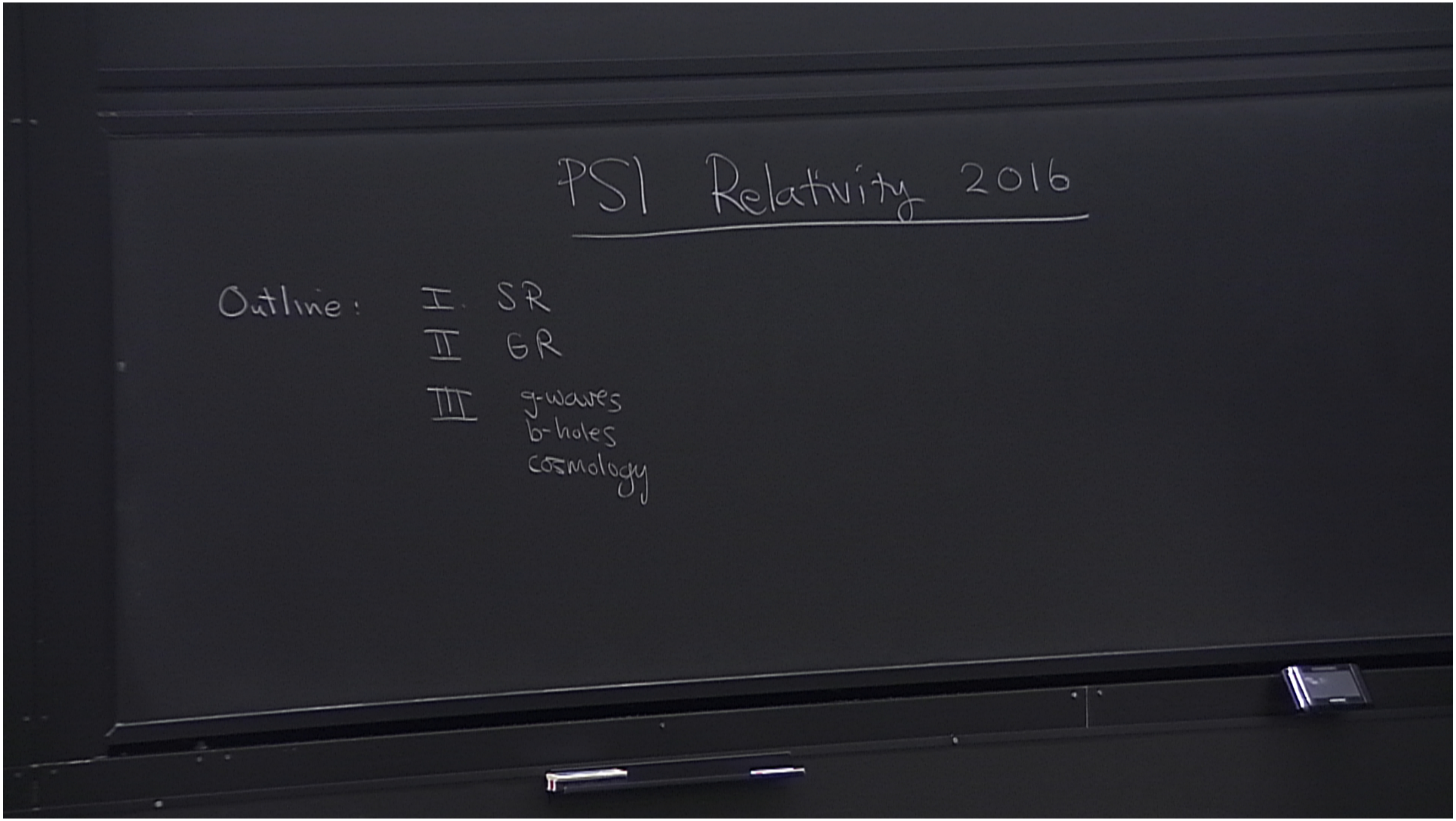
Title: PSI 2016/2017 Relativity - Lecture 1

Date: Sep 06, 2016 09:00 AM

URL: http://pirsa.org/16090001

Abstract:


PSI Relativity 2016
Outline: I. SR
II GR
III g-waves
b-holes
cosmology

PSI Relativity 2016

Outline:

I. SR

II. GR

III. g-waves
b-holes
cosmology

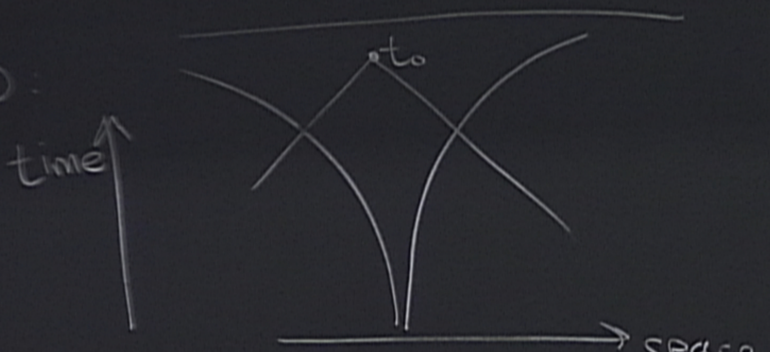
Recommend:

S. Carroll, Spacetime + Geometry, Pearson 2003

A. Zee, Einstein Gravity in a Nutshell, Princeton, 2013

L. Hughston + P. Tod, Introduction to GR, LMS, 1991.

LIGO:



$M = \frac{E}{c^2}$

strong gravity: $\frac{\phi}{c^2} \sim 1$

Newtonian potential

to probe scale r, need $\lambda < r$

$E \sim \frac{hc}{\lambda} > \frac{hc}{r} \rightarrow \frac{\phi}{c^2} \sim \frac{GE}{c^4 r} > \frac{Gh}{c^3 r^2} \sim$

strong gravity:

$$\frac{\phi}{c^2} \sim 1$$

Newtonian potential

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to probe scale r, need $\lambda < r$

$$E \sim \frac{hc}{\lambda} > \frac{hc}{r} \rightarrow \frac{\phi}{c^2} \sim \frac{GE}{c^4 r} > \frac{Gh}{c^3 r^2} \sim 1 \quad \text{for } r = \left(\frac{Gh}{c^3}\right)^{1/2} \equiv \text{Planck length.}$$

– smallest scale possible to probe geometry, $\ell_p = 1.6 \times 10^{-35}$ m.

Newtonian potential

strong gravity: $\frac{\phi}{c^2} \sim 1$

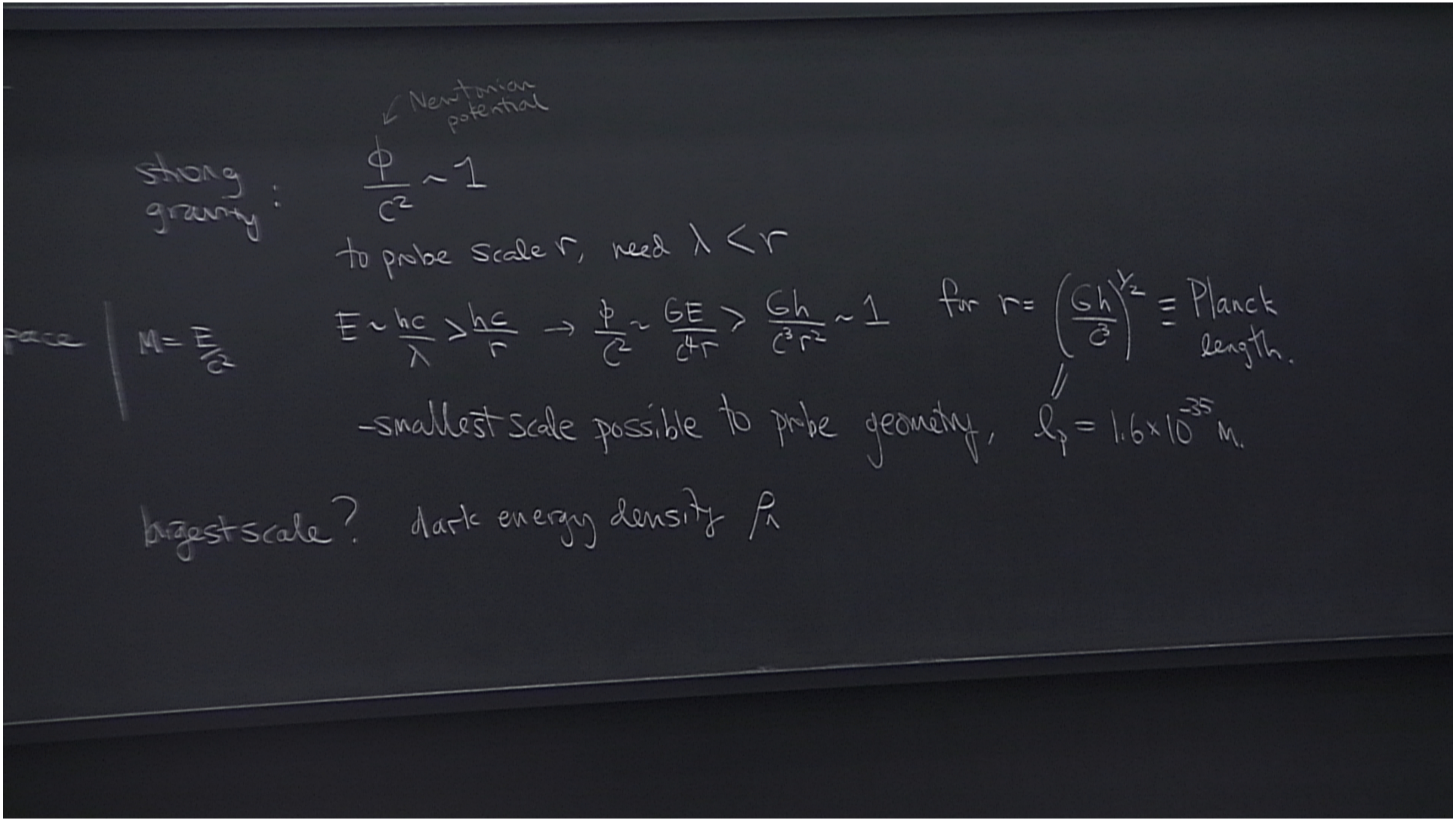
to probe scale r, need $\lambda < r$

space | $M = E/c^2$

$E \sim \frac{hc}{\lambda} > \frac{hc}{r} \rightarrow \frac{\phi}{c^2} \sim \frac{GE}{c^4 r} > \frac{Gh}{c^3 r^2} \sim 1$ for $r = \left(\frac{Gh}{c^3}\right)^{1/2}$

-smallest scale possible to probe geometry, $\ell_p = 1.6\times$

largest scale?


Newtonian potential
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-smallest scale possible to probe geometry, $l_p = 1.6 \times 10^{-35}$ m.
biggest scale? dark energy density ρ_Λ

r, need $\lambda < r$

$\rightarrow \frac{\phi}{c^2} \sim \frac{GE}{c^4 r} > \frac{Gh}{c^3 r^2} \sim 1$ for $r = \left(\frac{Gh}{c^3}\right)^{1/2} \equiv$ Planck length.

le possible to probe geometry, $\ell_p = 1.6 \times 10^{-35}$ m.

gy density ρ_Λ defines a scale. $\frac{G\rho_\Lambda r^2}{c^4} \sim 1$ $\ell_\Lambda \sim \frac{c^2}{\sqrt{G\rho_\Lambda}} \sim 17.4 \times 10^9$ lt yrs.

$\sim 1.65 \times 10^{26}$ m

r, need $\lambda < r$

$\rightarrow \frac{\phi}{c^2} \sim \frac{GE}{c^4 r} > \frac{G\hbar}{c^3 r^2} \sim 1$ for $r = \left(\frac{G\hbar}{c^3}\right)^{1/2} \equiv$ Planck length.

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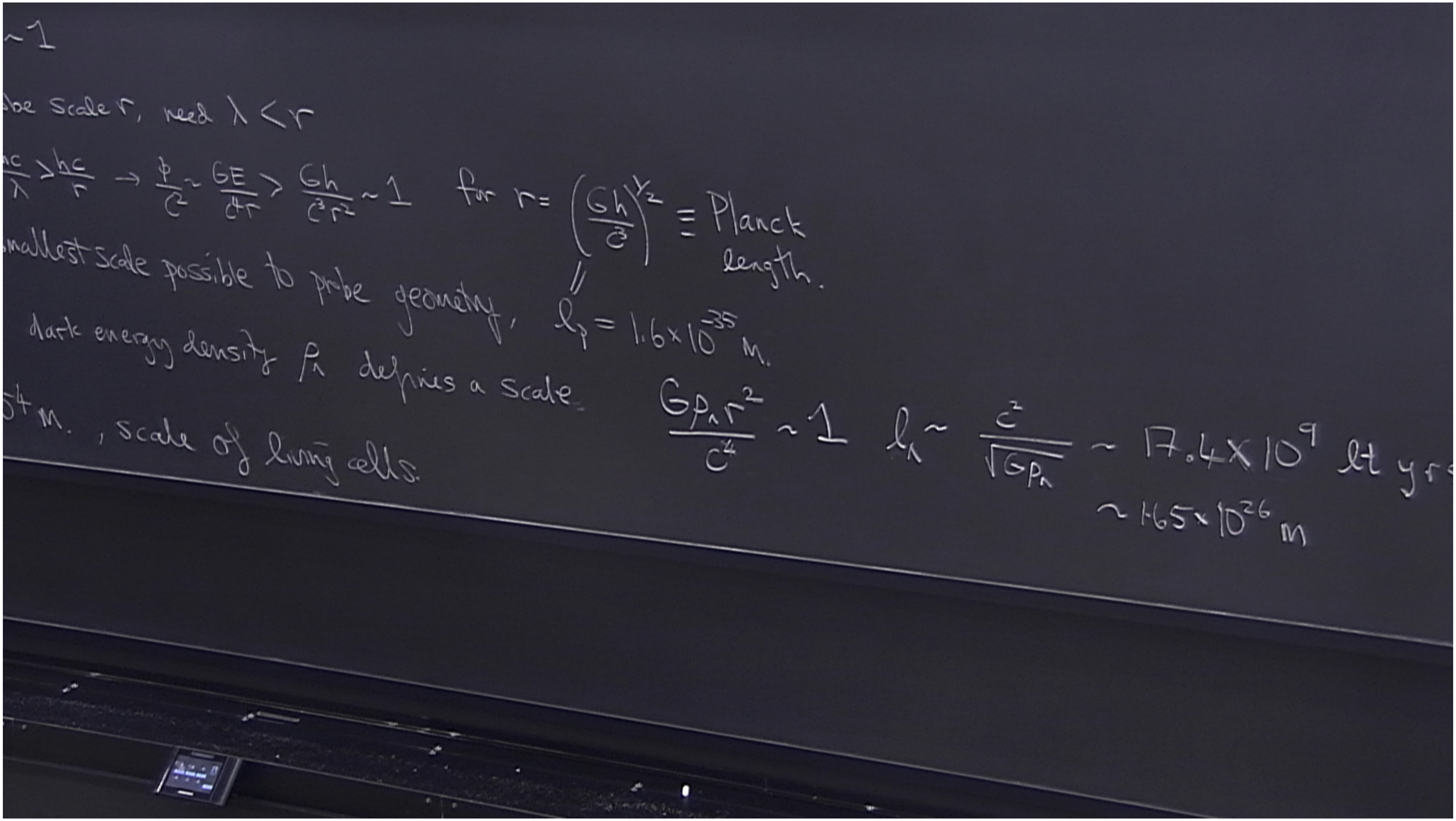
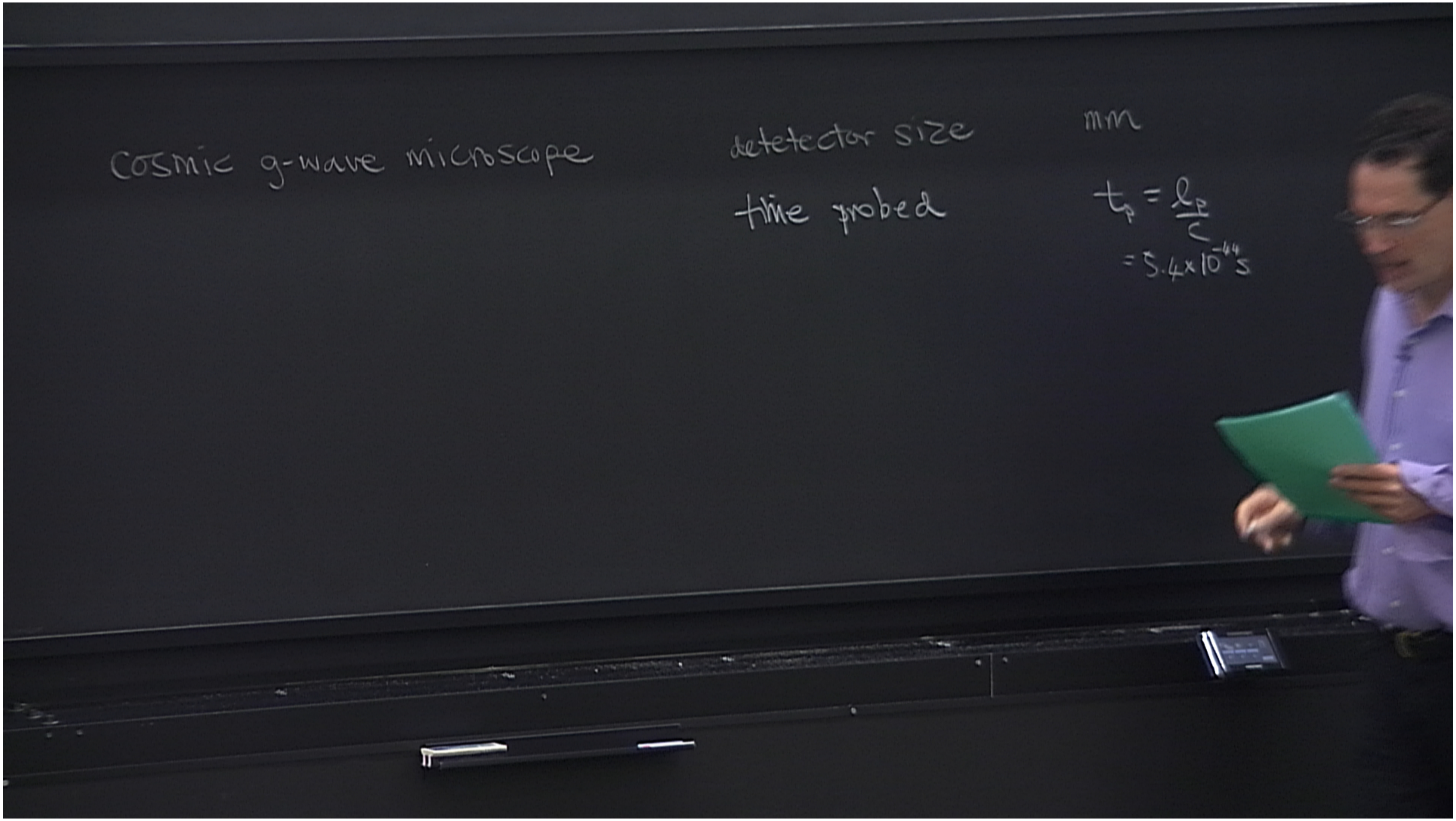
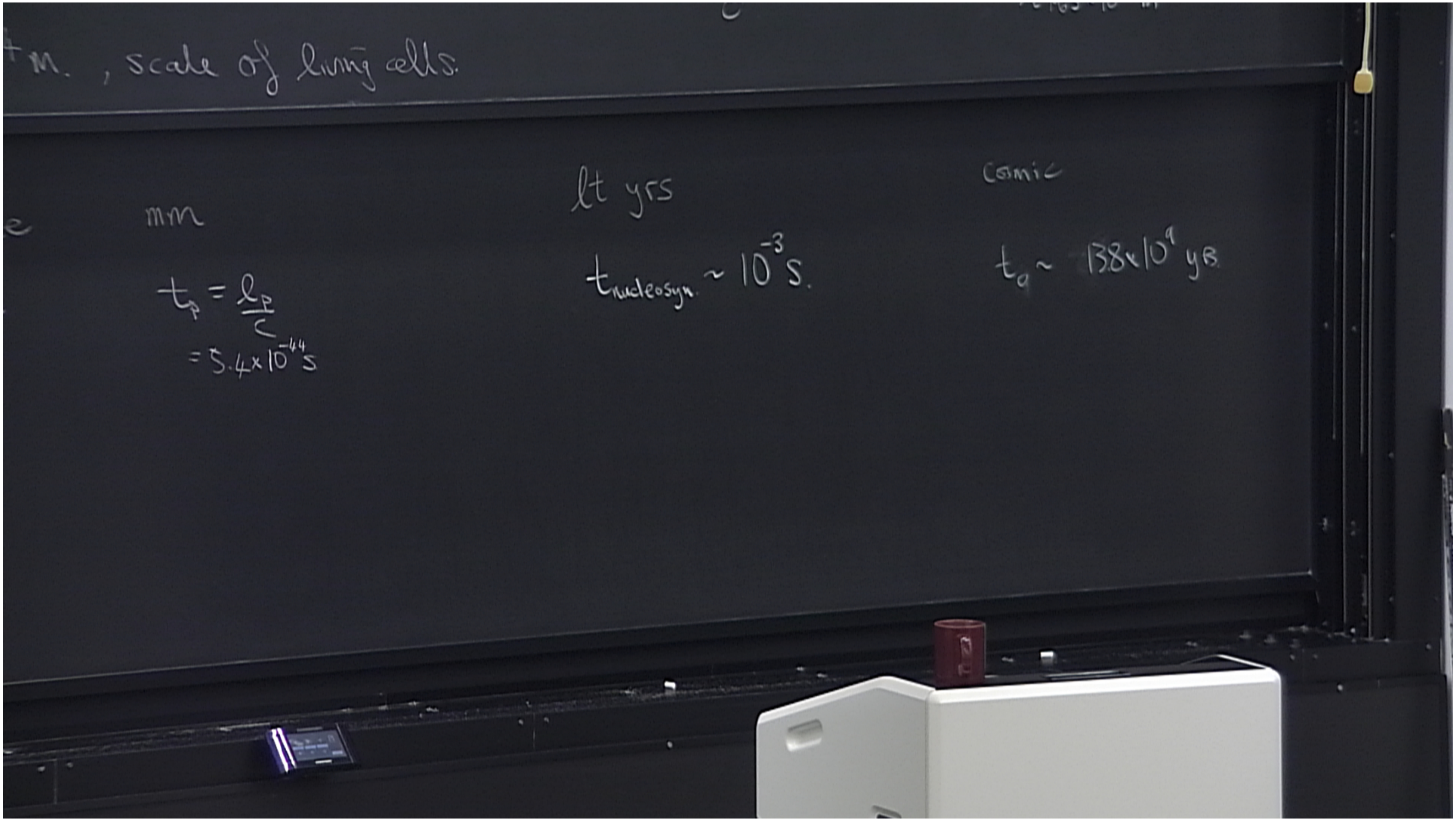

~1
be scale r, need $\lambda < r$
$\frac{hc}{\lambda} > \frac{hc}{r} \rightarrow \frac{\Phi}{c^2} \sim \frac{GE}{c^4 r} > \frac{Gh}{c^3 r^2} \sim 1$ for $r = \left(\frac{Gh}{c^3}\right)^{1/2} \equiv$ Planck length.
mallest scale possible to probe geometry, $l_p = 1.6 \times 10^{-35}$ m.
dark energy density ρ_Λ defines a scale
$\frac{G\rho_\Lambda r^2}{c^4} \sim 1$ $l_\Lambda \sim \frac{c^2}{\sqrt{G\rho_\Lambda}} \sim 17.4 \times 10^9$ lt yr
$\sim 1.65 \times 10^{26}$ m
4 m. , scale of living cells.


cosmic g-wave microscope
detetector size
mm
time probed
$t_p = \frac{l_p}{c}$
$= 5.4 \times 10^{-44}$ s


m. , scale of living cells.
mm
lt yrs
cosmic
$t_p = \frac{l_p}{c} = 5.4 \times 10^{-44}$ s
$t_{nucleosyn} \sim 10^{-3}$ s.
$t_0 \sim 13.8 \times 10^9$ yrs

Maxwell 1860's

ρ_e = electric charge density

$$Q = \int_V d^3x \, \rho_e(\vec{x})$$

Electric + Magnetic Fields

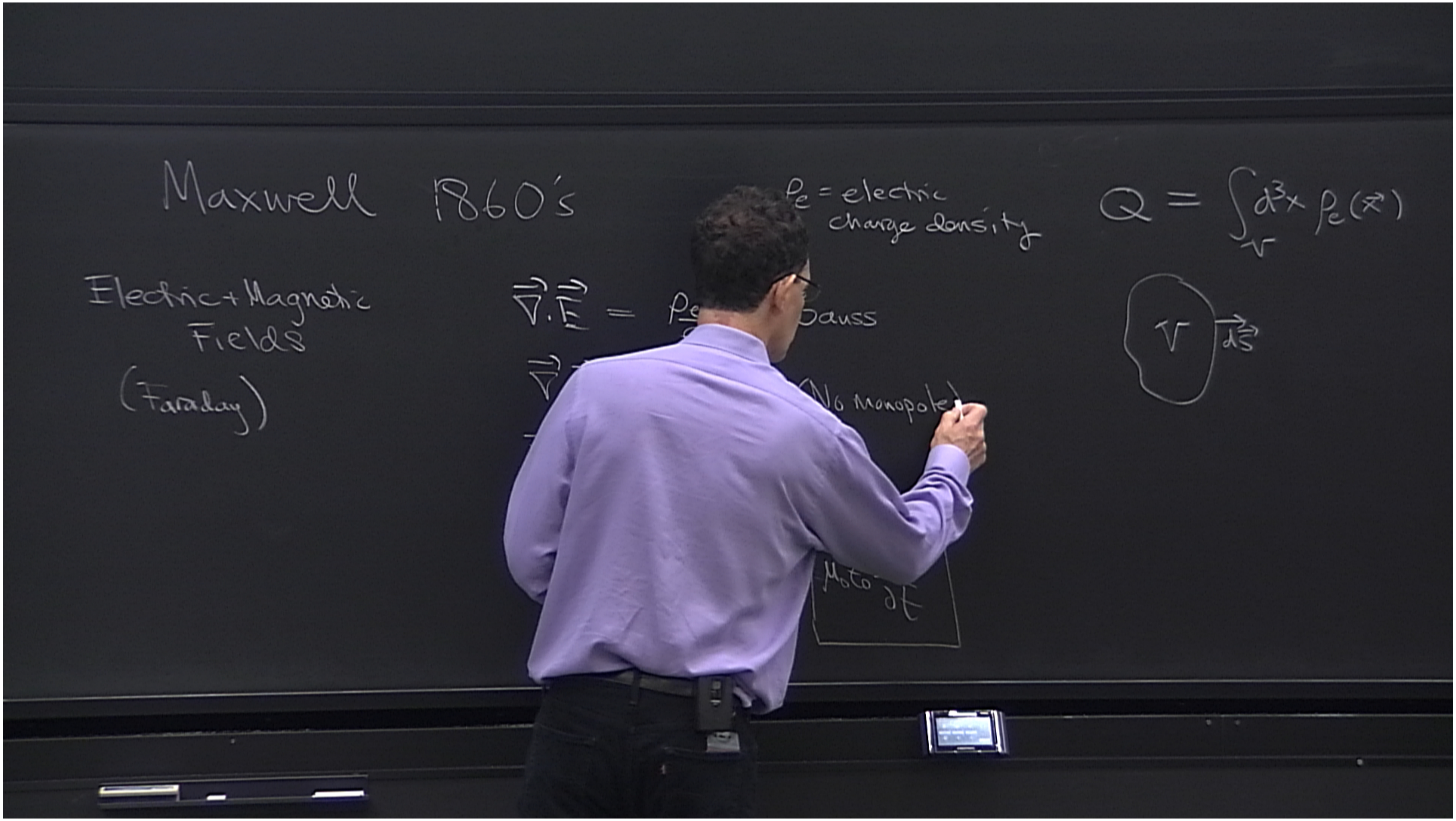
(Faraday)

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho_e}{\epsilon_0} \quad \text{Gauss}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \wedge \vec{E} = -\frac{\partial B}{\partial t}$$

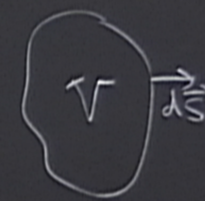
$$\vec{\nabla} \wedge \vec{B} = \mu_0 \vec{J}_e + \boxed{\mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}}$$


Maxwell 1860's
Electric + Magnetic Fields
(Faraday)
Gauss
(No monopole)
ρ_e = electric charge density
$Q = \int_V d^3x \, \rho_e(\vec{x})$
V
$\vec{dS}$

Maxwell 1860's

ρ_e = electric charge density

$$Q = \int_V d^3x \, \rho_e(\vec{x})$$



Electric + Magnetic Fields

(Faraday)

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho_e}{\epsilon_0} \qquad \text{Gauss}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \text{(No monopoles)}$$

$$\vec{\nabla}\wedge\vec{E} = -\frac{\partial B}{\partial t} \qquad \text{(Faraday)}$$

$$\vec{\nabla}\wedge\vec{B} = \underbrace{\mu_0 \vec{J}_e}_{\text{Ampere}} + \boxed{\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}}$$

Maxwell's "displacement current" added to make eq^{n}s consistent.

Ampere

consistent.

V/A^2

C^2/Nm^2

recall $\vec{\nabla}.\vec{E} = \partial_i E_i$ $\quad i = 1, 2, 3$, Einstein summatio(n)

(~~repeated~~ indices

$\partial_i = \frac{\partial}{\partial x^i}$

$(\vec{\nabla}\wedge\vec{E})_i = \epsilon_{ijk}\partial_j E_k$

ϵ_{ijk} totally antisymmetric, $\epsilon_{123} = +1$.

Check M.E.'s for consistency:

from definitions of ρ_e, $\vec{j}_e$, $\quad \frac{\partial \rho}{\partial t} + \vec{\nabla}.\vec{j}_e = 0$

Faraday: lhs $\vec{\nabla}\cdot(\vec{\nabla}\wedge\vec{E}) = \partial_i \epsilon_{ijk}\partial_j E_k = 0$ ✓

rhs $\vec{\nabla}.\frac{\partial\vec{B}}{\partial t} = 0$ by no. monopoles om definitions of $\rho_e, \vec{j}_e$, $\quad \frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{j}_e = 0$

(continuity eqn / consn. of electric charge)

$(\vec{\nabla} \wedge \vec{E}) = \partial_i \epsilon_{ijk} \partial_j E_k = 0 \quad \checkmark$

$\frac{\partial \vec{B}}{\partial t} = 0$ by no. monopoles.

0 rhs. $\mu_0 \vec{\nabla}\cdot\vec{j} \neq 0$ if $\frac{\partial \rho_e}{\partial t} \neq 0$.

$\vec{\nabla}\cdot\frac{\partial\vec{B}}{\partial t} = 0$ by no: monopoles.

$= 0$ rhs. $\mu_0\vec{\nabla}\cdot\vec{j} \neq 0$ if $\frac{\partial\rho_e}{\partial t} \neq 0$.

add displacement current $\mu_0\epsilon_0\frac{\partial\vec{\nabla}\cdot\vec{E}}{\partial t} \overset{\text{Gauss}}{=} \mu_0\frac{\partial\rho_e}{\partial t}$ $\Big\}$ $\mu_0\left(\vec{\nabla}\cdot\vec{j}_e + \frac{\partial\rho_e}{\partial t}\right) =$

set ρ_e and $\vec{j}_e$ to zero, there are still very interesting solutions:

rhs $\vec{\nabla}\cdot\frac{\partial \vec{B}}{\partial t} = 0$ by no. monopoles.

Ampere lhs: $\vec{\nabla}\cdot(\vec{\nabla}\wedge\vec{B}) = 0$ rhs. $\mu_0 \vec{\nabla}\cdot\vec{j} \neq 0$ if $\frac{\partial \rho_e}{\partial t} \neq 0$.

add displacement current $\mu_0 \epsilon_0 \frac{\partial \vec{\nabla}\cdot\vec{E}}{\partial t} \overset{\text{Gauss}}{=}$

<u>Amazing</u> consequence: even if we set ρ_e and $\vec{j}_e$ to zero, there are still

$$\mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \overset{\text{modified Ampere}}{=} \vec{\nabla}\wedge\frac{\partial \vec{B}}{\partial t} \overset{\text{Faraday}}{=} -\vec{\nabla}\wedge(\vec{\nabla}\wedge\vec{E})$$

no. monopoles

$\mu_0 \vec{\nabla}\cdot\vec{j} \neq 0$ if $\frac{\partial \rho_e}{\partial t} \neq 0$.

displacement current $\quad \mu_0 \epsilon_0 \frac{\partial \vec{\nabla}\cdot\vec{E}}{\partial t} \overset{\text{Gauss}}{=} \mu_0 \frac{\partial \rho_e}{\partial t}$

$\Big\}\ \mu_0 \left(\vec{\nabla}\cdot\vec{j}_e + \frac{\partial \rho_e}{\partial t} \right) = 0$

$\underbrace{\epsilon_{ijk}\,\epsilon_{klm}}_{\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}} \partial_l E_m$

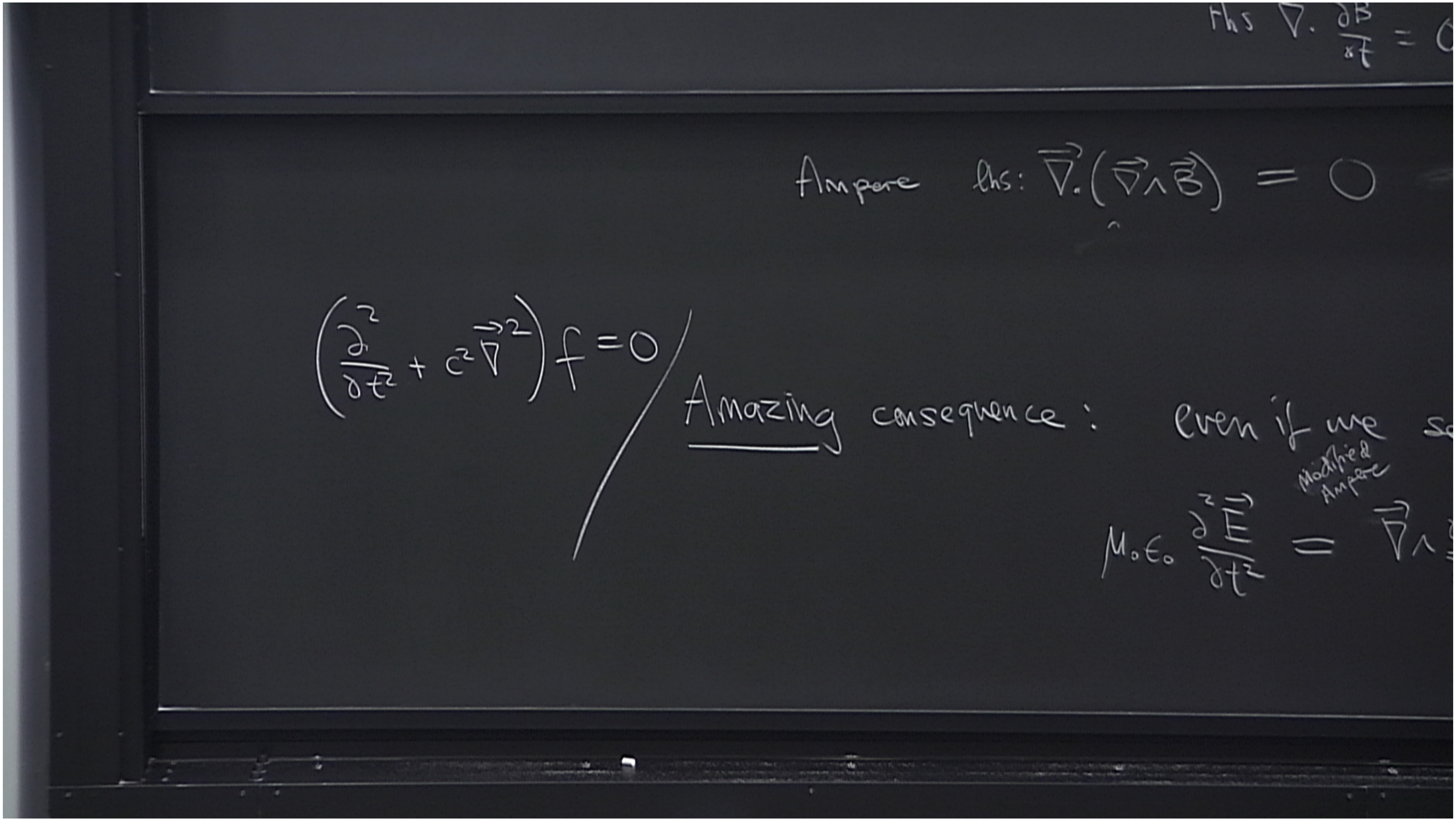
ρ_e and $\vec{j}_e$ to zero, there are still very interesting solutions:

Faraday

$= -\vec{\nabla}\wedge(\vec{\nabla}\wedge\vec{E}) = -\left(\vec{\nabla}(\cancel{\vec{\nabla}\cdot\vec{E}}) + \vec{\nabla}^2\vec{E}\right) = \vec{\nabla}^2\vec{E}$, i.e. wave eq^n for $\vec{E}$, with

$= 0$ if $\rho_e = 0$

Speed $c^2 = \frac{1}{\mu_0 \epsilon_0}$.


rhs $\nabla \cdot \frac{\partial B}{\partial t} = 0$
Ampere lhs: $\vec{\nabla} \cdot (\vec{\nabla} \wedge \vec{B}) = 0$
$\left(\frac{\partial^2}{\partial t^2} + c^2 \vec{\nabla}^2\right) f = 0$
Amazing consequence: even if we
Modified Ampere
$\mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = \vec{\nabla} \wedge$

if $\frac{\partial \rho_e}{\partial t} \neq 0$.

Gauss

$$\mu_0\epsilon_0 \frac{\partial \vec{\nabla}\cdot\vec{E}}{\partial t} = \mu_0 \frac{\partial \rho_e}{\partial t}$$

$$\mu_0\left(\vec{\nabla}\cdot\vec{j}_e + \frac{\partial \rho_e}{\partial t}\right) = 0$$

$$\underbrace{\epsilon_{ijk}\,\epsilon_{klm}}_{\delta_{il}\delta_{jm}-\delta_{im}\delta_{jl}}\partial_j\partial_l E_m$$

o, there are still very interesting solutions:

$$-\left(\vec{\nabla}\left(\cancel{\vec{\nabla}\cdot\vec{E}}\right)\right) + \vec{\nabla}^2\vec{E} = \vec{\nabla}^2\vec{E}$$, i.e. wave eq^n for $\vec{E}$, with

$= 0$ if $\rho = 0$

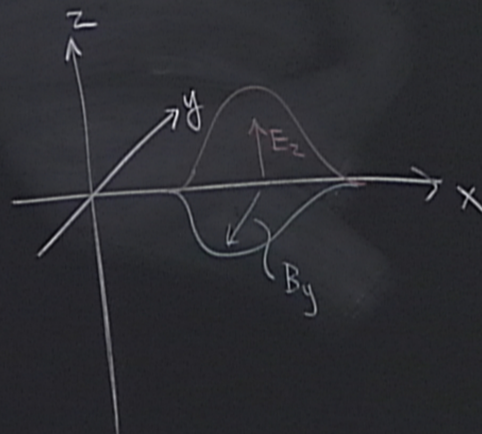
Speed $c^2 = \frac{1}{\mu_0\epsilon_0} \Rightarrow c \simeq 3\times10^8\ \mathrm{m\,s^{-1}}$

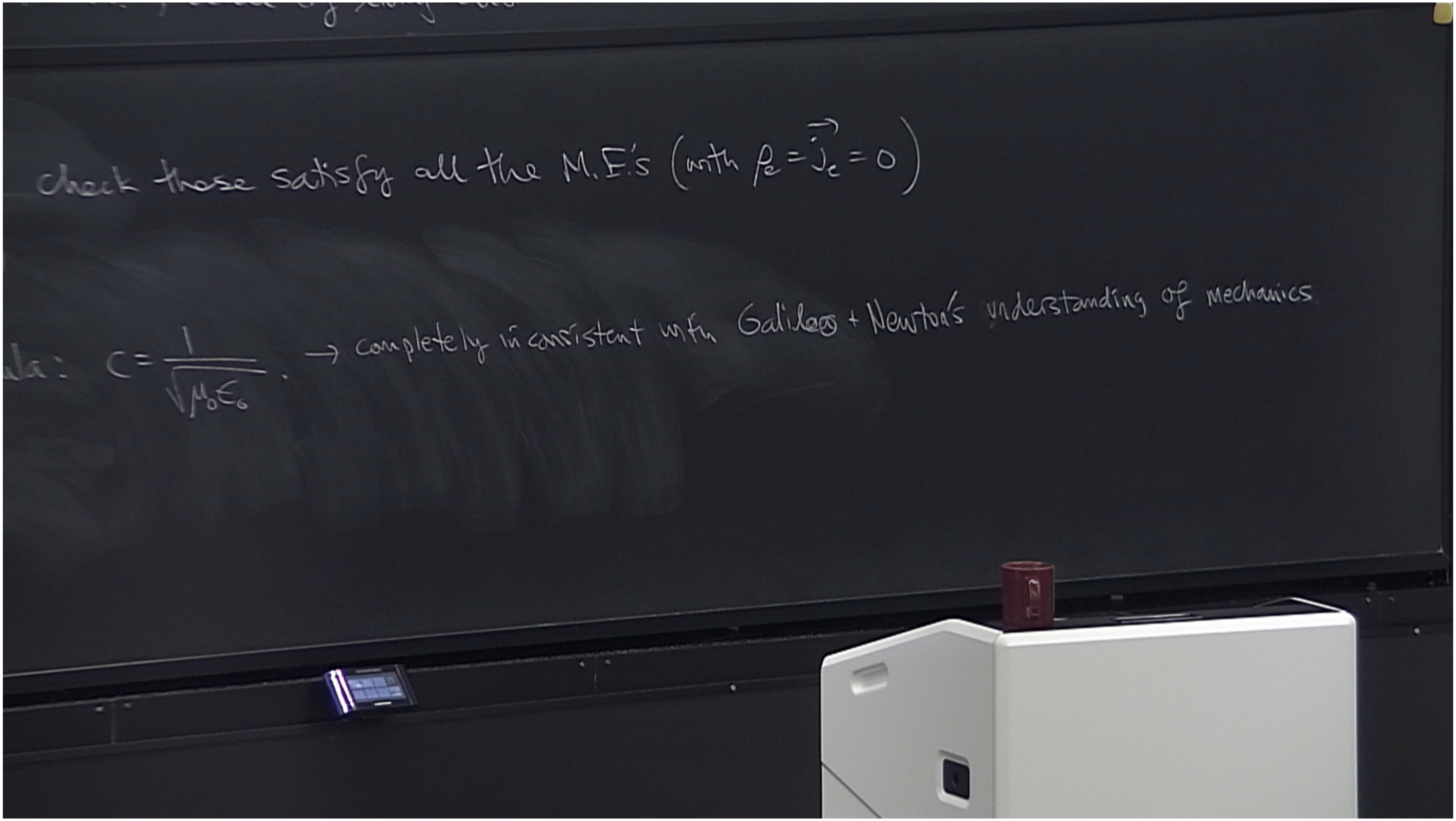
take $\sqrt{l_p l_\Lambda} \simeq 10^{[illegible]}$ m. , scale of living cells.

example: $\vec{E} = \left(0, 0, f(x-ct)\right)$

$\vec{B} = \left(0, -\frac{1}{c}f(x-ct), 0\right)$

check those satisfy all the M




check those satisfy all the M.E.'s (with $\rho_e = \vec{j}_e = 0$)
ula: $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$. → completely inconsistent with Galileo's + Newton's understanding of mechanics