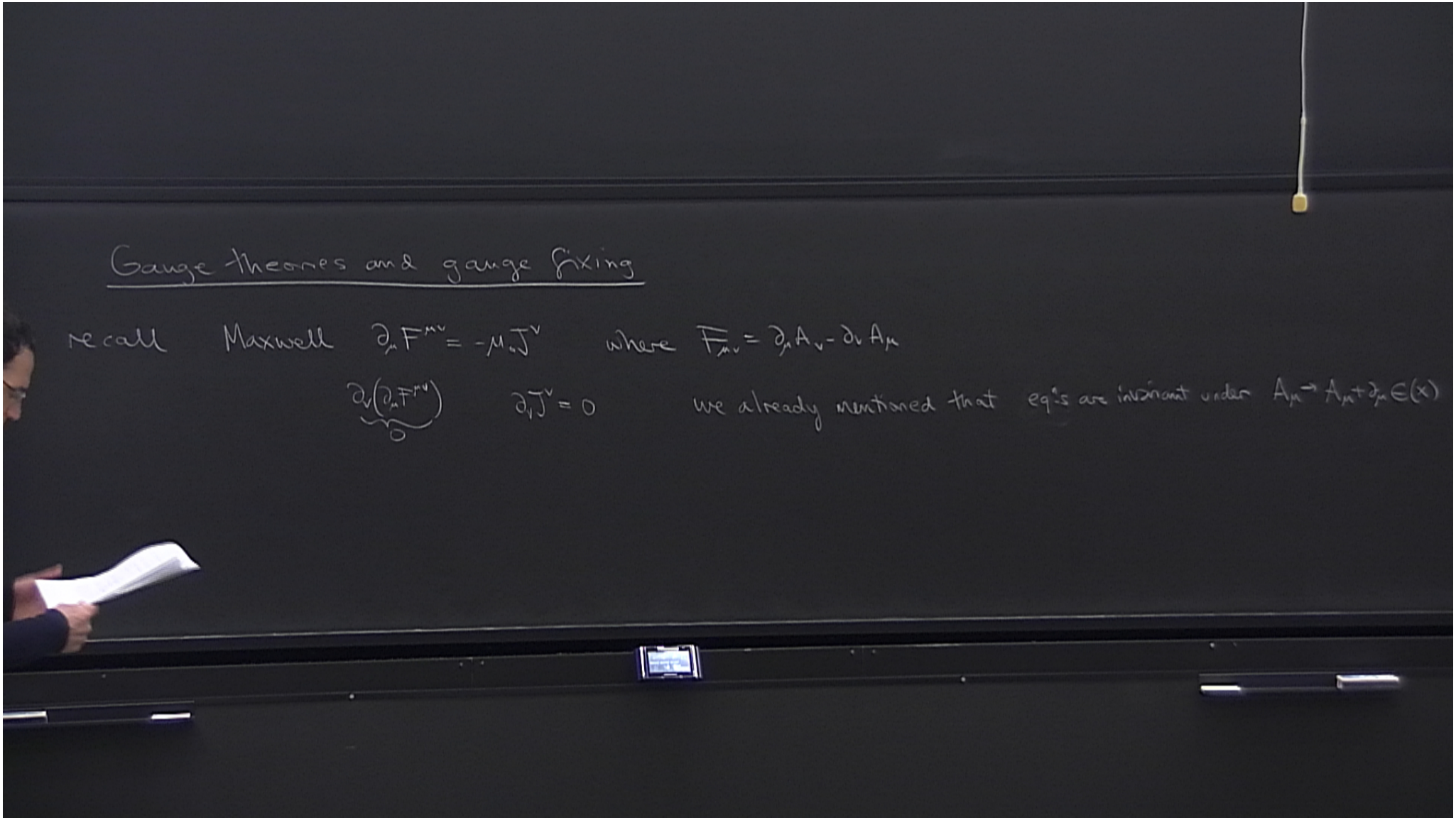
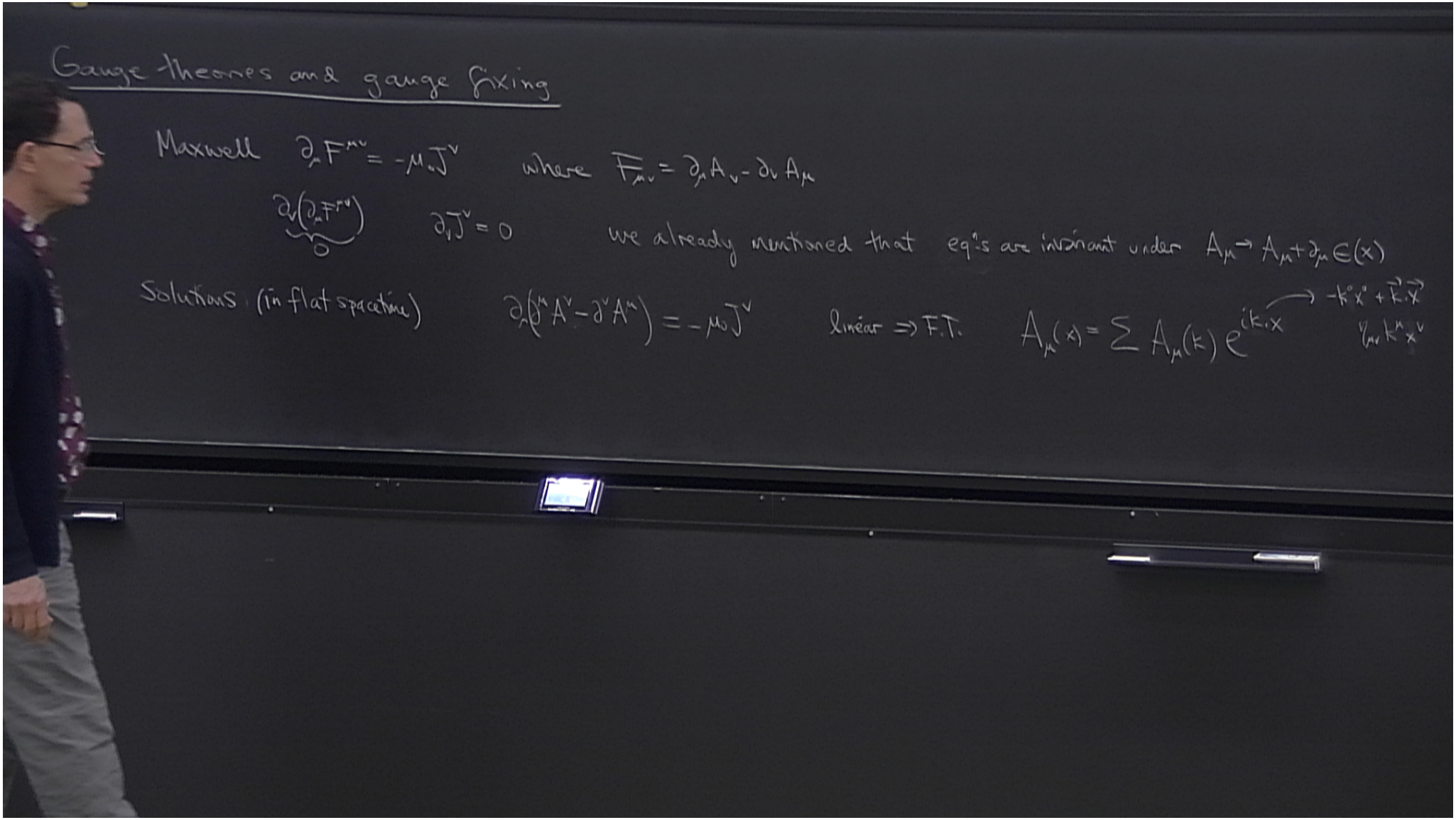
Title: PSI 2016/2017 Relativity - Lecture 9

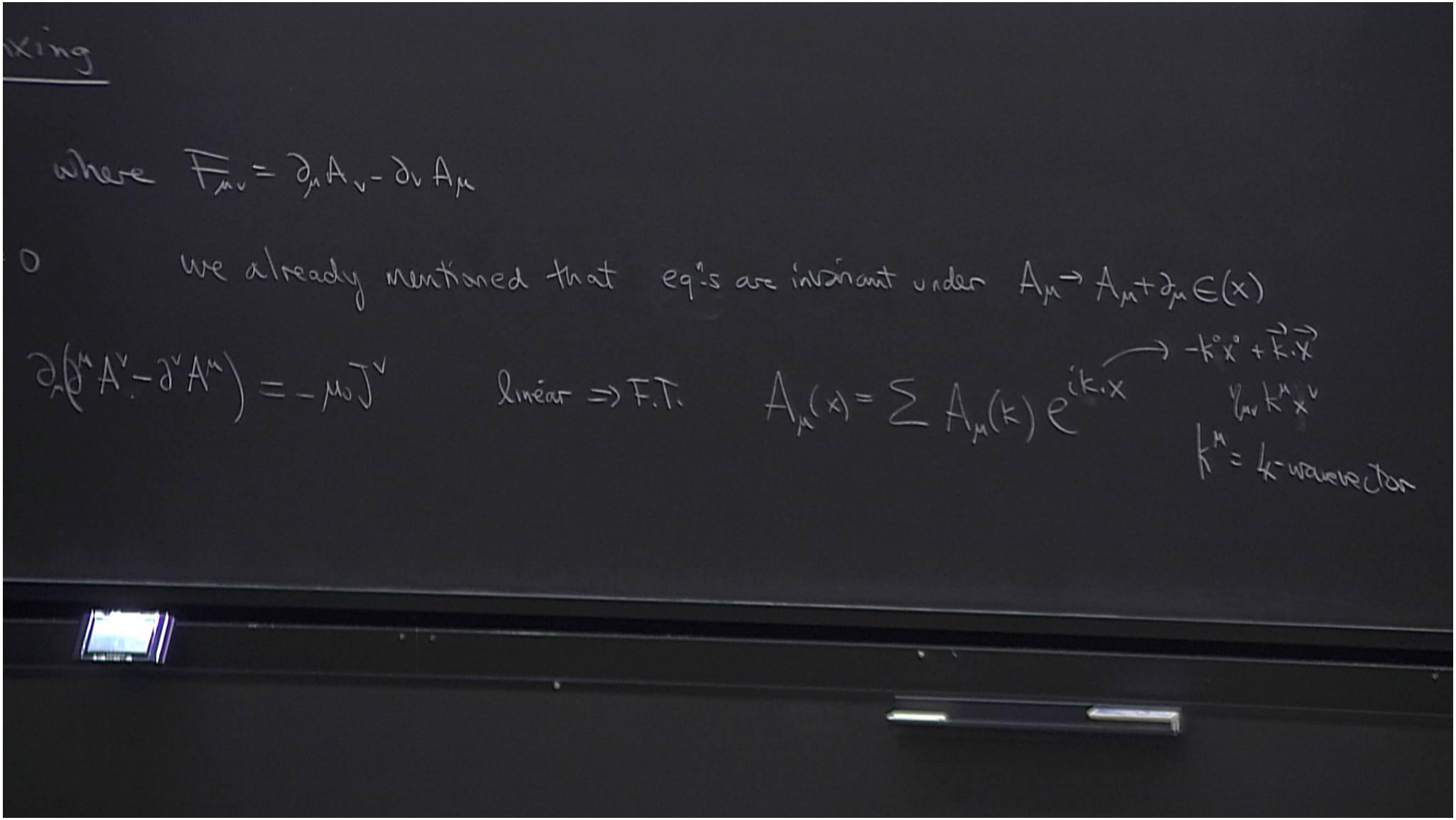
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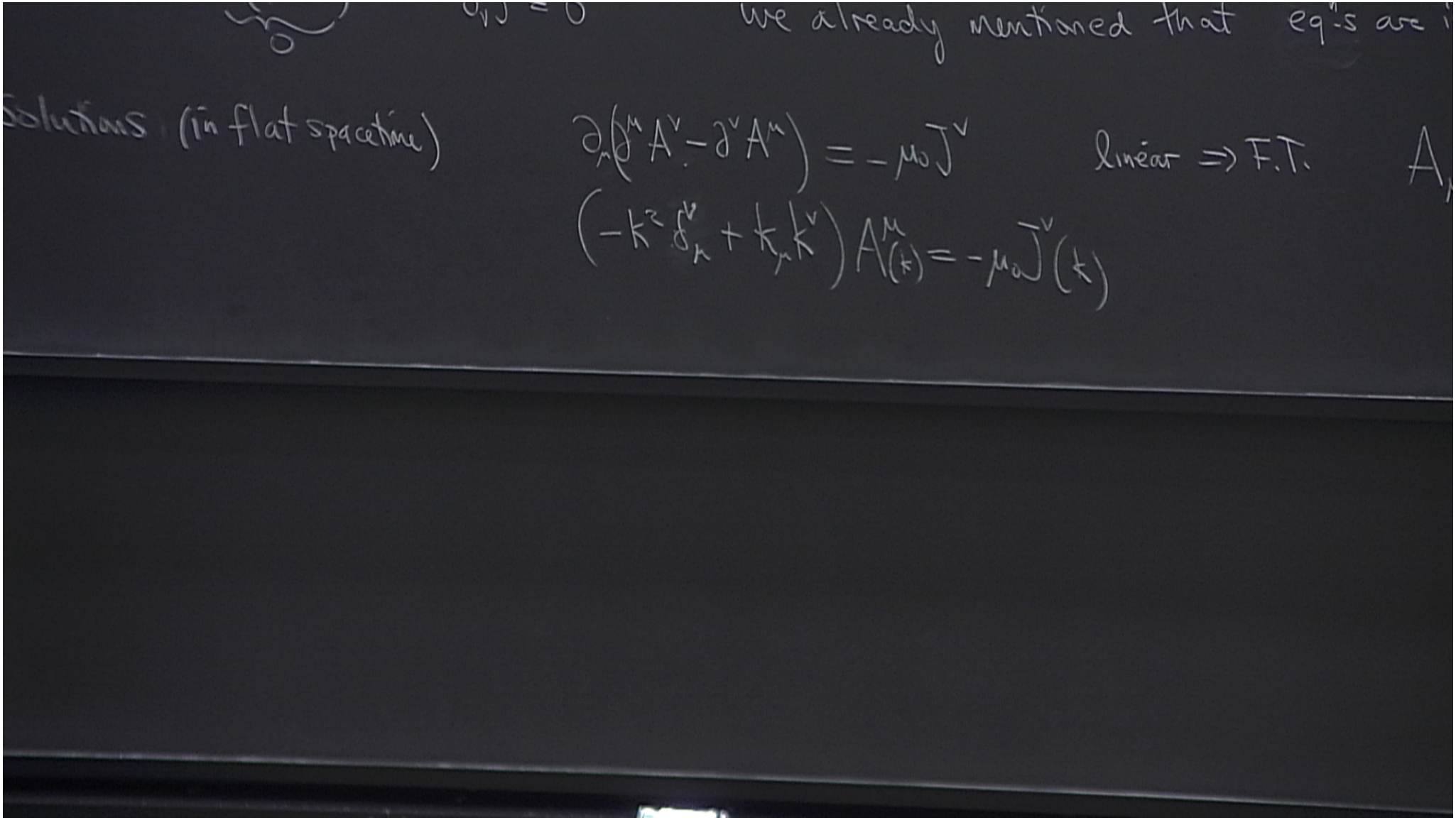
URL: http://pirsa.org/16090009

Abstract:


Gauge theories and gauge fixing
recall Maxwell $\partial_\mu F^{\mu\nu} = -\mu_0 J^\nu$ where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
$\partial_\nu(\partial_\mu F^{\mu\nu})$ $\partial_\nu J^\nu = 0$
we already mentioned that eq's are invariant under $A_\mu \to A_\mu + \partial_\mu \epsilon(x)$


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$\partial_\nu(\partial_\mu F^{\mu\nu}) = 0$ $\partial_\nu J^\nu = 0$
we already mentioned that eq's are invariant under $A_\mu \to A_\mu + \partial_\mu \epsilon(x)$
Solutions (in flat spacetime) $\partial_\mu(\partial^\mu A^\nu - \partial^\nu A^\mu) = -\mu_0 J^\nu$
linear $\Rightarrow$ F.T. $A_\mu(x) = \sum A_\mu(k) e^{ik\cdot x}$
$-k^0 x^0 + \vec{k}\cdot\vec{x}$
$\eta_{\mu\nu} k^\mu x^\nu$


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k^μ = 4-wavevector


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A_μ
$(-k^2\delta^\nu_\lambda + k_\lambda k^\nu)\tilde{A}^\mu_{(k)} = -\mu_0 \tilde{J}^\nu(k)$

$= -\mu_0 J$ where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

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$$\underbrace{\left(-k^2 \delta^\nu_\mu + k_\mu k^\nu\right)}_{4\times 4 \text{ matrix}} A^\mu(k) = -\mu_0 J^\nu(k)$$

4×4 matrix : invert to find $A^\mu(k)$.

$= -\mu_0 J^\nu$ where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

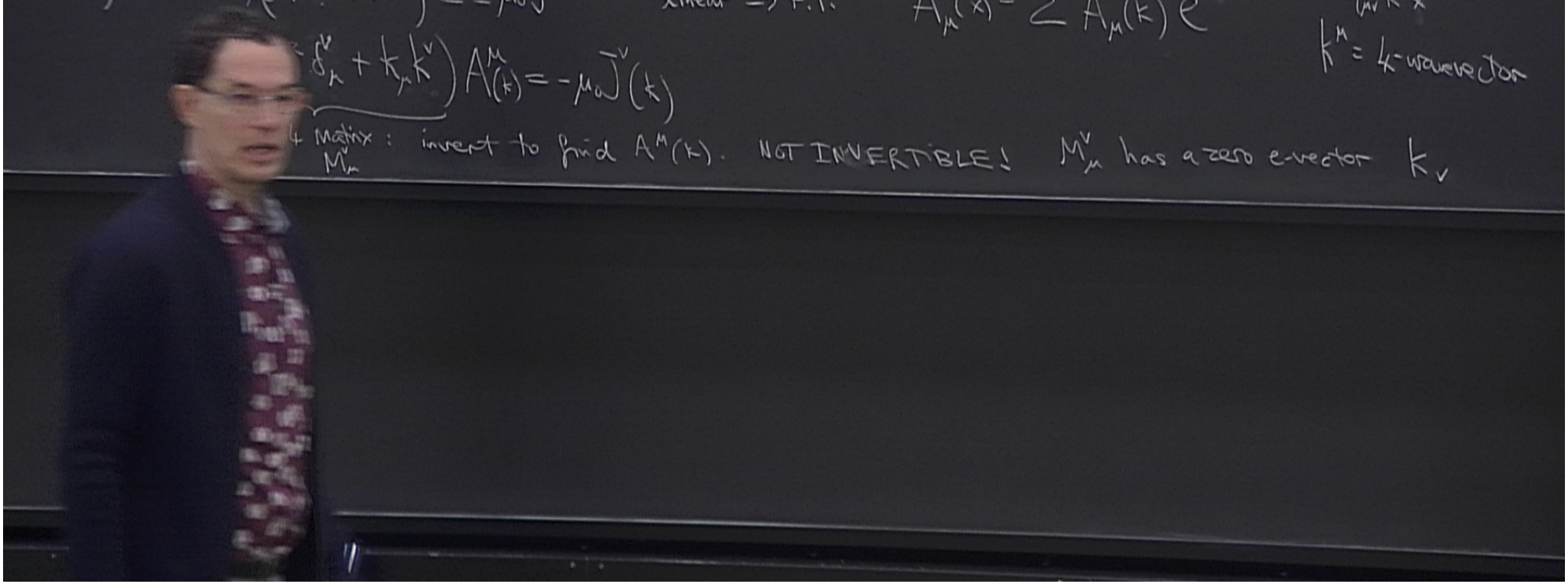
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$(\delta^\mu_\nu + k_\nu k^\nu) A^\nu(k) = -\mu_0 J^\nu(k)$

4 matrix M^μ_ν: invert to find $A^\mu(k)$. NOT INVERTIBLE! M^μ_ν has a zero e-vector k_ν



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In order to solve for $A^\mu(k)$, need to restrict $A^\mu(k)$ in some way. No surprise because A_μ was only unique up to g.trans. (*)

$A_\mu(k) \to A_\mu(k) + i k_\mu \epsilon(k)$

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Lorentz Gauge

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In Lorentz gauge, M.E.'s read $\partial^2 A^\nu = -\mu_0 J^\nu$ $\to$ Solve this via a Green's fn.

Naively A^ν represents 4 independent propagating degrees of freedom i.e. sol^{n}s of $\partial^2(\,) = 0$

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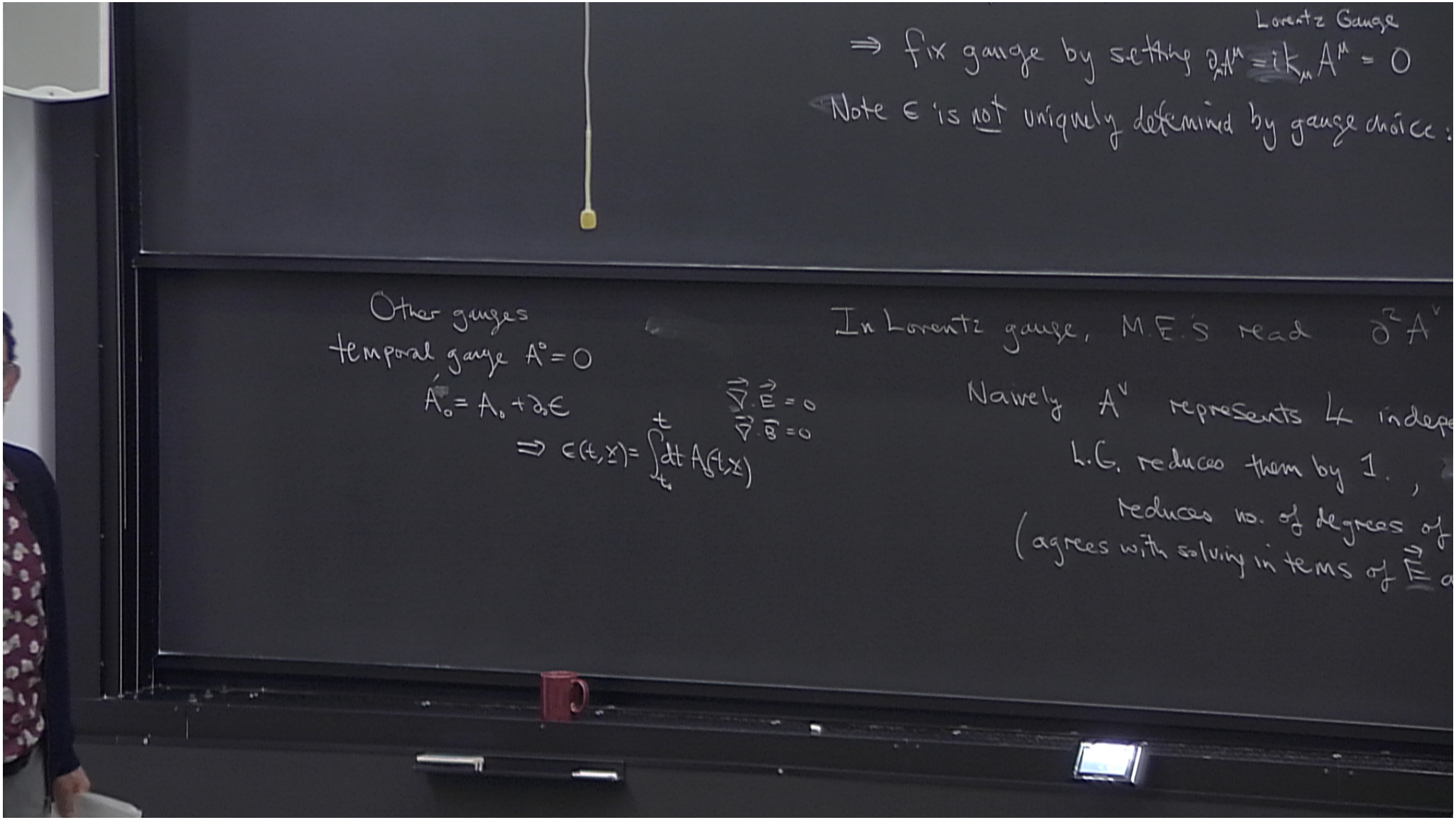
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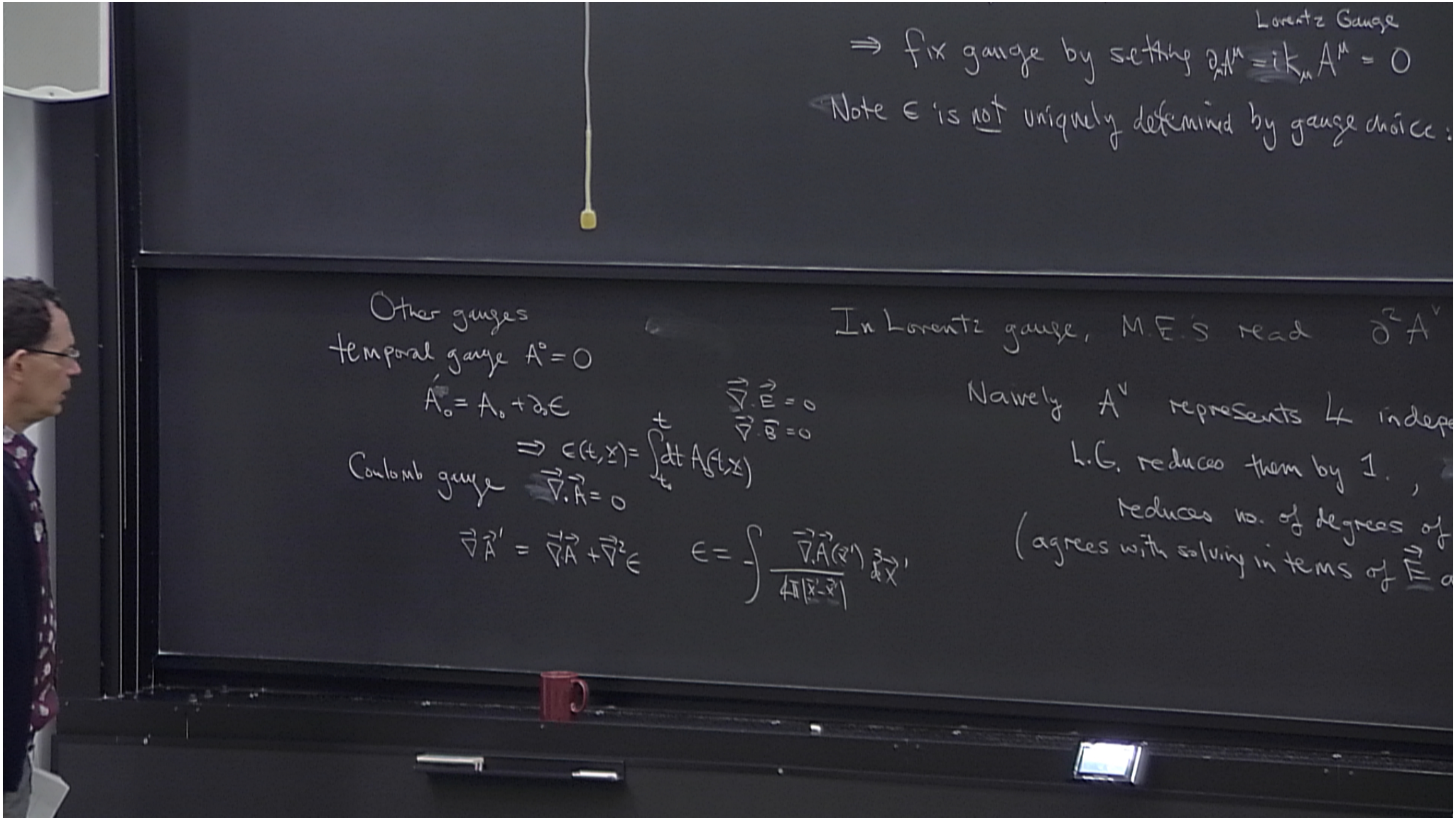
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reduces no. of degrees of freedom to 2.

(agrees with solving in terms of $\vec{E}$ and $\vec{B}$ directly)


Lorentz Gauge
⇒ fix gauge by setting $\partial_\mu A^\mu = ik_\mu A^\mu = 0$
Note ϵ is not uniquely determined by gauge choice:
Other gauges
temporal gauge $A^0 = 0$
$A'_0 = A_0 + \partial_0 \epsilon$
$\Rightarrow \epsilon(t,\vec{x}) = \int_{t_0}^{t} dt\, A_0(t,\vec{x})$
$\vec{\nabla}\cdot\vec{E} = 0$
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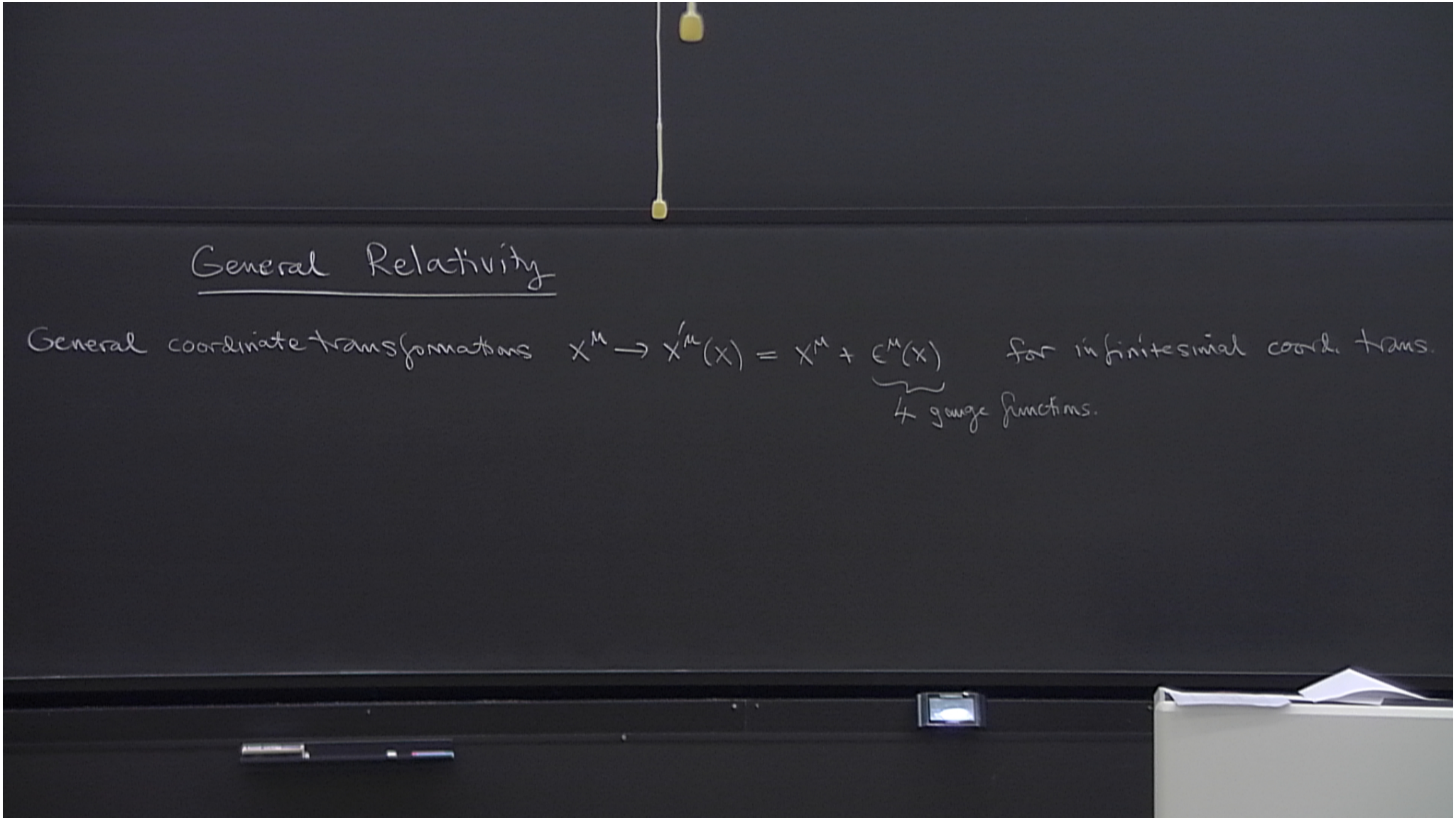

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Coulomb gauge $\vec{\nabla}\cdot\vec{A} = 0$
$\vec{\nabla}\cdot\vec{A}' = \vec{\nabla}\cdot\vec{A} + \vec{\nabla}^2\epsilon$
$\epsilon = -\int \frac{\vec{\nabla}\cdot\vec{A}(\vec{x}')}{4\pi|\vec{x}-\vec{x}'|} d^3x'$
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General Relativity

General coordinate transformations $x^\mu \to x'^\mu(x)$

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4 gauge functions.

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4 gauge transfm's.

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General coordinate transformations $x^\mu \to x'^\mu(x) = x^\mu + \underbrace{\epsilon^\mu(x)}_{4 \text{ gauge transf}^{n}\text{s.}}$ for infinitesimal coord. trans.

g. trans^{n}'s is $g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$

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$(*) \Rightarrow x^\mu = x'^\mu - \epsilon^\mu(x')$
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$g'_{\mu\nu}(x)$

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$g'_{\mu\nu}(x) = g'_{\mu\nu}(x') - \epsilon^\lambda g'_{\mu\nu,\lambda}(x')$

$g_{\mu\nu,\lambda}(x)$ to lowest order

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$- \epsilon^\mu(x')$

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$$g'_{\mu\nu}(x) = g'_{\mu\nu}(x') - \epsilon^\lambda g'_{\mu\nu,\lambda}(x')$$

$$\left(\delta^\alpha_\mu - \epsilon^\alpha{}_{,\mu}\right)\left(\delta^\beta_\nu - \epsilon^\beta{}_{,\nu}\right) g_{\alpha\beta} - \epsilon^\lambda g_{\mu\nu,\lambda}(x) \text{ to lowest order}$$

$$\Rightarrow \delta g_{\mu\nu} = g'_{\mu\nu}(x) - g_{\mu\nu}(x) = -\epsilon^\alpha{}_{,\mu} g_{\nu\alpha}(x) - \epsilon^\alpha{}_{,\nu} g_{\mu\alpha}(x) - \epsilon^\lambda g_{\mu\nu,\lambda}(x) = -\epsilon_{\mu;\nu} - \epsilon_{\nu;\mu}$$ Lie Derivative.

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c.f. $\delta A_\mu = \partial_\mu \epsilon$

Just as in EM, must fix a gauge

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Just as in EM, must fix a gauge : need 4 conditions to fix 4 ϵ^μ's.

$\Rightarrow \delta g_{\mu\nu} = g'_{\mu\nu}(x) - g_{\mu\nu}(x) = -\epsilon^{\alpha}{}_{,\mu}\, g_{\alpha\nu}(x) - \epsilon^{\alpha}{}_{,\nu}\, g_{\mu\alpha}(x) - \epsilon^{\lambda} g_{\mu\nu,\lambda}(x) = -\epsilon_{\mu;\nu} - \epsilon_{\nu;}$

c.f. $\delta A_\mu = \partial_\mu \epsilon$

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$\Gamma^{\lambda}_{\mu\nu}$

R ....

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$\Gamma^{\lambda}_{\mu\nu} = 0$

R ....

$g^{\mu\nu}\Gamma^{\lambda}_{\mu\nu}$

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because. $\Box\phi = g^{\mu\nu}\nabla_\mu\nabla_\nu\phi = g^{\mu\nu}\nabla_\mu\partial_\nu\phi = g^{\mu\nu}\left(\partial_\mu\partial_\nu - \Gamma^{\lambda}_{\nu\mu}\partial_\lambda\right)\phi$

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In particular, if $\phi = x^\mu$

$\Rightarrow \delta g_{\mu\nu} = g'_{\mu\nu}(x) - g_{\mu\nu}(x) = -\epsilon^{\alpha}{}_{,\mu}\, g_{\alpha\nu}(x) - \epsilon^{\alpha}{}_{,\nu}\, g_{\mu\alpha}(x) - \epsilon^{\lambda} g_{\mu\nu,\lambda}(x) = -\epsilon_{\mu;\nu} - \epsilon_{\nu;\mu}$ Lie Derivative.

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coords

In particular, if $\phi = x^\mu$, then $\Box x^\alpha = g^{\mu\nu}\partial_\mu\partial_\nu x^\alpha = 0$.

are harmonic fn's

$\Box x^\mu = 0$

Fix gauge by setting $\partial_\mu A^\mu = i k_\mu A^\mu = 0$ i.e. $\epsilon = \frac{i k_\mu A^\mu}{k^2}$

Note ϵ is not uniquely determined by gauge choice: only determined up to s

Einstein equations in vacuum $(T_{\mu\nu} = 0)$

Lorentz Gauge

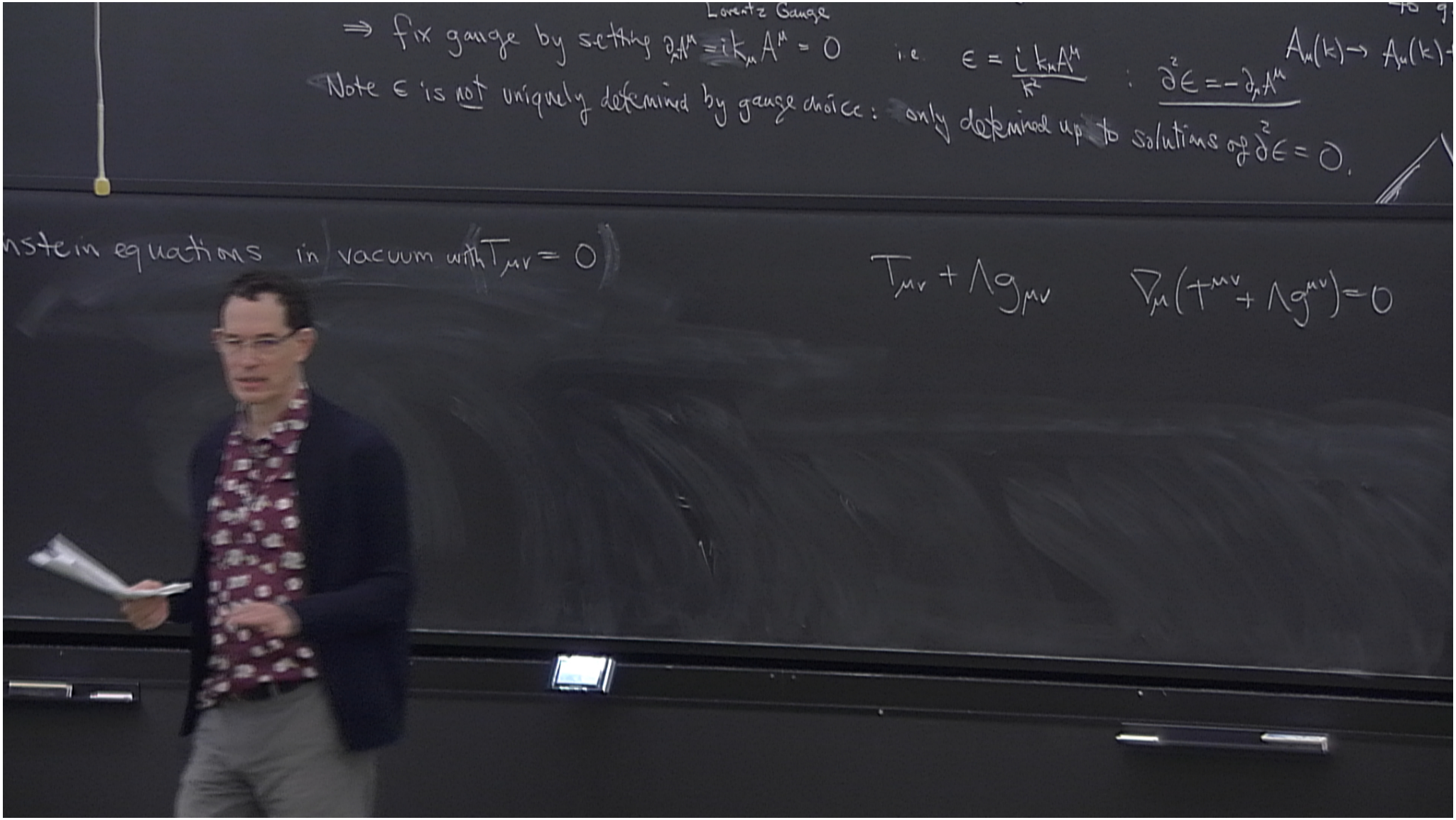
$\Rightarrow$ fix gauge by setting $\partial_\mu A^\mu = ik_\mu A^\mu = 0$ i.e. $\epsilon = \frac{ik_\mu A^\mu}{k^2}$ $\therefore \partial^2\epsilon = -\partial_\mu A^\mu$

$A_\mu(k) \to A_\mu(k) +$

Note ϵ is not uniquely determined by gauge choice: only determined up to solutions of $\partial^2\epsilon = 0$.

nstein equations in vacuum with $T_{\mu\nu} = 0$

$T_{\mu\nu} + \Lambda g_{\mu\nu}$ $\nabla_\mu(T^{\mu\nu} + \Lambda g^{\mu\nu}) = 0$



Lorentz Gauge

$\Rightarrow$ fix gauge by setting $\partial_\mu A^\mu = ik_\mu A^\mu = 0$ i.e. $\epsilon = \frac{i k_\mu A^\mu}{k^2}$: $\partial^2 \epsilon = -\partial_\mu$

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Einstein equations in vacuum with $T_{\mu\nu} = 0$

$\partial_\mu F^{\mu\nu} = 0 \Longleftrightarrow G_{\mu\nu} = 0$

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$\partial_\mu F^{\mu\nu} = 0 \Longleftrightarrow$ $G_{\mu\nu} = 0$

$= R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$

$\times g^{\mu\nu}$

$g^{\mu\nu} g_{\mu\nu} = \delta^\nu_\nu \Rightarrow R -$

$= 4$

Lorentz Gauge

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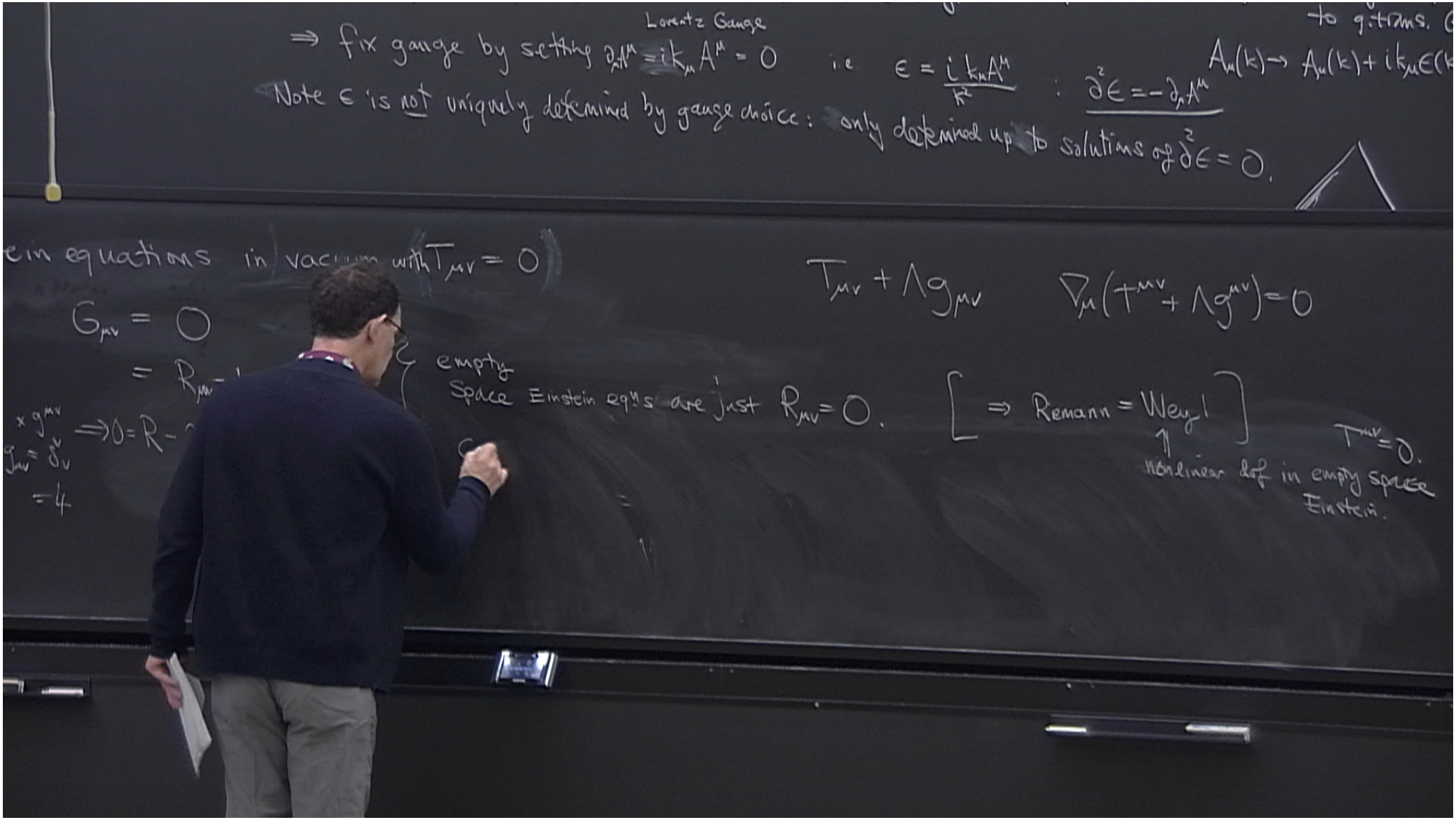
$= R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$

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$g^{\mu\nu} g_{\mu\nu} = \delta^\nu_\nu = 4$

$\Rightarrow 0 = R - 2R \Rightarrow R = 0.$

} empty space Einstein eq^{n}s are just $R_{\mu\nu} = 0$.


Lorentz Gauge
⇒ fix gauge by setting $\partial_\mu A^\mu = ik_\mu A^\mu = 0$ i.e. $\epsilon = \frac{i k_\mu A^\mu}{k^2}$: $\partial^2 \epsilon = -\partial_\mu A^\mu$
$A_\mu(k) \to A_\mu(k) + ik_\mu \epsilon(k$
Note ϵ is not uniquely determined by gauge choice: only determined up to solutions of $\partial^2 \epsilon = 0$.
to g.t.rms.
ein equations in vac with $T_{\mu\nu} = 0$
$G_{\mu\nu} = 0$
$= R_{\mu\nu} -$
empty space Einstein eq^{n}s are just $R_{\mu\nu} = 0$.
$T_{\mu\nu} + \Lambda g_{\mu\nu}$
$\nabla_\mu (T^{\mu\nu} + \Lambda g^{\mu\nu}) = 0$
$[\Rightarrow$ Remann = Weyl$]$
$T^{\mu\nu} = 0$.
nonlinear dof in empty space Einstein.
$\times g^{\mu\nu} \Rightarrow 0 = R -$
$= 4$

ein equations in vacuum with $T_{\mu\nu} = 0$ $\quad T_{\mu\nu} + \Lambda g_{\mu\nu}$ $\quad \nabla_\mu (T^{\mu\nu} + \Lambda g^{\mu\nu}) = 0$

$G_{\mu\nu} = 0$

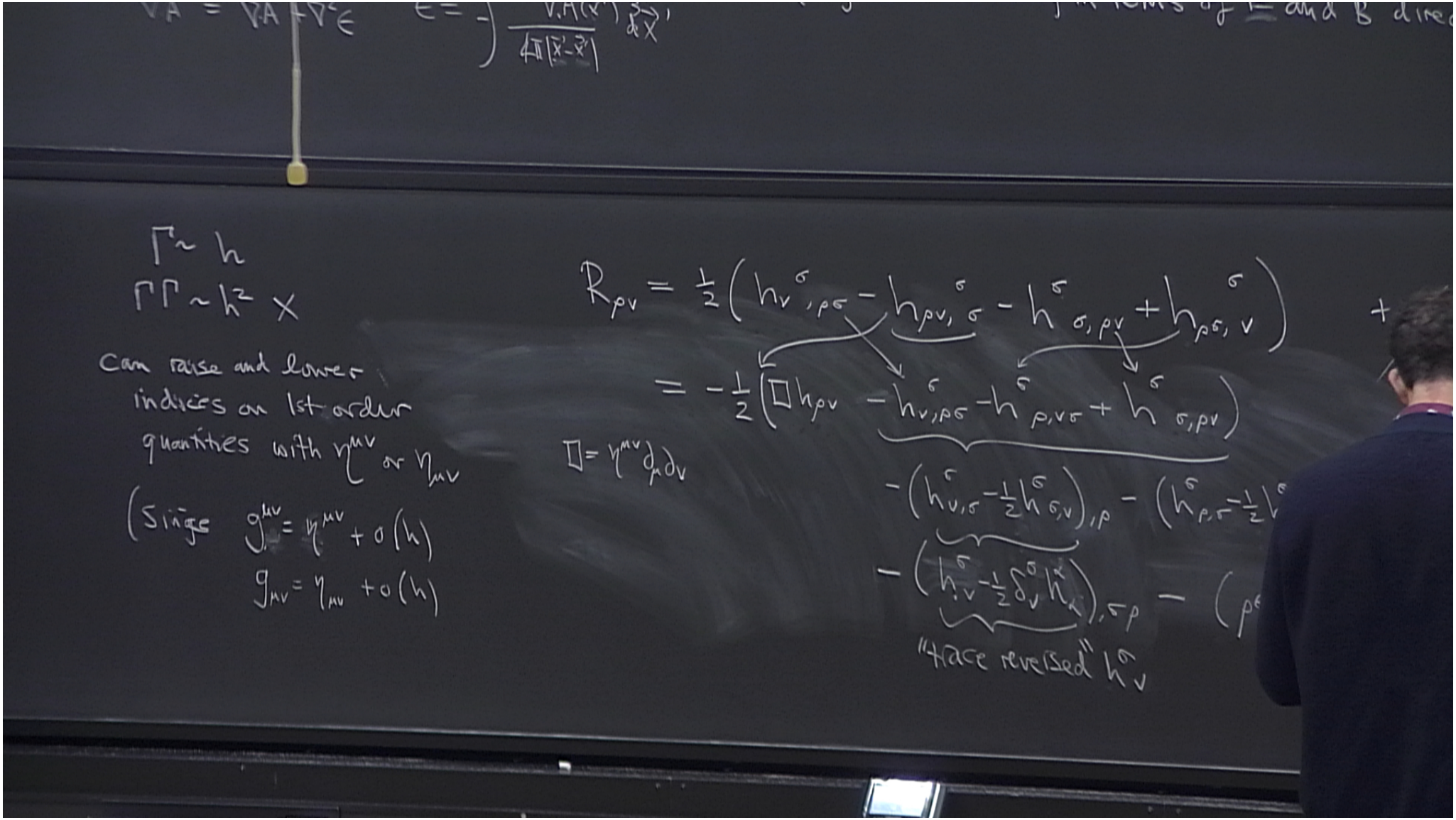
$= R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$

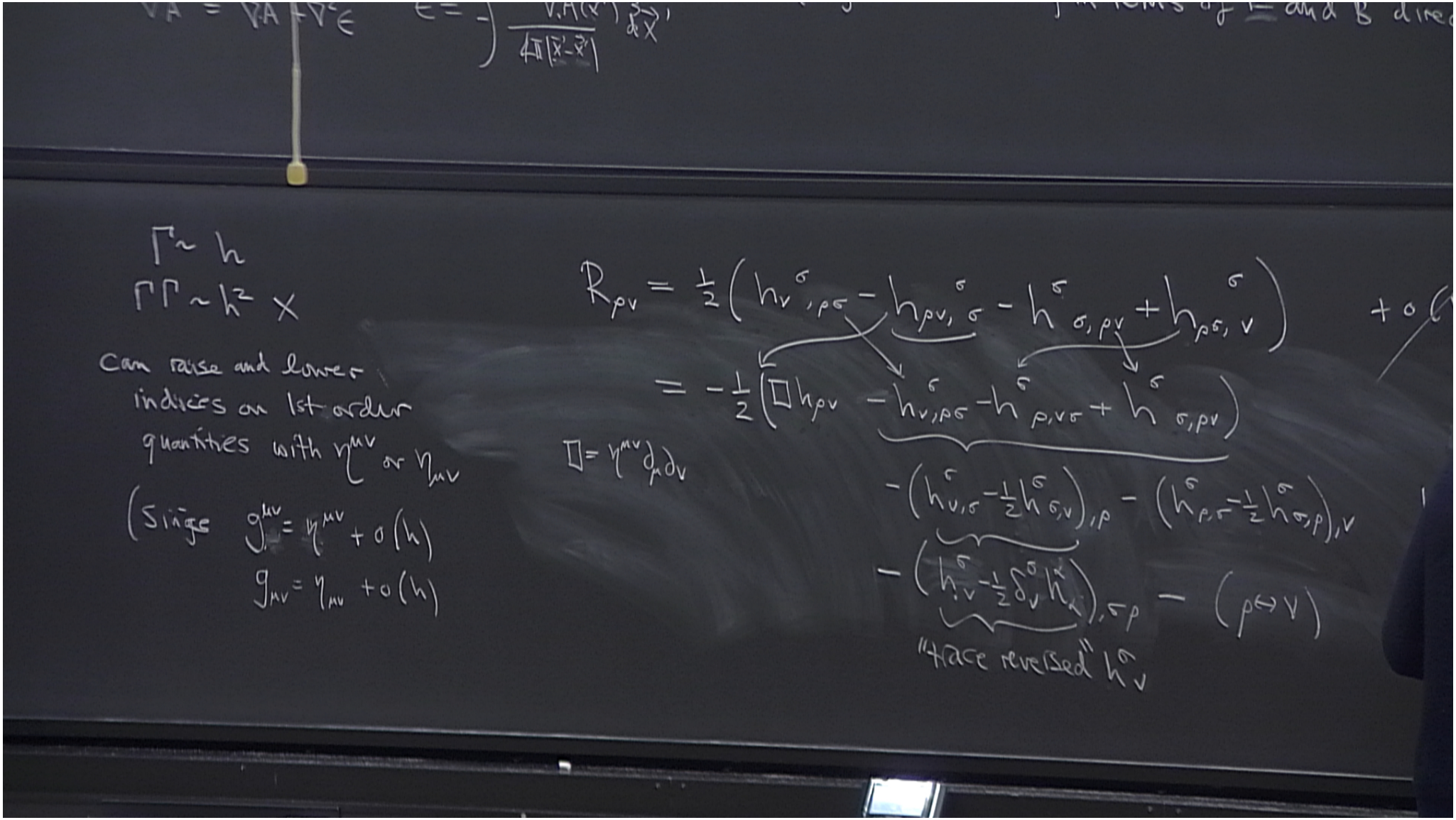
$\Rightarrow 0 = R - 2R \Rightarrow R = 0.$

} empty space Einstein eq^{n}s are just $R_{\mu\nu} = 0$. [$\Rightarrow$ Remann = Weyl ↑ nonlinear dof in empty space Einstein.] $T^{\mu\nu} = 0$.

Consider $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ $\quad (|h_{\mu\nu}| \ll 1)$

work to ~~first~~ order in $h_{\mu\nu}$. (linearized GR)


$\Gamma \sim h$
$\Gamma\Gamma \sim h^2$ X
Can raise and lower indices on 1st order quantities with $\eta^{\mu\nu}$ or $\eta_{\mu\nu}$
(Since $g^{\mu\nu} = \eta^{\mu\nu} + O(h)$
$g_{\mu\nu} = \eta_{\mu\nu} + O(h)$
$R_{\rho\nu} = \frac{1}{2}\left(h_{\nu}{}^{\sigma}{}_{,\rho\sigma} - h_{\rho\nu,}{}^{\sigma} - h^{\sigma}{}_{\sigma,\rho\nu} + h_{\rho\sigma,}{}^{\sigma}{}_{\nu} \right)$ +
$= -\frac{1}{2}\left(\Box h_{\rho\nu} - h_{\nu,\rho\sigma}{}^{\sigma} - h^{\sigma}{}_{\rho,\nu\sigma} + h^{\sigma}{}_{\sigma,\rho\nu} \right)$
$\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu$
$-\left(h^{\sigma}_{\nu,\sigma} - \frac{1}{2}h^{\sigma}_{\sigma,\nu}\right)_{,\rho} - \left(h^{\sigma}_{\rho,\sigma} - \frac{1}{2}h^{\sigma}_{\sigma,\rho}\right)_{,\nu}$
$-\left(h^{\sigma}_{\nu} - \frac{1}{2}\delta^{\sigma}_{\nu}h^{\kappa}_{\kappa}\right)_{,\sigma\rho} - (\rho\leftrightarrow\nu)$
"trace reversed" h^{σ}_{ν}


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$- \left(h^{\sigma}_{\nu} - \frac{1}{2}\delta^{\sigma}_{\nu} h^{\kappa}_{\kappa} \right)_{,\sigma\rho} - (\rho \leftrightarrow \nu)$
"trace reversed" h^{σ}_{ν}

$\ldots$ of E and B directly)

$$-h^{\sigma}_{\rho\nu,\sigma} - h^{\sigma}_{\sigma,\rho\nu} + h^{\sigma}_{\rho\sigma,\nu}\Big) \quad + o(h^2)$$

$$-h^{\sigma}_{\nu,\rho\sigma} - h^{\sigma}_{\rho,\nu\sigma} + h^{\sigma}_{\sigma,\rho\nu}\Big)$$

$$-\left(h^{\sigma}_{\nu,\sigma} - \tfrac{1}{2}h^{\sigma}_{\sigma,\nu}\right)_{,\rho} - \left(h^{\sigma}_{\rho,\sigma} - \tfrac{1}{2}h^{\sigma}_{\sigma,\rho}\right)_{,\nu}$$

$$-\left(\underbrace{h^{\sigma}_{\nu} - \tfrac{1}{2}\delta^{\sigma}_{\nu}h^{\kappa}_{\kappa}}_{\text{"trace reversed" } h^{\sigma}_{\nu}}\right)_{,\sigma\rho} - (\rho \leftrightarrow \nu)$$

In harmonic gauge, to first order in $h_{\mu\nu}$,

[Ex] $$\left(h^{\sigma}_{\nu} - \tfrac{1}{2}h^{\kappa}_{\kappa}\delta^{\sigma}_{\nu}\right)_{,\sigma} = 0$$

also called "TT" gauge in

transverse to k_σ / trace reversed.

$\nabla A = \nabla\cdot A + \nabla^2\epsilon \qquad \epsilon = -\int \frac{\nabla\cdot A(x')}{4\pi|\vec{x}-\vec{x}'|}\, d^3x'$

$\Gamma \sim h$

$\Gamma\Gamma \sim h^2$ ✗

Can raise and lower indices on 1st order quantities with $\eta^{\mu\nu}$ or $\eta_{\mu\nu}$

(Since $g^{\mu\nu} = \eta^{\mu\nu} + O(h)$, $g_{\mu\nu} = \eta_{\mu\nu} + O(h)$)

$$R_{\rho\nu} = \tfrac{1}{2}\left(h_\nu{}^\sigma{}_{,\rho\sigma} - h_{\rho\nu,}{}^\sigma - h^\sigma{}_{\sigma,\rho\nu} + h_{\rho\sigma,}{}^\sigma{}_\nu\right) + \cdots$$

$$= -\tfrac{1}{2}\Big(\Box h_{\rho\nu} - h^\sigma_{\nu,\rho\sigma} - h^\sigma_{\rho,\nu\sigma} + h^\sigma_{\sigma,\rho\nu}\Big) - \Big(h^\sigma_{\nu,\sigma} - \tfrac{1}{2}h^\sigma_{\sigma,\nu}\Big)_{,\rho} - \Big(h^\sigma_{\rho,\sigma} - \tfrac{1}{2}h^\sigma_{\sigma,\rho}\Big)_{,\nu} - \Big(h^\sigma_{\nu} - \tfrac{1}{2}\delta^\sigma_\nu h^\kappa_\kappa\Big)_{,\sigma\rho} - (\rho \leftrightarrow \nu)$$

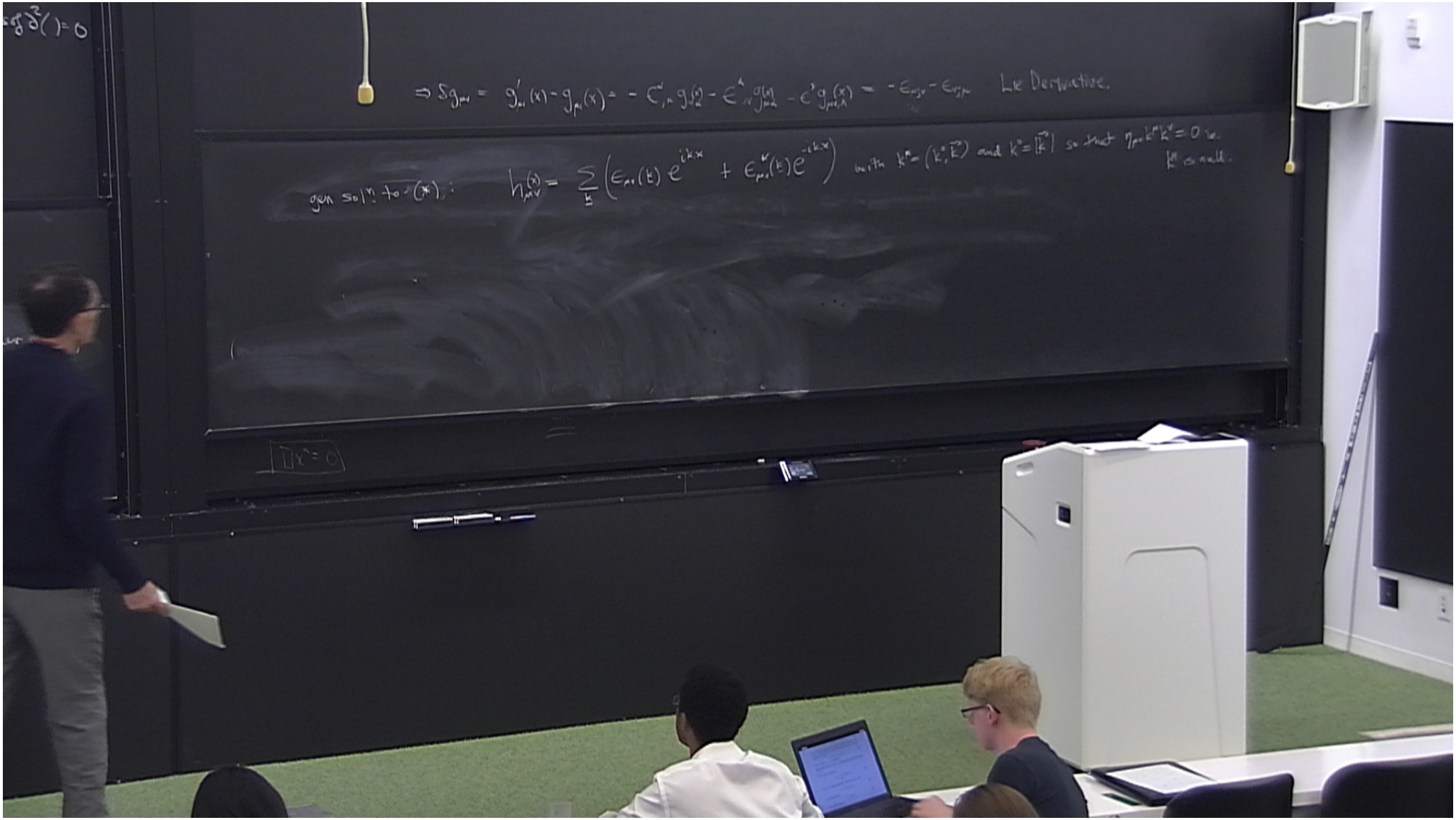
$\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu$

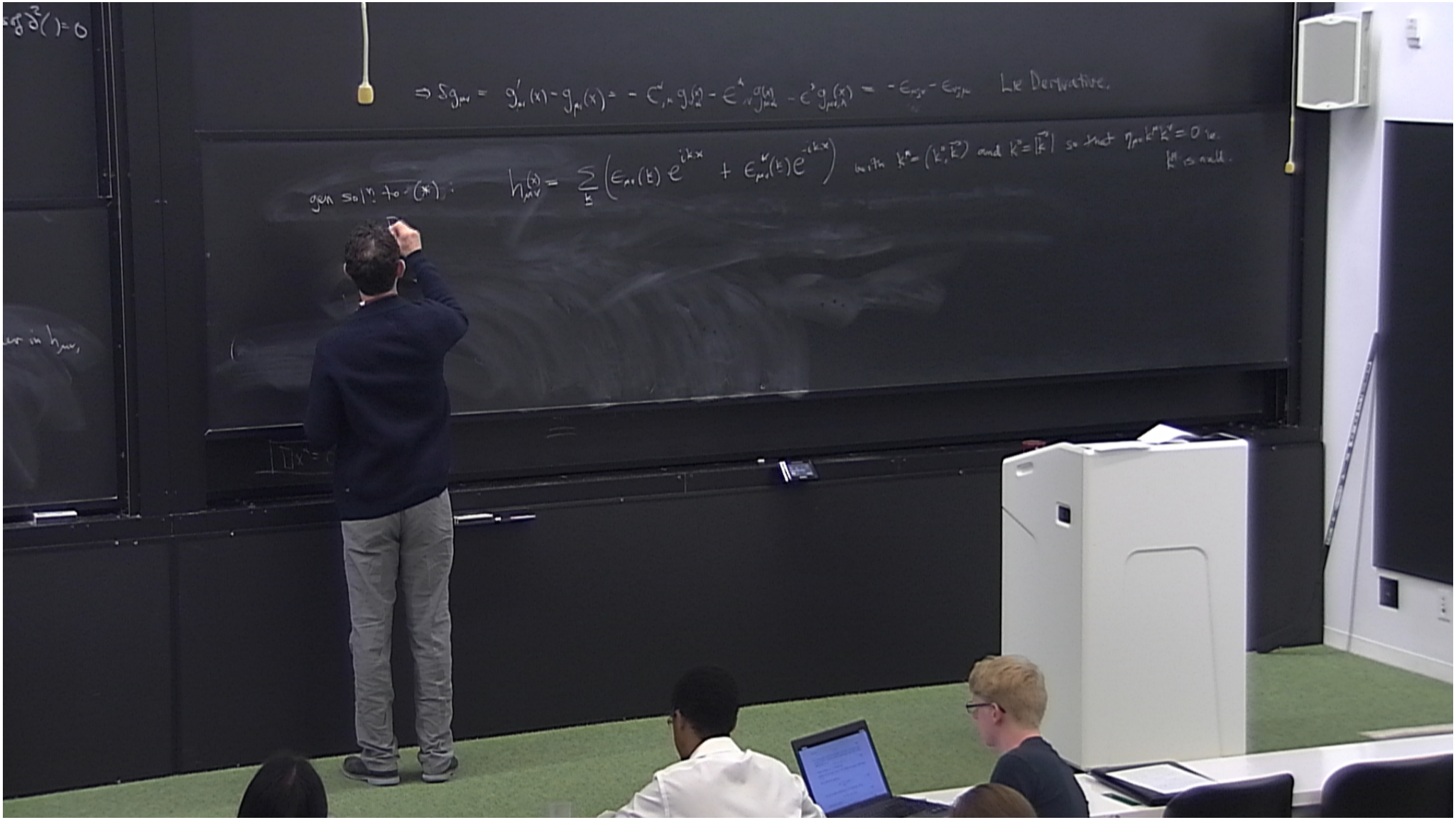
"trace reversed" h^σ_ν

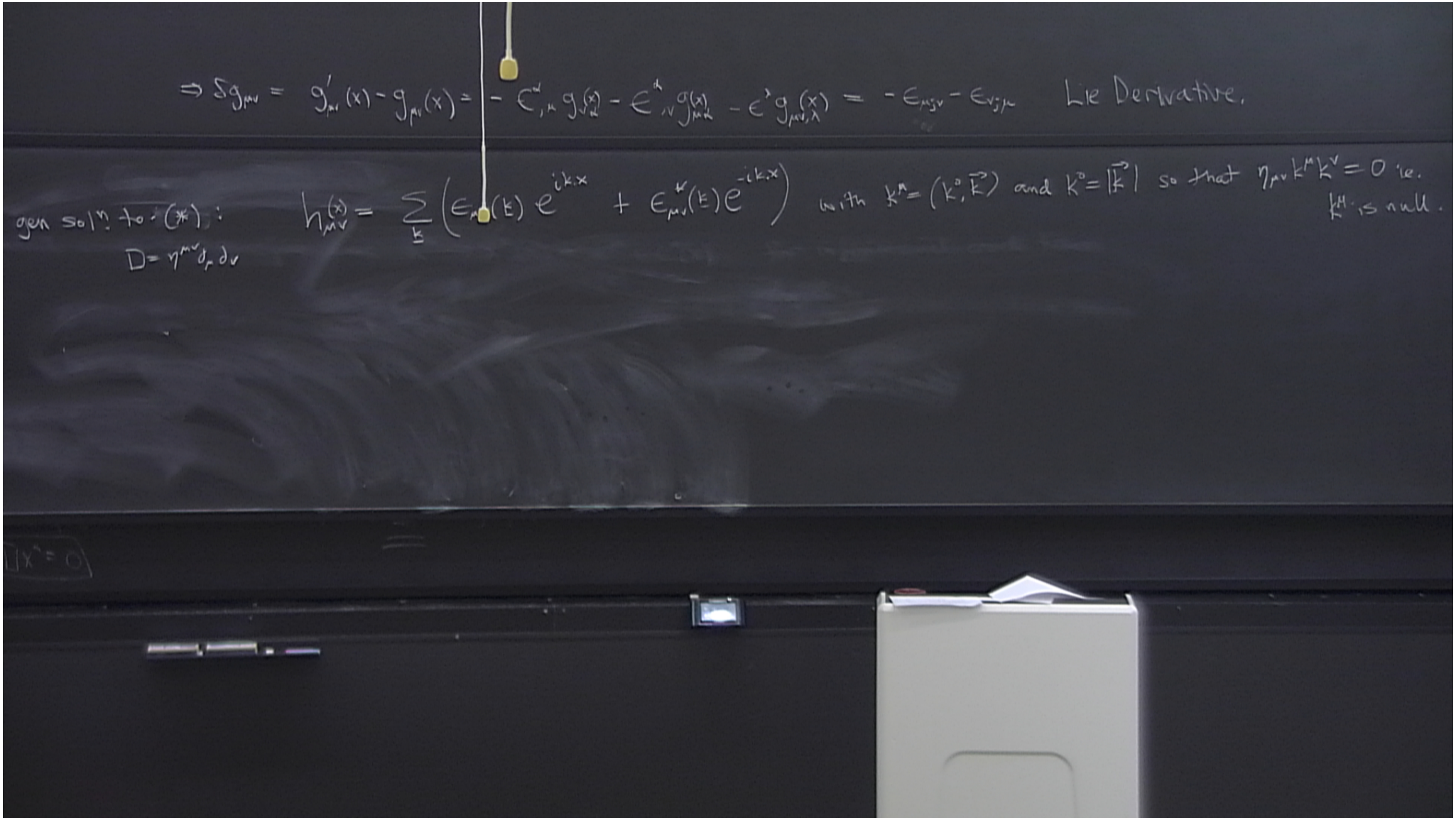
in TT gauge

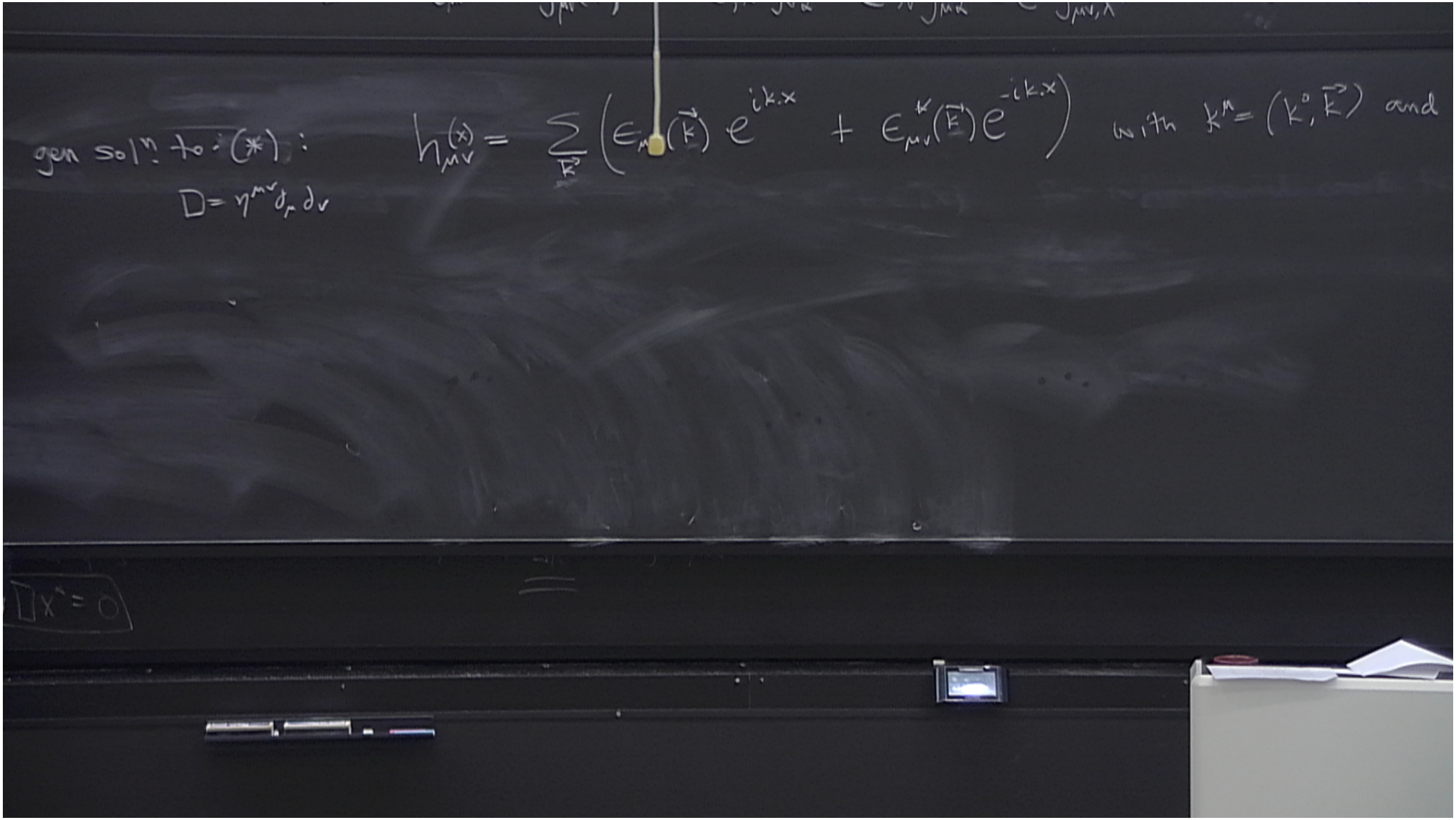
$\Rightarrow \Box h_{\mu\nu} = 0$

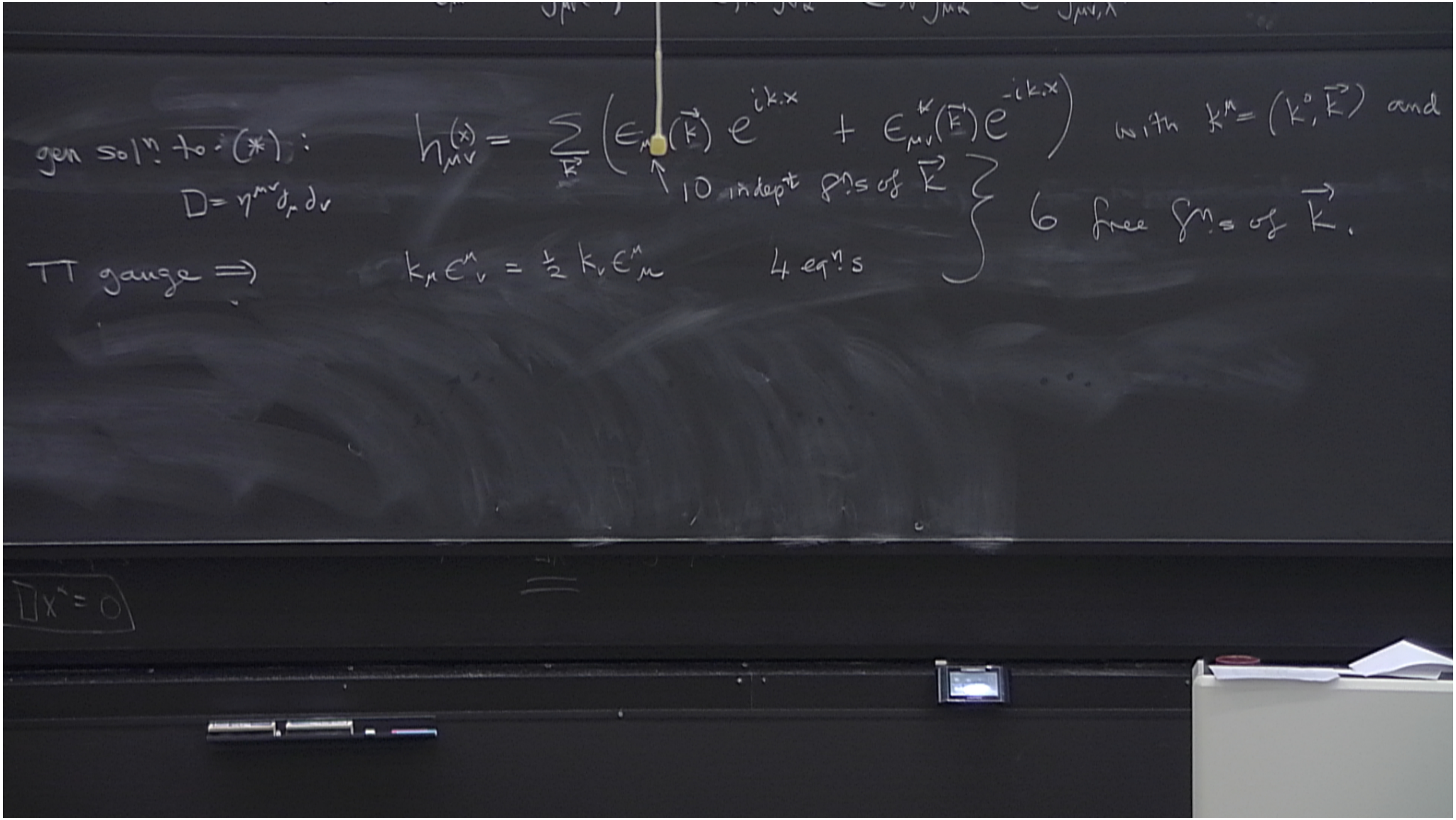
$\Rightarrow$ waves travelling at speed of light.

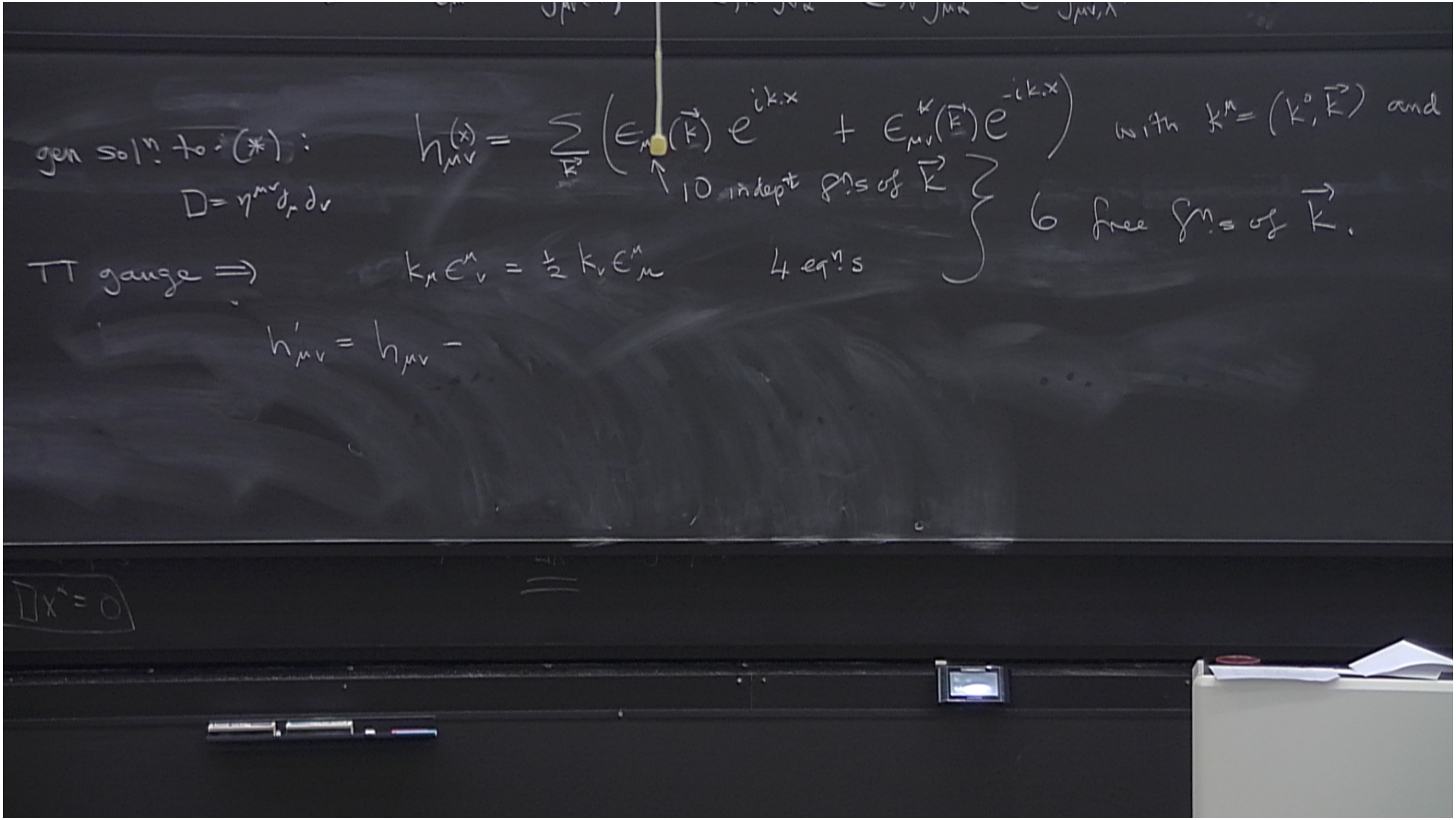

$\Rightarrow \delta g_{\mu\nu} = g'_{\mu\nu}(x) - g_{\mu\nu}(x) = -\epsilon^{\alpha}{}_{,\mu} g_{\alpha\nu} - \epsilon^{\alpha}{}_{,\nu} g_{\mu\alpha} - \epsilon^{\alpha} g_{\mu\nu,\alpha}(x) = -\epsilon_{\mu;\nu} - \epsilon_{\nu;\mu}$ Lie Derivative.
gen sol'n to (*): $h^{(x)}_{\mu\nu} = \sum_{\vec{k}} \left(\epsilon_{\mu\nu}(k)\, e^{ikx} + \epsilon^{*}_{\mu\nu}(k) e^{-ikx} \right)$ with $k^{\mu} = (k^0, \vec{k})$ and $k^0 = |\vec{k}|$ so that $\eta_{\mu\nu} k^{\mu} k^{\nu} = 0$ i.e. k^{μ} is null.


$\Rightarrow \delta g_{\mu\nu} = g'_{\mu\nu}(x) - g_{\mu\nu}(x) = -\epsilon^{\alpha}{}_{,\mu} g_{\alpha\nu}(x) - \epsilon^{\alpha}{}_{,\nu} g_{\mu\alpha}(x) - \epsilon^{\alpha} g_{\mu\nu,\alpha}(x) = -\epsilon_{\mu;\nu} - \epsilon_{\nu;\mu}$ Lie Derivative.
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$D = \eta^{\mu\nu} \partial_{\mu} \partial_{\nu}$
$\partial_i x^i = 0$


gen soln to (*):
$h^{(x)}_{\mu\nu} = \sum_{\vec{k}} \left(\epsilon_{\mu\nu}(\vec{k})\, e^{ik.x} + \epsilon^{*}_{\mu\nu}(\vec{k})\, e^{-ik.x} \right)$ with $k^\mu = (k^0, \vec{k})$ and
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$D = \eta^{\mu\nu}\partial_\mu\partial_\nu$
10 indept fn's of $\vec{k}$
6 free fn's of $\vec{k}$.
TT gauge $\Rightarrow$ $k_\mu \epsilon^\mu{}_\nu = \frac{1}{2} k_\nu \epsilon^\mu{}_\mu$ 4 eqn's
$\Box X^\mu = 0$


gen soln to (*):
$h^{(x)}_{\mu\nu} = \sum_{\vec{k}} \left(\epsilon_{\mu\nu}(\vec{k})\, e^{ik.x} + \epsilon^{*}_{\mu\nu}(\vec{k}) e^{-ik.x} \right)$ with $k^\mu = (k^0, \vec{k})$ and
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$h'_{\mu\nu} = h_{\mu\nu} -$
$\Box x^\mu = 0$

$\Rightarrow \delta g_{\mu\nu} = g'_{\mu\nu}(x) - g_{\mu\nu}(x) = -\epsilon^{\alpha}{}_{,\mu}\, g_{\alpha\nu}(x) - \epsilon^{\alpha}{}_{,\nu}\, g_{\mu\alpha}(x) - \epsilon^{\lambda} g_{\mu\nu,\lambda}(x) = -\epsilon_{\mu;\nu} - \epsilon_{\nu;\mu}$ (#) Lie Derivative.

$\cdot(*)$: $h^{(x)}_{\mu\nu} = \sum_{\vec{k}} \left(\epsilon_{\mu\nu}(\vec{k})\, e^{ik.x} + \epsilon^{*}_{\mu\nu}(\vec{k})\, e^{-ik.x} \right)$ with $k^{\mu} = (k^0, \vec{k})$ and $k^0 = |\vec{k}|$ so that $\eta_{\mu\nu} k^{\mu} k^{\nu} =$

$= \eta^{\mu\nu} \partial_{\mu} \partial_{\nu}$

10 independent fn's of $\vec{k}$ — 6 free fn's of $\vec{k}$.

$\Rightarrow \quad k_{\mu} \epsilon^{\mu}{}_{\nu} = \frac{1}{2} k_{\nu} \epsilon^{\mu}{}_{\mu}$ 4 eq^{n}s

$h'_{\mu\nu} = h_{\mu\nu} - \epsilon_{\mu,\nu} - \epsilon_{\nu,\mu}$ (from (#), ignoring terms of order $\epsilon.h$ ie treat ϵ as of the same order as h)

$\Rightarrow \delta g_{\mu\nu} = g'_{\mu\nu}(x) - g_{\mu\nu}(x) = -\epsilon^{\alpha}{}_{,\mu}\, g^{(x)}_{\alpha\nu} - \epsilon^{\alpha}{}_{,\nu}\, g^{(x)}_{\mu\alpha} - \epsilon^{\lambda} g^{(x)}_{\mu\nu,\lambda} \equiv -\epsilon_{\mu;\nu}$

gen soln. to : (*) :

$h^{(x)}_{\mu\nu} = \sum_{\vec{k}} \left(\epsilon_{\mu\nu}(\vec{k})\, e^{ik.x} + \epsilon^{*}_{\mu\nu}(\vec{k})\, e^{-ik.x} \right)$ with $k^{\mu} = (k$

Polarization tensor.

10 indept fn's of $\vec{k}$

$D = \eta^{\mu\nu}\partial_{\mu}\partial_{\nu}$

6 free fn's of

TT gauge $\Rightarrow$ $k_{\mu}\epsilon^{\mu}{}_{\nu} = \frac{1}{2} k_{\nu}\epsilon^{\mu}{}_{\mu}$ 4 eqn's

Gauge transform

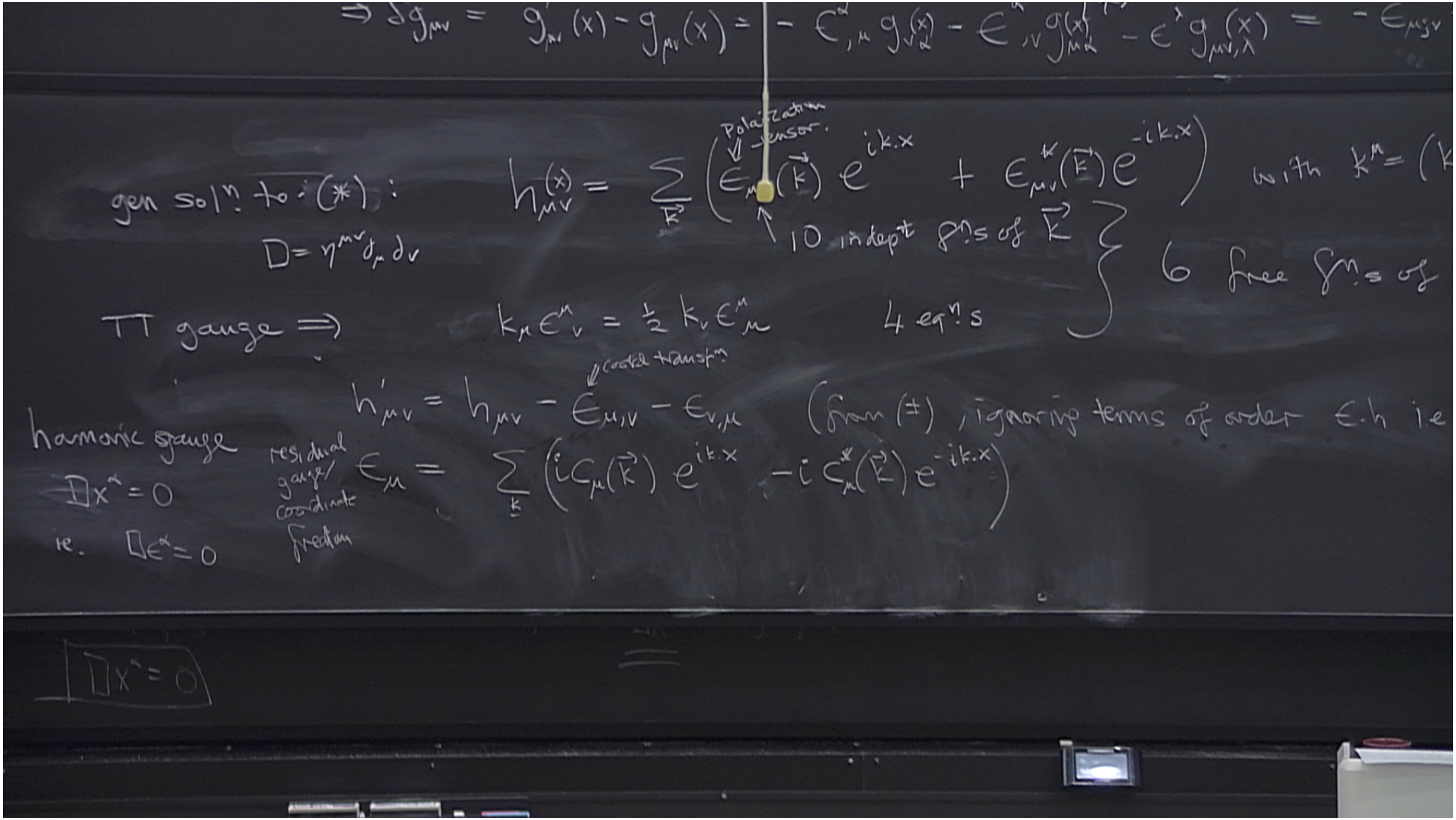
$h'_{\mu\nu} = h_{\mu\nu} - \epsilon_{\mu,\nu} - \epsilon_{\nu,\mu}$ (from (*), ignoring terms of order $\epsilon.h$ i.e

harmonic gauge

$\Box x^{\mu} = 0$

$\epsilon_{\mu} =$

$\Box x^{\mu} = 0$


$\Rightarrow \delta g_{\mu\nu} = g'_{\mu\nu}(x) - g_{\mu\nu}(x) = -\epsilon^{\alpha}{}_{,\mu} g_{\nu\alpha}(x) - \epsilon^{\alpha}{}_{,\nu} g_{\mu\alpha}(x) - \epsilon^{\lambda} g_{\mu\nu,\lambda}(x) \equiv -\epsilon_{\mu;\nu}$
gen soln to (*):
$D = \eta^{\mu\nu}\partial_\mu\partial_\nu$
$h^{(x)}_{\mu\nu} = \sum_{\vec{k}} \left(\epsilon_{\mu\nu}(\vec{k})\, e^{ik.x} + \epsilon^{*}_{\mu\nu}(\vec{k}) e^{-ik.x} \right)$
Polarization tensor.
with $k^\mu = (k$
10 indept fn's of $\vec{k}$
6 free fn's of
TT gauge $\Rightarrow$
$k_\mu \epsilon^{\mu}{}_{\nu} = \frac{1}{2} k_\nu \epsilon^{\mu}{}_{\mu}$
4 eqn's
$h'_{\mu\nu} = h_{\mu\nu} - \epsilon_{\mu,\nu} - \epsilon_{\nu,\mu}$
Gauge transf^n
(from (*), ignoring terms of order $\epsilon.h$ i.e
harmonic gauge
$\Box x^\mu = 0$
ie. $\Box \epsilon^\mu = 0$
residual gauge/ coordinate freedom
$\epsilon_\mu = \sum_{\vec{k}} \left(i C_\mu(\vec{k})\, e^{ik.x} - i C^{*}_{\mu}(\vec{k}) e^{-ik.x} \right)$
$\Box x^\mu = 0$

$\ldots g^{(x)}_{\nu\lambda} - \epsilon_{,\nu} g^{(x)}_{\mu\lambda} - \epsilon^{\lambda} g^{(x)}_{\mu\nu,\lambda} = -\epsilon_{\mu;\nu} - \epsilon_{\nu;\mu}$ Lie Derivative.

$e^{ik.x} + \epsilon^{*}_{\mu\nu}(\vec{k}) e^{-ik.x}$) with $k^{\mu} = (k^0, \vec{k})$ and $k^0 = |\vec{k}|$ so that $\eta_{\mu\nu} k^{\mu} k^{\nu} = 0$ i.e. k^{μ} is null.

indept fn's of $\vec{k}$ } 6 free fn's of $\vec{k}$. BUT have 4 free fn's of $\vec{k}$ in residual

4 eqn's gauge freedom $\Rightarrow$ 2 propagating, physical d.o.f. in g-waves.

from (*), ignoring terms of order $\epsilon.h$ i.e treat ϵ as of the same order as h)

$\epsilon^{*}_{\mu\nu}(\vec{k}) e^{-ik.x}$)