Title: Ising anyons in frustration-free Majorana-dimer models

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Abstract: <p>Dimer models have long been a fruitful playground for understanding topological physics. We introduce a new class -- termed Majorana-dimer models -- where the dimers represent pairs of Majorana modes. We find that the simplest examples of such systems realize an intriguing, intrinsically fermionic phase of matter that can be viewed as the product of a chiral Ising theory, which hosts deconfined non-Abelian Ising quasiparticles, and a topological (p âˆ’ ip) superconductor. While the bulk anyons are described by a single copy of the Ising theory, the edge remains fully gapped. Consequently, this phase can arise in exactly solvable, frustration-free models. We present parent Hamiltonians for this phase and unambiguously identify the topological order from entanglement measurement of the ground-state manifold on torus.</p>

Ising Anyons in Majorana-dimer Models

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arXiv:1605.06125

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Station Q

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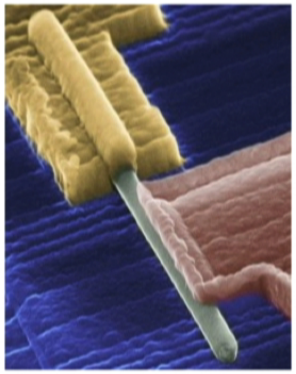
Majorana Modes

$$\frac{1}{2}(\gamma_1 + i\gamma_2) = c^\dagger$$

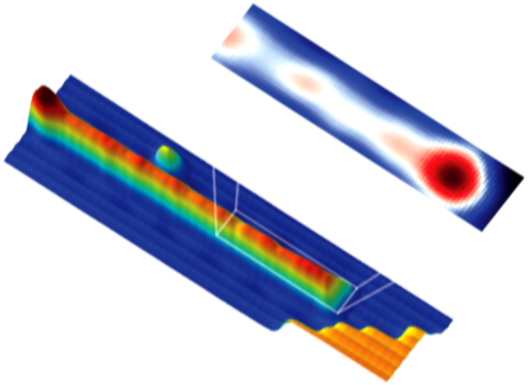
=

$$1 - 2c^\dagger c = i\gamma_1\gamma_2 = \pm 1$$

Majoranas in Real Life



Kouwenhoven lab

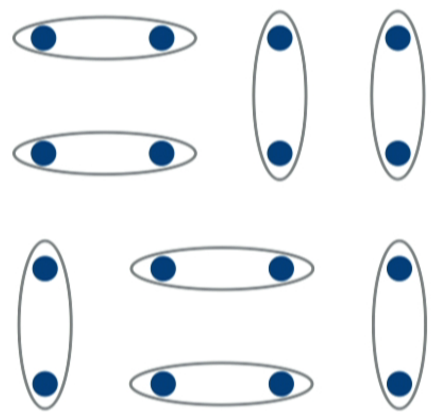


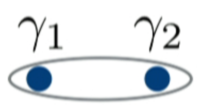
Yazdani lab

Many Majoranas

$$\{\gamma_i, \gamma_j\} = 2\delta_{ij}$$

Many Majoranas

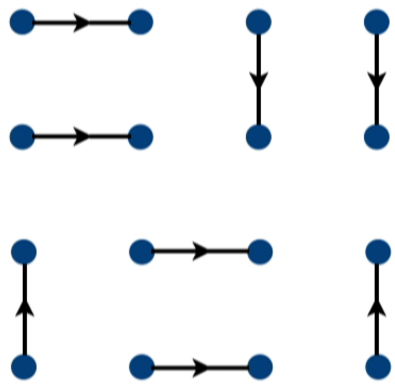


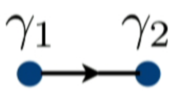


$$i\gamma_1\gamma_2 = \pm 1$$

$$1 - 2c^\dagger c = i\gamma_1\gamma_2 = \pm 1$$

Many Majoranas



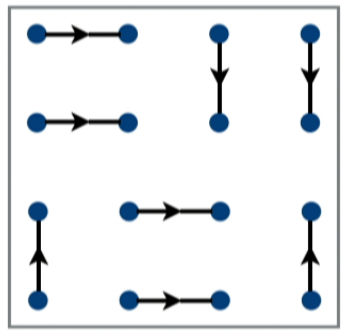
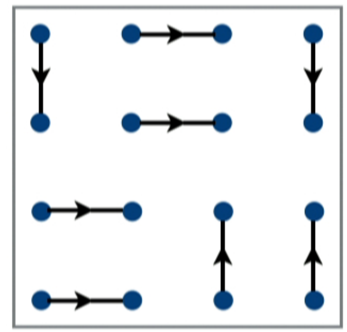


$$i\gamma_1\gamma_2 = 1$$

Majorana dimer

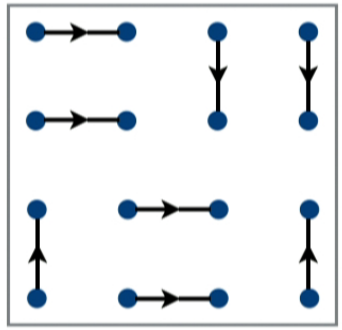
Majorana RVB?

Majorana RVB?

$|\mathrm{RVB}\rangle \overset{?}{=}$  $+$  $+\cdots$

Majorana RVB?

$|\mathrm{RVB}\rangle \stackrel{?}{=}$



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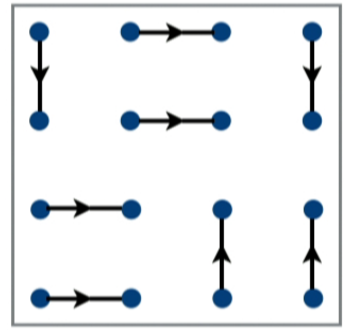

$+\cdots$

Problem: each dimer config. must have the same total fermion parity

Counting Fermion Parity: Clockwise-Odd Rule

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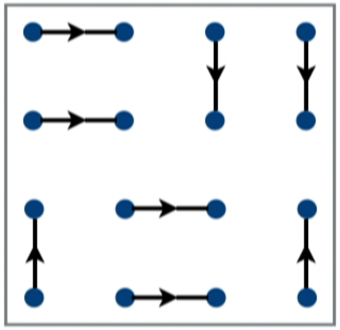

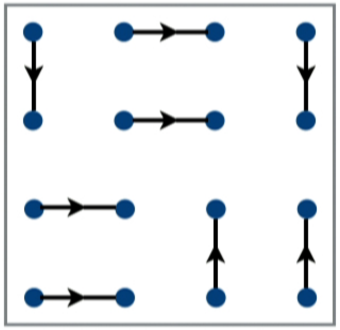

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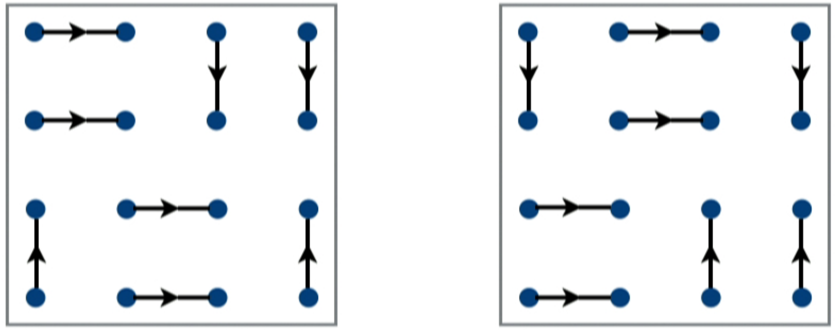

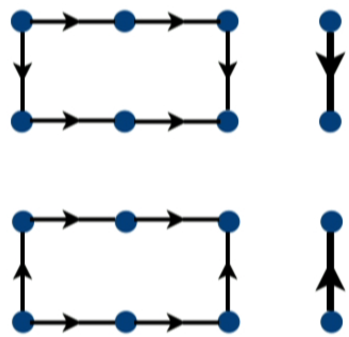

Transition graph

Kasteleyn Orientations

Edge orientations such that all loops satisfy the clockwise-odd rule

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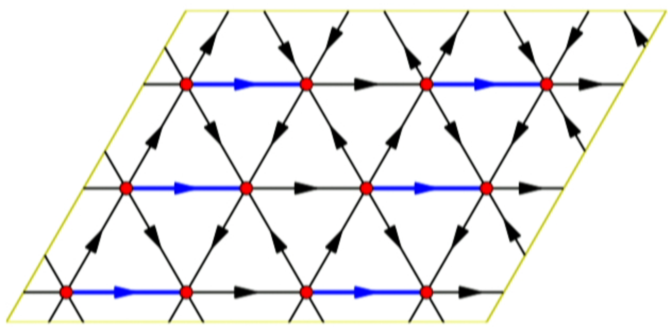
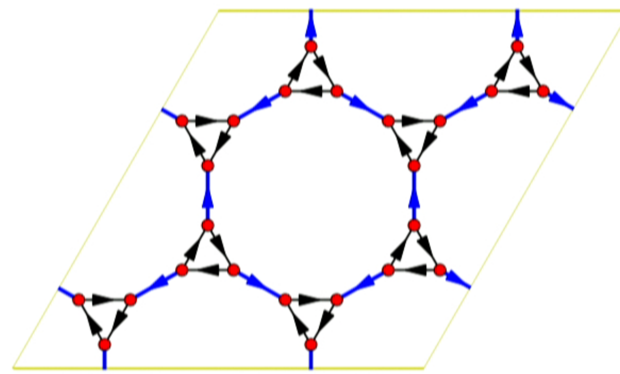
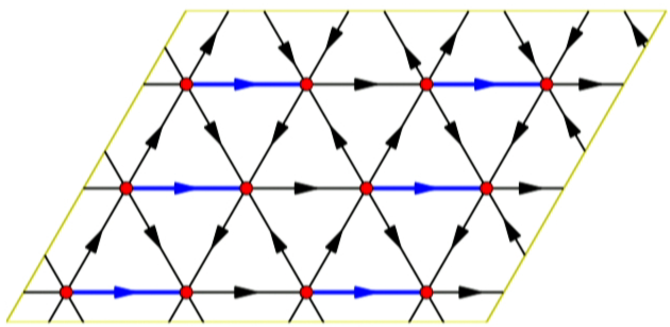
Edge orientations such that all loops satisfy the clockwise-odd rule

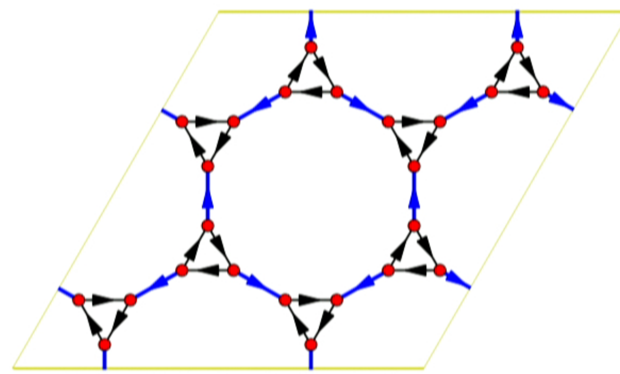




Kasteleyn Orientations

Edge orientations such that all loops satisfy the clockwise-odd rule

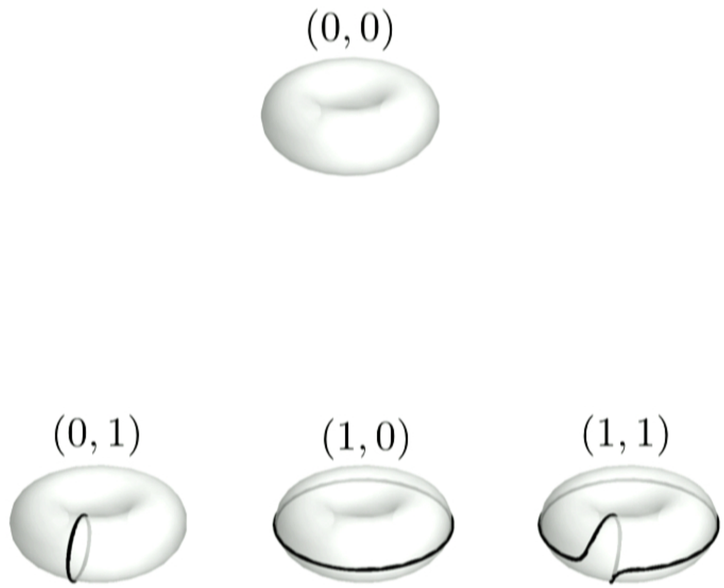




All planar graphs have Kasteleyn orientations

(with even number of vertices)

Kasteleyn Orientations on a Torus



Kasteleyn Orientations on a Torus

Clockwise-odd

$(-1)^{N_f} = 1$

$(0, 0)$

Clockwise-even

$(-1)^{N_f} = -1$

$(0, 1)$

$(1, 0)$

$(1, 1)$

Model I: Triangular Lattice

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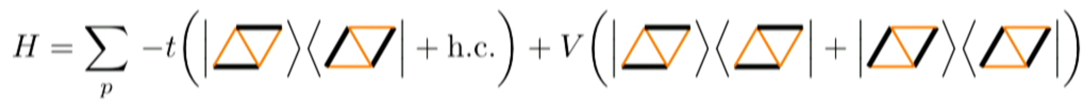
Rokhsar-Kivelson model



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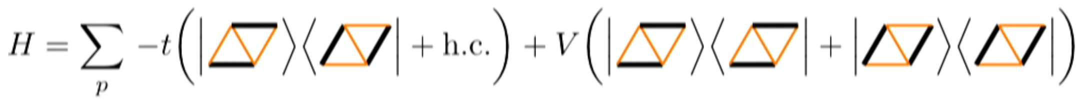
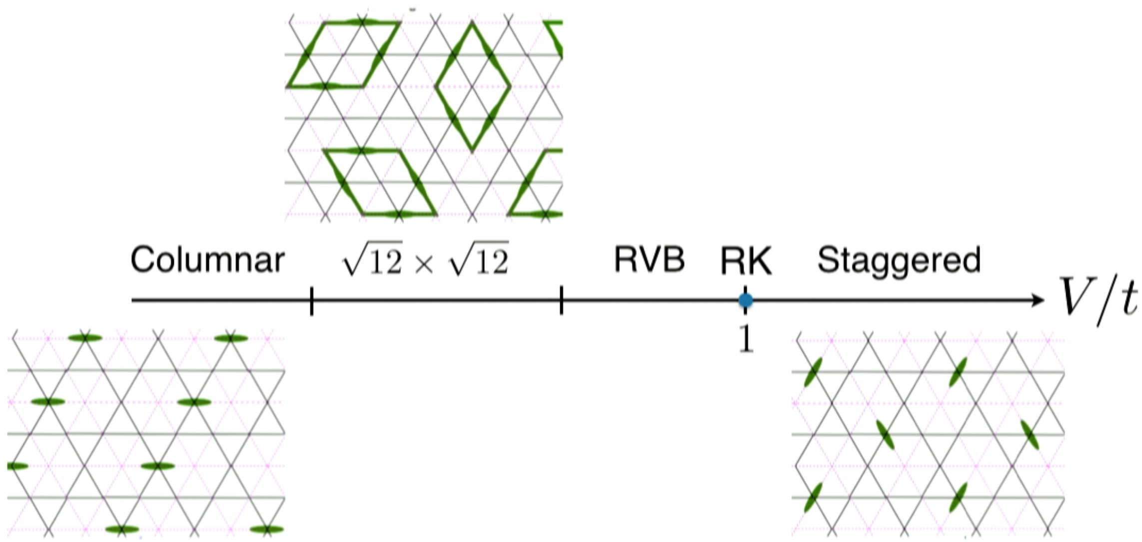
Rokhsar-Kivelson model





Moessner '01

Non-Orthogonality

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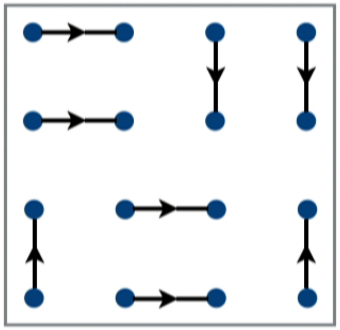
Majorana dimers do not form an orthogonal basis



Hilbert space: Majoranas on sites

Bosonic dimers on bonds

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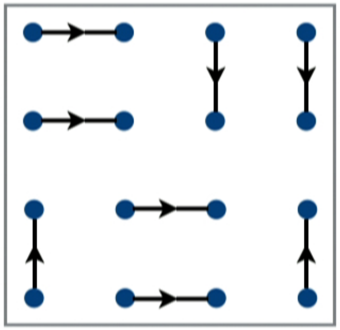


Hilbert space: Majoranas on sites / Bosonic dimers on bonds

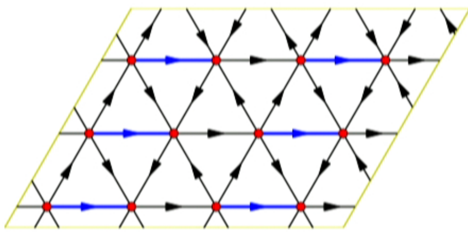
Restricted Hilbert space $\mathcal{H}_r$: $|D\rangle \otimes |F(D)\rangle$

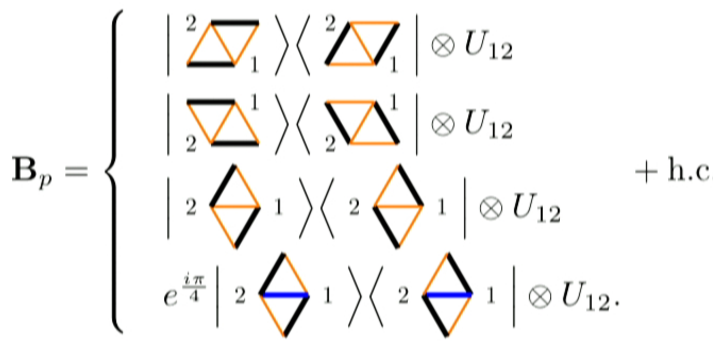
Majorana-Rokhsar-Kivelson Model

Dimer flips are accompanied by braiding Majoranas

Majorana-Rokhsar-Kivelson Model

Dimer flips are accompanied by braiding Majoranas





$$U_{12} = \frac{1 + s_{12}\gamma_1\gamma_2}{\sqrt{2}}$$

Spectrum

Open boundary condition

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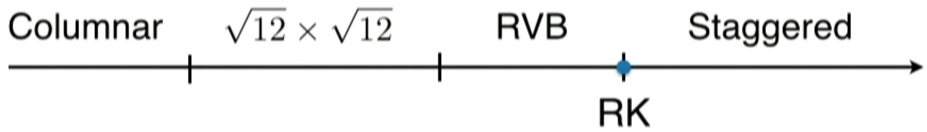
Open boundary condition

Spectrum of Majorana-dimer model = Spectrum of RK model



Spectrum

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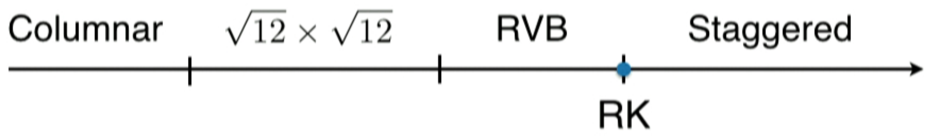
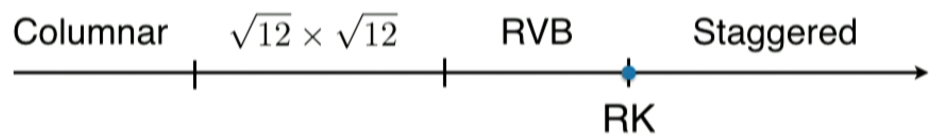


No gapless edge modes

Spectrum

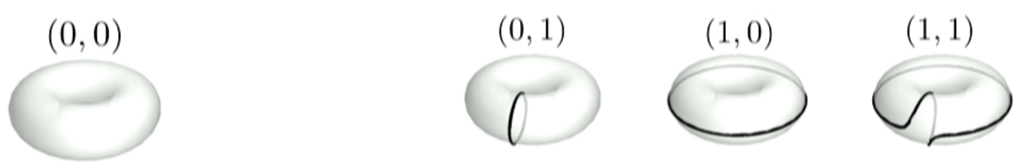
Open boundary condition

Spectrum of Majorana-dimer model = Spectrum of RK model



No gapless edge modes

4x4 torus



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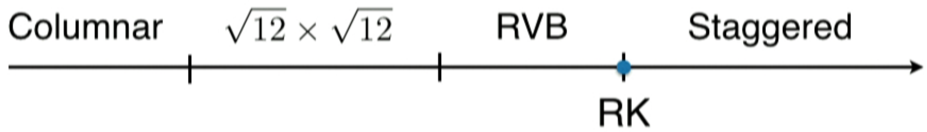


No gapless edge modes

4x4 torus

$(0, 0)$

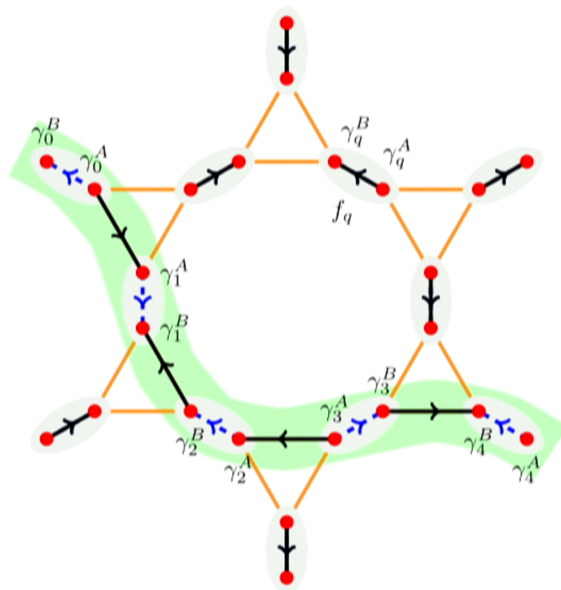
$(0, 1)$ $(1, 0)$ $(1, 1)$

$$E = 0.14t$$

6-fold degeneracy

Exactly zero energy
(RVB/Staggered)

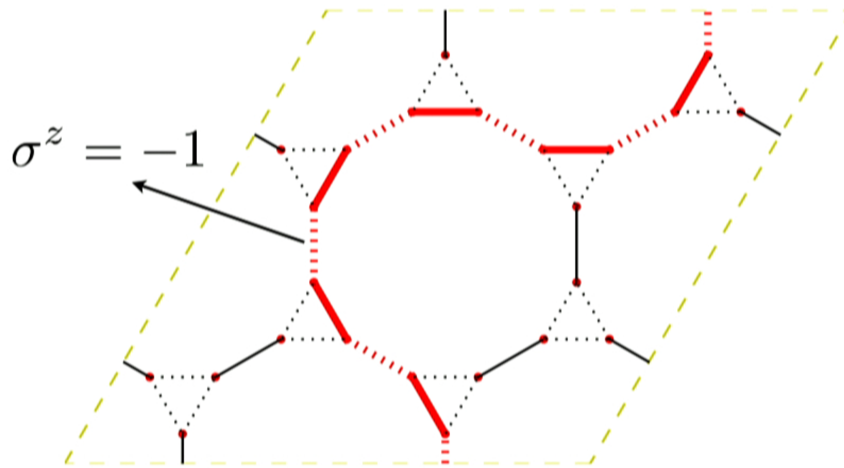
Model II: Fisher Lattice



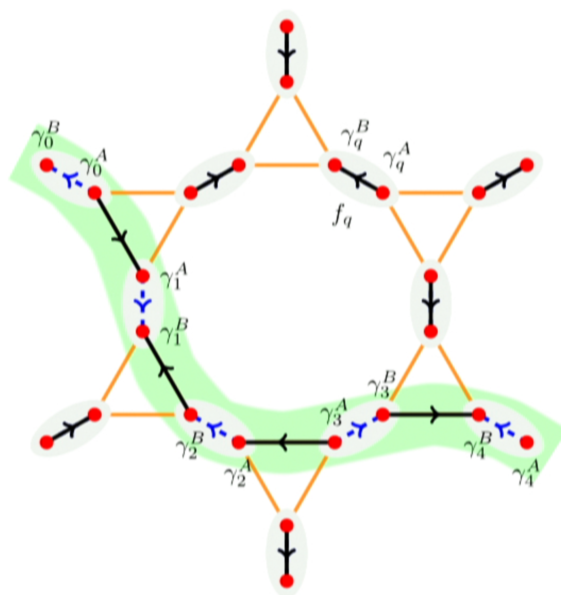
Reference state: $i\gamma_q^A\gamma_q^B = 1$

Loop Basis

Dimers on Fisher lattice ⟷ Loops on honeycomb lattice



Loops Decorated by Majorana Chains



The wave function

The wave function

$$|\mathrm{GS}\rangle = \sum_D e^{i\varphi_D} |D\rangle |F(D)\rangle$$

Resolve the phase ambiguity by dimer flipping:

$$D_0 \xrightarrow{\text{Plaquette flips}} D$$

$$|D_0\rangle|F(D_0)\rangle \xrightarrow{\mathbf{B}_{p_1}\mathbf{B}_{p_2}\cdots\mathbf{B}_{p_L}} |D\rangle|F(D)\rangle$$

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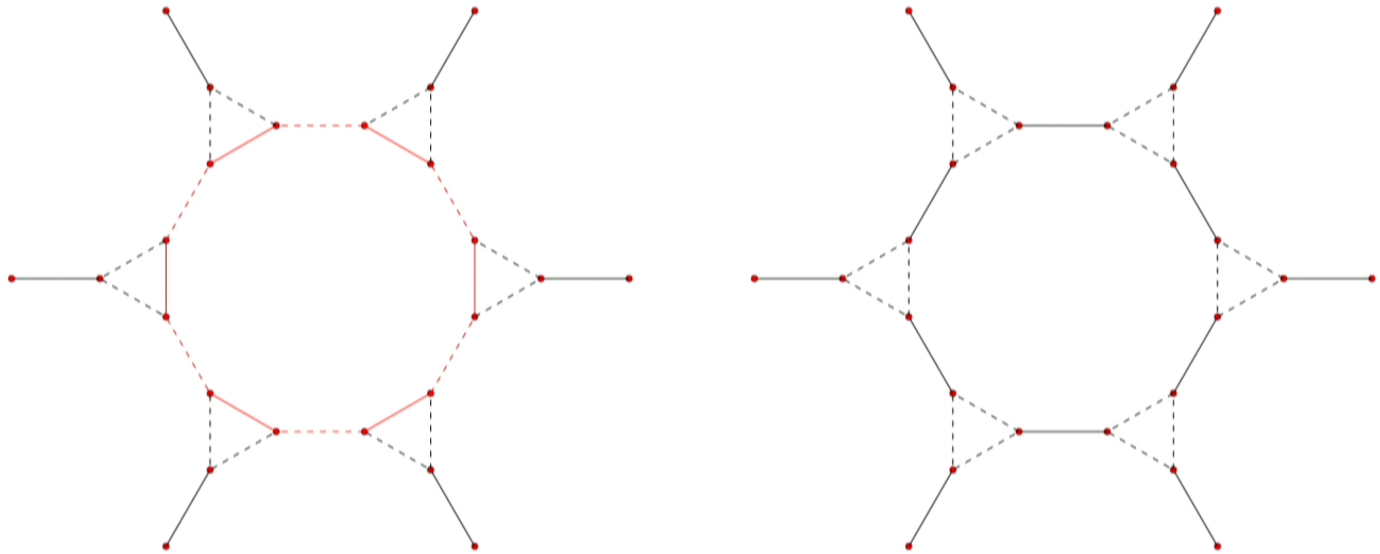
$$\mathbf{B}_p \mathbf{B}_{p'} = \mathbf{B}_{p'} \mathbf{B}_p \qquad \mathbf{B}_p^2 = 1$$

Plaquette Flip



Exactly Solvable Hamiltonian

Hamiltonian in $\mathcal{H}_r$

$$H = -\sum_p \mathbf{B}_p$$

$$[\mathbf{B}_p, \mathbf{B}_{p'}] = 0, \mathbf{B}_p^2 = 1$$

Exactly Solvable Hamiltonian

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$$\mathbf{B}_p = \left(\prod_{e \in p} \sigma_e^x\right) \otimes \mathcal{B}_p$$

$$\langle F(D_p)|\mathcal{B}_p|F(D)\rangle = \frac{\langle F(D_p)|F(D)\rangle}{\left|\langle F(D_p)|F(D)\rangle\right|}$$

Exactly Solvable Hamiltonian

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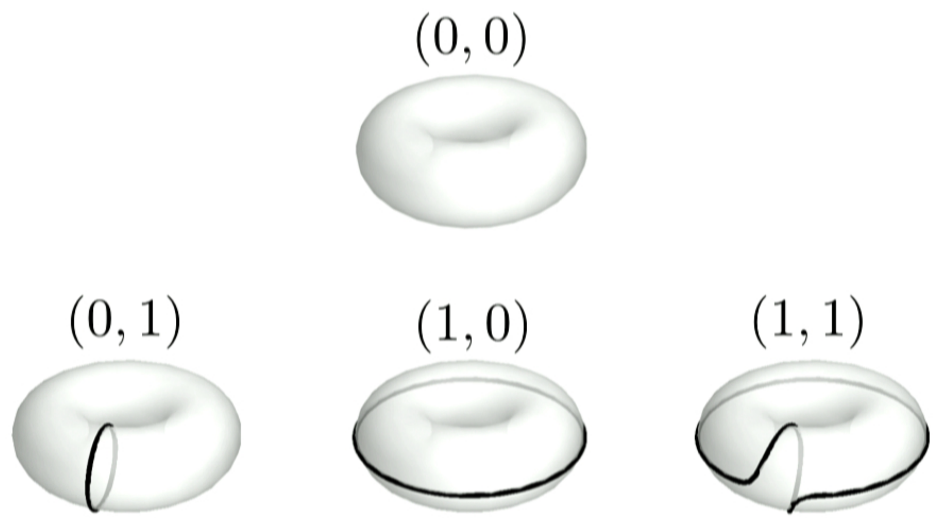
Fully gapped spectrum (even with boundaries!)

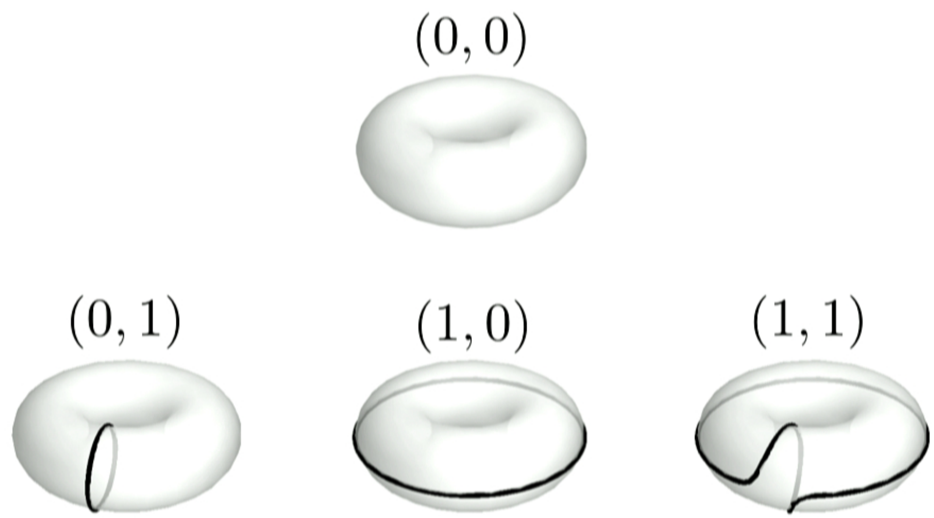
GSD on Torus (PBC)



GSD on Torus (PBC)

$$(-1)^{N_f} = 1$$

$$(-1)^{N_f} = -1$$



GSD on Torus (PBC)

$(0,0)$

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$(0,1)$

$(1,0)$

$(1,1)$

$(-1)^{N_f} = -1$

$$\prod_p \mathbf{B}_p = -(-1)^{N_f}$$

GSD on Torus (PBC)

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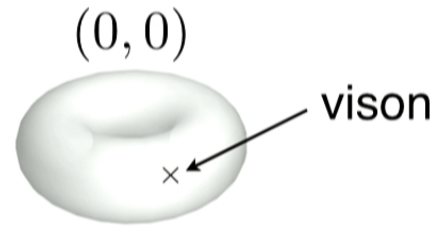

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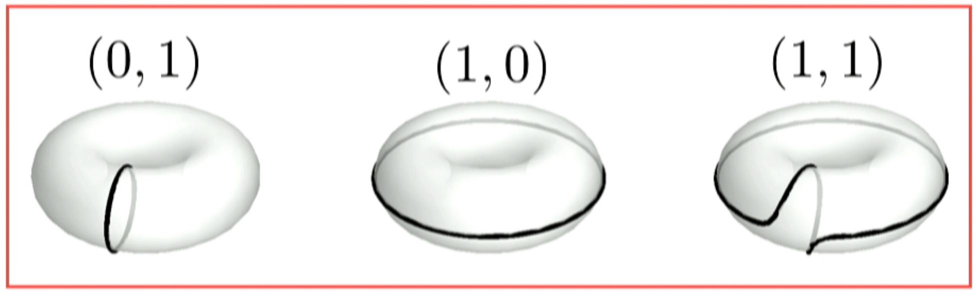

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Entanglement Characterization

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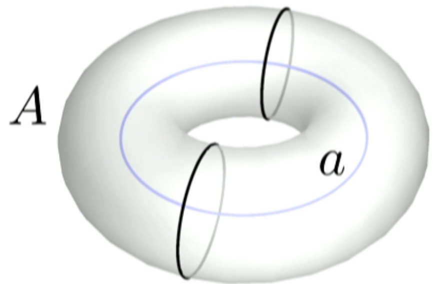
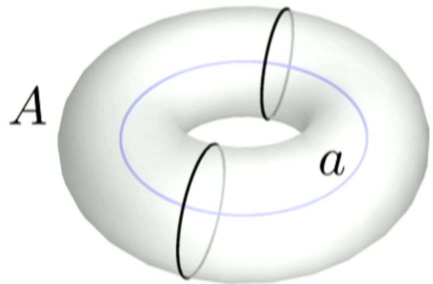


$$S_A = \alpha L - \gamma$$

Entanglement Characterization

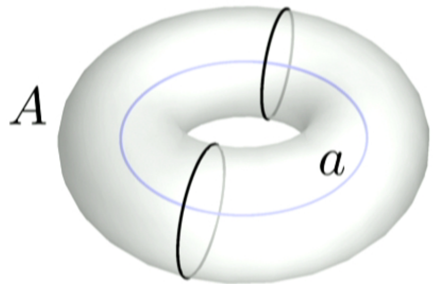


$$S_A = \alpha L - \gamma$$

Minimally entangled states:

$$\gamma = 2 \ln \frac{\mathcal{D}}{d_a}$$

Entanglement Characterization



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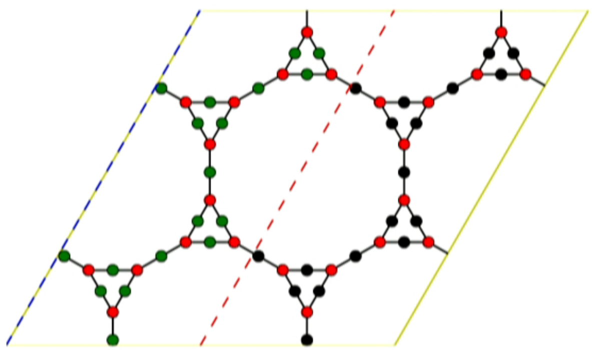
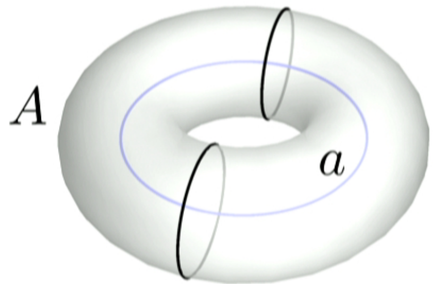


$$|1\rangle = \frac{1}{\sqrt{2}}(|1,0\rangle - e^{\frac{3i\pi}{8}}|1,1\rangle)$$

$$|2\rangle = \frac{1}{\sqrt{2}}(|1,0\rangle + e^{\frac{3i\pi}{8}}|1,1\rangle)$$

$$|3\rangle = |0,1\rangle$$

Entanglement Characterization



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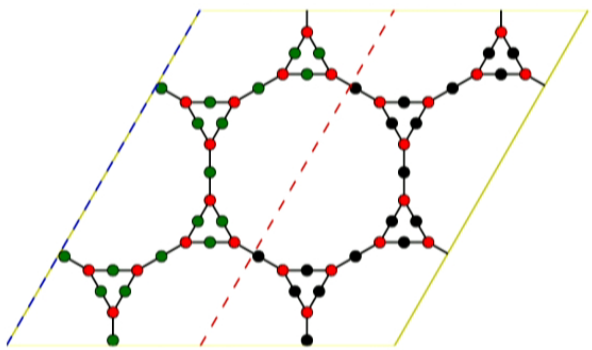

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$$S_1 = S_2 = 3\ln 2, S_3 = 4\ln 2 \implies d_1 = d_2 = \frac{d_3}{\sqrt{2}}$$

Identifying the Topological Order

- Three ground states on torus with odd fermion parity
- Topological entanglement entropy $d_1 = d_2 = \frac{d_3}{\sqrt{2}}$
- Can have a fully gapped boundary

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Ising$^{(n)}$

	1	σ	ψ
d	1	$\sqrt{2}$	1
θ	1	$e^{\frac{i\pi n}{8}}$	-1

$$c_- = \frac{n}{2}$$

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Ising$^{(n)}$ $\otimes$ $(p_x - ip_y)^n$

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Short-ranged entangled state

$c_- = \frac{n}{2}$ $\qquad$ $c_- = -\frac{n}{2}$

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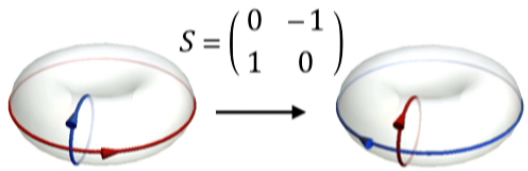
Short-ranged entangled state

$c_- = \frac{n}{2}$

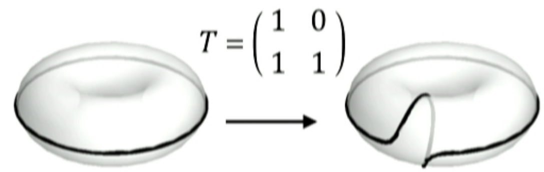
$c_- = -\frac{n}{2}$

$n = 1, 3, 5, 7$

Modular Transformations



$$S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

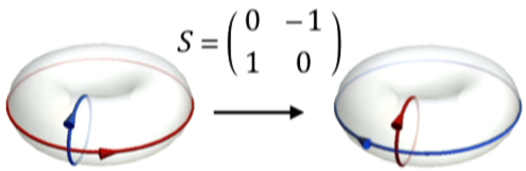


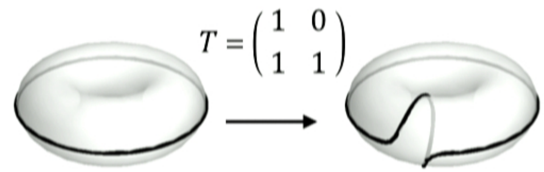
$$T = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

Modular Transformations

S, T matrices uniquely determine the topological order in bosonic systems

Modular Transformations





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Expectations:

$$S = S_{\mathrm{Ising}^{(n)}} \otimes S_{(p_x - ip_y)^n}$$

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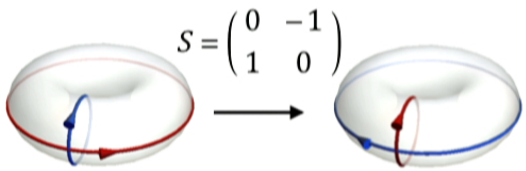

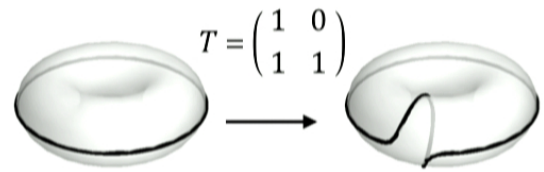

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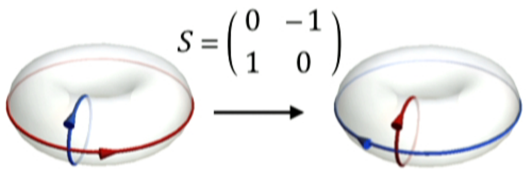

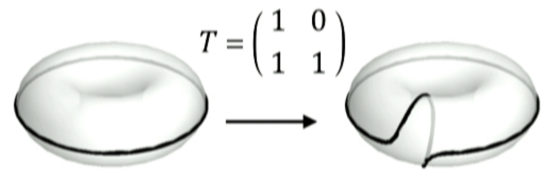

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$$S_{(p_x - ip_y)^n} = e^{\frac{\pi i n}{4}}$$

$$T_{(p_x - ip_y)^n} = e^{-\frac{\pi i n}{12}}$$

You and Cheng '15

Modular Transformations

Extracting modular matrices from rotations:

Zhang PRB'12, '15; Cincio PRL'13; …

Modular Transformations

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Consistent with n=1

Zhang PRB'12, '15; Cincio PRL'13; …

Generalizations: 8-fold ways

1. "Semionic" variant

$$|\mathrm{GS}\rangle = \sum_{D} (-1)^{n(D)} |D\rangle |F(D)\rangle$$

Gu PRB'14

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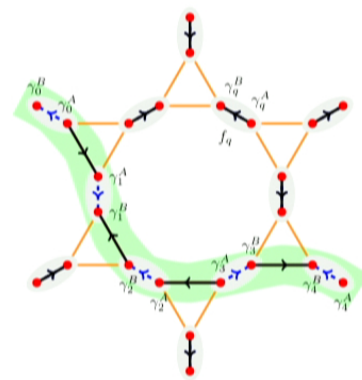
2. Multiple Majoranas per site



2 Majoranas $\longrightarrow$ $\mathrm{U}(1)_4 \times (p_x - ip_y)^2$

"Fermionic toric code"

Gu PRB'14

8-fold Ways and Fermion SPT Phases

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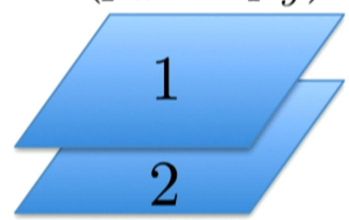
Free-fermion realization

$$(p_x + ip_y)^n$$



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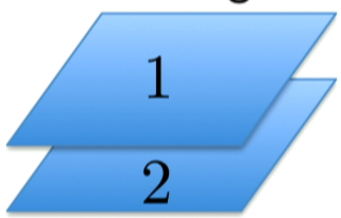
8-fold Ways and Fermion SPT Phases

Gauging

8-fold ways ⟷ Z_2 fermion SPTs

Free-fermion realization

Ising$^{(n)}$



$(p_x - ip_y)^n$

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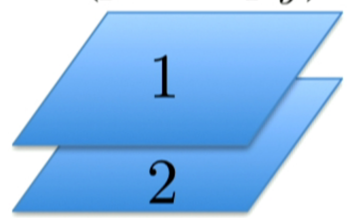

$(p_x - ip_y)^n$

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Summary

- We introduce Majorana-dimer models and construct a RVB-type state.
- Kasteleyn orientations on the lattice are necessary.
- Exactly solvable parent Hamiltonians are found.
- The topological order can be described as Ising x (p-ip)