

Title: PSI 2016/2017 Relativity - Lecture 5

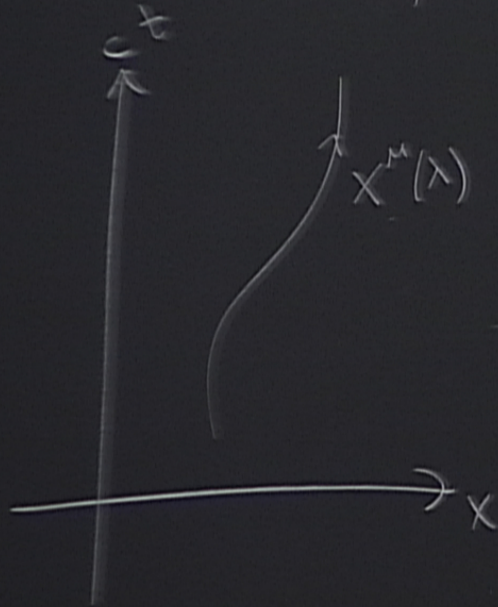
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Abstract:

$$A_\mu, F_{\mu\nu}(x) = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$A'_\mu = A_\mu + \partial_\mu \chi$$



coordinates on spacetime
redundant by

$$x'^\mu = \Lambda^\mu_\nu x^\nu + a^\mu$$

Poincaré transfm's.

Some quantities are invariant, e.g.

$$ds^2 = dx'^\mu dx'^\nu \eta_{\mu\nu} = dx^\mu dx^\nu \eta_{\mu\nu} \quad \text{line element.}$$

$$x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu} + a^{\mu}$$

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$$ds^2 = dx'^{\mu} dx'^{\nu} \eta_{\mu\nu} = dx^{\mu} dx^{\nu} \eta_{\mu\nu} \quad \text{line element.}$$

$$\eta_{\mu\nu} \Lambda^{\mu}_{\alpha} \Lambda^{\nu}_{\beta} = \eta_{\alpha\beta}$$

Consider a general coordinate transformation $x'^{\mu}(x)$.

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$$dx'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\alpha}} dx^{\alpha}$$

$$\frac{\partial}{\partial x'^{\mu}} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial}{\partial x^{\alpha}}$$

Jacobian matrix $\frac{\partial x'^{\mu}}{\partial x^{\alpha}}$ should be nonsingular

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Consider a general coordinate transformation $x'^{\mu}(x)$.

$$dx'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\alpha}} dx^{\alpha}$$

$$\frac{\partial}{\partial x'^{\mu}} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial}{\partial x^{\alpha}}$$

Jacobian matrix $\frac{\partial x'^{\mu}}{\partial x^{\alpha}}$ is 1-1. Show

$$\text{If we impose } (*) \text{ then } \eta_{\gamma\delta} = \eta_{\alpha\beta} \frac{\partial x^{\alpha}}{\partial x'^{\gamma}} \frac{\partial x^{\beta}}{\partial x'^{\delta}}$$

Jacobian matrix $\frac{\partial x'^{\mu}}{\partial x^{\alpha}}$ should be nonsingular in order that the
 is 1-1.

$$\Rightarrow \frac{\partial}{\partial x^{\epsilon}} : 0 = \eta_{\alpha\beta} \frac{\partial^2 x'^{\alpha}}{\partial x^{\epsilon} \partial x^{\delta}} \frac{\partial x'^{\beta}}{\partial x^{\delta}} + \eta_{\alpha\beta} \frac{\partial x'^{\alpha}}{\partial x^{\epsilon}} \frac{\partial^2 x'^{\beta}}{\partial x^{\delta} \partial x^{\delta}} + \eta_{\alpha\beta} \frac{\partial x'^{\alpha}}{\partial x^{\epsilon}} \frac{\partial^2 x'^{\beta}}{\partial x^{\delta} \partial x^{\delta}} - \eta_{\alpha\beta} \frac{\partial^2 x'^{\alpha}}{\partial x^{\epsilon} \partial x^{\delta}} \frac{\partial x'^{\beta}}{\partial x^{\delta}} - \eta_{\alpha\beta} \frac{\partial x'^{\alpha}}{\partial x^{\epsilon}} \frac{\partial^2 x'^{\beta}}{\partial x^{\delta} \partial x^{\delta}}$$

Cancel

$\frac{\partial X}{\partial X^\alpha}$ should be nonsingular in order

$$\begin{aligned}
 & \eta_{\alpha\beta} \frac{\partial^2 X'^\alpha}{\partial X^\mu \partial X^\nu} \frac{\partial X'^\beta}{\partial X^\epsilon \partial X^\delta} + \eta_{\alpha\beta} \frac{\partial^2 X'^\alpha}{\partial X^\mu \partial X^\nu} \frac{\partial^2 X'^\beta}{\partial X^\epsilon \partial X^\delta} \\
 & + \eta_{\alpha\beta} \frac{\partial^2 X'^\alpha}{\partial X^\mu \partial X^\nu} \frac{\partial X'^\beta}{\partial X^\epsilon \partial X^\delta} + \eta_{\alpha\beta} \frac{\partial X'^\alpha}{\partial X^\mu \partial X^\nu} \frac{\partial^2 X'^\beta}{\partial X^\epsilon \partial X^\delta} \\
 & - \eta_{\alpha\beta} \frac{\partial^2 X'^\alpha}{\partial X^\mu \partial X^\nu} \frac{\partial X'^\beta}{\partial X^\epsilon \partial X^\delta} - \eta_{\alpha\beta} \frac{\partial X'^\alpha}{\partial X^\mu \partial X^\nu} \frac{\partial^2 X'^\beta}{\partial X^\epsilon \partial X^\delta}
 \end{aligned}$$

Cancel $\Rightarrow \eta_{\alpha\beta} \frac{\partial^2 X'^\alpha}{\partial X^\mu \partial X^\nu} \frac{\partial X'^\beta}{\partial X^\epsilon \partial X^\delta} = 0$

order that the coordinate transformation

$$\Rightarrow \boxed{x'^{\alpha} = \Lambda^{\alpha}_{\beta} x^{\beta} + a^{\alpha}}$$

Poincaré

(sub back into (1) to show Λ must be LT)

Cancel

$\Rightarrow$

$$\eta_{\alpha\beta} \frac{\partial^2 x'^{\alpha}}{\partial x^{\mu} \partial x^{\nu}} = 0$$

$$\Rightarrow \frac{\partial^2 x'^{\alpha}}{\partial x^{\mu} \partial x^{\nu}} = 0$$

coordinate transformation $x'^{\mu}(x)$.

$$= \frac{\partial x'^{\mu}}{\partial x^{\alpha}} dx^{\alpha}$$

$$= \frac{\partial x'^{\mu}}{\partial x'^{\alpha}} \frac{\partial x^{\alpha}}{\partial x'^{\mu}} dx^{\alpha}$$

Jacobian matrix $\frac{\partial x'^{\mu}}{\partial x^{\alpha}}$ should be nonsingular in order to be 1-1.

$$= \eta_{\alpha\beta} \frac{\partial x'^{\alpha}}{\partial x^{\epsilon}} \frac{\partial x'^{\beta}}{\partial x^{\delta}} \Rightarrow \frac{\partial}{\partial x^{\epsilon}} : 0 = \eta_{\alpha\beta} \frac{\partial^2 x'^{\alpha}}{\partial x^{\epsilon} \partial x^{\delta}} \frac{\partial x'^{\beta}}{\partial x^{\delta}} + \eta_{\alpha\beta} \frac{\partial x'^{\alpha}}{\partial x^{\delta}} \frac{\partial^2 x'^{\beta}}{\partial x^{\epsilon} \partial x^{\delta}} + (\delta \leftrightarrow \epsilon) + \eta_{\alpha\beta} \frac{\partial x'^{\alpha}}{\partial x^{\epsilon}} \frac{\partial^2 x'^{\beta}}{\partial x^{\delta} \partial x^{\delta}} - (\delta \leftrightarrow \epsilon) - \eta_{\alpha\beta} \frac{\partial^2 x'^{\alpha}}{\partial x^{\delta} \partial x^{\delta}} \frac{\partial x'^{\beta}}{\partial x^{\epsilon}} - \eta_{\alpha\beta} \frac{\partial x'^{\alpha}}{\partial x^{\delta}} \frac{\partial^2 x'^{\beta}}{\partial x^{\delta} \partial x^{\epsilon}}$$

($\neq$)

cancel

$$dx'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\alpha}} dx^{\alpha}$$

$$\frac{\partial}{\partial x'^{\mu}} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial}{\partial x^{\alpha}}$$

Jacobian matrix $\frac{\partial x'^{\mu}}{\partial x^{\alpha}}$ should be
is 1-1.

impose (x) , then $\eta_{\gamma\delta} = \eta_{\alpha\beta} \frac{\partial x^{\alpha}}{\partial x'^{\gamma}} \frac{\partial x^{\beta}}{\partial x'^{\delta}} \Rightarrow \frac{\partial}{\partial x^{\epsilon}} : 0 = \eta_{\alpha\beta} \frac{\partial^2 x^{\alpha}}{\partial x'^{\gamma} \partial x'^{\delta}} \frac{\partial x^{\beta}}{\partial x'^{\epsilon}} +$
 $+ (\delta \leftrightarrow \epsilon) +$
 $- (\delta \leftrightarrow \epsilon) - \eta_{\alpha\beta} \frac{\partial^2 x^{\alpha}}{\partial x'^{\gamma} \partial x'^{\delta}} \frac{\partial x^{\beta}}{\partial x'^{\epsilon}} \leftarrow \text{cancel}$
 $\uparrow$
 $f(x)$
 $(\neq)$

nonsingular in order that the coordinate transformation

$$\Rightarrow \boxed{x'^{\alpha} = \Lambda^{\alpha}_{\beta} x^{\beta} + a^{\alpha}}$$

Poincaré (sub back into $\square$)
to show Λ must be LT

$$\eta_{\alpha\beta} \frac{\partial x^{\alpha}}{\partial x^{\epsilon}} \frac{\partial x^{\beta}}{\partial x^{\delta}} \xrightarrow{\text{cancel}} \eta_{\alpha\beta} \frac{\partial x^{\alpha}}{\partial x^{\epsilon}} \frac{\partial x^{\beta}}{\partial x^{\delta}}$$

$$\Rightarrow \eta_{\alpha\beta} \frac{\partial x^{\alpha}}{\partial x^{\epsilon}} \frac{\partial x^{\beta}}{\partial x^{\delta}} = 0$$

$$\Rightarrow \frac{\partial x^{\alpha}}{\partial x^{\epsilon}} \frac{\partial x^{\alpha}}{\partial x^{\delta}} = 0$$

$$\frac{\partial}{\partial X'^{\mu}} = \frac{\partial X}{\partial X'^{\mu}} \frac{\partial}{\partial X}$$

dilatation

$$X'^{\alpha} = \lambda X^{\alpha}$$

+ 4 Special conformal
trans.

If we impose (X) , then $\eta_{\alpha\beta} = \eta_{\alpha\beta} \frac{\partial}{\partial X}$
 $\uparrow$
 $f(X)$

Let us consider a more general theory, where
to be any 4×4 symmetric, invertible matrix of
value of $g_{\mu\nu}$ in some particular state of the theory.

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$g^{\mu\nu} g_{\nu\alpha} = \delta^\mu_\alpha$$

where metric $\eta_{\mu\nu} \rightarrow g_{\mu\nu}(x)$, with $g_{\mu\nu}$ allowed
to be a function of x . Then think of $\eta_{\mu\nu}$ as representing the local
theory.

is a curved manifold.



a more general theory, where metric $g_{\mu\nu}$
is a symmetric, invertible matrix function of x . Then think
in some particular state of the theory.

Geometrically, we say spacetime is a curved manifold.

Physics will be invariant under general coordinate transformations.

etime such that $g_{\mu\nu} = \eta_{\mu\nu}$ and

$$\frac{\partial}{\partial x^\alpha} g_{\mu\nu} \equiv g_{\mu\nu,\alpha} = 0 \quad \text{at that point.}$$

of x . Then think of $\eta_{\mu\nu}$ as representing the local

manifold.

transformations.



coordinates

of any point in spacetime such that $g_{\mu\nu} = \eta_{\mu\nu}$ and

$$\frac{\partial}{\partial x^\alpha} g_{\mu\nu} \equiv g_{\mu\nu,\alpha} = 0 \quad \text{at that point.}$$

(more convenient to think of $X^\alpha(x')$ rather than $X^\alpha(x)$).

always choose coordinates in the neighbourhood of any point

Proof:
$$g'_{\mu\nu}(x') = g_{\alpha\beta}(x) \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} \quad \left(\text{more convenient} \right)$$

imagine the point concerned is at $x^\alpha = x'^\alpha = 0$ (to simplify the algebra)

$$g_{\alpha\beta}(x)$$

Always choose coordinates in the neighbourhood of any point in

Proof: $g'_{\mu\nu}(x') = g_{\alpha\beta}(x) \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu}$ (more convenient)

imagine the point concerned is at $x^\alpha = x'^\alpha = 0$ (to simplify the algebra)

Assume
 $g_{\alpha\beta}(x)$ has
a Taylor series

$$g_{\alpha\beta}(x) = g_{\alpha\beta}(0) + g_{\alpha\beta,\lambda}(0)x^\lambda + \frac{1}{2}g_{\alpha\beta,\lambda\sigma}(0)x^\lambda x^\sigma + \dots$$

and that

$$x^\alpha(x') = K^\alpha_\gamma x'^\gamma + \frac{1}{2}L^\alpha_{\gamma\delta} x'^\gamma x'^\delta + \frac{1}{6}M^\alpha_{\gamma\delta\lambda} x'^\gamma x'^\delta x'^\lambda + \dots$$

ose coordinates in the neighbourhood of any point in spacetime

$$g'_{\mu\nu}(x') = g_{\alpha\beta}(x) \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} \quad (*) \quad \left(\text{more convenient to think of} \right)$$

the point concerned is at $x^\alpha = x'^\alpha = 0$ (to simplify the algebra)

$$g_{\alpha\beta}(x) = g_{\alpha\beta}(0) + g_{\alpha\beta,\lambda}(0) x^\lambda + \frac{1}{2} g_{\alpha\beta,\lambda\sigma}(0) x^\lambda x^\sigma + \dots$$

$$x^\alpha(x') = K^\alpha_{\gamma} x'^\gamma + \frac{1}{2} L^\alpha_{\gamma\delta} x'^\gamma x'^\delta + \frac{1}{6} M^\alpha_{\gamma\delta\lambda} x'^\gamma x'^\delta x'^\lambda + \dots$$

$$\eta = K^T g K : \text{ solve this via } K = OL, \text{ } O \text{ is orthogonal,}$$

$$O^T g O = \Lambda = \text{diagonal} = \begin{pmatrix} -l_0^2 & & & \\ & l_1^2 & & \\ & & l_2^2 & \\ & & & l_3^2 \end{pmatrix} \left(\begin{array}{l} \text{have to} \\ \text{assume } g \text{ has} \\ 1 \text{ negative, } 3 \\ \text{positive evals} \end{array} \right)$$

$$L = \begin{pmatrix} \frac{1}{l_0} & & & \\ & \frac{1}{l_1} & & \\ & & \frac{1}{l_2} & \\ & & & \frac{1}{l_3} \end{pmatrix} \Rightarrow \underline{K^T g K = \eta}$$

$l_0, l_1, l_2, l_3 \text{ real.}$

to (*) and set $g_{\mu\nu}(0) = \eta_{\mu\nu}$:

$$\Rightarrow \eta_{\mu\nu} = g_{\alpha\beta}(0) K_{\mu}^{\alpha} K_{\nu}^{\beta} \quad \text{ie.} \quad \eta = K^T g K \quad ; \text{ solve this}$$

$$O^T g O = \Lambda =$$

$$\left(\begin{array}{c} 6 \quad 4 \quad 6 = \underline{\underline{16}} \\ \downarrow \downarrow \downarrow \\ K \Rightarrow O \Lambda \quad \text{with } \Lambda \\ \text{a Lorentz trans}^n, \text{ we} \\ \text{still get } g' = \eta \end{array} \right)$$

$$L = \begin{pmatrix} \frac{1}{\ell_0} & \cdot & \cdot & \cdot \\ \cdot & \frac{1}{\ell_1} & \cdot & \cdot \\ \cdot & \cdot & \frac{1}{\ell_2} & \cdot \\ \cdot & \cdot & \cdot & \frac{1}{\ell_3} \end{pmatrix} \Rightarrow$$

$$(0) = \eta_{\mu\nu} :$$

$$K^\alpha_\mu K^\beta_{\nu} \quad \text{ie.} \quad K = OL, \quad O \text{ is}$$

$$\Lambda^\top \underbrace{L^\top D L}_\eta \Lambda = \eta$$

$$\eta = K^\top g K \quad ; \text{ solve this via } K = OL, \quad O \text{ is}$$

$$O^\top g O = D = \text{diagonal} = \begin{pmatrix} -l_0^2 & & & \\ & l_1^2 & & \\ & & l_2^2 & \\ & & & l_3^2 \end{pmatrix}$$

$64 \quad 6 = 16$
 $\downarrow \downarrow \downarrow$
 $\Rightarrow OL\Lambda$ with Λ
 a Lorentz transⁿ, we
 still get $g' = \eta$

then
choose

$$L = \begin{pmatrix} \frac{1}{l_0} & & & \\ & \frac{1}{l_1} & & \\ & & \frac{1}{l_2} & \\ & & & \frac{1}{l_3} \end{pmatrix}$$

$$\Rightarrow \underline{K^\top g K = \eta}$$

choose coordinates in the neighbourhood of any point in space

$$: \quad g'_{\mu\nu}(x') = g_{\alpha\beta}(x) \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} \quad (*) \quad \left(\text{more convenient to think} \right)$$

the point concerned is at $x^\alpha = x'^\alpha = 0$ (to simplify the algebra)

some
 $g_{\alpha\beta}(x)$ has
 Taylor series

$$g_{\alpha\beta}(x) = g_{\alpha\beta}(0) + g_{\alpha\beta,\lambda}(0) (x-a)^\lambda + \frac{1}{2} g_{\alpha\beta,\lambda\sigma}(0) (x-a)^\lambda (x-a)^\sigma + \dots$$

and that

$$x^\alpha(x) = K^\alpha_\gamma x'^\gamma + \frac{1}{2} L^\alpha_{\gamma\delta} x'^\gamma x'^\delta + \frac{1}{6} M^\alpha_{\gamma\delta\lambda} x'^\gamma x'^\delta x'^\lambda + \dots$$

$$g_{\mu\nu}(x') = g_{\alpha\beta}^{(x)} \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} \quad (*) \quad (\text{more convenient to think of } x(x))$$

concerned is at $x^\alpha = x'^\alpha = 0$ (to simplify the algebra)

$$g_{\alpha\beta}(x) = g_{\alpha\beta}(0) + g_{\alpha\beta,\lambda}(0) \frac{(x-a)^\lambda}{(x-a)^\lambda} + \frac{1}{2} g_{\alpha\beta,\lambda\sigma}(0) \frac{(x-a)^\lambda (x-a)^\sigma}{(x-a)^\lambda (x-a)^\sigma} + \dots$$

$$x^\alpha(x) = K^\alpha_\gamma x'^\gamma + \frac{1}{2} L^\alpha_{\gamma\delta} x'^\gamma x'^\delta + \frac{1}{6} M^\alpha_{\gamma\delta\lambda} x'^\gamma x'^\delta x'^\lambda + \dots \quad (\dagger)$$

and set $g_{\mu\nu}(0) = \eta_{\mu\nu}$:

$$\eta_{\mu\nu} = g_{\alpha\beta}(0) K^\alpha_\mu K^\beta_\nu \quad \text{ie.} \quad \eta = K^T g K$$

$$\Lambda^T \underbrace{L^T D L}_{\eta} \Lambda = \eta$$

solve this via

$$O^T g O = D = \text{diagonal}$$

$$\begin{matrix} 6 & 4 & 6 & = & \underline{\underline{16}} \\ \downarrow & \downarrow & \downarrow & & \end{matrix}$$

Sup
a Taylor series
and that

$$X^\alpha(x') = K^\alpha_\gamma X'^\gamma + \frac{1}{2} L^\alpha_{\gamma\delta} X'^\gamma X'^\delta + \frac{1}{6} M^\alpha_{\gamma\delta\lambda} X'^\gamma X'^\delta X'^\lambda + \dots$$

Sub these into (*) and set $g'_{\mu\nu}(0) = \eta_{\mu\nu}$:

$$(*) \Rightarrow \eta_{\mu\nu} = g_{\alpha\beta}(0) K^\alpha_\mu K^\beta_\nu \quad \text{i.e.} \quad \eta = K^T g K$$

$$\frac{\partial x}{\partial x'} \frac{\partial}{\partial x'} g(x)$$

$$g(0) \frac{\partial x}{\partial x'} \frac{\partial x}{\partial x'}$$

$\nearrow L$
 $\nearrow K$

$$\left(K \Rightarrow OL\Lambda \text{ with } \Lambda \text{ a Lorentz trans.}, \text{ we still get } g' = \eta \right) \quad \text{then choose } L = \begin{pmatrix} 1 & & \\ & \gamma & \\ & & \gamma v \\ & & & \gamma v \end{pmatrix}$$

$6 \quad 4 \quad 6 = 16$
 $\downarrow \quad \downarrow \quad \downarrow$

+ to show that we can choose c.t. ($\neq$). st. $\frac{\partial}{\partial x'^{\alpha}} g'_{\mu\nu}(x') = 0$

Calculate $g'_{\mu\nu,\lambda}(0) = g_{\alpha\beta,\delta}^{(0)} K_{\lambda}^{\delta} K_{\mu}^{\alpha} K_{\nu}^{\beta} + g_{\alpha\beta}(0) (L_{\mu\lambda}^{\alpha} K_{\nu}^{\beta} + K_{\mu}^{\alpha} L_{\nu\lambda}^{\beta})$

Can we choose $\underline{L_{\mu\lambda}^{\alpha}}$ so that r.h.s. $\stackrel{\uparrow}{=} 0$? Yes, since $g_{\alpha\beta}^{(0)}$

$g_{\alpha\beta,\delta}^{(0)} \Rightarrow \frac{D(D+1)}{2} \cdot D$ parameters.
 $L_{\mu\lambda}^{\alpha} \rightarrow$

a point concerned is at $x^\alpha = x'^\alpha = 0$ (to simplify the algebra)

$$g_{\alpha\beta}(x) = g_{\alpha\beta}(0) + g_{\alpha\beta,\lambda}(0)(x-a)^\lambda + \frac{1}{2} g_{\alpha\beta,\lambda\sigma}(0)(x-a)^\lambda (x-a)^\sigma + \dots$$

$$x^\alpha(x) = K^\alpha_\gamma x'^\gamma + \frac{1}{2} L^\alpha_{\gamma\delta} x'^\gamma x'^\delta + \frac{1}{6} M^\alpha_{\gamma\delta\lambda} x'^\gamma x'^\delta x'^\lambda + \dots \quad (*)$$

into (*) and set $g'_{\mu\nu}(0) = \eta_{\mu\nu}$:

$$\Rightarrow \eta_{\mu\nu} = g_{\alpha\beta}(0) K^\alpha_\mu K^\beta_\nu \quad \text{ie.} \quad \eta = K^T g K \quad ; \text{ solve}$$

$$\Lambda^T \underbrace{L^T D L}_{\eta} \Lambda = \eta$$

$$\begin{matrix} 6 & 4 & 6 & = & \underline{\underline{16}} \\ \downarrow & \downarrow & \downarrow & & \end{matrix}$$

$$O^T g O =$$

to show that we can choose C.V. (1) $\partial x^\alpha / \partial \lambda$

Calculate $g'_{\mu\nu,\lambda}(0) = g_{\alpha\beta,\delta}^{(0)} K_\lambda^\delta K_\mu^\alpha K_\nu^\beta + g_{\alpha\beta}(0) (L_{\mu\lambda}^\alpha K_\nu^\beta)$

Can we choose $\underline{L_{\mu\lambda}^\alpha}$ so that r.h.s. = 0? Yes, since

$g_{\alpha\beta,\delta}^{(0)} \Rightarrow \frac{D(D+1)}{2} \cdot D$ parameters.

$L_{\mu\lambda}^\alpha \rightarrow \frac{D(D+1)}{2} \cdot D$ parameters.

Continue to second derivatives: try to set $\frac{\partial^2 g'_{\mu\nu}}{\partial x'^\alpha \partial x'^\beta}$

obstruction is the Riemann tensor

lhs $\Rightarrow \frac{\partial^2 g'(x')}{\partial x'^\alpha \partial x'^\beta} \Big|_{x'=0}$ rhs $\Rightarrow$ known. + $M \times (\text{known})$

$$M^{\alpha}_{\delta\delta\lambda} \Rightarrow \frac{D(D+1)(D+2)}{6} D$$

Symmetric terms of δ, δ, λ .

independent components

but $\frac{\partial^2 g'_{\mu\nu}}{\partial x'^\alpha \partial x'^\beta}(0)$

$$\overline{\partial x^\alpha \partial x^\beta}$$

$M \times (\text{known})$: can we solve this for M ?

at $\partial_\alpha \partial_\beta g_{\mu\nu}(0)$ has $\left(\frac{D(D+1)}{2}\right)^2$ independent components:

$$\begin{array}{c} g_{\mu\nu}(\phi) \\ \downarrow \\ \frac{D(D+1)}{2} \end{array}$$

e fewer available parameters than there are

residual # = $\left(\frac{D(D+1)}{2}\right)^2 - \frac{D^2(D+1)(D+2)}{6} = \frac{D^2}{6}$
which cannot
in general
be set zero

exactly the number of independent components of a

ters than there are $g'_{\mu\nu,\alpha\beta}$ so cannot set them zero.

$$\frac{D^2(D+1)(D+2)}{6} = \frac{D^2(D^2-1)}{12}, \text{ which will turn out to be}$$

of independent components of a tensor, called the Riemann tensor.

$$= K_{\alpha\beta} X^{\alpha} X^{\beta} + \frac{1}{2} L_{\alpha\beta\gamma} X^{\alpha} X^{\beta} X^{\gamma} + \frac{1}{6} M_{\alpha\beta\gamma\delta} X^{\alpha} X^{\beta} X^{\gamma} X^{\delta} + \dots \quad (\neq)$$

fewer available parameters than there are $g'_{\mu\nu, \alpha\beta}$ so ca

actual # = $\left(\frac{D(D+1)}{2}\right)^2 - \frac{D^2(D+1)(D+2)}{6} = \frac{D^2(D^2-1)}{12}$, which will be
 cannot
 general
 be set zero

exactly the number of independent components of a tensor, called the R

In fact, for any curved manifold, we can choose coordinates in the nhd of any point st.

$$g_{\mu\nu}(x) = \eta_{\mu\nu} - \frac{1}{3} R_{\mu\alpha\nu\beta} X^{\alpha} X^{\beta} + o(x^3) \dots \quad [\text{Rie}]$$

zero.

$$\frac{1(D+2)}{6} = \frac{D^2(D^2-1)}{12}, \text{ which will turn out to be}$$

ident components of a tensor, called the Riemann tensor.

coordinates in the nhd of any point s.t.

$$g_{\mu\nu}(x) = \eta_{\mu\nu} - \frac{1}{3} R_{\mu\alpha\nu\beta}^{(0)} x^\alpha x^\beta + O(x^3) \dots \quad \left[\text{"Riemann normal" coords} \right]$$