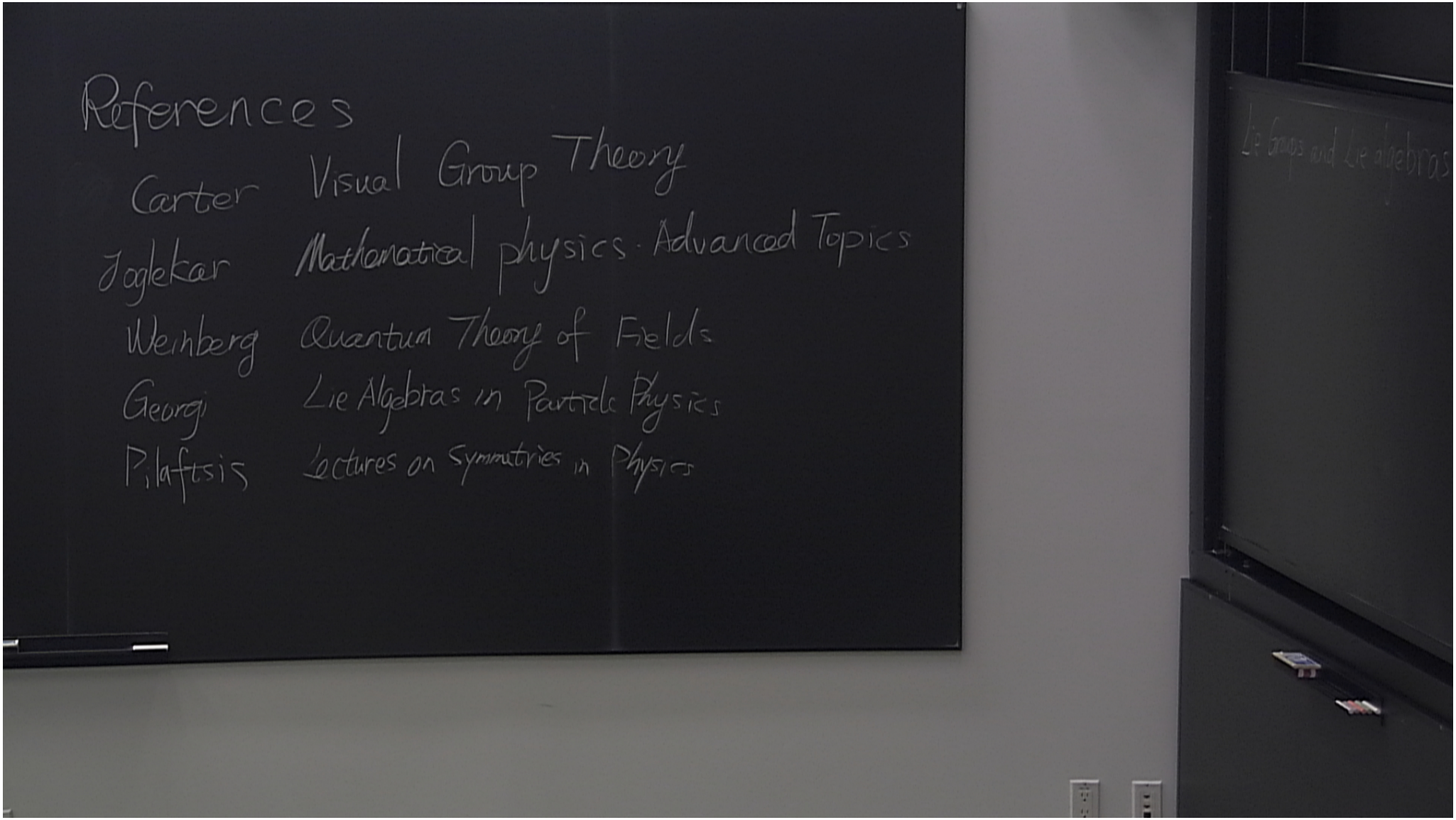
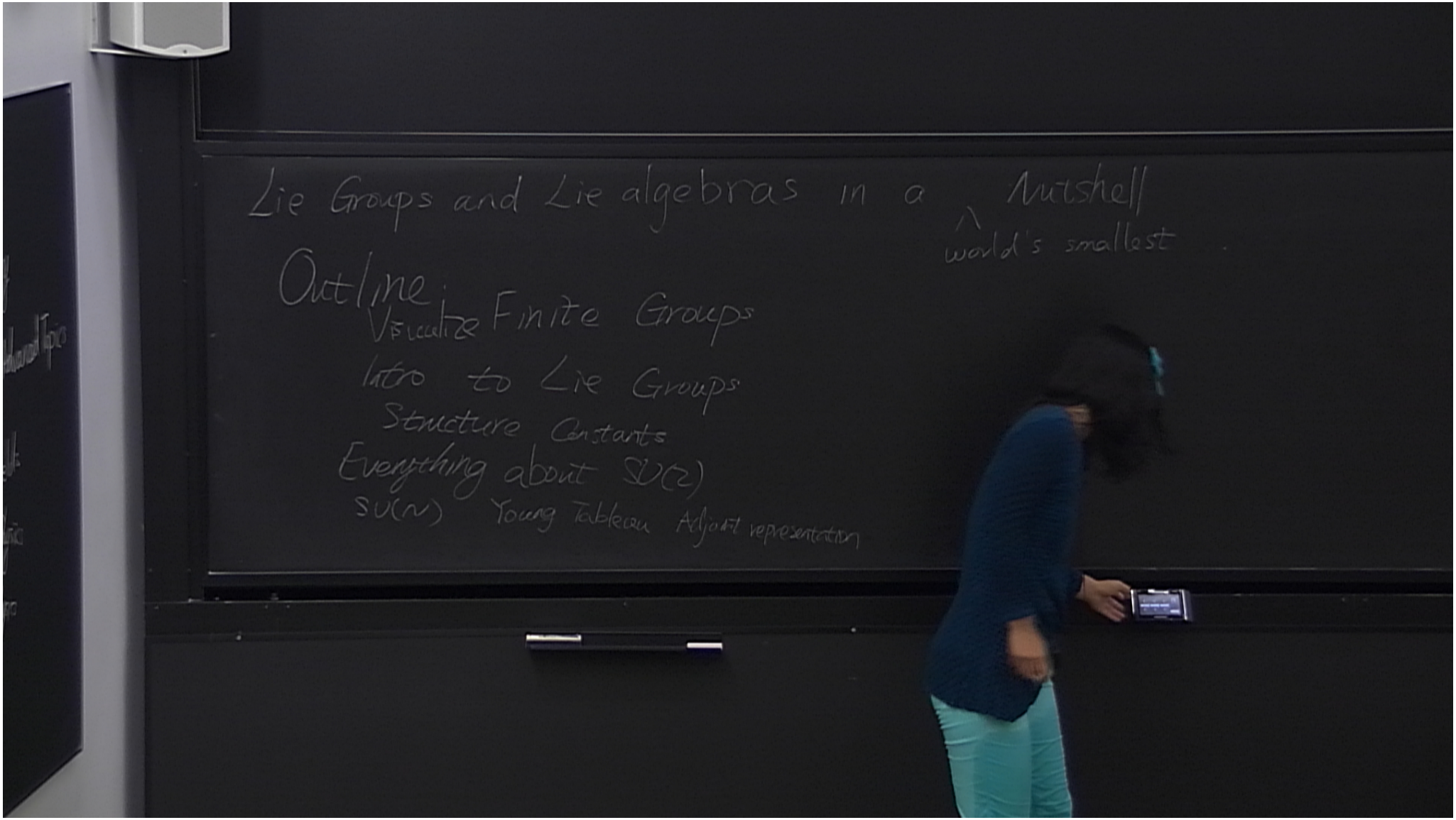
Title: Front End - Lie Groups and Lie Algebras - 1

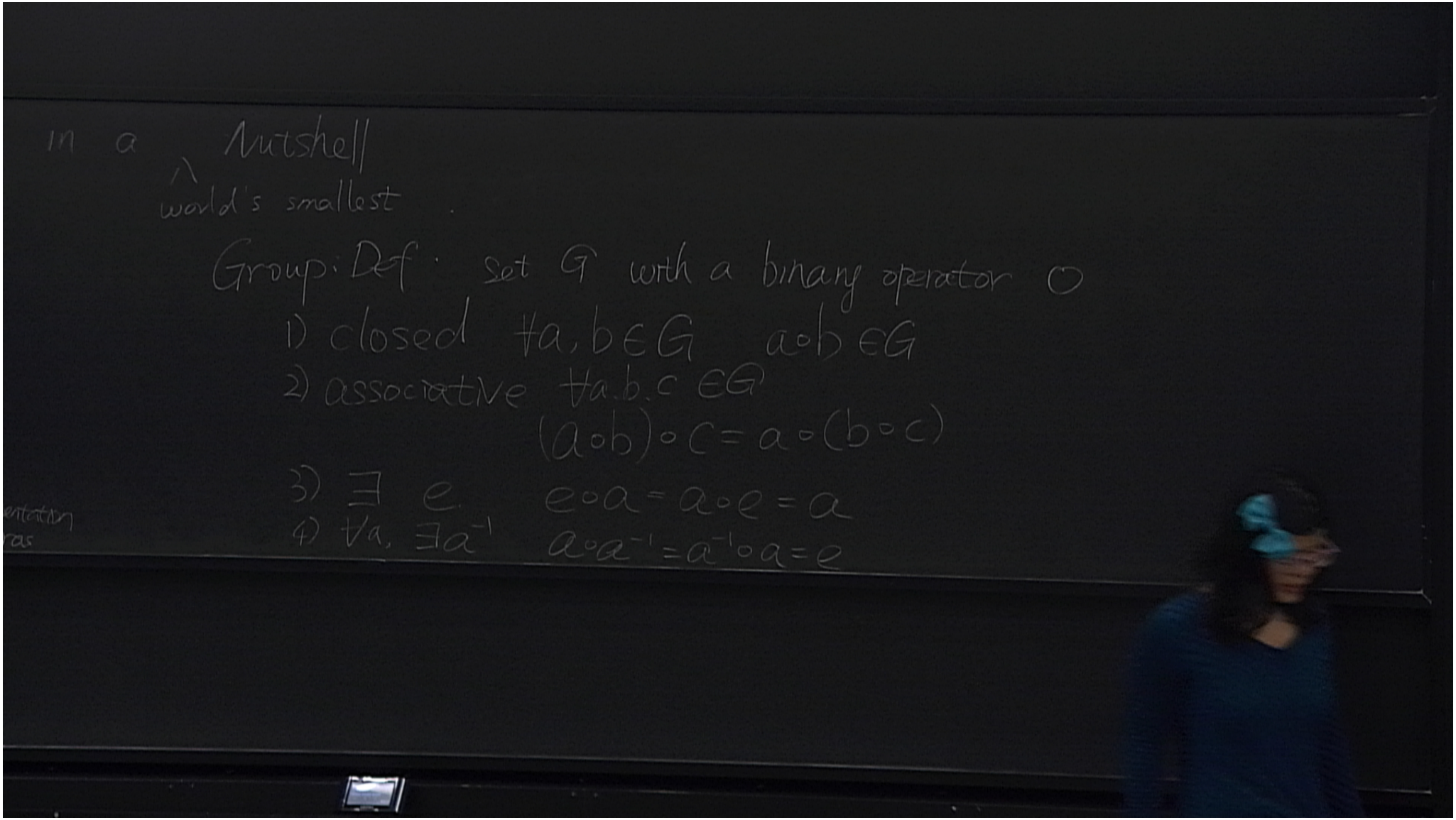
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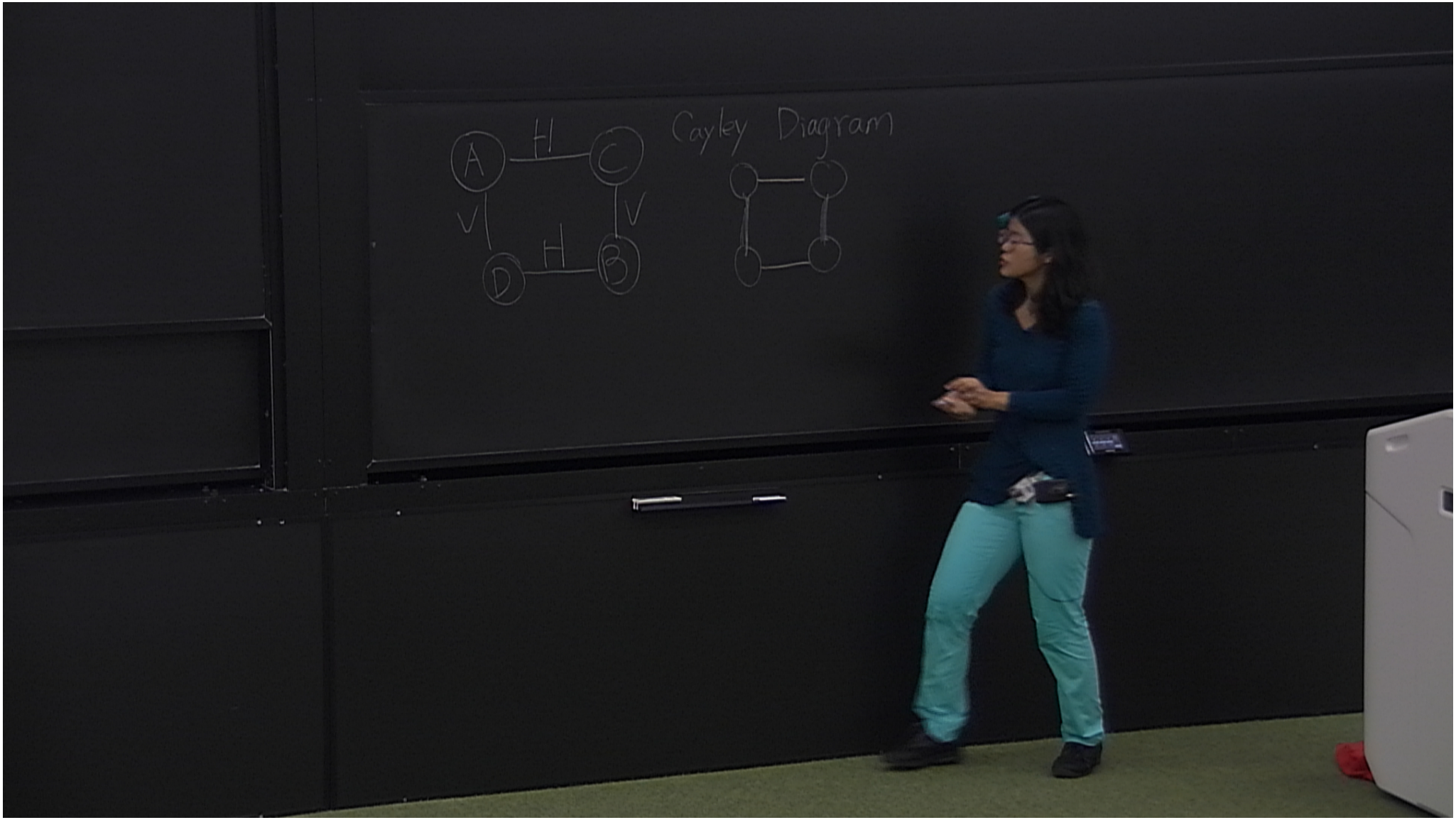
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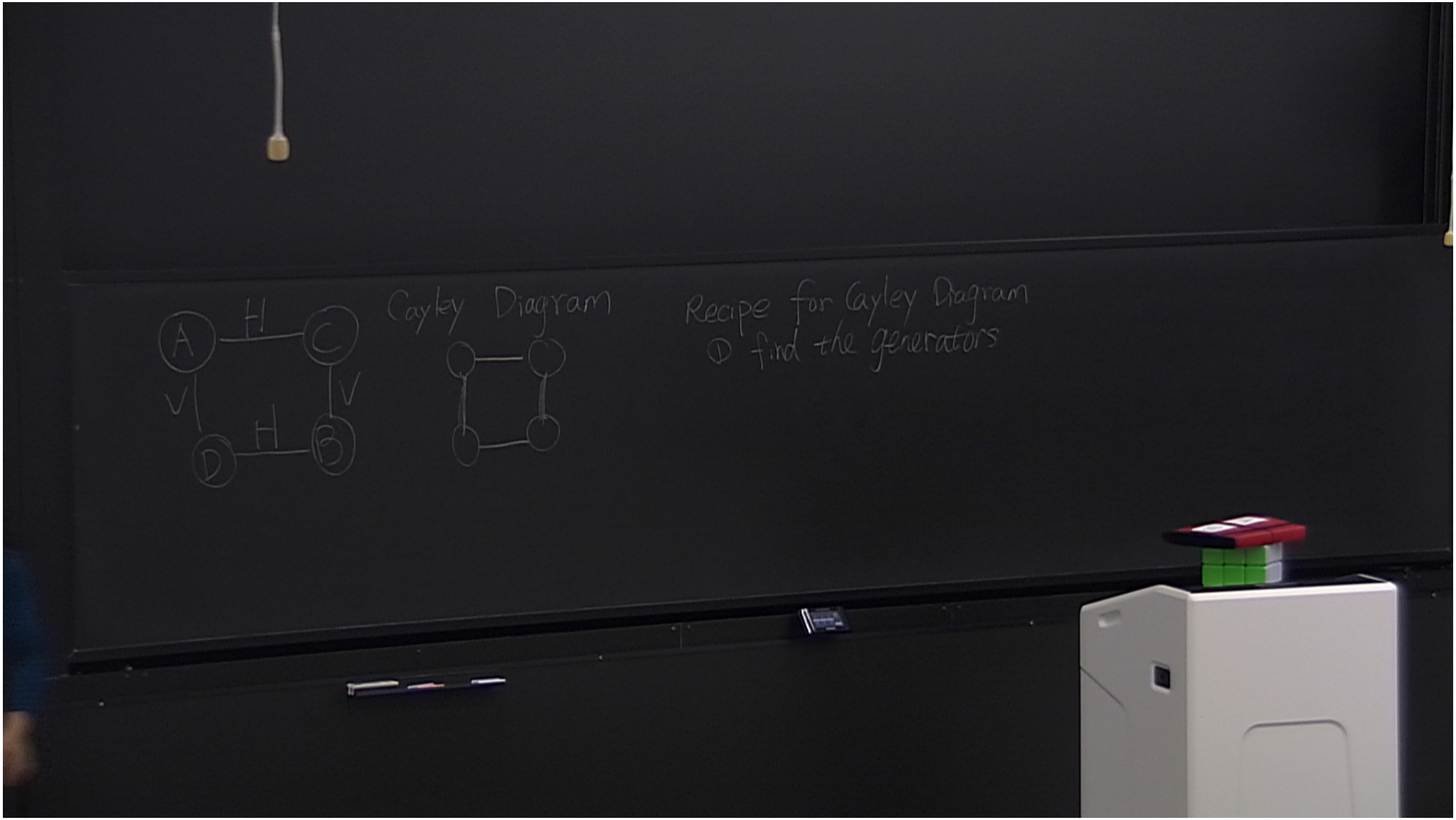
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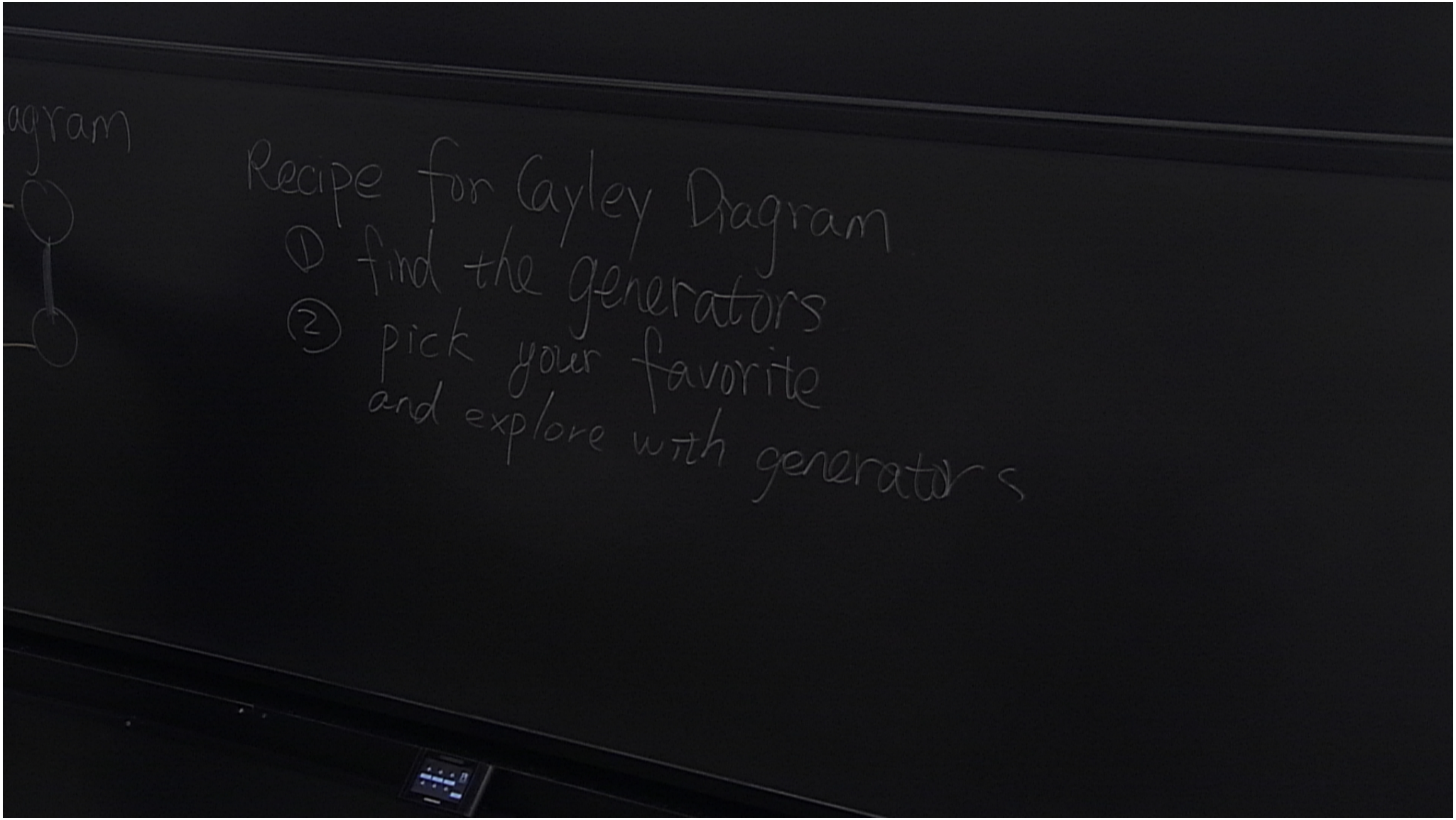

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Lie Groups and Lie algebras

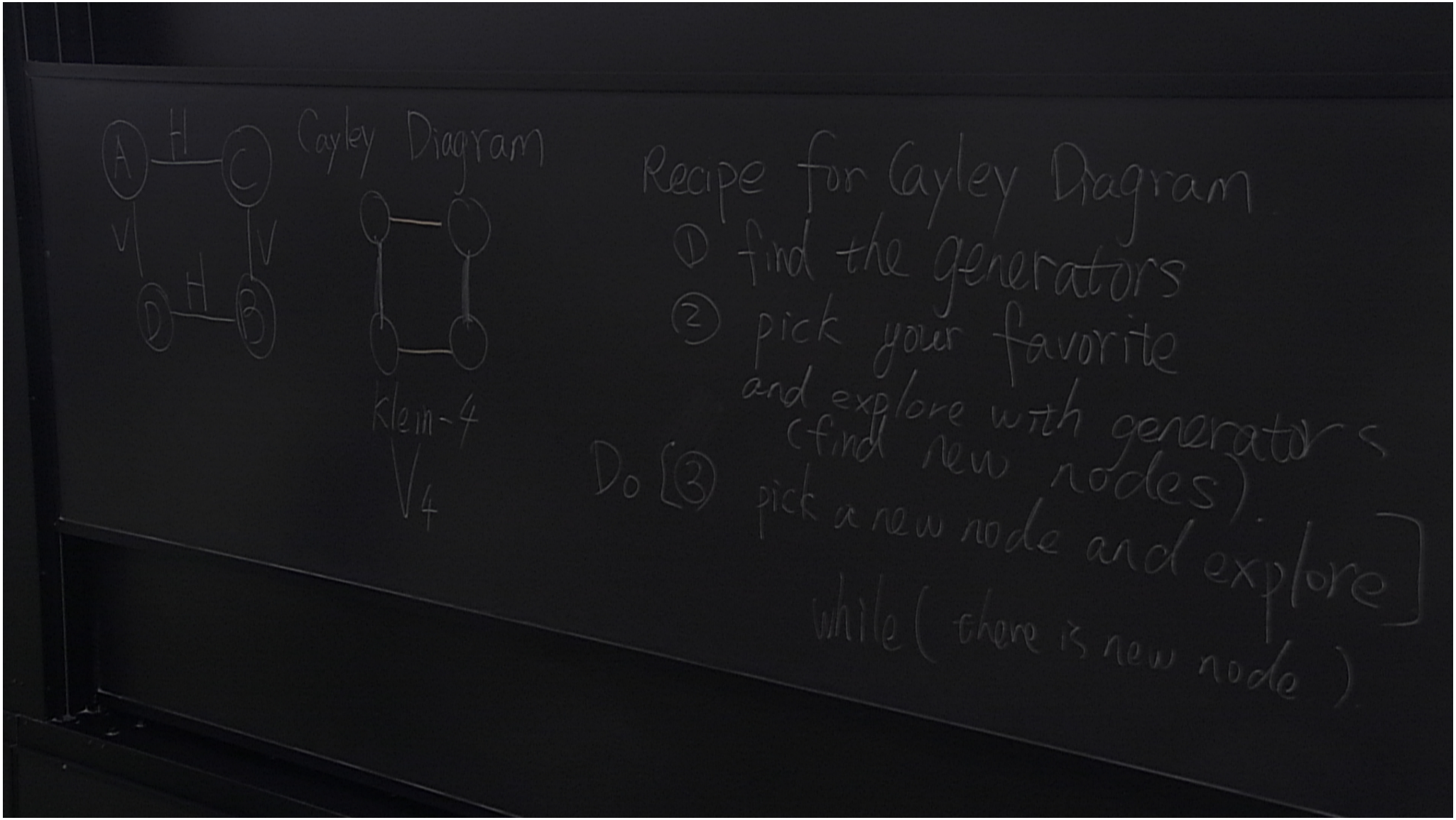

Lie Groups and Lie algebras in a Nutshell
world's smallest
Outline:
Visualize Finite Groups
Intro to Lie Groups
Structure Constants
Everything about SU(2)
SU(N) Young Tableau Adjoint representation


in a Nutshell
world's smallest
Group: Def: set G with a binary operator ∘
1) closed ∀a, b ∈ G a∘b ∈ G
2) associative ∀a.b.c ∈ G
(a∘b)∘c = a∘(b∘c)
3) ∃ e e∘a = a∘e = a
4) ∀a, ∃a⁻¹ a∘a⁻¹ = a⁻¹∘a = e

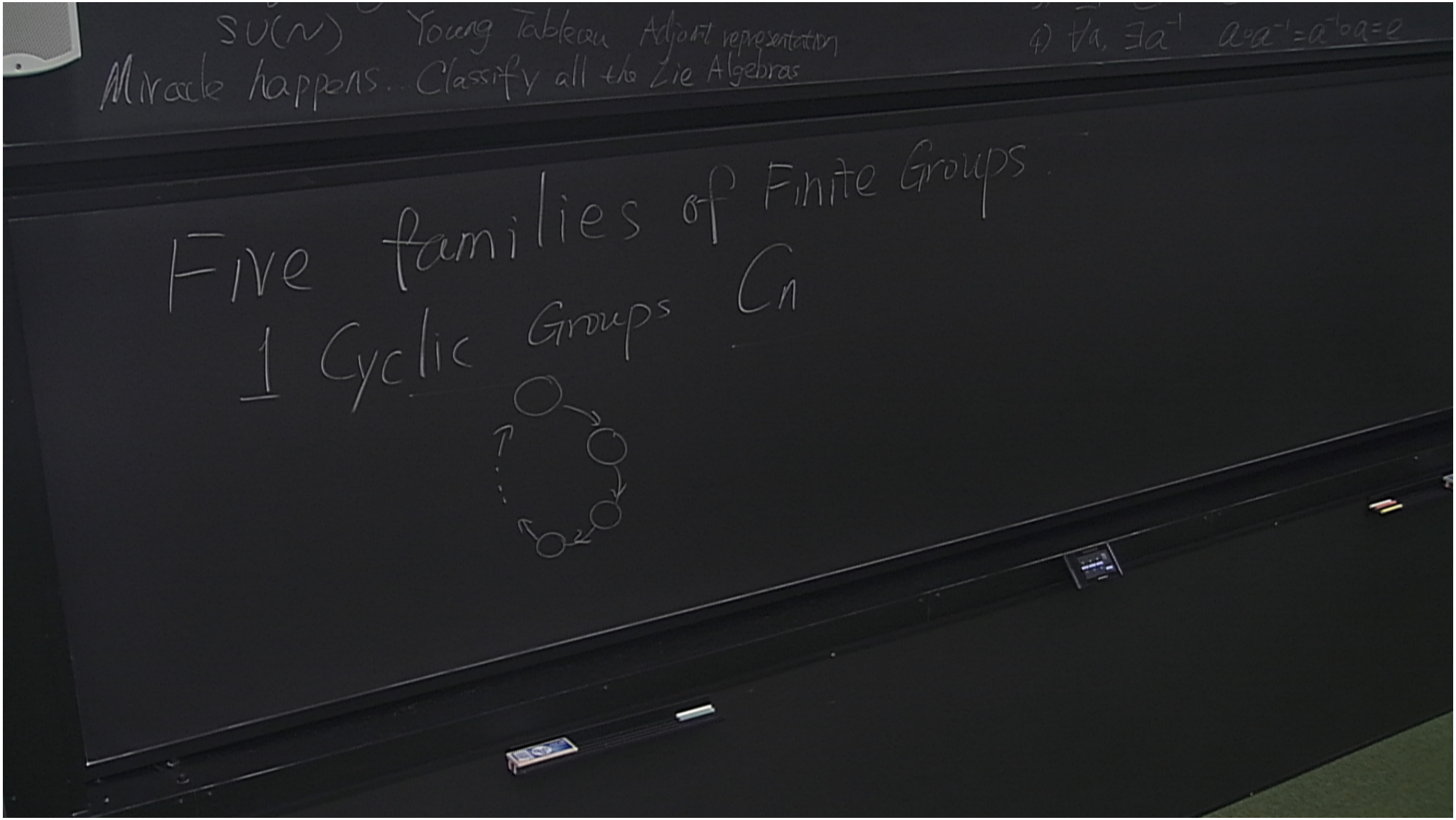

A
H
C
V
V
H
D
B
Cayley Diagram

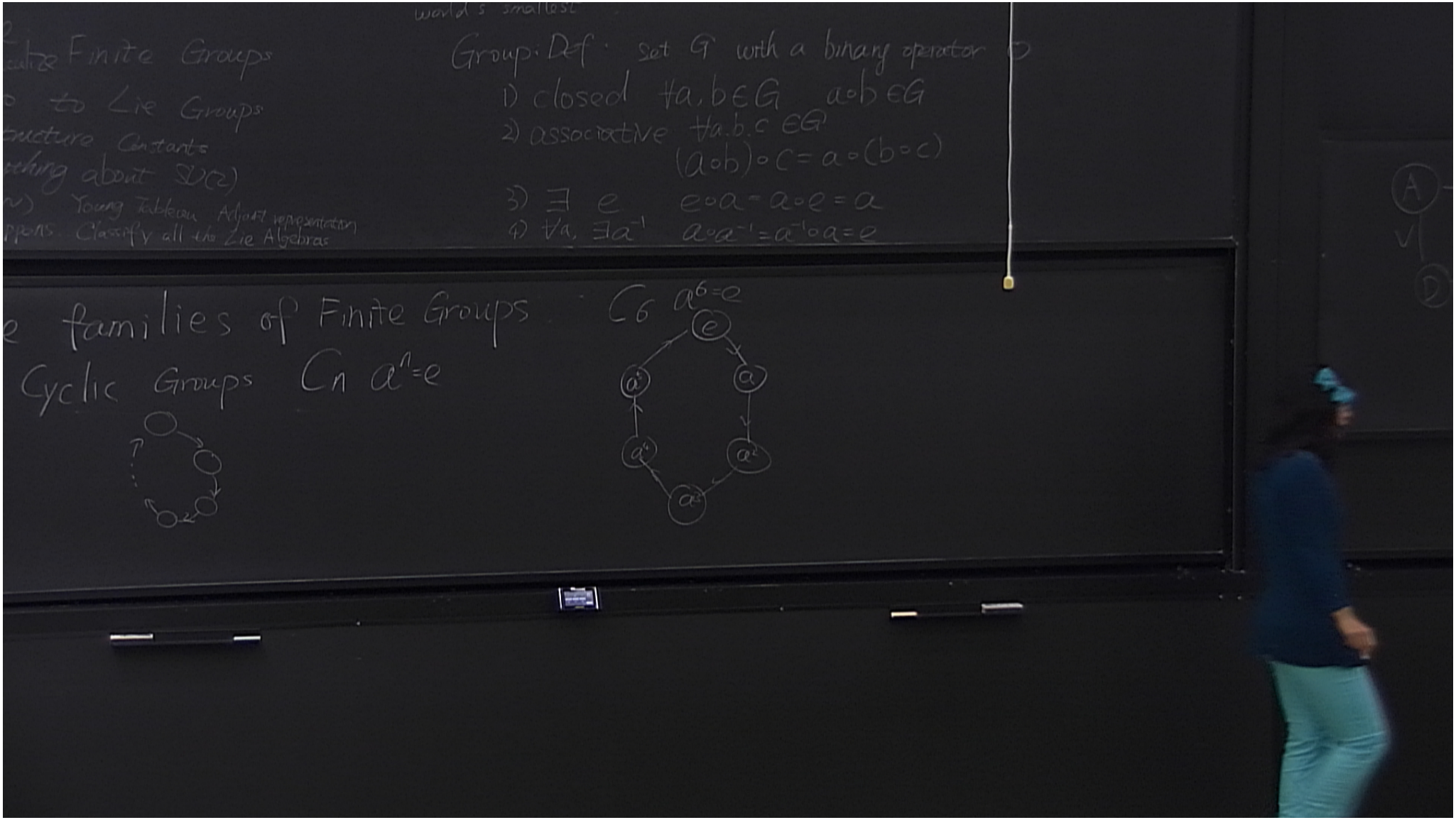

A
H
C
V
V
H
D
B
Cayley Diagram
Recipe for Cayley Diagram
① find the generators


agram
Recipe for Cayley Diagram
① find the generators
② pick your favorite
and explore with generators


A
H
C
V
D
H
B
Cayley Diagram
Klein-4
V_4
Recipe for Cayley Diagram
① find the generators
② pick your favorite
and explore with generators
(find new nodes)
Do [③ pick a new node and explore]
while (there is new node)

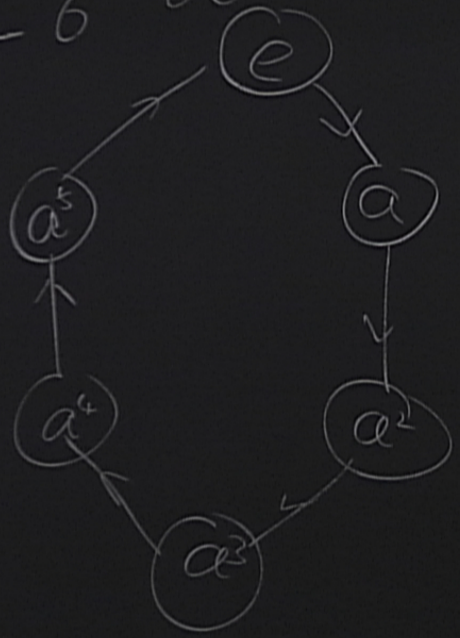

erators
es)
explore]
node)

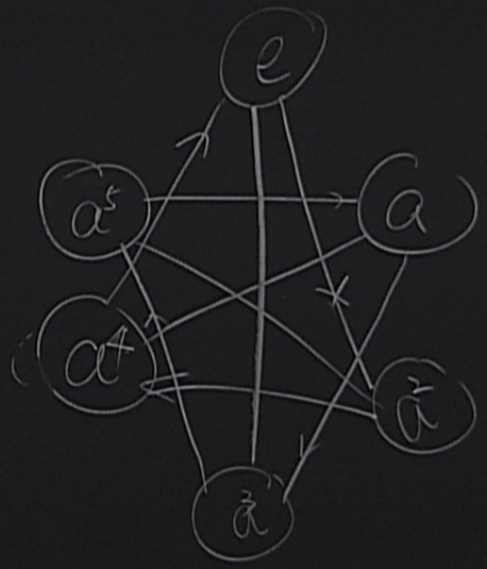

SU(N) Young Tableau Adjoint representation
Miracle happens... Classify all the Lie Algebras
4) ∀a, ∃a⁻¹ a∘a⁻¹=a⁻¹∘a=e
Five families of Finite Groups
1 Cyclic Groups Cn


Finite Groups
to Lie Groups
structure Constants
thing about SU(2)
Young Tableau Adjoint representation
ppns. Classify all the Lie Algebras
Group. Def. set G with a binary operator
1) closed ∀a,b∈G a∘b∈G
2) associative ∀a,b,c∈G
(a∘b)∘c = a∘(b∘c)
3) ∃ e e∘a = a∘e = a
4) ∀a, ∃a⁻¹ a∘a⁻¹ = a⁻¹∘a = e
families of Finite Groups
Cyclic Groups Cn aⁿ=e
C6 a⁶=e
e
a
a²
a³
a⁴
a⁵

$\forall a,\ \exists a^{-1}\quad a\circ a^{-1}=a^{-1}\circ a=e$

$C_6\quad a^6=e$

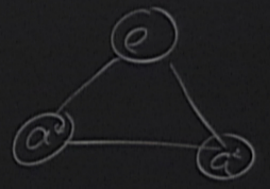




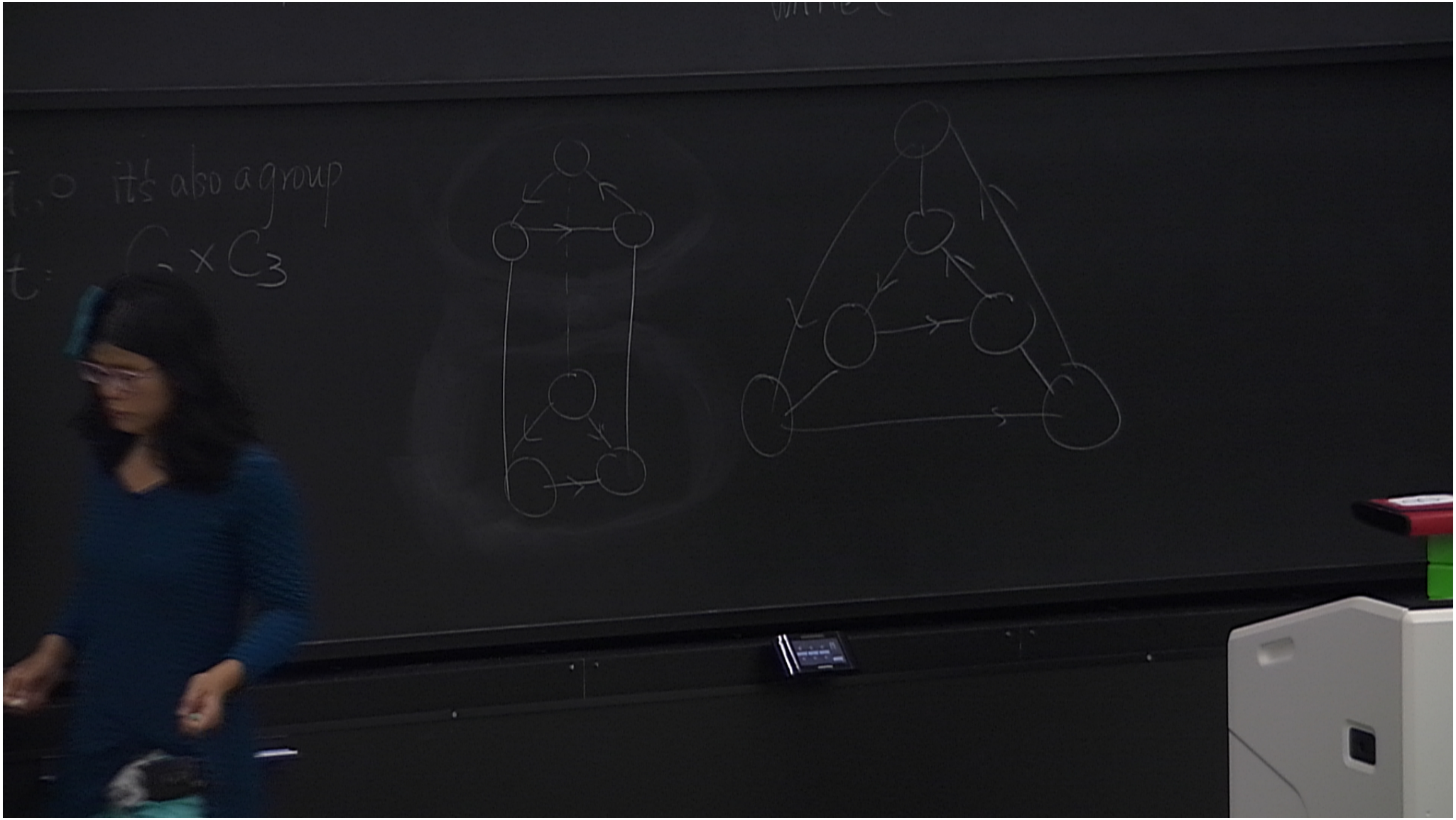
$\{a^2, a^3\}$.



C_2



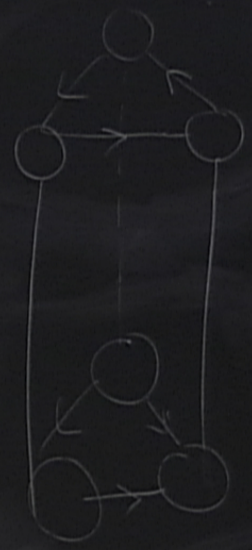
C_3


it's also a group
$C_2 \times C_3$

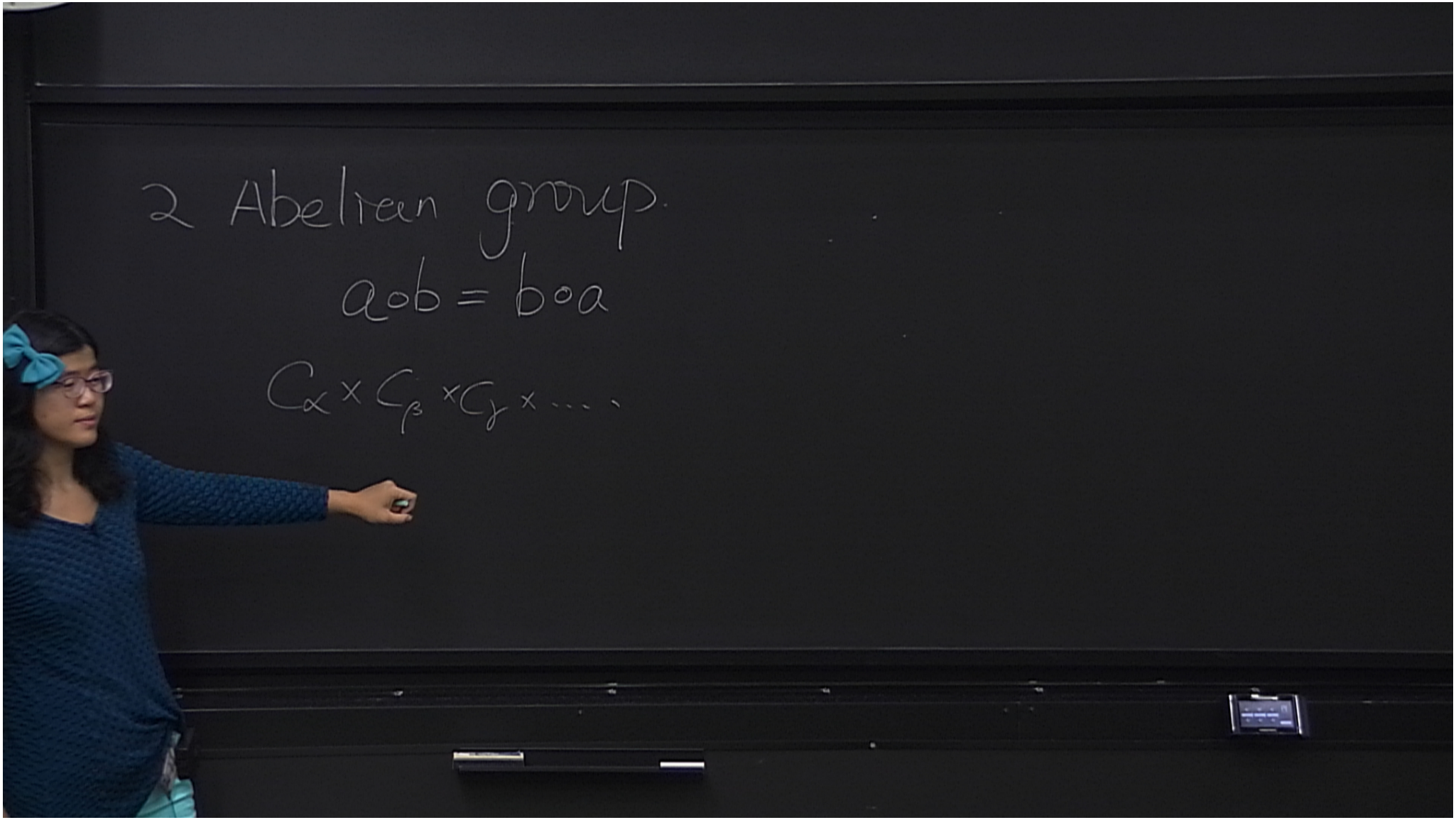
subgroup: $A \subset G$, o it's also a group

direct product: $C_2 \times C_3$

$C_{mn} = C_m \times C_n$






2 Abelian group
a∘b = b∘a
$C_\alpha \times C_\beta \times C_\gamma \times \cdots$

3. Dehydral Group D_n Polygon-groups

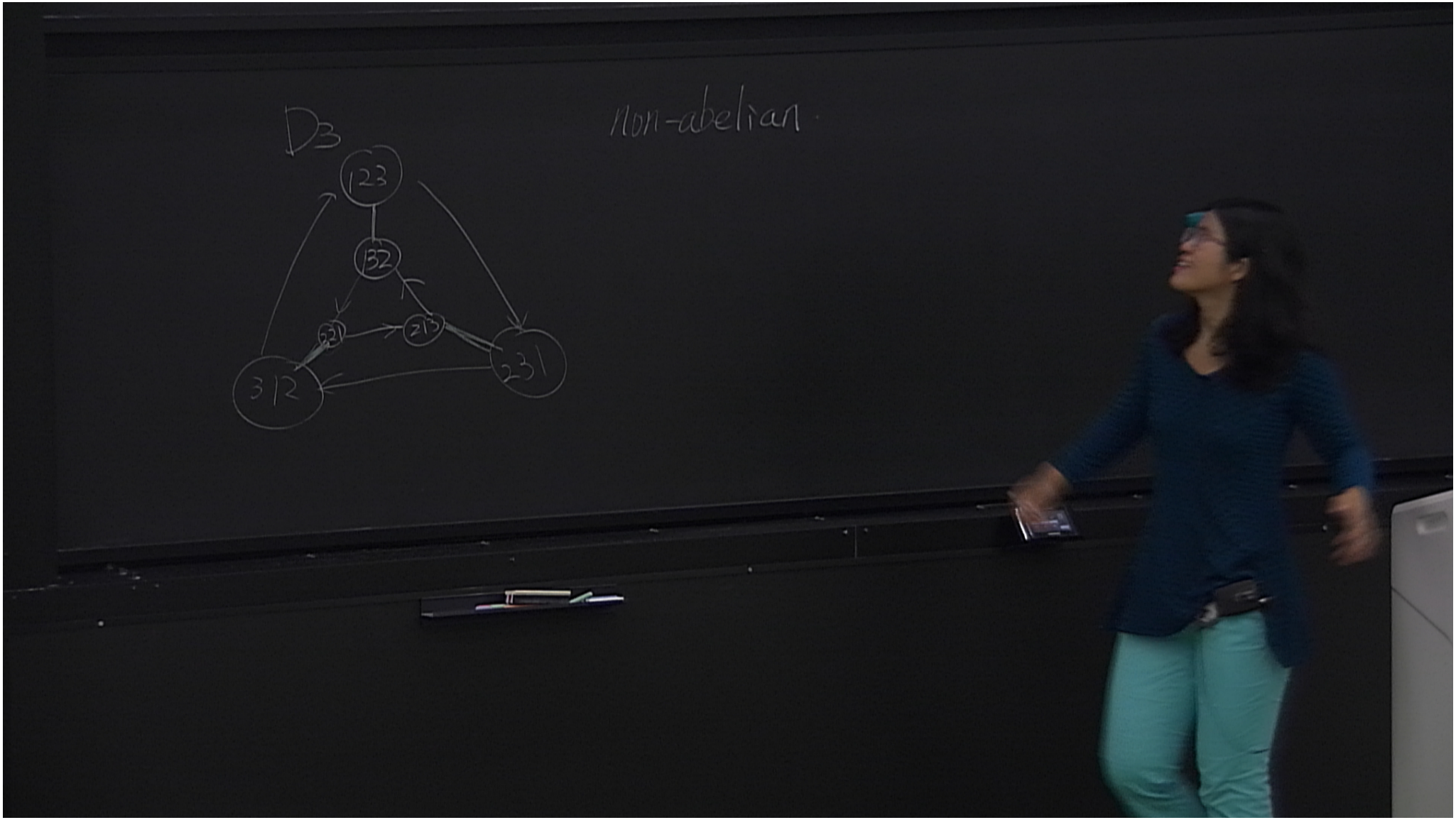
operators: rotate by $\frac{360^\circ}{n}$

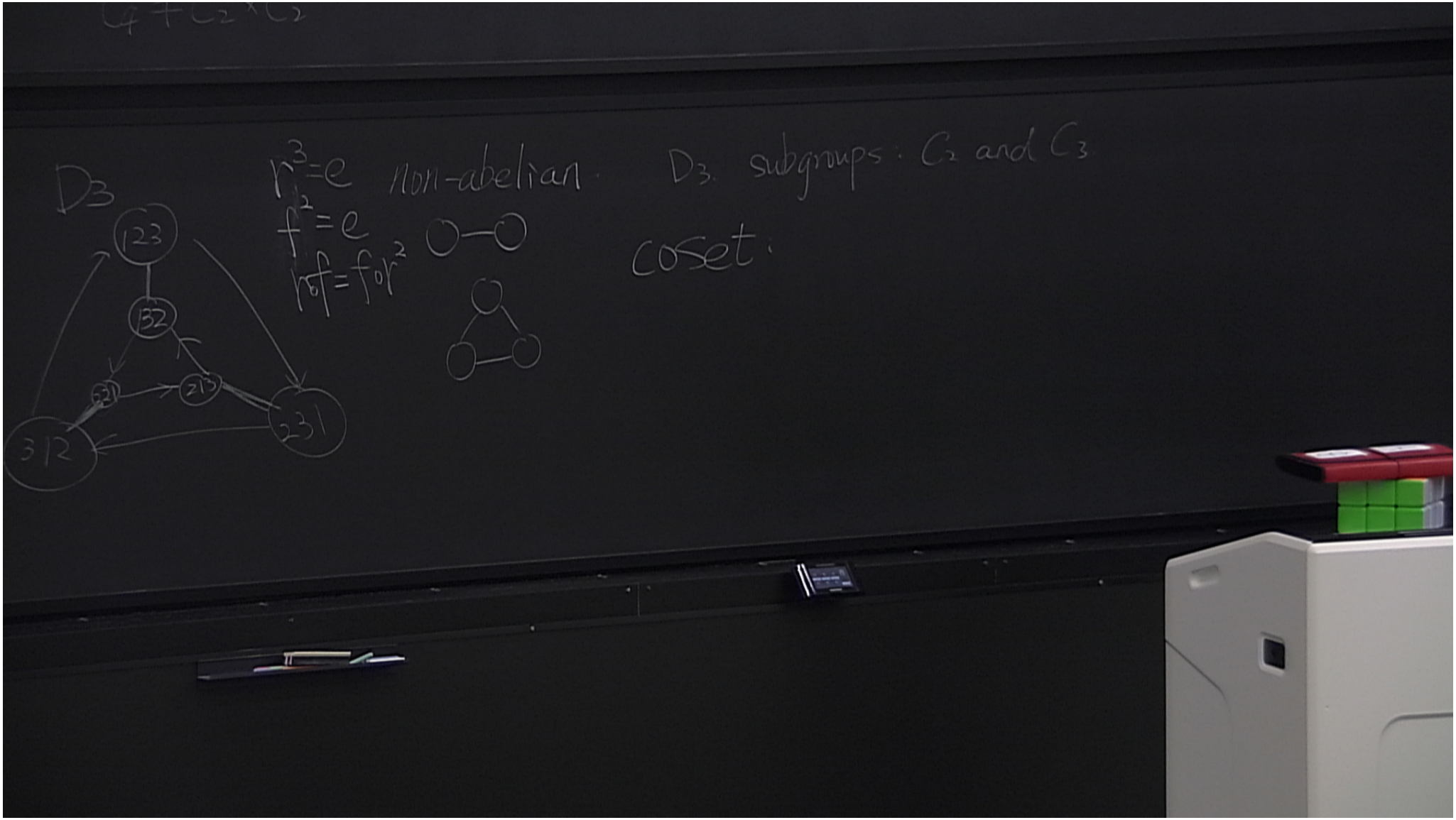
or flip

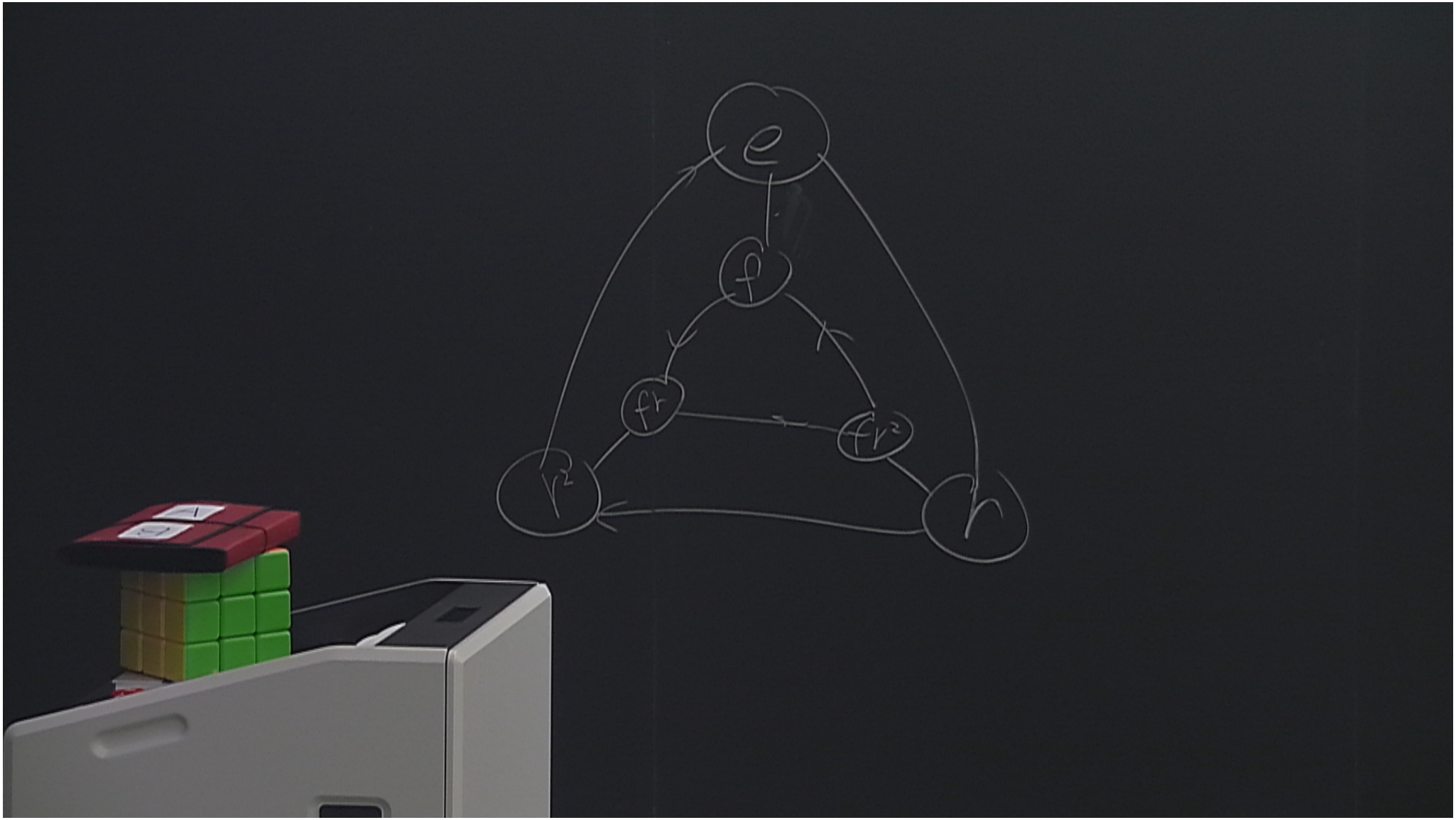
$n=1$ point uni-gon $D_1 = C_2$

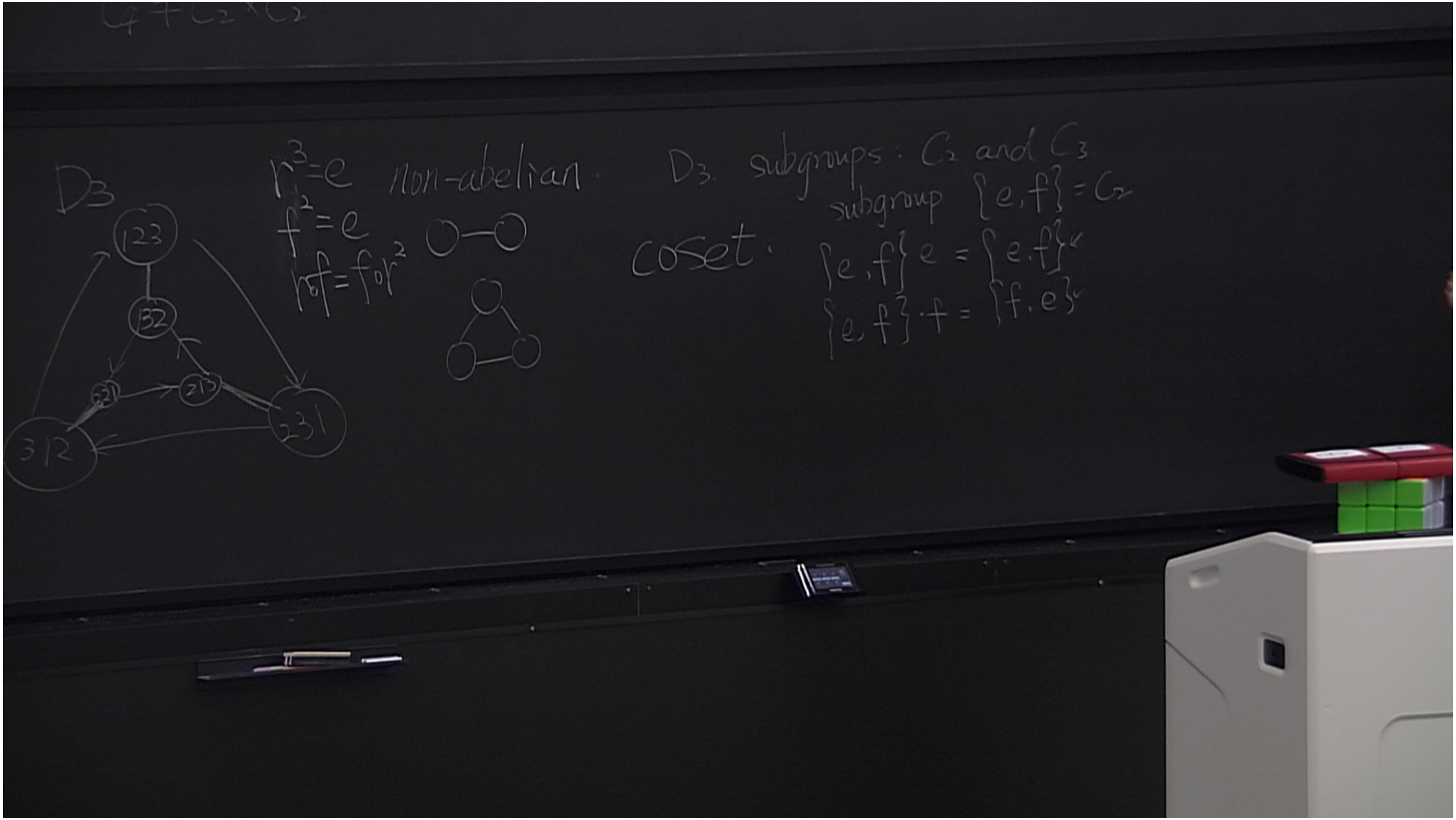
$n=2$ line bi-gon $D_2 = C_2 \times C_2 = V_4$

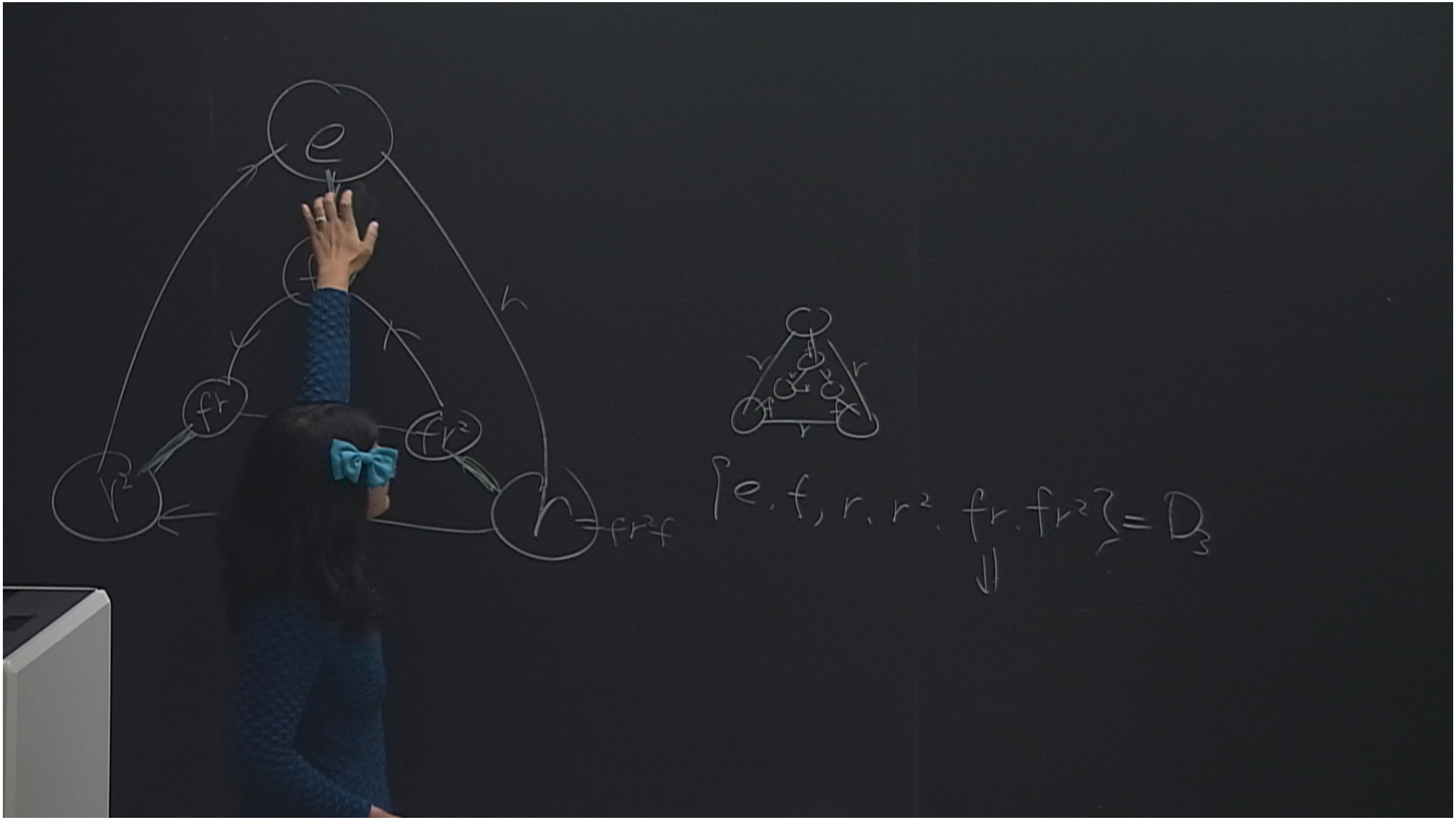
$n=3$ triangle

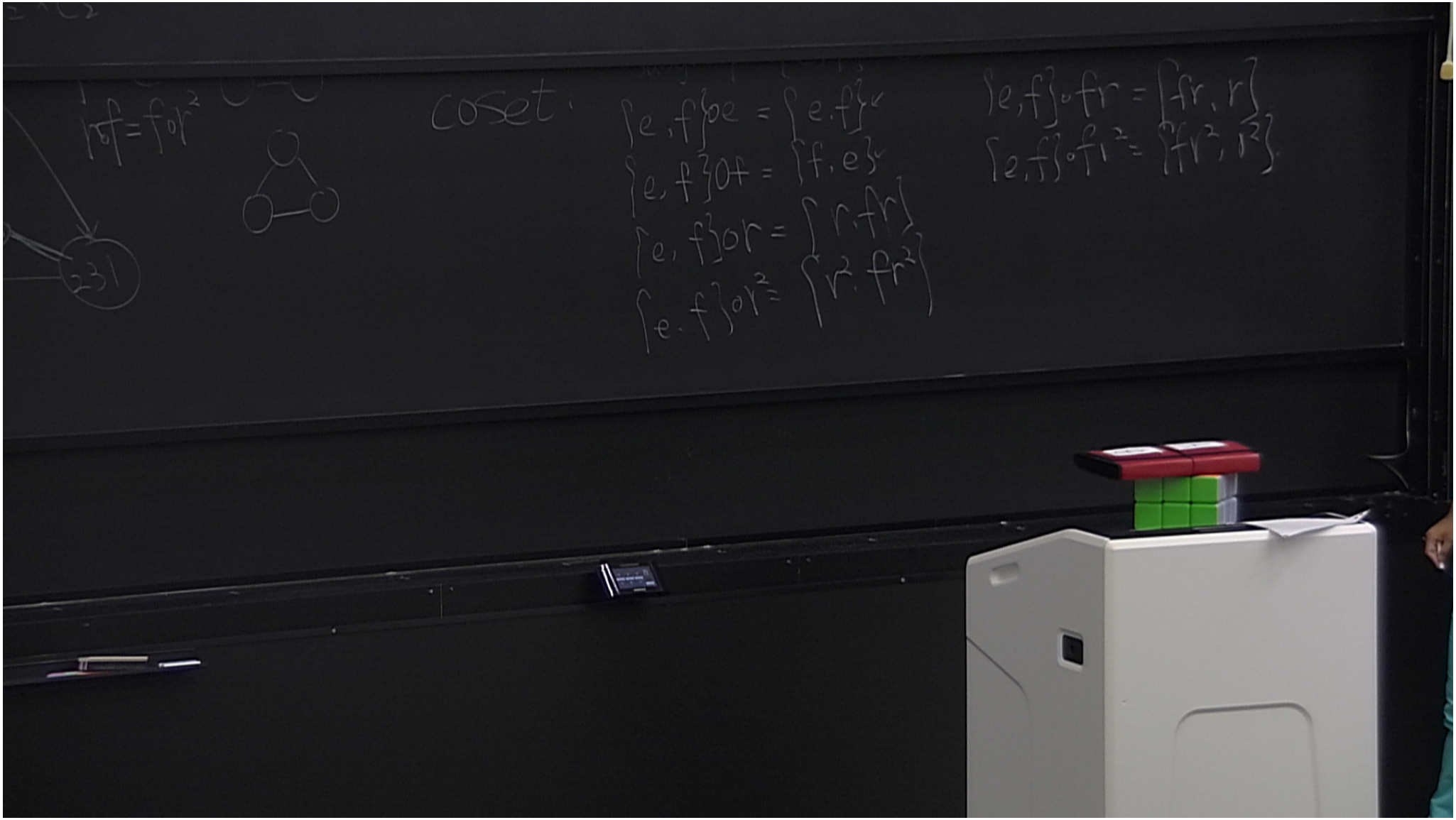

D3
123
132
213
231
312
non-abelian

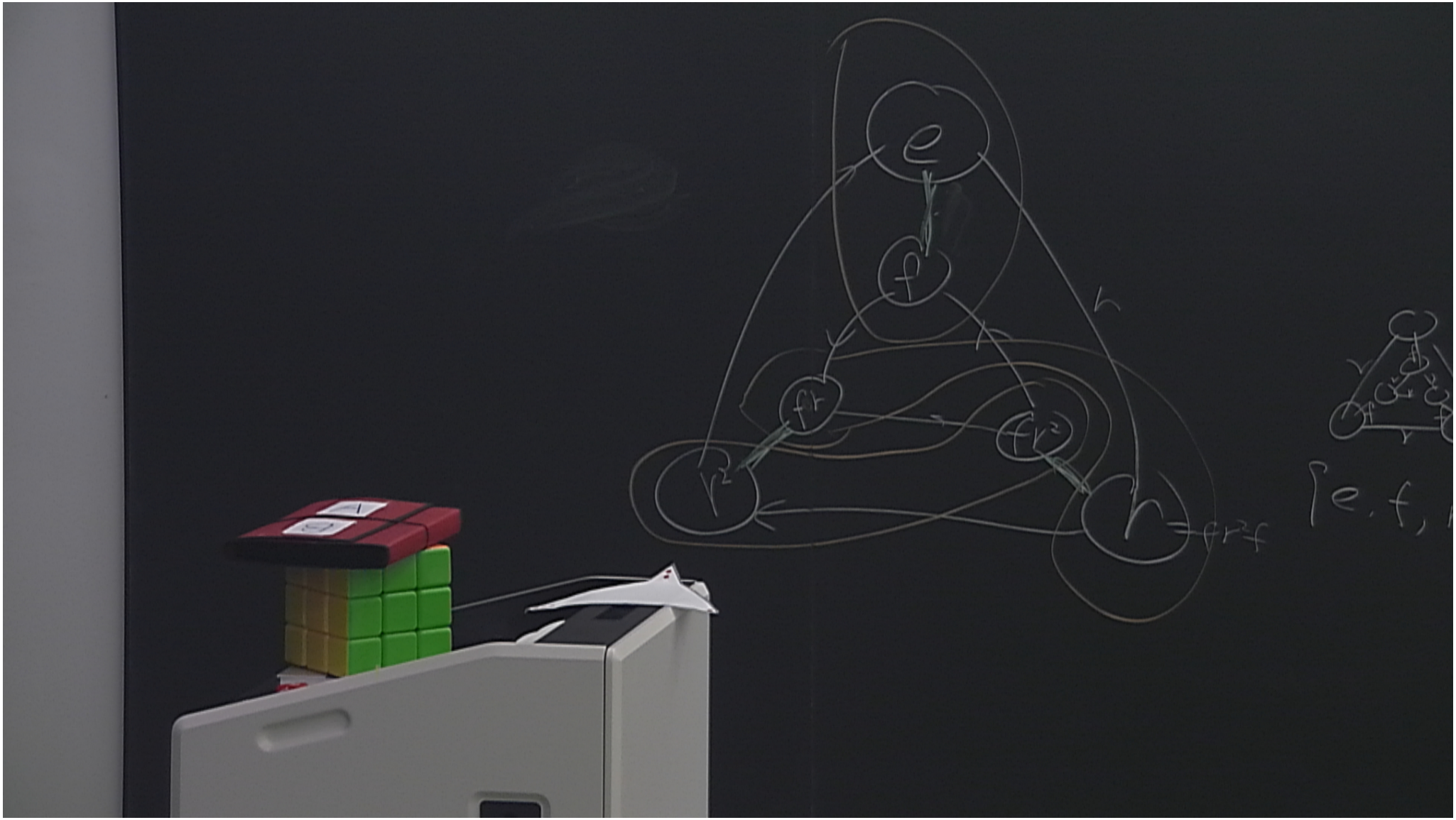

$C_4 + C_2 \times C_2$
D_3
123
132
321
213
312
231
$r^3 = e$ non-abelian
$f^2 = e$
$rf = fr^2$
D_3 subgroups: C_2 and C_3
coset:

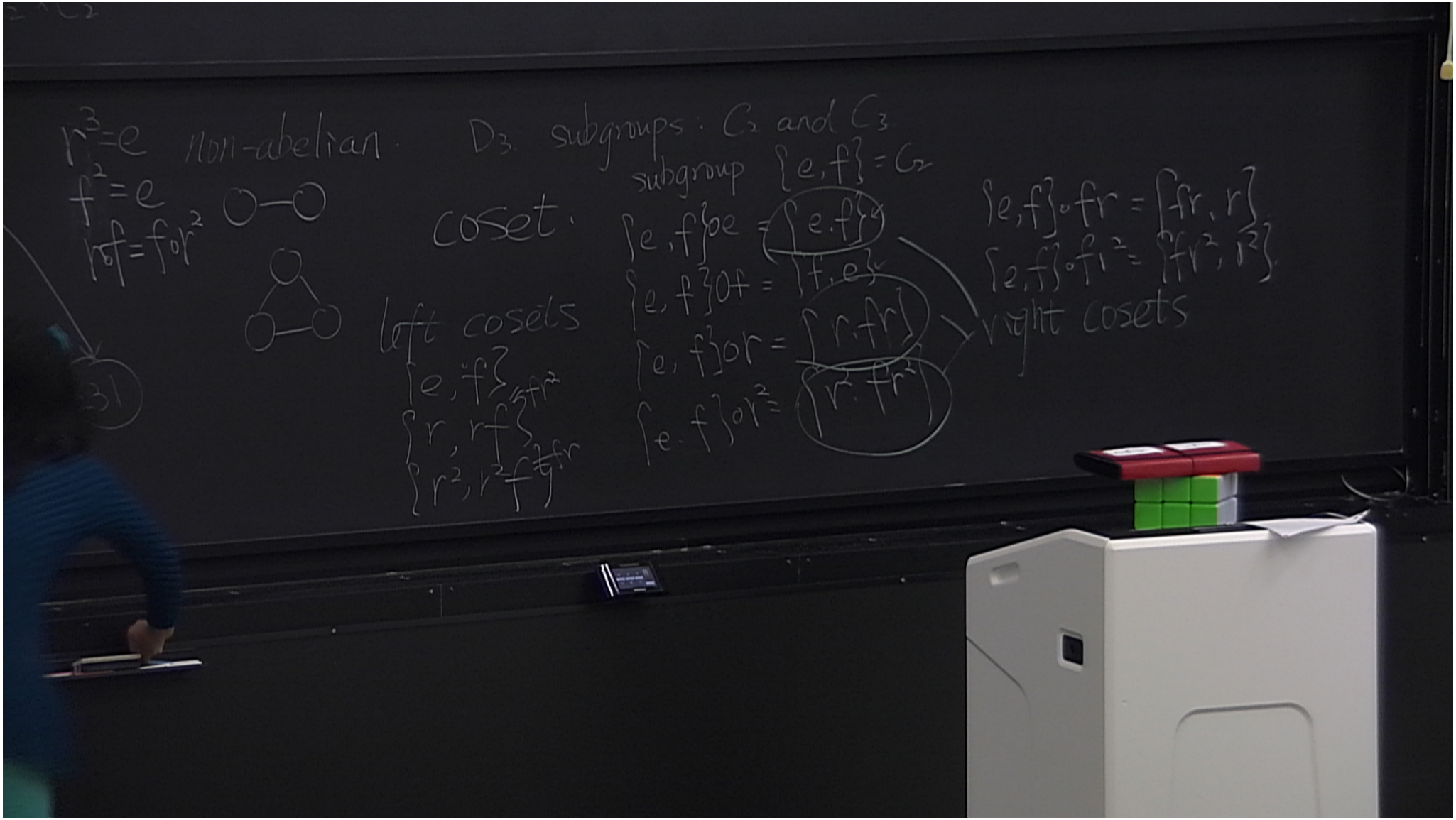

e
f
fr
fr²
r²
r

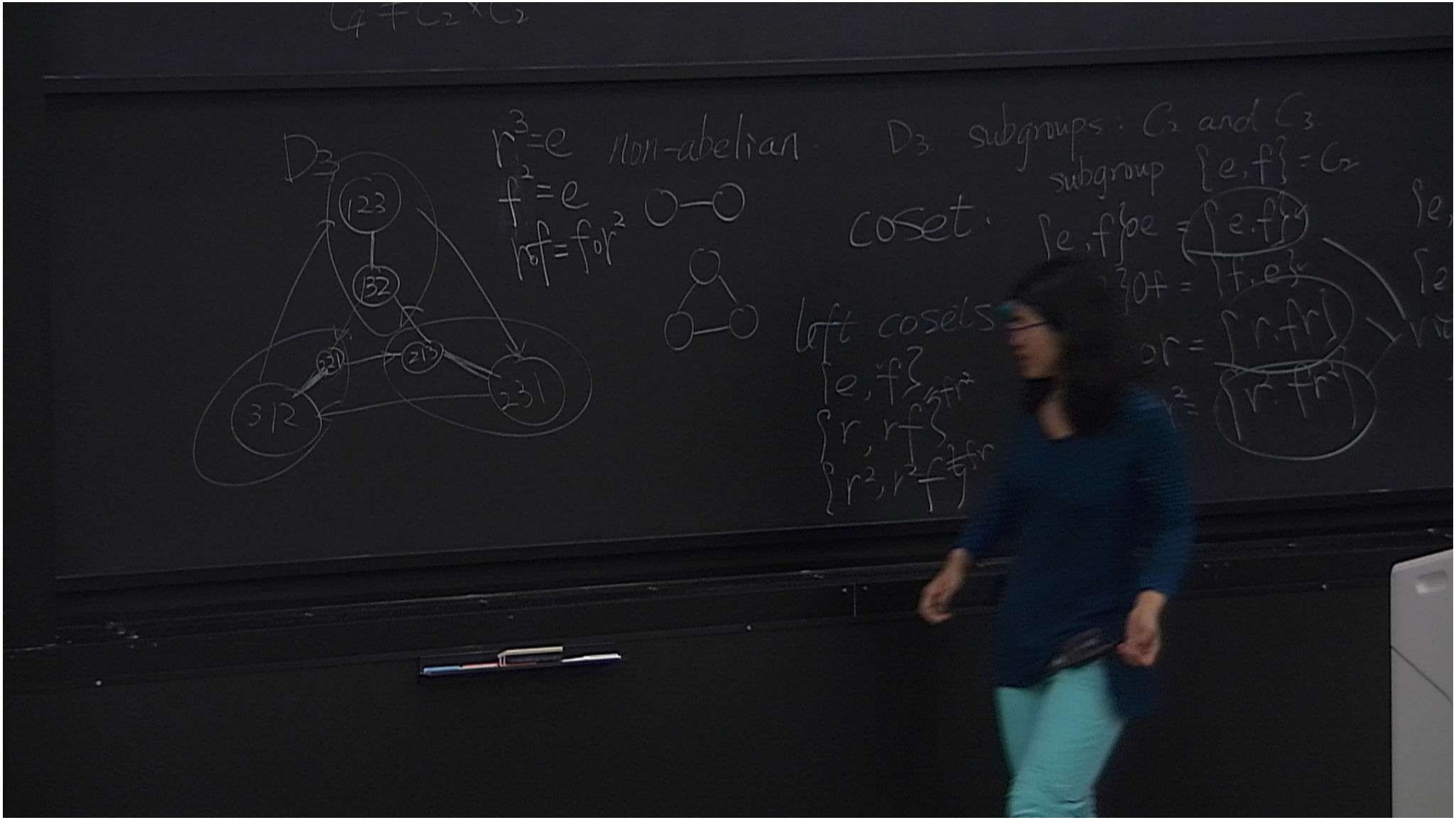

$C_4 + C_2 \times C_2$
D_3
123
132
321
213
312
231
$r^3 = e$ non-abelian
$f^2 = e$
$rf = fr^2$
D_3 subgroups: C_2 and C_3
coset:
subgroup $\{e, f\} = C_2$
$\{e, f\}e = \{e, f\}$
$\{e, f\} \cdot f = \{f, e\}$

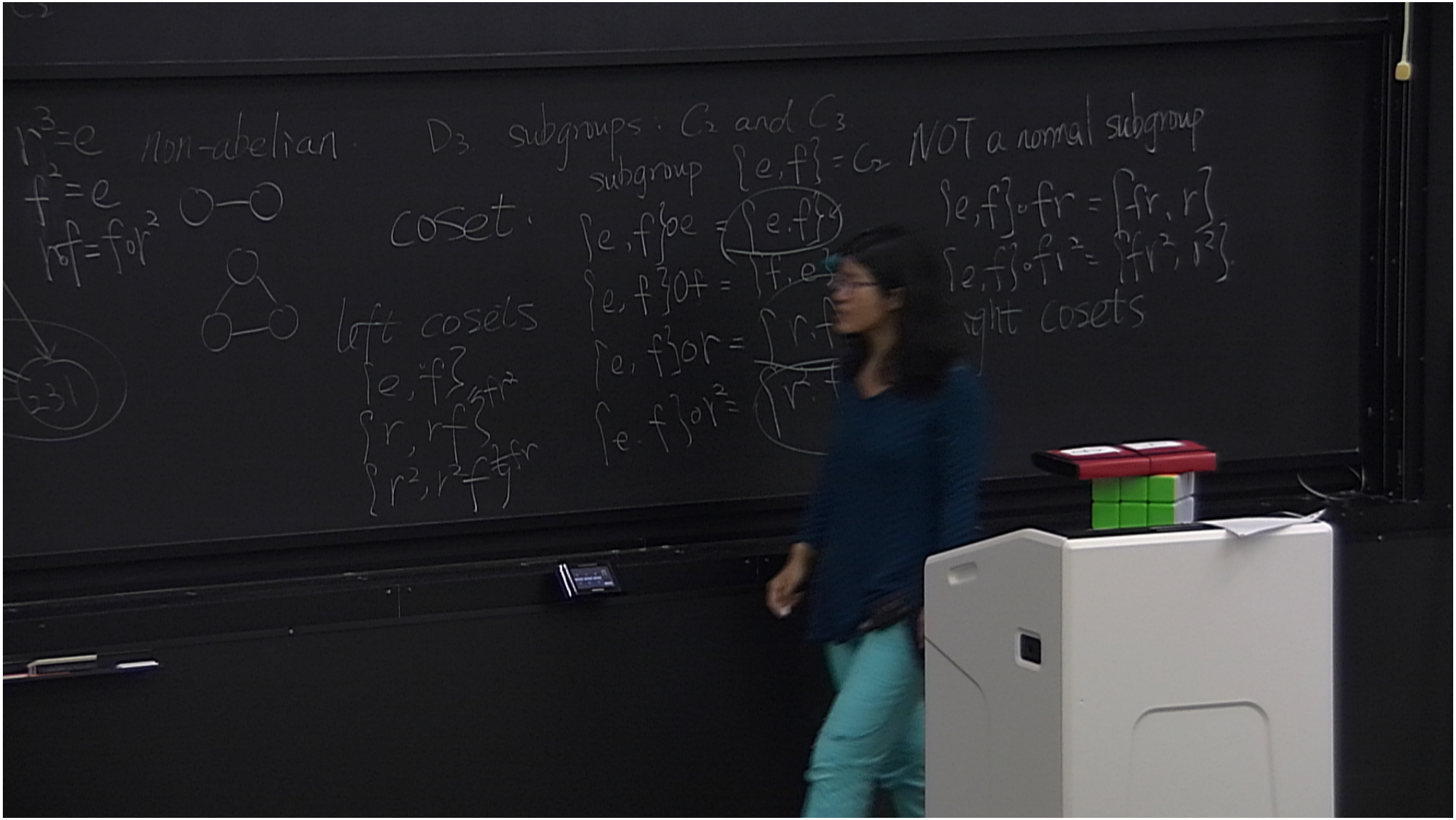

e
r
r²
fr
fr²
f
fr²f
{e, f, r, r², fr, fr²} = D₃

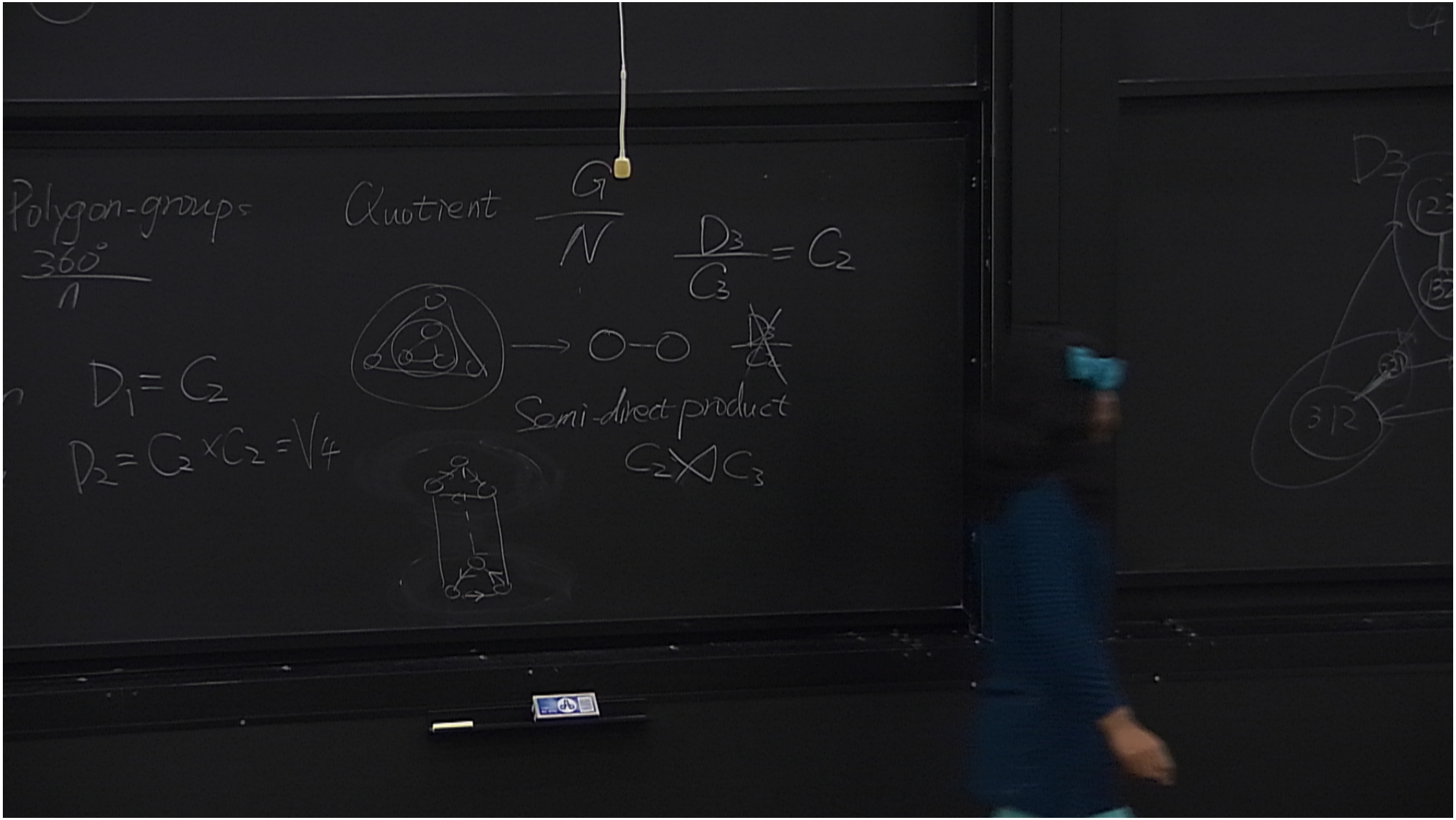

rf = fr²
coset.
{e, f}e = {e, f}
{e, f}f = {f, e}
{e, f}r = {r, fr}
{e, f}r² = {r², fr²}
{e, f}fr = {fr, r}
{e, f}fr² = {fr², r²}
231

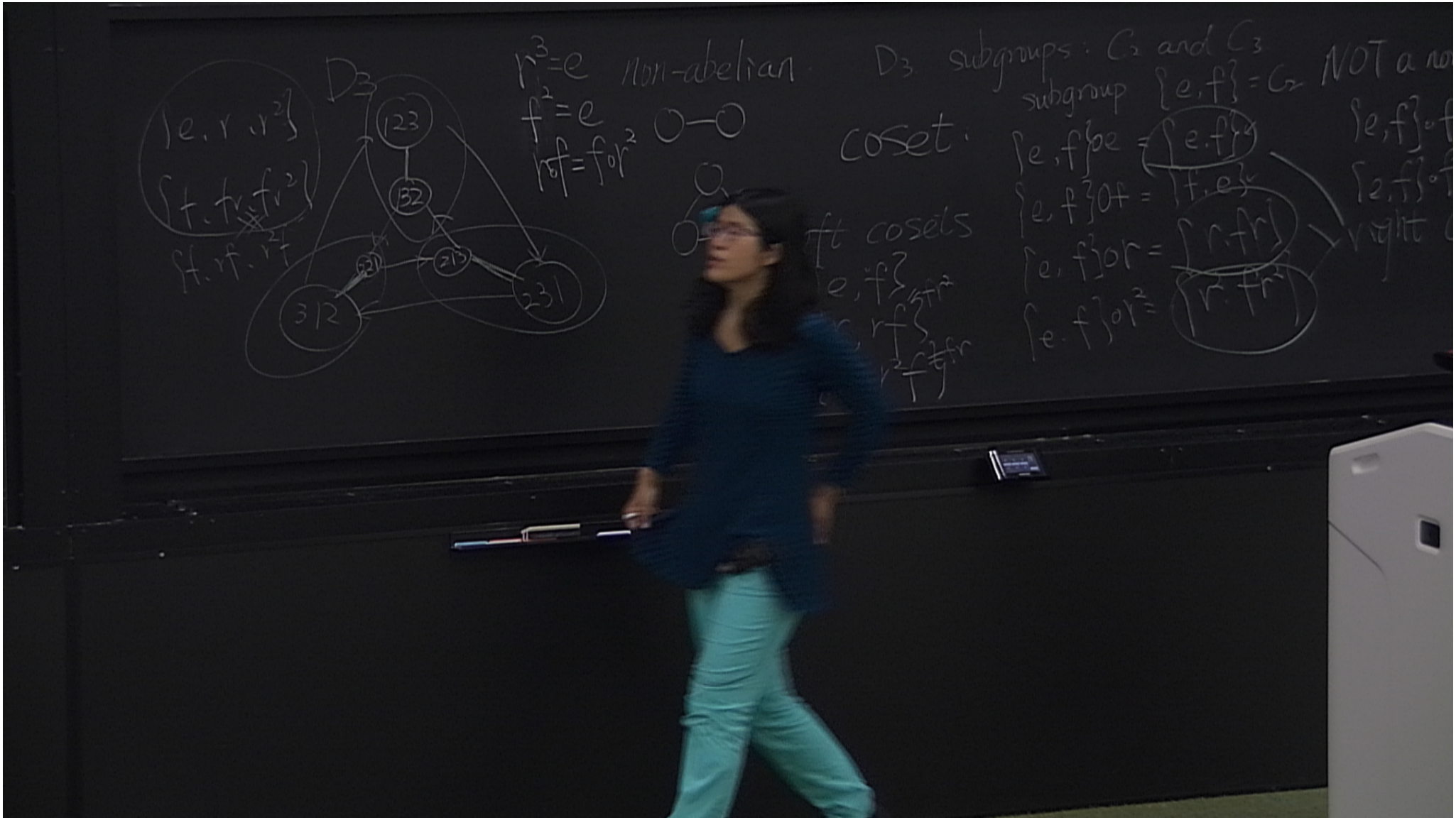

e
f
r
r²
r³
[e, f,


$r^3=e$ non-abelian
$f^2=e$
$rf=fr^2$
D3 subgroups: C2 and C3
coset.
left cosets
right cosets
subgroup {e,f} = C2
{e,f}e = {e,f}
{e,f}f = {f,e}
{e,f}r = {r,fr}
{e,f}r² = {r,fr²}
{e,f}·fr = {fr,r}
{e,f}·fr² = {fr²,r²}
{e,f}
{r,rf}
{r²,r²f}


$C_4 \neq C_2 \times C_2$
D_3
123
132
321
213
312
231
$r^3 = e$ non-abelian
$f^2 = e$
$rf = fr^2$
D_3 subgroups: C_2 and C_3
coset.
subgroup $\{e, f\} = C_2$
$\{e, f\}e = \{e, f\}$
left cosets
$\{e, f\}$
$\{r, rf\}$
$\{r^2, r^2f\}$


$r^3=e$ non-abelian
$f^2=e$
$rf=fr^2$
231
D_3 subgroups: C_2 and C_3
coset:
left cosets
$\{e,f\}$
$\{r,rf\}$
$\{r^2,r^2f\}$
subgroup $\{e,f\}=C_2$
$\{e,f\}e=\{e,f\}$
$\{e,f\}f=\{f,e\}$
$\{e,f\}r=\{r,$
$\{e,f\}r^2=\{r,$
NOT a normal subgroup
$\{e,f\}\cdot fr=\{fr,r\}$
$\{e,f\}\cdot fr^2=\{fr^2,r^2\}$
right cosets


Polygon-groups
360°/n
Quotient
G/N
D3/C3 = C2
D1 = C2
D2 = C2 × C2 = V4
Semi-direct product
C2 ⋊ C3


$r^3=e$ non-abelian
$f^2=e$
$rf=fr^2$
D_3
D_3 subgroups: C_2 and C_3
coset:
subgroup $\{e,f\}=C_2$
NOT a no
$\{e,r,r^2\}$
$\{f,rf,r^2f\}$
123
132
213
231
312
321
$\{e,f\}e=\{e,f\}$
$\{e,f\}f=\{f,e\}$
$\{e,f\}r=\{r,fr\}$
$\{e,f\}r^2=\{r^2,fr^2\}$