Title: Hydrodynamic theory of fluctuating stripes

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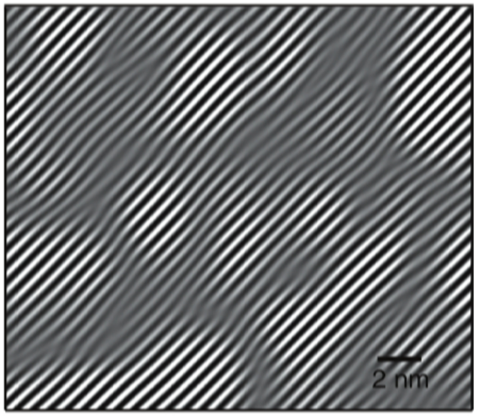
Abstract: I will present a hydrodynamic description of matter in a charge density wave (or "smectic") phase. As in superfluids, the spontaneous breaking of a continuous symmetry -- here translations in one direction -- adds a Goldstone phase to the usual long lived hydrodynamic variables. This phase propagates as a highly anisotropic "second sound" mode at low energies, affecting properties such as transport. Phase fluctuations, due to proliferating dislocations, give a finite life-time to certain collective modes, which can be experimentally probed e.g. by measuring ultrasound attenuation. Using the memory matrix, the hydrodynamic approach predicts sound attenuation to be proportional to the shear viscosity of the normal (non-smectic) state.

Charge Density Wave (CDW)



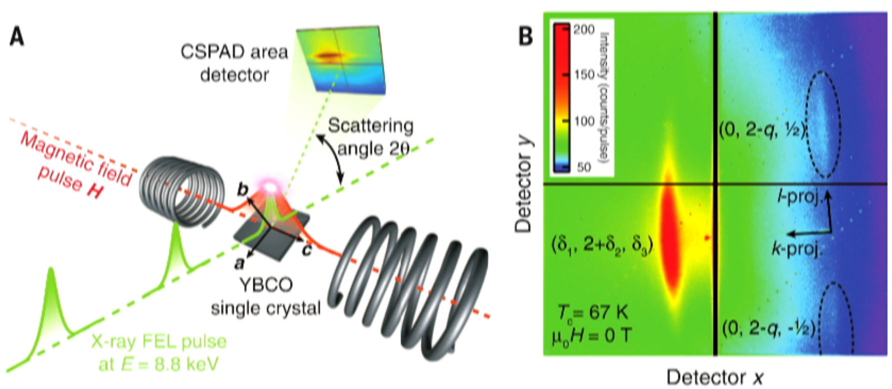
[Mesaros, Fujita et. al. (2011)]

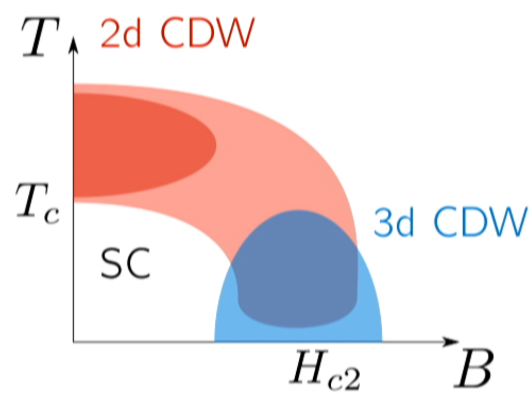
Spontaneous formation of stripes of charge density $\langle \rho \rangle = \rho_0 \cos(Q \cdot x)$

Incommensurate: spontaneously breaks translations in one direction

Can be measured e.g. with x-ray scattering or nuclear magnetic resonance

Ubiquitous in cuprates – CDW orders competes with superconducting order





[Gerber, Jang et. al. (2015)] on underdoped YBCO

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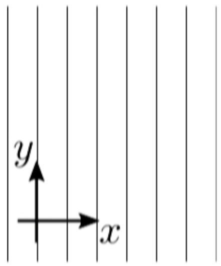
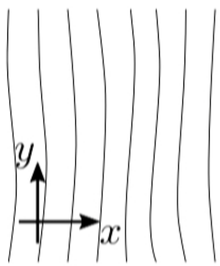
   - $\not{P}_x \Rightarrow$ Goldstone ϕ, $\rightsquigarrow$ anisotropic second sound
2. **Momentum Relaxation**
   - $\dot{P} = 0 \Rightarrow \sigma_{\mathrm{dc}} = \infty$, but unlike clean metals $\dot{P} = \mathcal{O}(\epsilon) \not\Rightarrow$ Drude conductivity
3. **Phase Relaxation**
   - Proliferation of dislocations

Constitutive Relations

$\langle \phi \rangle = 0$



$\delta\phi$



CDW spontaneously breaks translations: $\mathbb{R}/\mathbb{Z} \cong U(1)$

Goldstone non-linearly realizes translations: $\phi \to \phi - \Delta x$

Its canonical conjugate is the associated charge density: $[\phi(\mathbf{x}), T_{0x}(\mathbf{y})] = i\delta^2(\mathbf{x}-\mathbf{y})$.

It is a hydro variable and as such can appear in any constitutive relation. As for superfluids, $\partial_x \phi \sim \mathcal{O}(\partial^0)$.

Note that $\phi \xrightarrow{\mathsf{R}_x} -\phi$, so

$$j_x = \rho v_x + \cancel{\partial_x \phi} + \gamma_1 \partial_x^2 \phi + \dots$$

$$\text{but:} \quad T_{xx} = p - \rho_\phi \partial_x \phi + \dots$$

Constitutive relations are constrained by rotations (isotropic) or at least reflections R_x, R_y (nematic) and T (Onsager relations).

Josephson Relation

Conservation laws $[\partial_\mu j^\mu = \partial_\mu T^{\mu\nu} = 0]$ + Constitutive relations give hydrodynamic equations of motion for the variables $\delta\rho$, $\delta\epsilon = \delta T_{00}$, δT_{0i}

Need an extra equation for ϕ: "Josephson" equation. For superfluids,

$$H \supset \int d^d x \, \mu\rho \;\Rightarrow\; \dot{\phi} = -\mu + \ldots$$

Here $H \supset \int d^d x \, v_i T_{0i}$ so

$$\dot{\phi} = -v_x + \gamma_1 \partial_x \mu + \gamma_2 \partial_x T + \ldots$$

$\rightsquigarrow$ hydrodynamic equations give Green's functions [Kadanoff, Martin (1963)]

Constitutive Relations

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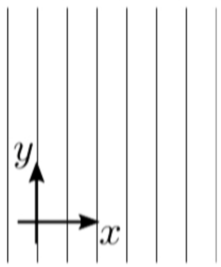

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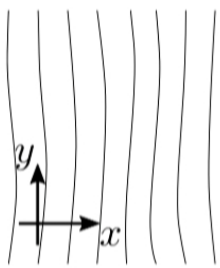

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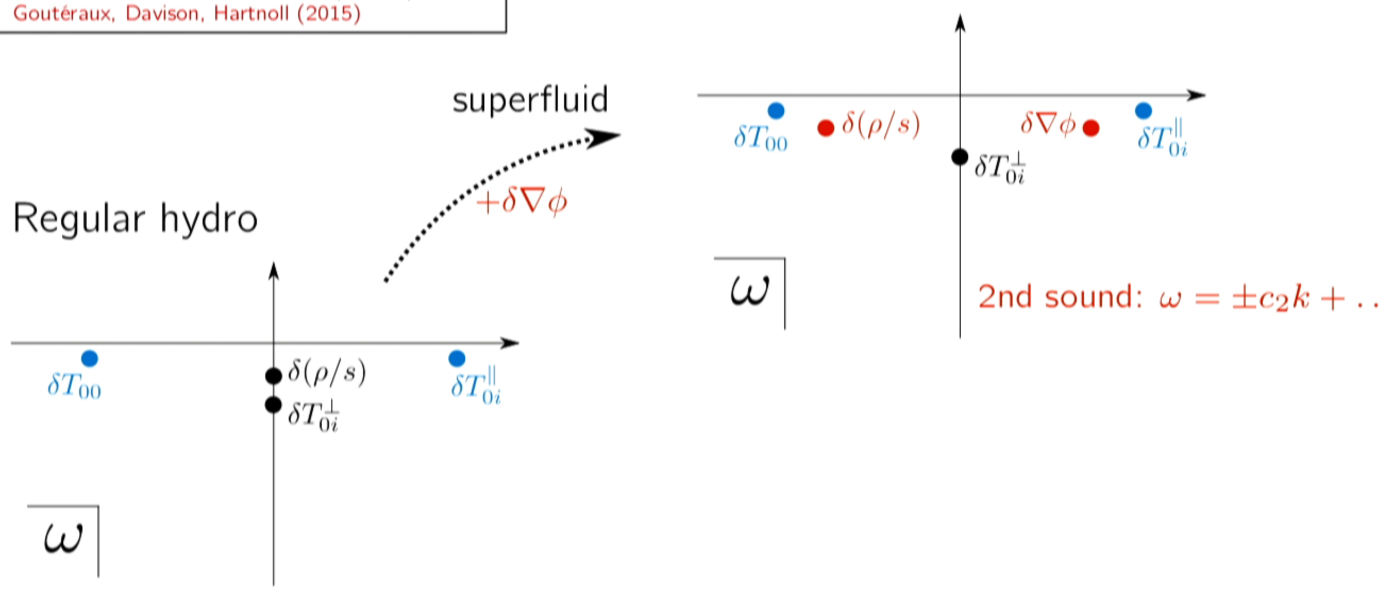
Hydrodynamic Modes

"Diagonal charges" δQ have a single pole in $G^R(\omega, k)$

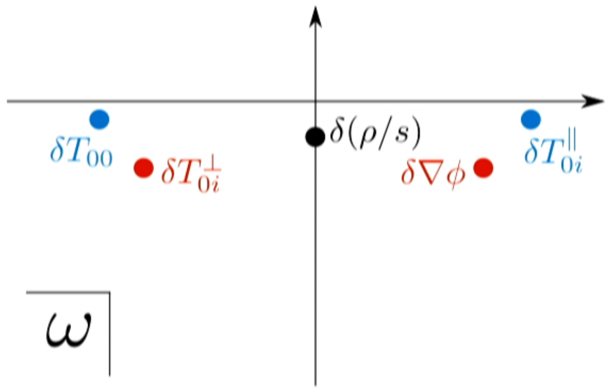
Goutéraux, Davison, Hartnoll (2015)

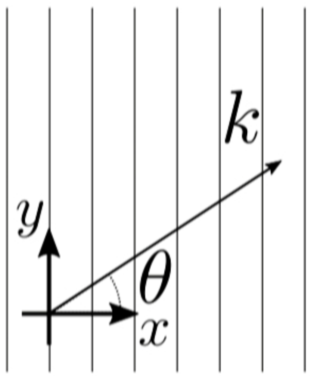


First sound: $\omega = \pm c_1 k + \ldots$

2 diffusive modes: $\omega = -iDk^2 + \ldots$

Hydrodynamic Modes





Anisotropic dispersion relation

First sound:

$$c_1^2(\theta) \simeq \frac{\epsilon + p + \rho_\phi \cos^4\theta}{\chi_{PP}}$$

Second sound:

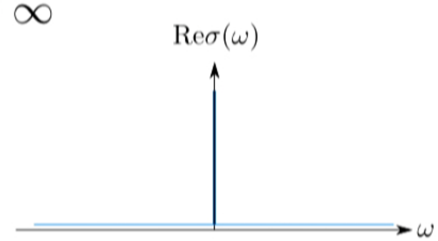
$$c_2^2(\theta) \simeq \frac{\rho_\phi \cos^2\theta \sin^2\theta}{\chi_{PP}}$$

Contents

Momentum Relaxation and Transport

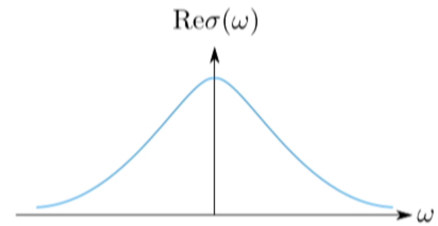
Translation invariant systems at finite density have $\sigma_{\mathrm{dc}} = \infty$

$$\begin{aligned} \dot{P} &= 0 \\ \chi_{PJ} &= \rho \end{aligned} \quad \Longrightarrow \quad \sigma(\omega) = \frac{\chi_{PJ}^2}{\chi_{PP}} \frac{\mathrm{i}}{\omega + \mathrm{i}0^+} + \dots$$



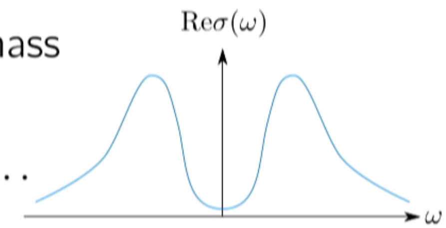
Weak translation breaking (disorder, lattice) relaxes pole

$$\dot{P} = \mathcal{O}(\epsilon) \quad \Longrightarrow \quad \sigma(\omega) = \frac{\chi_{PJ}^2}{\chi_{PP}} \frac{\mathrm{i}}{\omega + \mathrm{i}\Gamma} + \dots$$



CDW have a different pole structure due to Goldstone mass

$$\begin{aligned} &\dot{P} = \mathcal{O}(\epsilon) \\ &\text{CDW } \phi \end{aligned} \quad \Longrightarrow \quad \sigma(\omega) = \frac{\chi_{PJ}^2}{\chi_{PP}} \frac{\mathrm{i}\omega}{\omega(\omega + \mathrm{i}\Gamma) - \frac{m^2 \rho_\phi}{\chi_{PP}}} + \dots$$



Lee, Rice, Anderson (1974)

Memory Matrix

For a given translation breaking mechanism $\Delta H = \int d^d x \, V(x)\mathcal{O}(x)$, memory matrix gives a formula for momentum relaxation rate

$$\Gamma \simeq \frac{1}{\chi_{PP}} \int \frac{d^d k}{(2\pi)^d} V_k V_{-k} k^2 \lim_{\omega \to 0} \frac{\mathrm{Im}\, G^R_{\mathcal{O}\mathcal{O}}(\omega, k)}{\omega} + O(V^4)$$

[Hartnoll, Herzog (2008); Hartnoll, Hofman (2012); Lucas, Sachdev, Schalm (2014); Lucas, Sachdev (2015) ...]

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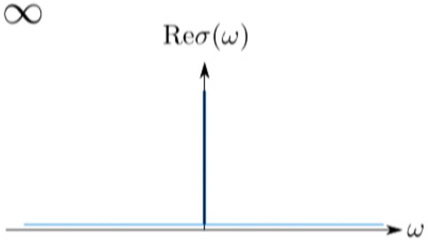

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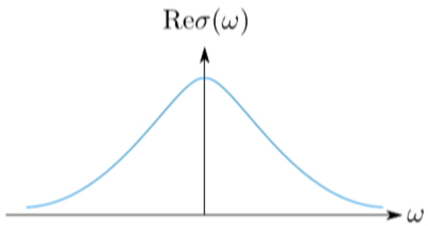

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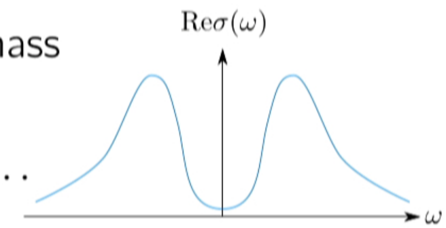

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   - Proliferation of dislocations

Phase Relaxation

There is another operator whose conservation protects $\omega = 0$ poles:

$$W = \int d^2x \, \partial_x \phi \,,$$

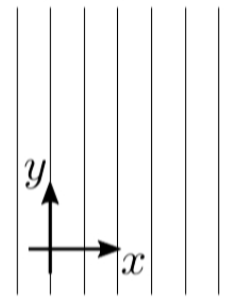
total "compression" of the CDW (winding of the phase ϕ).

Unlike P, conservation of W does not cause infinite DC conductivities, because it cannot carry current ($\chi_{WJ} = 0$ by R$_x$ or T).

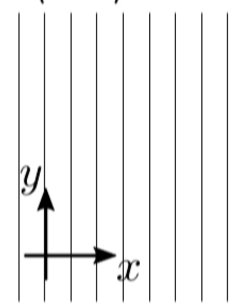
- What physics controls the relaxation scale $\langle \dot{W} \rangle \sim \Gamma' \langle W \rangle$?
- How can Γ' be measured?

[In superfluids: relaxation of $J_\phi = \int d^2x \, \nabla\phi$ controls pole structure of $\sigma(\omega)$ Davison, LVD, Goutéraux, Hartnoll PRB **94** (2016)]

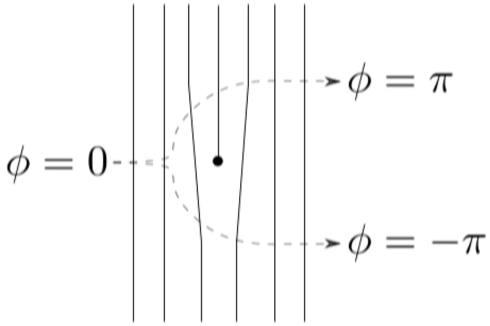
$\langle W \rangle = 0$

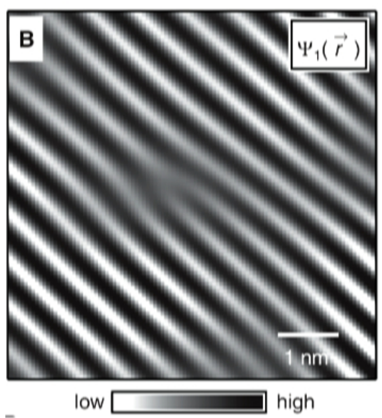


$\langle W \rangle > 0$



Defects





[Mesaros, Fujita et. al. (2011)]

Vortices for the phase ϕ = dislocations

Large gradient $\nabla\phi$ near dislocation site too costly
$\rightsquigarrow$ dislocation core is in normal (symmetric) state

$\Rightarrow$ no Goldstone in dislocation core

The best "winding" operator we can define is

$$W = \int_{\mathbb{R}^2 \setminus \text{cores}} d^2x \, \partial_x \phi \, ,$$

but this operator is not exactly conserved

$$H \supset \frac{\chi_{PP}^{-1}}{2} \int d^2x \, (T_{0x})^2$$

$$\Rightarrow \quad \dot{W} = i[H, W] = \chi_{PP}^{-1} \int_{\text{cores}} d^2x \, \partial_x T_{0x}$$

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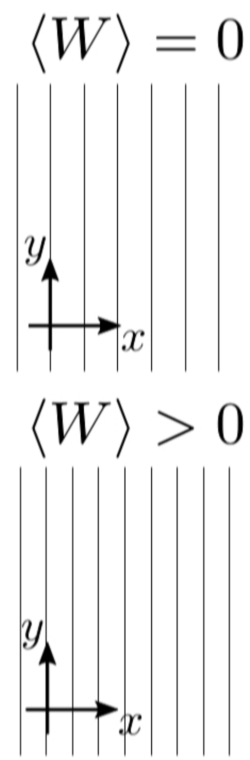


Memory matrix

$$\boxed{\dot{W} = i[H, W] = \chi^{-1}_{PP} \int_{\text{cores}} d^2x\, \partial_x T_{0x}}$$

Memory matrix then provides a formula for the phase relaxation

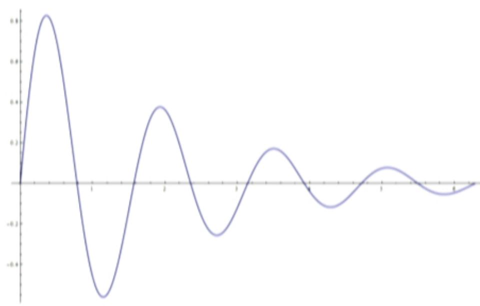
$$\Gamma' \simeq \rho_\phi \lim_{\omega \to 0} \frac{\operatorname{Im} G^R_{\dot{W}\dot{W}}(\omega)}{\omega} \sim \rho_\phi\, k^2_{\text{core}} \lim_{\omega \to 0} \frac{\operatorname{Im} G^R_{T_{0x}T_{0x}}(\omega, k_{\text{core}})}{\omega}$$

For large vortices $k_{\text{core}} < \Lambda_{\text{hyd}} \sim 1/\xi_{\text{mfp}}$, the Green's function can be evaluated with hydrodynamics in the normal state:

$$\Gamma' = \frac{\rho_\phi}{2\eta_n} \cdot A_{\text{core}} n_{\text{dis}}$$

shear viscosity of the normal state

How to measure Γ'?

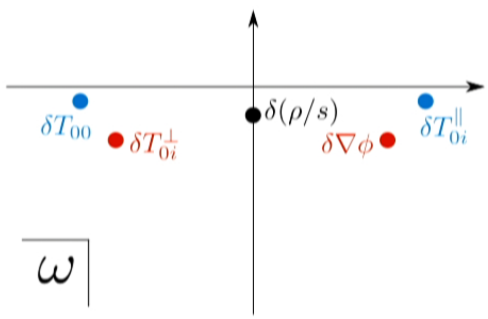


Γ' not directly tied to thermoelectric transport

Instead, more closely related to response to stress

Simple experimentally accessible observable:
sound attenuation

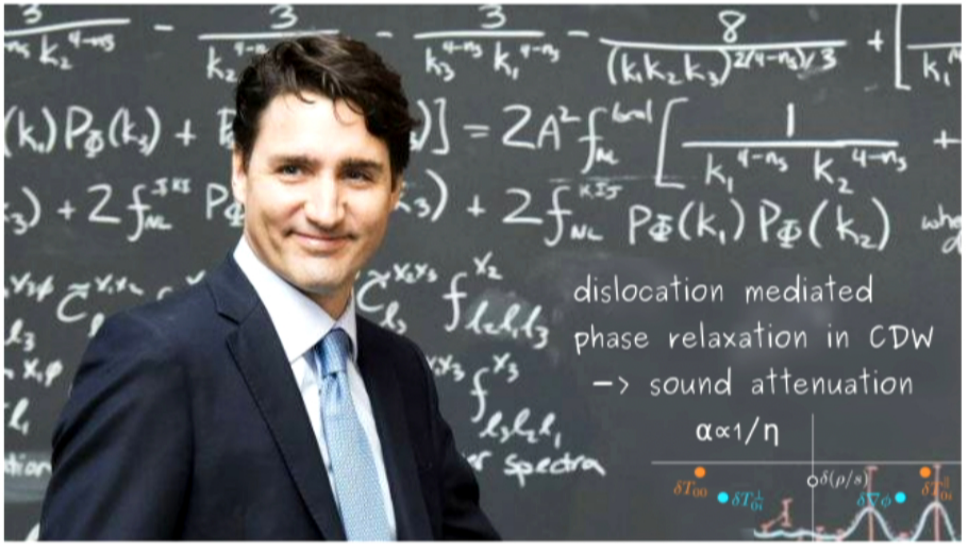
$$\omega = \pm ck - i\alpha$$



Both hydrodynamic sound modes get attenuated
$\alpha_1, \alpha_2 \sim \Gamma'$

- Experimentally, phonon sound mode is probed $\rightsquigarrow$ couples to hydrodynamics
- Competing attenuations: momentum relaxation Γ, diffusive part of sound modes $\alpha \sim k^2$, ...

Thank you!


dislocation mediated
phase relaxation in CDW
-> sound attenuation
α∝1/η