Title: Near-horizon instability of rapidly rotating black holes

Date: Aug 18, 2016 01:00 PM

URL: http://pirsa.org/16080091

Abstract: <p>Aretakis' discovery of a horizon instability of extremal black holes came as something of a surprise given earlier proofs that individual frequency modes are bounded. Is this kind of instability invisible to frequency-domain analysis? The answer is no: We show that the horizon instability can be recovered in a mode analysis as a branch point at the horizon frequency. We use the approach to generalize to nonaxisymmetric gravitational perturbations and reveal that certain Weyl scalars are unbounded in time on the horizon. We will also discuss new results showing how the instability manifests for *nearly* extremal black holes: long-lived quasinormal modes collectively give rise to a transient period of growth near the horizon. This period lasts arbitrarily long in the extremal limit, reproducing the Aretakis instability precisely on the horizon. We interpret these results in terms of near-horizon geometry and discuss potential astrophysical implications.</p>

Near-Horizon Instability of Rapidly Rotating Black Holes

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arXiv:1606.08506 arXiv:1608.04739

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Motivation

- In 2010 Aretakis proved that extremal black holes are linearly unstable. His proof
    - restricted to axisymmetric perturbations,
    - involved conserved quantities on the horizon,
    - and applied only to precisely extremal black holes.
- Since his discovery, despite much work, research has largely been within his initial restrictions

Our work

- We
    - *recover* the Aretakis instability in a mode expansion (as a branch point at the superradiant bound),
    - *extend* to nonaxisymmetric perturbations and reveal curvature growth,
    - and *connect* to the long-lived QNMs of near-extremal Kerr which exhibit a transient instability that limits to Aretakis.

Background

Kerr black holes

- The Kerr solution is the unique stationary asymptotically flat vacuum solution
- Two parameters: spin and mass
- Killing fields ∂_v and ∂_φ. Generators $\xi = \partial_v + \Omega_H \partial_\varphi$
- Type D
  - Shear-free principal null congruence
  - Radiative perturbations are encoded in a single scalar field

Extremal black holes

- Parameters are saturated (a=M in Kerr)
- Enhanced symmetries (Kerr/CFT)
  - SL(2,R) x U(1)
- Vanishing surface gravity (zero Hawking temperature)
  - redshift effect $e^{-v\sigma} \to 1$
- Conserved quantities
  - infinite number of charges (Aretakis constants)

Extremal limits

- The limit to extremality isn't unique.
- Letting $a \to M$ at fixed Kerr coordinates $\Longrightarrow$extremal Kerr
- Letting $a \to M$ at fixed "scaled coordinates" $\Longrightarrow$ NHEK spacetime with AdS asymptotics and enhanced symmetry
- The limits are singularly related but give smooth metrics in each region.

Perturbations of Kerr

- Teukolsky master equation $L_s[\Omega_s] = 0$
- Ω_s defined in a tetrad regular on $\mathcal{H}$
- *Sectors*
  - Gravitational $\Omega_{\pm 2} \leftrightarrow \{\Psi_4, \Psi_0\}$,
  - Electromagnetic $\Omega_{\pm 1} \leftrightarrow \{\phi_2, \phi_0\}$,
  - Scalar $\Omega_0 \leftrightarrow \Phi$

Green function for Ω_s

spheroidal harmonics (SL problem)

$$G = \frac{1}{2\pi} \sum_{m\ell} e^{im(\varphi-\varphi')} \int_{-\infty+ic}^{\infty+ic} {}_s f_{\ell m\omega}(\theta,\theta')\, {}_s\tilde{g}_{\ell m\omega}(x,x') e^{-i\omega(v-v')} d\omega$$

transfer function

inverse **Laplace Transform**

Stability of Extremal Kerr

- Consider Kerr geometry with a=M and evolve small amplitude initial data (e.g. massless scalar).
- Does the solution grow large enough to back-react?
  - Mode stability (Whiting, 1989) forbids exponentially growing modes, but doesn't prove boundedness of generic perturbations.
  - **Aretakis** (2010) shows axisymmetric perturbations decay but transverse derivatives *blow up* polynomially on the event horizon. Lucietti and Reall extend to gravity (2012).

Mode analysis deficiency?

- Is the Aretakis horizon instability invisible to a mode analysis?
- No. The derivative instability at the horizon is recovered as a branch point in the complex frequency plane.
- The mode technique allows us to predict the growth of *all* modes.

The Laplace transform and linear stability

$$\Phi = G \circ (\mathrm{data})$$

- The late-time linear response is dictated by singular points of its transfer (Green) function in the complex frequency plane.
  - *Mode stability* - lack of singular points in the upper half plane
    - No exponentially growing modes.

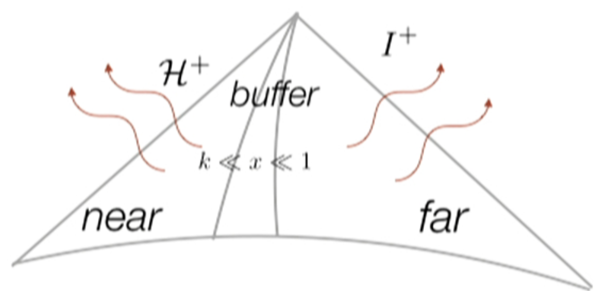
Matched asymptotic expansion

- Construct $\tilde{g}_{\ell m\omega}$ from homogeneous solutions via the method of *matched asymptotic expansions*.
    - valid for $k \equiv \omega - m/2 \ll 1$

Asymptotic regions

- Near zone $x \ll 1$
- Far zone $x \gg k$

Horizon at $x = 0$



Extremal Kerr transfer function

$$\tilde{g}_{\ell m\omega}(k \to 0) \sim u(x') \left[\frac{(-ik)^{-H+i\mu-s}}{\mathcal{S}k(-ik)^{-2H} + \mathcal{U}} \right] e^{i\omega x} \sum_{j=0}^{\infty} A_j(\omega) \left(\frac{x}{ik} \right)^j .$$

generates growth

- The branch point at the superradiant bound frequency k=0 determines the late-time solution.
- The character of the singularity is determined by the *conformal weight H*.

Conformal Weight

- The conformal weight labels representations of the near-horizon symmetry group, SL(2,R).

- At the superradiant bound, the highest weight solution has conformal weight given by

$$H \text{ is } \begin{cases} = 1/2 + ib, & |m| \gtrsim .74\ell \qquad \text{(dominant)}, \\ > 1, \ \& \notin \mathbb{Z}, & 0 < |m| \lesssim .74\ell \quad \text{(subdominant)}, \\ = \ell + 1, & m = 0 \qquad \text{(axisymmetry)}. \end{cases}$$

Growth rates on the event horizon

- *Nonaxisymmetric* perturbations (compact data off $\mathcal{H}$)

- Complex H $\boxed{\Omega_s^{(n)} \simeq v^{n+s-1/2}, \quad v \to \infty}$

- Real H $\Omega_s^{(n)} \simeq v^{n+s-H}, \quad v \to \infty.$ $\boxed{H > 1}$

$A \simeq B$ meaning asymptotic equality up to a bounded function

Growth rates on the event horizon (s=2)

- Axisymmetric perturbations (compact far-horizon data)

when $n = \ell - 1, \ell - 2$

$$\Psi_4^{(n)} \simeq v^{-2}, \quad v \to \infty$$

otherwise

$$\Psi_4^{(n)} \simeq v^{n-\ell}, \quad v \to \infty$$

Consistent with Lucietti and Reall's weak estimates for gravity.
Jump predicted by Aretakis.

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The *nonaxisymmetric* modes yield the dominate horizon growth.

Infalling observers (earth probes) measure:

$$\text{energy density} \simeq v$$

$$\text{tidal fields} \simeq v^{3/2}$$

Explanation of derivative growth

- Off the horizon, the (scalar) field decays slower (1/v). The jump in the decay *at* the horizon explains the derivative growth.

simple example $f(v,x) = e^{-vx}$

[more generally any self-similar field]

Near-extremal Kerr

Near-extremal Kerr black holes

- Small but finite surface gravity

$$\sigma = 2\sqrt{1 - a^2/M^2} \ll 1$$

- Near horizon modes

$$\bar{\omega} = \frac{2M(\omega - m\Omega_H)}{\sigma}$$

Long-lived quasinormal modes

- Extended "ringing" of nearly extremal Kerr (Detweiler, 1980)

$$\omega_n = m\Omega_H + O(\sigma)$$

  - The collective excitation of weakly damped overtones slows decay (transiently) from exponential to power law. (Glampedakis and Andersson 2008, Yang et al 2013, Burko and Khanna 2016)
  - The modes vanish in the extremal limit (no real, non-superradiant QNMs) and the *branch point* "emerges" at the superradiant bound.

Near horizon QNM response

- We perform overtone sum for dominant modes (complex *H*)
  - At early times on the horizon

$$\Omega_s^{(d)} \simeq v^{d-1/2+s}, \quad v \ll 1/\sigma,$$

  - and slightly off the horizon

$$\Omega_s^{(d)} \simeq v^{d-1/2+s} \left(1 + \frac{xv}{2}\right)^{-1/2-s}, \quad v \ll 1/\sigma, \, x \ll \sigma,$$

Transient instability

Fields *grow* until $v_{\max} = \sigma^{-1}$

then decay exponentially

Physical effects - scalars (s=0)

- Amplification of fields

$$\Omega_s^{(d)} \simeq \sigma^{1/2-s-d} \quad \text{for} \quad x \sim \sigma.$$

- Large energy density
  - Quadratic $T_{\mu\nu}$ implies

$$\rho_{\rm obs} = T_{\mu\nu} u^\mu u^\nu \sim \sigma^{-1} \to \infty.$$

Physical effects - E&M (|s|=1)

- Amplification of fields

$$\Omega_s^{(d)} \simeq \sigma^{1/2-s-d} \quad \text{for} \quad x \sim \sigma.$$

- **Electromagnetic field**
  - Charge particles experience large Lorentz forces

$$F_{\mu\nu} u^\mu \simeq \sigma^{-1/2}$$

  - Possible non-linear QED effects (Schwinger pairs)

Physical effects - gravity (|s|=2)

- Amplification of fields

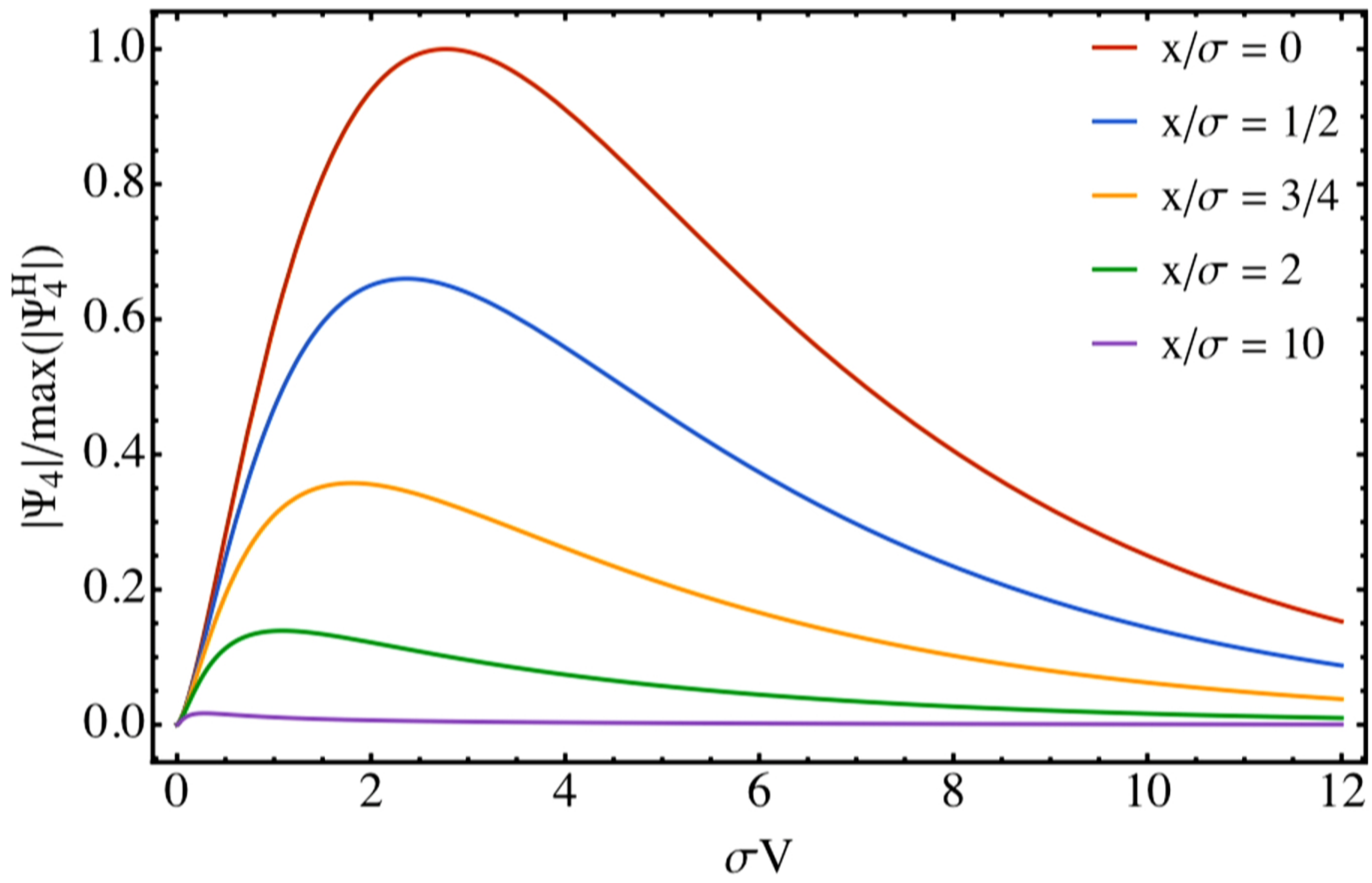
$$\Omega_s^{(d)} \simeq \sigma^{1/2-s-d} \quad \text{for} \quad x \sim \sigma.$$

- **Gravitational fields** $\Omega_2 \propto \Psi_4 = C_{n\bar{m}n\bar{m}} \simeq \sigma^{-3/2}$

$$\Omega_{-2} \propto \Psi_0 = C_{\ell m \ell m} \simeq \sigma^{1/2}$$

- Enhancement of tidal fields (probably minor relative to background tides)

Transient Instability



Summary and rules of thumb

- Potentials

$$(\Phi, A_\mu, h_{\mu\nu}) \sim \sigma^{1/2}$$

- Fields

$$(\nabla_\mu \Phi, F_{\mu\nu}, \Gamma^\alpha{}_{\mu\nu}) \sim \sigma^{-1/2}$$

- Curvatures

$$(\nabla_\mu \nabla_\nu \Phi, d \star F, C_{\mu\nu\alpha\beta}) \sim \sigma^{-3/2}$$

$$v_{\max} \sim \sigma^{-1}$$

Limiting geometries - earth adapted limit

- Far horizon limit
  - Fix ingoing Kerr coordinates $x^\mu = (v, x, \theta, \varphi)$ and take $\sigma \to 0$
  - Faithful to earthlings and the probes they drop into the black hole
  - Puts physically interesting unstable orbits (IBCOs) on the horizon with an infinite boost.

Limiting geometries - Miller adapted limit

- Near horizon limit
  - Fix ingoing scaled coordinates

$$\bar{x}^\mu = (\sigma v, x/\sigma, \theta, \varphi - v)$$

and take $\sigma \to 0$

  - Faithful to Millerites and the probes they drop into the black hole but puts earth at infinity.
  - The near horizon region is singularly related to the far horizon region in the extremal limit!

Simple explanation - singular limits

- Fields with a smooth near-horizon limit look singular to far-zone observers.

- Far-horizon adapted (regular) initial data *generically* excites near horizon modes

  - => sufficiently high-order derivatives blow up as measure by far probes.

derivatives => $\partial_x = \sigma^{-1}\partial_{\bar{x}}$

Near horizon/far horizon cross-talk

Far horizon initial data excites near horizon modes

$$G_{\mathrm{NHM}} = \sum_{\ell m} \sigma^{1/2-s-i\delta+im} \mathcal{G}_{\ell m}(\bar{x}^{\mu}, x^{\mu\prime}),$$

Self-similar scaling predicted by near horizon symmetry

Turbulence criterion

- Non-linear resonances among the integer-spaced near horizon QMNs leads to a turbulent-esque cascade

- Yang et.al (2015) estimated a Reynolds number assuming that the driving perturbation is the *lowest* overtone.

    - The coherent excitation will likely lower the Reynolds number

    - Extrapolating the scaling of Psi4

$$\Psi_4 \simeq \sigma^{-3/2} \implies h \simeq \sigma^{1/2} \implies R \simeq \sigma^{-1/2} \to \infty$$

Open questions

- **Extremal Kerr**
  - What are the rates when we consider initial data that penetrates the horizon?
  - Can we find nonaxisymmetric conserved quantities on the horizon? If so, are they related to asymptotic symmetries?
  - Is there a chiral CFT analog of the instability?

Open questions

- **Non-extremal Kerr**
  - What happens non-linearly?
  - Are there observational consequences of the transient phase?
    - Synchrotron signatures in emission lines?
    - Turbulent gravitational waveforms?
    - ???