

Title: PSI 2016/2017 Relativity - Lecture 11

Date: Sep 20, 2016 09:00 AM

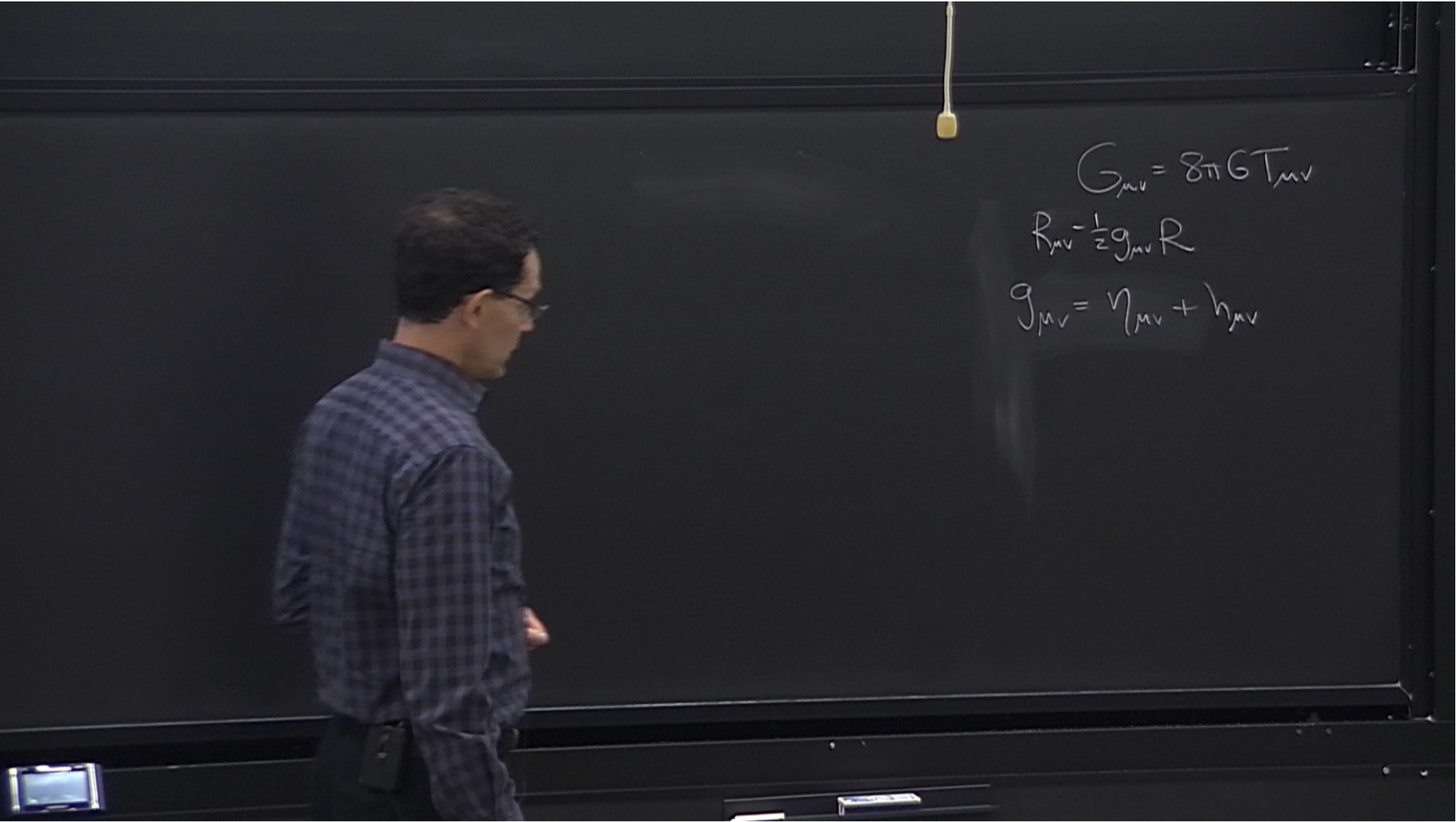
URL: <http://pirsa.org/16090011>

Abstract:

$$\begin{aligned}
 R_{\mu\nu} &= -\frac{1}{2}\Box h_{\mu\nu} + \frac{1}{2}(h_{\mu\sigma}^{{};\sigma} - \frac{1}{2}h_{\sigma\mu}^{{};\sigma})_{;\nu} + \frac{1}{2}(h_{\nu\sigma}^{{};\sigma} - \frac{1}{2}h_{\sigma\nu}^{{};\sigma})_{;\mu} \\
 &= 8\pi G(T_{\mu\nu} + E_{\mu\nu}) + o(h^3)
 \end{aligned}$$

$$\langle E_{\mu\nu} \rangle = \frac{1}{32\pi G} \left\langle \partial_\mu h_{\rho\sigma} \partial_\nu h^{\rho\sigma} - \frac{1}{2} \partial_\mu h \partial_\nu h - \partial_\mu h^{\rho\sigma} (\partial_\rho h_{\nu\sigma} + \partial_\sigma h_{\mu\nu}) \right\rangle$$

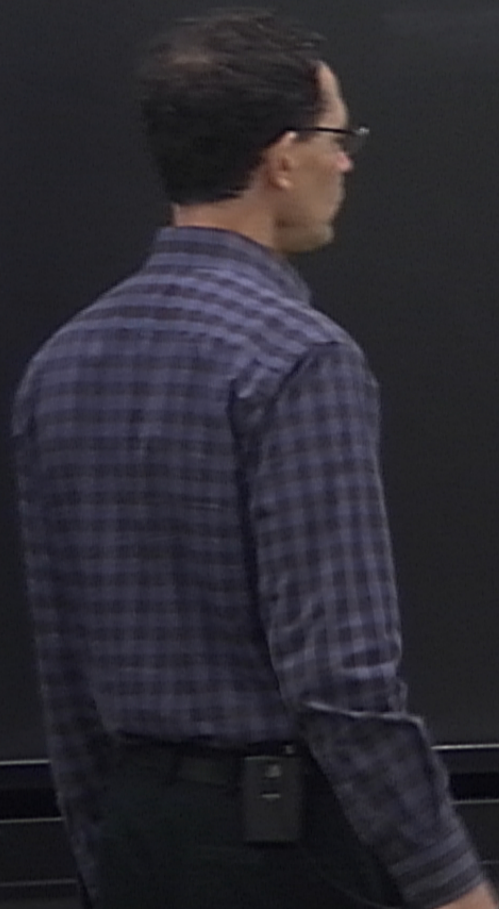
$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$
$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$$



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$$\hat{O}h = 8\pi G T$$

↑
no $h's$

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$\uparrow$
 no $\hbar$'s

$$\begin{aligned}
 G_{\mu\nu} &= 8\pi G T_{\mu\nu} \\
 R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \\
 g_{\mu\nu} &= \eta_{\mu\nu} + h_{\mu\nu} \\
 \hat{O} h &= 8\pi G T \\
 &\quad \uparrow \\
 &\quad \text{no } h's \\
 \rightarrow h_{ij} &\sim \frac{G}{r^{d/2}} \int T^{00} x^i x^j
 \end{aligned}$$

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 g_{\mu\nu} &= \eta_{\mu\nu} + h_{\mu\nu} \\
 \hat{O} h &= 8\pi G T \quad \uparrow \text{no } h's \\
 \underline{c=1} \quad \rightarrow h_{ij} &\sim \frac{G}{r} \int \frac{d^3x}{d^3x} T^{00} x^i x^j d^3x \\
 &\quad \frac{GM}{r}
 \end{aligned}$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

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$$\rightarrow h_{ij} \sim \frac{G}{r} \int \frac{d^3x}{d^3x} T^{00} x^i x^j d^3x$$

$$\frac{GM}{r} \text{ dimless}$$

$$G_{\mu\nu}^{(1)} \quad R_{\mu\nu}^{(1)} = -\frac{1}{2} \square h_{\mu\nu} + \frac{1}{2} \underbrace{\left(h_{\mu,\sigma}^\sigma - \frac{1}{2} h_{\sigma,\mu}^\sigma \right)_{,\nu}}_{\partial^2 = \partial_\rho \partial^\rho} + \frac{1}{2} \underbrace{\left(h_{\nu,\sigma}^\sigma - \frac{1}{2} h_{\sigma,\nu}^\sigma \right)_{,\mu}}_{\text{vanish in harmonic gauge}}$$

$$R_{\mu\nu}^{(1)} - \frac{1}{2} \eta_{\mu\nu} R^{(1)} = 8\pi G (T_{\mu\nu} + E_{\mu\nu}) + o(h^3)$$

$$\langle E_{\mu\nu} \rangle = \frac{1}{32\pi G} \left\langle \partial_\mu h_{\rho\sigma} \partial_\nu h^{\rho\sigma} - \frac{1}{2} \partial_\mu h \partial_\nu h - \partial_\mu h^{\rho\sigma} (\partial_\rho h_{\nu\sigma} + \partial_\sigma h_{\nu\rho}) \right\rangle$$

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$$G_{\mu\nu}^{(1)} \quad R_{\mu\nu}^{(1)} = -\frac{1}{2} \square h_{\mu\nu} + \frac{1}{2} \underbrace{\left(h_{\mu\sigma}^{\sigma} - \frac{1}{2} h_{\sigma,\mu}^{\sigma} \right)_{,\nu}}_{\partial^2 = \partial_\mu \partial^\mu} + \frac{1}{2} \underbrace{\left(h_{\nu,\sigma}^{\sigma} - \frac{1}{2} h_{\sigma,\nu}^{\sigma} \right)_{,\mu}}_{\text{vanish in harmonic gauge}}$$

$$R_{\mu\nu}^{(1)} - \frac{1}{2} \eta_{\mu\nu} R^{(1)} = 8\pi G (T_{\mu\nu} + E_{\mu\nu}) + o(h^3)$$

$$+ R_{\mu\nu}^{(2)} - \frac{1}{2} \eta_{\mu\nu} R^{(2)} \quad \langle E_{\mu\nu} \rangle = \frac{1}{32\pi G} \left\langle \partial_\mu h_{\rho\sigma} \partial_\nu h^{\rho\sigma} - \frac{1}{2} \partial_\mu h \partial_\nu h - \partial_\mu h^{\rho\sigma} (\partial_\rho h_{\nu\sigma} + \partial_\sigma h_{\nu\rho}) \right\rangle$$

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$$R_{\mu\nu}^{(1)} - \frac{1}{2} \eta_{\mu\nu} R^{(1)} = 8\pi G (T_{\mu\nu} + E_{\mu\nu}) + o(h^3)$$

$$\left(\begin{array}{l} + R_{\mu\nu}^{(2)} \\ - \frac{1}{2} \eta_{\mu\nu} R^{(1)} \\ - \frac{1}{2} \eta_{\mu\nu} R^{(2)} \end{array} \right) \quad \langle E_{\mu\nu} \rangle = \frac{1}{32\pi G} \left\langle \partial_\mu h_{\rho\sigma} \partial_\nu h^{\rho\sigma} - \frac{1}{2} \partial_\mu h \partial_\nu h - \partial_\rho h^{\rho\sigma} (\partial_\mu h_{\nu\sigma} + \partial_\nu h_{\mu\sigma}) \right\rangle$$

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$$R_{\mu\nu}^{(1)} - \frac{1}{2} \eta_{\mu\nu} R^{(1)} = 8\pi G (T_{\mu\nu} + E_{\mu\nu}) + o(h^3)$$

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$(\partial h)^2$
 $h \partial^2 h$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$c=1$ $\hat{O} h = 8\pi G T$
↑
no h 's

$$\rightarrow h_{ij} \sim \frac{G}{r} \int \frac{d^3x}{r} T^{00} x^i x^j d^3x$$

$\frac{GM}{r}$ dimless

$$G_{\mu\nu}^{(1)} \quad R_{\mu\nu}^{(1)} = -\frac{1}{2} \square h_{\mu\nu} + \frac{1}{2} \underbrace{\left(h_{\mu,\sigma}^\sigma - \frac{1}{2} h_{\sigma,\mu}^\sigma \right)_\nu + \frac{1}{2} \left(h_{\nu,\sigma}^\sigma - \frac{1}{2} h_{\sigma,\nu}^\sigma \right)_\mu}_{\text{vanish in harmonic gauge}}$$

can check that $\partial_\mu F^{\mu\nu} = 0$

$$R_{\mu\nu}^{(1)} - \frac{1}{2} \eta_{\mu\nu} R^{(1)} = 8\pi G (T_{\mu\nu} + E_{\mu\nu}) + o(h^3)$$

$$\left(\begin{array}{l} + R_{\mu\nu}^{(2)} \\ - \frac{1}{2} \eta_{\mu\nu} R^{(1)} \\ - \frac{1}{2} \eta_{\mu\nu} R^{(2)} \end{array} \right) \quad \langle E_{\mu\nu} \rangle = \frac{1}{32\pi G} \left\langle \partial_\mu h_{\rho\sigma} \partial_\nu h^{\rho\sigma} - \frac{1}{2} \partial_\mu h \partial_\nu h - \partial_\rho h^{\rho\sigma} (\partial_\mu h_{\nu\sigma} + \partial_\nu h_{\mu\sigma}) \right\rangle$$

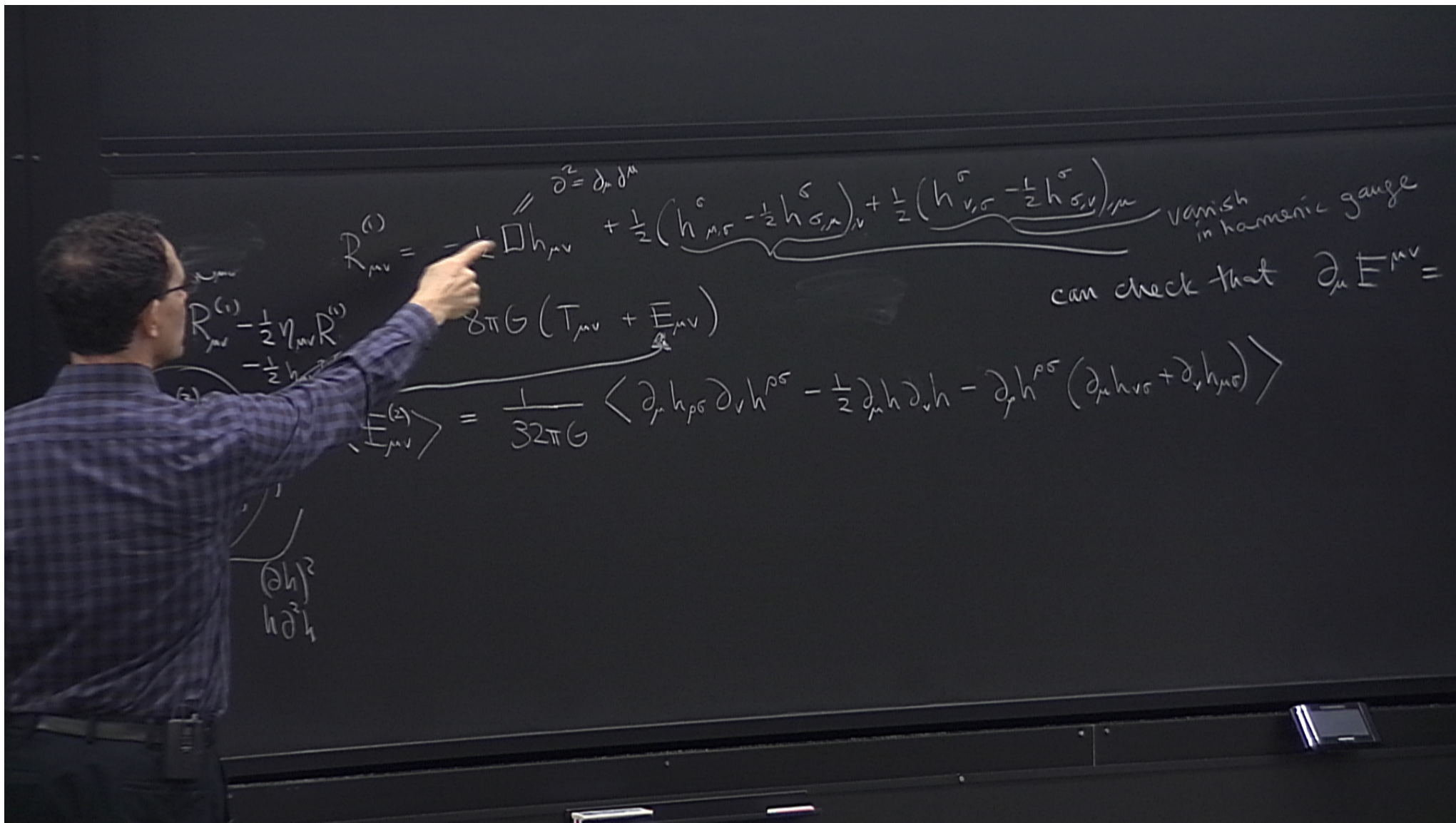
$(\partial h)^2$
 $h \partial^2 h$

$\left(\frac{\delta S}{\delta h_{\mu\nu}} \right)_{,\mu}$ vanish in harmonic gauge

can check that $\partial_\mu F^{\mu\nu} = 0$

[Can easily check

$(\partial_\mu h_{\nu\sigma} + \partial_\nu h_{\mu\sigma})$



$$R^{(1)} = \eta^{\alpha\beta} R_{\alpha\beta}^{(1)} \quad R_{\mu\nu}^{(1)} = -\frac{1}{2} \square h_{\mu\nu} + \frac{1}{2} \underbrace{\left(h_{\mu,\sigma}^\sigma - \frac{1}{2} h_{\sigma,\mu}^\sigma \right)}_{\partial^2 = \partial_\mu \partial^\mu} + \frac{1}{2} \underbrace{\left(h_{\nu,\sigma}^\sigma - \frac{1}{2} h_{\sigma,\nu}^\sigma \right)}_{\text{vanish in harmonic gauge}}_{,\mu}$$

can check that $\partial_\mu E^{\mu\nu} =$

$$R_{\mu\nu}^{(1)} - \frac{1}{2} \eta_{\mu\nu} R^{(1)} = 8\pi G (T_{\mu\nu} + E_{\mu\nu})$$

$$\left(\begin{array}{l} + R_{\mu\nu}^{(2)} \\ - \frac{1}{2} \eta_{\mu\nu} R^{(1)} \\ - \frac{1}{2} \eta_{\mu\nu} R^{(2)} \end{array} \right) \langle E_{\mu\nu}^{(2)} \rangle = \frac{1}{32\pi G} \left\langle \partial_\mu h_{\rho\sigma} \partial_\nu h^{\rho\sigma} - \frac{1}{2} \partial_\mu h \partial_\nu h - \partial_\mu h^{\rho\sigma} (\partial_\nu h_{\rho\sigma} + \partial_\nu h_{\sigma\rho}) \right\rangle$$

$(\partial h)^2$
 $h \partial^2 h$

$(S_{\mu\nu})_{\rho\sigma}$ vanish in harmonic gauge
can check that $\partial_\mu F^{\mu\nu} = 0$

$$(\partial_\mu h_{\nu\rho} + \partial_\nu h_{\mu\rho}) >$$

[Can easily check that $\partial_\mu (G^{\mu\nu}) = 0$.]

$\left(\frac{5}{2} \right)_{\mu\nu}$ vanish in harmonic gauge
 can check that $\partial_\mu F^{\mu\nu} = 0$

$$(\partial_\mu h_{\nu\rho} + \partial_\nu h_{\mu\rho}) >$$

$$\nabla_\mu G^{\mu\nu} = 0 \Rightarrow \nabla_\mu G^{\mu\nu} = 0$$

[Can easily check that $\partial_\mu (G^{\mu\nu}) = 0$.]

$\left(\frac{\delta S}{\delta h_{\mu\nu}} \right)_{,\mu}$ vanish in harmonic gauge
 can check that $\partial_\mu F^{\mu\nu} = 0$

$$(\partial_\mu h_{\nu\rho} + \partial_\nu h_{\mu\rho}) >$$

$$\nabla_\mu G^{\mu\nu} = 0 \Rightarrow \partial_\mu \underline{G^{\mu\nu}_{(1)}} = 0 \text{ at 1st order in } h$$

[Can easily check that $\partial_\mu (G^{\mu\nu}_{(1)}) = 0$.]

$$\bigcirc \quad \circ$$

$$\frac{1}{2} m \dot{r}^2 - \frac{GM}{r}$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\underline{c=1} \quad \boxed{\hat{\mathcal{O}} h = 8\pi G T}$$

↑
no h 's

$$\rightarrow h_{ij} \sim \frac{G}{r^{d-2}} \int T^{00} x^i x^j d^3x$$

$\frac{GM}{r}$ dimless

$$\bigcirc \quad \circ$$

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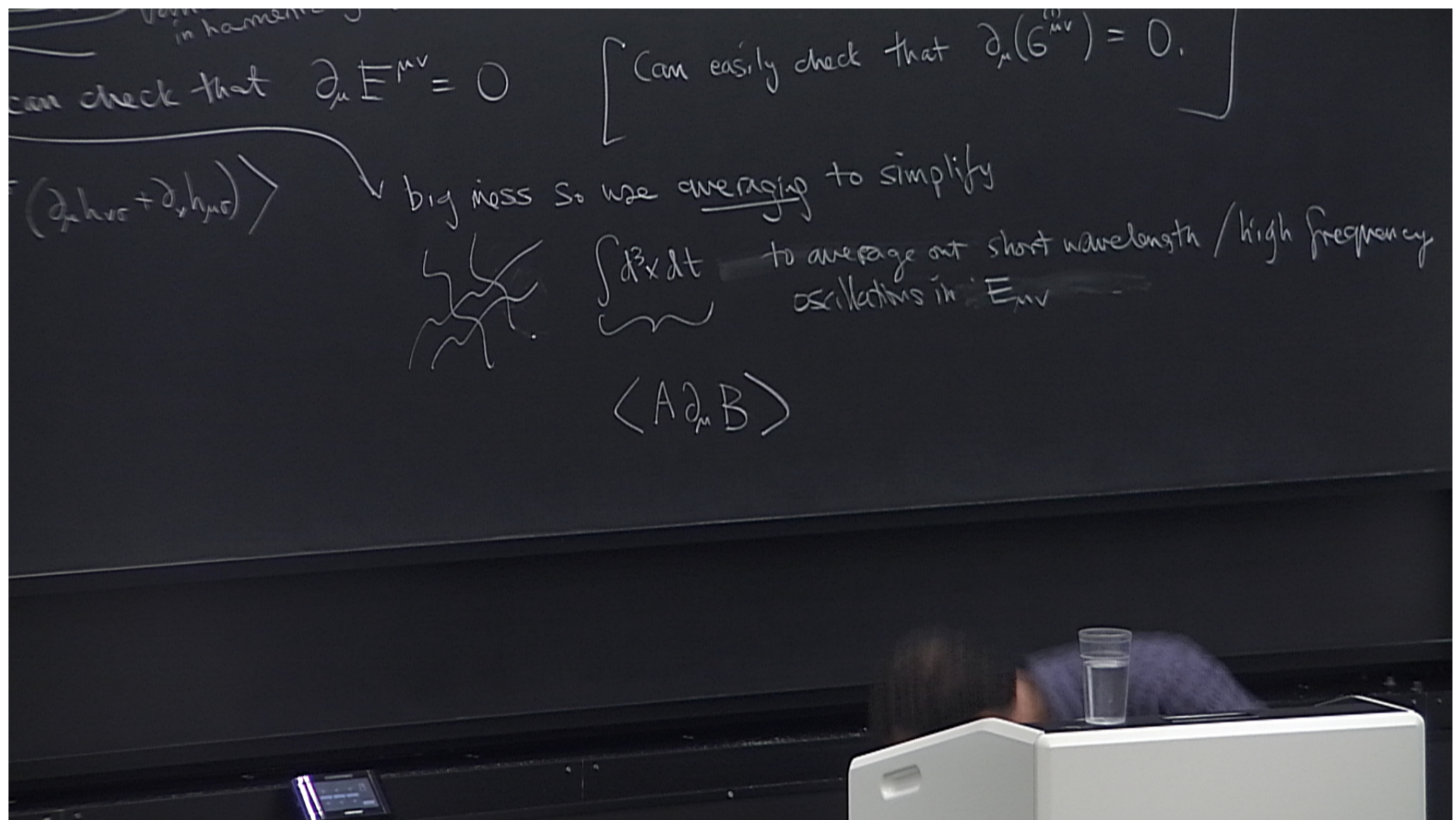
$$\frac{GM}{r} \text{ dimless}$$

$$R^{(1)} = \eta^{\alpha\beta} R_{\alpha\beta}^{(1)} \quad R_{\mu\nu}^{(1)} = -\frac{1}{2} \square h_{\mu\nu} + \frac{1}{2} \underbrace{\left(h_{\mu,\sigma}^\sigma - \frac{1}{2} h_{\sigma,\mu}^\sigma \right)}_{\partial^2 = \partial_\mu \partial^\mu} + \frac{1}{2} \underbrace{\left(h_{\nu,\sigma}^\sigma - \frac{1}{2} h_{\sigma,\nu}^\sigma \right)}_{\text{vanish in harmonic gauge}}_{,\mu}$$

$$\underbrace{R_{\mu\nu}^{(1)} - \frac{1}{2} \eta_{\mu\nu} R^{(1)}}_{-\frac{1}{2} h_{\mu\nu} R^{(1)}} = 8\pi G (T_{\mu\nu} + E_{\mu\nu}) \quad \text{Note } E_{\mu\nu} \text{ not a tensor}$$

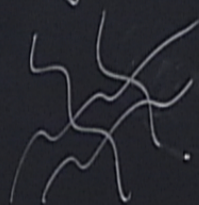
can check that $\partial_\mu E^{\mu\nu} = 0$

$$\underbrace{\left(\begin{array}{l} +R_{\mu\nu}^{(2)} \\ -\frac{1}{2} h_{\mu\nu} R^{(1)} \\ -\frac{1}{2} \eta_{\mu\nu} R^{(2)} \end{array} \right)}_{\substack{(\partial h)^2 \\ h \partial^2 h}} \langle E_{\mu\nu}^{(2)} \rangle = \frac{1}{32\pi G} \left\langle \partial_\mu h_{\rho\sigma} \partial_\nu h^{\rho\sigma} - \frac{1}{2} \partial_\mu h \partial_\nu h - \partial_\mu h^{\rho\sigma} (\partial_\rho h_{\nu\sigma} + \partial_\sigma h_{\mu\nu}) \right\rangle$$



can check that $\partial_\mu E^{\mu\nu} = 0$ [Can easily check that $\partial_\mu (G^{\mu\nu}) = 0$.]

$\langle \partial_\mu h_{\nu\sigma} + \partial_\nu h_{\mu\sigma} \rangle$ big mess so use averaging to simplify



$$\int d^3x dt$$

to average out short wavelength / high frequency oscillations in $E_{\mu\nu}$

$$\langle \partial_\mu (AB) \rangle = 0 \Rightarrow \underline{\langle A \partial_\mu B \rangle = - \langle \partial_\mu A \cdot B \rangle}$$

$$R^{(1)} = \eta^{\alpha\beta} R_{\alpha\beta}^{(1)} \quad R_{\mu\nu}^{(1)} = -\frac{1}{2} \Box h_{\mu\nu} + \frac{1}{2} \left(h_{\mu,\sigma} - \frac{1}{2} h_{\sigma,\mu} \right)_{,\nu} + \frac{1}{2} \left(h_{\nu,\sigma} - \frac{1}{2} h_{\sigma,\nu} \right)_{,\mu}$$

vanish in harmonic gauge

$$R_{\mu\nu}^{(1)} - \frac{1}{2} \eta_{\mu\nu} R^{(1)} = 8\pi G (T_{\mu\nu} + E_{\mu\nu})$$

Note $E_{\mu\nu}$ not a tensor can check that $\partial_\mu E^{\mu\nu} =$

$$\langle E_{\mu\nu}^{(2)} \rangle = \frac{1}{32\pi G} \left\langle \partial_\mu h_{\rho\sigma} \partial_\nu h^{\rho\sigma} - \frac{1}{2} \partial_\mu h \partial_\nu h - \partial_\rho h^{\rho\sigma} (\partial_\mu h_{\nu\sigma} + \partial_\nu h_{\mu\sigma}) \right\rangle$$

big

$\frac{(\partial h)^2}{h^3 \partial^3 h}$

$\frac{h^2 \partial^2 h}{\partial h \partial h}$

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(Arnold p. 310)

big

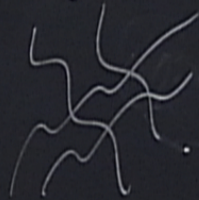
$\frac{(\partial h)^2}{h^3 \partial^3 h}$

$\frac{h^2 \partial^2 h}{\partial h \partial h}$

can check that $\partial_\mu E^{\mu\nu} = 0$ [Can easily check that $\partial_\mu (G^{\mu\nu}) = 0$.]

$(\partial_\mu h_{\nu\sigma} + \partial_\nu h_{\mu\sigma})$ $\rightarrow$ big mess so use averaging to simplify

(Griffiths p. 310)



$$\int d^3x dt$$

to average out short wavelength / high frequency oscillations in $E_{\mu\nu}$

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vanish in harmonic gauge

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(Arnold p. 310) big

$$\begin{aligned} &+ R_{\mu\nu}^{(2)} - \frac{1}{2} \eta_{\mu\nu} R^{(2)} - \frac{1}{2} h_{\mu\nu} R^{(1)} \\ &- \frac{1}{2} \eta_{\mu\nu} R^{(2)} \end{aligned}$$

$(\partial h)^2$ $h \partial^2 h$
 $h \partial^3 h$ $\partial h \partial h$

$$h \equiv h_{\mu\nu} \eta^{\mu\nu} = h^\lambda{}_\lambda$$

$$O \circ$$

$$\frac{1}{2} m \dot{r}^2 = \frac{GM}{r}$$

$$T_{\mu\nu} \sim F_{\mu}^{\alpha} F_{\nu\alpha} + \dots$$

$$\mu = 0$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\underline{c=1} \quad \boxed{\hat{O}h = 8\pi G T}$$

↑
no h 's

$$\rightarrow h_{ij} \sim \frac{G}{r^2} \int T^{00} x^i x^j d^3x$$

$\frac{GM}{r}$ dimensionless

$$0 \quad \circ$$

$$\frac{1}{2} m \dot{r}^2 - \frac{GM}{r}$$

$$T_{\mu\nu} \sim F_{\mu}^{\alpha} F_{\nu\alpha} + \dots$$

$$\partial_{\mu} T^{\mu\nu} = 0$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\underline{c=1} \quad \boxed{\hat{O}h = 8\pi G T}$$

↑
no $\hbar$'s

$$\rightarrow h_{ij} \sim \frac{G}{r} \int T^{00} x^i x^j d^3x$$

$$\frac{GM}{r} \text{ dimensionless}$$

$$0 \quad \circ$$

$$\frac{1}{2} m \dot{r}^2 - \frac{GM}{r}$$

$$T_{\mu\nu}^m \sim F_{\mu}^{\alpha} F_{\nu\alpha} + \dots$$

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$$\rightarrow h_{ij} \sim \frac{G}{r} \int T^{00} x^i x^j d^3x$$

$$\frac{GM}{r} \text{ dimless}$$

$$R^{(1)} = \eta^{\mu\nu} R_{\mu\nu}^{(1)} = -\frac{1}{2} \square h_{\mu\nu} + \frac{1}{2} (\eta^{\mu\sigma} \partial_\mu \partial_\nu h^\alpha{}_\alpha - \partial_\mu \partial_\nu h^\alpha{}_\alpha)$$

$$R_{\mu\nu}^{(1)} - \frac{1}{2} \eta_{\mu\nu} R^{(1)} = 8\pi G (T_{\mu\nu} + E_{\mu\nu})$$

Note: $E_{\mu\nu}$ not a tensor can check that $\partial_\mu E^{\mu\nu}$

$$\langle E_{\mu\nu}^{(2)} \rangle = \frac{1}{32\pi G} \langle \partial_\mu h_{\rho\sigma} \partial_\nu h^{\rho\sigma} - \frac{1}{2} \partial_\mu h \partial_\nu h - \partial_\mu h^{\rho\sigma} (\partial_\rho h_{\nu\sigma} + \partial_\sigma h_{\nu\rho}) \rangle$$

(Carroll p. 310)

$$\begin{aligned} &+ R_{\mu\nu}^{(2)} \\ &- \frac{1}{2} \eta_{\mu\nu} R^{(2)} \\ &- \frac{1}{2} \eta_{\mu\nu} R^{(2)} \end{aligned}$$

$$\begin{aligned} &(\partial h)^2 & \overbrace{h \partial^2 h} \\ &h \partial^3 h & \partial h \partial h \\ &h \equiv h_{\mu\nu} \eta^{\mu\nu} = h^\lambda{}_\lambda \end{aligned}$$

TT gauge

$$R^{(1)} = \eta^{\mu\nu} \partial_\mu \partial_\nu h_{\mu\nu} - \frac{1}{2} \square h_{\mu\nu} + \frac{1}{2} (\eta_{\mu\sigma} \partial_\mu \partial_\nu h^{\sigma\nu} - \eta_{\nu\sigma} \partial_\mu \partial_\nu h^{\mu\sigma})$$

$$R_{\mu\nu}^{(1)} - \frac{1}{2} \eta_{\mu\nu} R^{(1)} = 8\pi G (T_{\mu\nu} + E_{\mu\nu})$$

Note: $E_{\mu\nu}$ not a tensor can check that $\partial_\mu E^{\mu\nu}$

$$\langle E_{\mu\nu}^{(2)} \rangle = \frac{1}{32\pi G} \langle \partial_\mu h_{\rho\sigma} \partial_\nu h^{\rho\sigma} - \frac{1}{2} \partial_\mu h \partial_\nu h - \partial_\rho h^{\rho\sigma} (\partial_\mu h_{\nu\sigma} + \partial_\nu h_{\mu\sigma}) \rangle$$

(Carrillo p. 310)

$$+ R_{\mu\nu}^{(2)} - \frac{1}{2} \eta_{\mu\nu} R^{(2)} - \frac{1}{2} \eta_{\mu\nu} R^{(1)2}$$

$$\begin{matrix} (\partial h)^2 & h^2 \partial^2 h \\ h \partial^3 h & \partial h \partial h \end{matrix}$$

$$h \equiv h_{\mu\nu} \eta^{\mu\nu} = h^\lambda{}_\lambda$$

TT gauge : $h = 0$

$$R^{(1)} = \eta^{\mu\nu} \partial_\mu \partial_\nu h_{\mu\nu} - \frac{1}{2} \square h_{\mu\nu} + \frac{1}{2} (\partial_\mu h_{\nu\sigma} + \partial_\nu h_{\mu\sigma} - \partial_\sigma h_{\mu\nu})$$

$$R_{\mu\nu}^{(1)} - \frac{1}{2} \eta_{\mu\nu} R^{(1)} = 8\pi G (T_{\mu\nu} + E_{\mu\nu})$$

Note: $E_{\mu\nu}$ not a tensor. can check that $\partial_\mu E^{\mu\nu}$

$$\langle E_{\mu\nu}^{(2)} \rangle = \frac{1}{32\pi G} \langle \partial_\mu h_{\rho\sigma} \partial_\nu h^{\rho\sigma} - \frac{1}{2} \partial_\mu h \partial_\nu h - \partial_\rho h^{\rho\sigma} (\partial_\mu h_{\nu\sigma} + \partial_\nu h_{\mu\sigma}) \rangle$$

(Arnold p. 310)

$$\begin{aligned} &+ R_{\mu\nu}^{(2)} \\ &- \frac{1}{2} \eta_{\mu\nu} R^{(2)} \\ &- \frac{1}{2} \eta_{\mu\nu} R^{(2)} \end{aligned}$$

$$\begin{aligned} &(\partial h)^2 & h^2 \partial^2 h \\ &h \partial^3 h & \partial h \partial h \\ &h \equiv h_{\mu\nu} \eta^{\mu\nu} = h^\lambda{}_\lambda \end{aligned}$$

TT gauge : $h = 0, \partial_\rho h^{\rho\sigma} = 0$ $\vec{k}$ by $\vec{k}$.

$$R^{(1)} = \eta^{\mu\nu} \partial_\mu \partial_\nu h_{\mu\nu} - \frac{1}{2} \square h_{\mu\nu} + \frac{1}{2} (\partial_\mu h_{\nu\sigma} + \partial_\nu h_{\mu\sigma} - \partial_\sigma h_{\mu\nu})$$

$$R^{(1)} - \frac{1}{2} \eta_{\mu\nu} R^{(1)} = 8\pi G (T_{\mu\nu} + E_{\mu\nu})$$

Note: $E_{\mu\nu}$ not a tensor can check that $\partial_\mu E^{\mu\nu}$

$$\langle E_{\mu\nu}^{(2)} \rangle = \frac{1}{32\pi G} \langle \partial_\mu h_{\rho\sigma} \partial_\nu h^{\rho\sigma} - \frac{1}{2} \partial_\mu h \partial_\nu h - \partial_\mu h^{\rho\sigma} (\partial_\rho h_{\nu\sigma} + \partial_\sigma h_{\nu\rho}) \rangle$$

(Arnold p. 310)

$$\begin{aligned} &+ R^{(2)} \\ &- \frac{1}{2} \eta_{\mu\nu} R^{(1)} \\ &- \frac{1}{2} \eta_{\mu\nu} R^{(2)} \end{aligned}$$

$$\begin{aligned} &(\partial h)^2 & h \partial^2 h \\ &h \partial^3 h & \partial h \partial h \end{aligned}$$

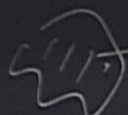
$$h \equiv h_{\mu\nu} \eta^{\mu\nu} = h^\lambda{}_\lambda$$

TT gauge : $h = 0, \partial_\mu h^{\mu\nu} = 0$ $\vec{k}$ by $\vec{k}_\perp$

$$\Rightarrow \langle E_{\mu\nu}^{(2)} \rangle = \frac{1}{32\pi G} \langle \partial_\mu h_{\rho\sigma}^{\text{TT}} \partial_\nu h^{\text{TT}\rho\sigma} \rangle$$

A hand-drawn diagram on a chalkboard. On the left, there is a hand-drawn irregular shape representing a region, with the coordinates $(1,1)$ written inside. An arrow points from this region to the right, where the metric tensor $h_{ij}^{TT}(z-t)$ is written. Above the arrow, the coordinate transformation $z = x^3$ is written.

$$z = x^3$$
$$h_{ij}^{TT}(z-t)$$



$$z = x^3$$

$$h_{ij}^{\pm\pm}(z-t) \Rightarrow \begin{aligned} \partial_t h_{ij} &= -\partial_z h_{ij} \\ \partial_0 h_{ij} &= -\partial_3 h_{ij} \end{aligned}$$

$$z = x^3$$

$$h_{ij}^{TT}(z-t) \Rightarrow \begin{aligned} \partial_t h_{ij} &= -\partial_z h_{ij} \\ \partial_0 h_{ij} &= -\partial_3 h_{ij} \end{aligned}$$

$$\partial_t T^{00} = -\partial_z T^{0z}$$

$$\partial_0 \int d^3x T^{00} = -\int T^{0i} dS^i$$

$E^{03} = \text{rate of loss of energy per area}$

$$= -E_{03} = -\frac{1}{32\pi G} \langle \partial_0 h_{ij}^{TT} \partial_3 h_{ij}^{TT} \rangle = \frac{1}{32\pi G} \langle \partial_0 h_{ij}^{TT} \partial_0 h_{ij}^{TT} \rangle$$

for area

$$\langle \partial_3 h_{ij}^{\text{TT}} \rangle = \frac{1}{32\pi G} \langle \partial_0 h_{ij}^{\text{TT}} \partial_0 h_{ij}^{\text{TT}} \rangle = \frac{1}{32\pi G} \langle \dot{h}_{11}^2 + \dot{h}_{22}^2 + 2\dot{h}_{12}^2 \rangle$$

$$\text{In TT gauge } h_{22} = -h_{11}$$

per area

$$\langle \partial_3 h_{ij}^{\text{TT}} \rangle = \frac{1}{32\pi G} \langle \partial_0 h_{ij}^{\text{TT}} \partial_0 h_{ij}^{\text{TT}} \rangle = \frac{1}{32\pi G} \left(\dot{h}_{11}^2 + \dot{h}_{22}^2 + 2\dot{h}_{12}^2 \right)$$

$$= \frac{1}{32\pi G} \frac{2}{4} (\dot{h}_{11} - \dot{h}_{22})^2$$

In TT gauge $h_{22} = -h_{11}$

per area

$$\langle \partial_3 h_{ij}^{\text{TT}} \rangle = \frac{1}{32\pi G}$$

$$= \frac{1}{32\pi G} \langle \dot{h}_{11}^2 + \dot{h}_{22}^2 + 2\dot{h}_{12}^2 \rangle$$

$$= \frac{1}{32\pi G} \langle \frac{2}{4} (\dot{h}_{11} - \dot{h}_{22})^2 + 2\dot{h}_{12}^2 \rangle$$

$$= \frac{1}{16\pi G} \langle \frac{1}{4} (\dot{h}_{11} - \dot{h}_{22})^2 + \dot{h}_{12}^2 \rangle$$

per area

$$\langle \dot{h}_{ij}^{\text{T T}} \rangle = \frac{1}{32\pi G} \langle \partial_0 \dot{h}_{ij}^{\text{T T}} \partial_0 \dot{h}_{ij}^{\text{T T}} \rangle = \frac{1}{32\pi G} \langle \dot{h}_{11}^2 + \dot{h}_{22}^2 + 2\dot{h}_{12}^2 \rangle$$

$$= \frac{1}{16\pi G} \left(\frac{1}{4} (\dot{h}_{11} - \dot{h}_{22})^2 + \dot{h}_{12}^2 \right)$$

recall $\overset{\text{trace reverse}}{\bar{h}_{ij}} = -\frac{2G}{r} \frac{d^2}{dt^2} \int d^3\vec{x}' T^{00}(t-r, \vec{x}') x'^i x'^j$

recall $\overset{\text{trace reverse}}{\bar{h}_{ij}} = - \frac{2G}{r} \left[\frac{d^2}{dt^2} \int d^3\vec{x}' T^{00}(t, \vec{x}') x'^i x'^j \right]_{t \rightarrow t-r}$

recall $\overset{\text{trace reverse}}{\bar{h}_{ij}} = -\frac{2G}{r} \left[\frac{d^2}{dt^2} \int d^3\vec{x}' T^{00}(t, \vec{x}') x'^i x'^j \right]_{t \rightarrow t-r}$

$$\frac{1}{3} Q^{ij} + \frac{1}{3} \int T^{00} \frac{\vec{x}^i \vec{x}^j}{x^3} \delta^{ij}$$

recall $\overset{\text{trace reverse}}{\bar{h}_{ij}} = - \frac{2G}{r} \left[\frac{d^2}{dt^2} \int d^3\vec{x}' T^{00}(t, \vec{x}') x'^i x'^j \right]_{t \rightarrow t-r}$

$$Q^{ij} = \int d^3x' (3x'^i x'^j - \delta^{ij} x'^2) T^{00}$$

quadrupole
moment

$$\left. \begin{matrix} \vec{x}^i \vec{x}^j \end{matrix} \right\} t \rightarrow t-r$$

$$\vec{x}^i \vec{x}^j = \delta^{ij}$$

$$T^{00}$$

quadrupole
moment

} only the traceless part contributes to the traceless part
of h_{ij} , when go to TT gauge, that
is all that remains.

$$\frac{2G}{r} \left[\frac{d^2}{dt^2} \int d^3\vec{x}' T^{00}(t, \vec{x}') x'^i x'^j \right]$$

$\frac{1}{3} Q^{ij} + \frac{1}{3} \int T^{00} \frac{x'^i x'^j}{x'^2} d^3x'$

$Q^{ij} = \int d^3x' (3x'^i x'^j - \delta^{ij} x'^2) T^{00}$

$t \rightarrow t-r$
creates an h_{ij} which
gets gauged away

quadrupole
moment

} only the traceless part contributes to the traceless part
of h_{ij} , when go to TT gauge, that
is all that remains.

recall $\bar{h}_{ij} = - \frac{2G}{r} \left[\frac{d^2}{dt^2} \int d^3x' T^{00}(t, \vec{x}') x'^i x'^j \right]$

$\nearrow$ trace reverse

$\nearrow$ $t \rightarrow t-r$ creates an h_{ij} which gets gauged away

$\frac{1}{3} Q^{ij} + \frac{1}{3} \int T^{00} \frac{1}{x^2} \delta^{ij}$

$Q^{ij} = \int d^3x' (3x'^i x'^j - \delta^{ij} x'^2) T^{00}$

quadrupole moment } on

h_{TT}
gauge

$\bar{h}_{ij} = h_{ij}^{TT}$

recall $\overset{\text{trace reverse}}{\bar{h}_{ij}} = -\frac{2G}{r} \left[\frac{d^2}{dt^2} \int d^3\vec{x}' T^{00}(t, \vec{x}') x'^i x'^j \right]$

$\frac{1}{3}Q^{ij} + \frac{1}{3} \int T^{00} \frac{x'^i x'^j}{x'^2} d^3x'$ $\left. \begin{array}{l} t \rightarrow t-r \\ \text{creates an } h_{ij} \text{ which} \\ \text{gets gauged away} \end{array} \right\}$

$Q^{ij} = \int d^3x' (3x'^i x'^j - \delta^{ij} x'^2) T^{00}$ $\left. \begin{array}{l} \text{quadrupole} \\ \text{moment} \end{array} \right\} \text{ on}$

in TT
gauge

$$\bar{h}_{ij} = \overset{\text{TT}}{h}_{ij} = -\frac{2G}{3r} \ddot{Q}_{ij}(t-r)$$

$$\chi^i \chi^j$$

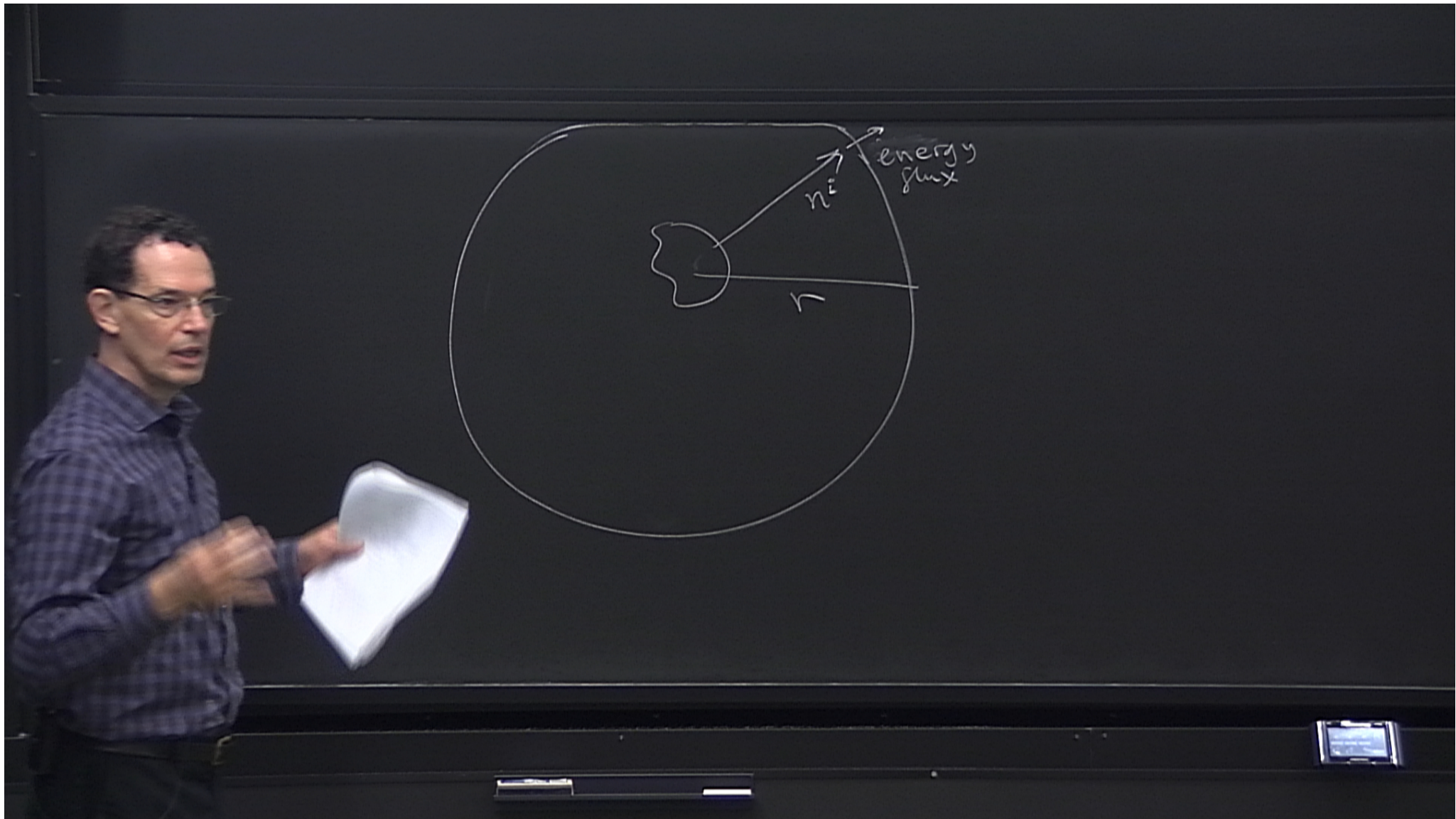
$t \rightarrow t-r$
creates an h_{ij} which
gets gauged away

$$\chi^i \chi^j = \delta^{ij}$$

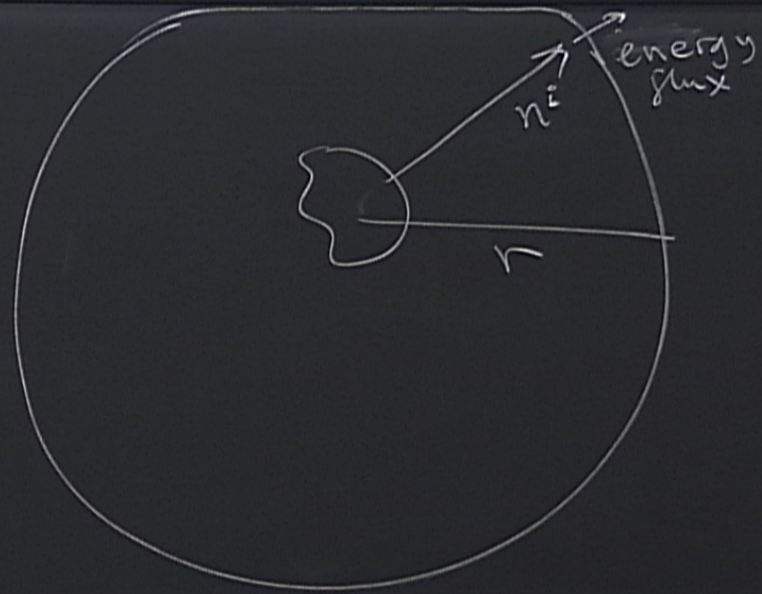
$$T_{00}$$

quadrupole
moment

} only the traceless part contributes to the traceless part
of h_{ij} , when go to TT gauge, that
is all that remains.

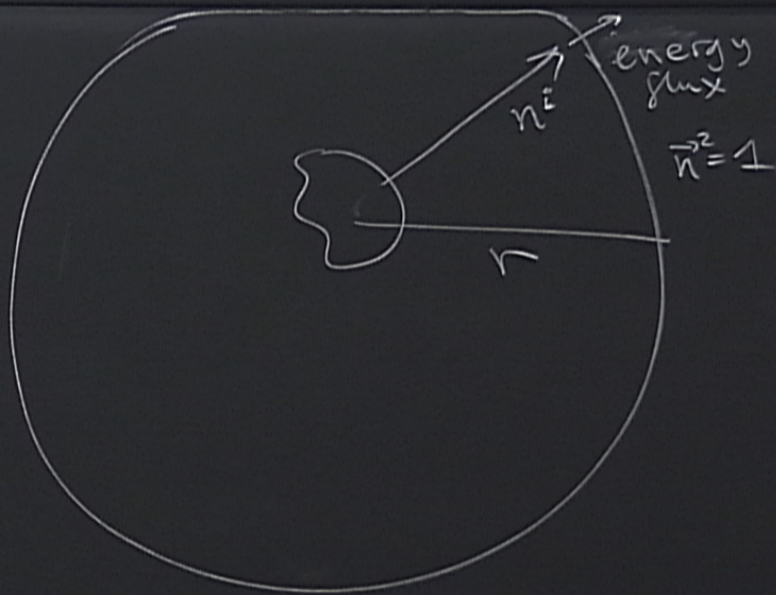


Want to
do
this :



We need T^{oi}_{ni}

Want to
do
this :



We need

for area

$$\langle h_{ij}^{\text{TT}} \rangle = \frac{1}{32\pi G} \langle \partial_0 h_{ij}^{\text{TT}} \partial_0 h_{ij}^{\text{TT}} \rangle = \frac{1}{32\pi G} \langle \dot{h}_{11}^2 + \dot{h}_{22}^2 + 2\dot{h}_{12}^2 \rangle$$

$$= \frac{1}{16\pi G} \left(\frac{2}{4} (\dot{h}_{11} - \dot{h}_{22})^2 + \dot{h}_{12}^2 \right)$$

*

$$x^i x^j$$

$$\delta_{ij}$$

which
get away

the traceless part contributes to the traceless part

recall

$$\bar{h}_{ij} = -\frac{2G}{r} \left[\frac{d^2}{dt^2} \int d^3\vec{x}' T^{00}(t, \vec{x}') x'^i x'^j \right]$$

$t \rightarrow t-r$
creates an h_{ij} which
gets gauged away

$$\frac{1}{3} Q^{ij} + \frac{1}{3} \int T^{00} \frac{x'^i x'^j}{x'^2} d^3x'$$

$$Q^{ij} = \int d^3x' (3x'^i x'^j - \delta^{ij} x'^2) T^{00}$$

quadrupole moment } only the traceless part of h_{ij} , which is all.

in TT gauge

$$\bar{h}_{ij} = \boxed{h_{ij}^{TT} = -\frac{2G}{3r} \ddot{Q}_{ij}(t-r)}$$

on z axis, from (*),

$$E^{03} = \frac{G}{36\pi r^2} \left(\ddot{Q}_{12}^2 + \frac{1}{4} (\ddot{Q}_{11} - \ddot{Q}_{22})^2 \right)$$

recall

trace reverse

$$\bar{h}_{ij} = -\frac{2G}{r} \left[\frac{d^2}{dt^2} \int d^3\vec{x}' T^{00}(t, \vec{x}') x'^i x'^j \right]$$

$t \rightarrow t-r$ creates an h_{ij} which gets gauged away

$$\frac{1}{3} Q^{ij} + \frac{1}{3} \int T^{00} \frac{1}{x^2} \delta^{ij}$$

$$Q^{ij} = \int d^3x' (3x'^i x'^j - \delta^{ij} x'^2) T^{00}$$

quadrupole moment } only the traceless part of h_{ij} , which is all.

in TT gauge

$$\bar{h}_{ij} = \boxed{h_{ij}^{TT} = -\frac{2G}{3r} \ddot{Q}_{ij}(t-r)}$$

on z axis, from (*)

$$E^{03} = \frac{16}{36\pi r^2} \left(\ddot{Q}_{12}^2 + \frac{1}{4} (\ddot{Q}_{11} - \ddot{Q}_{22})^2 \right)$$

$\frac{1}{16} \cdot \frac{4}{9} = \frac{1}{36}$

we need $T^{oi}_{ni} = \frac{G}{36\pi r^2} [A \ddot{Q}_{ke} \ddot{Q}_{ke}]$

we need $T^{oi}_{ni} = \frac{G}{36\pi r^2} [A \ddot{Q}_{ke} \ddot{Q}_{ke} + B \ddot{Q}_{ke} \ddot{Q}_{km} n^e n^m]$

we need $T^{oi}_{ni} = \frac{G}{36\pi r^2} [A \ddot{Q}_{ke} \ddot{Q}_{ke} + B \ddot{Q}_{ke} \ddot{Q}_{km} n^e n^m + C (\ddot{Q}_{ke} n^k n^e)^2]$

We need $T^{oi} n^i = \frac{G}{36\pi r^2} [A \ddot{Q}_{ke} \ddot{Q}_{ke} + B \ddot{Q}_{ke} \ddot{Q}_{km} n^e n^m + C (\ddot{Q}_{ke} n^k n^e)^2]$

we know answer for $n^i = (0, 0, 1)$

We need $T^{oi} n^i = \frac{G}{36\pi r^2} \left[A \ddot{Q}_{ke} \ddot{Q}_{ke} + B \ddot{Q}_{ke} \ddot{Q}_{km} n^e n^m + C (\ddot{Q}_{ke} n^k n^e)^2 \right]$

we know answer for $n^i = (0, 0, 1)$: this is enough to fix A, B and C .

We need $T^{oi} n^i = \frac{G}{36\pi r^2} \left[A \ddot{Q}_{ke} \ddot{Q}_{ke} + B \ddot{Q}_{ke} \ddot{Q}_{km} n^e n^m + C (\ddot{Q}_{ke} n^k n^e)^2 \right]$

we know answer for $n^i = (0, 0, 1)$: this is enough to fix A, B and C .

We need $T^{oi} n^i = \frac{G}{36\pi r^2} [A \ddot{Q}_{ke} \ddot{Q}_{ke} + B \ddot{Q}_{ke} \ddot{Q}_{km} n^k n^m + C$
 $(\neq)$

$$\ddot{Q}_{ij} \equiv D_{ij}$$

We know answer for $n^i = (0, 0, 1)$: this is enough to fix A, B

$$\begin{pmatrix} \neq \\ \text{for} \\ \vec{n} = (0, 0, 1) \end{pmatrix} = A (D_{11}^2 + D_{22}^2 + D_{33}^2 + 2D_{12}^2 + 2D_{13}^2 + 2D_{23}^2)$$

We need $T^{oi} n^i = \frac{G}{36\pi r^2} [A \ddot{Q}_{ke} \ddot{Q}_{ke} + B \ddot{Q}_{ke} \ddot{Q}_{km} n^k n^m + C \ddot{Q}_{ke} \ddot{Q}_{km} n^k n^m] \quad (\neq)$

$\ddot{Q}_{ij} \equiv D_{ij}$

We know answer for $n^i = (0, 0, 1)$: this is enough to fix A, B, C

$\left(\begin{smallmatrix} \neq \\ \text{for} \\ \vec{n} = (0, 0, 1) \end{smallmatrix} \right)$

$$= A (D_{11}^2 + D_{22}^2 + D_{33}^2 + 2D_{12}^2 + 2D_{13}^2 + 2D_{23}^2) + B (D_{13}^2 + D_{23}^2 + D_{33}^2) + C D_{33}^2$$

We need $T^{oi} n^i = \frac{G}{36\pi r^2} \left[A \ddot{Q}_{ke} \ddot{Q}_{ke} + B \ddot{Q}_{ke} \ddot{Q}_{km} n^k n^m + C (\ddot{Q}_{ke} n^k n^e)^2 \right]$

D_{ij} we know answer for $n^i = (0, 0, 1)$: this is enough to fix A, B and C.

$$= A (D_{11}^2 + D_{22}^2 + D_{33}^2 + 2D_{12}^2 + 2D_{13}^2 + 2D_{23}^2) = D_{12}^2 + \frac{1}{4}(D_{11} - D_{22})^2$$

$$+ B (D_{13}^2 + D_{23}^2 + D_{33}^2) + C D_{33}^2$$

$\vec{x}$
 $\vec{x}^2 = 1$

$$\ddot{Q}_{ij} \equiv D_{ij}$$

$$\left(\begin{smallmatrix} + \\ - \end{smallmatrix} \right)_{\vec{n}=(0,0,1)}$$

Use

$$D_{11} + D_{22} + D_{33} = 0$$

$$\rightarrow D_{33}^2 = D_{11}^2 + D_{22}^2 + 2D_{11}D_{22}$$

We need $T^{oi} n_i = \frac{G}{36\pi r^2} [A \ddot{Q}_{kl} \ddot{Q}_{kl} + B \ddot{Q}_{kl} \ddot{Q}_{km}]$

We know answer for $n^i = (0,0,1)$: this is enon

$$= A (D_{11}^2 + D_{22}^2 + D_{33}^2 + 2D_{12}^2 + 2D_{13}^2 + 2D_{23}^2) + B (D_{13}^2 + D_{23}^2 + D_{33}^2) + C D_{33}^2$$

$$= D_{11}^2 (2A+B+C) + D_{11} D_{22} (2A+2B+2C)$$

We need $T^{oi} n^i = \frac{G}{36\pi r^2} \left[A \ddot{Q}_{ke} \ddot{Q}_{ke} + B \ddot{Q}_{ke} \ddot{Q}_{km} n^k n^m + C (\ddot{Q}_{ke} n^k n^e)^2 \right]$

we know answer for $n^i = (0, 0, 1)$: this is enough to fix A, B and C .

$$A (D_{11}^2 + D_{22}^2 + D_{33}^2 + 2D_{12}^2 + 2D_{13}^2 + 2D_{23}^2) = D_{12}^2 + \frac{1}{4}(D_{11} - D_{22})^2$$

$$+ B (D_{13}^2 + D_{23}^2 + D_{33}^2) + C D_{33}^2$$

$$= D_{11}^2 (2A + B + C) + D_{11} D_{22} (2A + 2B + 2C) + D_{12}^2 2A + D_{23}^2 (2A + B)$$

We need $T^{oi} n^i = \frac{G}{36\pi r^2} [A \ddot{Q}_{ke} \ddot{Q}_{ke} + B \ddot{Q}_{ke} \ddot{Q}_{km} n^k n^m + C (\ddot{Q}_{ke} n^k n^e)]$
 $(\neq)$

$\ddot{Q}_{ij} \equiv D_{ij}$

We know answer for $n^i = (0, 0, 1)$: this is enough to fix A, B and C.

$(\neq)$
 $\text{for } \vec{n} = (0, 0, 1)$

$$= A (D_{11}^2 + D_{22}^2 + D_{33}^2 + 2D_{12}^2 + 2D_{13}^2 + 2D_{23}^2) = \underbrace{D_{12}^2} + \frac{1}{4} (D_{11} - D_{22})^2$$

$$+ B (D_{13}^2 + D_{23}^2 + D_{33}^2) + C D_{33}^2$$

Use

$D_{11} + D_{22} + D_{33} = 0$

$\rightarrow D_{33}^2 = D_{11}^2 + D_{22}^2 + 2D_{11}D_{22}$

$= \underbrace{D_{11}^2} (2A + B + C) + D_{11} D_{22} (2A + 2B + 2C) + \underbrace{D_{11}^2} 2A + \underbrace{D_{22}^2} (2A + B)$
 $\Rightarrow \underbrace{2A = 1}_{A = \frac{1}{2}} \quad \underbrace{B = -2A}_{= -1} \quad \underbrace{2A + B + C = \frac{1}{4}}_{\Rightarrow C = \frac{1}{4}}$

We need $E^{0i} n^i = \frac{G}{36\pi r^2} [A \ddot{Q}_{ke} \ddot{Q}_{ke} + B \ddot{Q}_{ke} \ddot{Q}_{km} n^k n^m + C \ddot{Q}_{ke} \ddot{Q}_{km} n^k n^m] \quad (\neq)$

We know answer for $n^i = (0, 0, 1)$: this is enough to fix A, B and C

$$= A (D_{11}^2 + D_{22}^2 + D_{33}^2 + 2D_{12}^2 + 2D_{13}^2 + 2D_{23}^2) = \underbrace{D_{12}^2}_{\substack{(\neq) \\ n^i = (0,0,1)}} + \frac{1}{4} (D_{11} - D_{22})^2 + B (D_{13}^2 + D_{23}^2 + D_{33}^2) + C D_{33}^2$$

$$= (2A + B + C) + D_{11} D_{22} \left(\overset{-1/2}{2A + 2B + 2C} \right) + \underbrace{D_{11}^2 2A}_{\substack{(\neq) \\ n^i = (0,0,1)}} + \underbrace{D_{23}^2 (2A + B)}_{\substack{(\neq) \\ n^i = (0,0,1)}}$$

$$\begin{aligned} A &= 1 & B &= -2A \\ A &= \frac{1}{2} & &= -1 \end{aligned}$$

$$\begin{aligned} 2A + B + C &= \frac{1}{4} \\ \Rightarrow C &= \frac{1}{4} \end{aligned}$$

$$\rightarrow D_{33} = D_{11} + D_{22} + 2D_{11}D_{22}$$

$$\Rightarrow 2A = 1 \quad B = -2A \quad 2A + B + C = \frac{1}{4}$$

$$A = \frac{1}{2} \quad = -1$$

$$\Rightarrow C = \frac{1}{4}$$

$$\Rightarrow \text{in a direction } n^i, \quad E^{oi} n^i = \frac{G}{36\pi r^2} \left[\frac{1}{2} \ddot{Q}_{ij} \ddot{Q}_{ij} - \ddot{Q}_{km} \ddot{Q}_{km} n^k n^m + \frac{1}{4} (\ddot{Q}_{km} n^k n^m)^2 \right]$$

$$\Rightarrow \text{total emitted power} \quad P = \frac{dE}{dt} = r^2 \int d\Omega E^{oi} n^i$$

$$\rightarrow D_{33} = D_{11} + D_{22} + 2D_{11}D_{22}$$

$$\Rightarrow 2A = 1$$

$$B = -2A$$

$$2A + B + C = \frac{1}{4}$$

$$A = \frac{1}{2}$$

$$= -1$$

$$\Rightarrow C = \frac{1}{4}$$

$$\Rightarrow \text{in a direction } \mathbf{n}^i, \quad E^{oi} n^i = \frac{G}{36\pi r^2} \left[\frac{1}{2} \ddot{Q}_{ij} \ddot{Q}_{ij} - \ddot{Q}_{ke} \ddot{Q}_{km} n^e n^m + \frac{1}{4} (\ddot{Q}_{ke} n^k n^e)^2 \right]$$

$$\Rightarrow \text{total emitted power} \quad P = \frac{dE}{dt} = r^2 \int d\Omega E^{oi} n^i$$

$$\text{use } \int d\Omega = 4\pi$$

$$\int d\Omega n^e n^m =$$

$$\rightarrow D_{33} = D_{11} + D_{22} + 2D_{11}D_{22}$$

$$\Rightarrow 2A = 1 \quad B = -2A \quad 2A + B + C = \frac{1}{4}$$

$$A = \frac{1}{2} \quad = -1 \quad \Rightarrow C = \frac{1}{4}$$

$$\Rightarrow \text{in a direction } \mathbf{n}^i, \quad E^{oi} n^i = \frac{G}{36\pi r^2} \left[\frac{1}{2} \ddot{Q}_{ij} \ddot{Q}_{ij} - \ddot{Q}_{ka} \ddot{Q}_{km} n^k n^m + \frac{1}{4} (\ddot{Q}_{ka} n^k n^a)^2 \right]$$

$$\Rightarrow \text{total emitted power} \quad P = \frac{dE}{dt} = r^2 \int d\Omega E^{oi} n^i$$

$$\mathbf{n}^i = (\sin\theta \cos\varphi, \sin\theta \sin\varphi, \cos\theta)$$

$$\text{use } \int d\Omega = 4\pi$$

$$\int d\Omega n^k n^m =$$

$$\rightarrow D_{33} = D_{11} + D_{22} + 2D_{11}D_{22}$$

$$\Rightarrow 2A = 1$$

$$B = -2A$$

$$2A + B + C = \frac{1}{4}$$

$$A = \frac{1}{2}$$

$$= -1$$

$$\Rightarrow C = \frac{1}{4}$$

$$\Rightarrow \text{in direction } \mathbf{n}^i, \quad E^{oi} n^i = \frac{G}{36\pi r^2} \left[\frac{1}{2} \ddot{Q}_{ij} \ddot{Q}_{ij} - \ddot{Q}_{ka} \ddot{Q}_{km} n^k n^m + \frac{1}{4} (\ddot{Q}_{ka} n^k n^a)^2 \right]$$

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$$\int d\Omega n^k n^m = \frac{\delta^{km}}{3} 4\pi$$

$$\rightarrow D_{33} = D_{11} + D_{22} + 2D_{11}D_{22}$$

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$$\Rightarrow \text{in a direction } \mathbf{n}^i, \quad E^{oi} n^i = \frac{G}{36\pi r^2} \left[\frac{1}{2} \ddot{Q}_{ij} \ddot{Q}_{ij} - \ddot{Q}_{ka} \ddot{Q}_{km} n^k n^m + \frac{1}{4} (\ddot{Q}_{ka} n^k n^a)^2 \right]$$

$$\Rightarrow \text{total emitted power} \quad \frac{dE}{dt} = r^2 \int d\Omega E^{oi} n^i$$

$$\mathbf{n}^i = (\sin\theta \cos\phi, \sin\theta \sin\phi, \cos\theta)$$

$$\Omega = 4\pi$$

$$\int_{\Omega} n^m = \frac{8\pi}{3}$$

(by using rotational invariance
 $\int n^i n^j d\Omega = \frac{4\pi}{3} \delta^{ij}$
 - only calc trace)

$$\rightarrow D_{33} = D_{11} + D_{22} + 2D_{11}D_{22}$$

$$\Rightarrow 2A = 1 \quad B = -2A \quad 2A + B + C = \frac{1}{4}$$

$$A = \frac{1}{2} \quad = -1 \quad \Rightarrow C = \frac{1}{4}$$

$$\Rightarrow \text{in direction } \mathbf{n}^i, \quad E^{oi} n^i = \frac{G}{36\pi r^2} \left[\frac{1}{2} \ddot{Q}_{ij} \ddot{Q}_{ij} - \ddot{Q}_{ka} \ddot{Q}_{km} n^k n^m + \frac{1}{4} (\ddot{Q}_{ka} n^k n^a)^2 \right]$$

total emitted power

$$P = \frac{dE}{dt} = r^2 \int d\Omega E^{oi} n^i$$

$$\mathbf{n}^i = (\sin\theta \cos\phi, \sin\theta \sin\phi, \cos\theta)$$

$$\text{use } \int d\Omega = 4\pi$$

$$\int d\Omega n^k n^m = \frac{\delta^{km}}{3} 4\pi$$

$$\int d\Omega n^k n^m n^a n^b = \frac{4\pi}{15} (\delta^{km} \delta^{ab} + \delta^{ka} \delta^{bm} + \delta^{kb} \delta^{am})$$

(by using rotational invariance
 $\int n^i n^j d\Omega = \frac{4\pi}{3} \delta^{ij}$
 - only calc trace)

$$\rightarrow D_{33} = D_{11} + D_{22} + 2D_{11}D_{22}$$

$$\Rightarrow 2A = 1 \quad B = -2A \quad 2A + B + C = \frac{1}{4}$$

$$A = \frac{1}{2} \quad = -1 \quad \Rightarrow C = \frac{1}{4}$$

$\Rightarrow$ in a direction n^i

$$E^{oi} n^i = \frac{G}{36\pi r^2} \left[\frac{1}{2} \ddot{Q}_{ij} \ddot{Q}_{ij} - \ddot{Q}_{ka} \ddot{Q}_{km} n^k n^m + \frac{1}{4} (\ddot{Q}_{ka} n^k n^a)^2 \right]$$

$\Rightarrow$ tot a power

$$P = \frac{dE}{dt} = r^2 \int d\Omega E^{oi} n^i$$

$n^i = (\sin\theta \cos\phi, \sin\theta \sin\phi, \cos\theta)$

use $\int d\Omega = 4\pi$

$$\int d\Omega n^l n^m = \frac{\delta^{lm}}{3} 4\pi$$

(by using rot. invariance
 $\int n^i n^j d\Omega = \delta^{ij} \int d\Omega$
 - only calc trace)

$$\int d\Omega n^l n^m n^a n^b = \frac{1}{15} (\delta^{lm} \delta^{ab} + \delta^{la} \delta^{mb} + \delta^{lb} \delta^{ma})$$

$$\rightarrow D_{33} = D_{11} + D_{22} + 2D_{11}D_{22}$$

$$\Rightarrow 2A = 1 \quad B = -2A$$

$$A = \frac{1}{2} \quad = -1$$

$$2A + B + C = \frac{1}{4}$$

$$\Rightarrow C = \frac{1}{4}$$

$$\Rightarrow \text{in a direction } \mathbf{n}^i, \quad E^{oi} n^i = \frac{G}{36\pi r^2} \left[\frac{1}{2} \ddot{Q}_{ij} \ddot{Q}_{ij} - \ddot{Q}_{ke} \ddot{Q}_{km} n^e n^m + \frac{1}{4} (\ddot{Q}_{ke} n^k n^e)^2 \right]$$

$$\Rightarrow \text{total emitted power} \quad P = \frac{dE}{dt} = r^2 \int d\Omega E^{oi} n^i$$

$$\mathbf{n}^x = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

$$\text{use } \int d\Omega = 4\pi$$

$$\int d\Omega n^e n^m = \frac{\delta^{em}}{3} 4\pi$$

(by using rot. invariance
 $\int n^i n^j \delta^{ij}$
 - only calc trace)

$$\int d\Omega n^e n^m n^a n^b = \# (\delta^{em} \delta^{ab} + \delta^{ea} \delta^{mb} + \delta^{eb} \delta^{ma})$$

$$\text{trace lhs and r.h.s.} \quad \text{lhs} = 4\pi$$

$$\text{rhs} = \# \cdot 15$$

$$\left[-\ddot{Q}_{ke} \ddot{Q}_{km} n^e n^m + \frac{1}{4} (\ddot{Q}_{ke} n^k n^e)^2 \right]$$

$\ddot{Q}_{ke} n^k$

(by using rot. invariance
 $\int n^i n^j \delta_{ij}$
 - only calc trace)
 $\int g_{ab} g^{ab} + g_{ab} g^{ba} + g_{aa} g^{aa}$)

$$\text{trace } g_{ab} \text{ and } g^{ab} \quad \text{lhs} = 4\pi \Rightarrow \# = \frac{4\pi}{15}$$

$$\text{rhs} = \# \cdot 15$$

$$\int d\Omega n^l n^m = \frac{\delta^{lm}}{3} 4\pi$$

(by using $\int n^i n^j d\Omega = \frac{4\pi}{3} \delta^{ij}$
- only calc trace)

$$\int d\Omega n^l n^m n^a n^b = \# (\delta^{lm} \delta^{ab} + \delta^{la} \delta^{mb} + \delta^{lb} \delta^{ma})$$

trace $l=m$ and $a=b$
rhs = $\# \cdot 15$

$$\Rightarrow P = \frac{G}{45} \ddot{Q}_{kl} \ddot{Q}_{kl}$$

$$\int d\Omega n^l n^m = \frac{\delta^{lm}}{3} 4\pi \quad \left(\begin{array}{l} \text{by using } \int n^i n^j \\ \text{- only calc trace} \end{array} \right)$$

$$\int d\Omega n^l n^m n^a n^b = \# \left(\delta^{lm} \delta^{ab} + \delta^{la} \delta^{mb} + \delta^{lb} \delta^{ma} \right)$$

trace l,m and a,b l
 $\text{rhs} = \# \cdot 15$

$$\Rightarrow \frac{G}{45} \ddot{Q}_{kl} \ddot{Q}_{kl}$$

$$\frac{G}{9} \left(\frac{1}{2} - \frac{1}{3} \right)$$

$$\int d\Omega n^l n^m = \frac{\delta^{lm}}{3} 4\pi \quad \left(\begin{array}{l} \text{by using} \\ \int n^i n^j \\ \text{only calc trace} \end{array} \right)$$

$$\ddot{Q}_{ab} \ddot{Q}_{ab} \int d\Omega n^l n^m n^a n^b = \# \left(\delta^{lm} \delta^{ab} + \delta^{la} \delta^{mb} + \delta^{lb} \delta^{ma} \right) \quad \begin{array}{l} \text{trace } l,m \text{ and } a,b \\ \text{rhs} = \# \cdot 15 \end{array}$$

$$\Rightarrow P = \frac{G}{45} \ddot{Q}_{kl} \ddot{Q}_{kl}$$

$$\frac{G}{9} \left(\frac{1}{2} - \frac{1}{3} + \frac{1}{4} \cdot \frac{1}{15} \cdot 2 \right)$$

$$\frac{1}{30} (15 - 10 + 1)$$

$$\frac{1}{5}$$

$$\int d\Omega n^l n^m = \frac{\delta^{lm}}{3} 4\pi \quad \left(\begin{array}{l} \text{by using } \int n^i n^j \\ \text{only calc trace} \end{array} \right)$$

$$\ddot{Q}_{ab} \ddot{Q}_{ab} \int d\Omega n^l n^m n^a n^b = \# \left(\delta^{lm} \delta^{ab} + \delta^{la} \delta^{mb} + \delta^{lb} \delta^{ma} \right) \quad \begin{array}{l} \text{trace } l,m \text{ and } a,b \\ \text{rhs} = \# \cdot 15 \end{array}$$

$$\Rightarrow P = \frac{G}{45} \ddot{Q}_{kl} \ddot{Q}_{kl} \quad \frac{G}{9} \left(\frac{1}{2} - \frac{1}{3} + \frac{1}{4} \cdot \frac{1}{15} \cdot 2 \right)$$

$$\frac{1}{30} (15 - 10 + 1) = \frac{1}{5}$$

Landau-Lifshitz Quadrupole Radiation formula.

e.g. use this for LIGO's black holes (only approximate!)



$$\frac{dE}{dt}$$

e.g. use this for LIGO's black holes (only approximate!)



$$\frac{dE}{dt}$$

e.g. use this for LIGO's black holes (only approximate!)



$$\frac{dE}{dt}$$

e.g. use this for LIGO's black holes (only approximate!)



$$\frac{dE}{dt} = \frac{G \ddot{Q}_{ke} \ddot{Q}_{ke}}{45 c^5}$$

$$Q_{ij} = \int \rho_{\text{matter}} (x^i x^j - \delta^{ij} r^2) d^3x \quad [Q] =$$

$$\left[\frac{GM}{Lc^2} \right] \sim 1$$

$$[] \sim \left[\frac{G M L^2 M L^2}{c^5} \right]$$

e.g. use this for LIGO's black holes (only approximate!)

$$\frac{dE}{dt} = \frac{G \ddot{Q}_{kl} \ddot{Q}_{kl}}{45 c^5}$$

$$Q^{ij} = \int \rho_{\text{matter}} (x^i x^j - \delta^{ij} r^2) d^3x \quad [Q] = ML^2$$

$$\left[\frac{GM}{Lc^2} \right] \sim 1$$

$$[] \sim \left[\frac{G M L^2 M L^2}{c^5} \right] \sim \left[\frac{M L^5}{c^3} \right]$$

$$\left[\frac{dE}{dt} \right] = \left[\frac{M c^3}{L} \right]$$

e.g. use this for LIGO's black holes (only approximate!)



$$\frac{dE}{dt} = \frac{G \ddot{Q}_{ke} \ddot{Q}_{ke}}{45 c^5}$$

$$Q^{ij} = \int \rho_{\text{matter}} (x^i x^j - \delta^{ij} r^2) d^3x \quad [Q] = ML^2$$

$$\left[\frac{GM}{Lc^2} \right] \sim 1$$

$$[] \sim \left[\frac{G M L^2 M L^2}{c^5 T^6} \right] \sim \left[\frac{M L^5}{c^3 T^6} \right] \sim \left[\frac{M c^2}{T} \right]$$

$$\left[\frac{dE}{dt} \right] = \left[\frac{M c^2}{T} \right]$$

approximate !)

$$-s_{ij})d\vec{x} \quad [Q] = ML^2$$

timescale
for
decay

$$\left[\frac{Mc^2}{T} \right]$$

for 2 black holes of $30M_\odot$ each

$$\frac{1}{Mc^2} \frac{d(Mc^2)}{dt} = \frac{1}{Mc^2} GM^2 r^4$$

e.g. use this for LIGO's black holes (only approximate!)



$$\frac{dE}{dt} = \frac{G \ddot{Q}_{ke} \ddot{Q}_{ke}}{45 c^5}$$

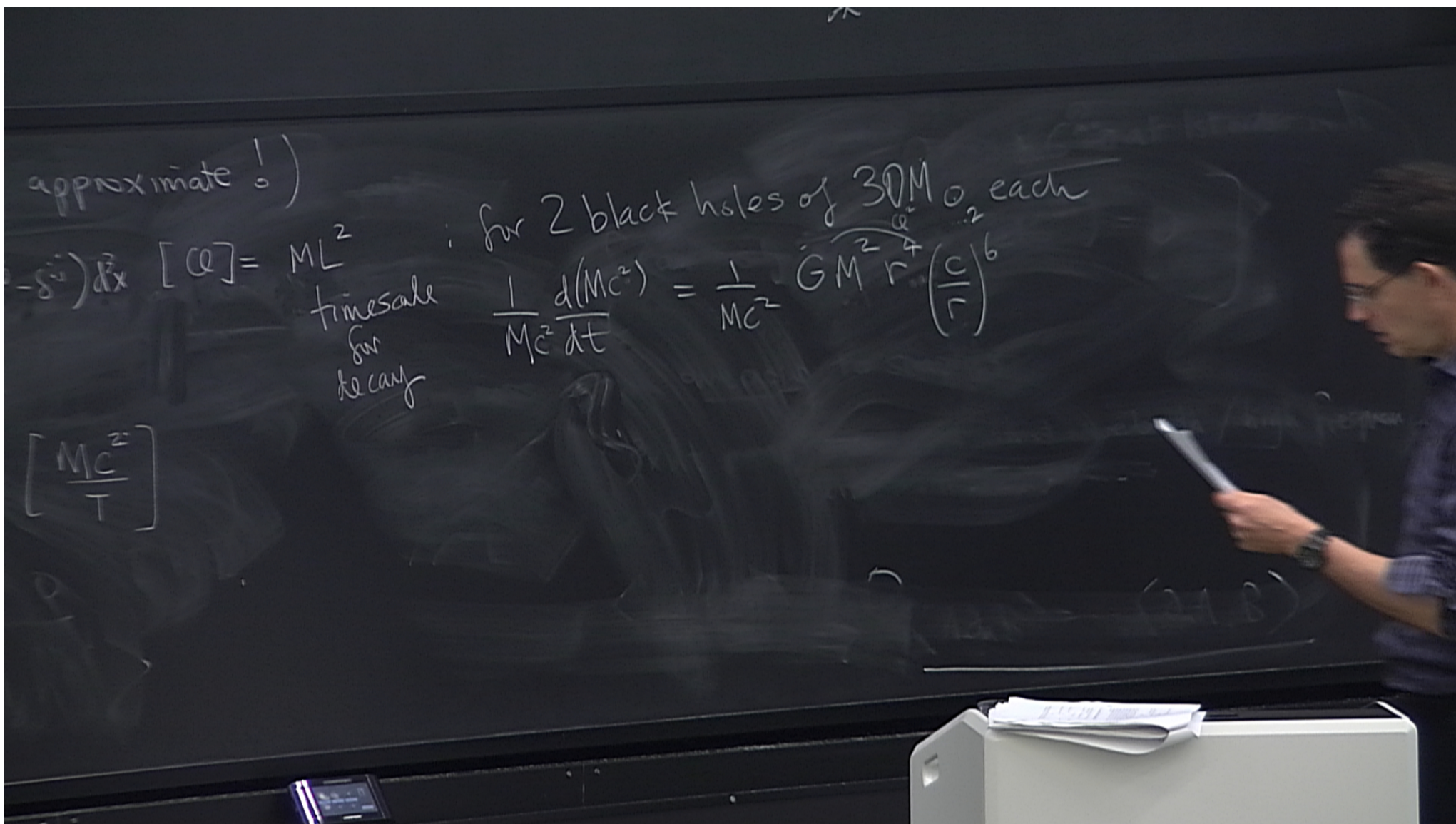
$$Q^{ij} = \int \rho_{\text{mass}} (x^i x^j - \delta^{ij} r^2) d^3x \quad [Q] = ML^2$$

timescale
for decay

$$\left[\frac{GM}{Lc^2} \right] \sim 1$$

$$Q \sim M r^2 \quad [] \sim \left[\frac{G M L^2 M L^2}{c^5 T^6} \right] \sim \left[\frac{M L^5}{c^3 T^6} \right] \sim \left[\frac{M c^2}{T} \right]$$

$$\left[\frac{dE}{dt} \right] = \left[\frac{M c^2}{T} \right]$$



approximate !)

$-8\pi G \rho \vec{x} \cdot d\vec{x} \quad [Q] = ML^2$
 timescale
 for
 decay

$\left[\frac{Mc^2}{T} \right]$

for 2 black holes of $30M_{\odot}$ each

$$\frac{1}{Mc^2} \frac{d(Mc^2)}{dt} = \frac{1}{Mc^2} \frac{GM^2}{r^4} \left(\frac{c}{r} \right)^6 = \frac{GM^2}{r^2 c}$$

approximate !)

$-8\pi G \rho$ $[Q] = ML^2$
 timescale
 for
 decay

$\left[\frac{Mc^2}{T} \right]$

for 2 black holes of $30M_{\odot}$ each

$$\frac{1}{Mc^2} \frac{d(Mc^2)}{dt} = \frac{1}{Mc^2} \frac{GM^2}{c^5} r^4 \left(\frac{c}{r} \right)^6 = \frac{GM^2}{r^2} c$$

approximate !)

$-g_{ij} dx^i dx^j$ $[Q] = ML^2$
 timescale
 for
 decay

$\left[\frac{Mc^2}{T} \right]$

for 2 black holes of $30 M_{\odot}$ each

$$\frac{1}{Mc^2} \frac{d(Mc^2)}{dt} = \frac{1}{Mc^2} \frac{GM^2}{c^5} r^4 \left(\frac{c}{r} \right)^6 = \frac{GM}{cr^2}$$

e.g. use this for LIGO's black holes (only approximate!)



$$\frac{dE}{dt} = \frac{G \ddot{Q}_{ke} \ddot{Q}_{ke}}{45 c^5}$$

$$Q^{ij} = \int \rho_{\text{mass}} (x^i x^j - \delta^{ij} r^2) d^3x \quad [Q] = ML^2$$

timescale for decay

$$\left[\frac{GM}{Lc^2} \right] \sim 1$$

$$Q \sim M r^2$$

$$\frac{GM}{r^2} \sim 1$$

$$[] \sim \left[\frac{G M L^2 M L^2}{c^5 T^6} \right] \sim \left[\frac{M L^5}{c^3 T^6} \right] \sim \left[\frac{M c^2}{T} \right]$$

$$\left[\frac{dE}{dt} \right] = \left[\frac{M c^2}{T} \right]$$

approximate !)

$$-s_{ij} dx^i dx^j [Q] = ML^2$$

timescale
for
decay

$$\left[\frac{Mc^2}{T} \right]$$

for 2 black holes of $30M_{\odot}$ each

$$\frac{1}{Mc^2} \frac{d(Mc^2)}{dt} = \frac{1}{Mc^2} \frac{GM^2}{c^5} r^4 \left(\frac{c}{r} \right)^6 = \frac{GM}{cr^2} = \frac{c}{r} = \frac{1}{\text{light xing time}}$$

$$\frac{GM}{c^2} = 1$$

approximate !)

$-s_{ij} dx^j$ $[Q] = ML^2$
 timescale
 for
 decay

$\left[\frac{Mc^2}{T} \right]$

What is r ?

for 2 black holes of $30M_{\odot}$ each

$$\frac{1}{Mc^2} \frac{d(Mc^2)}{dt} = \frac{1}{Mc^2} \frac{GM^2}{c^5} r^4 \left(\frac{c}{r} \right)^6 = \frac{GM}{cr^2} = \frac{c}{r} = \frac{1}{\text{light}}$$

$\frac{GM}{c^2} = 1$

convention

Solar mass: $r_s = 2 \frac{GM_{\odot}}{c^2} = 3 \text{ km.}$

approximate !)

$-8\pi G \rho$ $[Q] = ML^2$
 timescale
 for
 decay

$\left[\frac{Mc^2}{T} \right]$

What is r ?

for 2 black holes of $30 M_{\odot}$ each

$$\frac{1}{Mc^2} \frac{d(Mc^2)}{dt} = \frac{1}{Mc^2} \frac{GM^2}{c^5} r^{-4} \left(\frac{c}{r} \right)^6 = \frac{GM}{cr^2} = \frac{c}{r} = \frac{1}{\text{light} \times \text{time}}$$

$\frac{GM}{c^2} = 1$

Solar mass: $r_s = 2 \frac{GM_{\odot}}{c^2} = 3 \text{ km.}$

$30 M_{\odot} \sim 100 \text{ km} \sim 10^5 \text{ m.}$

approximate !)

$-s_{ij} dx^j$ $[Q] = ML^2$
 timescale for decay

$\left[\frac{Mc^2}{T} \right]$

What is r ?

for 2 black holes of $30 M_{\odot}$ each

$\frac{1}{Mc^2} \frac{d(Mc^2)}{dt} = \frac{1}{Mc^2} \frac{GM^2}{c^5} r^{-4} \left(\frac{c}{r} \right)^6 = \frac{GM}{cr^2} = \frac{c}{r} =$

$\frac{GM}{c^2} = 1$

convention

Solar mass: $r_s = 2 \frac{GM_{\odot}}{c^2} = 3 \text{ km.}$

$30 M_{\odot} \sim 100 \text{ km} \sim 10^5 \text{ m.} \Rightarrow \text{timescale } \frac{c}{r} \sim$

approximate !)

$$-s_{ij} dx^j [Q] = ML^2$$

timescale for decay

$$\left[\frac{Mc^2}{T} \right]$$

What is r ?

for 2 black holes of $30 M_{\odot}$ each

$$\frac{1}{Mc^2} \frac{d(Mc^2)}{dt} = \frac{1}{Mc^2} \frac{GM^2}{c^5} r^{-4} \left(\frac{c}{r} \right)^6 = \frac{GM}{cr^2} = \frac{c}{r} = \frac{1}{\text{light xing time}}$$

$$\frac{GM}{c^2} = 1$$

Solar mass: $r_s = 2 \frac{GM_{\odot}}{c^2} = 3 \text{ km.}$

$30 M_{\odot} \sim 100 \text{ km} \sim 10^5 \text{ m.} \Rightarrow \text{timescale } \frac{c}{r} \sim 10^{-3} \text{ s} //$

e.g. use this for LIGO



$$\frac{dE}{dt} = \frac{G \ddot{Q}_{ke} \ddot{Q}_{ke}}{45 c^5}$$

$$Q_{ij} = \int \rho_{mass} (2x^i x^j - \delta^{ij} r^2) d^3x \quad [Q] = ML^2$$

timescale for decay

$$\left[\frac{GM}{Lc^2} \right] \sim 1$$

$$Q \sim M r^2$$

$$\frac{GM}{r^3} \sim$$

$$[\] \sim \left[\frac{G M L^2 M L^2}{c^5 T^6} \right] \sim \left[\frac{M L^5}{c^3 T^6} \right] \sim \left[\frac{M c^2}{T} \right]$$

What is r?

$$\left[\frac{dE}{dt} \right] = \left[\frac{M c^2}{T} \right]$$

$$\frac{1}{3} \overline{Q_{ij} \ddot{Q}_{ij}}$$

$$E_{\text{axis, perm}} = \frac{1}{36\pi r^2}$$

$$\frac{1}{16} \cdot \frac{4}{9} = \frac{1}{36}$$

e.g. use this for LIGO



$$\frac{dE}{dt} = \frac{G \ddot{Q}_{ke} \ddot{Q}_{ke}}{45 c^5}$$

$$Q_{ij} = \int \rho_{mass} (2x^i x^j - \delta_{ij} r^2) d^3x \quad [Q] = ML^2$$

timescale for decay

$$\left[\frac{GM}{Lc^2} \right] \sim 1$$

$$Q \sim M r^2$$

$$\frac{GM}{r^3} \sim 1$$

$$[] \sim \left[\frac{G M L^2 M L^2}{c^5 T^6} \right] \sim \left[\frac{M L^5}{c^3 T^6} \right] \sim \left[\frac{M c^2}{T} \right]$$

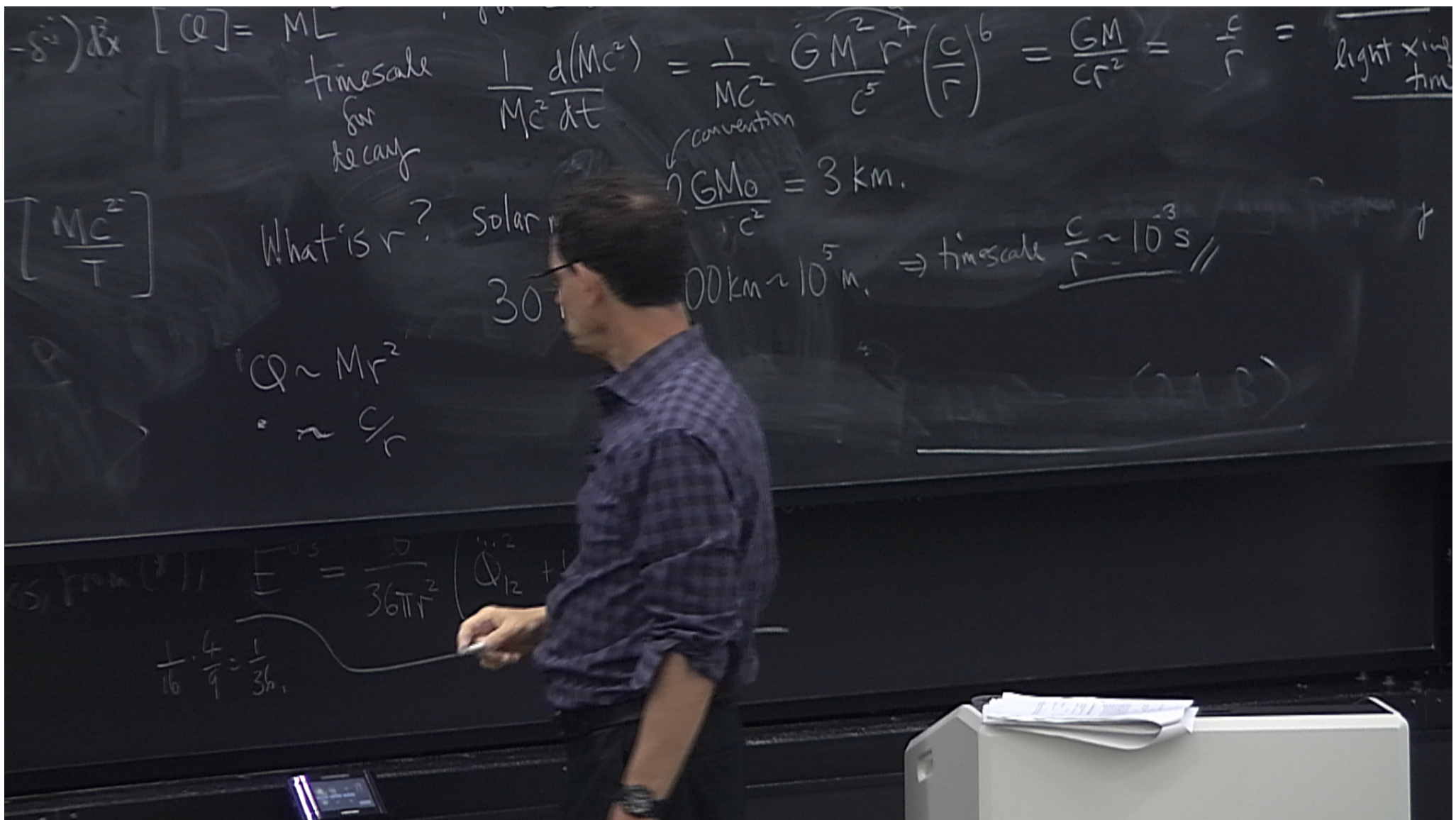
What is r?

$$\left[\frac{dE}{dt} \right] = \left[\frac{M c^2}{T} \right]$$

$$\frac{1}{35} Q_{ij}$$

$$E = \frac{5}{36 \pi^2} \dots$$

$$\frac{1}{16} \cdot \frac{4}{9} = \frac{1}{36}$$



e.g. use this for LIGO



$$\frac{dE}{dt} = \frac{G \ddot{Q}_{ke} \ddot{Q}_{ke}}{45 c^5}$$

$$Q^i_j = \int \rho_{mass} (2x^i x^j - \delta^{ij}) d^3x \quad [Q] = ML^2$$

timescale for decay

$$\left[\frac{GM}{Lc^2} \right] \sim 1$$

$$Q \sim M r^2$$

$$\frac{GM}{r^3} \sim 1$$

$$[] \sim \left[\frac{G M L^2 M L^2}{c^5 T^6} \right] \sim \left[\frac{M L^5}{T^6} \right] \sim \left[\frac{M c^2}{T} \right]$$

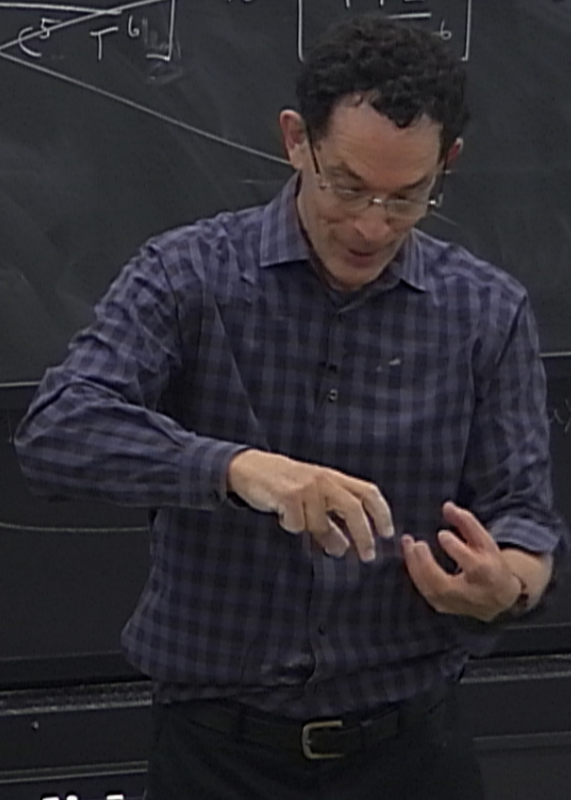
$$\left[\frac{dE}{dt} \right] = \left[\frac{M c^2}{T} \right]$$

What is r?



$$Q \sim M r^2$$

$$\sim \frac{c}{r}$$



$$E = \frac{1}{36\pi r^2}$$

$$\frac{1}{16} \cdot \frac{4}{9} = \frac{1}{36}$$

e.g. use this for LIGO



$$\boxed{\frac{dE}{dt} = \frac{G \ddot{Q}_{ke} \ddot{Q}_{ke}}{45 c^5}}$$

$$Q^i = \int \rho_m (2x^i x^i - \delta^{ii}) d^3x \quad [Q] = ML^2$$

timescale for decay

$$\left[\frac{GM}{Lc^2} \right] \sim 1$$

$$Q \sim Mr^2$$

$$\frac{GM}{r^3} \sim 1$$

$$v^2 = \frac{GM}{r}$$

$$[] \sim \left[\frac{GML^2 ML^2}{c^5 T^6} \right] \sim \left[\frac{ML^5}{c^3 T^6} \right] \sim \left[\frac{Mc^2}{T} \right]$$

$$\left[\frac{dE}{dt} \right] = \left[\frac{Mc^2}{T} \right]$$



What is r?

$$Q \sim Mr^2$$

$$\sim \frac{c}{r}$$

$$\frac{1}{35} Q_{ij}$$

$$E^{00} = \frac{5}{36\pi r^2}$$

$$\frac{1}{16} \cdot \frac{4}{9} = \frac{1}{36}$$

e.g. use this for LIGO



$$\boxed{\frac{dE}{dt} = \frac{G \ddot{Q}_{ke} \ddot{Q}_{ke}}{45 c^5}}$$

$$Q^i = \int \rho_{mass} (2x^i x^i - \delta^{ij}) d^3x \quad [Q] = ML^2$$

timescale for decay

$$\left[\frac{GM}{Lc^2} \right] \sim 1$$

$$Q \sim Mr^2$$

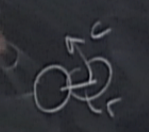
$$\frac{GM}{r^2} \sim 1$$

$$[] \sim \left[\frac{GML^2 ML^2}{c^5 T^6} \right] \sim \left[\frac{M}{c} \right] \left[\frac{Mc^2}{T} \right]$$

What is r?

$$V^2 = \frac{GM}{r} = c^2 \quad \left[\frac{dE}{dt} \right] = \left[\frac{Mc^2}{T} \right]$$

gravity strong



$$Q \sim Mr^2$$

$$\sim \frac{c}{r}$$

$$E = \frac{5}{36\pi r^2}$$

$$\frac{1}{16} \cdot \frac{4}{9} = \frac{1}{36}$$