

Title: PSI 2016/2017 Relativity - Lecture 10

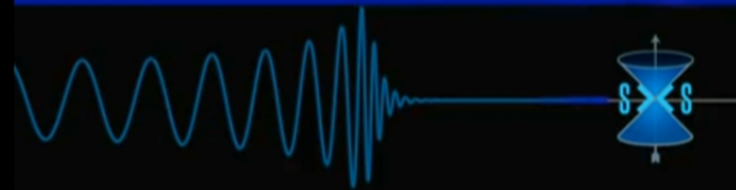
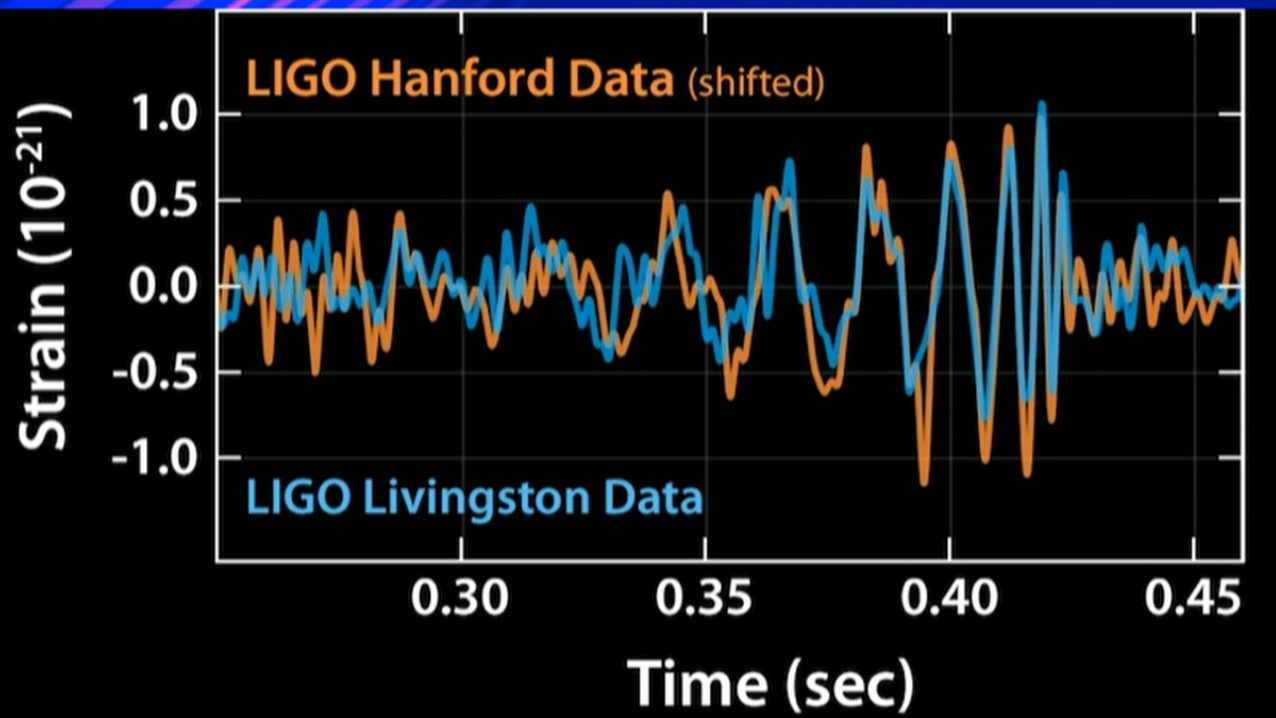
Date: Sep 19, 2016 09:00 AM

URL: <http://pirsa.org/16090010>

Abstract:

0.09s

discovery of the century



Shocks in the Early Universe

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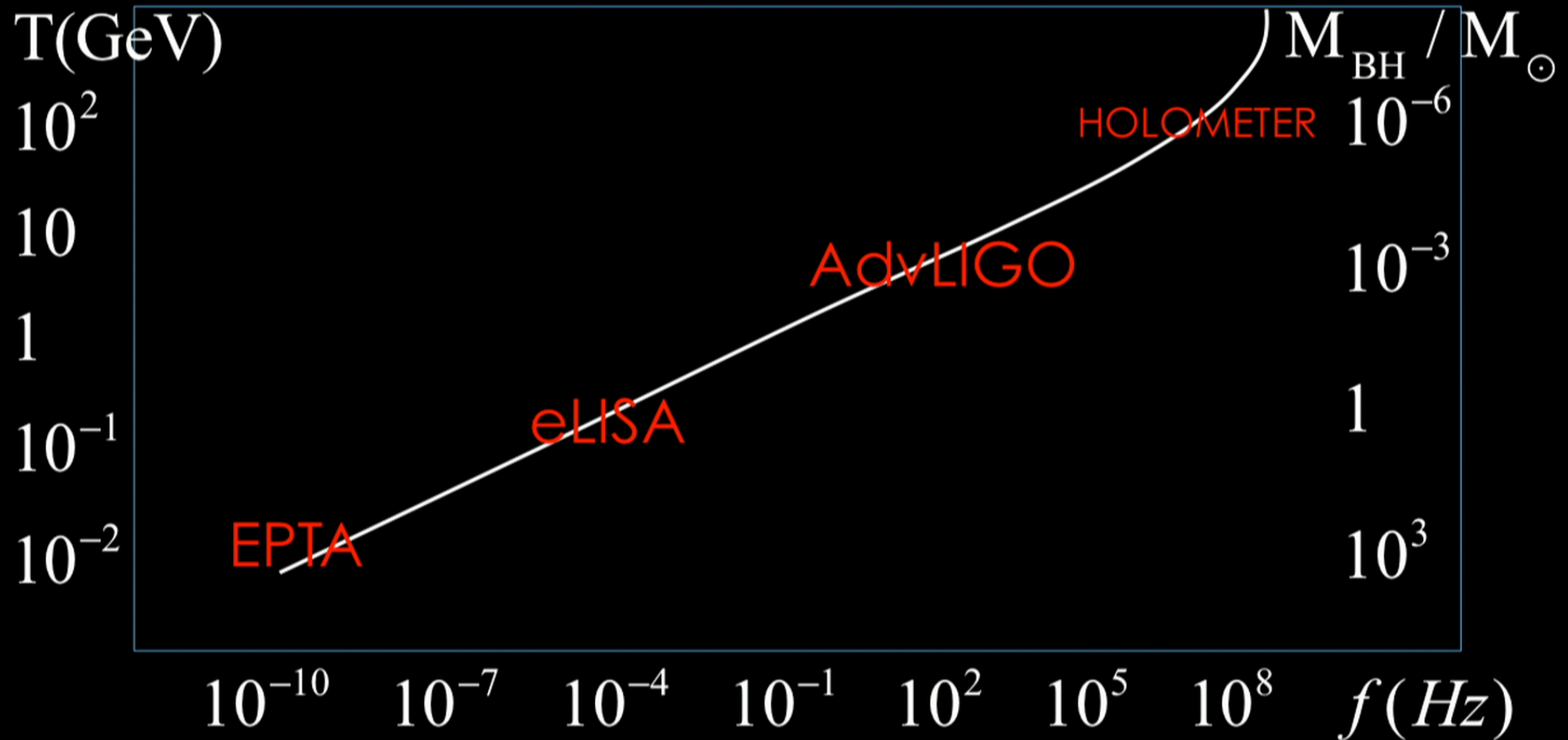
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(Received 21 October 2015; revised manuscript received 16 August 2016)

We point out a surprising consequence of the usually assumed initial conditions for cosmological perturbations. Namely, a spectrum of Gaussian, linear, adiabatic, scalar, growing mode perturbations not only creates acoustic oscillations of the kind observed on very large scales today, it also leads to the production of shocks in the radiation fluid of the very early Universe. Shocks cause departures from local thermal equilibrium as well as create vorticity and gravitational waves. For a scale-invariant spectrum and standard model physics, shocks form for temperatures $1 \text{ GeV} < T < 10^7 \text{ GeV}$. For more general power spectra, such as have been invoked to form primordial black holes, shock formation and the consequent gravitational wave emission provide a signal detectable by current and planned gravitational wave experiments, allowing them to strongly constrain conditions present in the primordial Universe as early as 10^{-30} sec after the big bang.

$$\Omega_{gw}(f) \approx \# \varepsilon^4 \Omega_{rad}; h \approx \# 10^{-10} \varepsilon^2 (\text{MeV} / T_s) \text{ at horizon}$$



$$\int -\frac{1}{4} F^2 \quad \sum [\partial_\mu + A_\mu, \partial_\nu + A_\nu] \frac{1}{Tr} (F_{\mu\nu} F^{\mu\nu})$$

$$A_\mu \rightarrow A_\mu + \partial_\mu \epsilon$$

$F_{\mu\nu}$ invariant

$$g_{\mu\nu} \rightarrow g_{\mu\nu} - \nabla_\mu \epsilon_\nu - \nabla_\nu \epsilon_\mu \quad \text{invariants are quantities like } R, R^{\mu\nu} R_{\mu\nu}, R^{\mu\nu\rho\lambda} R_{\mu\nu\rho\lambda} \text{ etc.}$$

field strength $\sim R_{\mu\nu\rho\lambda}$

actions $\int \sqrt{-g} R$ not $\int \sqrt{-g} R^{\mu\nu\rho\lambda} R_{\mu\nu\rho\lambda}$

Gravitational waves

$$R_{\mu\nu} = 0$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

}

$$R_{\mu\nu} = -\frac{1}{2} \square h_{\mu\nu} = 0 \quad \text{in harmonic gauge}$$

$$\partial_\mu (h^\mu{}_\nu - \frac{1}{2} \delta^\mu_\nu h^\lambda{}_\lambda) = 0$$

harmonic gauge

$$\partial_\mu \left(h^\mu_\nu - \frac{1}{2} \delta^\mu_\nu h^\lambda_\lambda \right) = 0$$

$\uparrow$

$g_{\mu\nu} \rightarrow h_{\mu\nu} - \epsilon_{\mu,\nu} - \epsilon_{\nu,\mu}$, preserves if $\square \epsilon_\nu = 0$.

$$\partial_\mu \left(h^\mu_\nu - \frac{1}{2} \delta^\mu_\nu h^\lambda_\lambda \right) = 0$$

age

$\epsilon_{\mu\nu} = \epsilon_{\nu\mu}$

preserves

if $D\epsilon_\nu = 0$.

(obvious from $Dx^\mu = 0$).

Gravitational waves

$$R_{\mu\nu} = 0$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$R_{\mu\nu} = -\frac{1}{2} \Box h_{\mu\nu} = 0$$

in harmonic gauge ∂_μ

residual gauge freedom: $h_{\mu\nu} \rightarrow h_{\mu\nu} - \epsilon_{\mu,\nu} - \epsilon_{\nu,\mu}$

$$R_{\mu\nu} = -\frac{1}{2} \Box h_{\mu\nu} = 0$$

in harmonic gauge

$$\partial_\mu \left(h^\mu_\nu - \frac{1}{2} \delta^\mu_\nu h^\lambda_\lambda \right)$$

residual gauge freedom:

$$h_{\mu\nu} \rightarrow h_{\mu\nu} - \epsilon_{\mu,\nu} - \epsilon_{\nu,\mu}, \text{ preserves}$$

write $\epsilon_\mu = \sum_{\vec{k}} i c_\mu(\vec{k}) e^{i\vec{k} \cdot \vec{x}}$

$$k^2 = -k^0^2 + \vec{k}^2 = 0$$

Notes

$$\left. \begin{aligned} R_{\mu\nu} &= -\frac{1}{2} \Box h_{\mu\nu} = 0 \quad \text{in harmonic gauge} \\ \text{residual gauge freedom: } h_{\mu\nu} &\rightarrow h_{\mu\nu} - \epsilon_{\mu,\nu} - \epsilon_{\nu,\mu}, \text{ preserves} \end{aligned} \right\}$$

cont. $\epsilon_\mu = \sum_{\vec{k}} i c_\mu(\vec{k}) e^{ik \cdot x}$ $k^2 = -k^0^2 + \vec{k}^2 = 0$

free to transform $\epsilon_{\mu\nu} \rightarrow \epsilon_{\mu\nu} + k_\mu \epsilon_\nu + k_\nu \epsilon_\mu$.

Gravitational waves

$$R_{\mu\nu} = 0$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

work to first order
in h .

$$R_{\mu\nu}^{(1)} = -\frac{1}{2} \square h_{\mu\nu} = 0$$

residual gauge freedom: $h_{\mu\nu}$

$$\text{write } \epsilon_{\mu} = \sum_{\vec{k}} i c_{\mu}(\vec{k}) e^{i \vec{k} \cdot \vec{x}}$$

free to transform $\epsilon_{\mu\nu} \rightarrow \epsilon_{\mu\nu}$

$$\square \equiv g^{\mu\nu} \nabla_{\mu} \nabla_{\nu}$$

3d spherical

$$ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2$$

$$\vec{\nabla}^2 = \partial_r^2 + \frac{1}{r^2} \partial_{\theta}^2 + \frac{1}{r^2 \sin^2 \theta} \partial_{\varphi}^2$$

in harmonic gauge $\partial_\mu (h^\mu_\nu - \frac{1}{2} \delta^\mu_\nu h^\lambda_\lambda) = 0$

trans: $h_{\mu\nu} \rightarrow h_{\mu\nu} - \epsilon_{\mu,\nu} - \epsilon_{\nu,\mu}$, preserves if $\square \epsilon_\nu = 0$. (obvious from $\square x^\mu = 0$)

$k^2 = -k^0^2 + \vec{k}^2 = 0$

$\epsilon_{\mu\nu} \rightarrow \underline{\epsilon_{\mu\nu} + k_\mu \epsilon_\nu + k_\nu \epsilon_\mu}$ while preserving harmonic gauge.

positive z direction, i.e., $k^\mu = (k, 0, 0, k)$, $k_\mu = (-k, 0, 0, k)$ $(\eta_{\mu\nu})$

$$\frac{\partial_\mu h^\mu_\nu}{2\hbar} = \frac{1}{2} k (-\epsilon_{00} + \epsilon_{11} + \epsilon_{22} + \epsilon_{33})$$

the z direction, i.e., $k^\mu = (k, 0, 0, k)$, $k_\mu = (-k, 0, 0, k)$ $(\eta_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix})$

$\partial_\mu h^\mu_\nu$

$$\begin{aligned} \nu=0 \quad k(\epsilon_{00} + \epsilon_{30}) &= -\frac{1}{2}k(-\epsilon_{00} + \epsilon_{11} + \epsilon_{22} + \epsilon_{33}) \\ \nu=3 \quad k(\epsilon_{03} + \epsilon_{33}) &= \frac{1}{2}k(-\epsilon_{00} + \epsilon_{11} + \epsilon_{22} + \epsilon_{33}) \\ \nu=1 \quad k(\epsilon_{01} + \epsilon_{31}) &= 0 \\ \nu=2 \quad k(\epsilon_{02} + \epsilon_{32}) &= 0 \end{aligned}$$

$$\left\{ \begin{aligned} \epsilon_{01} &= -\epsilon_{31} \\ \epsilon_{02} &= -\epsilon_{32} \\ \epsilon_{03} &= -\frac{1}{2}(\epsilon_{00} + \epsilon_{33}) \\ \epsilon_{11} &= -\epsilon_{22} \end{aligned} \right.$$

Still have freedom to do harmonic coord transfs : $\epsilon'_\mu = \epsilon_\mu + \dots$

$$\epsilon'_{00} = \epsilon_{00} - 2kC_0$$

$$\epsilon'_{13} = \epsilon_{13} + kC_1$$

$$\epsilon'_{23} = \epsilon_{23} + kC_2$$

$$\epsilon'_{33} = \epsilon_{33} + 2kC_3$$

} pick C_μ so that these

$\Rightarrow$ The only nonzero components of the polarization tensor are $\epsilon_{\mu\nu} =$

pick c_μ so that these are all zero.

tensor are $\epsilon_{\mu\nu} =$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \epsilon_{11} & \epsilon_{12} & 0 \\ 0 & \epsilon_{12} & -\epsilon_{11} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

so that these are all zero

$$k^\mu \epsilon_{\mu\nu} = 0 \quad ; \quad k^\mu = (k, 0, 0, k)$$

$$\epsilon_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \epsilon_{11} & \epsilon_{12} & 0 \\ 0 & \epsilon_{12} & -\epsilon_{11} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

i.e. transverse traceless.

m as representations of $O(2)$.

$$\begin{pmatrix} \epsilon_{11} & \epsilon_{12} \\ \epsilon_{21} & \epsilon_{22} \end{pmatrix} \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

$$\begin{bmatrix} \epsilon_{22} = -\epsilon_{11} \\ \epsilon_{21} = \epsilon_{12} \end{bmatrix}$$

}

$$\epsilon_R \equiv \epsilon_{11} - i\epsilon_{12}$$

$$\epsilon_L = \epsilon_{11} + i\epsilon_{12}$$

transform as $e^{\pm i\theta}$

$$\epsilon'_R = e^{i\theta} \epsilon_R$$

$$\epsilon'_L = e^{-i\theta} \epsilon_L$$

of $O(2)$.

$$\begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$$

}

$$\epsilon_R \equiv \epsilon_{11} - i\epsilon_{12}$$

$$\epsilon_L \equiv \epsilon_{11} + i\epsilon_{12}$$

transform as $\overset{\text{spin } 2}{2i\theta}$

$$\epsilon'_R = e^{2i\theta} \epsilon_R$$

$$\epsilon'_L = e^{-2i\theta} \epsilon_L$$

$$\epsilon'_R, \epsilon'_L$$

are the helicity $+2, -2$ components of a
massless d.o.f with spin 2.

$$\epsilon_{jn}$$

$$\epsilon O^T$$

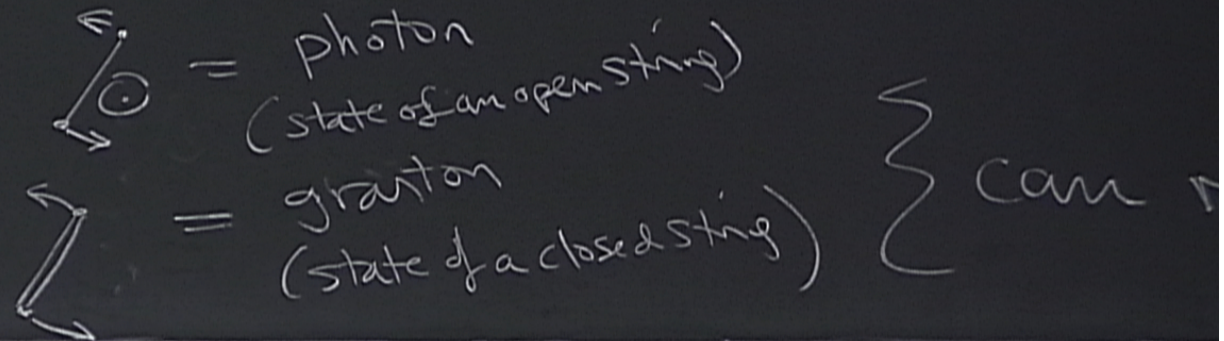
$$\begin{bmatrix} \epsilon_{22} = -\epsilon_{11} \\ \epsilon_{21} = \epsilon_{12} \end{bmatrix}$$

ϵ'_R, ϵ'_L
are the
Mass

$\left\{ \text{can repeat this for } A_\mu \text{ in EM, find it has spin } 1 \right\}$

$$\text{ie. } \epsilon = O \in O^T$$

String theory:



Emission of gravitational waves

want
to solve

$$\mathcal{R}_{\mu\nu}^{(1)} = -\frac{1}{2} \overset{\text{flat space}}{\square} h_{\mu\nu} = 8\pi G (\overset{\text{trace}}{T_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T^\lambda{}_\lambda})$$

treat as if in flat
space.
ie. $GT \sim h$

— the weak-field, linearized approximation.

(fails when gravity is important in determining the structure of the source).

only OK for "stiff" sources (in which grav. is unimportant).

Emission of gravitational waves

solve

$$R_{\mu\nu}^{(1)} = -\frac{1}{2} \square h_{\mu\nu} = 8\pi G \underbrace{\left(T_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T^\lambda{}_\lambda \right)}_{\frac{1}{M_{\text{Pl}}^2}}$$

↓ flat space

recall $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h^\lambda{}_\lambda$ (trace reversed $h_{\mu\nu}$)

treat as if in f
ie. $GT \sim$
- the
(fails wh
only

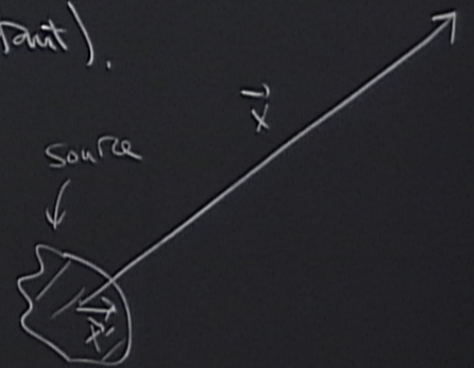
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$$\bar{h}_{\mu\nu}(t, \vec{x}) = -4G \int \frac{d^3\vec{x}'}{|\vec{x} - \vec{x}'|} T_{\mu\nu}(t - |\vec{x} - \vec{x}'|, \vec{x}')$$



Emission of gravitational waves

want
to solve

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

$$\Rightarrow -R = 8\pi G T^\lambda{}_\lambda$$

$$R_{\mu\nu} = 8\pi G \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T^\lambda{}_\lambda \right)$$

$$g_{\mu\nu} g^{\mu\nu} = 4$$

$$R_{\mu\nu}^{(1)} = -\frac{1}{2} \overset{\text{flat space}}{\square} h_{\mu\nu} = 8\pi G \left(T_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T^\lambda{}_\lambda \right)$$

$$\bar{h}^\lambda{}_\lambda = -h^\lambda{}_\lambda \text{ in 4d.}$$

$$\frac{1}{M_{\text{Pl}}^2}$$

recall $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h^\lambda{}_\lambda$ (trace reversed $h_{\mu\nu}$)

$$\Rightarrow -\frac{1}{2} \square \bar{h}_{\mu\nu} = 8\pi G T_{\mu\nu}$$

solution
familiar
from
EM

treat as
ie.

(fa

$\bar{h}$

gravitational waves

$$= 8\pi G \left(T_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T^\lambda{}_\lambda \right)$$

$\frac{1}{M_{\text{Pl}}^2}$

(trace reversed $h_{\mu\nu}$)

$$\bar{h}_{\mu\nu} = 8\pi G T_{\mu\nu}$$

solution
familiar
from
EM

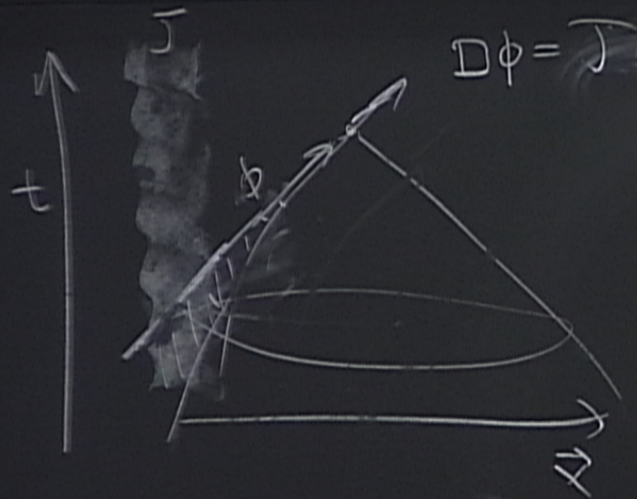
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$$\bar{h}_{\mu\nu}(t, \vec{x}) = -4G \int \frac{d^3\vec{x}'}{|\vec{x} - \vec{x}'|} T_{\mu\nu}(t - |\vec{x} - \vec{x}'|, \vec{x}')$$

$$\partial A_\mu = -J_\mu$$



Morse Feshbach
Math Methods

harmonic coord +

$$\begin{aligned} \epsilon'_{00} &= \epsilon_{00} - 2kC_0 \\ \epsilon'_{13} &= \epsilon_{13} + kC_1 \\ \epsilon'_{23} &= \epsilon_{23} + kC_2 \\ \epsilon'_{33} &= \epsilon_{33} + 2kC_3 \end{aligned}$$

$\Rightarrow$ The only nonzero components of the polarization tensor

gravitational waves

$$\pi G (T_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T^\lambda{}_\lambda)$$

$$\frac{1}{M_{\text{Pl}}^2}$$

space reversed $h_{\mu\nu}$

solution
familiar
from
EM

$$8\pi G T_{\mu\nu}$$

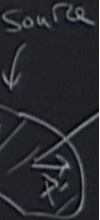
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$$\bar{h}_{\mu\nu}(t, \vec{x}) = -4G \int \frac{d^3\vec{x}'}{|\vec{x} - \vec{x}'|} T_{\mu\nu}(t - |\vec{x} - \vec{x}'|, \vec{x}')$$

$$\square A_\mu = -J_\mu$$



$$|\vec{x} - \vec{x}'| = \sqrt{(\vec{x} - \vec{x}')^2} = \sqrt{\vec{x}^2 - 2\vec{x} \cdot \vec{x}' + \vec{x}'^2} \quad r \equiv |\vec{x}| \gg |\vec{x}'|$$

$$\approx \frac{r}{\left(1 - \frac{\vec{x} \cdot \vec{x}'}{r^2} \dots\right)}$$

$$\bar{h}_{\mu\nu} \simeq -\frac{4G}{r} \int d^3\vec{x}' T_{\mu\nu}(t-r, \vec{x}')$$

Spatial components $\bar{h}_{ke} = -\frac{4G}{r} \int d^3\vec{x}' T_{ij}(t-r, \vec{x}')$

can show that $\int d^3\vec{x}' T_{ij}(t, \vec{x}') = \frac{1}{2} \frac{d^2}{dt^2} \int d^3\vec{x}' x^i x^j T(t, \vec{x}')$

$$d^3\vec{x}' T_{\mu\nu}(t-r, \vec{x}')$$

follows from

$$= -\frac{4G}{r} \int d^3\vec{x}' T_{ij}(t-r, \vec{x}')$$

$$\int d^3\vec{x}' T_{ij}(t, \vec{x}') = \frac{1}{2} \frac{d^2}{dt^2} \int d^3\vec{x}' x^i x^j T^{00}(t, \vec{x}')$$

follows from $\partial_\mu T^{\mu\nu} = 0$

$$\partial_0 T^{00} + \partial_i T^{i0} = 0$$

$$\partial_0 T^{0j} + \partial_i T^{ij} = 0$$

$$\left. \begin{array}{l} \partial_0 T^{00} + \partial_i T^{i0} = 0 \\ \partial_0 T^{0j} + \partial_i T^{ij} = 0 \end{array} \right\} \partial_0^2 T^{00} = \partial_i \partial_j T^{ij}$$

follows from $\partial_\mu T^{\mu\nu} = 0$

$$\partial_0 T^{00} + \partial_i T^{i0} =$$

$$\partial_0 T^{0j} + \partial_i T^{ij} =$$

$$= \frac{1}{2} \frac{d^2}{dt^2} \int d^3\vec{x}' x^i x^j T^{00}(t, \vec{x}')$$

$$= \frac{1}{2} \int d^3\vec{x} x^i x^j \partial_\ell \partial_m T^{\ell m}(t, \vec{x})$$

$$\stackrel{\text{Ex}}{=} \int d^3\vec{x}' T^{ij}(t, \vec{x}').$$

$$\Rightarrow \bar{h}_{\mu\nu} = -\frac{2G}{r} \left[\frac{d^2}{dt^2} \int d^3\vec{x}' x'^{\mu} x'^{\nu} T^{00}(t, \vec{x}') \right]_{t \rightarrow t-r.}$$

follows from $\partial_\mu T^{\mu\nu} = 0$ Energy-momentum consⁿ

$$\left. \begin{aligned} \nu=0 \quad \partial_0 T^{00} + \partial_i T^{i0} &= 0 \\ \nu=i \quad \partial_0 T^{0i} + \partial_i T^{ij} &= 0 \end{aligned} \right\} \partial_0^2 T^{00} = \partial_i \partial_j T^{ij}$$

$$E(t, \vec{x}) = \int d^3\vec{x}' T^{00}(t, \vec{x}')$$