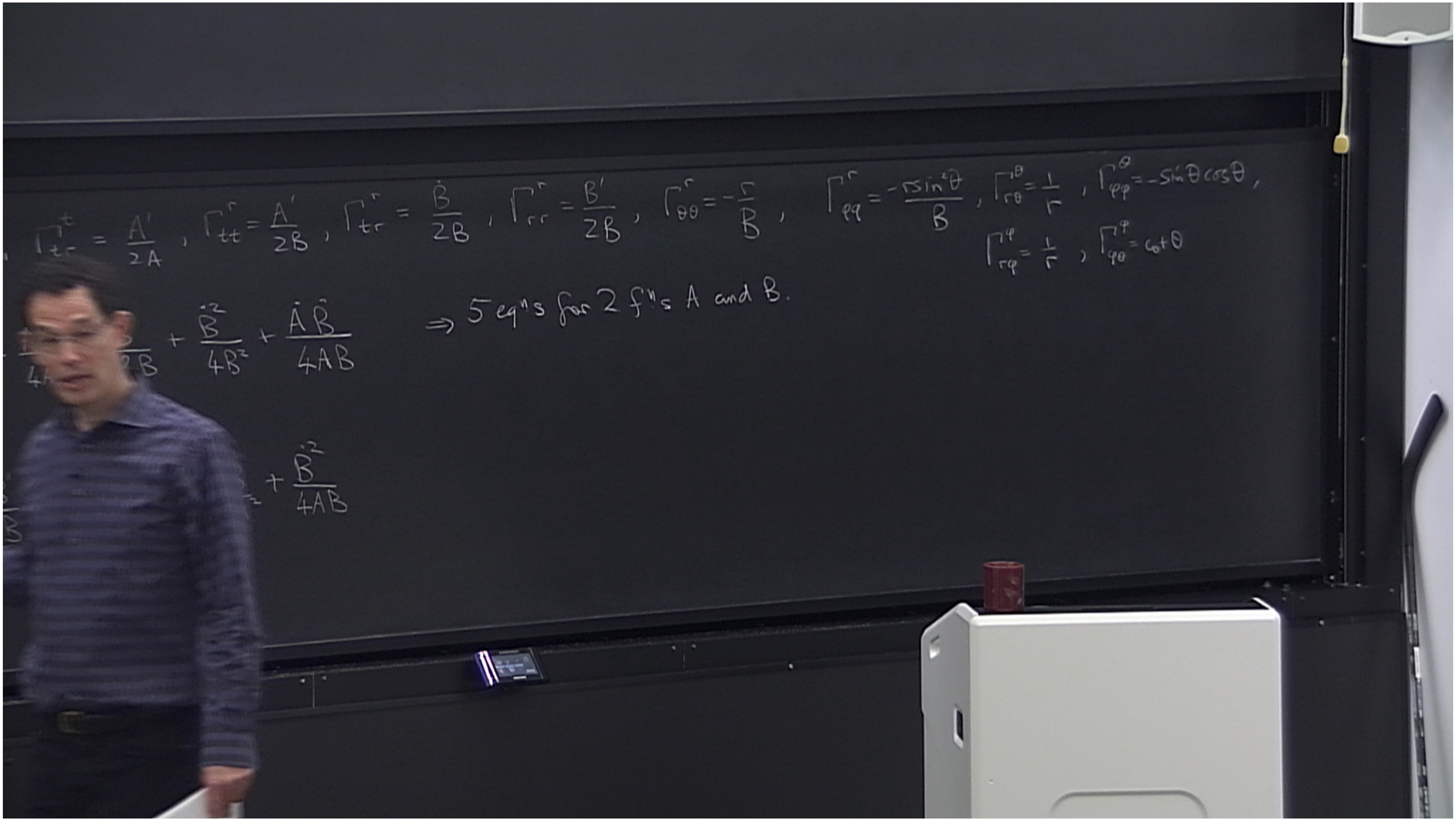
$$R_{\theta\theta} = 1 - \frac{1}{B} + \frac{rB'}{2B^2} - \frac{rA'}{2AB}$$

$$R_{\phi\phi} = \sin^2\theta \, R_{\theta\theta}$$


⇒ 5 eqⁿs for 2 fⁿs A and B.

$\frac{A'}{2A}$, $\Gamma^r_{tt} = \frac{A'}{2B}$, $\Gamma^r_{tr} = \frac{\dot{B}}{2B}$, $\Gamma^r_{rr} = \frac{B'}{2B}$, $\Gamma^r_{\theta\theta} = -\frac{r}{B}$, $\Gamma^r_{\varphi\varphi} = -\frac{r\sin^2\theta}{B}$, $\Gamma^\theta_{r\theta} = \frac{1}{r}$, $\Gamma^\theta_{\varphi\varphi} = -\sin\theta\cos\theta$,

$\Gamma^\varphi_{r\varphi} = \frac{1}{r}$, $\Gamma^\varphi_{\varphi\theta} = \cot\theta$

$$\frac{\ddot{B}}{2B} + \frac{\dot{B}^2}{4B^2} + \frac{\dot{A}\dot{B}}{4AB}$$

$\Rightarrow$ 5 eqⁿs for 2 fⁿs A and B.

$\Rightarrow$ Unique solution up to a single constant.

$$-\frac{\ddot{B}}{2A} + \frac{\dot{A}\dot{B}}{4A^2} + \frac{\dot{B}^2}{4AB}$$

Only nonzero Γ's and $R_{\mu\nu}$'s are

$$\Gamma^t_{tt} = \frac{\dot{A}}{2A}, \quad \Gamma^t_{rr} = \frac{\dot{B}}{2A}, \quad \Gamma^t_{tr} = \frac{A'}{2A}, \quad \Gamma^r_{tt} = \frac{A'}{2B}, \quad \Gamma^r_{tr} = \frac{\dot{B}}{2B}, \quad \Gamma^r_{rr} = \ldots$$

$$\left.\begin{aligned} \dot{} &\equiv \frac{\partial}{\partial t} \\ ' &\equiv \frac{\partial}{\partial r} \end{aligned}\right\}$$

$$R_{tt} = \frac{A''}{2B} - \frac{A'B'}{4B^2} + \frac{A'}{Br} - \frac{A'^2}{4AB} - \frac{\ddot{B}}{2B} + \frac{\dot{B}^2}{4B^2} + \frac{\dot{A}\dot{B}}{4AB} \quad (1)$$

$$R_{tr} = \frac{\dot{B}}{Br} \quad (2)$$

$$R_{rr} = -\frac{A''}{2A} + \frac{A'^2}{4A^2} + \frac{A'B'}{4AB} + \frac{B'}{Br} - \frac{\ddot{B}}{2A} + \frac{\dot{A}\dot{B}}{4A^2} + \frac{\dot{B}^2}{4AB} \quad (3)$$

$$R_{\theta\theta} = 1 - \frac{1}{B} + \frac{rB'}{2B^2} - \frac{rA'}{2AB} \quad (4)$$

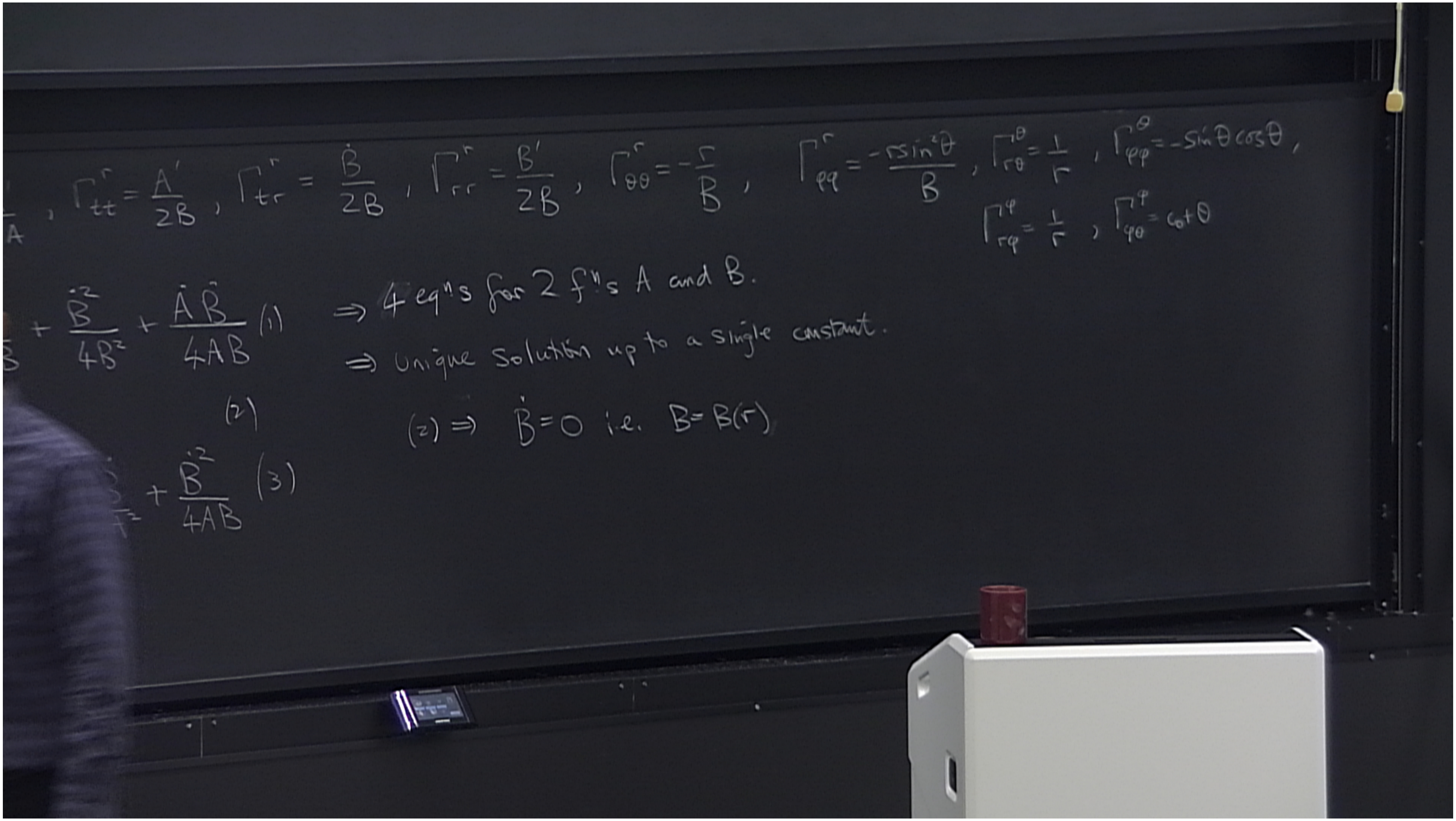
$$R_{\varphi\varphi} = \sin^2\theta \, R_{\theta\theta}$$

$$\frac{\partial}{\partial t} B(t,r) = 0$$

$\Rightarrow$ 4 eq^{n}s for

$\Rightarrow$ Unique So

(2) $\Rightarrow$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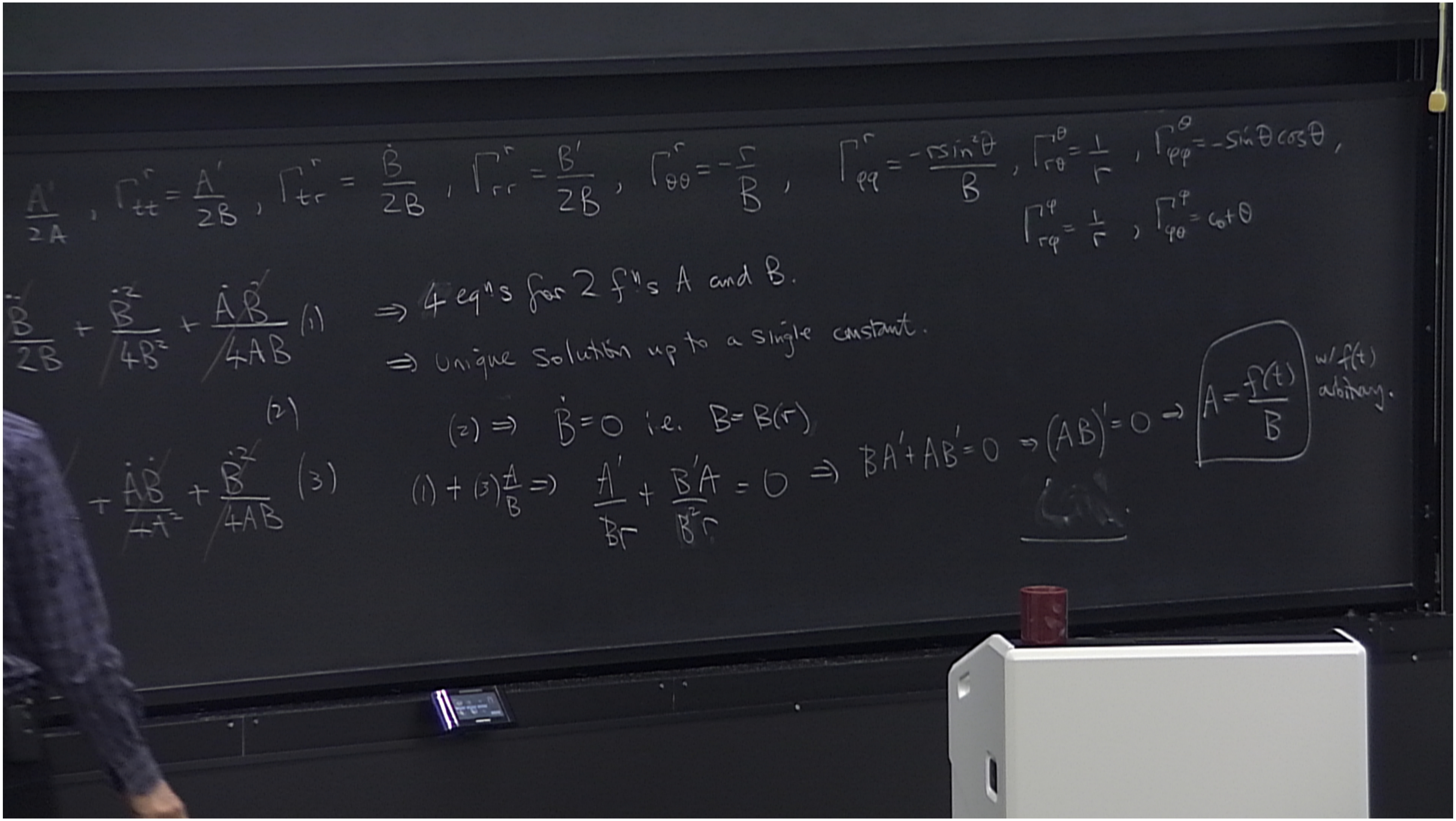
$\Gamma^\varphi_{r\varphi} = \frac{1}{r}$, $\Gamma^\varphi_{\varphi\theta} = \cot\theta$
$+ \frac{\dot{B}^2}{4B^2} + \frac{\dot{A}\dot{B}}{4AB}$ (1)
$\Rightarrow$ 4 eq^{n}s for 2 f^{n}s A and B.
$\Rightarrow$ Unique solution up to a single constant.
(2)
(2) $\Rightarrow$ $\dot{B} = 0$ i.e. $B = B(r)$
$+ \frac{\dot{B}^2}{4AB}$ (3)


$\frac{A'}{2A}$, $\Gamma^r_{tt} = \frac{A'}{2B}$, $\Gamma^r_{tr} = \frac{\dot B}{2B}$, $\Gamma^r_{rr} = \frac{B'}{2B}$, $\Gamma^r_{\theta\theta} = -\frac{r}{B}$, $\Gamma^r_{\varphi\varphi} = -\frac{r\sin^2\theta}{B}$, $\Gamma^\theta_{r\theta} = \frac{1}{r}$, $\Gamma^\theta_{\varphi\varphi} = -\sin\theta\cos\theta$,
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$\frac{\ddot B}{2B} + \cancel{\frac{\dot B^2}{4B^2}} + \cancel{\frac{\dot A \dot B}{4AB}}$ (1)
$\Rightarrow$ 4 eqⁿs for 2 fⁿs A and B.
$\Rightarrow$ Unique solution up to a single constant.
(2)
(2) $\Rightarrow$ $\dot B = 0$ i.e. $B = B(r)$
$+ \cancel{\frac{\dot A \dot B}{4A^2}} + \cancel{\frac{\dot B^2}{4AB}}$ (3)
$(1) + (3)\frac{A}{B} \Rightarrow \frac{A'}{Br} + \frac{B'A}{B^2 r} = 0 \Rightarrow BA' + AB' = 0 \Rightarrow (AB)' = 0 \rightarrow \boxed{A = \frac{f(t)}{B}}$ w/ f(t) arbitrary.

$\Gamma^{\varphi}_{r\varphi} = \frac{1}{r}$, $\Gamma^{\varphi}_{\varphi\theta} = \cot\theta$

$$\ldots + \frac{A'}{Br} - \frac{A'^2}{4AB} - \frac{\ddot{B}}{2B} + \frac{\dot{B}^2}{4B^2} + \frac{\dot{A}\dot{B}}{4AB} \quad (1)$$

$\Rightarrow$ 4 eqⁿs for 2 fⁿs A and B.

$\Rightarrow$ Unique solution up to a single constant.

(2)

$(2) \Rightarrow \dot{B} = 0$ i.e. $B = B(r)$

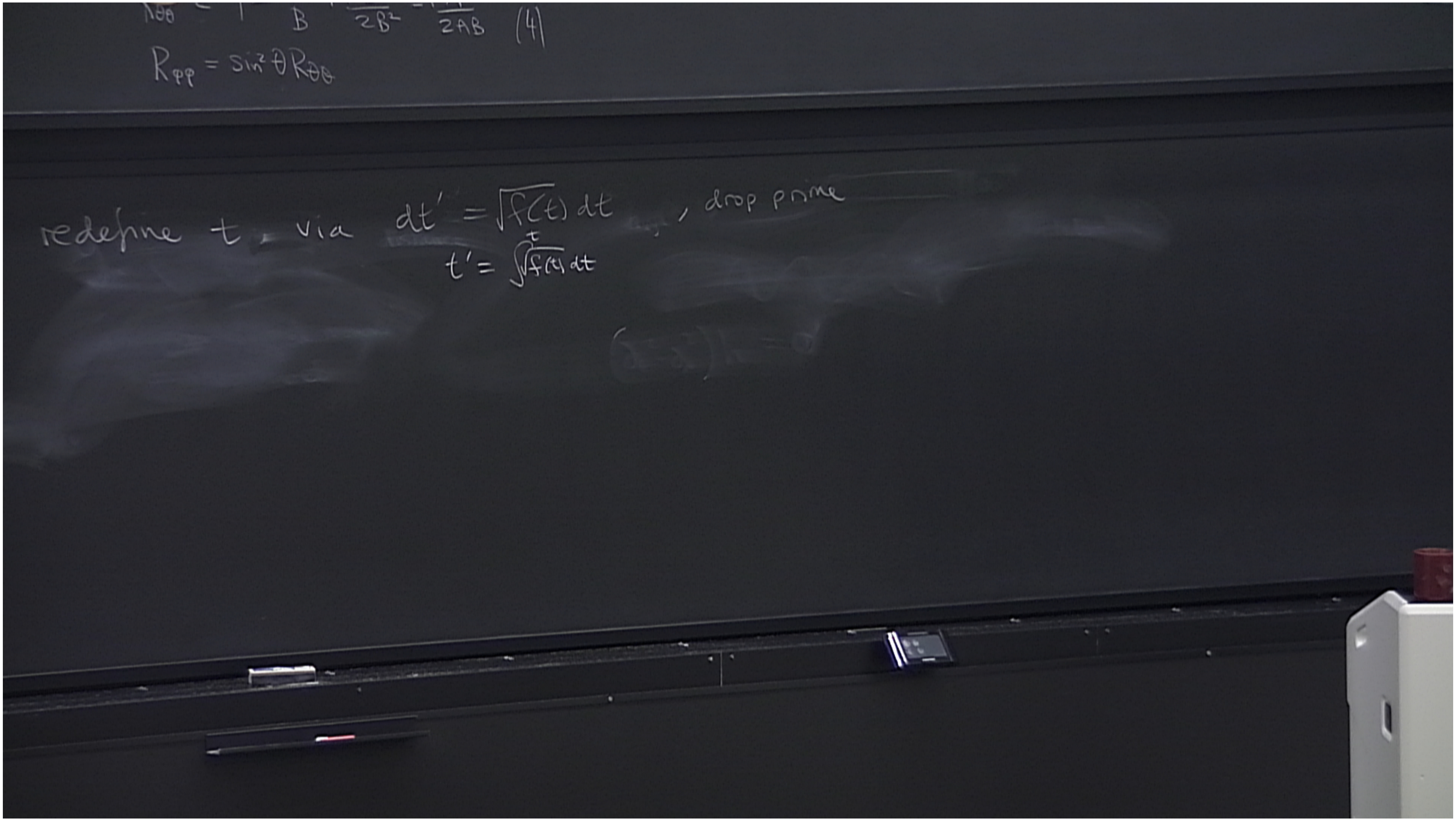
$$\frac{A'^2}{4A^2} + \frac{A'B'}{4AB} + \frac{B'}{Br} - \frac{\ddot{B}}{2A} + \frac{\dot{A}\dot{B}}{4A^2} + \frac{\dot{B}^2}{4AB} \quad (3)$$

$$(1) + (3)\frac{A}{B} \Rightarrow \frac{A'}{Br} + \frac{B'A}{B^2 r} = 0 \Rightarrow BA' + AB' = 0 \Rightarrow (AB)' = 0 \rightarrow \boxed{A = \frac{f(t)}{B}}$$

w/ $f(t)$ arbitrary.

$$\ldots + \frac{rB'}{2B^2} - \frac{rA'}{2AB} \quad (4)$$

...ia $dt'^2 = f(t)\,dt^2$, drop prime


B 2B² 2AB (4)
Rφφ = sin²θ Rθθ
redefine t via dt' = √f(t) dt , drop prime
t' = ∫^t √f(t) dt

$$R_{\theta\theta} = \ldots \frac{\ldots}{B} \ldots \frac{\ldots}{2B^2} - \frac{\ldots}{2AB} \quad (4)$$

$$R_{\varphi\varphi} = \sin^2\theta \, R_{\theta\theta}$$

redefine t via $dt' = \sqrt{f(t)}\, dt$, drop prime

$$t' = \int^{t} \sqrt{f(t)}\, dt$$

$$ds^2 = -\frac{dt^2}{B} + B\, dr^2 + r^2 d\Omega^2$$

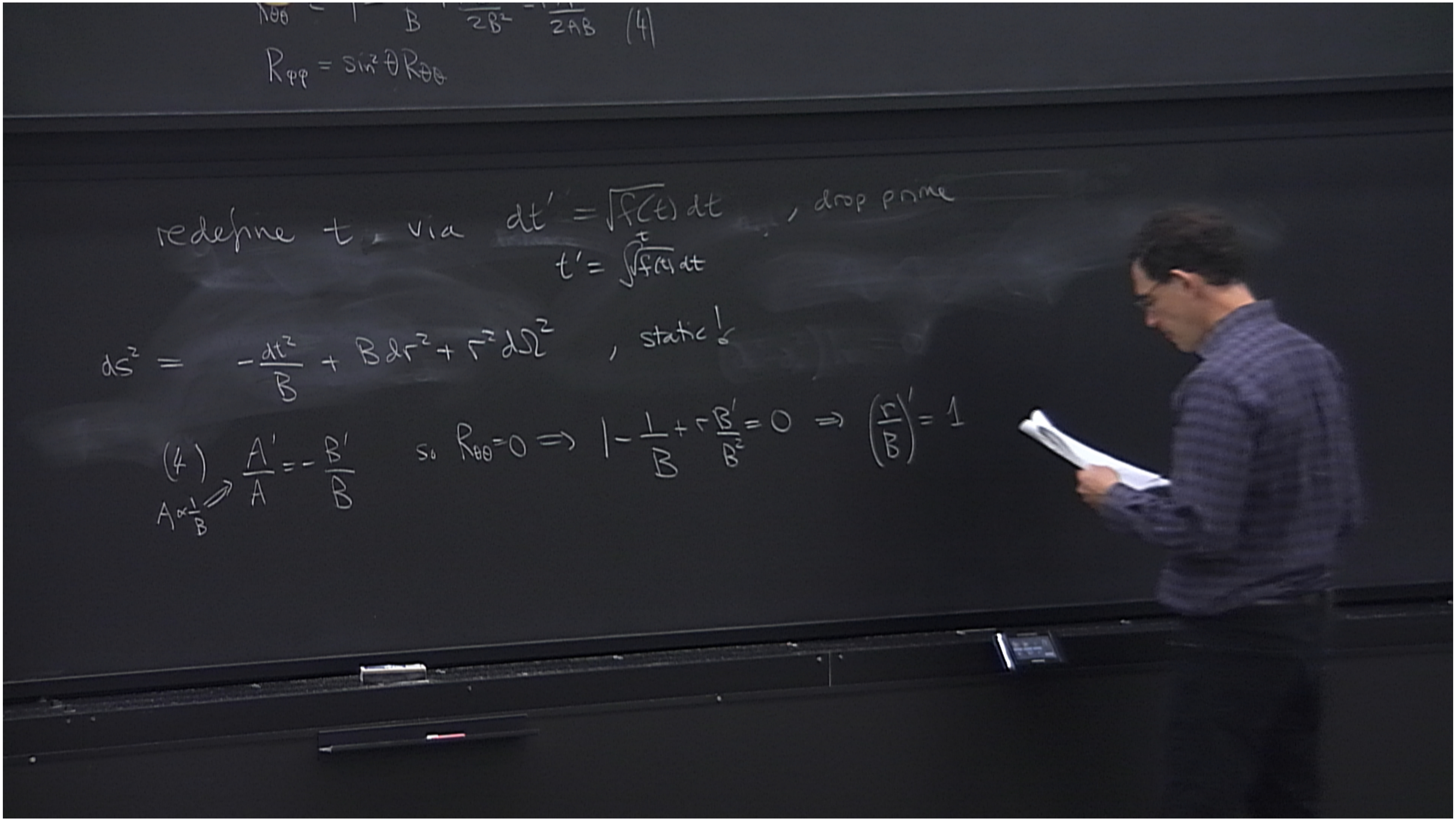
$R_{\theta\theta} = \ldots \frac{}{B} \ldots \frac{}{2B^2} - \frac{}{2AB}$ (4)

$R_{\varphi\varphi} = \sin^2\theta\, R_{\theta\theta}$

redefine t via $dt' = \sqrt{f(t)}\, dt$, drop prime

$$t' = \int^t \sqrt{f(t)}\, dt$$

$$ds^2 = -\frac{dt^2}{B} + B\, dr^2 + r^2 d\Omega^2 \quad , \text{ static!}$$


R_θθ = 1 − 1/B + rB′/2B² − rA′/2AB (4)
R_φφ = sin²θ R_θθ
redefine t via dt′ = √f(t) dt , drop prime
t′ = ∫√f(t) dt
ds² = −dt²/B + B dr² + r² dΩ² , static!
(4) A′/A = −B′/B
A ∝ 1/B
so R_θθ = 0 ⇒ 1 − 1/B + r B′/B² = 0 ⇒ (r/B)′ = 1

$R_{\theta\theta} = 1 - \ldots \frac{}{B} + \frac{}{2B^2} - \frac{}{2AB}$ (4)

$R_{\varphi\varphi} = \sin^2\theta\, R_{\theta\theta}$

redefine t via $dt' = \sqrt{f(t)}\, dt$, drop prime

$t' = \int^t \sqrt{f(t)}\, dt$

$ds^2 = -\frac{dt^2}{B} + B\, dr^2 + r^2 d\Omega^2$, static!

(4) $\frac{A'}{A} = -\frac{B'}{B}$ $\Rightarrow A \propto \frac{1}{B}$

So $R_{\theta\theta} = 0 \Rightarrow 1 - \frac{1}{B} + r\frac{B'}{B^2} = 0 \Rightarrow \left(\frac{r}{B}\right)' = 1 \Rightarrow \frac{r}{B} = r + \underline{k}$, with k a constant

$\Rightarrow \boxed{B = \frac{1}{1 + k/r}\ ,\ A = 1 + k/r}$

Schwarzschild metric.

— drop primes

$$-C(r,t)\,dt^2 + E(r,t)\,dr^2 - 2D(r,t)\,dr\,dt + r^2 d\Omega^2 = -\frac{dt'^2}{Cf^2} + \left(\frac{D^2}{C} + E\right) dr^2 + r^2 d\Omega^2$$

now remove cross term

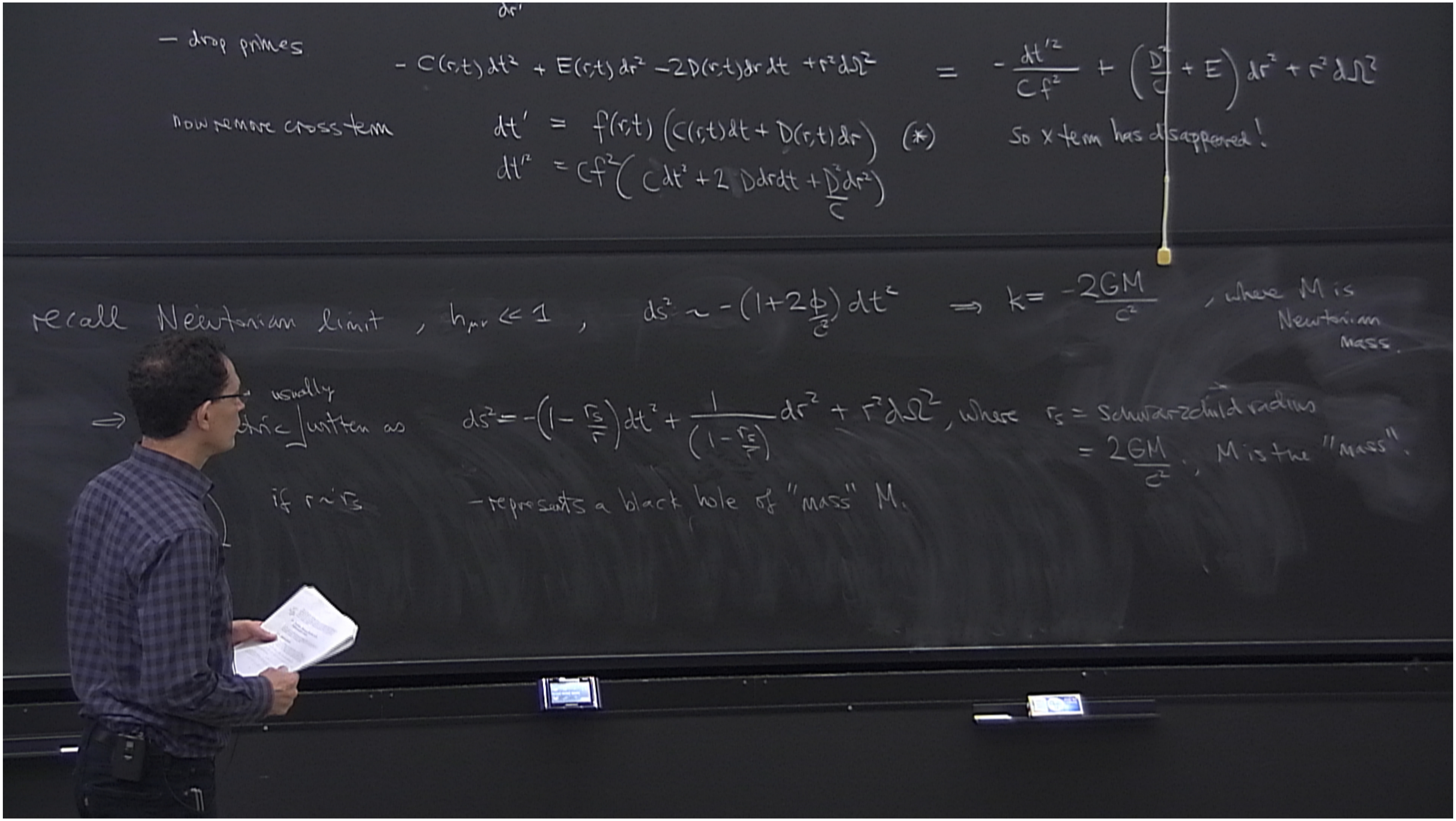
$$dt' = f(r,t)\left(C(r,t)\,dt + D(r,t)\,dr\right) \quad (*)$$

$$dt'^2 = Cf^2\left(C\,dt^2 + 2D\,dr\,dt + \frac{D^2}{C} dr^2\right)$$

So x term has disappeared!

recall Newtonian limit, $h_{\mu\nu} \ll 1$, $ds^2 \sim -\left(1 + \frac{2\phi}{c^2}\right) dt^2 \Rightarrow k = -\frac{2GM}{c^2}$, where M is Newtonian mass.

$\Rightarrow$ Schw metric usually written as $ds^2 = -\left(1 - \frac{r_s}{r}\right) dt^2 + \frac{1}{\left(1 - \frac{r_s}{r}\right)} dr^2 + r^2 d\Omega^2$, where r_s = Schwarzschild radius $= \frac{2GM}{c^2}$, M is the "mass".


— drop primes
$-C(r,t)\,dt^2 + E(r,t)\,dr^2 - 2D(r,t)\,dr\,dt + r^2 d\Omega^2 \;=\; -\frac{dt'^2}{Cf^2} + \left(\frac{D^2}{C} + E\right)dr^2 + r^2 d\Omega^2$
now remove cross term
$dt' = f(r,t)\left(C(r,t)\,dt + D(r,t)\,dr\right)$ (*)
So x term has disappeared!
$dt'^2 = Cf^2\left(C\,dt^2 + 2\,D\,dr\,dt + \frac{D^2}{C}dr^2\right)$
recall Newtonian limit, $h_{\mu\nu} \ll 1$, $ds^2 \sim -(1+2\frac{\phi}{c^2})\,dt^2 \Rightarrow k = -\frac{2GM}{c^2}$, where M is Newtonian mass.
$\Rightarrow$ metric usually written as $ds^2 = -\left(1-\frac{r_s}{r}\right)dt^2 + \frac{1}{\left(1-\frac{r_s}{r}\right)}dr^2 + r^2 d\Omega^2$, where r_s = Schwarzschild radius $= \frac{2GM}{c^2}$, M is the "mass".
if $r \sim r_s$ — represents a black hole of "mass" M.

-wnt prove

i.e. $\partial_r(fC) = \partial_t(fD)$ $(\ddagger)$

this is a first order p.d.e. for unknown function $f(t,r)$
(arbitrary thus far)

given $f(0,r)$, $\ddagger$ gives you $\partial_t f(0,r)$, $\partial_t(\ddagger) \Rightarrow \partial_t^2 f(0,r)$ etc.

$\Rightarrow f(t,r) = f(0,r) + t\,\partial_t f(t,r)\big|_{t=0} + \frac{t^2}{2}\partial_t^2 f(0,r)$ etc.

Event Horizon Telescope
(A. Broderick)

at center of Milky way is a giant black hole of mass $M = 4\times10^6 M_\odot$

ie. $\partial_r(fC) = \partial_t(fD)$ $(\ddagger)$

this is a first order p.d.e. for unknown function $f(t,r)$
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given $f(0,r)$, $\ddagger$ gives you $\partial_t f(0,r)$, $\partial_t(\ddagger) \Rightarrow \partial_t^2 f(0,r)$ etc.

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Event Horizon
Telescope
(A. Broderick)

at center of Milky way is a giant black hole of mass $M = 4\times10^6 M_\odot$

$\Rightarrow r_s \approx 10^{10}$ metres $= 10^7$ km.

i.e. $\partial_r(f\,C) = \partial_t(f\,D)$ (‡)

this is a first order p.d.e. for unknown function $f(t,r)$
(arbitrary - thus far)

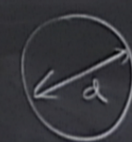
given $f(0,r)$, ‡ gives you $\partial_t f(0,r)$, ∂_t(‡) $\Rightarrow$ $\partial_t^2 f(0,r)$ etc.

$\Rightarrow f(t,r) = f(0,r) + t\,\partial_t f(t,r)\big|_{t=0} + \frac{t^2}{2}\partial_t^2 f(0,r)$ etc.

at center of Milky way is a giant black hole of mass $M = 4\times10^6 M_\odot$

$\Rightarrow r_s \approx 10^{10}$ metres $= 10^7$ km, distance $\sim 2\times10^{20}$ m $\Rightarrow$ $\theta \sim 10^{-10}$ radians

- just at limiting resolution of a telescope which is size of earth



, $\theta_{res} \sim \frac{\lambda}{d}$ — λ: radio waves; $d \sim 10^4$ km

~ just sufficient.

ie. $\partial_r(\ddagger C) = \partial_t(\ddagger D)$ $(\ddagger)$

this is a first order p.d.e. for unknown function $f(t,r)$
(arbitrary - thus far)

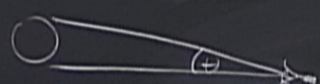
given $f(0,r)$, $\ddagger$ gives you $\partial_t f(0,r)$, $\partial_t(\ddagger) \Rightarrow \partial_t^2 f(0,r)$ etc.

$$\Rightarrow f(t,r) = f(0,r) + t\,\partial_t f(t,r)\big|_{t=0} + \frac{t^2}{2}\partial_t^2 f(0,r) \text{ etc.}$$

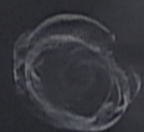
Event Horizon Telescope (A. Broderick)

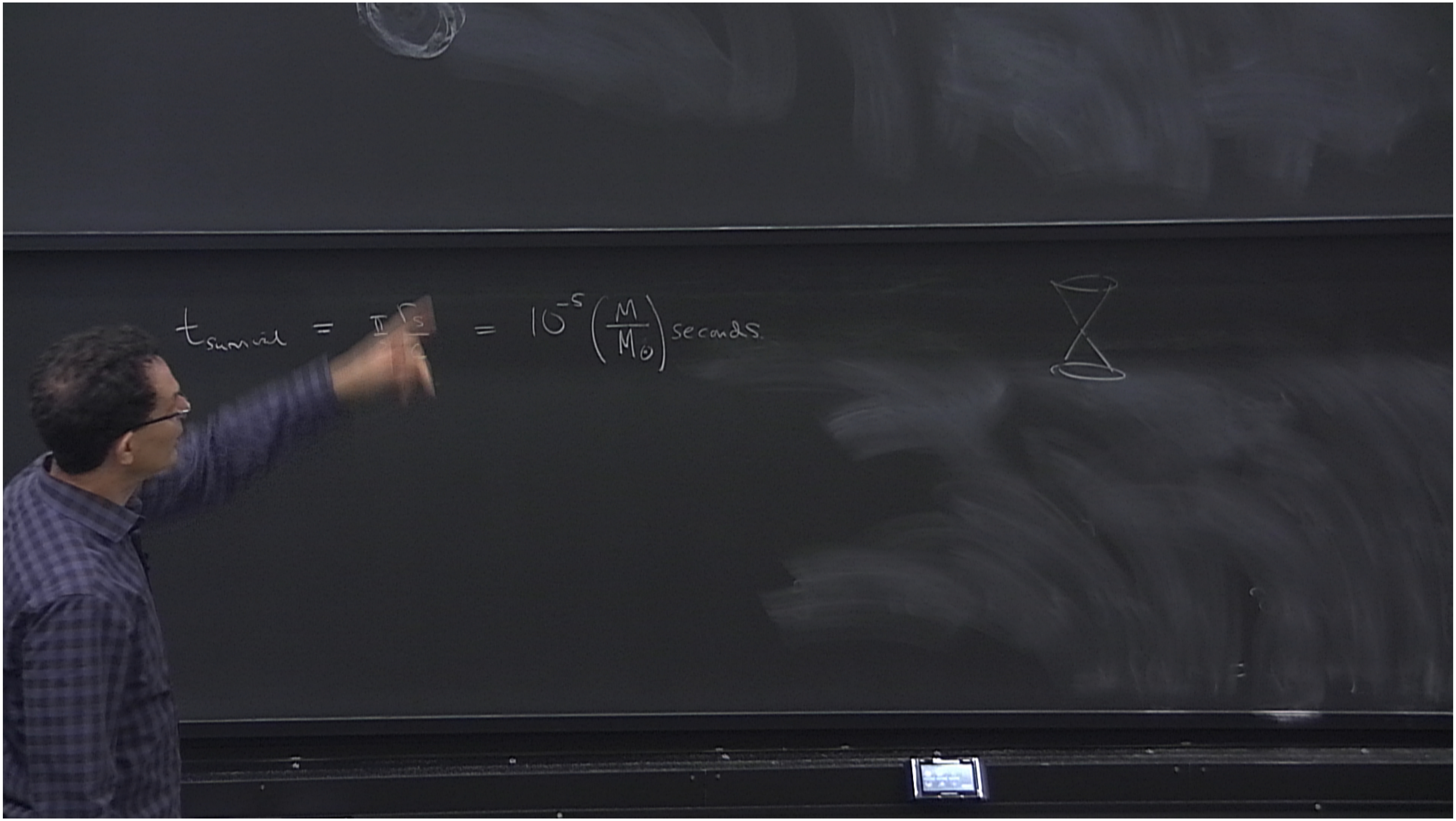
at center of Milky way is a giant black hole of mass $M = 4\times10^{6} M_\odot$

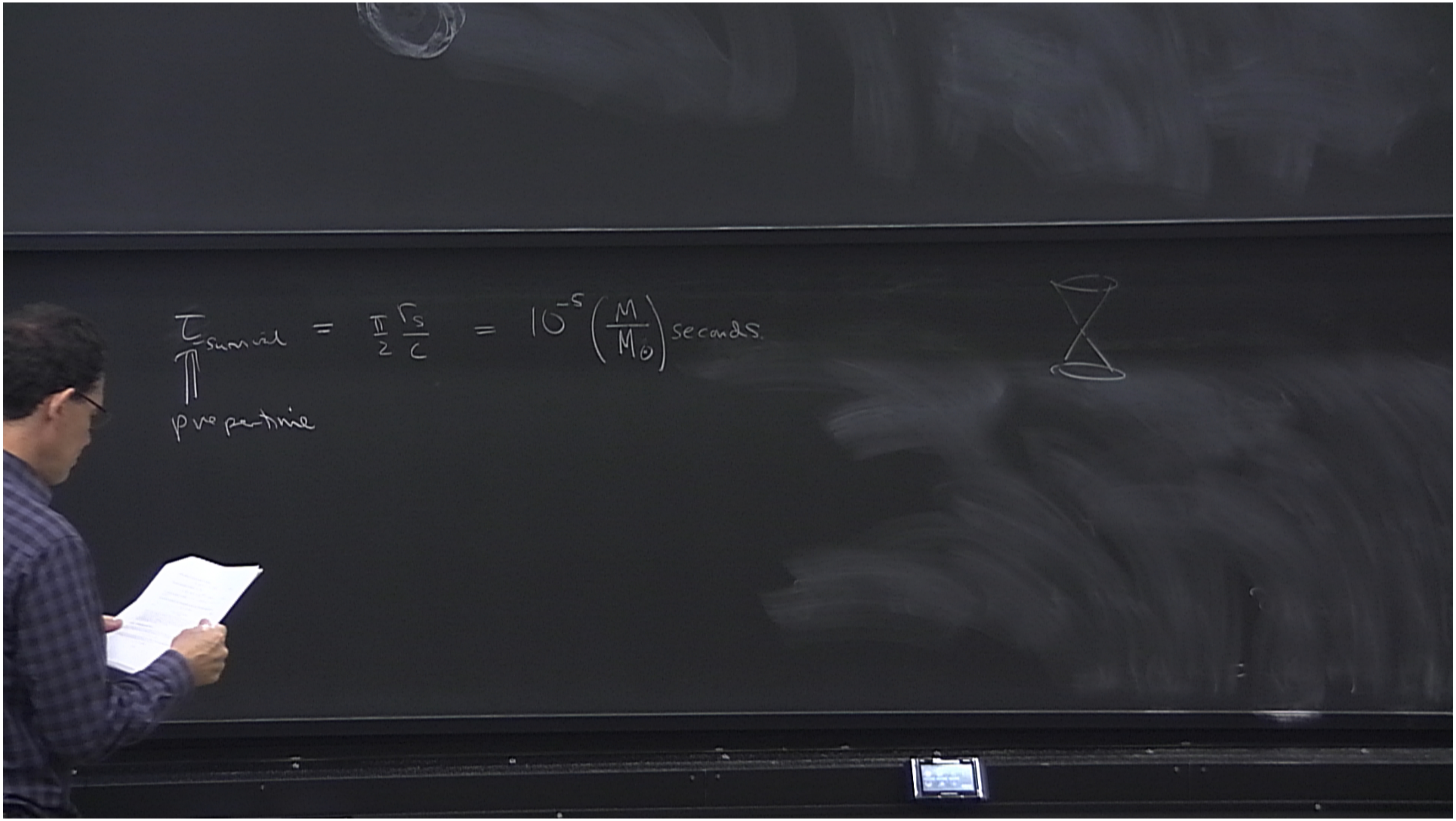
$\Rightarrow r_s \approx 10^{10}$ metres $= 10^{7}$ km. , distance ~ 2×10^{20} m $\Rightarrow \theta \sim 10^{-10}$ radians

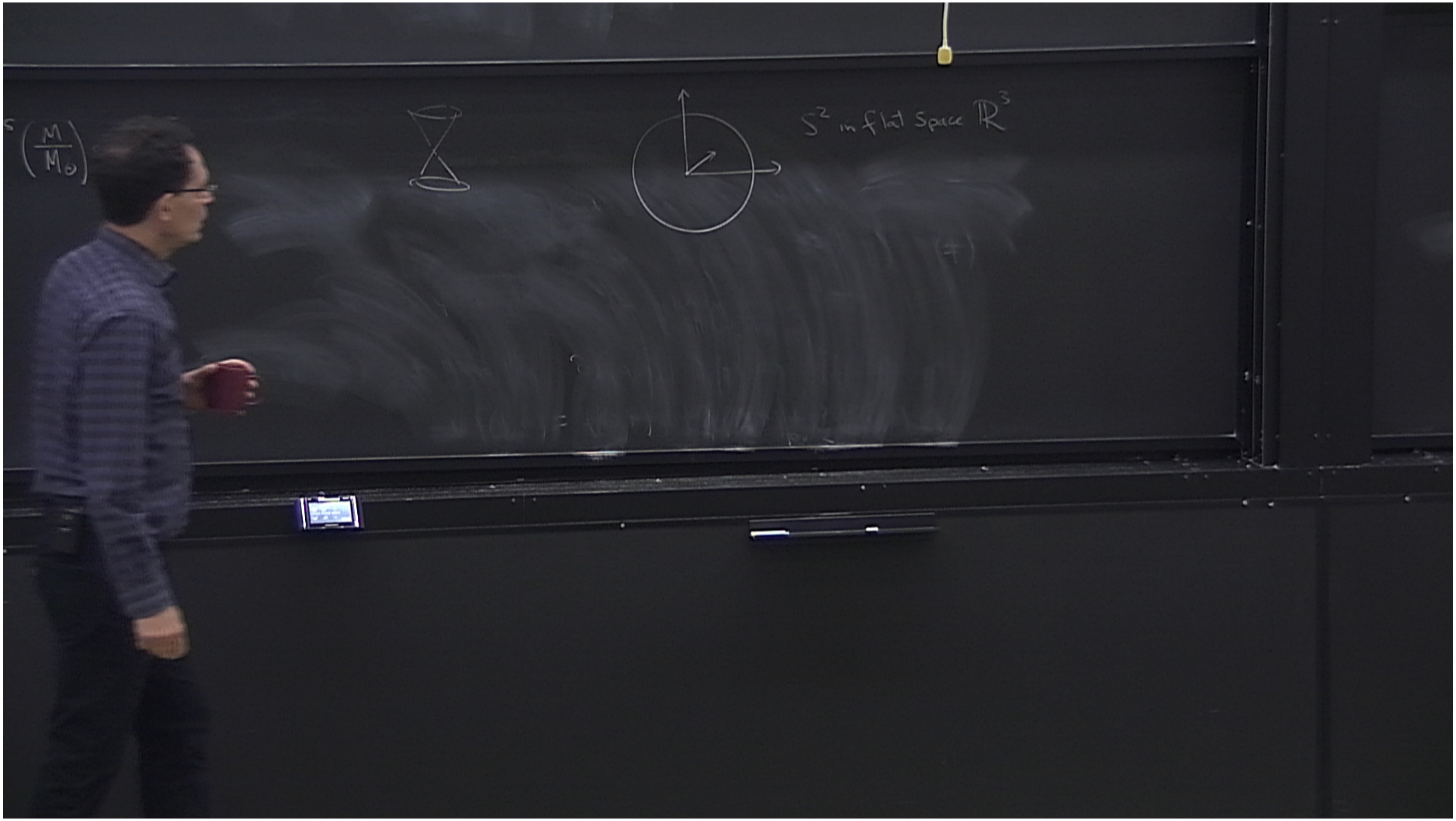


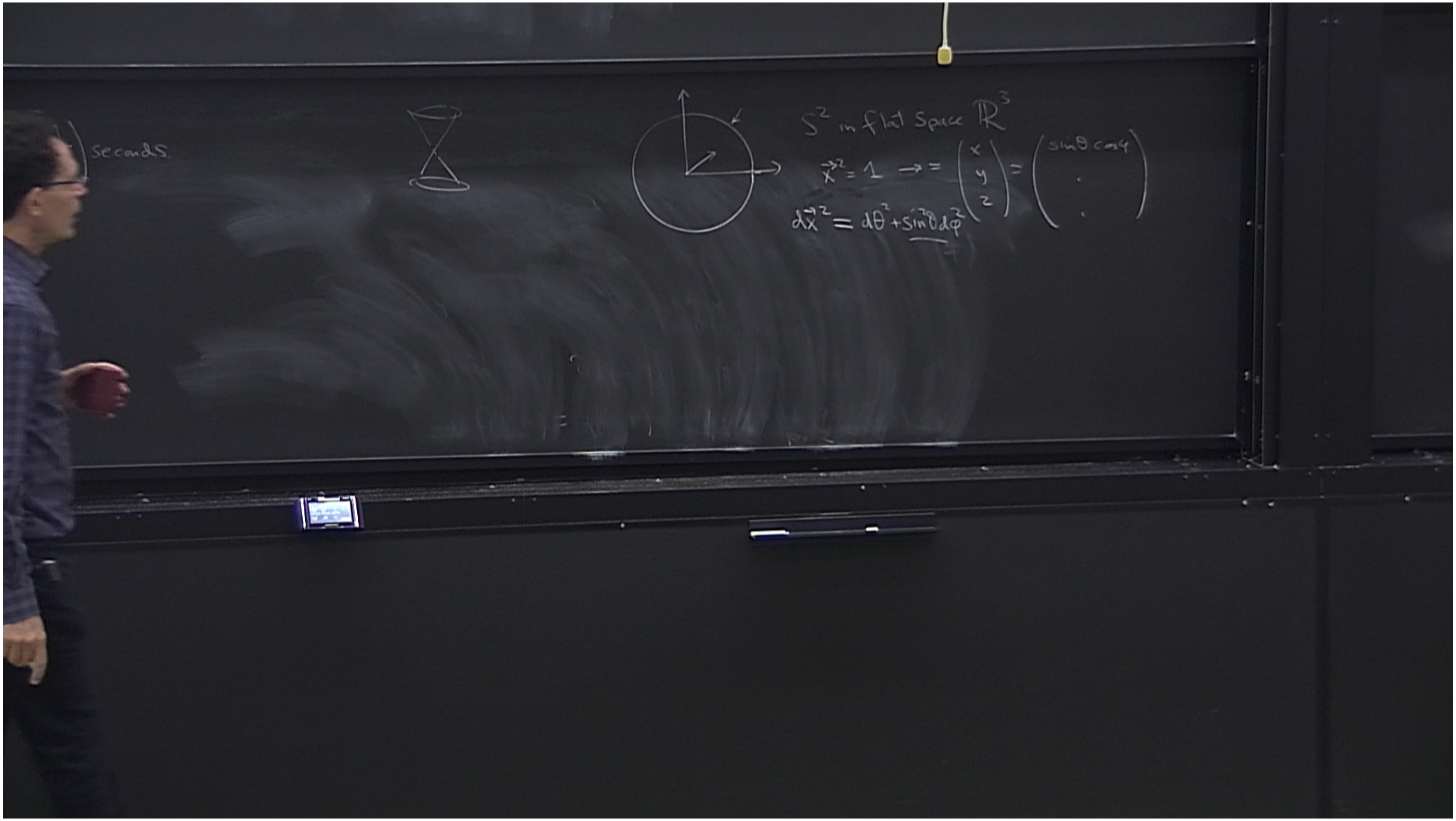
– just at limiting resolution of a telescope which is size of earth

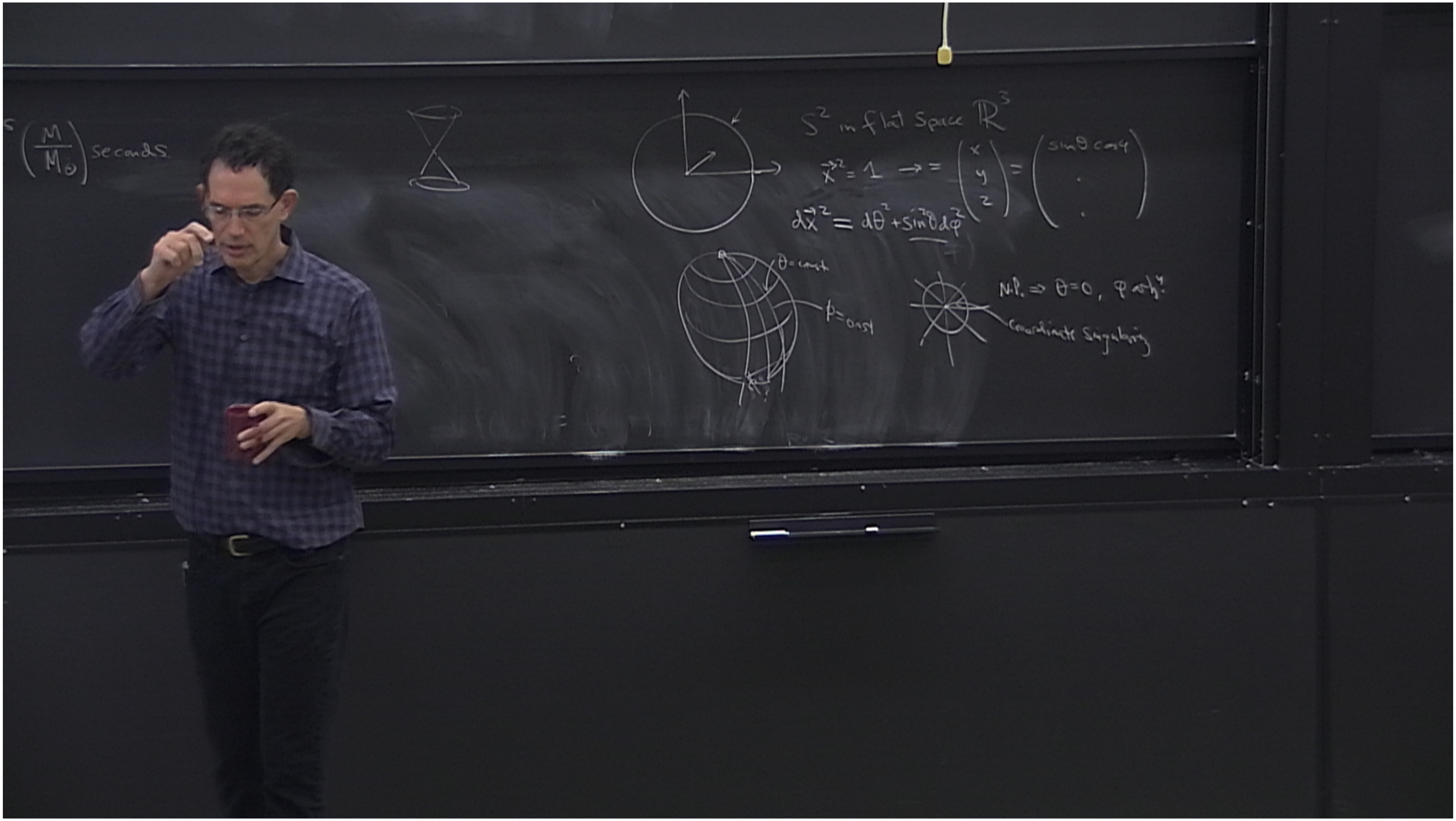



$t_{\text{survival}} =$
$= 10^{-5} \left(\frac{M}{M_{\odot}} \right)$ seconds

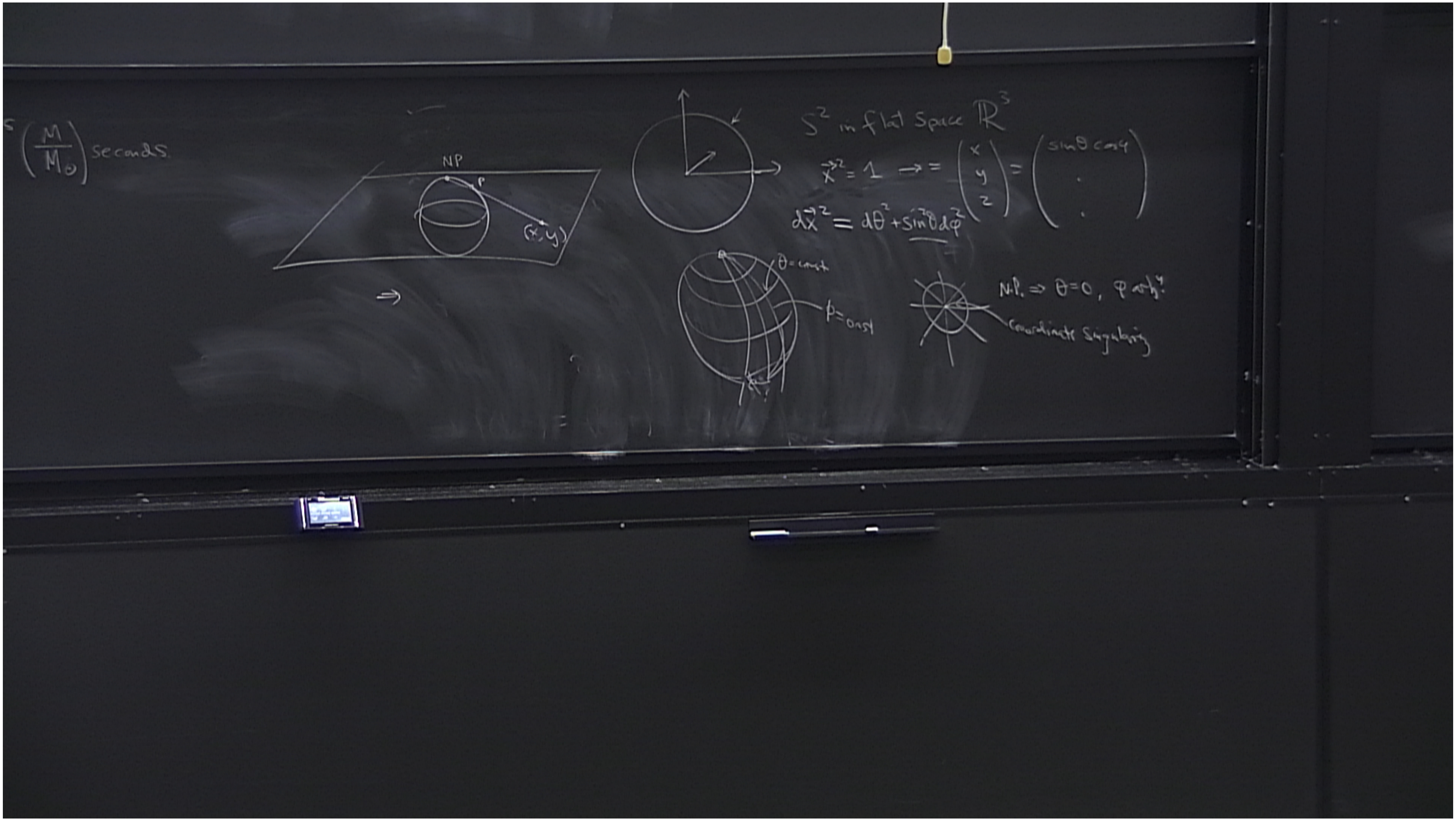

$\tau_{survival} = \frac{\pi}{2}\frac{r_s}{c} = 10^{-5}\left(\frac{M}{M_\odot}\right)$ seconds.
proper-time

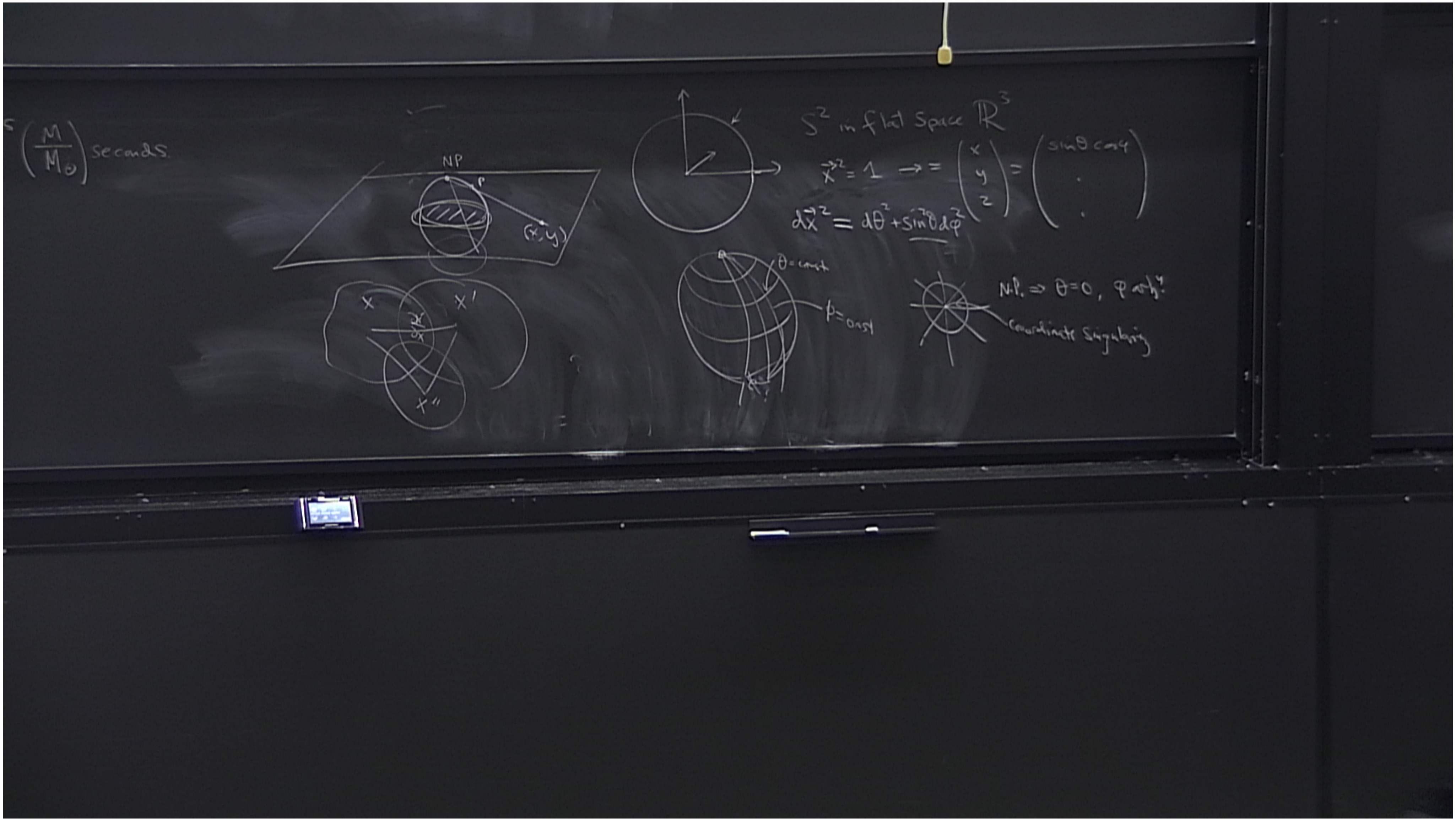

S² in flat space ℝ³


seconds
S² in flat space ℝ³
x⃗² = 1 → = (x y z) = (sinθ cosφ . .)
dx⃗² = dθ² + sin²θ dφ²



$\left(\frac{M}{M_\odot}\right)$ seconds
S^2 in flat space $\mathbb{R}^3$
$\vec{x}^2 = 1 \rightarrow = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\varphi \\ \cdot \\ \cdot \end{pmatrix}$
$d\vec{x}^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$
θ = const
φ = const
N.P. $\Rightarrow \theta = 0$, φ arb.
coordinate singularity


seconds
NP
(x, y)
S² in flat space ℝ³
θ = const
N.P. ⇒ θ = 0
Coordinate Singularity


seconds
NP
(x, y)
S² in flat space ℝ³
θ = const
N.P. ⇒ θ = 0
Coordinate Singularity