Title: Front End - Mathematica - 1

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Abstract:

Introduction to Mathematica

PSI Course 2016

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Basic Operations

In[1]:= `2 + 3`

Out[1]= 5

In[2]:= `Sqrt[4]`

Out[2]= 2

In[3]:= `Sqrt[2]`

Out[3]= $\sqrt{2}$

```
N[Sqrt[2]]
```

```
Pi
```

```
N[Pi]
```

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```
Out[4]= 1.41421

In[5]:= Pi

Out[5]= π

In[6]:= N[Pi]

Out[6]= 3.14159

In[7]:= N[Pi, 1000]

Out[7]= 3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534\
  211706798214808651328230664709384460955058223172535940812848111745028410270193852110555964462\
  294895493038196442881097566593344612847564823378678316527120190914564856692346034861045432664\
  821339360726024914127372458700660631558817488152092096282925409171536436789259036001133053054\
  882046652138414695194151160943305727036575959195309218611738193261179310511854807446237996274\
  956735188575272489122793818301194912983367336244065664308602139494639522473719070217986094370\
  277053921717629317675238467481846766940513200056812714526356082778577134275778960917363717872\
  146844090122495343014654958537105079227968925892354201995611212902196086403441815981362977477\
  130996051870721134999999837297804995105973173281609631859502445945534690830264252230825334468\
  503526193118817101000313783875288658753320838142061717766914730359825349042875546873115956286\
  388235378759375195778185778053217122680661300192787661119590921642019 9

10^100 + 10^101

x + 1

x = 5
```

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```
Out[5]= π

In[6]:= N[Pi]

Out[6]= 3.14159

In[7]:= N[Pi, 1000]

Out[7]= 3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534
211706798214808651328230664709384460955058223172535940812848111745028410270193852110555964462
294895493038196442881097566593344612847564823378678316527120190914564856692346034861045432664
821339360726024914127372458700660631558817488152092096282925409171536436789259036001133053054
882046652138414695194151160943305727036575959195309218611738193261179310511854807446237996274
956735188575272489122793818301194912983367336244065664308602139494639522473719070217986094370
277053921717629317675238467481846766940513200056812714526356082778577134275778960917363717872
146844090122495343014654958537105079227968925892354201995611212902196086403441815981362977477
130996051870721134999999837297804995105973173281609631859502445945534690830264252230825334468
503526193118817101000313783875288658753320838142061717766914730359825349042875546873115956286
3882353787593751957781857780532171226806613001927876611195909216420199

In[8]:= 10^100 + 10^101

Out[8]= 110 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000
000 000 000 000 000 000

x + 1

x = 5

x + 1
```

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```
000 000 000 000 000 000

In[9]:= x + 1

Out[9]= 1 + x

In[4]:= x = 5

Out[4]= 5

In[5]:= x + 1

Out[5]= 6

In[6]:= x =.

In[7]:= x

Out[7]= x

In[8]:= Prime[1 000 000]

Out[8]= 15 485 863

Solve[x^2 + 1 == 0, x]

Limit[1 / n, n → Infinity]

Sum[1 / 2^n, {n, 0, Infinity}]

? Sqrt

? Sum
```

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In[11]:= `Limit[1 / n, n → Infinity]`

Out[11]= 0

In[12]:= `Sum[1 / 2^n, {n, 0, Infinity}]`

Out[12]= 2

In[13]:= `? Sqrt`

Sqrt[z] or $\sqrt{z}$ gives the square root of z. >>

In[14]:= `? Sum`

Sum[f, {i, i_{max}}] evaluates the sum $\sum_{i=1}^{i_{max}} f$.

Sum[f, {i, i_{min}, i_{max}}] starts with $i = i_{min}$.
Sum[f, {i, i_{min}, i_{max}, di}] uses steps di.
Sum[f, {i, {i_1, i_2, ...}}] uses successive values i_1, i_2, ...

Sum[f, {i, i_{min}, i_{max}}, {j, j_{min}, j_{max}}, ...] evaluates the multiple sum $\sum_{i=i_{min}}^{i_{max}} \sum_{j=j_{min}}^{j_{max}} \ldots f$.

Sum[f, i] gives the indefinite sum $\sum_i f$. >>

Functions

paclet:ref/Sqrt 150%

Functions

```
In[15]:= f[x_] = Sin[x]
Out[15]= Sin[x]
```

```
f[1]

f[x]

f[q]

f[2 Pi]

a = 1

f[x_] = x + a

f[5]

a =.

f[5]

a = 1

g[x_] := x + a
```

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```
In[20]:= a = 1

Out[20]= 1

In[21]:= f[x_] = x + a

Out[21]= 1 + x

In[22]:= f[5]

Out[22]= 6

In[23]:= a =.

In[24]:= f[5]

Out[24]= 6

a = 1

g[x_] := x + a

g[5]

a =.

g[5]

1

1 &

1 &[3]
```

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```
In[21]:= f[x_] = x + a
Out[21]= 1 + x

In[22]:= f[5]
Out[22]= 6

In[23]:= a =.

In[24]:= f[5]
Out[24]= 6

In[25]:= a = 1
Out[25]= 1

In[26]:= g[x_] := x + a

In[27]:= g[5]
Out[27]= 6

In[28]:= a =.

In[29]:= g[5]
Out[29]= 5 + a

1
```

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```
In[26]:= g[x_] := x + a

In[27]:= g[5]

Out[27]= 6

In[28]:= a =.

In[29]:= g[5]

Out[29]= 5 + a

In[30]:= df := f'

In[31]:= f = Sin

Out[31]= Sin

In[32]:= df

Out[32]= Cos[#1] &

In[33]:= f = Sqrt

Out[33]= Sqrt

In[34]:= df
```

Out[34]= $\frac{1}{2\sqrt{\#1}}$ &

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Out[31]= Sin

```
In[32]:= df

Out[32]= Cos[#1] &

In[33]:= f = Sqrt

Out[33]= Sqrt

In[34]:= df
```

Out[34]= $\frac{1}{2\sqrt{\#1}}$ &

```
In[35]:= 1

Out[35]= 1

In[36]:= 1 &

Out[36]= 1 &

1 &[3]

f[x_] = 2 x

2 # &

2 # &[r]

f[r]
```

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```
Out[33]= Sqrt

In[34]:= df

Out[34]= 1/(2 Sqrt[#1]) &

In[35]:= 1

Out[35]= 1

In[36]:= 1 &

Out[36]= 1 &

In[37]:= 1 &[3]

Out[37]= 1

In[38]:= f[x_] = 2 x

Set::write : Tag Sqrt in √x_ is Protected. >>

Out[38]= 2 x

2 # &

2 # &[r]

f[r]

Array[2 &, {4, 4}] // MatrixForm
```

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```
Out[35]= 1

In[36]:= 1 &

Out[36]= 1 &

In[37]:= 1 &[3]

Out[37]= 1

In[39]:= f =.

In[40]:= f[x_] = 2 x

Out[40]= 2 x

In[41]:= 2 # &

Out[41]= 2 #1 &

In[42]:= 2 # &[r]

Out[42]= 2 r

In[43]:= f[r]

Out[43]= 2 r

Array[2 &, {4, 4}] // MatrixForm
```

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```
Out[35]= 1

In[36]:= 1 &

Out[36]= 1 &

In[37]:= 1 &[3]

Out[37]= 1

In[39]:= f =.

In[40]:= f[x_] = 2 x

Out[40]= 2 x

In[41]:= 2 # &

Out[41]= 2 #1 &

In[42]:= 2 # &[r]

Out[42]= 2 r

In[43]:= f[r]

Out[43]= 2 r

Array[2 &, {4, 4}] // MatrixForm
```

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```
Out[36]= 1 &

In[37]:= 1 &[3]

Out[37]= 1

In[39]:= f =.

In[40]:= f[x_] = 2 x

Out[40]= 2 x

In[41]:= 2 # &

Out[41]= 2 #1 &

In[42]:= 2 # &[r]

Out[42]= 2 r

In[43]:= f[r]

Out[43]= 2 r

In[44]:= w[n_] = D[x^n, x]
```

Out[44]= $n\, x^{-1+n}$

```
Array[2 &, {4, 4}] // MatrixForm
```

Internal Representation

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```
In[39]:= f =.

In[40]:= f[x_] = 2 x

Out[40]= 2 x

In[41]:= 2 # &

Out[41]= 2 #1 &

In[42]:= 2 # &[r]

Out[42]= 2 r

In[43]:= f[r]

Out[43]= 2 r

In[59]:= w[n_] = NSolve[x^n == 2, x]

NSolve::ifun :
  Inverse functions are being used by NSolve, so some solutions may not be found; use Reduce for complete solution information. >>
```

Out[59]= $\left\{\left\{x \to 1. \times 2.^{\frac{1}{n}}\right\}\right\}$

```
In[55]:= w[4]
```

Out[55]= $4\ x^3$

```
Array[2 &, {4, 4}] // MatrixForm
```

150%

In[40]:= `f[x_] = 2 x`

Out[40]= `2 x`

In[41]:= `2 # &`

Out[41]= `2 #1 &`

In[42]:= `2 # &[r]`

Out[42]= `2 r`

In[43]:= `f[r]`

Out[43]= `2 r`

In[60]:= `Array[2 &, {4, 4}] // MatrixForm`

Out[60]//MatrixForm=

$$\begin{pmatrix} 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \end{pmatrix}$$

Internal Representation

```
a + 2 b + 3 c // TreeForm
```

```
a + 2 b + 3 c // FullForm
```

150%

```
Out[41]= 2 #1 &

In[42]:= 2 # &[r]

Out[42]= 2 r

In[43]:= f[r]

Out[43]= 2 r

In[60]:= Array[2 &, {4, 4}] // MatrixForm

Out[60]//MatrixForm=
```

$$\begin{pmatrix} 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \end{pmatrix}$$

```
In[61]:= Sin'

Out[61]= Cos[#1] &

In[62]:= D[Sin[x], x]

Out[62]= Cos[x]
```

Internal Representation

```
a + 2 b + 3 c // TreeForm
```

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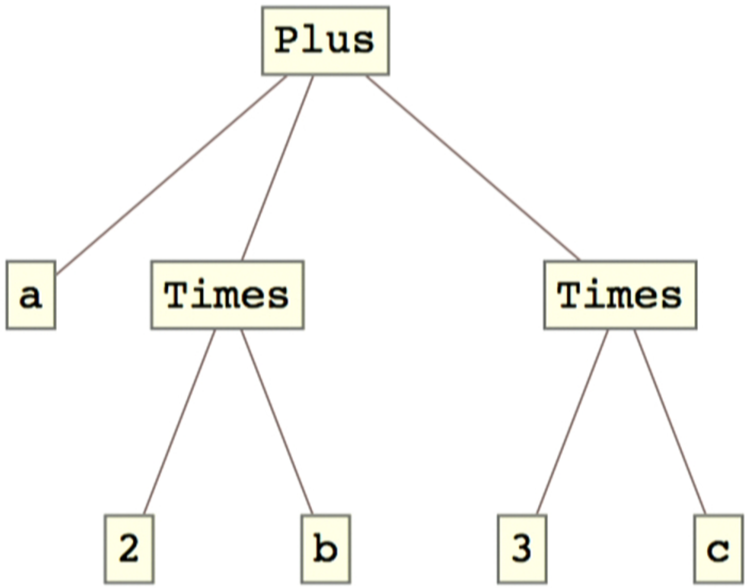
Out[65]= Function[x, Cos[x]]

Internal Representation

In[70]:= a + 2 b + 3 c // TreeForm

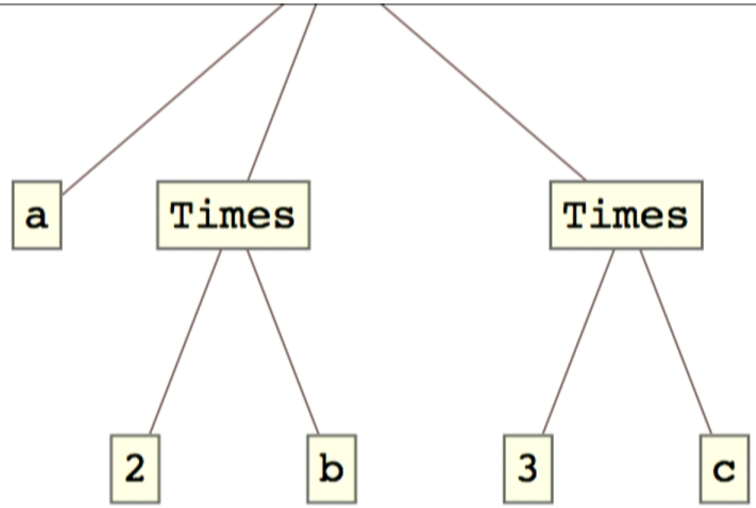
Out[70]//TreeForm=



a + 2 b + 3 c // FullForm

{1, 2, 3}

150%



In[71]:= a + 2 b + 3 c // FullForm

Out[71]//FullForm=

```
Plus[a, Times[2, b], Times[3, c]]
```

```
{1, 2, 3}

{1, 2, 3} + {a, b, c}

2 * {1, 2, 3}

Sqrt[{1, 2, 3}]

Map[(2 # + 1) &, {1, 2, 3}]

(2 # + 1) &[{1, 2, 3}]
```

150%

```
In[71]:= a + 2 b + 3 c // FullForm

Out[71]//FullForm=
         Plus[a, Times[2, b], Times[3, c]]

In[72]:= {1, 2, 3}

Out[72]= {1, 2, 3}

In[73]:= {1, 2, 3} + {a, b, c}

Out[73]= {1 + a, 2 + b, 3 + c}

In[74]:= 2 * {1, 2, 3}

Out[74]= {2, 4, 6}

In[75]:= Sqrt[{1, 2, 3}]
```

Out[75]= $\{1,\ \sqrt{2},\ \sqrt{3}\}$

```
Map[(2 # + 1) &, {1, 2, 3}]

(2 # + 1) &[{1, 2, 3}]

Table[2 i + 1, {i, 0, 3}]

Table[1 / (i + j + 1), {i, 0, 3}, {j, 0, 3}] // MatrixForm

ff[{1, 2}]
```

150%

In[74]:= 2 * {1, 2, 3}

Out[74]= {2, 4, 6}

In[75]:= Sqrt[{1, 2, 3}]

Out[75]= $\{1, \sqrt{2}, \sqrt{3}\}$

In[76]:= Map[(2 # + 1) &, {1, 2, 3}]

Out[76]= {3, 5, 7}

In[77]:= (2 # + 1) &[{1, 2, 3}]

Out[77]= {3, 5, 7}

In[78]:= Table[2 i + 1, {i, 0, 3}]

Out[78]= {1, 3, 5, 7}

```
Table[1 / (i + j + 1), {i, 0, 3}, {j, 0, 3}] // MatrixForm

ff[{1, 2}]

ff /@ {1, 2}

f =.

{{1, 2}, {3, 4}} // MatrixForm
```

150%

```
In[77]:= (2 # + 1) &[{1, 2, 3}]
Out[77]= {3, 5, 7}

In[78]:= Table[2 i + 1, {i, 0, 3}]
Out[78]= {1, 3, 5, 7}

In[79]:= Table[1 / (i + j + 1), {i, 0, 3}, {j, 0, 3}] // MatrixForm
Out[79]//MatrixForm=
```

$$\begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \end{pmatrix}$$

```
ff[{1, 2}]

ff /@ {1, 2}

f =.

{{1, 2}, {3, 4}} // MatrixForm

Flatten[{{1, 2}, {3, 4}}] // MatrixForm

v = {1, 2}

m = {{1, 2}, {3, 4}}
```

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```
In[87]:= Clear[f]

In[88]:= DSolve[f''[x] == x, f[x], x]
```

$$\text{Out[88]=}\ \left\{\left\{f[x] \to \frac{x^3}{6} + C[1] + x\,C[2]\right\}\right\}$$

```
In[90]:= DSolve[f''[x] == x, f, x]
```

$$\text{Out[90]=}\ \left\{\left\{f \to \text{Function}\left[\{x\}, \frac{x^3}{6} + C[1] + x\,C[2]\right]\right\}\right\}$$

```
In[91]:= f[1] = 1

Out[91]= 1

In[92]:= f[2] = 3

Out[92]= 3

In[93]:= f[3]

Out[93]= f[3]

In[94]:= f[2]

Out[94]= 3

Clear[f]

DSolve[{f'[x] == x, f[0] == 42, f[1] == 42}, f[x], x]
```

150%

```
Out[92]= 3

In[93]:= f[3]

Out[93]= f[3]

In[94]:= f[2]

Out[94]= 3

In[95]:= Clear[f]

DSolve[{f''[x] == x, f[0] == 42, f[1] == 42}, f[x], x]
```

Out[98]= $\left\{\left\{f[x] \to \frac{1}{6}\left(252 - x + x^3\right)\right\}\right\}$

```
In[99]:= (x + 1) (x - 1)
```

Out[99]= $(x-1)(x+1)$

```
(x + 1) (x - 1) // Simplify

x^2 - 1 // Factor

(x + 1) (x + 2) (x + 3) // Expand

Collect[(x + y) (x + y + 3), y]

expr = Normal[Series[1 / x, {x, 1, 4}]]

expr /. {x → epsilon + 1}
```

150%

```
Out[93]= f[3]

In[94]:= f[2]

Out[94]= 3

In[115]:= Clear[f]

In[116]:= sol = DSolve[{f''[x] == x, f[0] == 42, f[1] == 42}, f[x], x]
```

Out[116]= $\left\{\left\{f[x] \to \frac{1}{6}\left(252 - x + x^3\right)\right\}\right\}$

```
In[121]:= 2 x + 1 /. {1 → 15}

Out[121]= 15 + 2 x

f[x] /. sol[[1, 1]]
```

Out[117]= $f[x] \to \frac{1}{6}\left(252 - x + x^3\right)$

```
In[99]:= (x + 1) (x - 1)

Out[99]= (x - 1) (x + 1)

(x + 1) (x - 1) // Simplify

x^2 - 1 // Factor

(x + 1) (x + 2) (x + 3) // Expand
```

150%

```
In[123]:= f[x] /. sol[[1, 1]]

Out[123]= 1/6 (252 - x + x^3)

In[125]:= fsol[x_] = f[x] /. sol[[1, 1]]

Out[125]= 1/6 (252 - x + x^3)

In[126]:= (x + 1) (x - 1)

Out[126]= (-1 + x) (1 + x)

In[127]:= (x + 1) (x - 1) // Simplify

Out[127]= -1 + x^2

x^2 - 1 // Factor

(x + 1) (x + 2) (x + 3) // Expand

Collect[(x + y) (x + y + 3), y]

expr = Normal[Series[1 / x, {x, 1, 4}]]

expr /. {x → epsilon + 1}

Series[1 / (epsilon + 1), {epsilon, 0, 4}]

f[x_Integer] = x^2
```

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```
Out[129]= 6 + 11 x + 6 x^2 + x^3

In[130]:= Collect[(x + y) (x + y + 3), y]

Out[130]= 3 x + x^2 + (3 + 2 x) y + y^2

In[131]:= expr = Normal[Series[1 / x, {x, 1, 4}]]

Out[131]= 2 + (-1 + x)^2 - (-1 + x)^3 + (-1 + x)^4 - x

In[132]:= expr /. {x → epsilon + 1}

Out[132]= 1 - epsilon + epsilon^2 - epsilon^3 + epsilon^4

In[133]:= Series[1 / (epsilon + 1), {epsilon, 0, 4}]

Out[133]= 1 - epsilon + epsilon^2 - epsilon^3 + epsilon^4 + O[epsilon]^5

f[x_Integer] = x^2

f[2]

f[2.1]

2 /. (x_ → x^2)

expr

expr /. (x - 1 → epsilon)

Series[expr, {x, 1, 2}]
```

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```
ser = Series[1 / x, {x, 1, 4}]

ser // TreeForm

SeriesCoefficient[ser, 2]

√4

? Sqrt

Factor[x ^ 2 + 1, Extension → I]

? Factor
```

Graphics

```
Plot[Sin[x], {x, 0, 2 Pi}]

Plot[Sin[x], {x, 0, 2 Pi}, Frame → True, FrameStyle → Black, Background → RGBColor[0.8, 0.9, 1]]

ContourPlot[Sin[x] Cos[y], {x, 0, Pi}, {y, 0, 4 Pi}]

(x ^ 2 + x) /. x → y

(x ^ 2 + x) /. x_ → y

f[x_] = y

f[x ^ 2 + x]
```

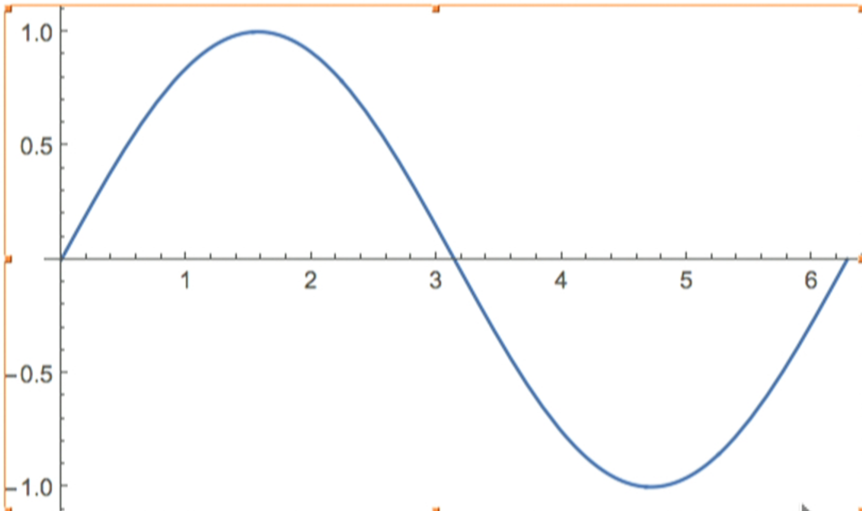
150%

```
? Factor
```

Graphics

In[145]:=
```
Plot[Sin[x], {x, 0, 2 Pi}]
```

Out[145]=




```
Plot[Sin[x], {x, 0, 2 Pi}, Frame → True, FrameStyle → Black, Background → RGBColor[0.8, 0.9, 1]]

ContourPlot[Sin[x] Cos[y], {x, 0, Pi}, {y, 0, 4 Pi}]

(x^2 + x) /. x → y

(x^2 + x) /. x_ → y
```

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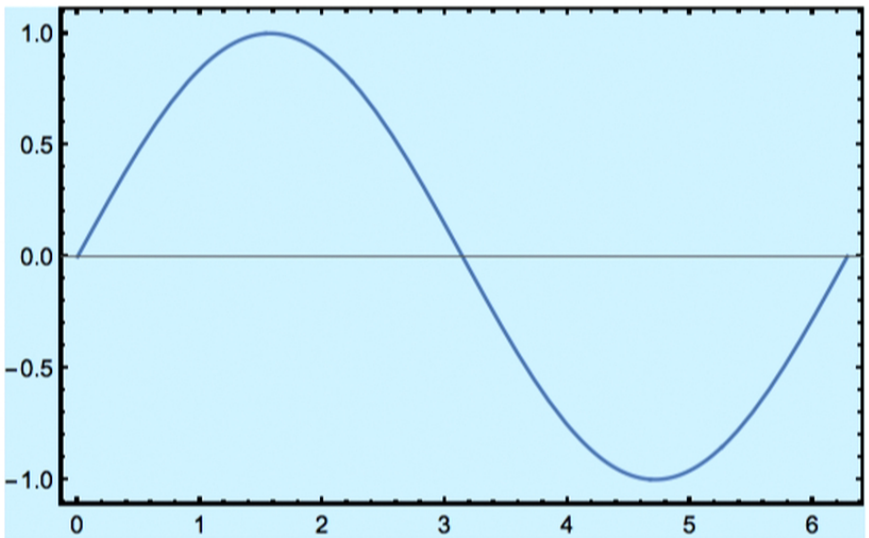
In[146]:=
```
Plot[Sin[x], {x, 0, 2 Pi}, Frame → True, FrameStyle → Black, Background → RGBColor[0.8, 0.9, 1]]
```

Out[146]=




```
ContourPlot[Sin[x] Cos[y], {x, 0, Pi}, {y, 0, 4 Pi}]

(x^2 + x) /. x → y

(x^2 + x) /. x_ → y

f[x_] = y
```

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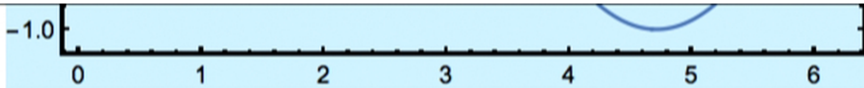
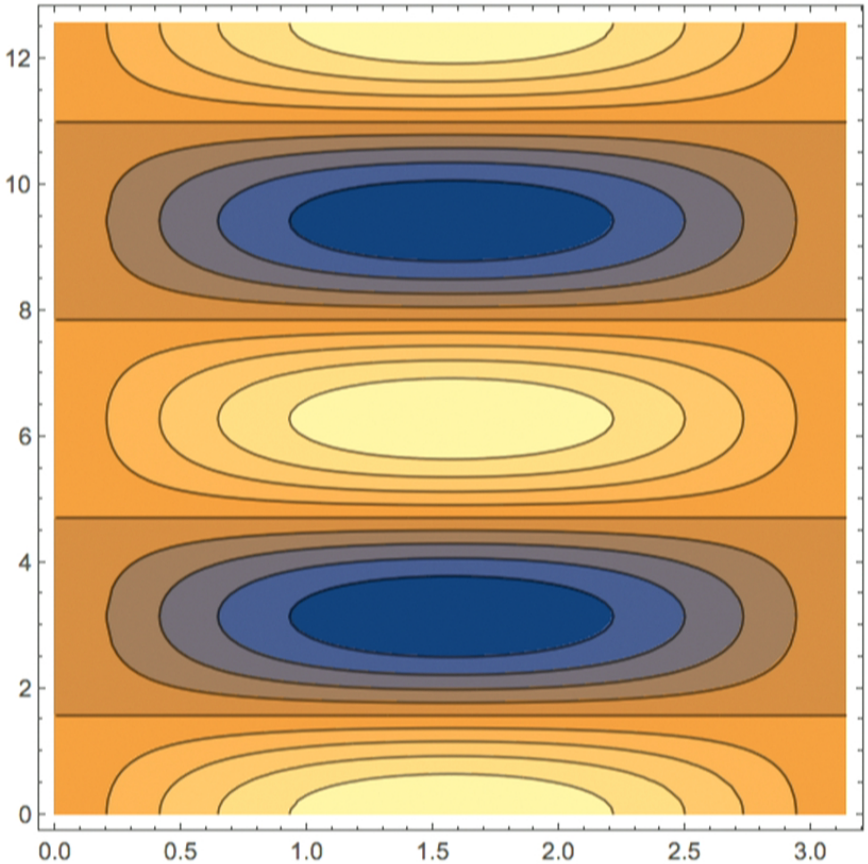


In[147]:= `ContourPlot[Sin[x] Cos[y], {x, 0, Pi}, {y, 0, 4 Pi}]`

Out[147]=



`(x^2 + x) /. x → y`

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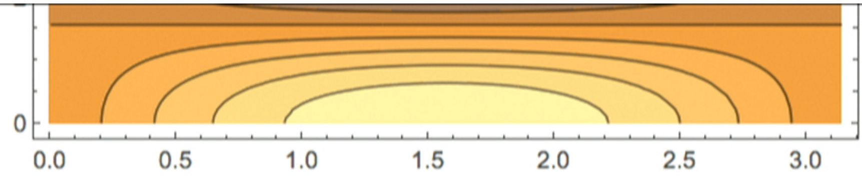

Factorial

```
In[148]:= fact[0] = 1

Out[148]= 1

In[149]:= fact[n_] := n fact[n - 1]

In[150]:= fact[100]

Out[150]= 93 326 215 443 944 152 681 699 238 856 266 700 490 715 968 264 381 621 468 592 963 895 217 599 993 229 915 608 
  941 463 976 156 518 286 253 697 920 827 223 758 251 185 210 916 864 000 000 000 000 000 000 000 000

fib[0] = 0

fib[1] = 1

fib[n_] := fib[n] = fib[n - 1] + fib[n - 2]

fib[10]

fib[1000]
```

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```
In[148]:= fact[0] = 1

Out[148]= 1

In[149]:= fact[n_] := n fact[n - 1]

In[150]:= fact[100]

Out[150]= 93 326 215 443 944 152 681 699 238 856 266 700 490 715 968 264 381 621 468 592 963 895 217 599 993 229 915 608 \
  941 463 976 156 518 286 253 697 920 827 223 758 251 185 210 916 864 000 000 000 000 000 000 000 000

In[151]:= fib[0] = 0

Out[151]= 0

In[152]:= fib[1] = 1

Out[152]= 1

In[153]:= fib[n_] := (*fib[n]=*) fib[n - 1] + fib[n - 2]

In[154]:= fib[10]

Out[154]= 55

In[155]:= fib[1000]
```

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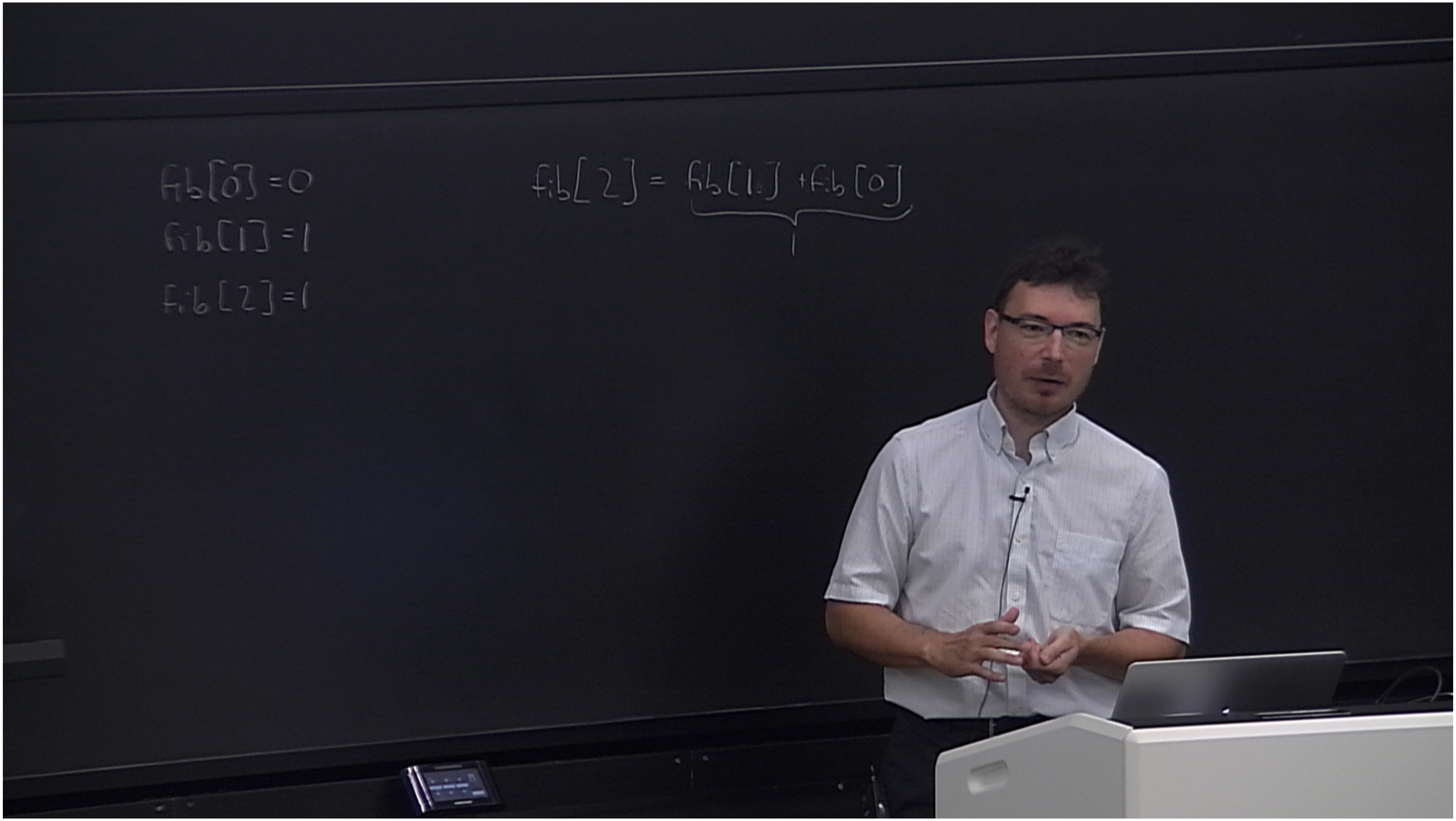
Out[148]= 1

```
In[149]:= fact[n_] := n fact[n - 1]

In[150]:= fact[100]

Out[150]= 93 326 215 443 944 152 681 699 238 856 266 700 490 715 968 264 381 621 468 592 963 895 217 599 993 229 915 608 \
  941 463 976 156 518 286 253 697 920 827 223 758 251 185 210 916 864 000 000 000 000 000 000 000 000

In[151]:= fib[0] = 0

Out[151]= 0

In[152]:= fib[1] = 1

Out[152]= 1

In[157]:= fib[n_] := fib[n] = fib[n - 1] + fib[n - 2]

In[158]:= fib[10]

Out[158]= 55

In[159]:= fib[1000]

Out[159]= 43 466 557 686 937 456 435 688 527 675 040 625 802 564 660 517 371 780 402 481 729 089 536 555 417 949 051 890 \
  403 879 840 079 255 169 295 922 593 080 322 634 775 209 689 623 239 873 322 471 161 642 996 440 906 533 187 938 \
  298 969 649 928 516 003 704 476 137 795 166 849 228 875
```

150%


fib[0] = 0
0
fib[1] = 1
1
fib[n_] := fib[n] = fib[n - 1] + fib[n - 2]
fib[10]
55
fib[1000]


fib[0] = 0
fib[1] = 1
fib[2] = 1
fib[2] = fib[1] + fib[0]

```
In[151]:= fib[0] = 0

Out[151]= 0

In[152]:= fib[1] = 1

Out[152]= 1

In[157]:= fib[n_] := fib[n] = fib[n - 1] + fib[n - 2]

In[158]:= fib[10]

Out[158]= 55

In[159]:= fib[1000]

Out[159]= 43 466 557 686 937 456 435 688 527 675 040 625 802 564 660 517 371 780 402 481 729 089 536 555 417 949 051 890 \
  403 879 840 079 255 169 295 922 593 080 322 634 775 209 689 623 239 873 322 471 161 642 996 440 906 533 187 938 \
  298 969 649 928 516 003 704 476 137 795 166 849 228 875

In[163]:= fib[3000]

Out[163]= 410 615 886 307 971 260 333 568 378 719 267 105 220 125 108 637 369 252 408 885 430 926 905 584 274 113 403 731 \
  330 491 660 850 044 560 830 036 835 706 942 274 588 569 362 145 476 502 674 373 045 446 852 160 486 606 292 497 \
  360 503 469 773 453 733 196 887 405 847 255 290 082 049 086 907 512 622 059 054 542 195 889 758 031 109 222 670 \
  849 274 793 859 539 133 318 371 244 795 543 147 611 073 276 240 066 737 934 085 191 731 810 993 201 706 776 838 \
  934 766 764 778 739 502 174 470 268 627 820 918 553 842 225 858 306 408 301 661 862 900 358 266 857 238 210 235 \
  802 504 351 951 472 997 919 676 524 004 784 236 376 453 347 268 364 152 648 346 245 840 573 214 241 419 937 917 \
  242 918 602 639 810 097 866 942 392 015 404 620 153 818 671 425 739 835 074 851 396 421 139 982 713 640 679 581 \
  178 458 198 658 692 285 968 043 243 656 709 796 000
```

150%