Title: First sound of zero temperature holographic superfluids

Date: Aug 24, 2016 11:35 AM

URL: http://pirsa.org/16080093

Abstract: Within the context of AdS/CFT, the gravity dual of an s-wave superfluid is given by scalar QED on an asymptotically AdS spacetime. While this conclusion is vastly based on numerical arguments, I will provide an analytical proof that this is indeed the case. In particular, I will present a technique which allows to explicitely compute the low-energy effective action for the boundary theory starting from the bulk system. This will be done for an arbitrary number of dimensions and an arbitrary potential. I will recover the known dispersion relation for conformal first sound.

FIRST SOUND OF ZERO TEMPERATURE HOLOGRAPHIC SUPERFLUIDS

Angelo Esposito

Low Energy Challenges for High Energy Physics II

Perimeter Institute for Theoretical Physics

August 2016, Waterloo, Canada

<u>Talk mostly based on:</u>

AE, S. Garcia-Saenz, R. Penco — arXiv:1606.03104

AE, S. Garcia-Saenz, A. Nicolis, R. Penco — work in progress

OUTLINE

- Review of EFT for superfluids and its gravity dual
- Setup and background equations of motion
- Quadratic action for phonons
  1. Linearized, low-energy equations of motion for the fluctuations
  2. The partially on-shell quadratic action
- Future plans and conclusions

A BRIEF REVIEW

The EFT for a superfluid

- An ***s-wave superfluid*** at $T = 0$ can be seen as a system where a spontaneously broken $U(1)$ symmetry is at finite density
- Its low energy description involves a single real field $\phi(x)$ such that:

$$\phi \xrightarrow{U(1)} \phi + \boxed{a} \qquad \langle\phi\rangle = \boxed{\mu t} \qquad \phi = \mu t + \boxed{\pi}$$

Shift under U(1)

Chemical potential

Goldstone boson = Phonon

- Most general action for the phonons: $S = \int d^D x\, P(X)$ with: $X = -\partial_\mu \phi \partial^\mu \phi$ (Mostly + metric)
- On the background: $X_{\text{bkg}} = \mu^2$
- The stress tensor is given by: $T_{\mu\nu} = 2\dfrac{\partial P}{\partial X}\partial_\mu\phi\partial_\nu\phi + \eta_{\mu\nu}P$
- ***Conformal symmetry*** constrains $P(X)$, and therefore the speed of sound to be:

$$\boxed{P(X) \propto X^{D/2}} \qquad \boxed{c_s^2 = \frac{1}{D-1}}$$

A BRIEF REVIEW

The gravity dual

- An alternative description of the same system involves a complex field $\Phi(x) = e^{i\phi(x)}$ and a shift in the time derivatives, such that:

$$\Phi \xrightarrow{U(1)} e^{ia}\Phi \qquad \partial_t \to \partial_t - \boxed{i\mu} \qquad \boxed{\langle \phi \rangle = 0}$$

Constant gauge field — No more VEV

- With this new language the holographic dual can be found easily:

scalar QED on asymptotically AdS space

[see e.g. Herzog, Kovtun, Son 0809.4870; Hartnoll, Herzog, Horowitz 0803.3295]

- There is no shortage of studies of this system in the literature. However, most of them are based on numerical arguments.
- We found a fairly simple method to ***explicitly*** *and* ***analytically*** *construct the* ***quadratic boundary action*** *for phonons starting from the bulk side.*
- Similar methods have been developed for the EFT of holographic fluids.

[Nickel, Son 1009.3094; de Boer, Heller, Pinzani-Fokeeva 1504.07616; Crossley, Glorioso, Liu, Wang 1504.07611]

OUR SETUP

- We consider the following action:

$$S = -\int d^{D+1}x\,\sqrt{-g}\left[|\partial\Phi - iqA\Phi|^2 + V\left(|\Phi|^2\right) + \frac{1}{4}F_{MN}F^{MN}\right] + S_{\text{c.t.}}$$

- We keep the potential completely general and of the form:

$$V\left(|\Phi|^2\right) = m^2|\Phi|^2 + \text{interaction terms}$$

- We want to study the ***zero temperature case*** ⟶ we work in the probe limit (no backreaction) with fixed AdS_{D+1} metric:

$$ds^2 = \frac{dx^\mu dx_\mu + du^2}{u^2} \qquad \begin{array}{l} u = \infty \rightsquigarrow \text{center of AdS} \\ u = 0 \rightsquigarrow \text{boundary} \end{array}$$

- **DISCLAIMER:** It has been shown that a non-backreacting AdS metric is not always a good description of the geometry of the ground state superfluid (IR sector not always conformally invariant).

  [see e.g. Horowitz, Roberts 0908.3677; Gubser, Nellore 0908.1972]

- We assume that our potential is such that this approximation is consistent and there is indeed **conformal symmetry** (e.g. free massless field, W-shaped potential at large charge).

THE BACKGROUND FIELDS

- *Background fields* ansatz: $\Phi \equiv \rho(u) \quad A_M \equiv \sqrt{2}\frac{\psi(u)}{u}\delta^0_M$

- Near-boundary behavior:

$$\rho = \rho_{(1)} u^{D-\Delta} + \rho_{(2)} u^{\Delta} + \cdots$$
$$\psi = \frac{\mu}{\sqrt{2}} u - \frac{\varepsilon}{\sqrt{2}} u^{D-1} + \cdots$$

Source of charged op. ($\rho_{(1)}$)

VEV of charge op. ($\rho_{(2)}$)

$\Delta(\Delta - D) = m^2$

Chemical pot. (μ)

U(1) charge density (ε)

- We take the counter term action to be:

$$S_{\text{c.t.}} = \lim_{u\to 0}\int d^D x \sqrt{-\gamma}\left[(\Delta - D)|\Phi|^2 - \frac{1}{2(2\Delta - D - 2)}\Phi\Box_\gamma\Phi^* + c.c.\right] + \cdots$$

and the boundary condition to be $\rho_{(1)} = 0$ ⟶ ***spontaneous breaking***

- The equations of motion for the background are then:

$$\rho'' - \frac{D-1}{u}\rho' - \frac{1}{u^2}V'(\rho^2)\,\rho + 2q^2\frac{\psi^2}{u^2}\rho = 0$$
$$\psi'' - \frac{D-1}{u}\psi' + \frac{D-1}{u^2}\psi - 2q^2\frac{\rho^2}{u^2}\psi = 0$$

THE QUADRATIC ACTION FOR PHONONS

Introducing the fluctuations

- Fluctuations of the fields around the background: $\Phi = (\rho + \sigma) e^{i\pi} \qquad A_M = \bar{A}_M + \alpha_M$
- Quadratic action:

$$S^{(2)} = -\int d^{D+1}x \sqrt{-g} \Big[\partial_M \sigma \partial^M \sigma + \rho^2 \partial_M \pi \partial^M \pi - 4q \bar{A}^M \rho \, \partial_M \pi \, \sigma - 2q\rho^2 \alpha^M \partial_M \pi$$
$$+ q^2 \bar{A}^M \bar{A}_M \sigma^2 + 4q^2 \rho \bar{A}_M \alpha^M \sigma + q^2 \rho^2 \alpha_M \alpha^M + (V' + 2\rho^2 V'') \, \sigma^2 + \frac{1}{4} f_{MN} f^{MN} \Big] + S^{(2)}_{\text{c.t.}}$$

Term of the c.t. action quadratic in the fluctuations

$f_{MN} = \partial_M \alpha_N - \partial_N \alpha_M$

- **Plan of action:**
  1. Write the equations of motion for the fluctuations to **lowest order in boundary derivatives**
  2. Solve them for all fluctuations but π, with vanishing double Dirichlet b.c.
  3. Find the resulting **partially on-shell action**
- We will see that, although the analytical expression for the background fields is not available, we can still compute the Goldstone action regardless of it details

THE QUADRATIC ACTION FOR PHONONS

The low-energy, linearized equations

- The low-energy expansion can be implemented with the following rules:

$$\sigma,\, \alpha_\mu,\, \partial_u \sim \mathcal{O}(1) \qquad \partial_\mu \sim \mathcal{O}(\epsilon) \qquad \pi,\, \alpha_u \sim \mathcal{O}(1/\epsilon)$$

- The resulting linearized equations are:

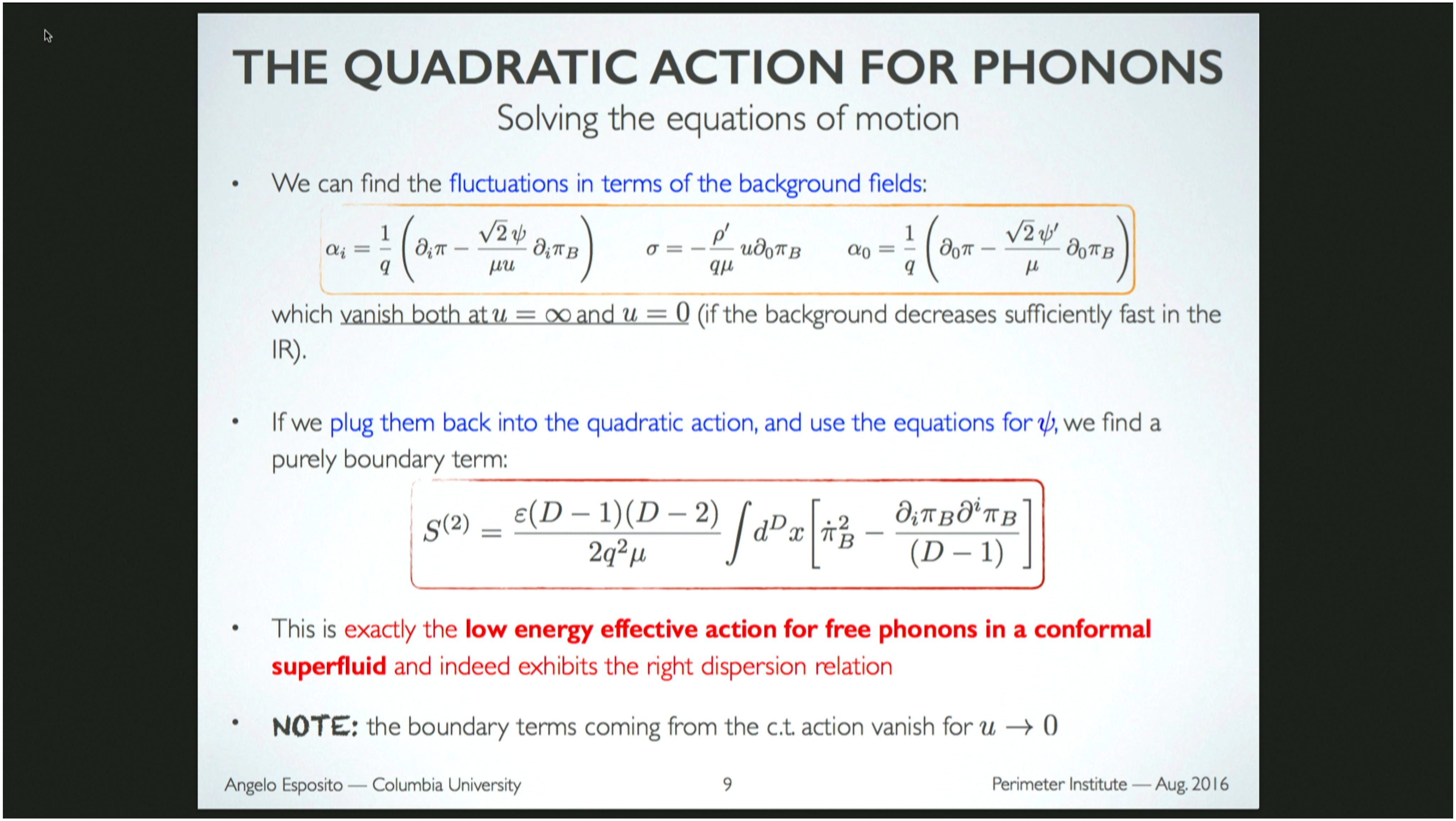
$$\sigma'' - \frac{(D-1)}{u}\sigma' - \frac{1}{u^2}\left(V' + 2\rho^2 V''\right)\sigma + 2\sqrt{2}\,\frac{q\rho\psi}{u}(q\alpha_0 - \partial_0\pi) + \frac{2q^2\psi^2}{u^2}\sigma = 0$$

$$(q\alpha_0 - \partial_0\pi)'' - \frac{(D-3)}{u}(q\alpha_0 - \partial_0\pi)' - 2\,\frac{q^2\rho^2}{u^2}(q\alpha_0 - \partial_0\pi) - 4\sqrt{2}\,\frac{q^3\rho\psi}{u^3}\sigma = 0$$

$$(q\alpha_i - \partial_i\pi)'' - \frac{(D-3)}{u}(q\alpha_i - \partial_i\pi)' - 2\,\frac{q^2\rho^2}{u^2}(q\alpha_i - \partial_i\pi) = 0$$

$$\pi' - q\alpha_u = 0$$

- **NOTE:** we did not fix any gauge
- The last equation tells us right away that: $\pi(u, x_\mu) = -q\int_u^\infty dw\, \alpha_u(w, x_\mu)$ with vanishing b.c. at the center of AdS
- We will see that ***the superfluid phonon*** corresponds to: $\boxed{\pi_B(x_\mu) = \pi(u=0, x_\mu)}$


THE QUADRATIC ACTION FOR PHONONS
Solving the equations of motion
We can find the fluctuations in terms of the background fields:
$$\alpha_i = \frac{1}{q}\left(\partial_i \pi - \frac{\sqrt{2}\,\psi}{\mu u}\,\partial_i \pi_B\right) \qquad \sigma = -\frac{\rho'}{q\mu}\,u\partial_0\pi_B \qquad \alpha_0 = \frac{1}{q}\left(\partial_0 \pi - \frac{\sqrt{2}\,\psi'}{\mu}\,\partial_0 \pi_B\right)$$
which vanish both at $u = \infty$ and $u = 0$ (if the background decreases sufficiently fast in the IR).
If we plug them back into the quadratic action, and use the equations for ψ, we find a purely boundary term:
$$S^{(2)} = \frac{\varepsilon(D-1)(D-2)}{2q^2\mu}\int d^D x\left[\dot{\pi}_B^2 - \frac{\partial_i \pi_B \partial^i \pi_B}{(D-1)}\right]$$
This is exactly the low energy effective action for free phonons in a conformal superfluid and indeed exhibits the right dispersion relation
NOTE: the boundary terms coming from the c.t. action vanish for $u \to 0$
Angelo Esposito — Columbia University
9
Perimeter Institute — Aug. 2016

DISCUSSION

- A few comments are now in order:
  1. Our argument does not apply to the $D = 2$ case since the asymptotic behavior of the fields is not regular anymore. However, we do not expect Goldstone bosons because of Coleman's theorem; [Anninos, Hartnoll, Iqbal 1005.1973]
  2. With our conventions $\varepsilon/\mu > 0$. For $D > 2$ this ensures that π_B is not ghost-like;
  3. The interpretation of the boundary Goldstone boson in terms of Wilson line of the radial component of the gauge field has been found is several other works:

     [see e.g. Nickle, Son 1009.3094 for Einstein-Maxwell;
     de Boer, Heller, Pinzani-Fokeeva 1504.07616 for ordinary fluids;
     Sakai, Sugimoto hep-th/0412141 for holographic QCD;
     Contino, Nomura, Pomarol hep-ph/0306259 for composite Higgs]
- We believe that if applied to other contexts, our method can help shed some light on some old issues as well as help in understanding the dual low energy theory of a given bulk system. In fact…

THE QUADRATIC ACTION FOR PHONONS

Solving the equations of motion

- We can find the fluctuations in terms of the background fields:

$$\alpha_i = \frac{1}{q}\left(\partial_i \pi - \frac{\sqrt{2}\,\psi}{\mu u}\,\partial_i \pi_B\right) \qquad \sigma = -\frac{\rho'}{q\mu}\,u\partial_0\pi_B \qquad \alpha_0 = \frac{1}{q}\left(\partial_0\pi - \frac{\sqrt{2}\,\psi'}{\mu}\,\partial_0\pi_B\right)$$

  which vanish both at $u = \infty$ and $u = 0$ (if the background decreases sufficiently fast in the IR).

- If we plug them back into the quadratic action, and use the equations for ψ, we find a purely boundary term:

$$S^{(2)} = \frac{\varepsilon(D-1)(D-2)}{2q^2\mu}\int d^D x\left[\dot{\pi}_B^2 - \frac{\partial_i\pi_B\partial^i\pi_B}{(D-1)}\right]$$

- This is exactly the **low energy effective action for free phonons in a conformal superfluid** and indeed exhibits the right dispersion relation

- **NOTE:** the boundary terms coming from the c.t. action vanish for $u \to 0$

FUTURE PLANS

- Directions for future work:

A. Holographic superfluids at finite temperature:

- The EFT for a superfluid at non-zero temperature has been developed quite a few years ago. [Nicolis 1108.2513]
- The gravity dual is considered to be scalar QED on Schwarzschild-AdS background.
- Numerical studies were not able to reproduce Landau's prediction for the relation between first and second sound at low temperature: $c_2^2 = c_1^2/(D-1) \qquad T \sim 0$
- This might be due to additional degrees of freedom. [Herzog, Yarom 0906.4810]
- Can we recover the known EFT using our method? If not, can we isolate the unexpected degrees of freedom?

B. Holographic solid:

- The EFT for a solid in D-dim. flat space has a global $ISO(D) \times ISO(D)$ symmetry spontaneously broken down to the diagonal subgroup. The group is non-compact $\longrightarrow$ no YM gauge theory can be built.
- However, if the solid lives on a D-dim. sphere the symmetry becomes $SO(D+1)$. The gravity dual of this system should be simply a non-abelian YM theory.
- We are employing the technique presented here to check if we can recover the known action for the solid phonons… stay tuned!

SUMMARY

To summarize:

- We showed a method to **recover the EFT of a superfluid** starting from the fluctuations of the bulk fields around their background
- The equations of motion for these fluctuations can be solved without knowing the explicit form for the background
- When the gauge is not fixed, the boundary Goldstone boson is the IR-to-UV Wilson line of the radial component of the gauge field
- Because it explicitly allows to build the action of the boundary EFT, ***we believe that this method has the potential to help us to understand other holographic systems***

THANKS FOR YOUR ATTENTION!