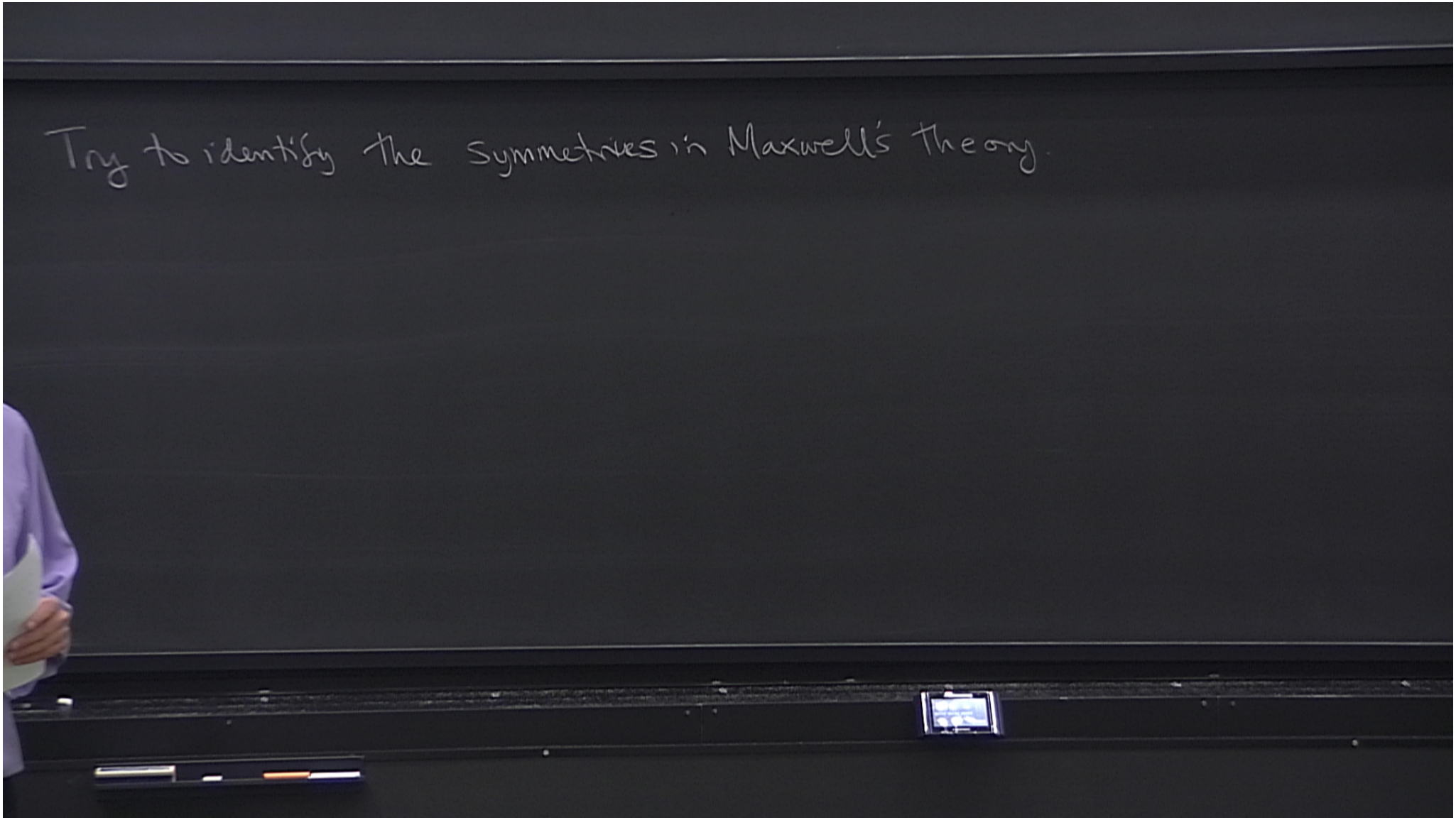
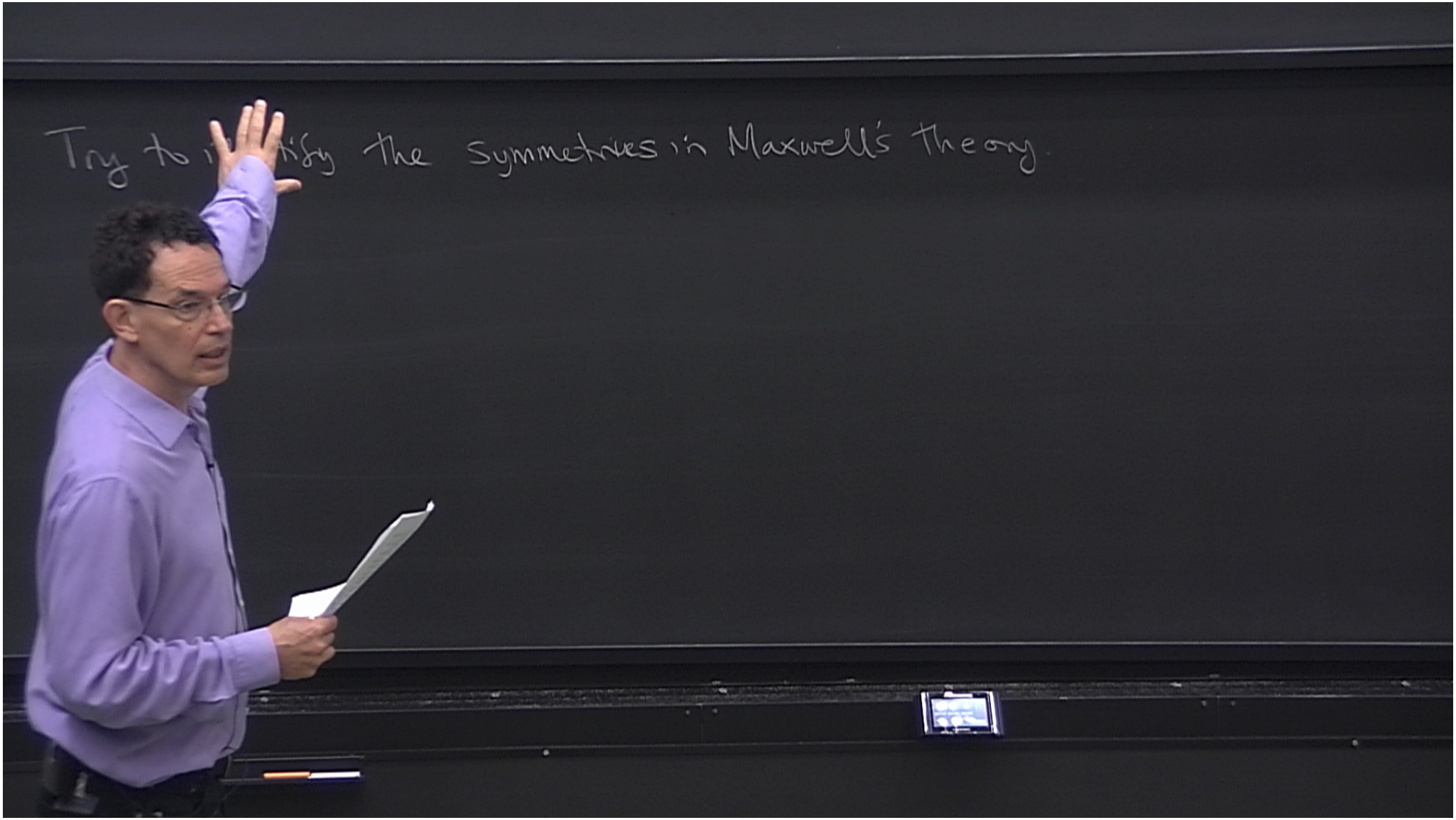
Title: PSI 2016/2017 Relativity - Lecture 2

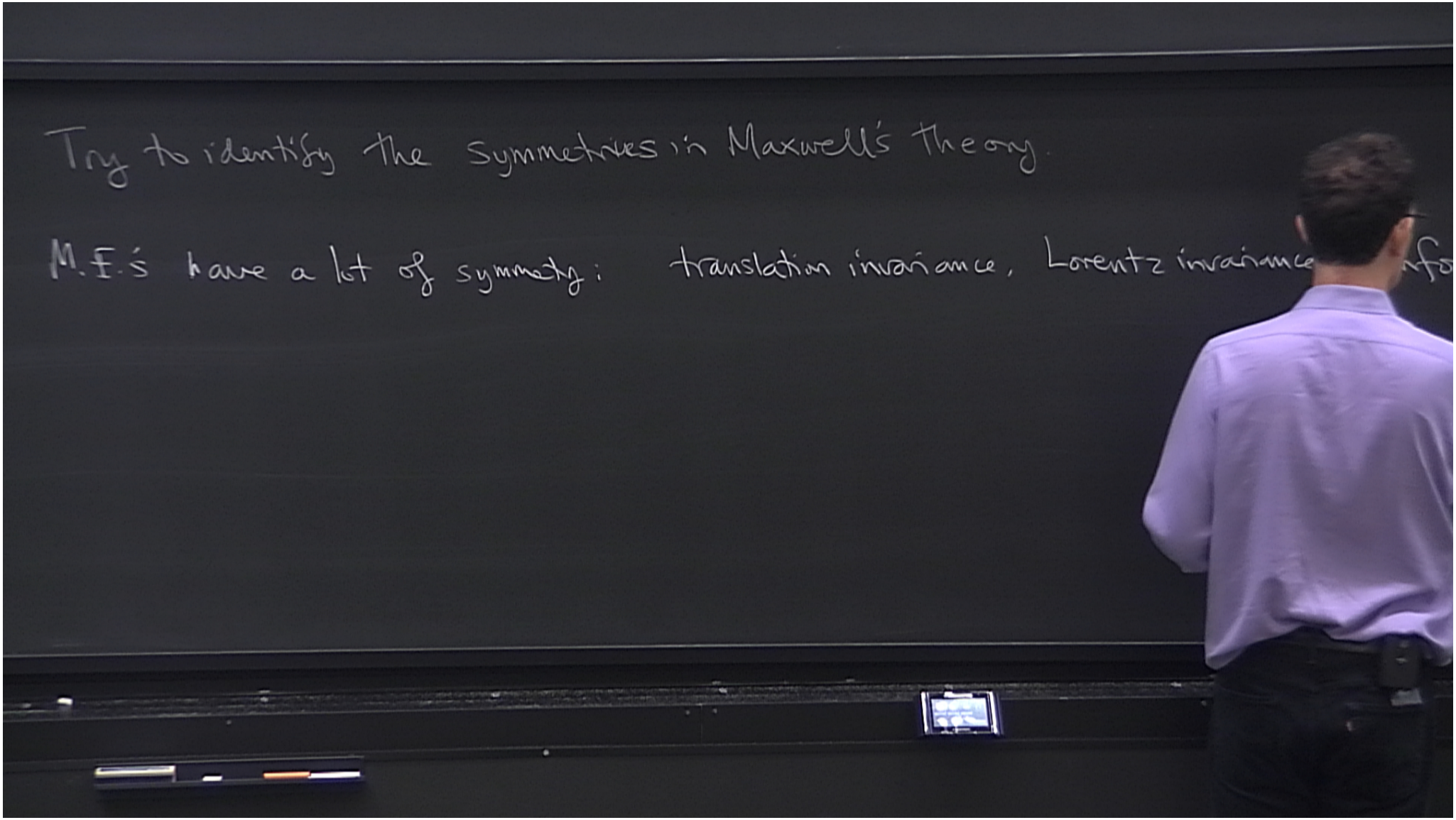
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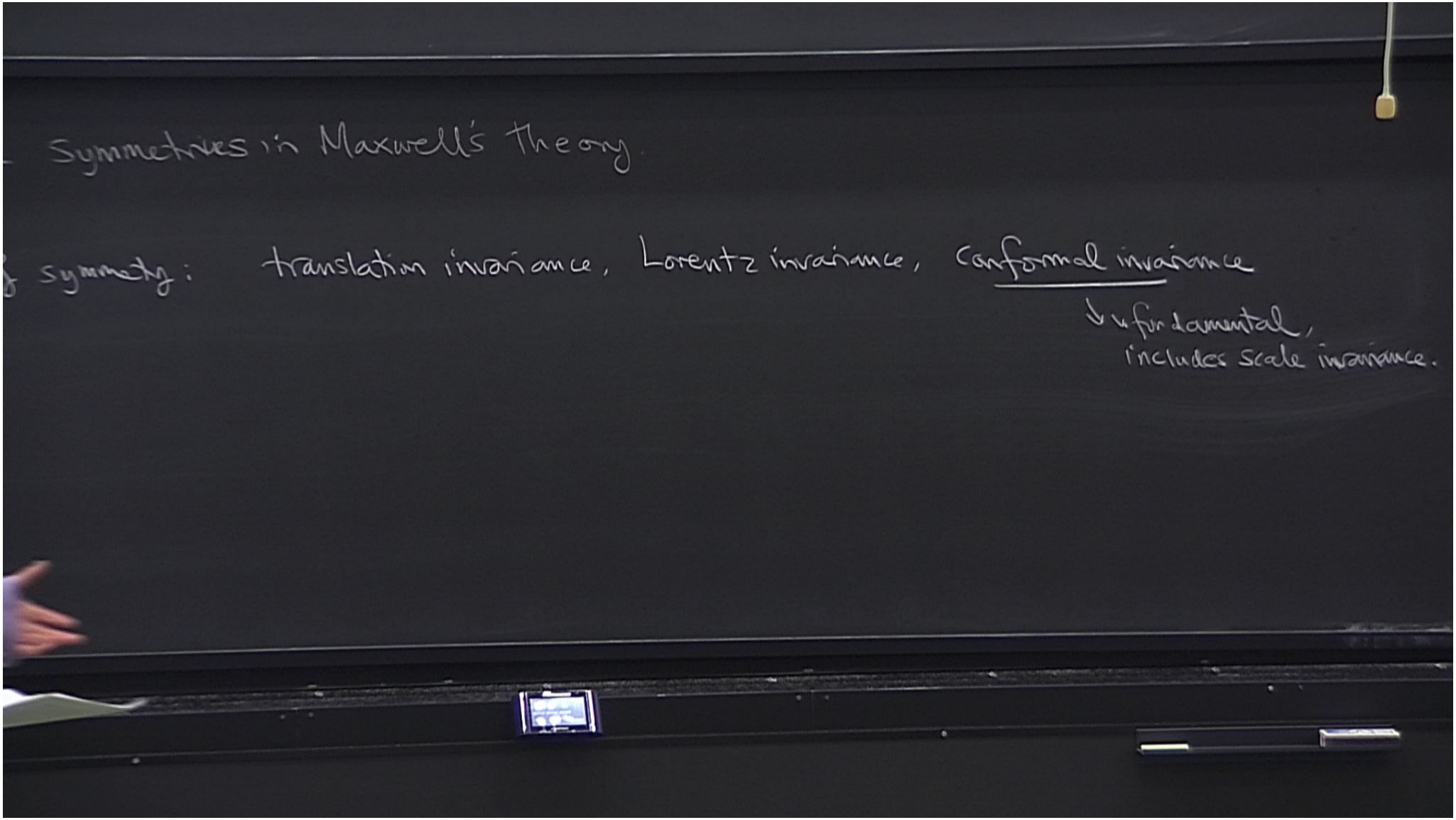
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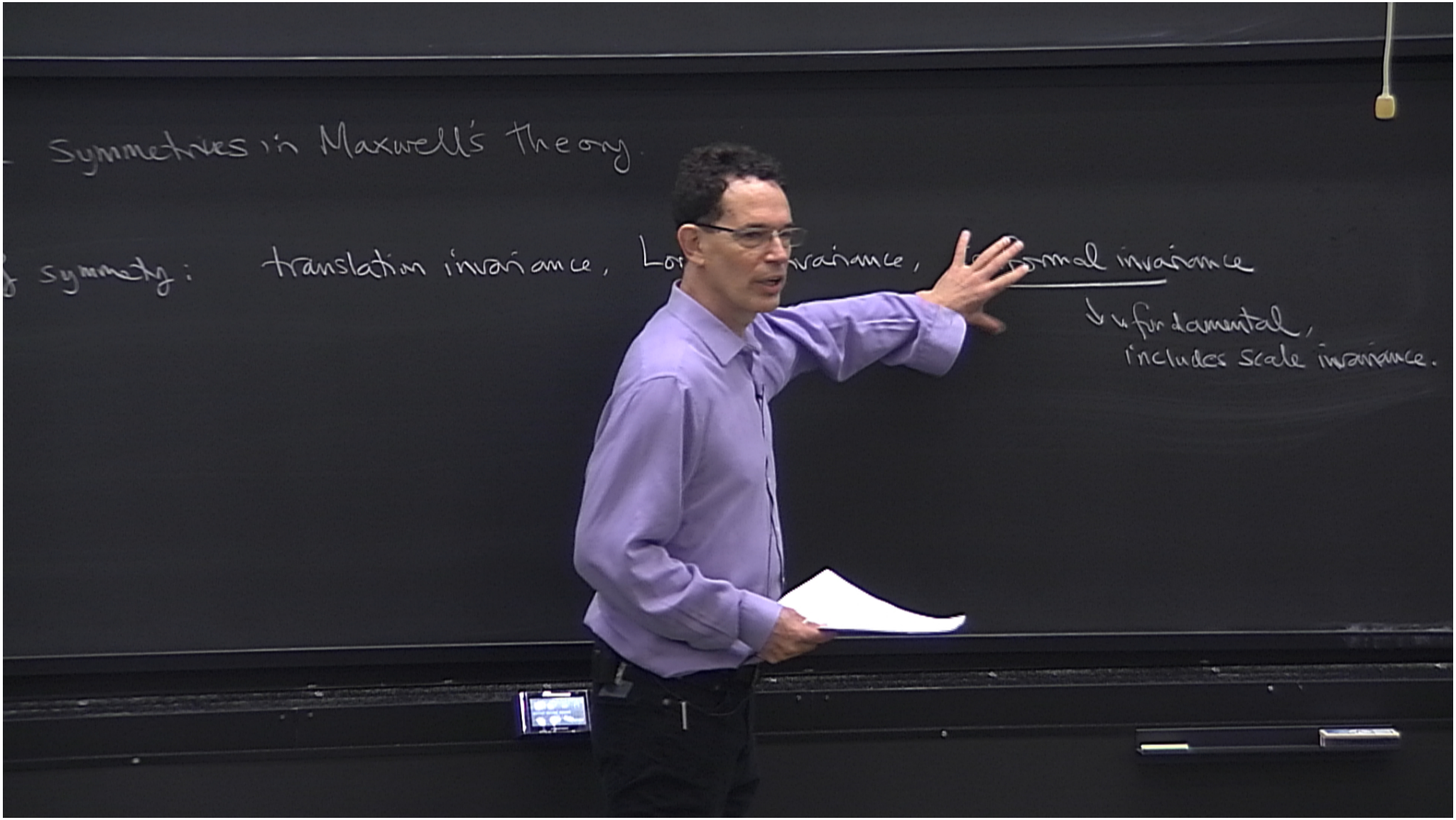
Abstract:

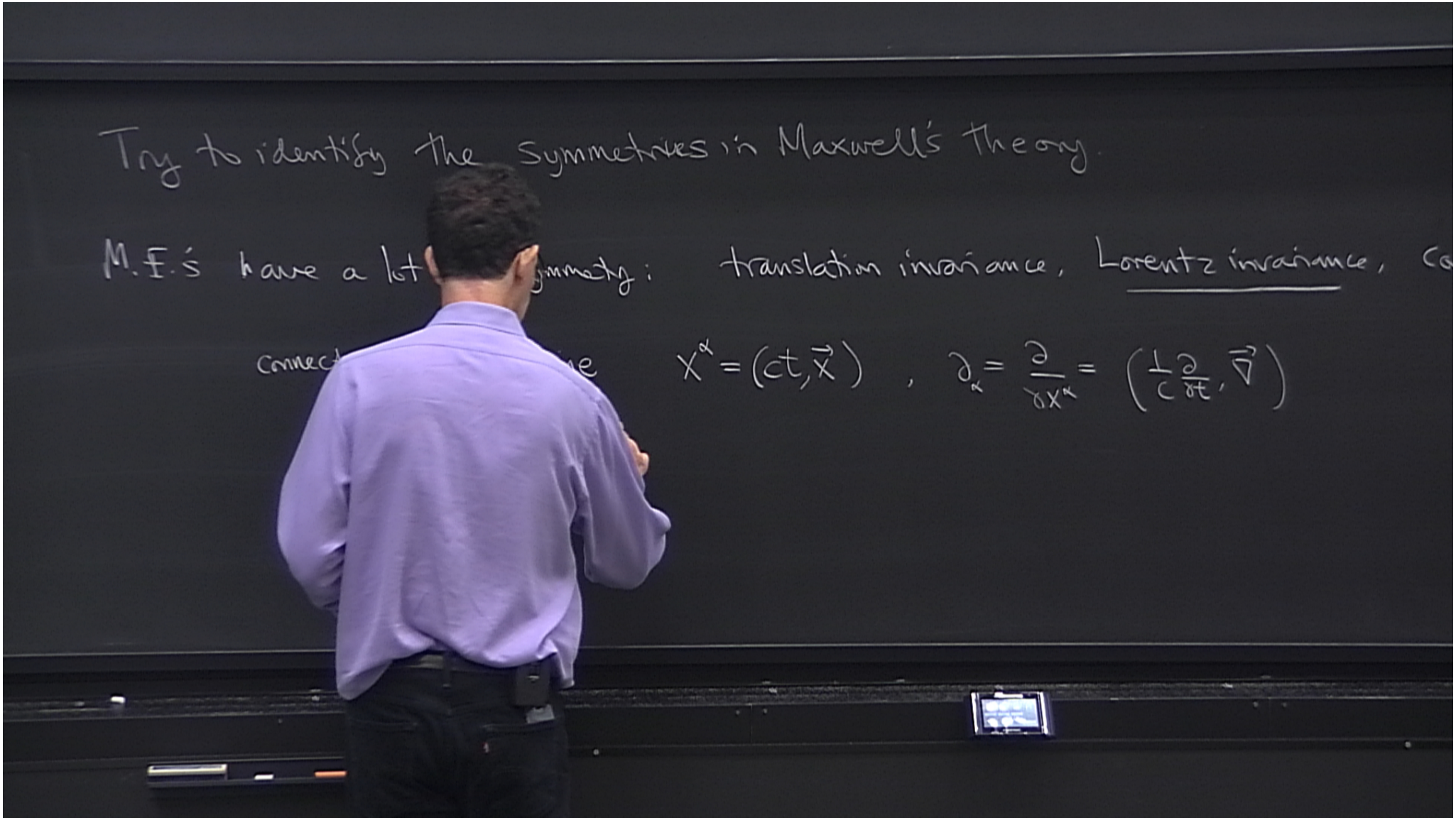

Try to identify the symmetries in Maxwell's theory.

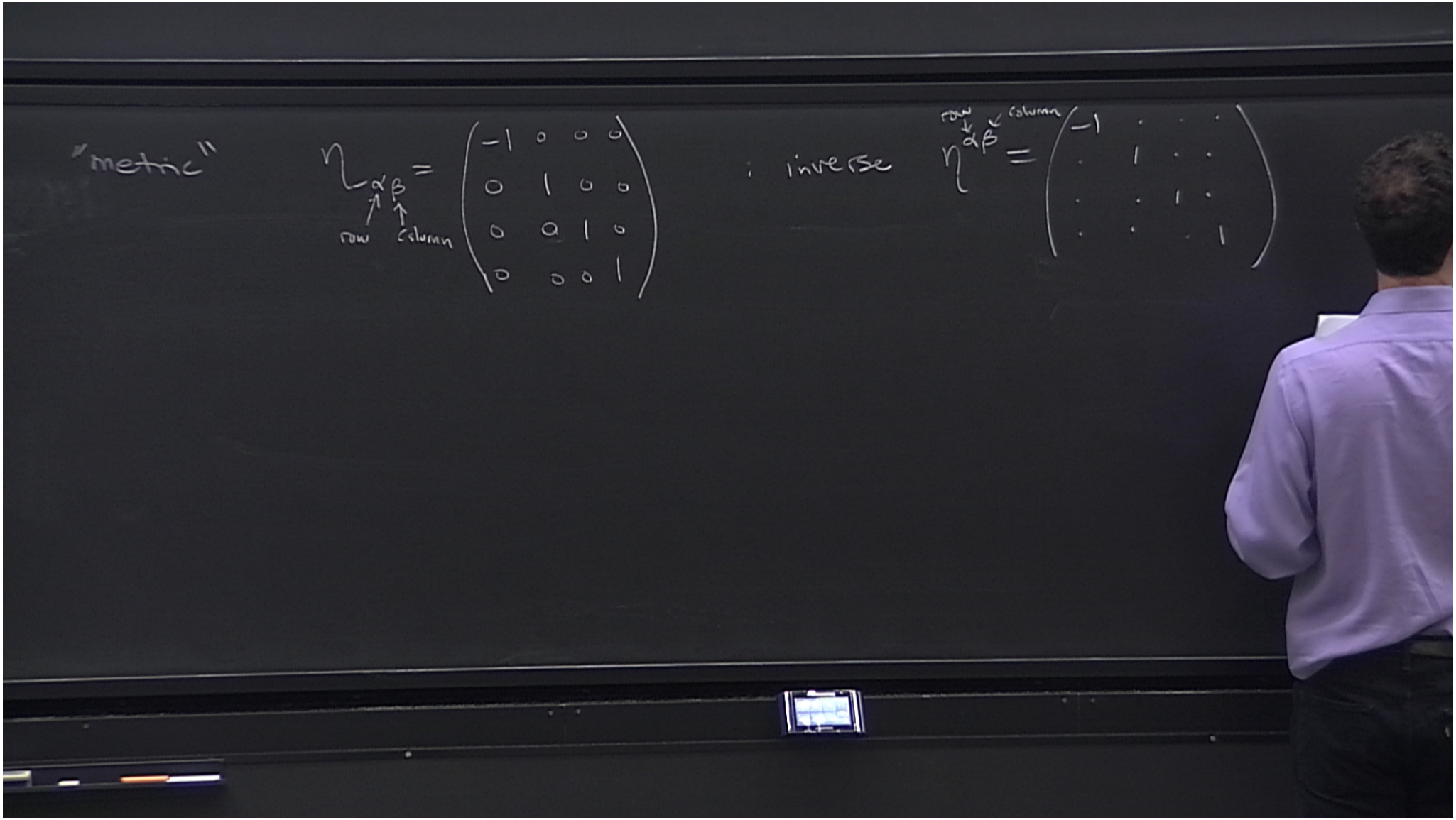

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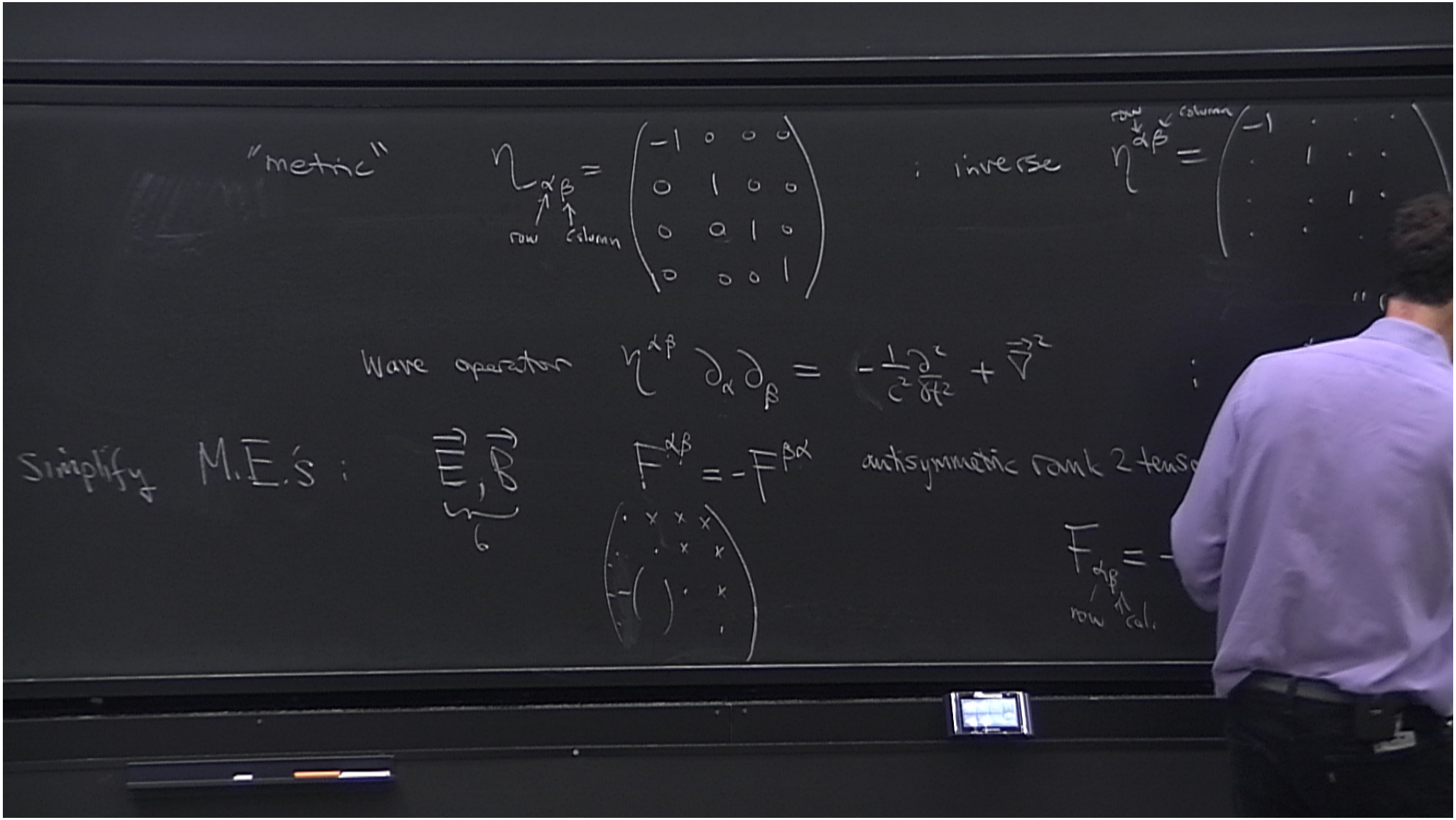

Try to identify the symmetries in Maxwell's theory.
M.E.'s have a lot of symmetry: translation invariance, Lorentz invariance


Symmetries in Maxwell's Theory
symmetry: translation invariance, Lorentz invariance, conformal invariance
↳ fundamental, includes scale invariance.


Symmetries in Maxwell's Theory
symmetry:
translation invariance,
invariance,
invariance
fundamental,
includes scale invariance.


Try to identify the symmetries in Maxwell's theory.
M.E.'s have a lot
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translation invariance, Lorentz invariance, Co
Connect
e
$x^\alpha = (ct, \vec{x})$, $\partial_\alpha = \frac{\partial}{\partial x^\alpha} = \left(\frac{1}{c}\frac{\partial}{\partial t}, \vec{\nabla}\right)$


"metric"
$\eta_{\alpha\beta} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
row
column
: inverse
$\eta^{\alpha\beta} = \begin{pmatrix} -1 & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & 1 \end{pmatrix}$
row
column

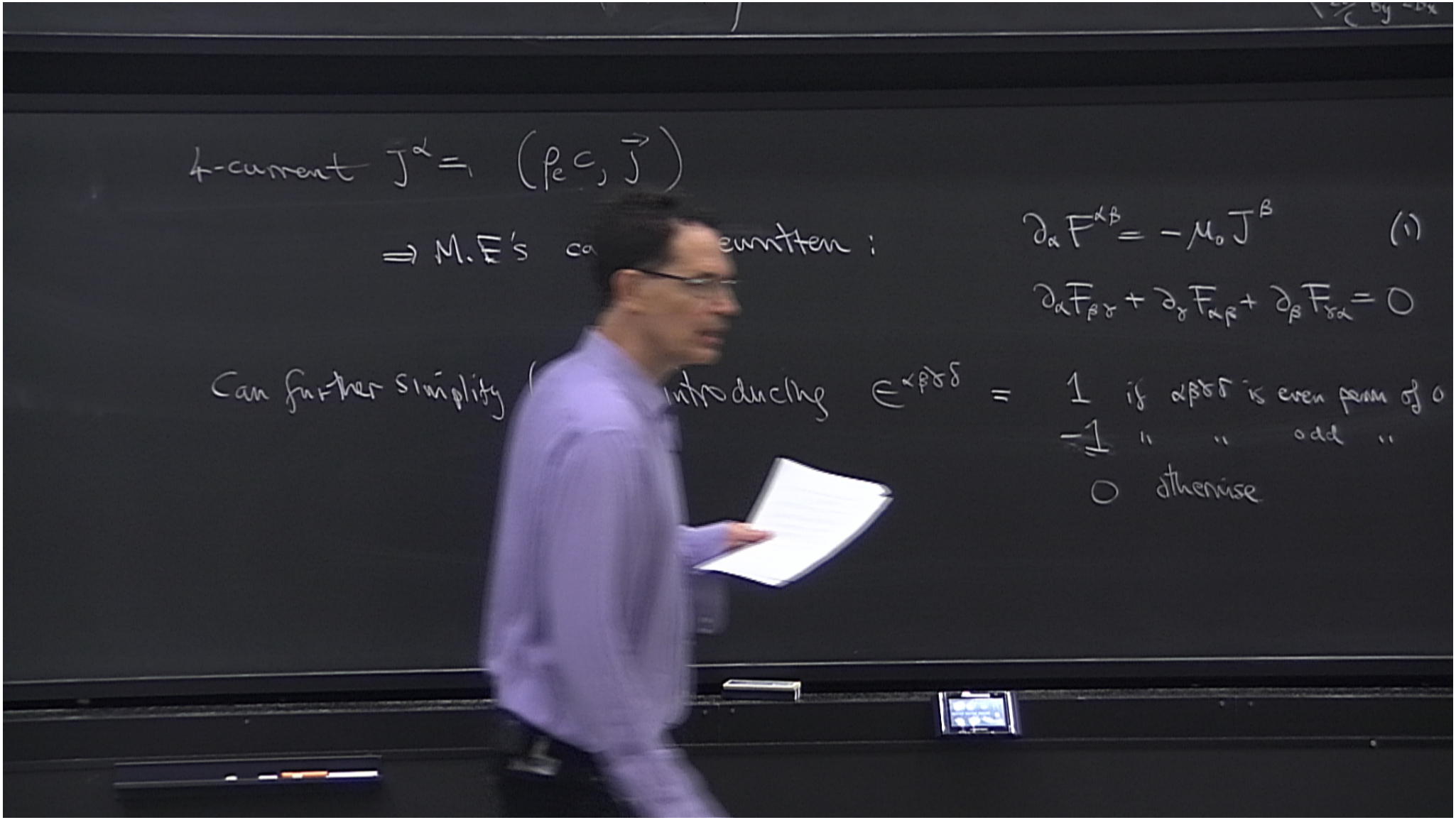

"metric"
row
column
: inverse
row
column
Wave operator
:
Simplify M.E.'s :
E, B
antisymmetric rank 2 tens
row
col.

4-current $J^\alpha = (\rho c, \vec{J})$

$\Rightarrow$ M.E's can be rewritten:

$$\partial_\alpha F^{\alpha\beta} = -\mu_0 J^\beta$$

$$\partial_\alpha F_{\beta\gamma} + \partial_\gamma F_{\alpha\beta} + \partial_\beta F_{\gamma\alpha} = 0$$


4-current $J^\alpha = (\rho_e c, \vec{J})$
$\Rightarrow$ M.E's ca eunitten :
$\partial_\alpha F^{\alpha\beta} = -\mu_0 J^\beta$ (1)
$\partial_\alpha F_{\beta\gamma} + \partial_\gamma F_{\alpha\beta} + \partial_\beta F_{\gamma\alpha} = 0$
Can further simplify (ntroducing $\epsilon^{\alpha\beta\gamma\delta}$ = 1 if $\alpha\beta\gamma\delta$ is even perm of 0
-1 " " odd "
0 otherwise

4-current $J^\alpha = (\rho_e c, \vec{J})$

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$$\partial_\alpha F^{\alpha\beta} = -\mu_0 J^\beta \quad (1)$$

$$\partial_\alpha F_{\beta\gamma} + \partial_\gamma F_{\alpha\beta} + \partial_\beta F_{\gamma\alpha} = 0 \quad (2)$$

Can further simplify (2) by introducing $\epsilon^{\alpha\beta\gamma\delta}$ =

1 if $\alpha\beta\gamma\delta$ is even perm of 0123

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(2) is then $\epsilon^{\alpha\beta\gamma\delta}\partial_\beta F_{\gamma\delta} = 0$.

(Ex).

$\alpha = 0 \Rightarrow$ no monopoles

$\alpha = i \Rightarrow$ Faraday

$\left(\frac{E_z}{c} \; B_y \; -\partial_x \; 0\right)$

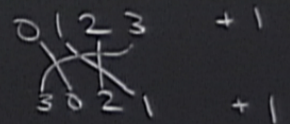
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$\partial_\alpha F_{\beta\gamma} + \partial_\gamma F_{\alpha\beta} + \partial_\beta F_{\gamma\alpha} = 0$ (2) (Ex)

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$J^\beta = 0$

$\partial^\alpha(2) \overset{(1)}{\Rightarrow} \partial^\alpha \partial_\alpha F_{\beta\gamma} = 0$.

hen $\epsilon^{\alpha\beta\gamma\delta}\partial_\beta F_{\gamma\delta} = 0$.

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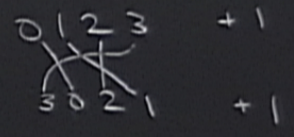
rewritten: $\partial_\alpha F^{\alpha\beta} = -\mu_0 J^\beta$ (1)

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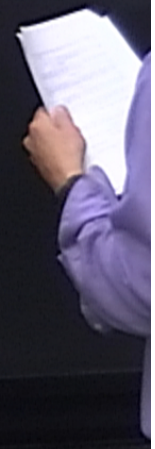
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take $\sqrt{l_p l_\Lambda} \simeq 10^{-4}$ m., scale of living cells.

Actually, $\epsilon^{\alpha\beta\gamma\delta}\partial_\beta F_{\gamma\delta} = 0$

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eg. $\epsilon^{\alpha\beta\gamma\delta}\partial_\beta F_{\gamma\delta} = 0$ implies (Poincare) that $F_{\alpha\beta}$ can be written in terms of a vector potential

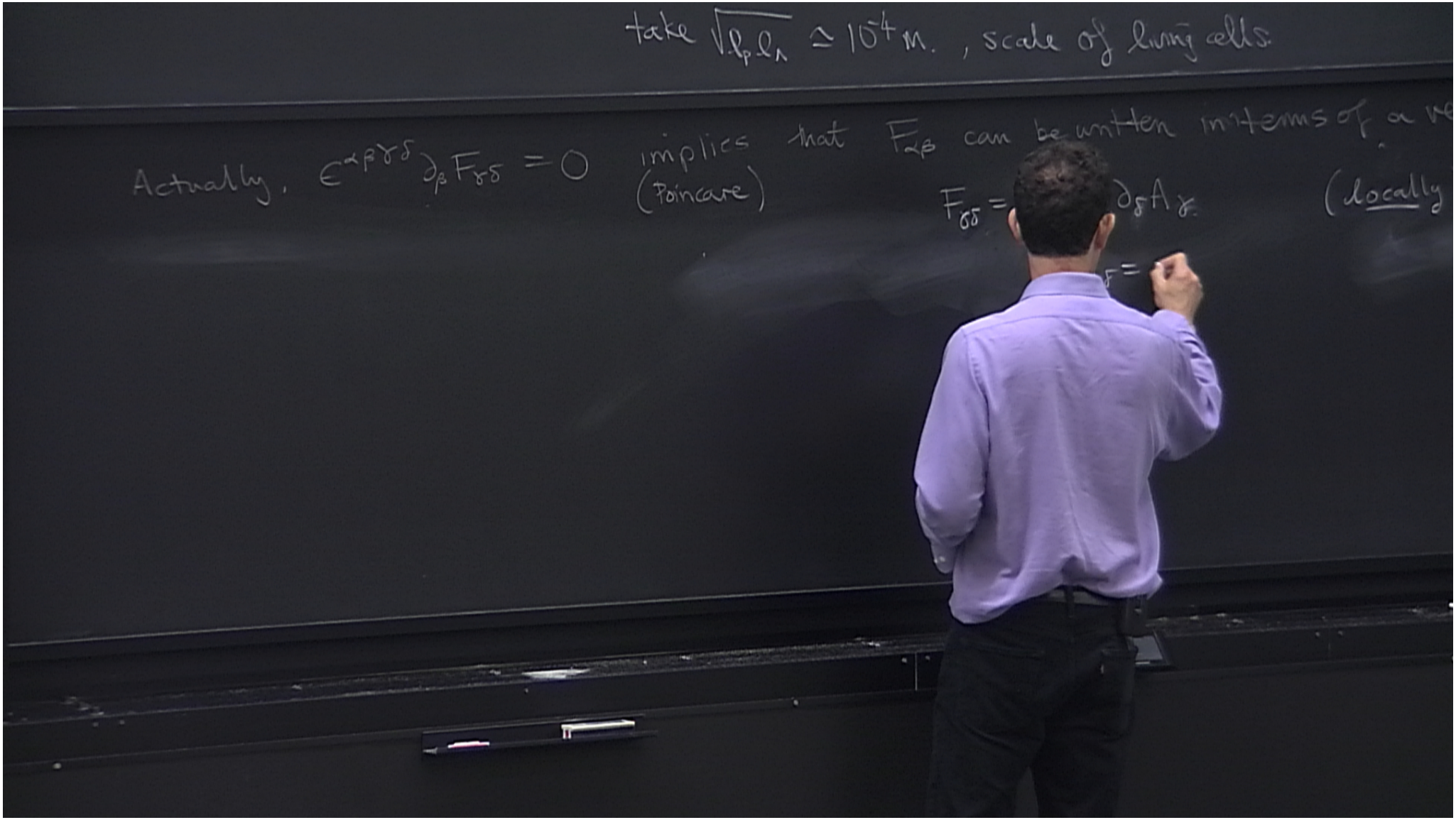
$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$$

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$F_{\alpha\beta} = \partial_\delta A_\gamma$ (locally

$F =$



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$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$ (locally)

A_β = vector potential.

Set $A^\mu = (\quad)$, $A_\mu = \left(-\frac{\phi}{c}, \vec{A}\right)$

$\Rightarrow$ $\vec{A}$

ϕ $-\frac{\partial}{\partial t}\vec{A}$

(usual expressions in electrodynamics)

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Notice that the 4-vector potential A_μ is [illegible]nique!

If we replace $A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu \chi$, with χ any fu

then $F_{\mu\nu} \rightarrow F'_{\mu\nu} =$ [illegible]

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The vector potential A_μ is not unique!

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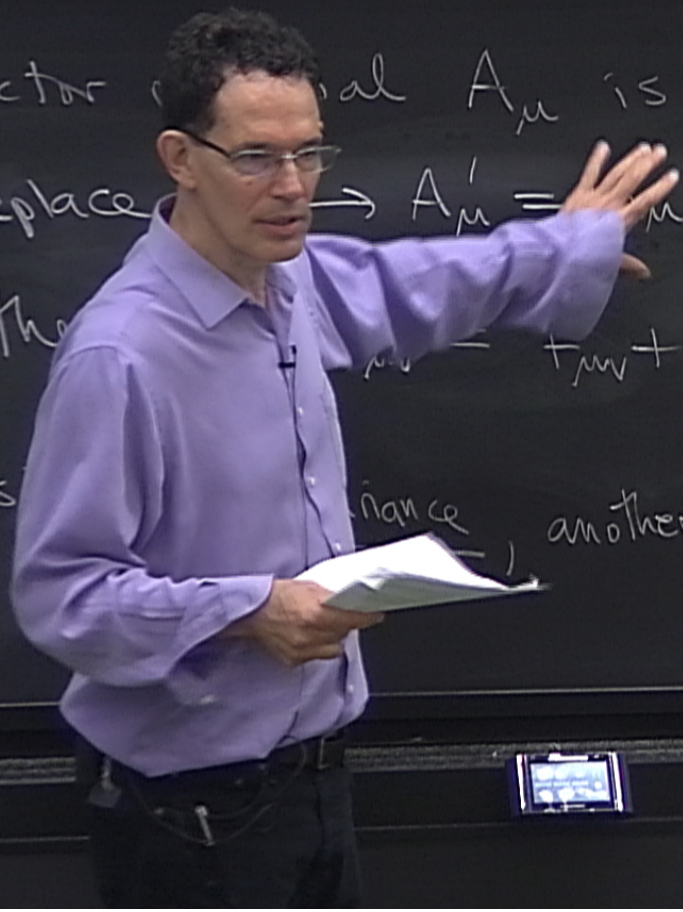
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this invariance, another (!) symmetry of Maxwell's equations.

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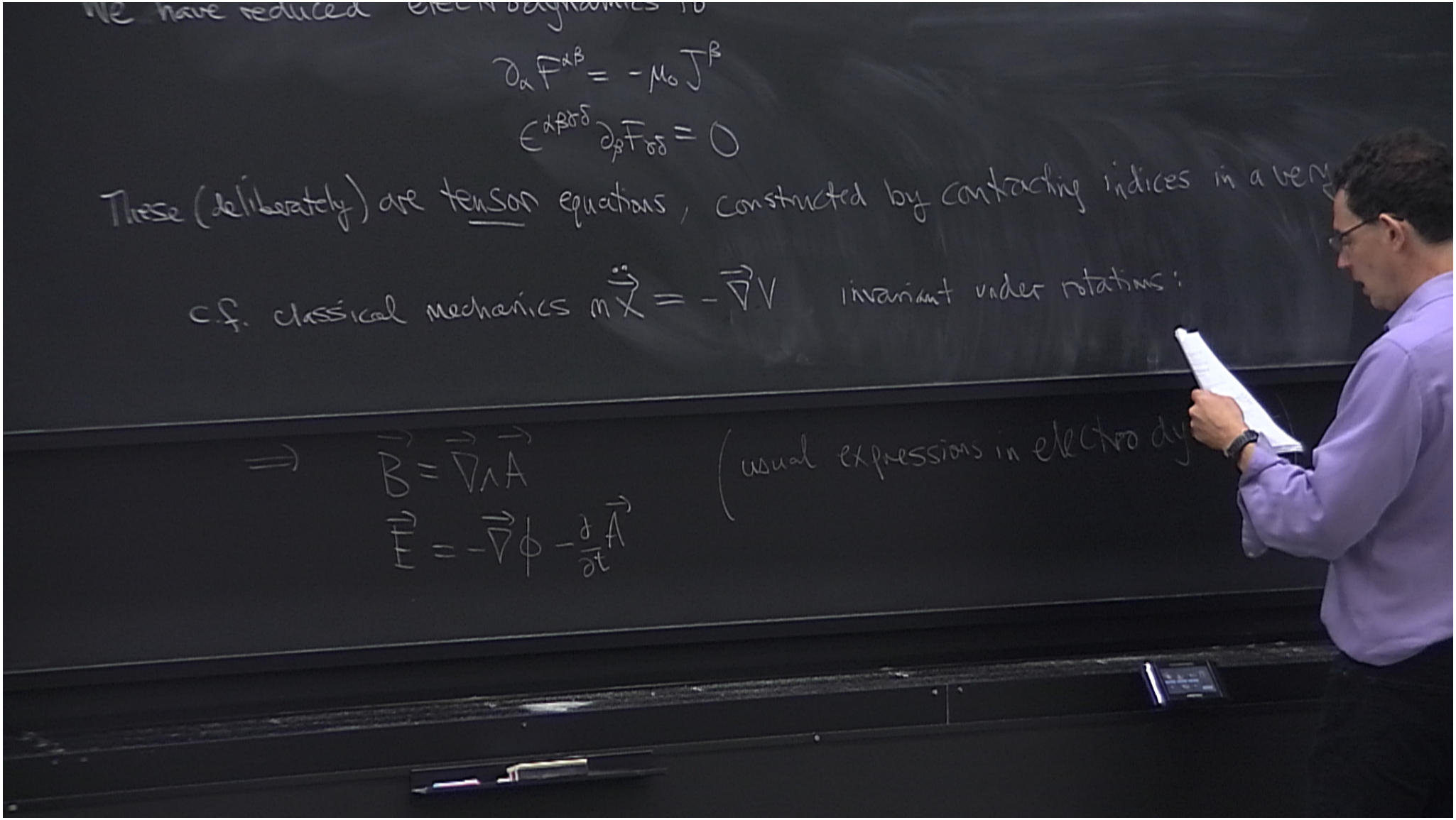
Lorentz invariance

We have reduced electrodynamics to

$$\partial_\alpha F^{\alpha\beta} = -\mu_0 J^\beta$$

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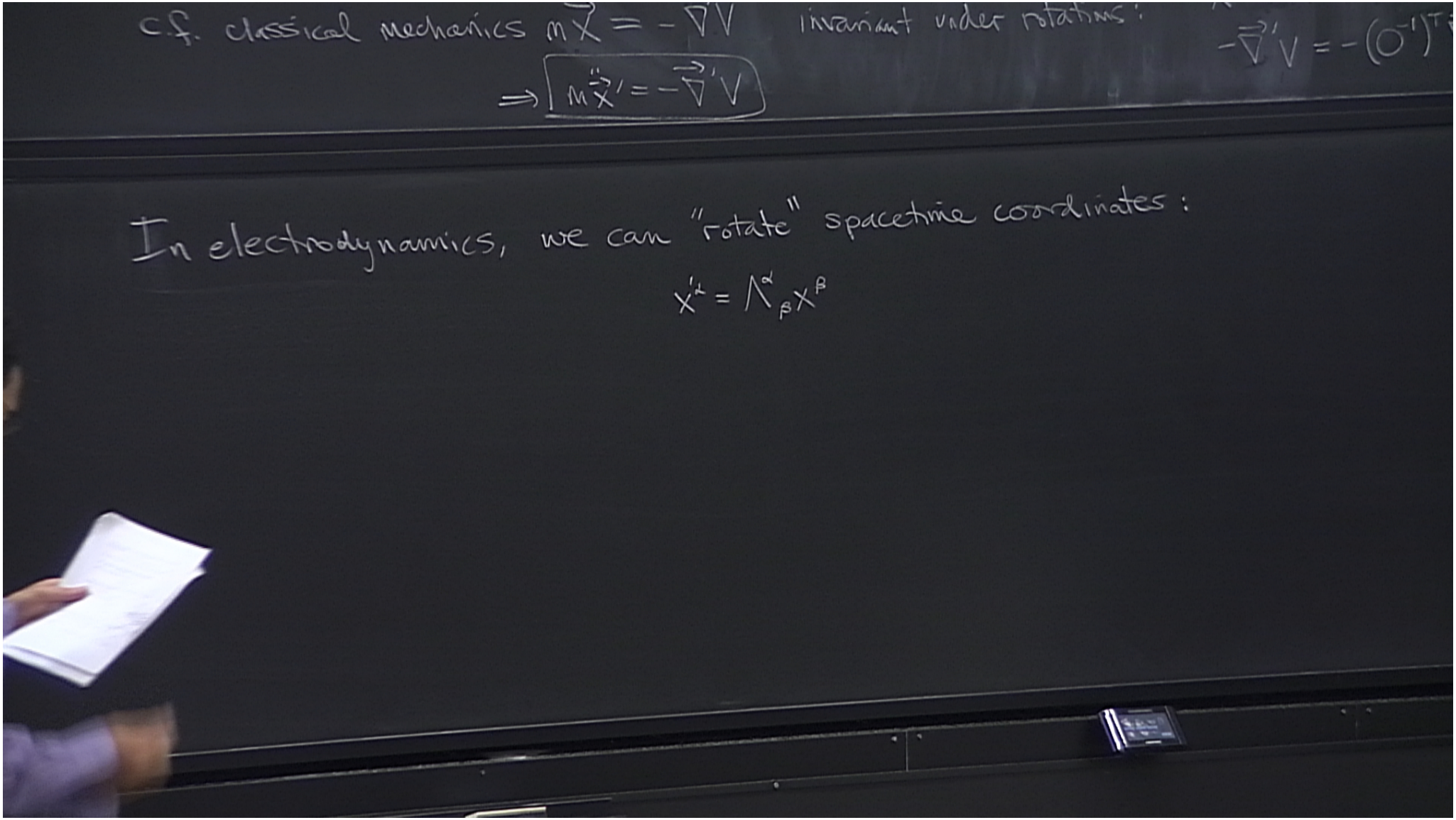
$-\vec{\nabla}'V = -(O^{-1})^T\vec{\nabla}$

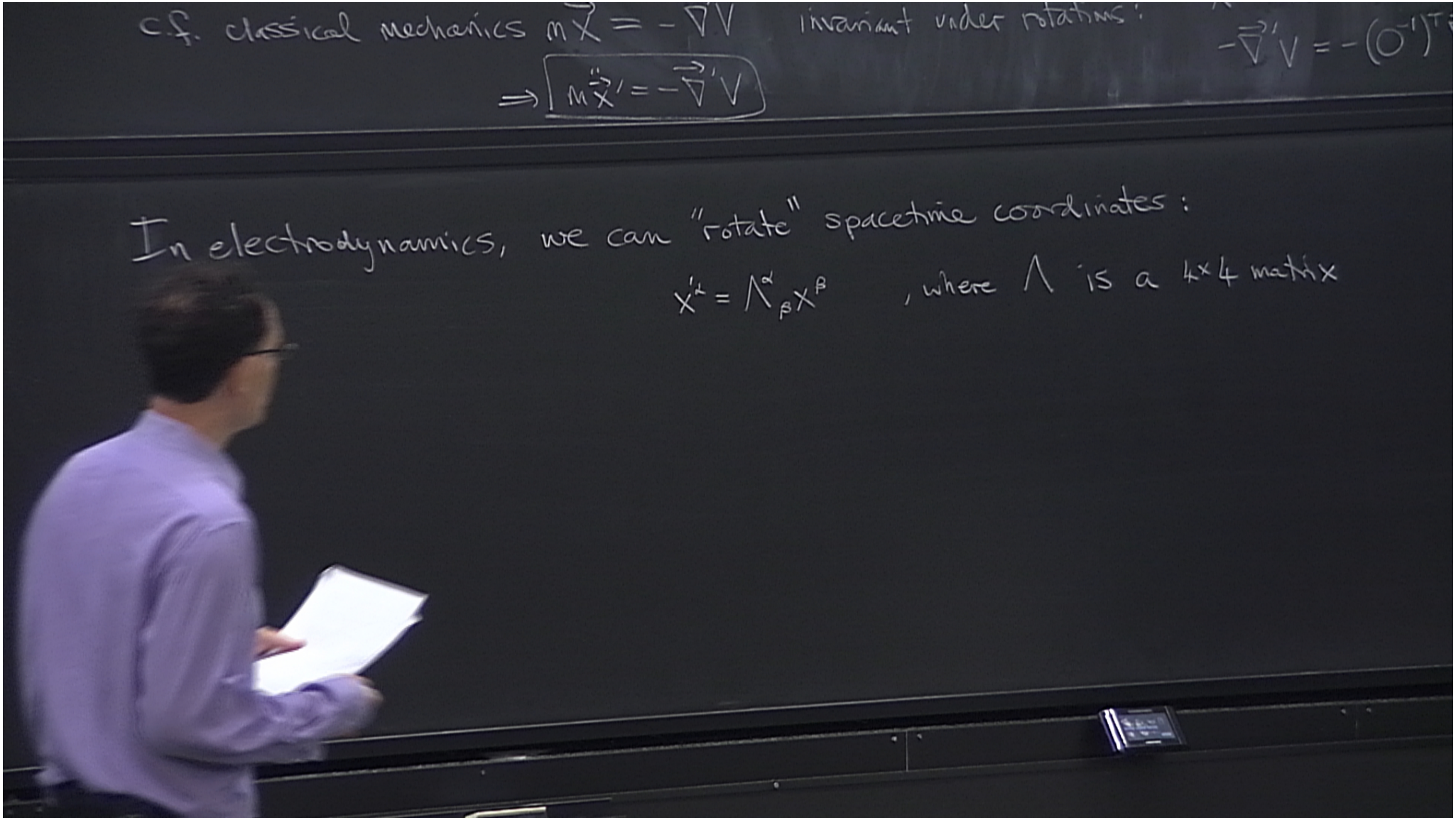
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In electrodynamics, we can "rotate" spacetime coordinates:
$x'^{\alpha} = \Lambda^{\alpha}_{\beta} x^{\beta}$


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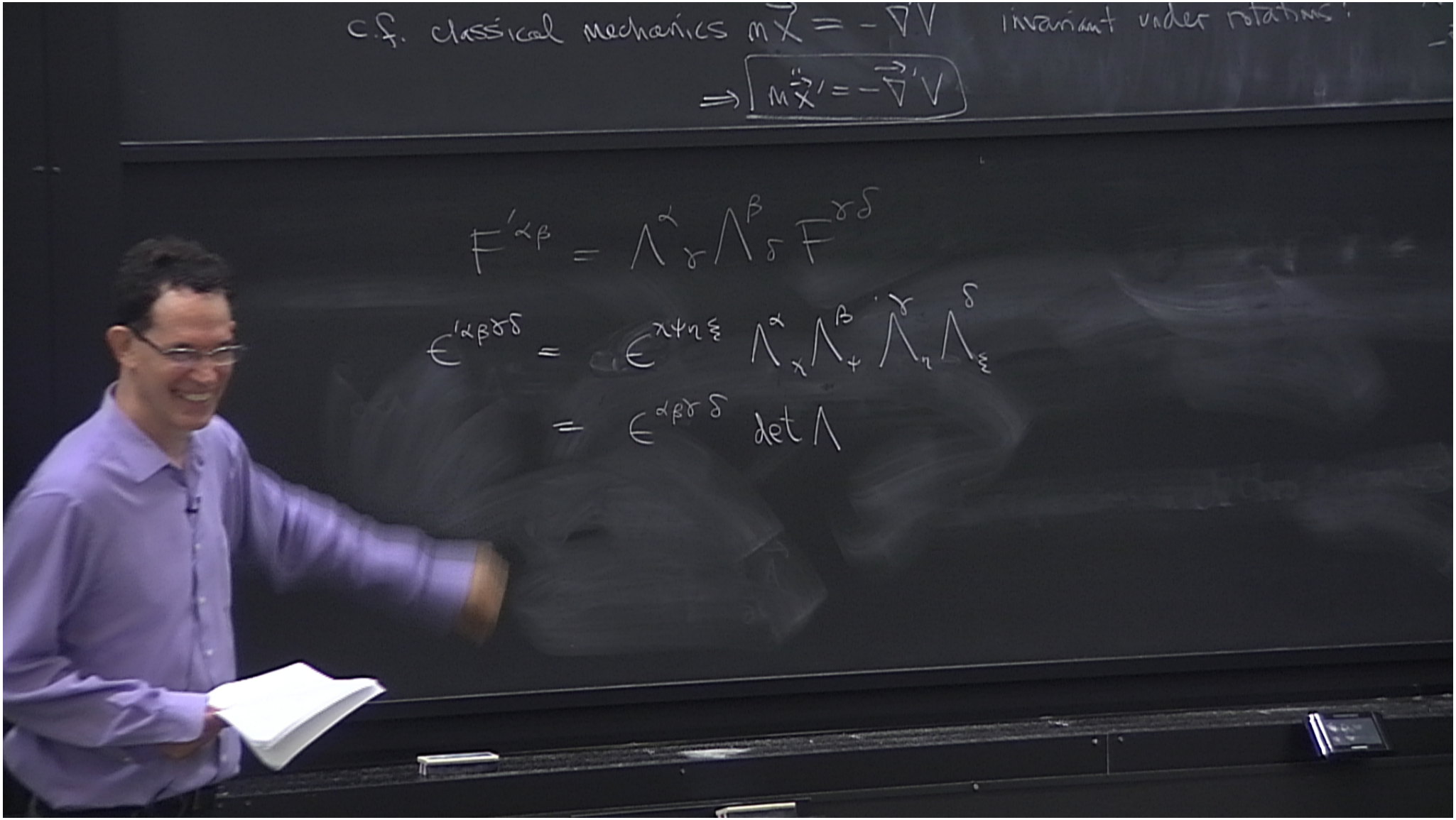
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In electrodynamics, we can "rotate" spacetime coordinates:

$$x'^\alpha = \Lambda^\alpha_{\ \beta} x^\beta \quad \text{, where } \Lambda \text{ is a } 4\times 4 \text{ matrix}$$

$$\Rightarrow \partial_\alpha = \frac{\partial x'^\beta}{\partial x^\alpha}\frac{\partial}{\partial x'^\beta} = \Lambda^\beta_{\ \alpha}\partial'_\beta$$

$$\Rightarrow \partial'_\alpha = \tilde{\Lambda}^\beta_{\ \alpha}\partial_\beta \quad \text{where } \tilde{\Lambda} \text{ is inverse of } \Lambda$$


c.f. classical mechanics $m\ddot{\vec{x}} = -\vec{\nabla}V$ invariant under rotations!
$\Rightarrow \boxed{m\ddot{\vec{x}}' = -\vec{\nabla}'V}$
$F'^{\alpha\beta} = \Lambda^{\alpha}_{\ \gamma}\Lambda^{\beta}_{\ \delta}F^{\gamma\delta}$
$\epsilon'^{\alpha\beta\gamma\delta} = \epsilon^{\chi\psi\eta\xi}\Lambda^{\alpha}_{\ \chi}\Lambda^{\beta}_{\ \psi}\Lambda^{\gamma}_{\ \eta}\Lambda^{\delta}_{\ \xi}$
$= \epsilon^{\alpha\beta\gamma\delta}\det\Lambda$

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Most important

c.f. classical mechanics $m\ddot{\vec{x}} = -\vec{\nabla} V$ invariant under rotations:

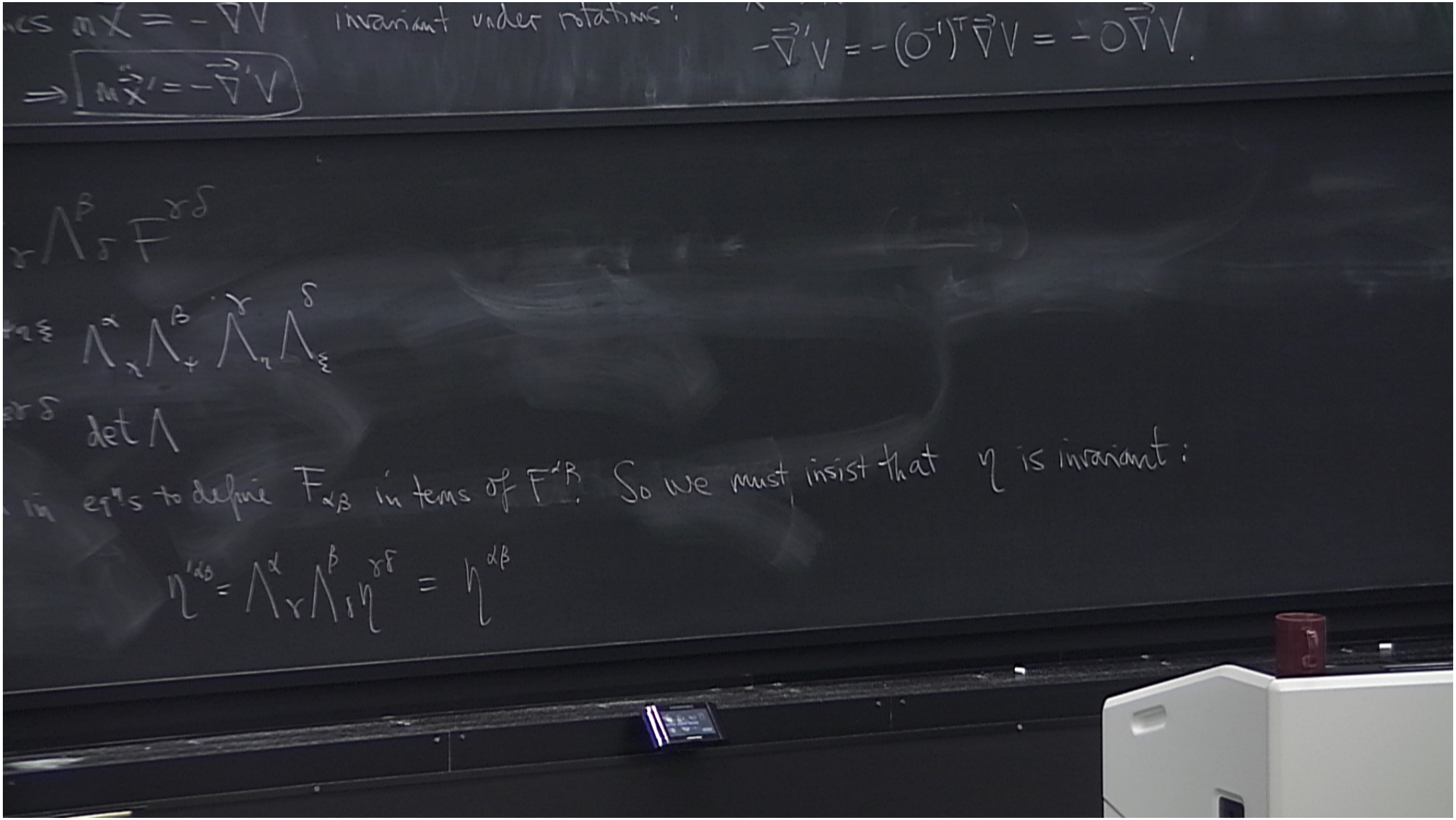
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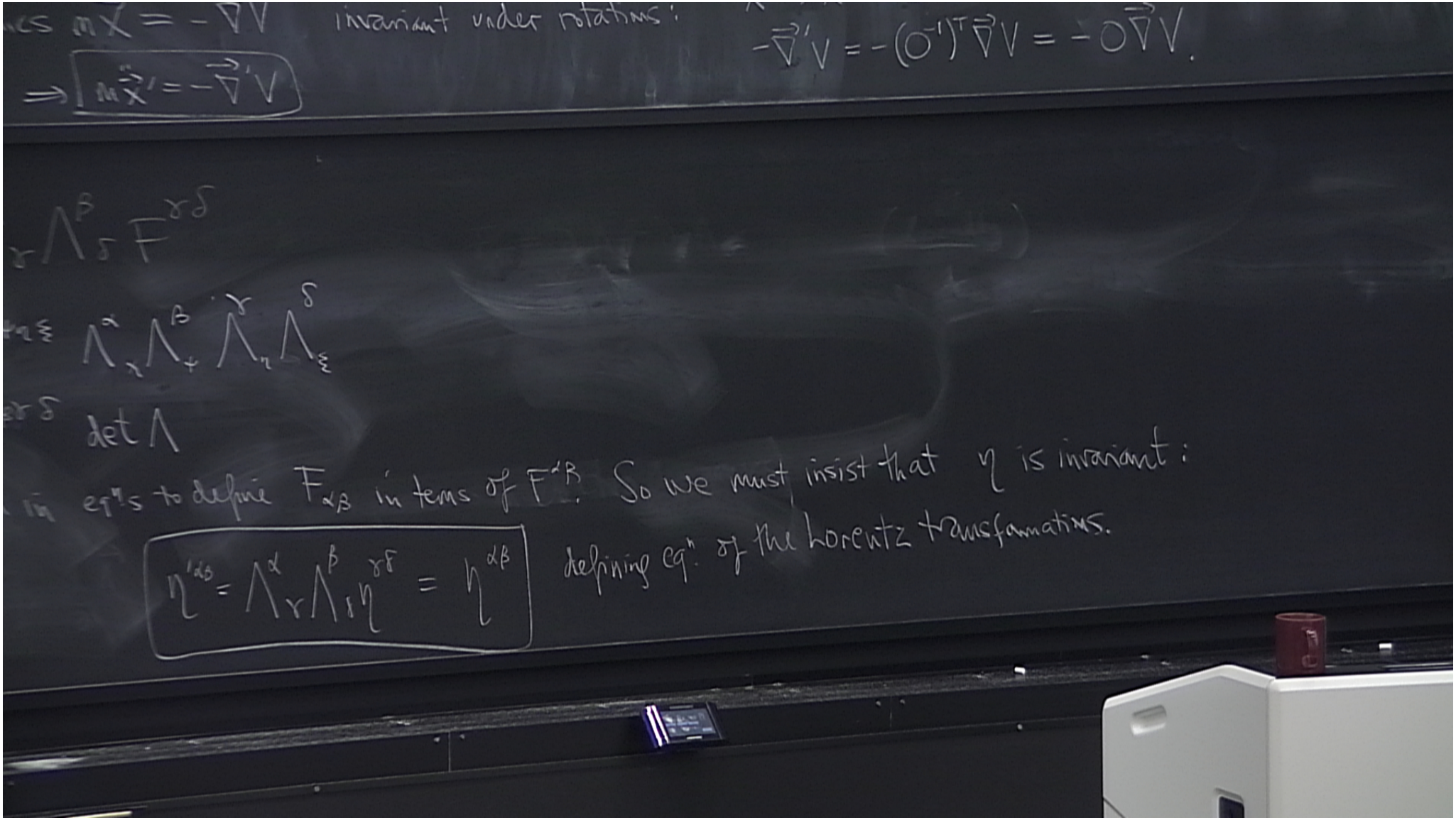
$$F'^{\alpha\beta} = \Lambda^{\alpha}_{\ \gamma} \Lambda^{\beta}_{\ \delta} F^{\gamma\delta}$$

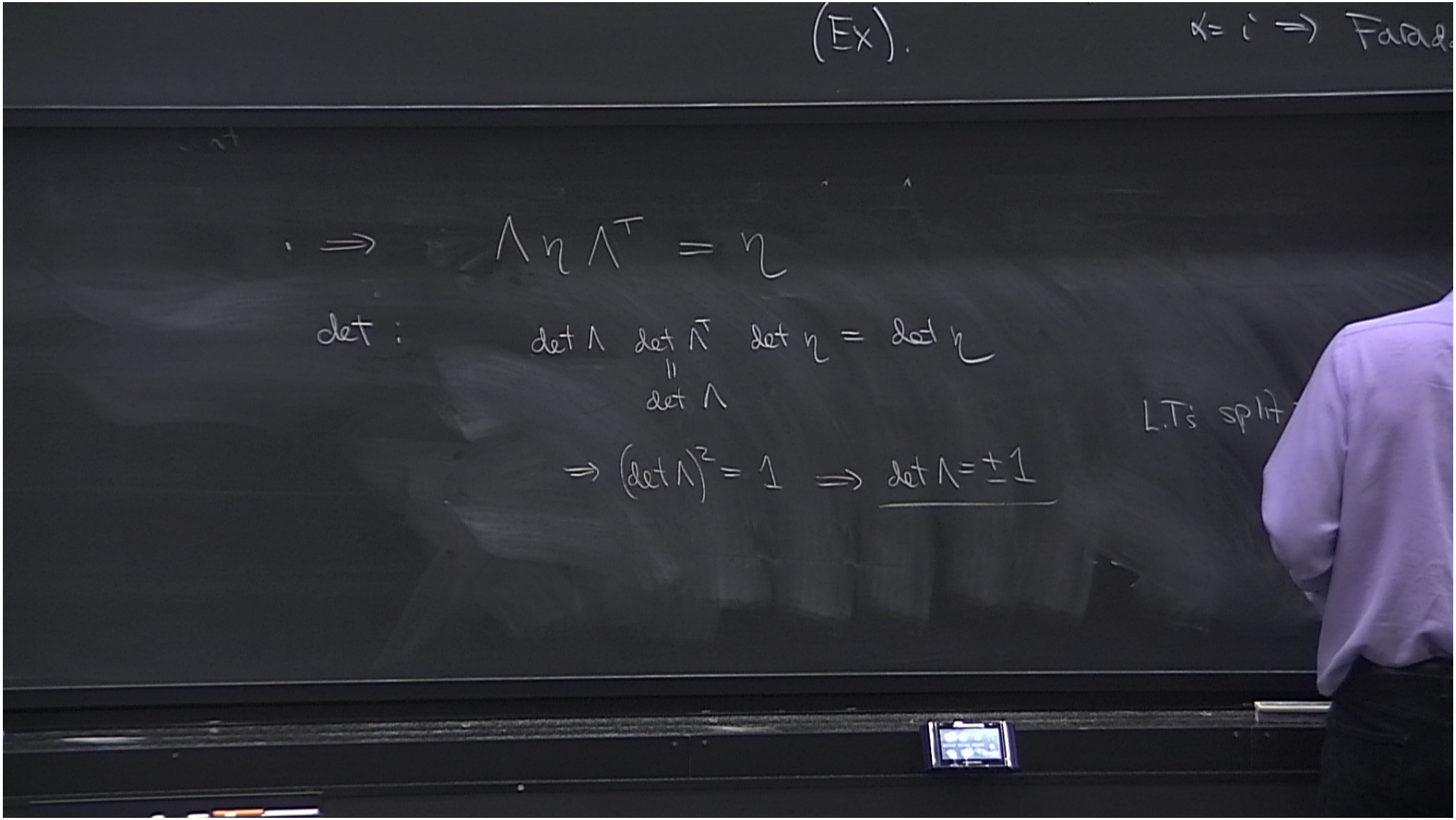
$$\epsilon'^{\alpha\beta\gamma\delta} = \epsilon^{\chi\phi\eta\xi} \Lambda^{\alpha}_{\ \chi} \Lambda^{\beta}_{\ \phi} \Lambda^{\gamma}_{\ \eta} \Lambda^{\delta}_{\ \xi}$$

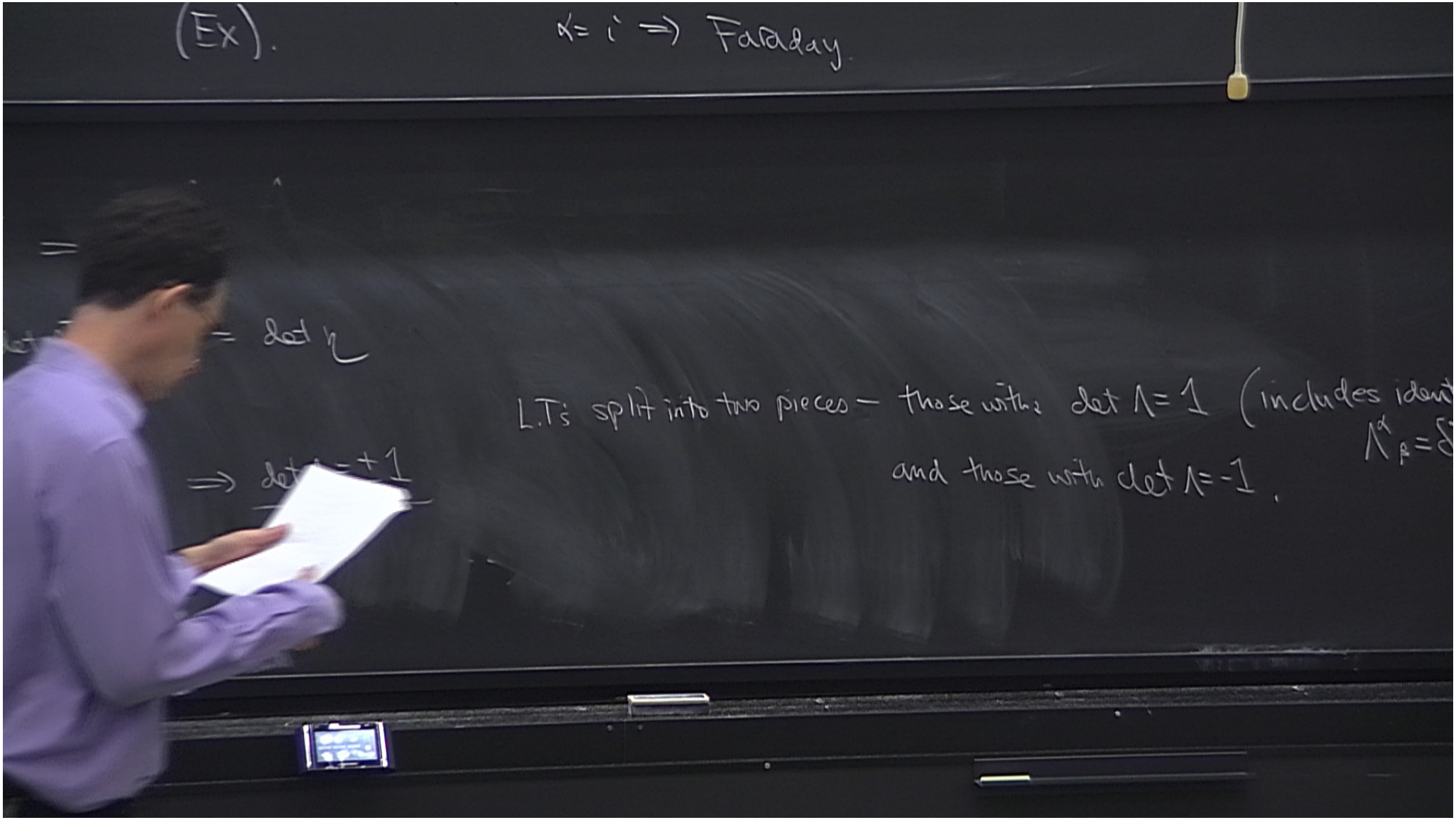
$$= \epsilon^{\alpha\beta\gamma\delta} \det \Lambda$$

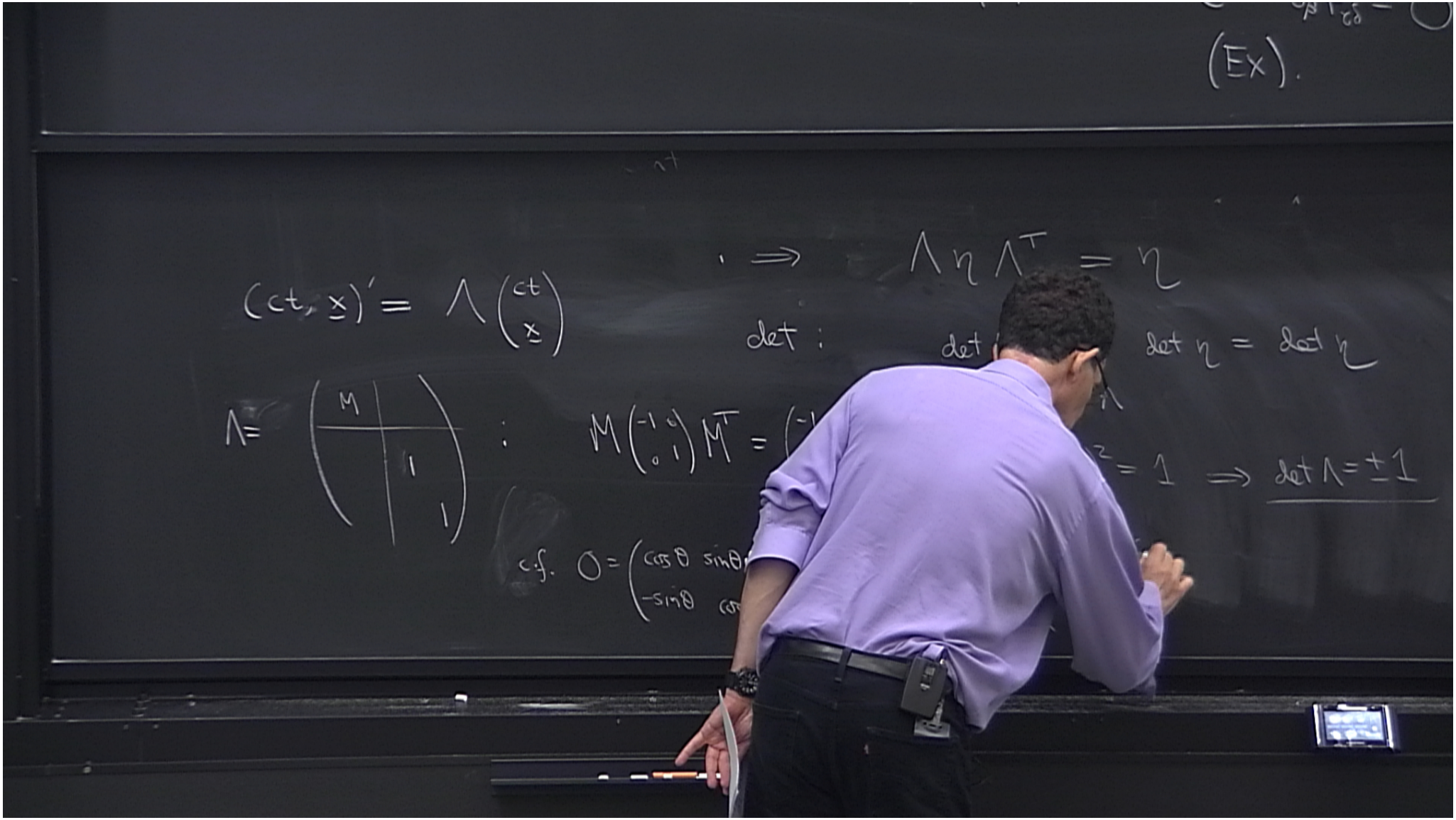
Most important : η is used in eq^{n}s to define $F_{\alpha\beta}$ in terms of $F^{\alpha\beta}$. So we must


invariant under rotations:
$m\ddot{\vec{x}} = -\vec{\nabla}V$
$\Rightarrow \boxed{m\ddot{\vec{x}}' = -\vec{\nabla}'V}$
$-\vec{\nabla}'V = -(O^{-1})^T\vec{\nabla}V = -O\vec{\nabla}V$
$\det\Lambda$
eq^{n}s to define $F_{\alpha\beta}$ in terms of $F^{\alpha\beta}$. So we must insist that η is invariant:
$\eta'^{\alpha\beta} = \Lambda^{\alpha}_{\ \gamma}\Lambda^{\beta}_{\ \delta}\eta^{\gamma\delta} = \eta^{\alpha\beta}$


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defining eq^n of the Lorentz transfamations.


(Ex).
$\Rightarrow \quad \Lambda \eta \Lambda^T = \eta$
det :
$\det\Lambda \; \det\Lambda^T \; \det\eta = \det\eta$
$\det\Lambda$
$\Rightarrow (\det\Lambda)^2 = 1 \Rightarrow \det\Lambda = \pm 1$


(Ex).
$\kappa = i \Rightarrow$ Faraday.
$= \det \Lambda$
$\Rightarrow \det \Lambda = +1$
L.T's split into two pieces — those with $\det \Lambda = 1$ (includes ident
and those with $\det \Lambda = -1$.


(Ex).
$(ct, \underline{x})' = \Lambda \begin{pmatrix} ct \\ \underline{x} \end{pmatrix}$
$\Rightarrow \quad \Lambda \eta \Lambda^T = \eta$
det :
$\det \eta = \det \eta$
$\Lambda = \begin{pmatrix} M & \\ & 1 \\ & & 1 \end{pmatrix}$: $M \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} M^T =$
$= 1 \Rightarrow \det \Lambda = \pm 1$
c.f. $O = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos \end{pmatrix}$

$= \Lambda \begin{pmatrix} ct \\ x \\ \vdots \end{pmatrix}$ $\Rightarrow \quad \Lambda \eta \Lambda^T = \eta$

$\det: \quad \det\Lambda \, \det\Lambda^T \, \det\eta = \det\eta$

$\det\Lambda = \det\Lambda$

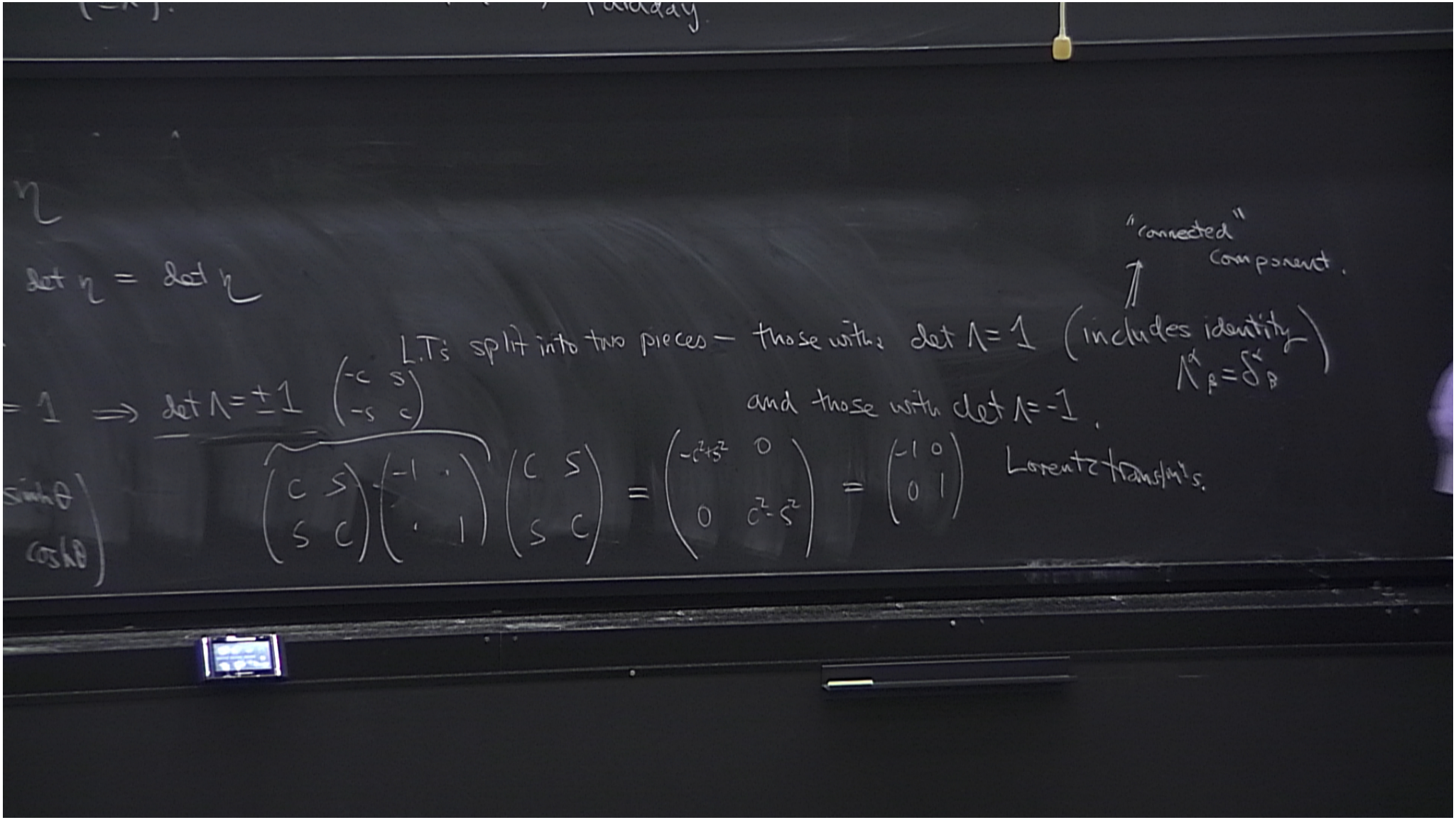
$\begin{pmatrix} 1 & & \\ & 1 & \\ & & \ddots \end{pmatrix}: \quad M\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}M^T = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow (\det\Lambda)^2 = 1 \Rightarrow \underline{\det\Lambda = \pm 1}$

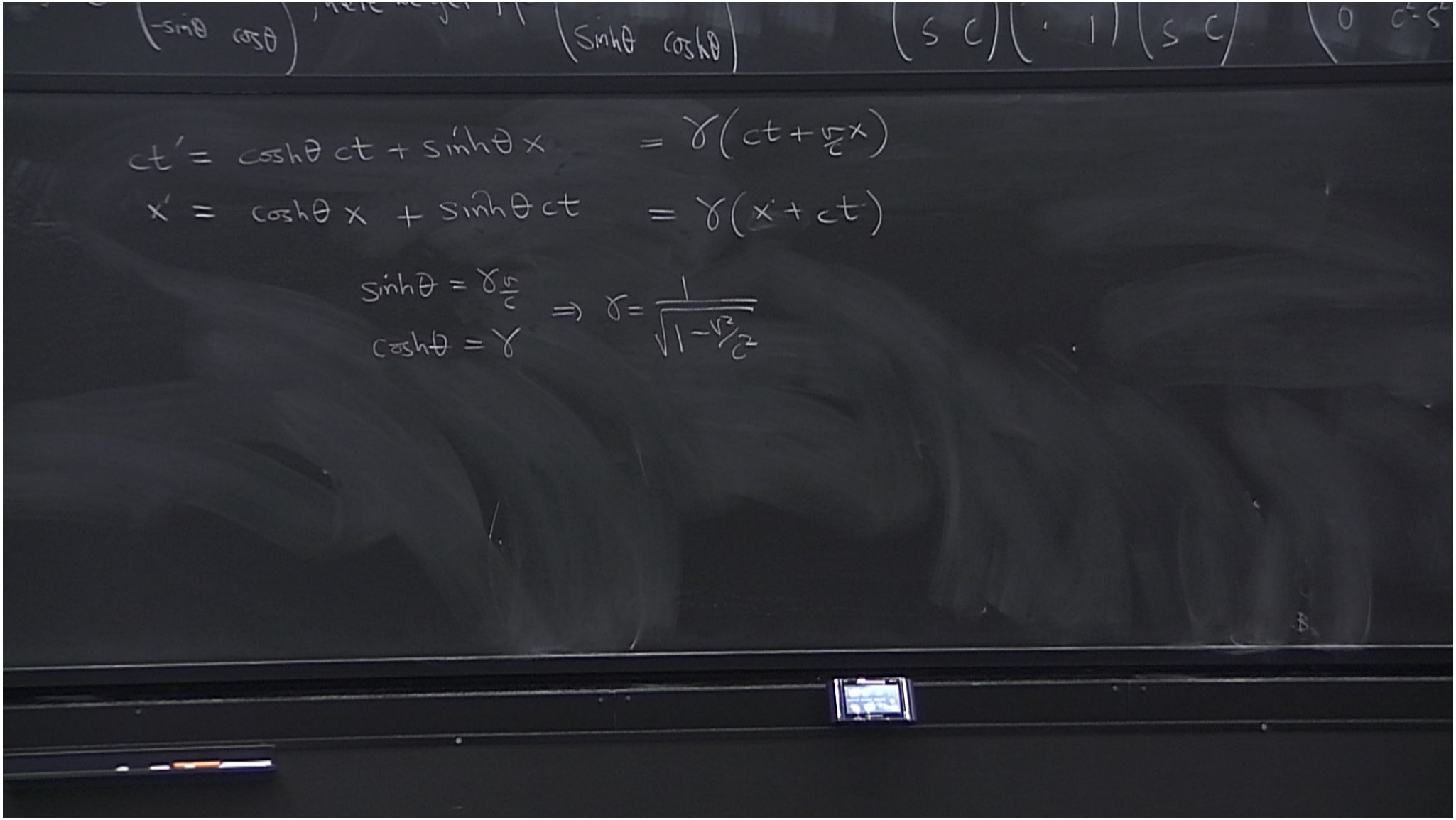
L.Ts split into two pieces

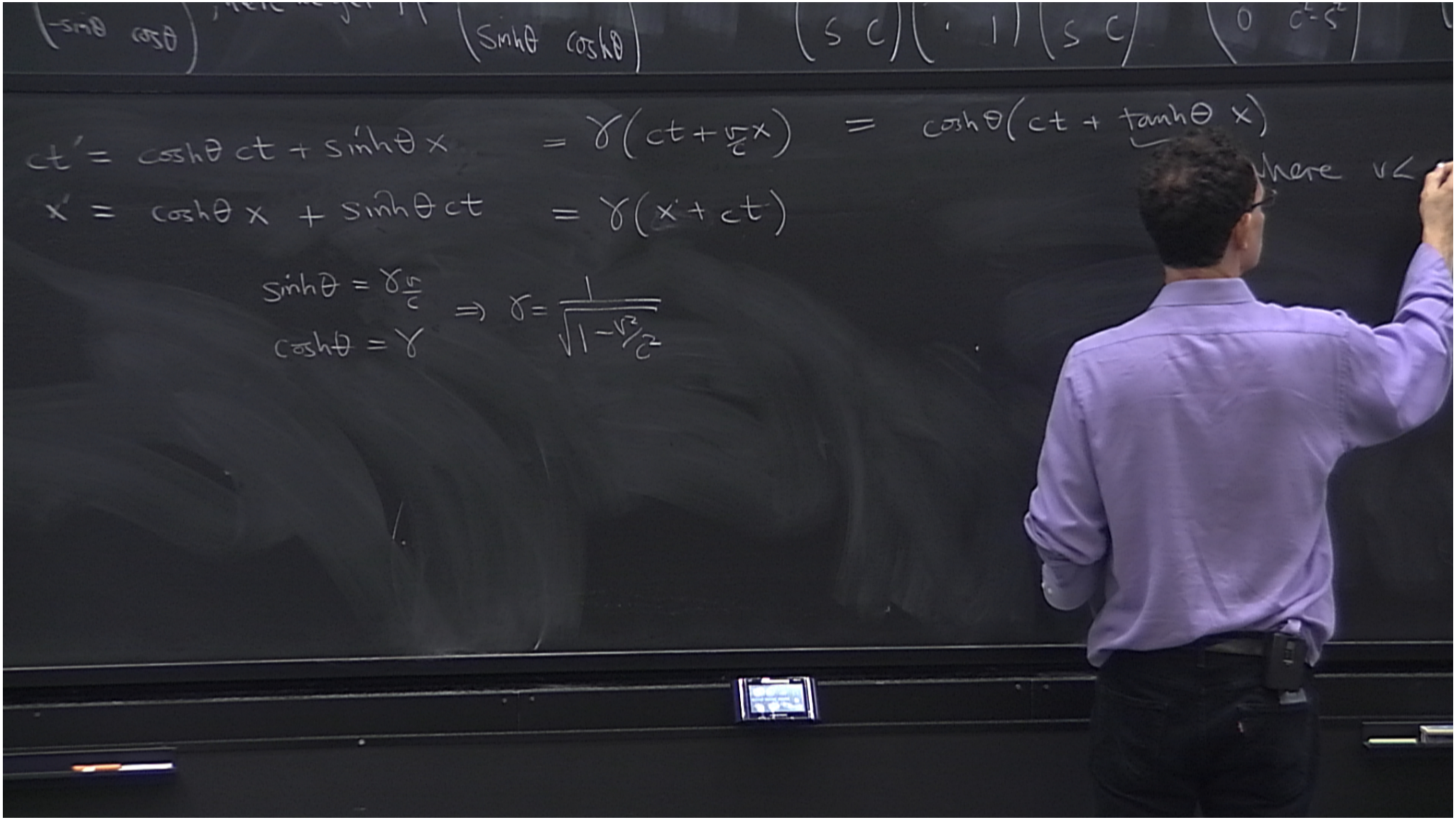
$\begin{pmatrix} -c & s \\ -s & c \end{pmatrix}$

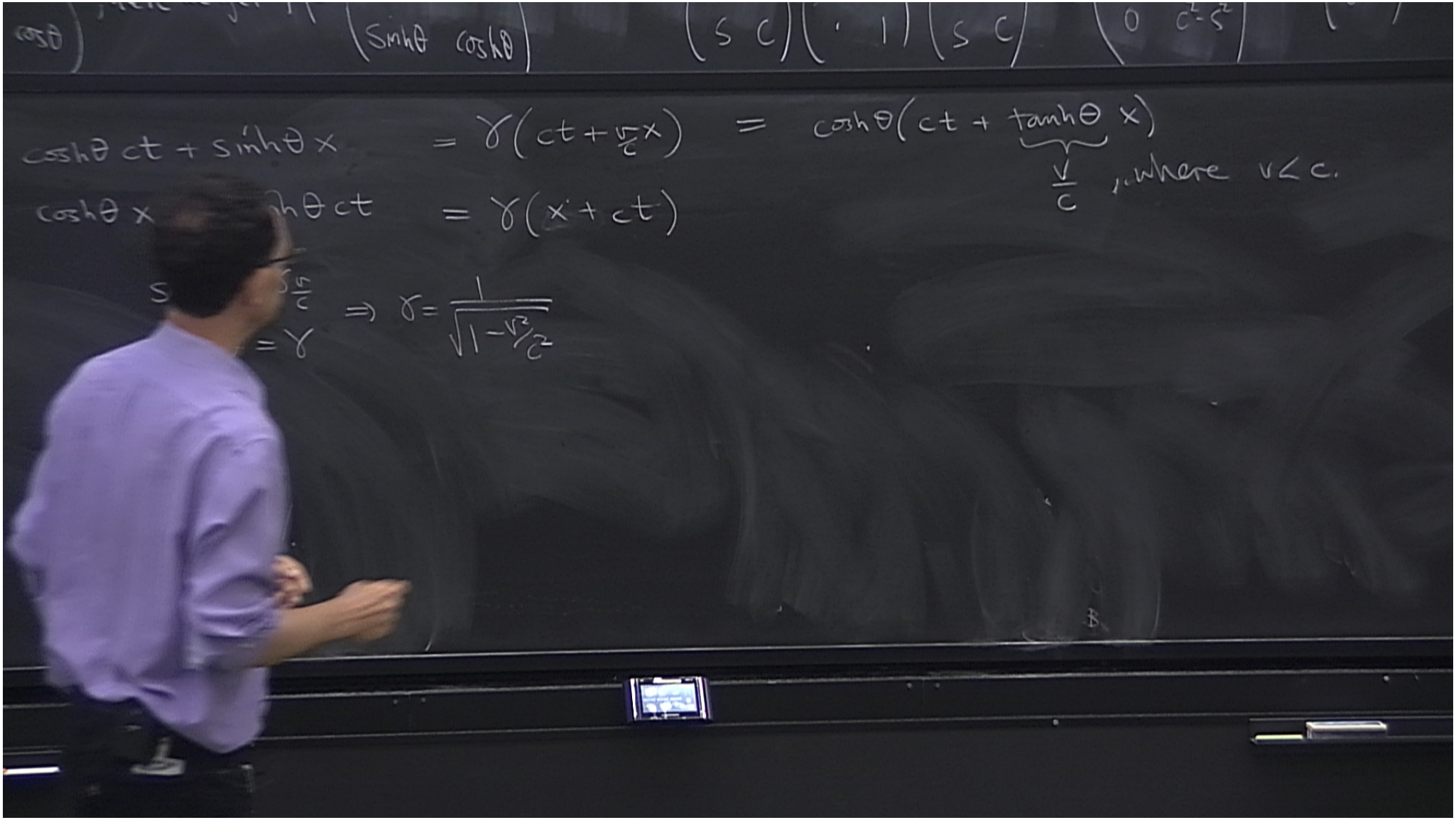
c.f. $O = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, here we get $M = \begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix}$

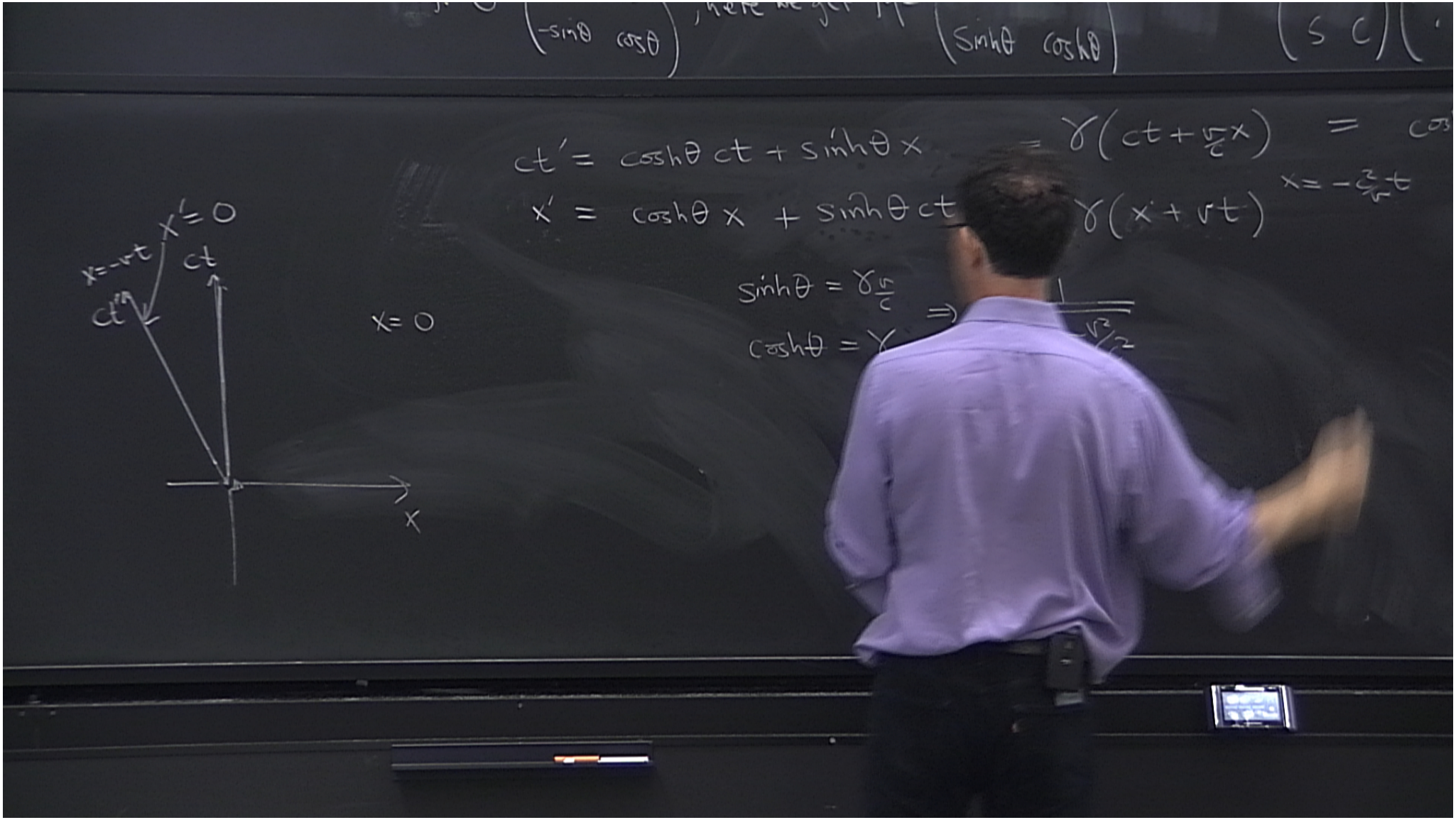
$\begin{pmatrix} c & s \\ s & c \end{pmatrix}\begin{pmatrix} -1 & & \\ & 1 & \\ & & \ddots \end{pmatrix}\begin{pmatrix} c & s \\ s & c \end{pmatrix} = \begin{pmatrix} \end{pmatrix}$


"connected" component.
L.T's split into two pieces — those with det Λ = 1 (includes identity $\Lambda^\alpha_\beta = \delta^\alpha_\beta$)
and those with det Λ = −1.
Lorentz transf'n's.


$\begin{pmatrix} -\sinh\theta & \cosh\theta \end{pmatrix}$
$\begin{pmatrix} \sinh\theta & \cosh\theta \end{pmatrix}$
$\begin{pmatrix} s & c \end{pmatrix}\begin{pmatrix} \cdot & 1 \end{pmatrix}\begin{pmatrix} s & c \end{pmatrix}$
$\begin{pmatrix} 0 & c^2-s^2 \end{pmatrix}$
$ct' = \cosh\theta\, ct + \sinh\theta\, x = \gamma\left(ct + \frac{v}{c}x\right)$
$x' = \cosh\theta\, x + \sinh\theta\, ct = \gamma(x + ct)$
$\sinh\theta = \gamma\frac{v}{c}$
$\cosh\theta = \gamma$
$\Rightarrow \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$


$\begin{pmatrix} -\sinh\theta & \cosh\theta \end{pmatrix}$
$\begin{pmatrix} \sinh\theta & \cosh\theta \end{pmatrix}$
$\begin{pmatrix} s & c \end{pmatrix}\begin{pmatrix} \cdot & 1 \end{pmatrix}\begin{pmatrix} s & c \end{pmatrix}$
$\begin{pmatrix} 0 & c^2-s^2 \end{pmatrix}$
$ct' = \cosh\theta\, ct + \sinh\theta\, x = \gamma\left(ct + \frac{v}{c}x\right) = \cosh\theta\,(ct + \tanh\theta\, x)$
$x' = \cosh\theta\, x + \sinh\theta\, ct = \gamma(x + ct)$
where $v <$
$\sinh\theta = \gamma\frac{v}{c}$
$\cosh\theta = \gamma$
$\Rightarrow \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$


$\begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix}$
$\begin{pmatrix} s & c \\ \cdot & 1 \end{pmatrix}$
$\begin{pmatrix} s & c \\ 0 & c^2-s^2 \end{pmatrix}$
$\cosh\theta\, ct + \sinh\theta\, x = \gamma(ct + \frac{v}{c}x) = \cosh\theta(ct + \tanh\theta\, x)$
$\frac{v}{c}$, where $v < c$.
$\cosh\theta\, x + \sinh\theta\, ct = \gamma(x + ct)$
$\frac{v}{c} = \gamma \Rightarrow \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$


$\begin{pmatrix} -\sinh\theta & \cosh\theta \end{pmatrix}$
$\begin{pmatrix} \sinh\theta & \cosh\theta \end{pmatrix}$
$\begin{pmatrix} s & c \end{pmatrix}$
$ct' = \cosh\theta\, ct + \sinh\theta\, x$
$= \gamma\left(ct + \frac{v}{c}x\right)$
$x' = \cosh\theta\, x + \sinh\theta\, ct$
$\gamma(x + vt)$
$x = -\frac{v}{c}t$
$\sinh\theta = \gamma\frac{v}{c}$
$\cosh\theta = \gamma$
$\Rightarrow$
$x' = 0$
$x = -vt$
ct
ct'
$x = 0$
x

$\begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix}$ $\begin{pmatrix} \cosh\theta & -\sinh\theta \\ -\sinh\theta & \cosh\theta \end{pmatrix}$

$ct' = \cosh\theta\, ct + \sinh\theta\, x = \gamma(ct + \frac{v}{c}x)$

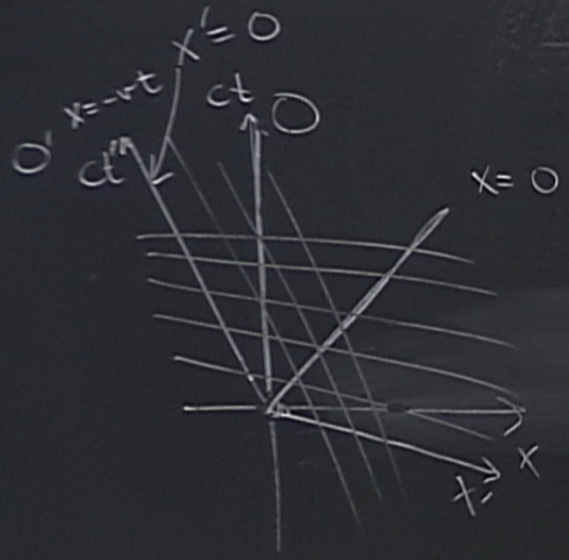
$x' = \cosh\theta\, x + \sinh\theta\, ct = \gamma(x + vt)$

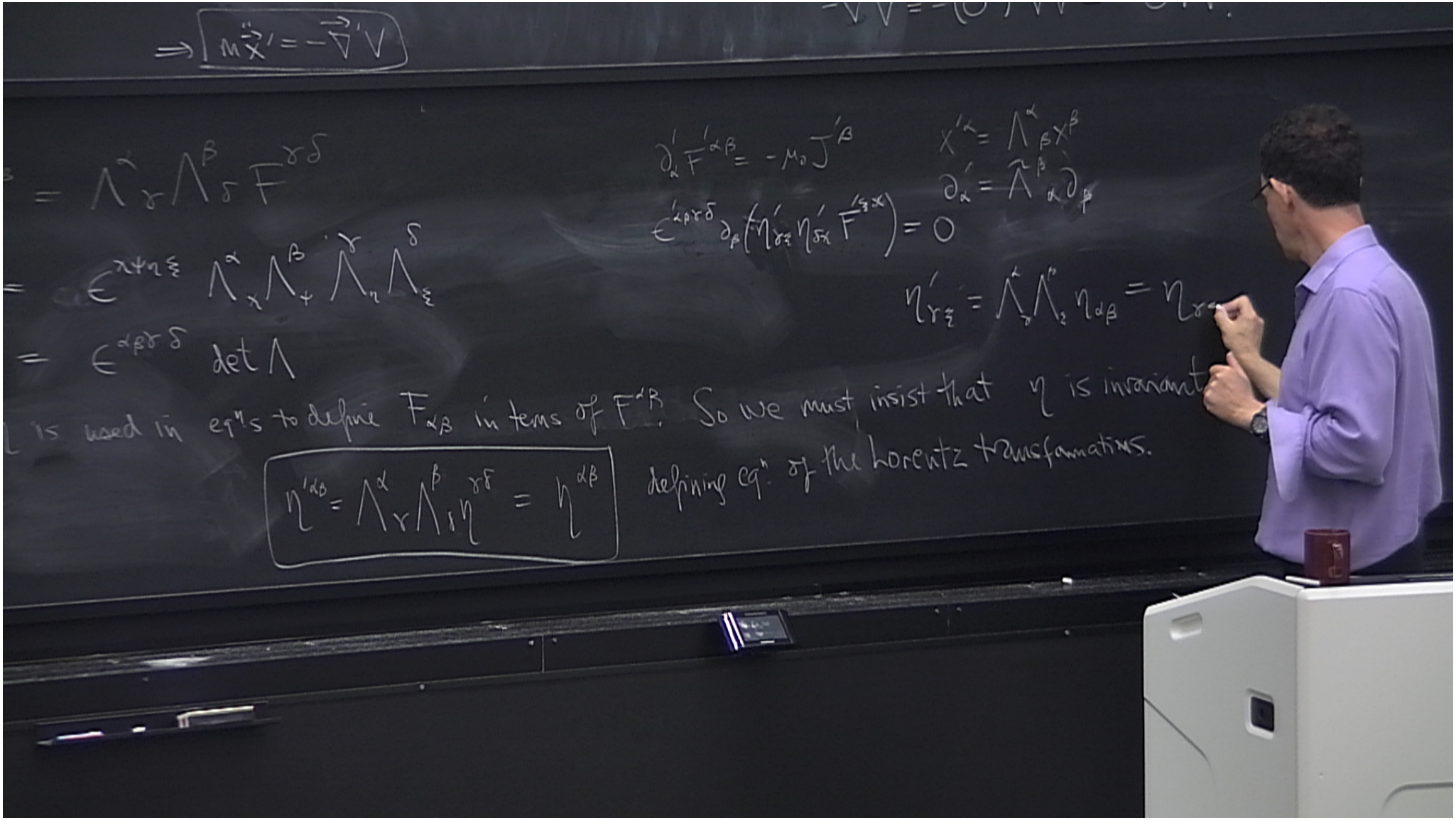
$x = -\frac{v}{c}t$

$ct = -\frac{v}{c}x$

$\sinh\theta = \gamma\frac{v}{c}$

$\cosh\theta = \gamma$ $\Rightarrow \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$




$\Rightarrow m\ddot{\vec{x}}' = -\vec{\nabla}' V$
$= \Lambda^{\alpha}_{\ \gamma} \Lambda^{\beta}_{\ \delta} F^{\gamma\delta}$
$= \epsilon^{\alpha\beta\gamma\delta} \det \Lambda$
$\partial'_\alpha F'^{\alpha\beta} = -\mu_0 J'^{\beta}$
$\epsilon'^{\alpha\beta\gamma\delta} \partial'_\beta \left(\eta'_{\gamma\xi} \eta'_{\delta\chi} F'^{\xi\chi} \right) = 0$
$x'^{\alpha} = \Lambda^{\alpha}_{\ \beta} x^{\beta}$
$\partial'_\alpha = \Lambda^{\beta}_{\ \alpha} \partial_\beta$
$\eta'_{\gamma\xi} = \Lambda^{\alpha}_{\ \gamma} \Lambda^{\beta}_{\ \xi} \eta_{\alpha\beta} = \eta_{\gamma\xi}$
η is used in eq^{n}s to define $F_{\alpha\beta}$ in terms of $F'^{\alpha\beta}$. So we must insist that η is invariant
$\eta'^{\alpha\beta} = \Lambda^{\alpha}_{\ \gamma} \Lambda^{\beta}_{\ \delta} \eta^{\gamma\delta} = \eta^{\alpha\beta}$
defining eq^{n} of the Lorentz transformations.