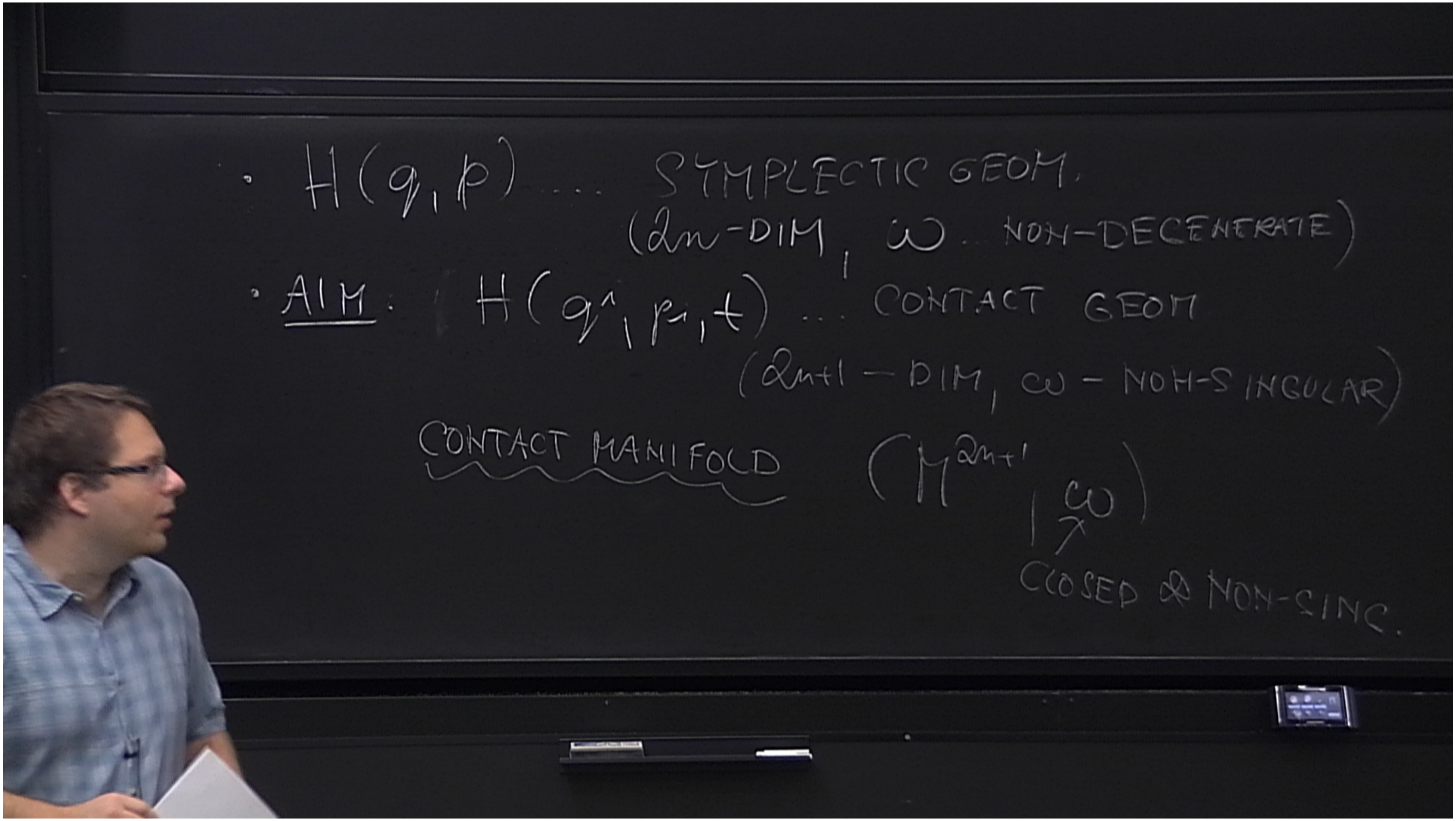
Title: Front End - Classical Mechanics 7

Date: Aug 19, 2016 10:30 AM

URL: http://pirsa.org/16080069

Abstract:


· H(q, p) … SYMPLECTIC GEOM.
(2n-DIM, ω … NON-DEGENERATE)
· AIM: H(q^, p, t) … CONTACT GEOM
(2n+1 – DIM, ω – NON-SINGULAR)
CONTACT MANIFOLD (M^2n+1, ω)
CLOSED & NON-SING.

ERATE)

SINGULAR)

NON-SING.

REMARKS: i) ODD-DIM V. OF SYMPL. G.

ii) CONTACT 1-FORM θ

$$\boxed{\omega = d\theta}$$

VORTEX (NULL) DIRECTION X: $\boxed{X \lrcorner \omega = 0}$

ERATE)

INGULAR)

N-SING.

REMARKS: i) ODD-DIM V. OF SYMPL. G.

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$$\boxed{\omega = d\theta}$$

VORTEX (NULL) DIRECTION X: $\boxed{X \lrcorner \omega = 0}$

SPEC: $\theta(X) = 1$ REEB VECTOR

ERATE)

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ON-SING.

REMARKS: i) ODD-DIM V. OF SYMPL. G.

ii) CONTACT 1-FORM θ

$$\boxed{\omega = d\theta}$$

VORTEX (NULL) DIRECTION X: $\boxed{X \lrcorner \omega = 0}$

SPEC: $\theta(X) = 1$ REEB VECTOR

iii) SPECIAL RIEMANNIAN MANIFOLDS

- KÄHLER $(M^{2n}, \omega \ldots$ KÄHLER 2-FORM$)$
- SASAKI $(M^{2n+1}, X \ldots$ REEB VECTOR$)$

R 2-FORM)

VECTOR)

iv) $\boxed{\mathcal{L}_X \omega = 0}$

$$= X \lrcorner \underbrace{d\omega}_{0} + d(\underbrace{X \lrcorner \omega}_{0}) = 0,$$

R 2-FORM)

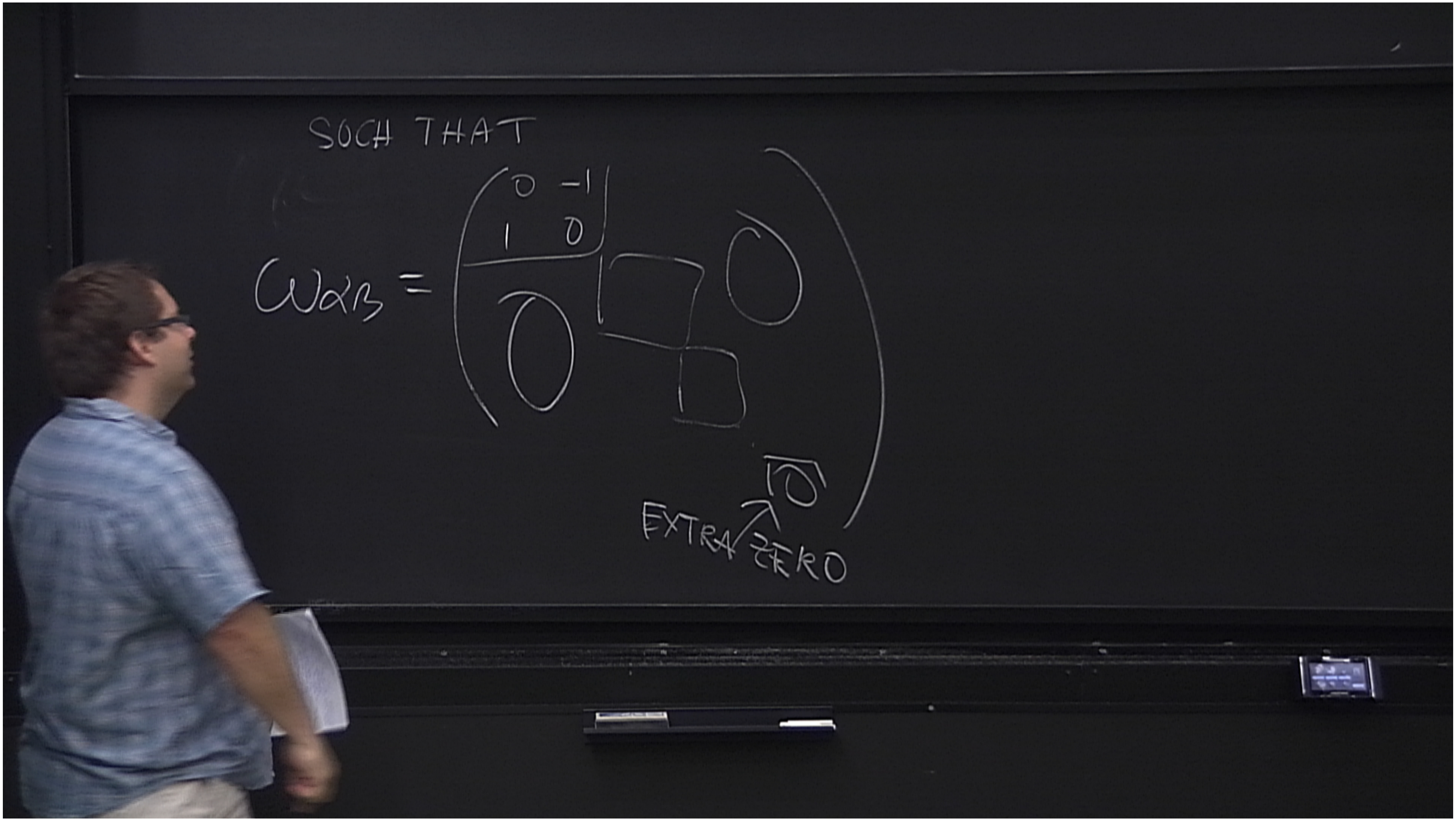
VECTOR)

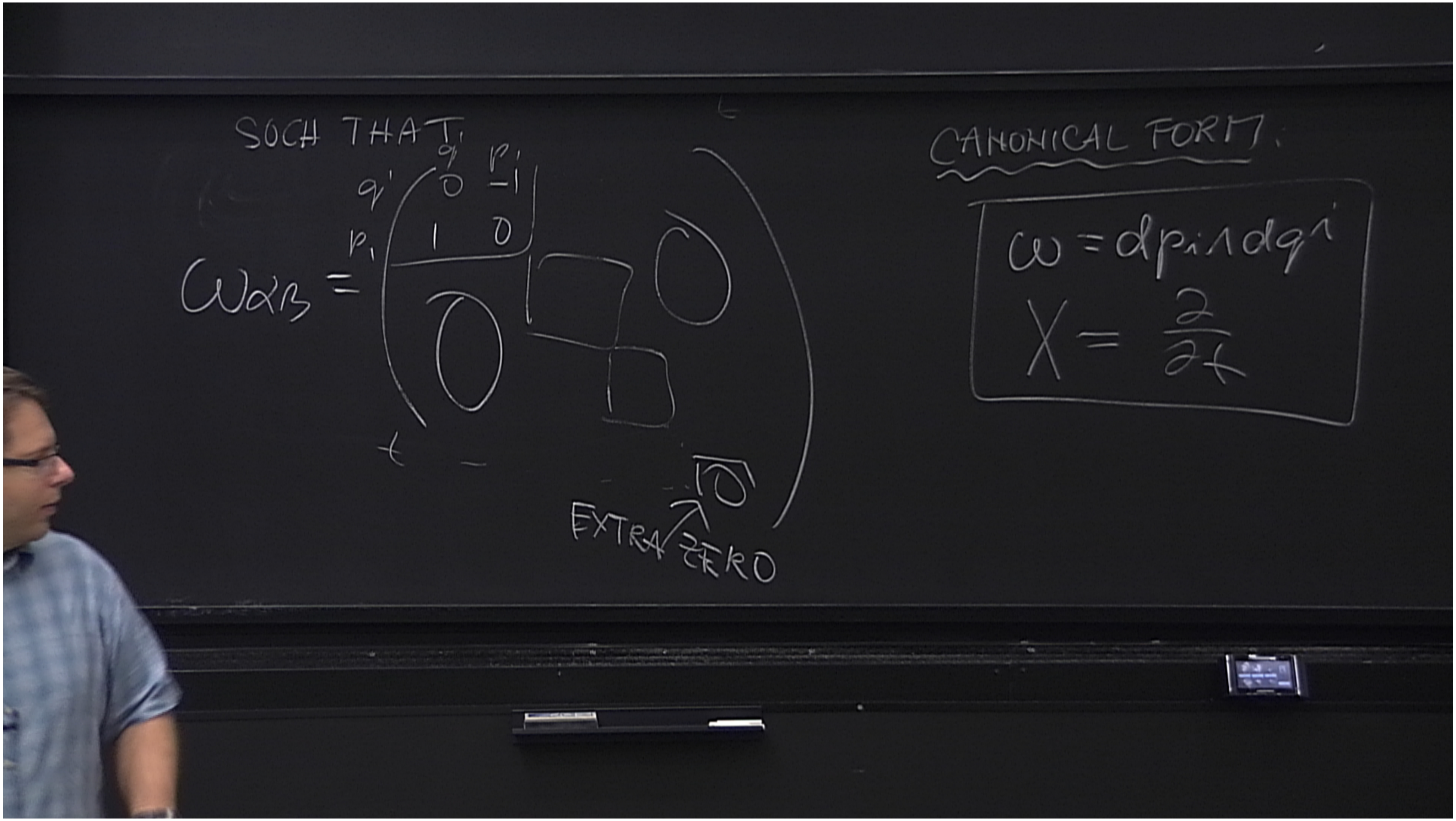
iv) $\boxed{\mathcal{L}_X \omega = 0}$

$$= X \lrcorner \underbrace{d\omega}_{0} + d(\underbrace{X \lrcorner \omega}_{0}) = 0,$$

v) DARBOUX THEOREM. ω CLOSED & NON-SING.

$\Rightarrow \exists$ COORDINATES (q^i, p_i, t)


SUCH THAT
$\omega_{\alpha\beta} =$
0 −1
1 0
0
0
EXTRA ZERO


SUCH THAT
ω_αβ =
CANONICAL FORM:
ω = dp_i ∧ dq^i
X = ∂/∂t
EXTRA ZERO

- HAMILTONIAN MECHANICS

DEF: EXTENDED PHASE SPACE: $M^{2n+1} = T^*C \times \mathbb{R}$

(q^i, p_i, t)

(q^i, p_i, t)

- A HAMILTONIAN : $H(q^i, p_i, t)$ DEFINES A CONTACT 1-FORM

$$\boxed{\Theta = p_i dq^i - H dt}$$

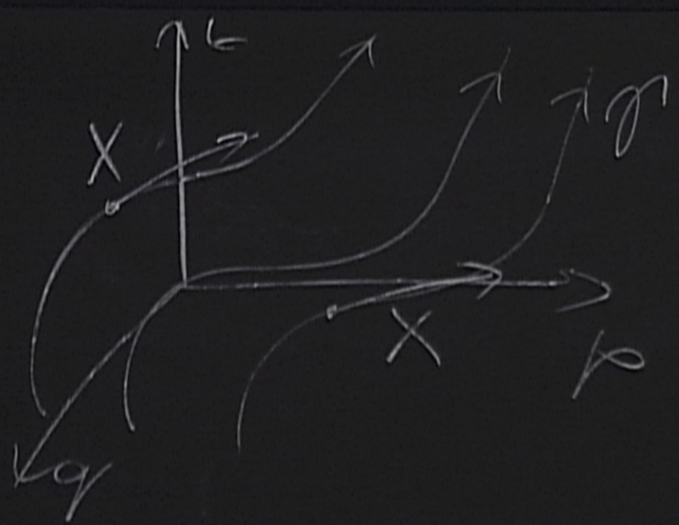
$\mathbb{R}$

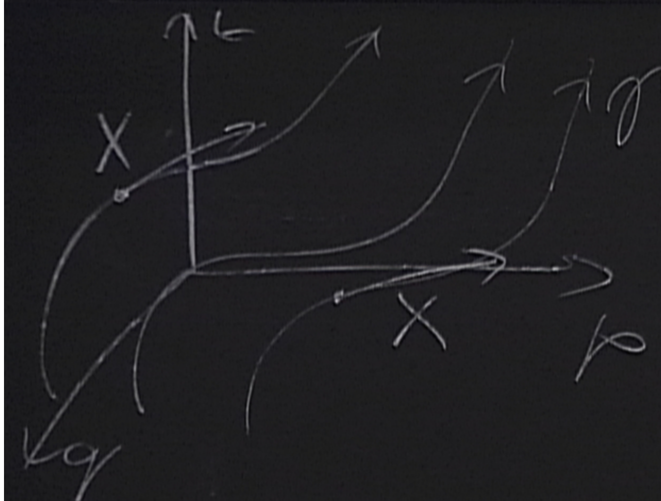
$$\Rightarrow \boxed{\omega = d\theta = dp_i \wedge dq^i - dH \wedge dt}$$

...CONTACK MANIFOLD $\left(M^{2n+1}, \omega\right)$

$\Rightarrow$ VORTEX DIRECTION .. DYNAMICAL VECTOR FIELD

$$\boxed{X = \frac{\partial H}{\partial p_j}\frac{\partial}{\partial q^j} - \frac{\partial H}{\partial q^j}\frac{\partial}{\partial p_j} + \frac{\partial}{\partial t}}$$

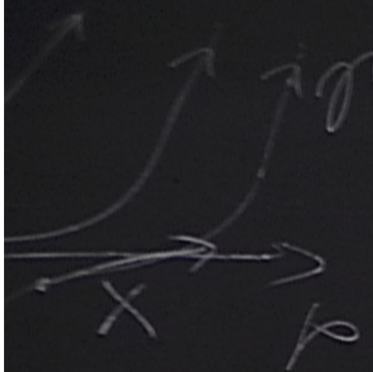

t
X
X
q
p
γ



$\gamma(\tau)$ ... INTEGRAL CURVE

$$\gamma(\tau) = \left(q^i(\tau), p_i(\tau), t(\tau)\right)$$

$$X = \frac{d}{d\tau} = \frac{dq^i}{d\tau}\frac{\partial}{\partial q^i} + \frac{dp_i}{d\tau}\frac{\partial}{\partial p_i} + \frac{dt}{d\tau}\frac{\partial}{\partial t}$$



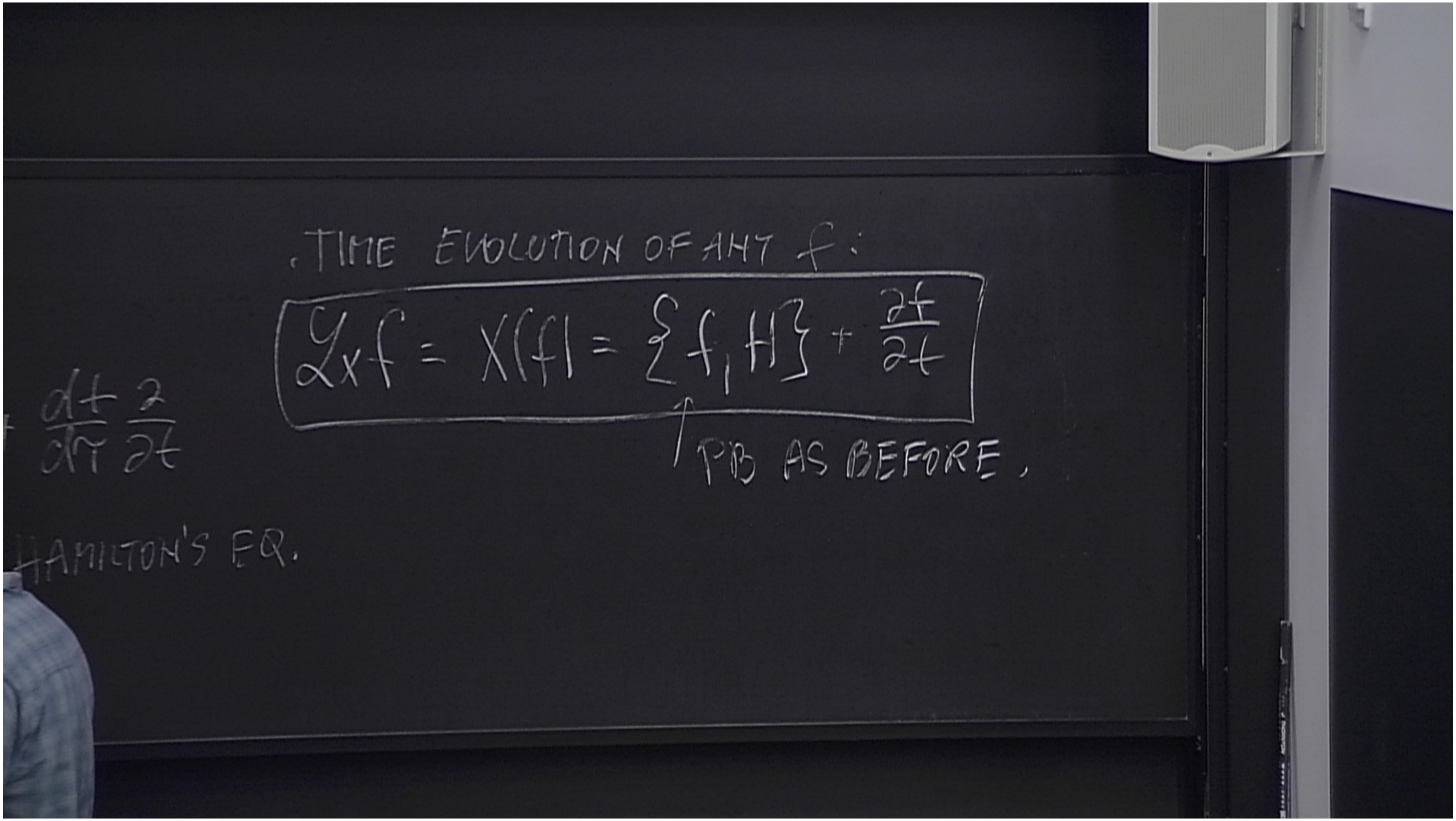
$\gamma(\tau)$ ... INTEGRAL CURVE

$$\gamma(\tau) = \left(q^i(\tau), p_i(\tau), t(\tau)\right)$$

$$X = \frac{d}{d\tau} = \frac{dq^i}{d\tau}\frac{\partial}{\partial q^i} + \frac{dp_i}{d\tau}\frac{\partial}{\partial p_i} + \frac{dt}{d\tau}\frac{\partial}{\partial t}$$

$$\boxed{\tau = t}$$

$$\boxed{\frac{\partial H}{\partial p_j} = \dot{q}^j, \quad \frac{\partial H}{\partial q^j} = -\dot{p}_j}$$


.TIME EVOLUTION OF ANY f:
$\boxed{\mathcal{L}_X f = X(f) = \{f, H\} + \frac{\partial f}{\partial t}}$
PB AS BEFORE.
$\frac{dt}{d\tau}\frac{\partial}{\partial t}$
HAMILTON'S EQ.

$$\frac{\partial H}{\partial p_j} = \dot{q}^j, \quad \frac{\partial H}{\partial q^j} = -\dot{p}_j$$

- CANONICAL TRANSFORMATION

$$Q^j = Q^j(q^i, p_i, t), \quad P_j = P_j(q^i, p_i, t)$$

$$\frac{\partial H}{\partial p_j} = \dot{q}^j, \qquad \frac{\partial H}{\partial q^j} = -\dot{p}_j$$

- CANONICAL TRANSFORMATION

DEF: $Q^j = Q^j(q^i, p_i, t)$, $P_j = P_j(q^i, p_i, t)$

CANONICAL.

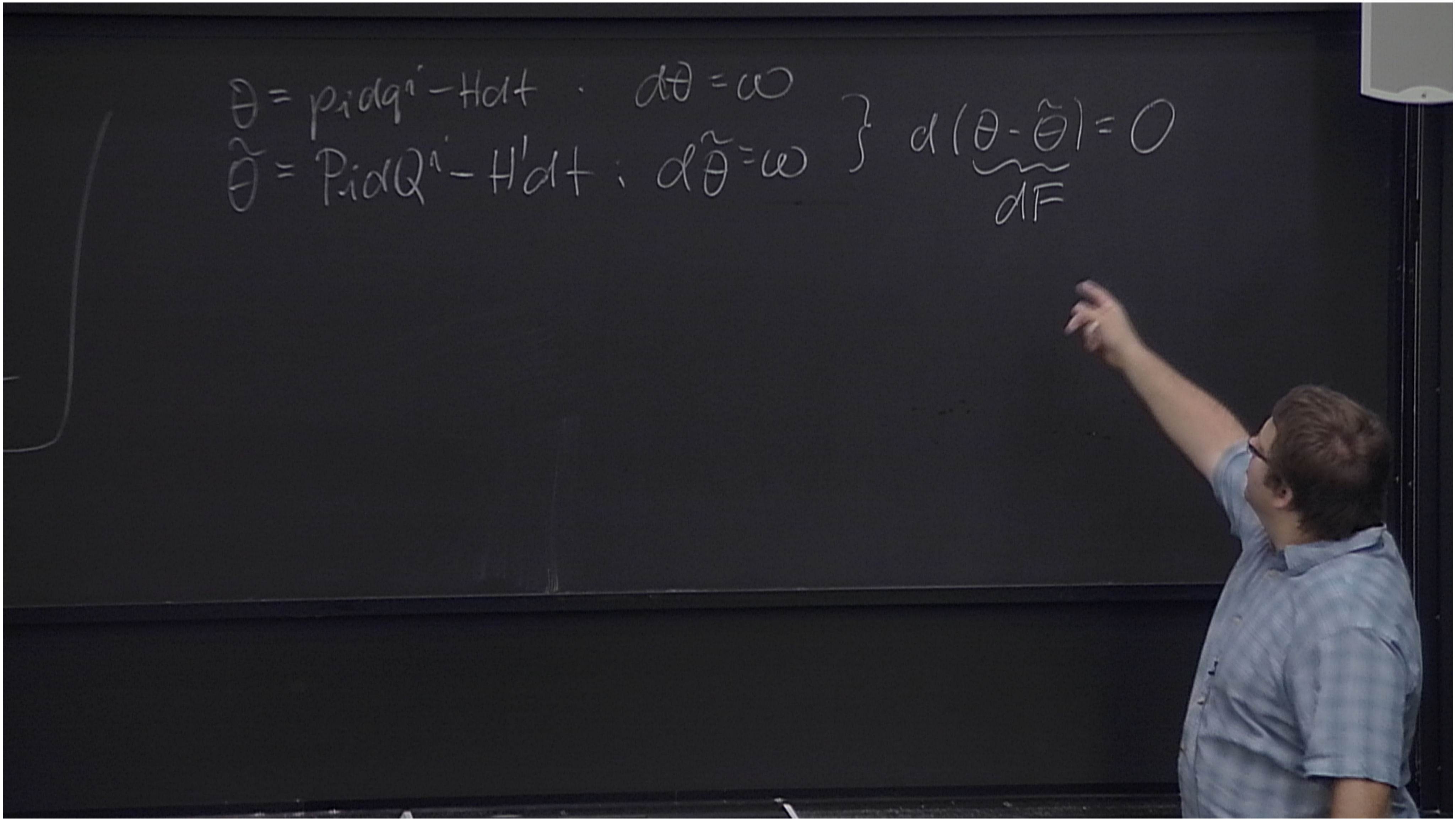
$$\frac{\partial H}{\partial p_j} = \dot{q}^j, \quad \frac{\partial H}{\partial q^j} = -\dot{p}_j$$

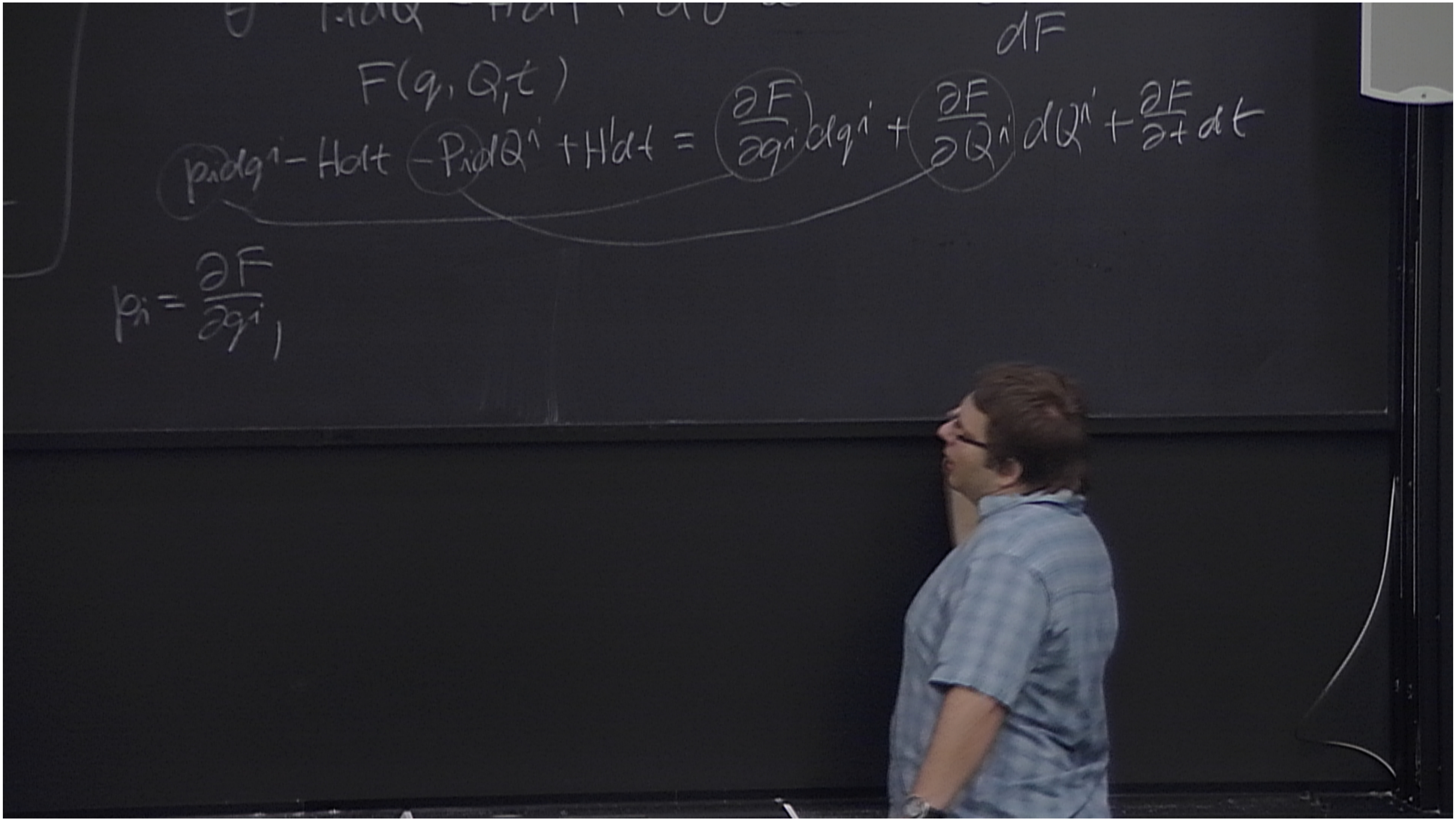
- CANONICAL TRANSFORMATION

DEF: $Q^j = Q^j(q^i, p_i, t), \quad P_j = P_j(q^i, p_i, t)$

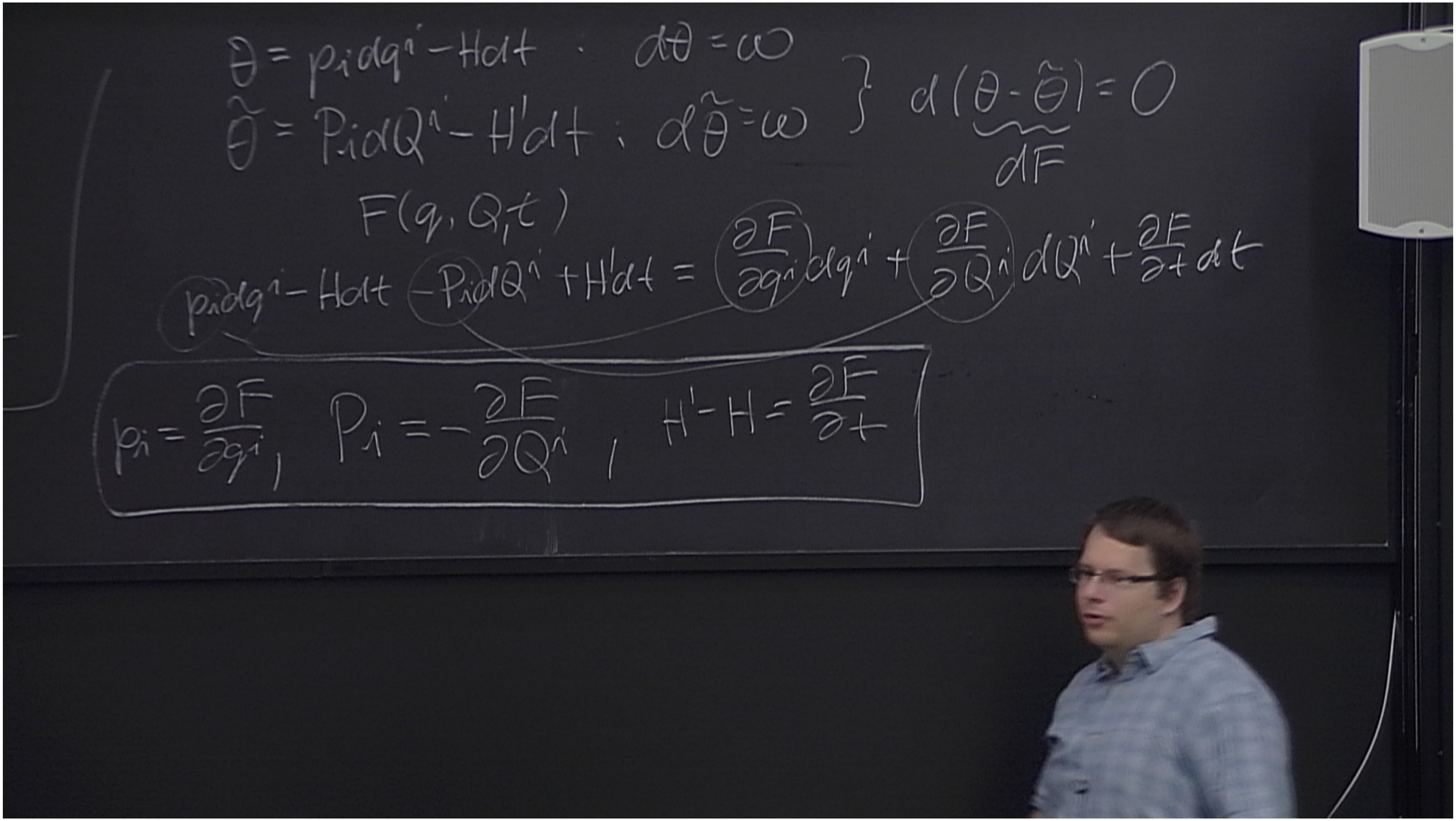
CANONICAL: $\exists\, H'$

$$\omega = dp_i \wedge dq^i - dH \wedge dt = dP_i \wedge dQ^i - dH' \wedge dt$$

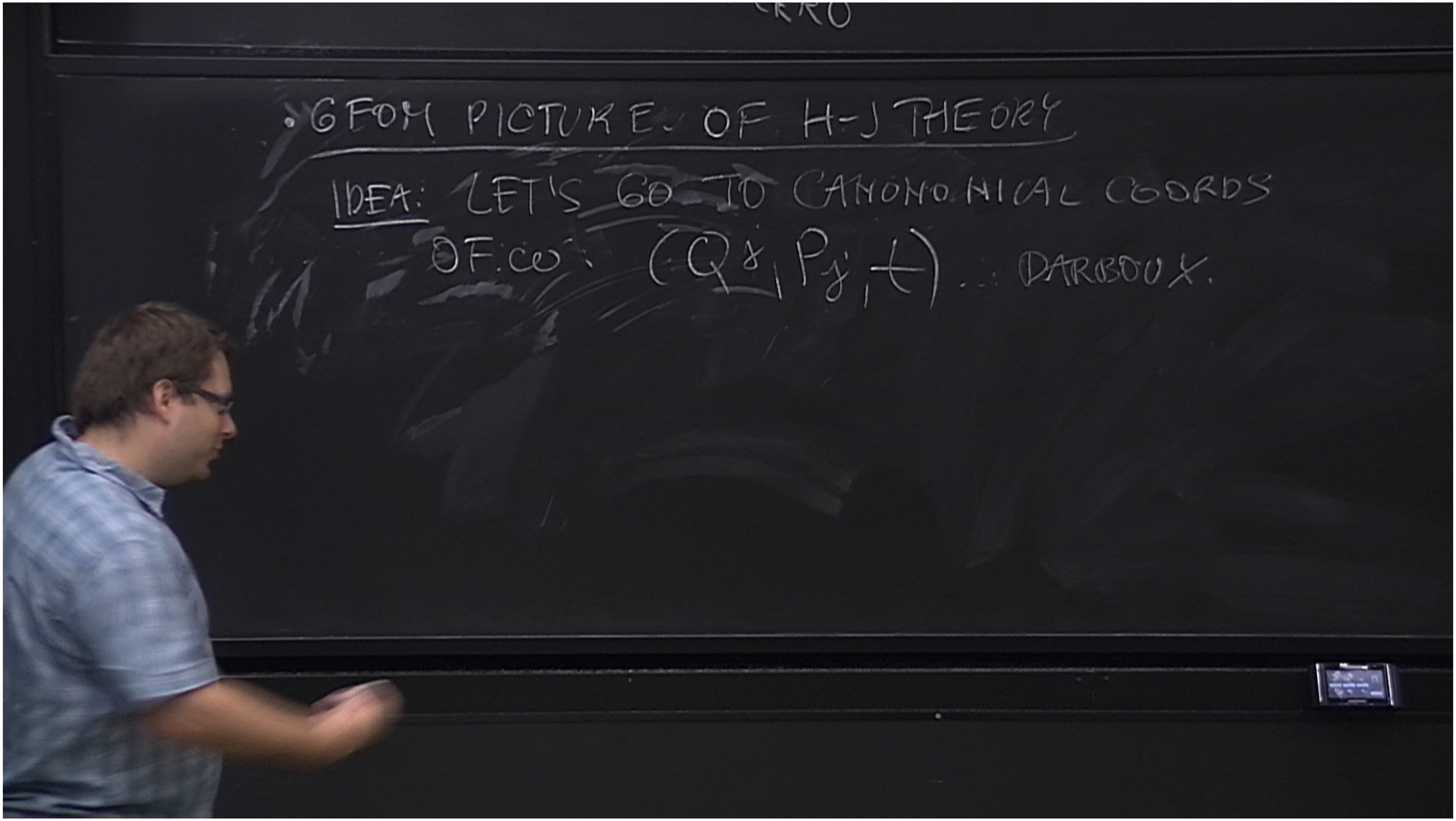

$\theta = p_i dq^i - H dt \; . \; d\theta = \omega$
$\tilde{\theta} = P_i dQ^i - H' dt \; ; \; d\tilde{\theta} = \omega$
$\} \; d(\underbrace{\theta - \tilde{\theta}}_{dF}) = 0$

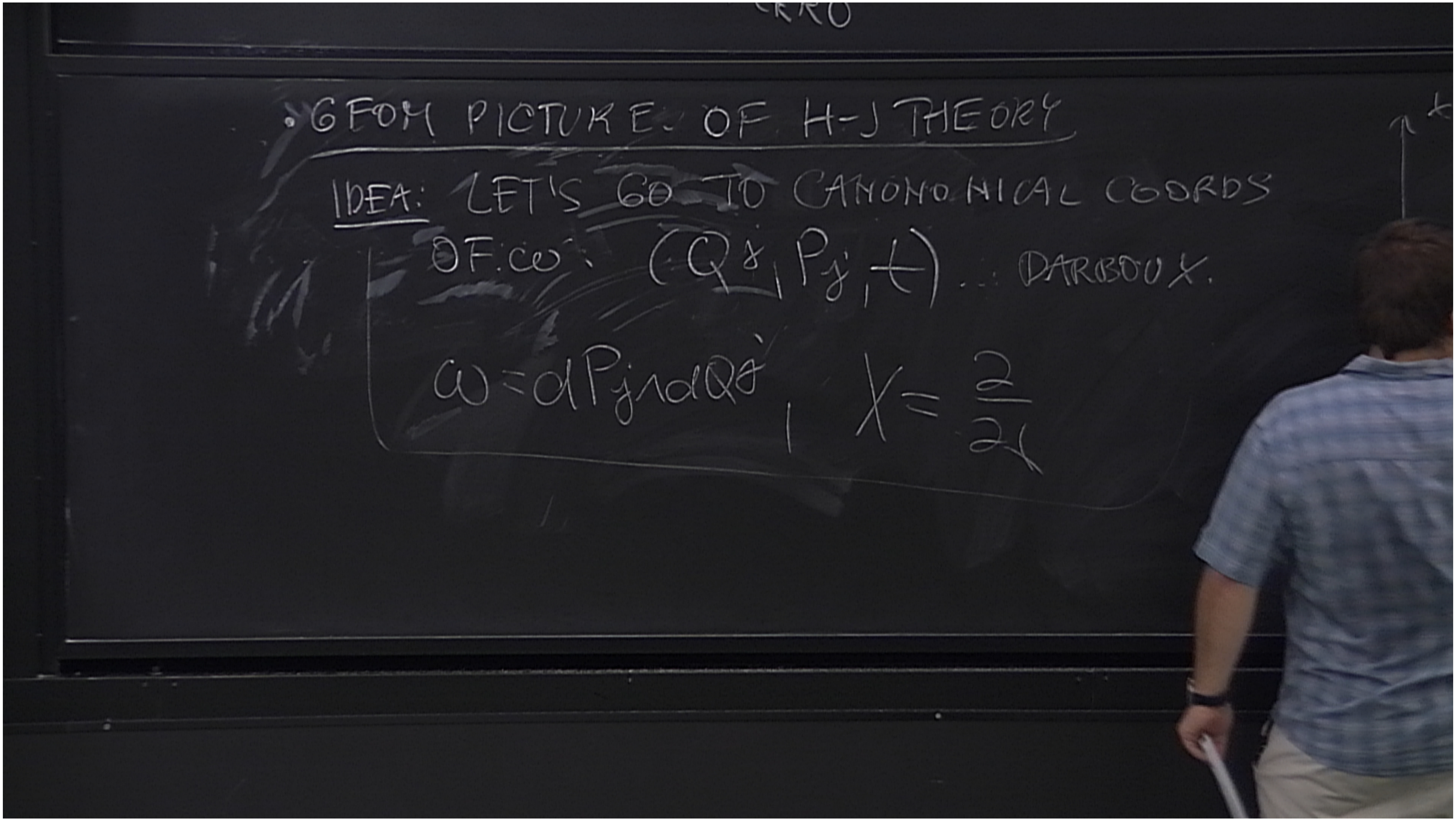


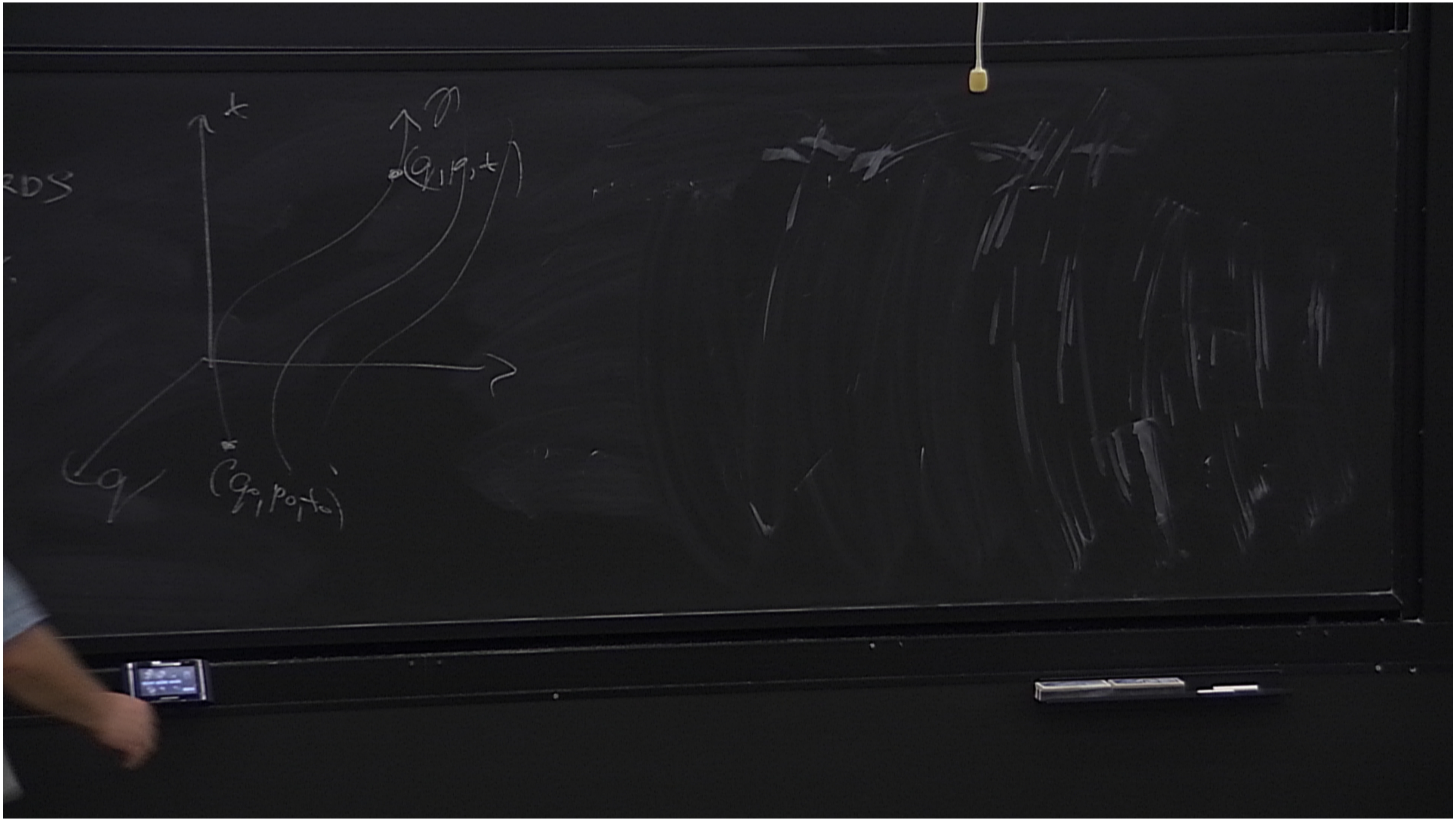
dF
$F(q, Q, t)$
$p_i dq^i - H dt - P_i dQ^i + H' dt = \frac{\partial F}{\partial q^i} dq^i + \frac{\partial F}{\partial Q^i} dQ^i + \frac{\partial F}{\partial t} dt$
$p_i = \frac{\partial F}{\partial q^i},$

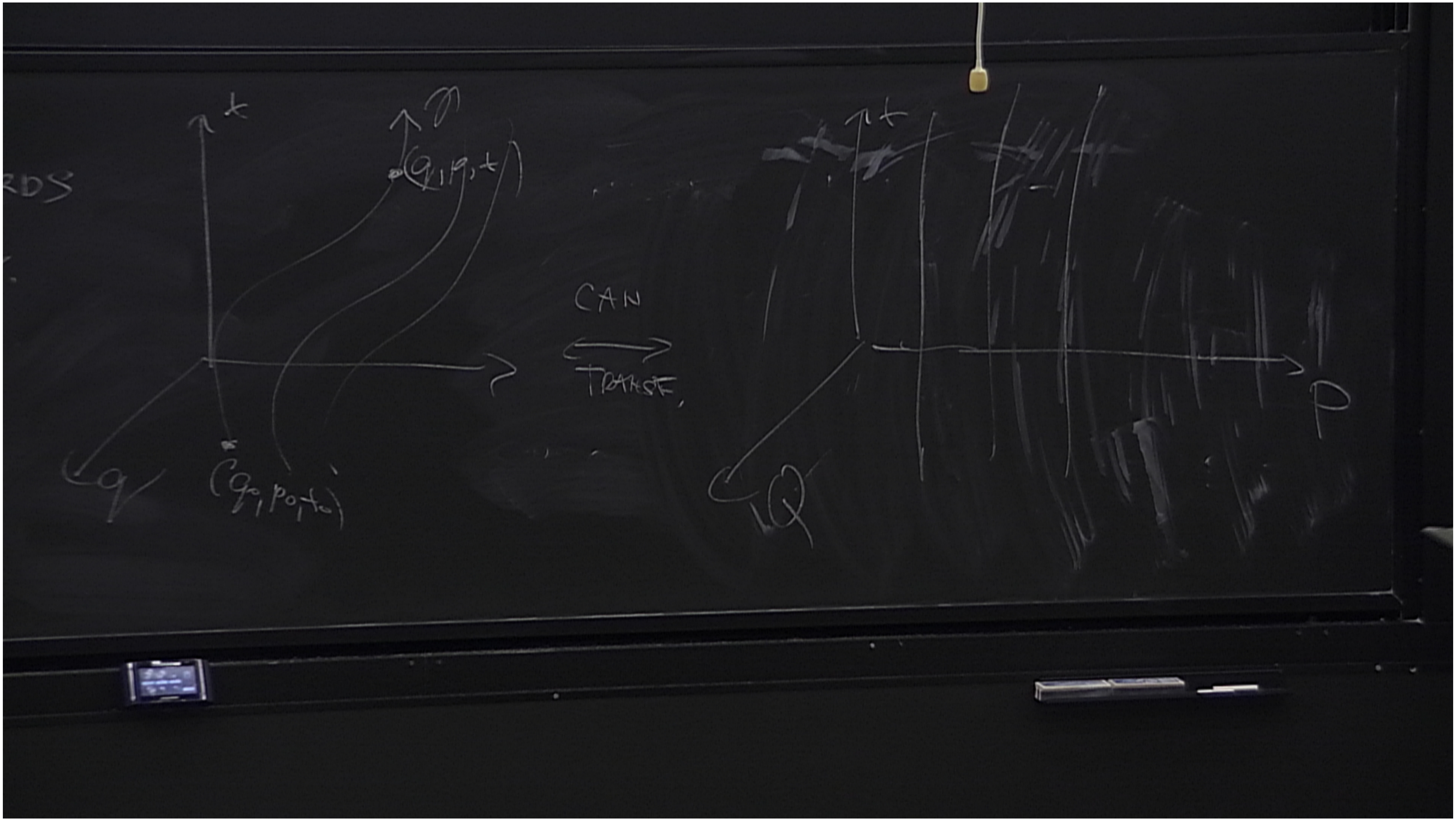


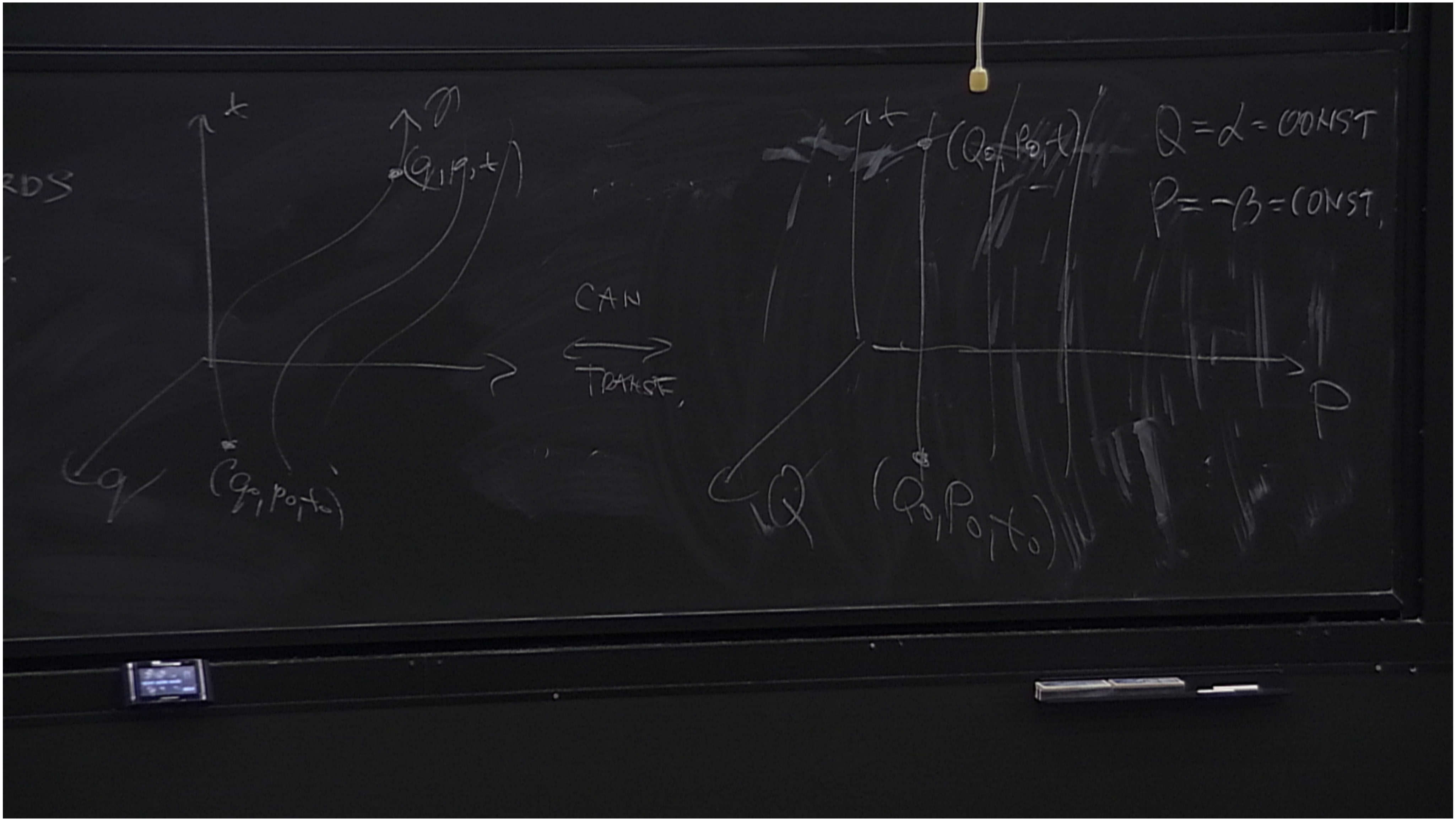
$\theta = p_i dq^i - H dt \quad . \quad d\theta = \omega$
$\tilde{\theta} = P_i dQ^i - H' dt \; ; \; d\tilde{\theta} = \omega$
$\} \; d(\underbrace{\theta - \tilde{\theta}}_{dF}) = 0$
$F(q, Q, t)$
$p_i dq^i - H dt - P_i dQ^i + H' dt = \frac{\partial F}{\partial q^i} dq^i + \frac{\partial F}{\partial Q^i} dQ^i + \frac{\partial F}{\partial t} dt$
$\boxed{p_i = \frac{\partial F}{\partial q^i}, \quad P_i = -\frac{\partial F}{\partial Q^i}, \quad H' - H = \frac{\partial F}{\partial t}}$

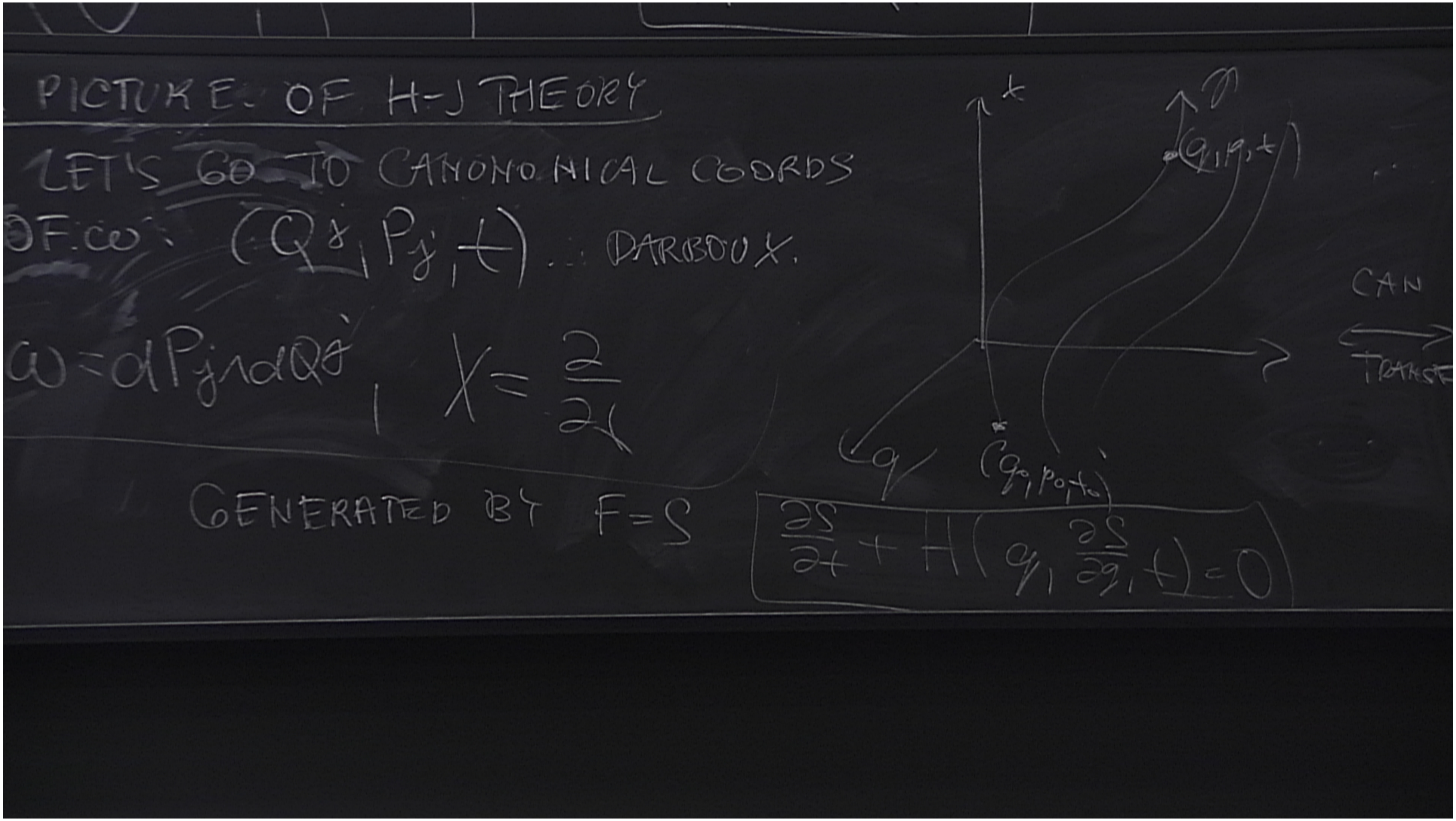

• GEOM PICTURE OF H-J THEORY
IDEA: LET'S GO TO CANONICAL COORDS
OF ω: (Q^i, P_j, t) ... DARBOUX.

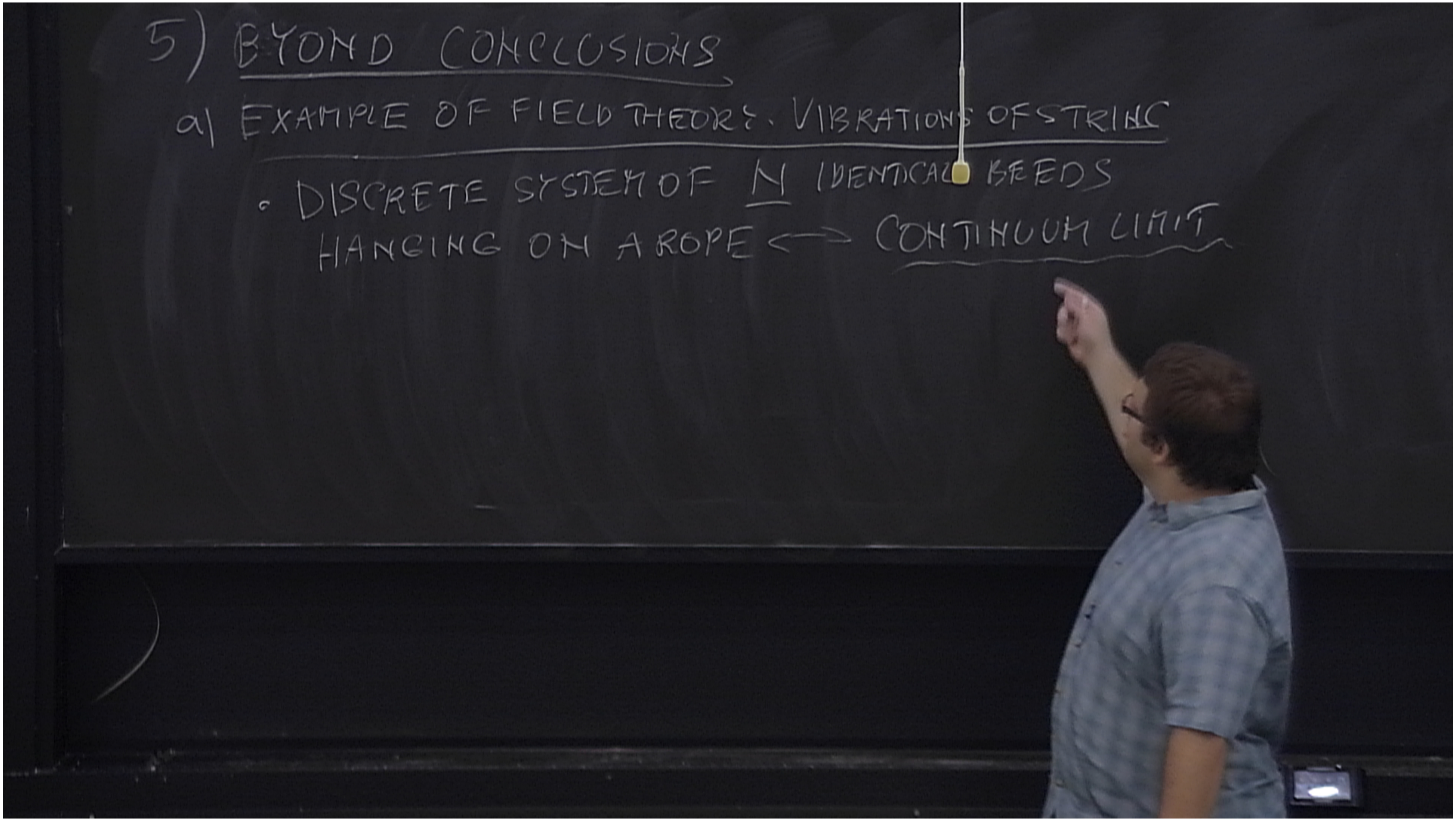

• GEOM PICTURE OF H-J THEORY
IDEA: LET'S GO TO CANONONICAL COORDS
OF ω: (Q^j, P_j, t) ... DARBOUX.
$\omega = dP_j \wedge dQ^j$, $X = \frac{\partial}{\partial t}$

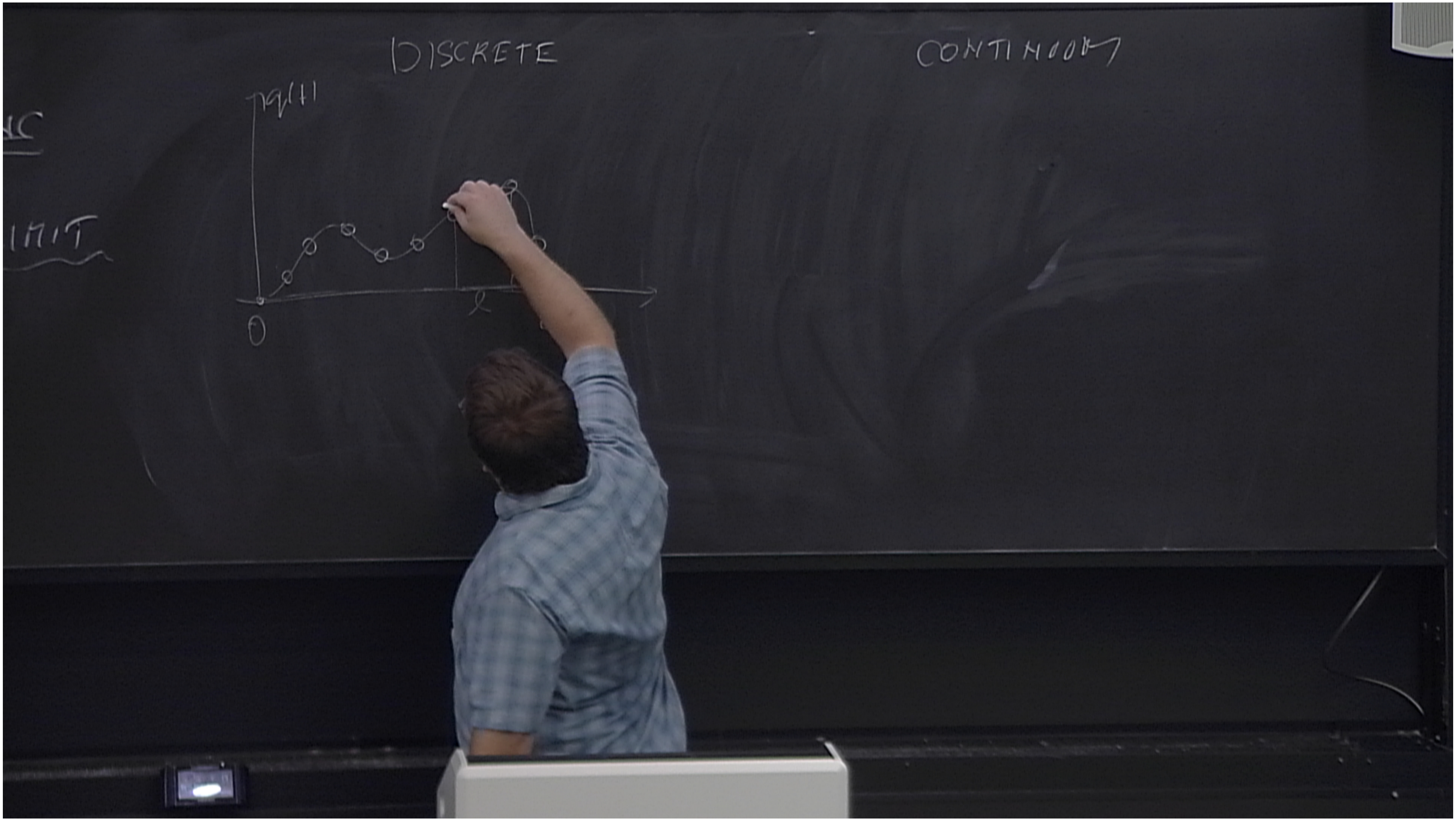

t
p
(q, p, t)
(q0, p0, t0)
q

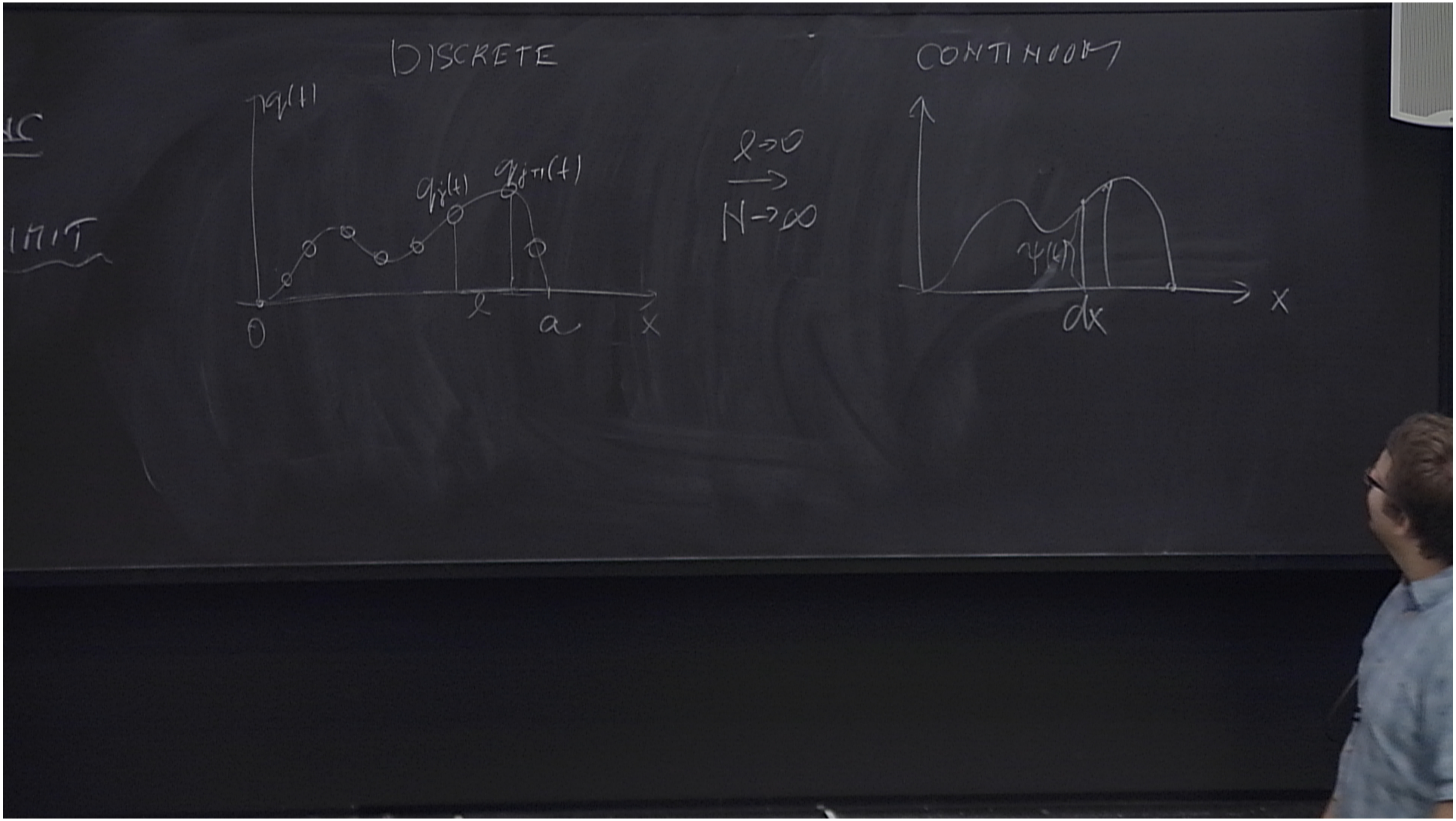

t
p
q
(q0, p0, t0)
CAN
TRANSF.
t
P
Q

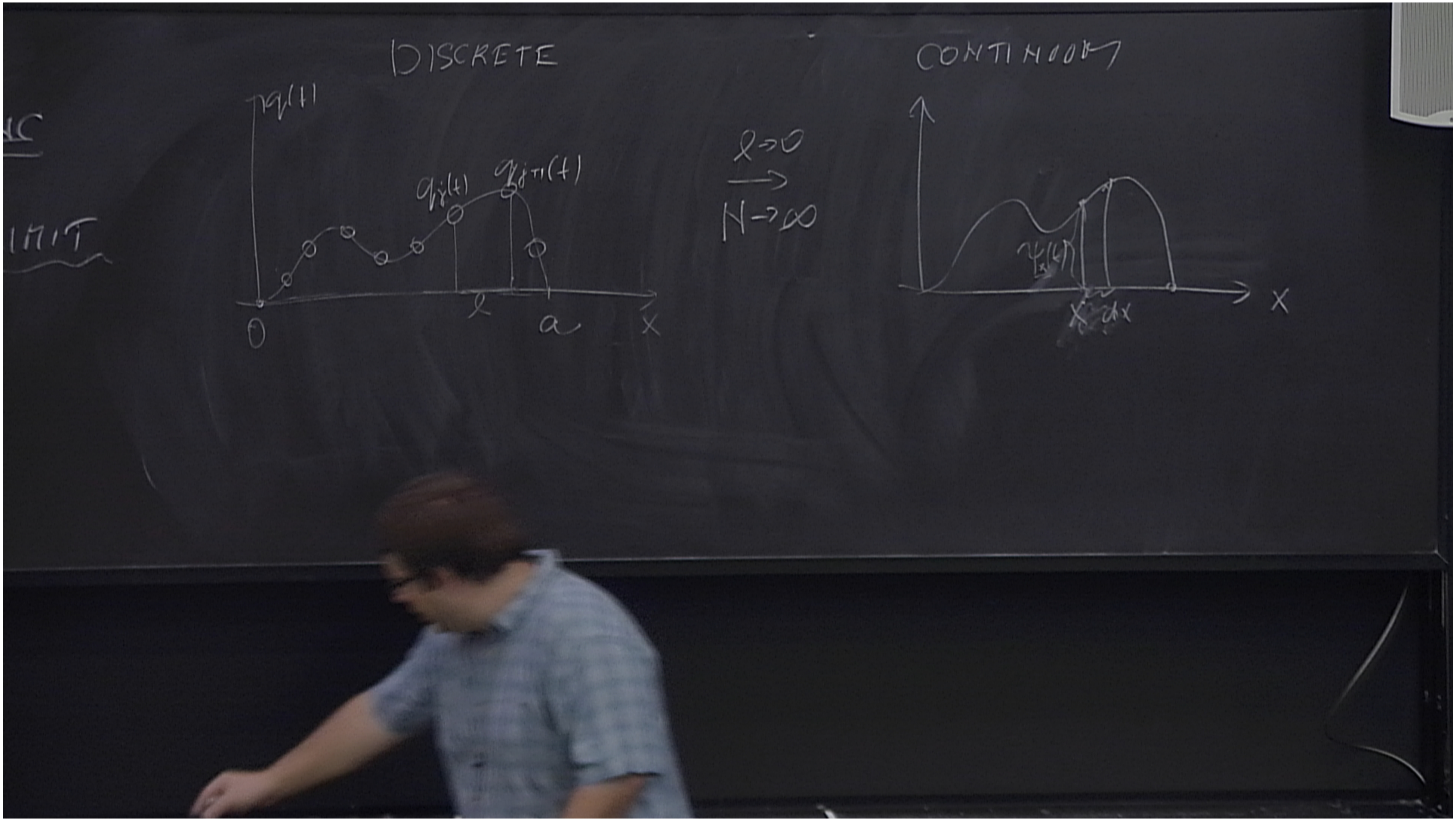

t
(q, p, t)
q
(q0, p0, t0)
CAN
TRANSF.
t
(Q0, P0, t)
Q = α = CONST
P = −β = CONST.
Q
P
(Q0, P0, t0)

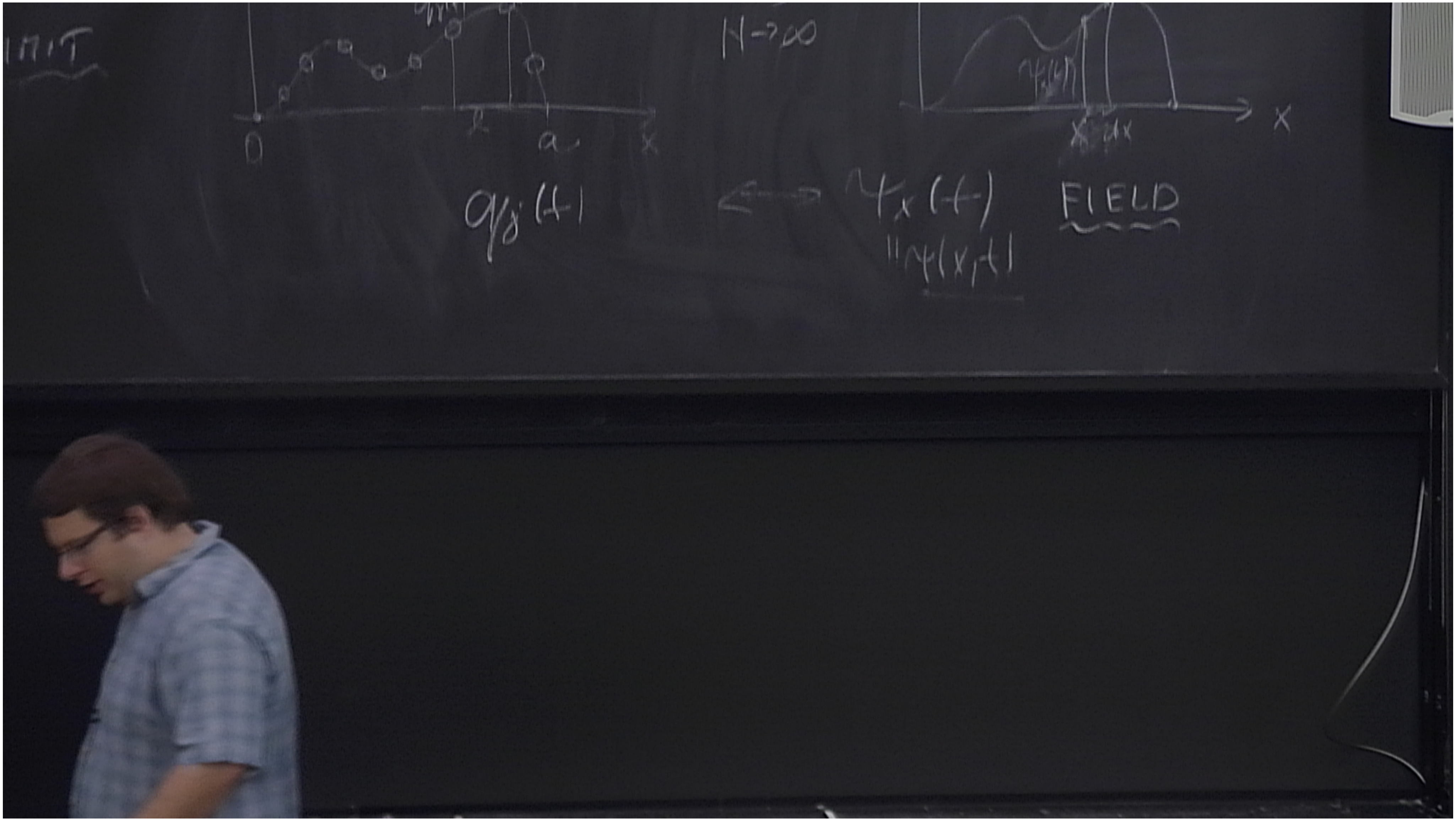

PICTURE OF H-J THEORY
LET'S GO TO CANONOMICAL COORDS
OF ω: (Q^j, P_j, t) ... DARBOUX.
$\omega = dP_j \wedge dQ^j$, $X = \frac{\partial}{\partial t}$
GENERATED BY $F = S$
$\frac{\partial S}{\partial t} + H\left(q, \frac{\partial S}{\partial q}, t\right) = 0$
t
p
(q, p, t)
q
(q_0, p_0, t_0)
CAN
TRANSF


5) BEYOND CONCLUSIONS
a) EXAMPLE OF FIELD THEORY: VIBRATIONS OF STRING
• DISCRETE SYSTEM OF N IDENTICAL BEEDS
HANGING ON A ROPE ⟷ CONTINUUM LIMIT


DISCRETE
CONTINUUM


DISCRETE
CONTINUUM
q(t)
$q_j(t)$
$q_{j+1}(t)$
0
ℓ
a
x
$\ell \to 0$
$N \to \infty$
$\psi(t)$
dx
x


DISCRETE
CONTINUUM
$q_j(t)$
$q_{j+1}(t)$
$l \to 0$
$N \to \infty$
0
l
a
X
$\psi_x(t)$
X
dx
X

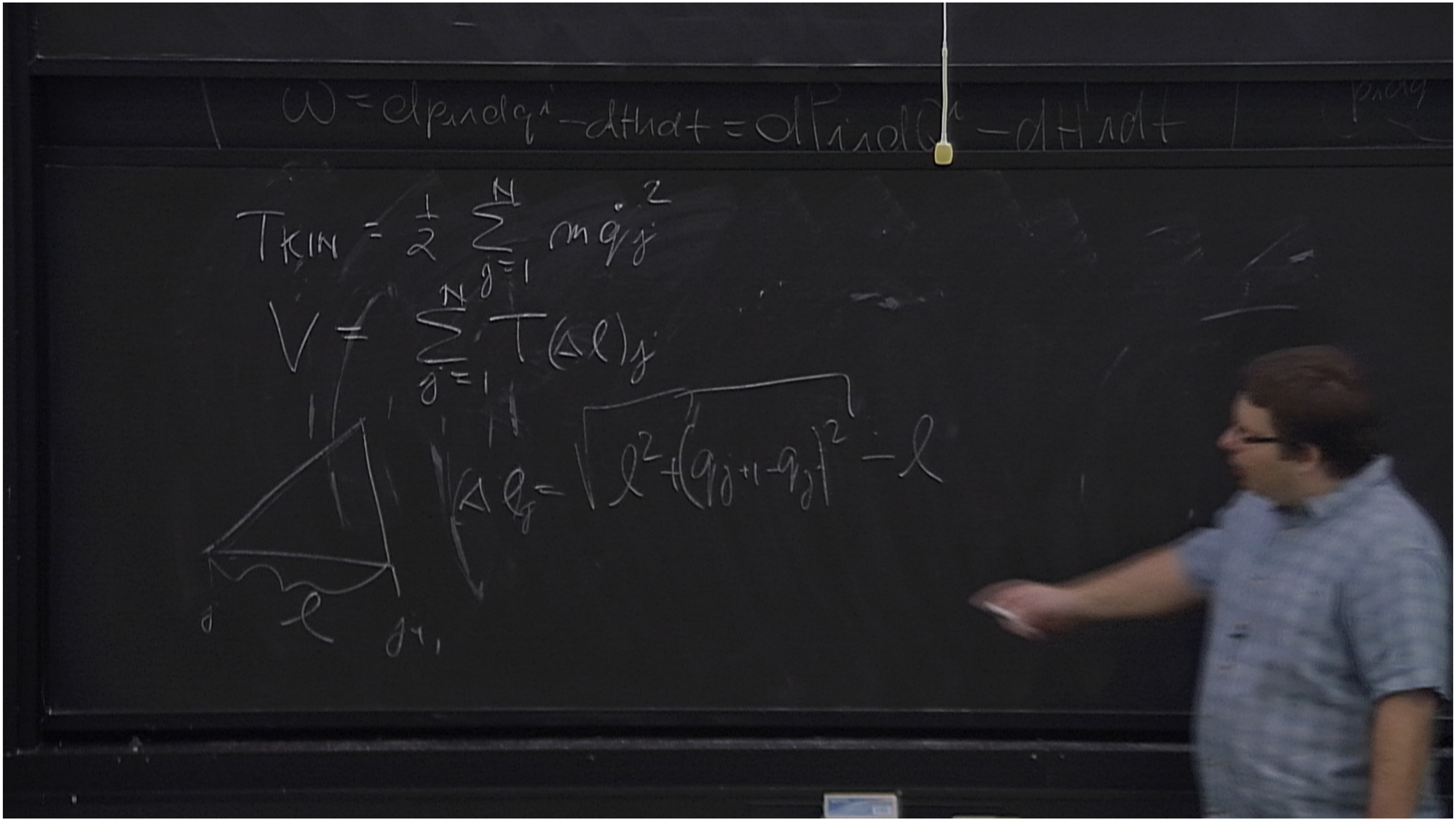


N→∞
$q_j(t)$
$\varphi_x(t)$
$\varphi(x,t)$
FIELD

$$\omega = dp_i \wedge dq^i - dH \wedge dt = dP_i \wedge dQ^i - dH' \wedge dt$$

$$T_{KIN} = \frac{1}{2} \sum_{j=1}^{N} m \dot{q}_j^{\,2}$$

$$V = \sum_{j=1}^{N} T$$

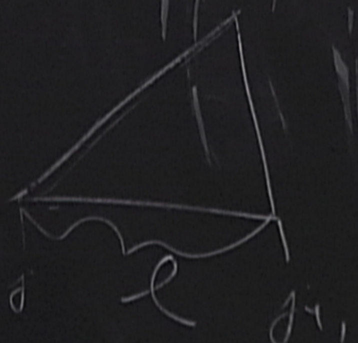

$\omega = dp_i \wedge dq^i - dH \wedge dt = dP_i \wedge dQ^i - dH' \wedge dt$
$T_{KIN} = \frac{1}{2} \sum_{j=1}^{N} m \dot{q}_j^2$
$V = \sum_{j=1}^{N} T (\Delta \ell)_j$
$(\Delta \ell)_j = \sqrt{\ell^2 + (q_{j+1} - q_j)^2} - \ell$
j
ℓ
$j+1$

$$\omega = dp_i \wedge dq^i - dH \wedge dt = dP_i \wedge dQ^i - dH' \wedge dt$$

$p \wedge dq$

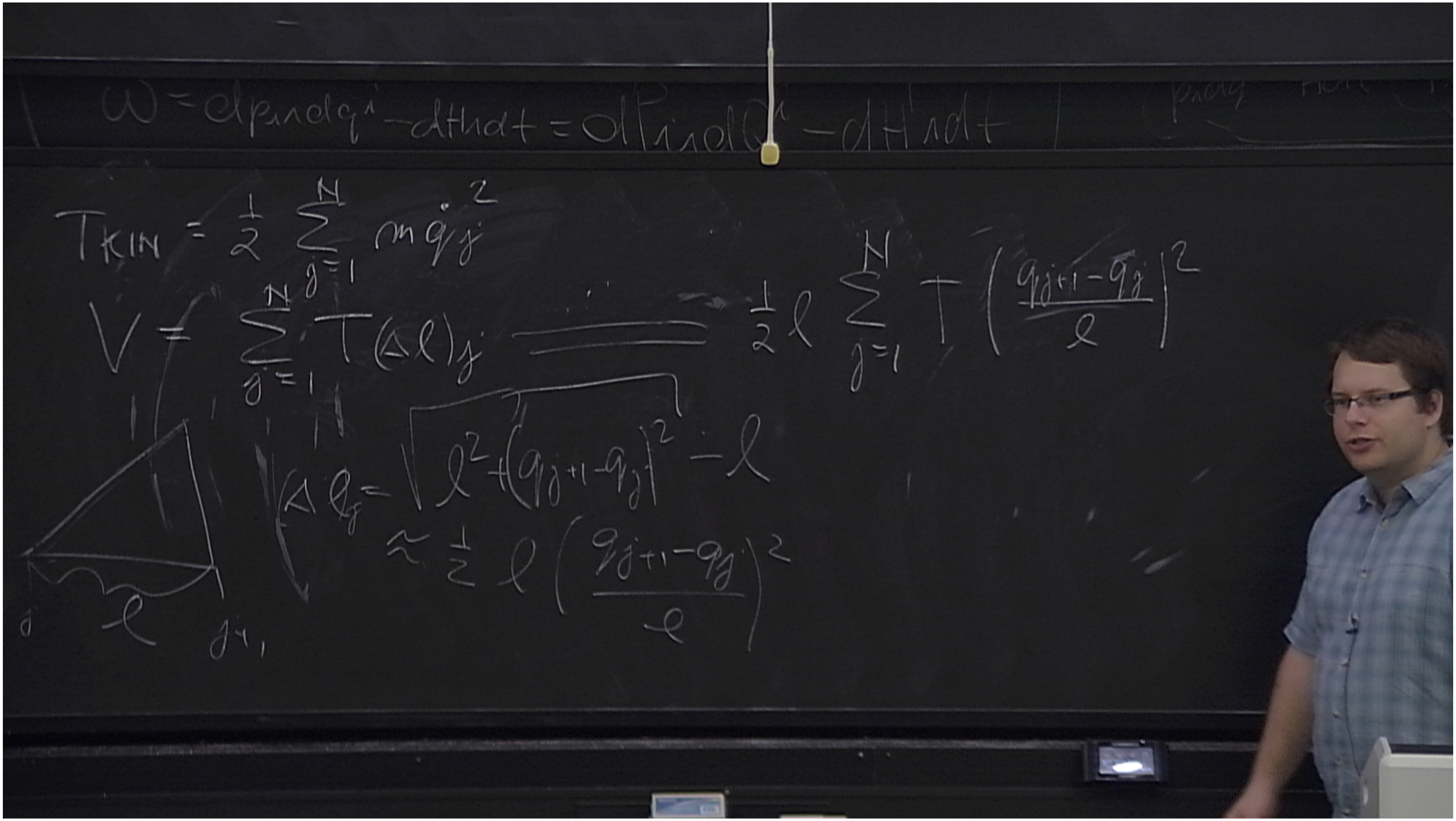
$$T_{KIN} = \frac{1}{2} \sum_{j=1}^{N} m \dot{q}_j^2$$

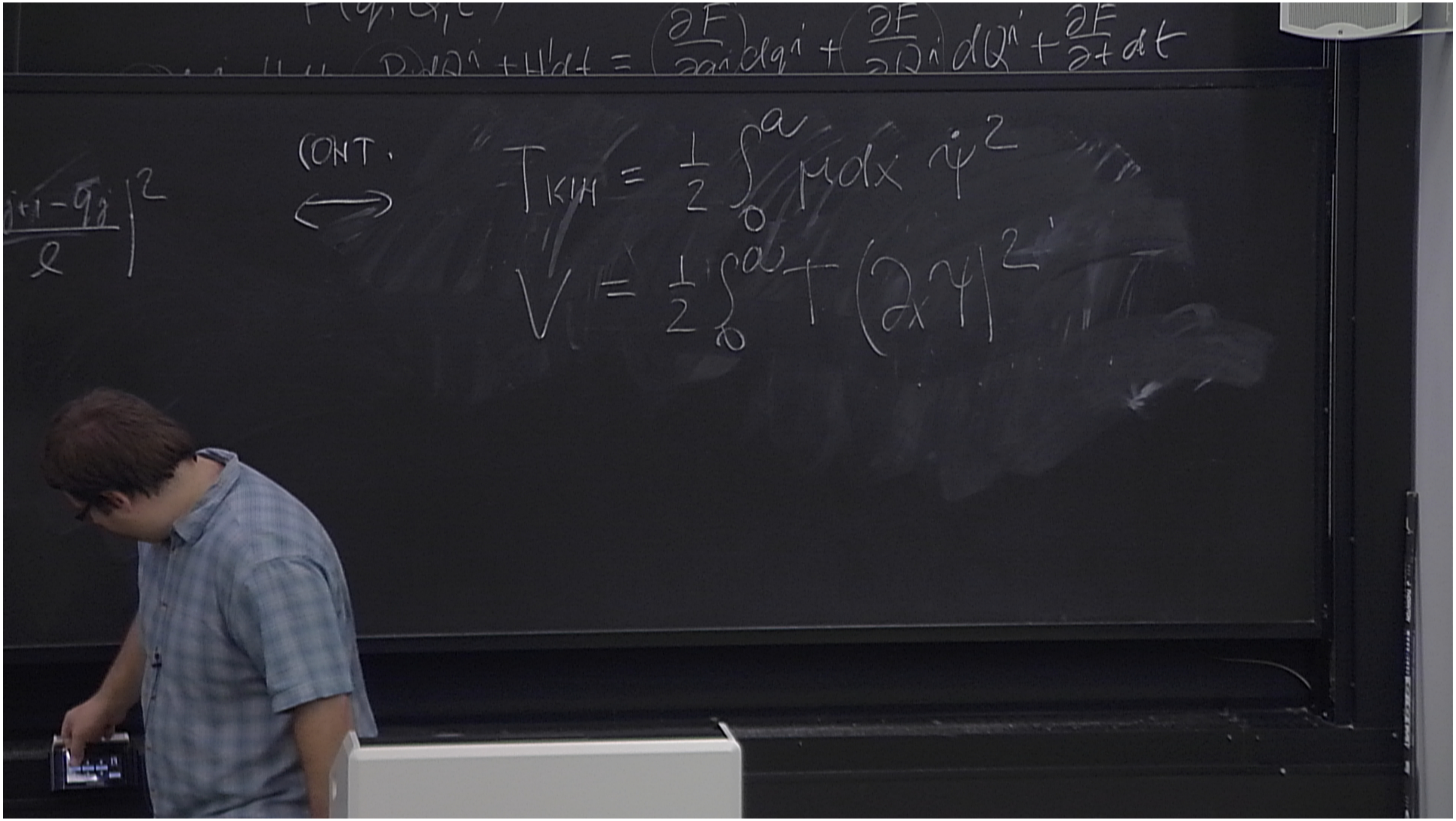
$$V = \sum_{j=1}^{N} T (\Delta l)_j$$

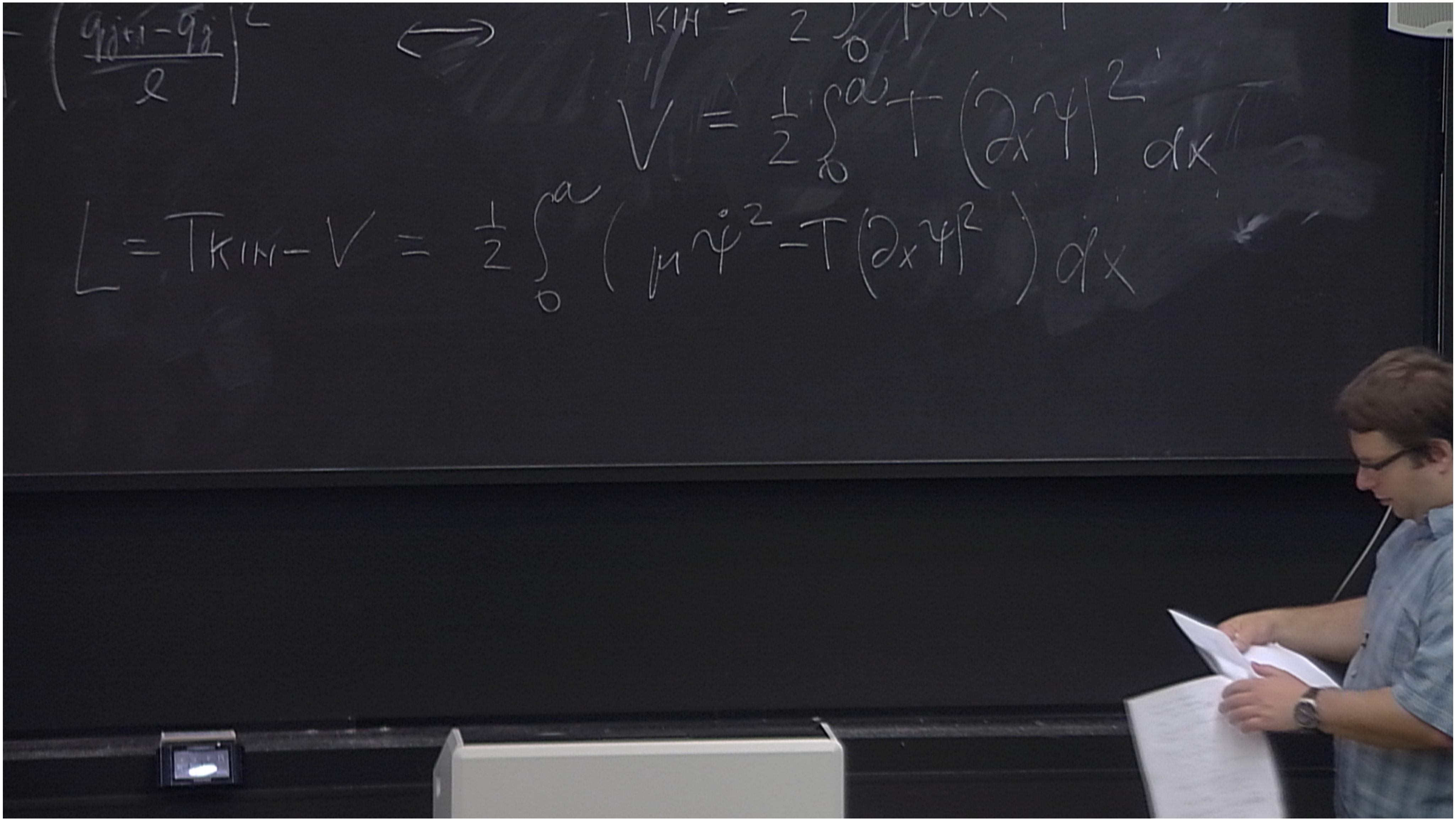


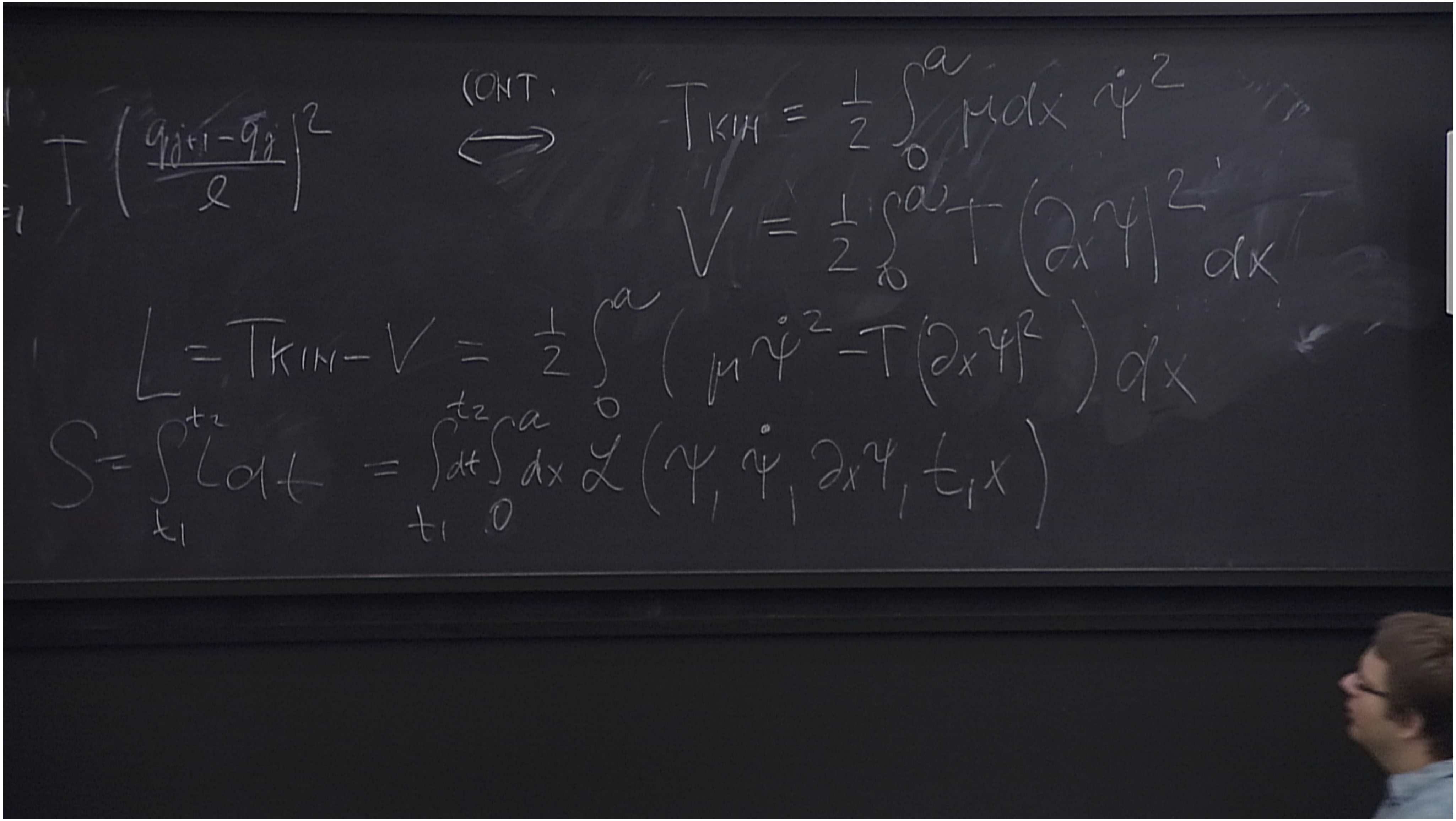
$$(\Delta l)_j = \sqrt{l^2 + (q_{j+1} - q_j)^2} - l$$

$$\approx \frac{1}{2} l \left(\frac{q_{j+1} - q_j}{l} \right)^2$$


$\omega = dp_i \wedge dq^i - dH \wedge dt = dP_i \wedge dQ^i - dH' \wedge dt$
$T_{KIN} = \frac{1}{2} \sum_{j=1}^{N} m \dot{q}_j^2$
$V = \sum_{j=1}^{N} T (\Delta \ell)_j = \frac{1}{2} \ell \sum_{j=1}^{N} T \left(\frac{q_{j+1} - q_j}{\ell} \right)^2$
$(\Delta \ell)_j = \sqrt{\ell^2 + (q_{j+1} - q_j)^2} - \ell$
$\approx \frac{1}{2} \ell \left(\frac{q_{j+1} - q_j}{\ell} \right)^2$
j
ℓ
$j+1$

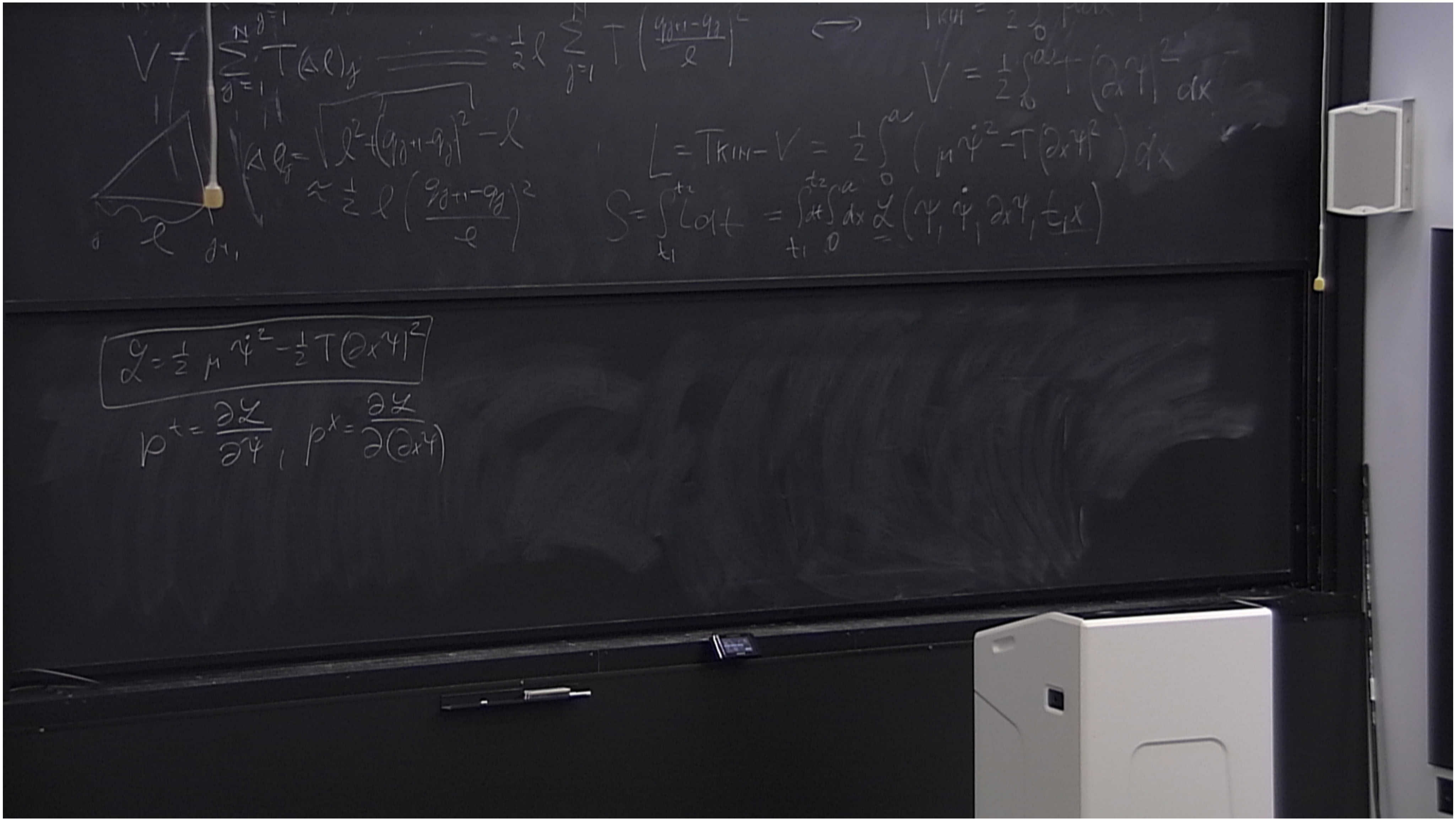

CONT.
$T_{KIN} = \frac{1}{2}\int_0^a \mu dx\, \dot{\psi}^2$
$V = \frac{1}{2}\int_0^a T\,(\partial_x \psi)^2$


$V = \frac{1}{2}\int_0^a T(\partial_x \psi)^2 dx$
$L = T_{kin} - V = \frac{1}{2}\int_0^a (\mu \dot{\psi}^2 - T(\partial_x \psi)^2) dx$

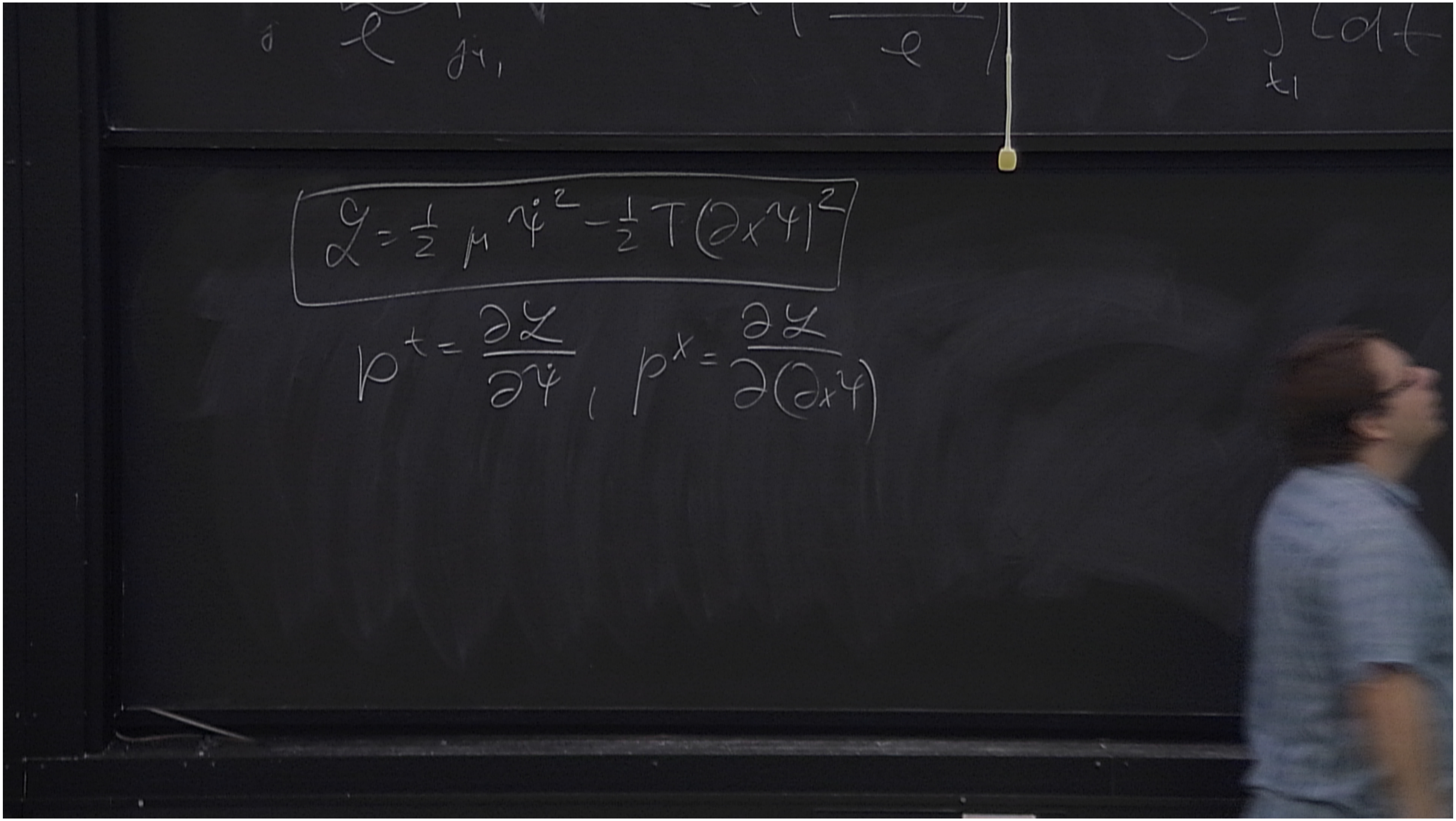

$T\left(\frac{q_{j+1}-q_j}{\ell}\right)^2$
CONT. $\longleftrightarrow$
$T_{KIN} = \frac{1}{2}\int_0^a \mu\, dx\, \dot{\psi}^2$
$V = \frac{1}{2}\int_0^a T(\partial_x\psi)^2\, dx$
$L = T_{KIN} - V = \frac{1}{2}\int_0^a \left(\mu\dot{\psi}^2 - T(\partial_x\psi)^2\right) dx$
$S = \int_{t_1}^{t_2} L\, dt = \int_{t_1}^{t_2} dt \int_0^a dx\, \mathcal{L}(\psi, \dot{\psi}, \partial_x\psi, t, x)$

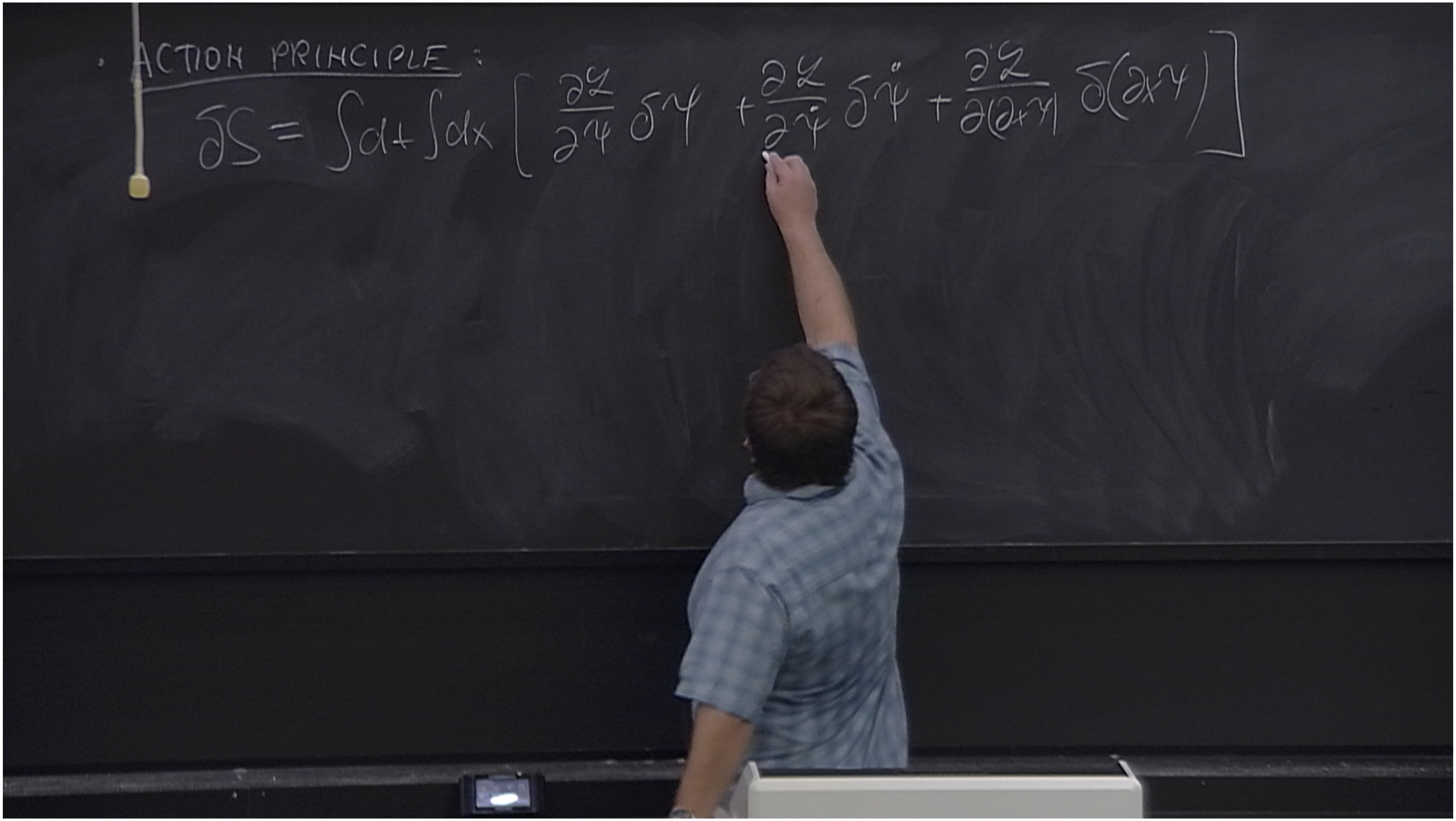
$$S = \int_{t_1} L\,dt$$

$$\boxed{\mathcal{L} = \frac{1}{2}\mu\dot{\psi}^2 - \frac{1}{2}T(\partial_x\psi)^2}$$

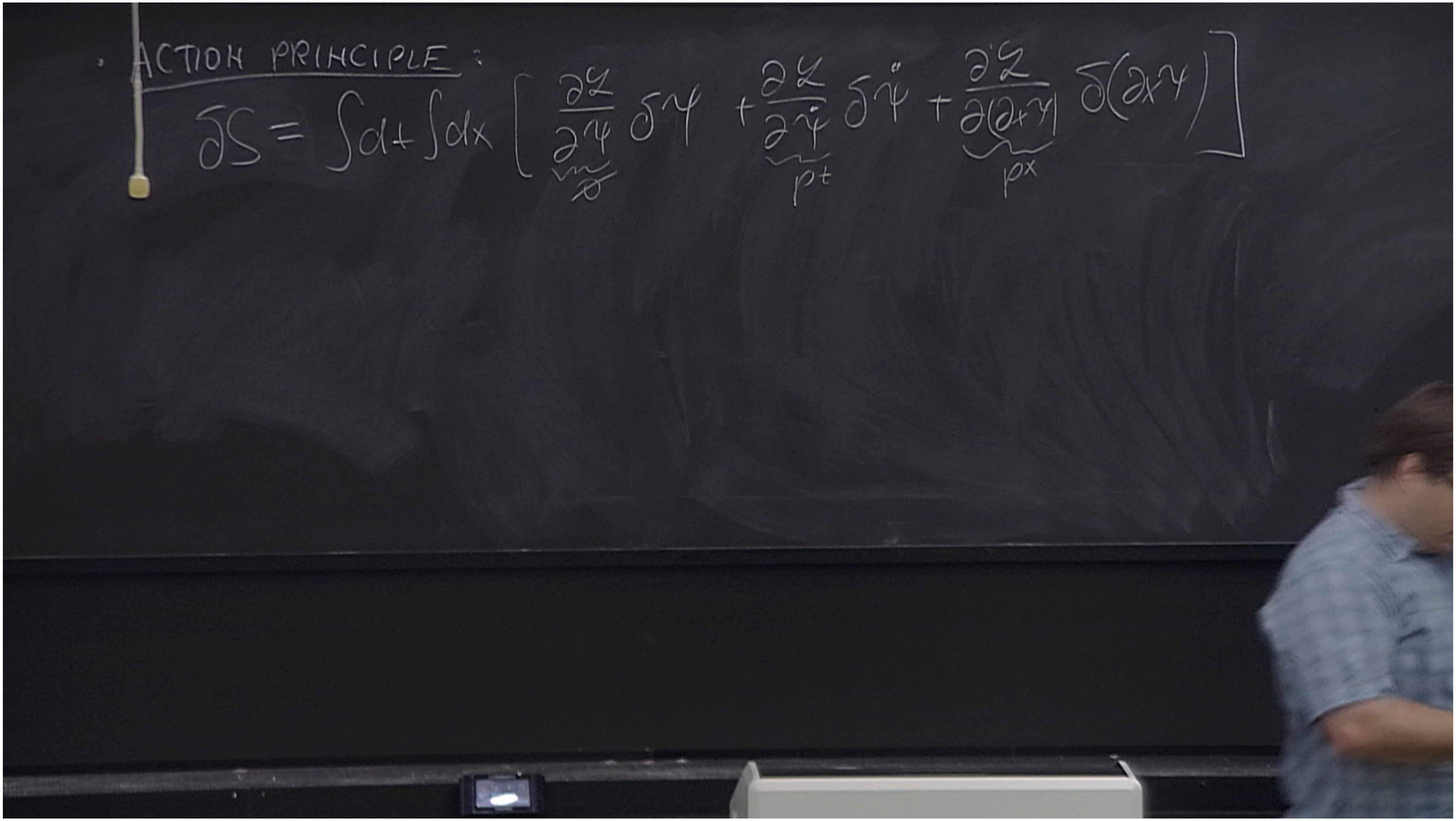


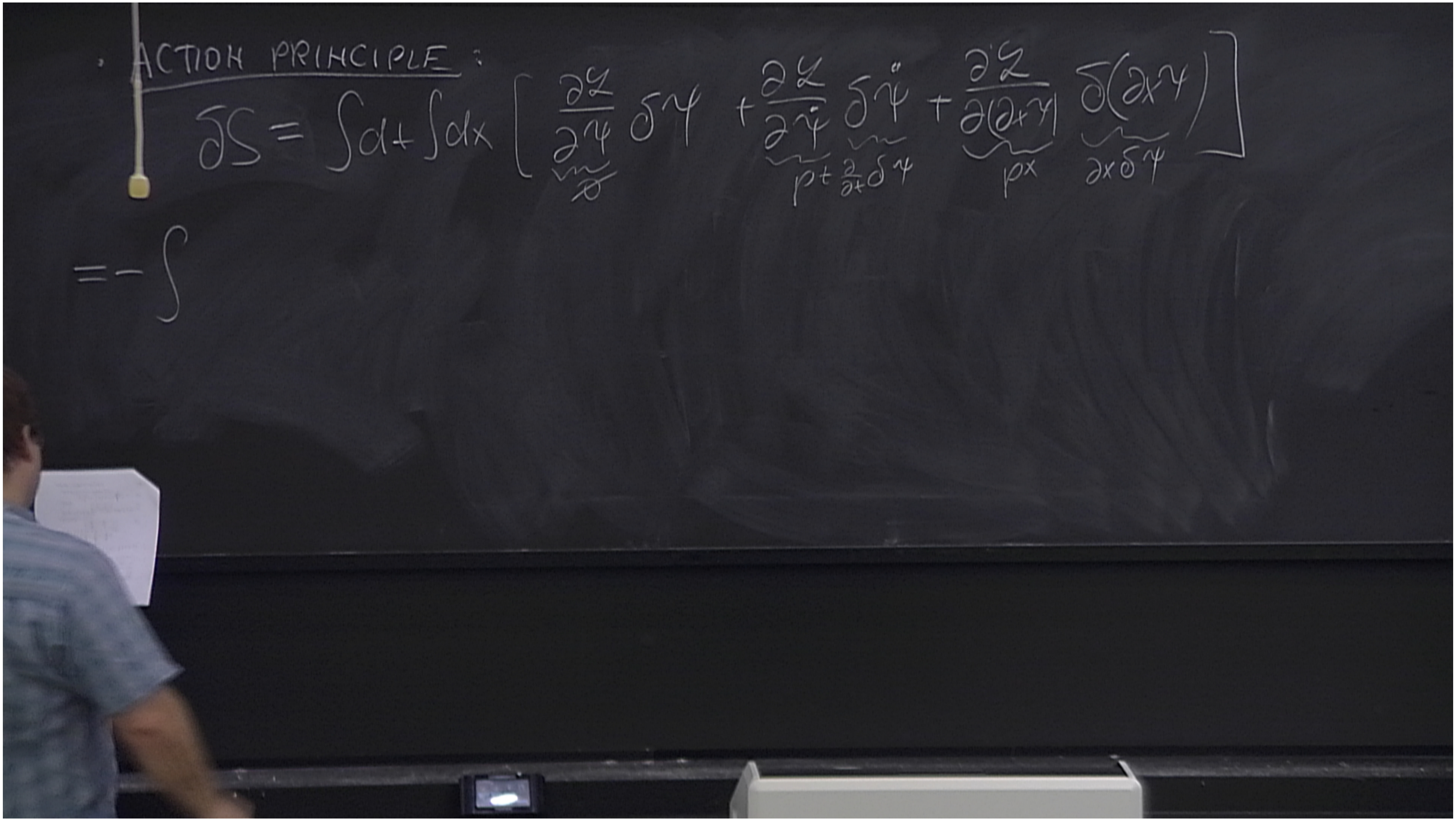
$V = \sum_{j=1}^{N} T(\Delta\ell)_j = \frac{1}{2}\ell \sum_{j=1}^{N} T\left(\frac{q_{j+1}-q_j}{\ell}\right)^2 \longleftrightarrow V = \frac{1}{2}\int_0^a T(\partial_x\psi)^2\,dx$
$\Delta\ell_j = \sqrt{\ell^2+(q_{j+1}-q_j)^2}-\ell \approx \frac{1}{2}\ell\left(\frac{q_{j+1}-q_j}{\ell}\right)^2$
$L = T_{KIN} - V = \frac{1}{2}\int_0^a \left(\mu\dot\psi^2 - T(\partial_x\psi)^2\right)dx$
$S = \int_{t_1}^{t_2} L\,dt = \int_{t_1}^{t_2} dt\int_0^a dx\,\mathcal{L}(\psi,\dot\psi,\partial_x\psi,t,x)$
$\boxed{\mathcal{L} = \frac{1}{2}\mu\dot\psi^2 - \frac{1}{2}T(\partial_x\psi)^2}$
$p^t = \frac{\partial\mathcal{L}}{\partial\dot\psi},\quad p^x = \frac{\partial\mathcal{L}}{\partial(\partial_x\psi)}$


$\boxed{\mathcal{L} = \frac{1}{2}\mu\dot{\psi}^2 - \frac{1}{2}T(\partial_x\psi)^2}$
$p^t = \frac{\partial\mathcal{L}}{\partial\dot{\psi}}, \quad p^x = \frac{\partial\mathcal{L}}{\partial(\partial_x\psi)}$



· ACTION PRINCIPLE:
$\delta S = \int dt \int dx \left[\frac{\partial \mathcal{L}}{\partial \varphi} \delta \varphi + \frac{\partial \mathcal{L}}{\partial \dot{\varphi}} \delta \dot{\varphi} + \frac{\partial \mathcal{L}}{\partial (\partial_x \varphi)} \delta (\partial_x \varphi) \right]$


ACTION PRINCIPLE:
$\delta S = \int dt \int dx \left[\frac{\partial \mathcal{L}}{\partial \varphi} \delta \varphi + \underbrace{\frac{\partial \mathcal{L}}{\partial \dot{\varphi}}}_{p^t} \delta \dot{\varphi} + \underbrace{\frac{\partial \mathcal{L}}{\partial (\partial_x \varphi)}}_{p^x} \delta (\partial_x \varphi) \right]$

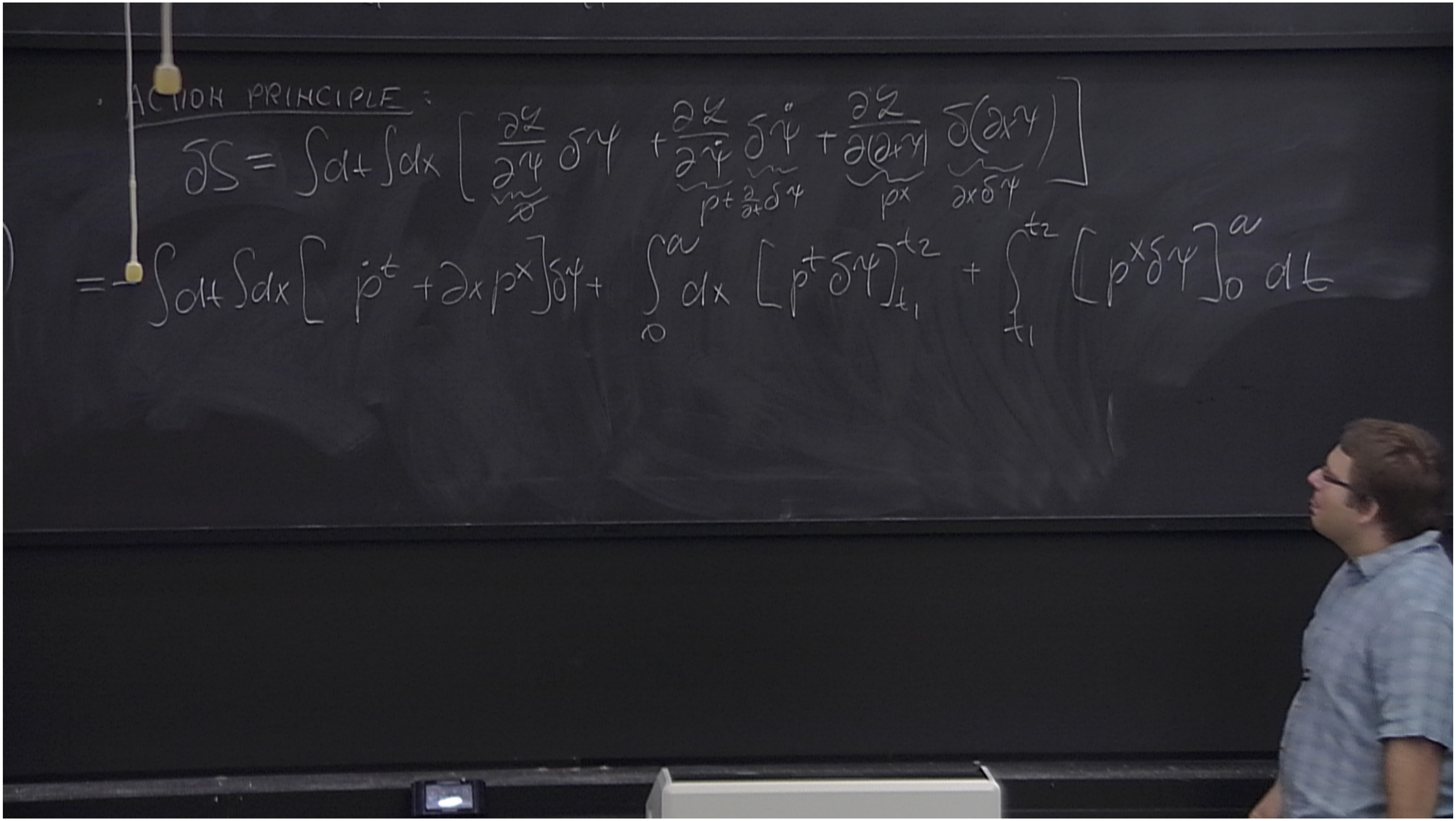


ACTION PRINCIPLE:
$\delta S = \int dt \int dx \left[\underbrace{\frac{\partial \mathcal{L}}{\partial \psi}} \delta \psi + \underbrace{\frac{\partial \mathcal{L}}{\partial \dot{\psi}}}_{p_t} \underbrace{\delta \dot{\psi}}_{\frac{\partial}{\partial t}\delta\psi} + \underbrace{\frac{\partial \mathcal{L}}{\partial (\partial_x \psi)}}_{p_x} \underbrace{\delta(\partial_x \psi)}_{\partial_x \delta \psi} \right]$
$= - \int$

• ACTION PRINCIPLE:

$$\delta S = \int dt \int dx \left[\frac{\partial \mathcal{L}}{\partial \varphi} \delta \varphi + \underbrace{\frac{\partial \mathcal{L}}{\partial \dot{\varphi}}}_{p^t} \underbrace{\delta \dot{\varphi}}_{\frac{\partial}{\partial t} \delta \varphi} + \underbrace{\frac{\partial \mathcal{L}}{\partial (\partial_x \varphi)}}_{p^x} \underbrace{\delta (\partial_x \varphi)}_{\partial_x \delta \varphi} \right]$$

$$= - \int dt \int dx \left[\dot{p}^t + \partial_x p^x \right] +$$



ACTION PRINCIPLE:
$\delta S = \int dt \int dx \left[\underbrace{\frac{\partial \mathcal{L}}{\partial \psi}}_{\varnothing} \delta\psi + \underbrace{\frac{\partial \mathcal{L}}{\partial \dot\psi}}_{p^t} \underbrace{\delta\dot\psi}_{\frac{\partial}{\partial t}\delta\psi} + \underbrace{\frac{\partial \mathcal{L}}{\partial(\partial_x\psi)}}_{p^x} \underbrace{\delta(\partial_x\psi)}_{\partial_x\delta\psi} \right]$
$= -\int dt \int dx \left[\dot p^t + \partial_x p^x \right] \delta\psi + \int_0^a dx \left[p^t \delta\psi \right]_{t_1}^{t_2} + \int_{t_1}^{t_2} \left[p^x \delta\psi \right]_0^a dt$

• ACTION PRINCIPLE:

$$\delta S = \int dt \int dx \left[\frac{\partial \mathcal{L}}{\partial \psi} \delta\psi + \underbrace{\frac{\partial \mathcal{L}}{\partial \dot{\psi}}}_{p^t} \underbrace{\delta\dot{\psi}}_{\frac{\partial}{\partial t}\delta\psi} + \underbrace{\frac{\partial \mathcal{L}}{\partial(\partial_x \psi)}}_{p^x} \underbrace{\delta(\partial_x \psi)}_{\partial_x \delta\psi} \right]$$

$$= -\int_{t_1}^{t_2} dt \int_0^a dx \left[\dot{p}^t + \partial_x p^x \right] \delta\psi + \int_0^a dx \left[p^t \delta\psi \right]_{t_1}^{t_2} + \int_{t_1}^{t_2} \left[p^x \delta\psi \right]_0^a dt$$

· ACTION PRINCIPLE :

$$\delta S = \int dt \int dx \left[\underbrace{\frac{\partial \mathcal{L}}{\partial \psi}}_{\emptyset} \delta\psi + \underbrace{\frac{\partial \mathcal{L}}{\partial \dot\psi}}_{p^t} \underbrace{\delta\dot\psi}_{\frac{\partial}{\partial t}\delta\psi} + \underbrace{\frac{\partial \mathcal{L}}{\partial(\partial_x\psi)}}_{p^x} \underbrace{\delta(\partial_x\psi)}_{\partial_x \delta\psi} \right]$$

$$= -\int_{t_1}^{t_2} dt \int_0^a dx \left[\underbrace{\dot p^t + \partial_x p^x}_{\emptyset} \right] \delta\psi + \int_0^a dx \left[p^t \delta\psi \right]_{t_1}^{t_2} + \int_{t_1}^{t_2} \left[p^x \delta\psi \right]_0^a dt$$

$$\boxed{\mathcal{L} = \frac{1}{2}\mu\dot{\psi}^2 - \frac{1}{2}T(\partial_x\psi)^2}$$

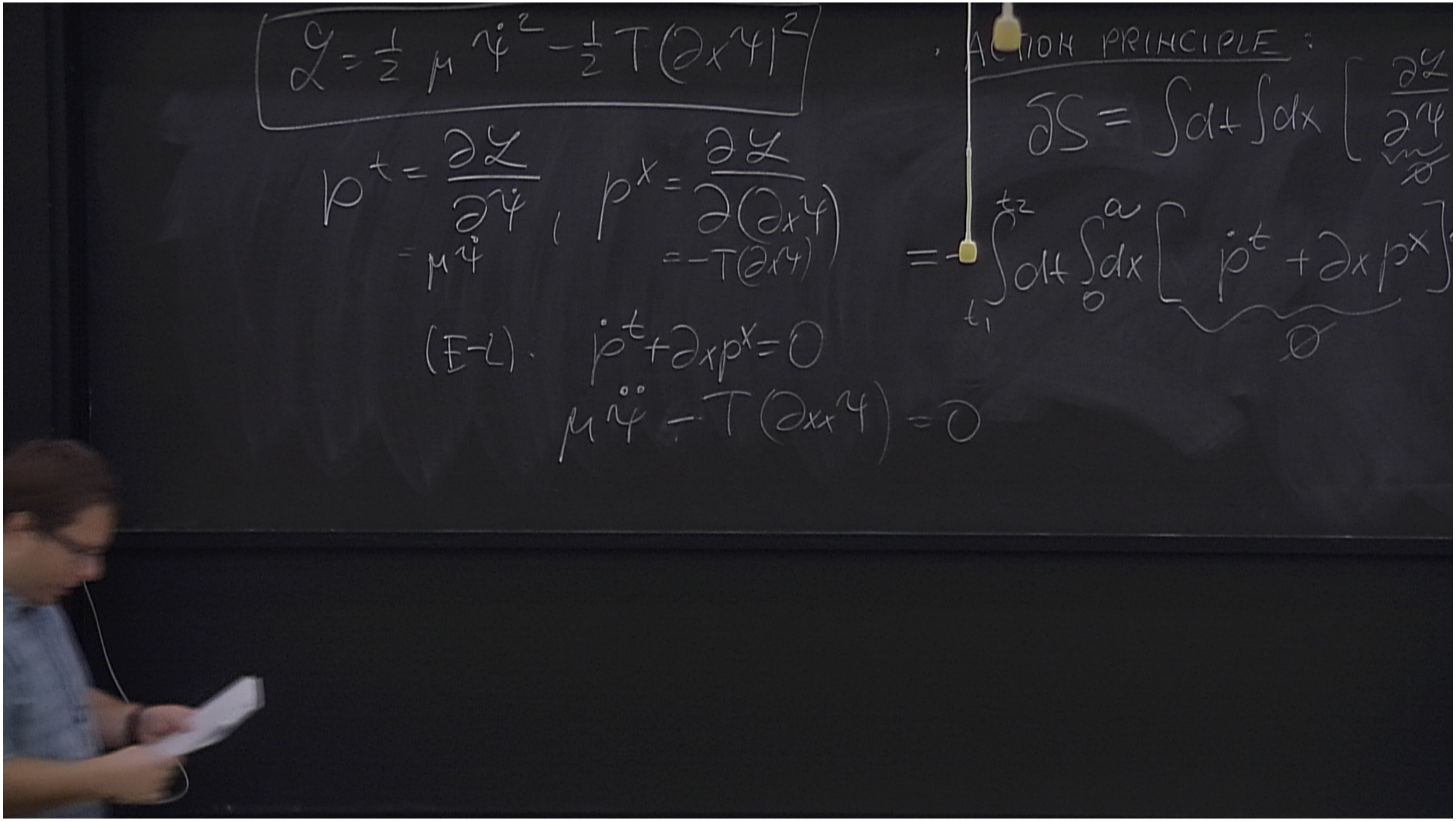
$$p^t = \frac{\partial\mathcal{L}}{\partial\dot{\psi}}, \quad p^x = \frac{\partial\mathcal{L}}{\partial(\partial_x\psi)}$$

$$(E\text{-}L): \quad \dot{p}^t + \partial_x p^x = 0$$

ACTION PRINCIPLE:

$$\delta S = \int dt \int dx \left[\underbrace{\frac{\partial\mathcal{L}}{\partial\psi}}_{0} \right.$$

$$= -\int_{t_1}^{t_2} dt \int_0^a dx \Big[\underbrace{\dot{p}^t + \partial_x p^x}_{0} \Big]$$


$\mathcal{L} = \frac{1}{2}\mu\dot{\psi}^2 - \frac{1}{2}T(\partial_x\psi)^2$
$p^t = \frac{\partial\mathcal{L}}{\partial\dot{\psi}} = \mu\dot{\psi}$, $p^x = \frac{\partial\mathcal{L}}{\partial(\partial_x\psi)} = -T(\partial_x\psi)$
(E-L): $\dot{p}^t + \partial_x p^x = 0$
$\mu\ddot{\psi} - T(\partial_{xx}\psi) = 0$
ACTION PRINCIPLE:
$\delta S = \int dt \int dx \left[\frac{\partial\mathcal{L}}{\partial\psi} \right.$
$= -\int_{t_1}^{t_2} dt \int_0^a dx \left[\dot{p}^t + \partial_x p^x \right]$

$$\boxed{\ldots = \frac{1}{2}\mu\dot{\psi}^2 - \frac{1}{2}T(\partial_x\psi)^2}$$

$$p^t = \frac{\partial\mathcal{L}}{\partial\dot{\psi}} = \mu\dot{\psi}, \quad p^x = \frac{\partial\mathcal{L}}{\partial(\partial_x\psi)} = -T(\partial_x\psi)$$

(E-L): $\dot{p}^t + \partial_x p^x = 0$

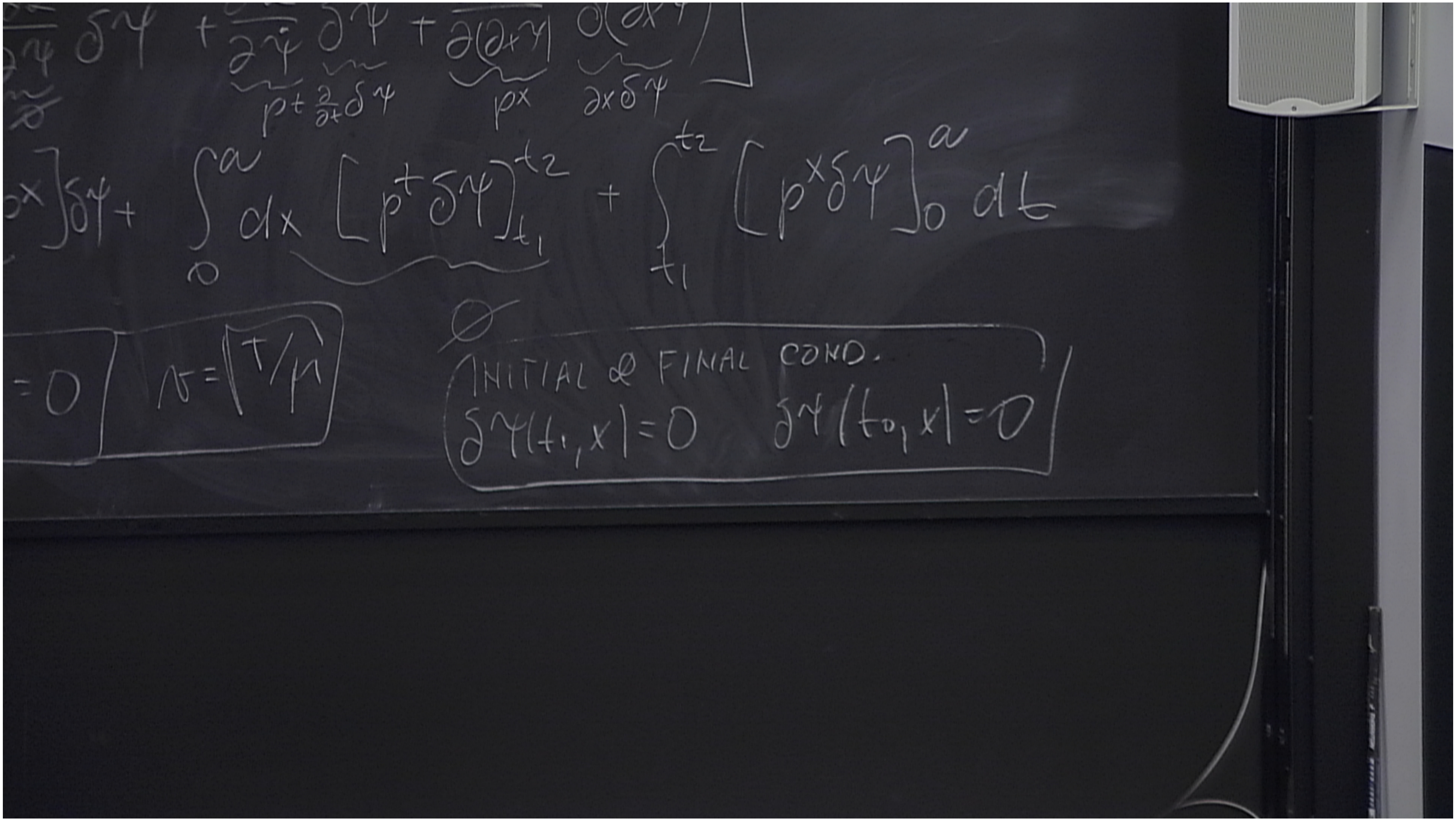
$$\mu\ddot{\psi} - T(\partial_{xx}\psi) = 0$$

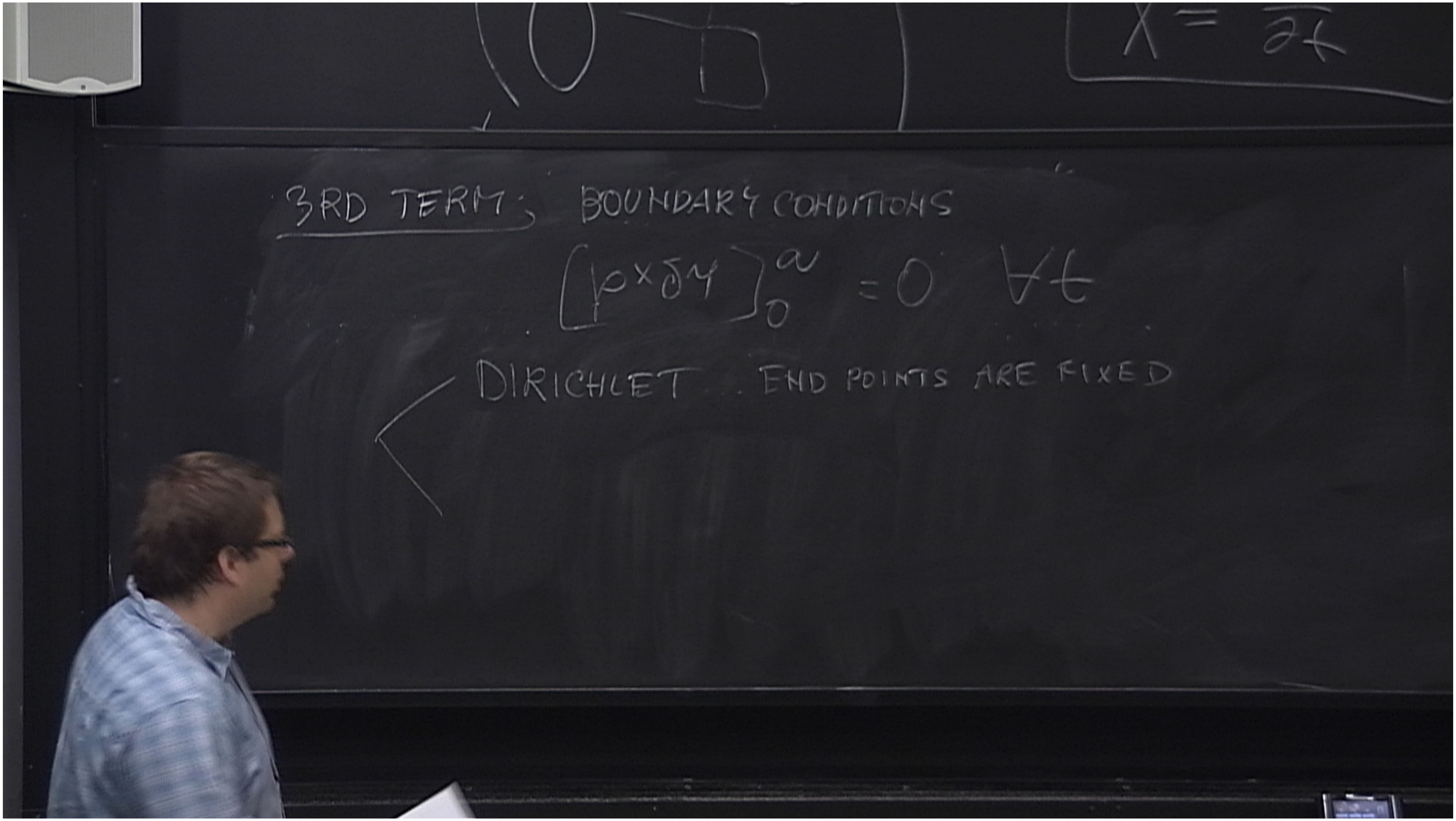
ACTION PRINCIPLE:

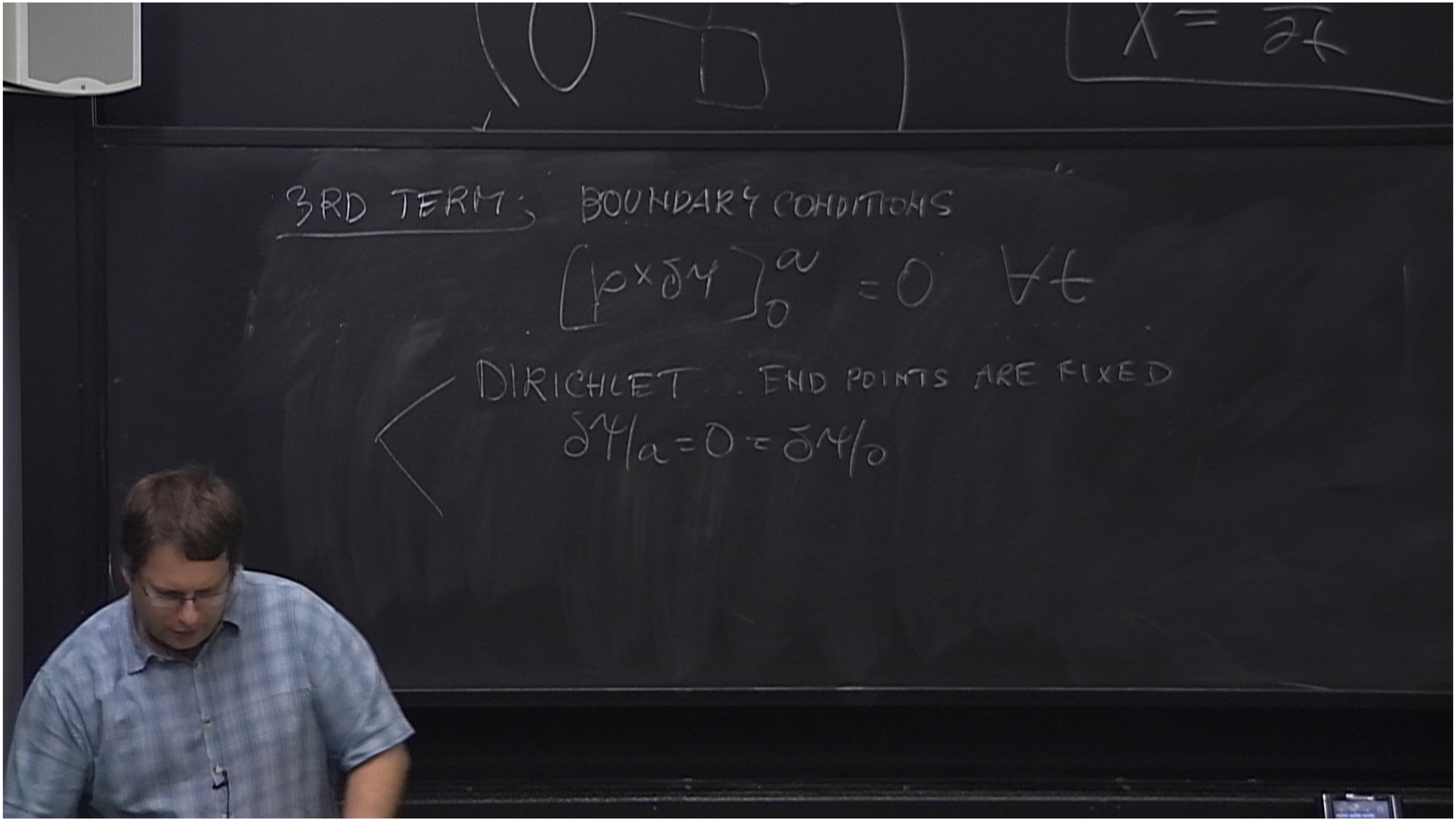
$$\delta S = \int dt \int dx \left[\underbrace{\frac{\partial\mathcal{L}}{\partial\psi}}_{0}\delta\psi + \underbrace{\frac{\partial\mathcal{L}}{\partial\dot{\psi}}\delta\dot{\psi}}_{p^t \frac{\partial}{\partial t}\delta\psi} + \ldots \right.$$

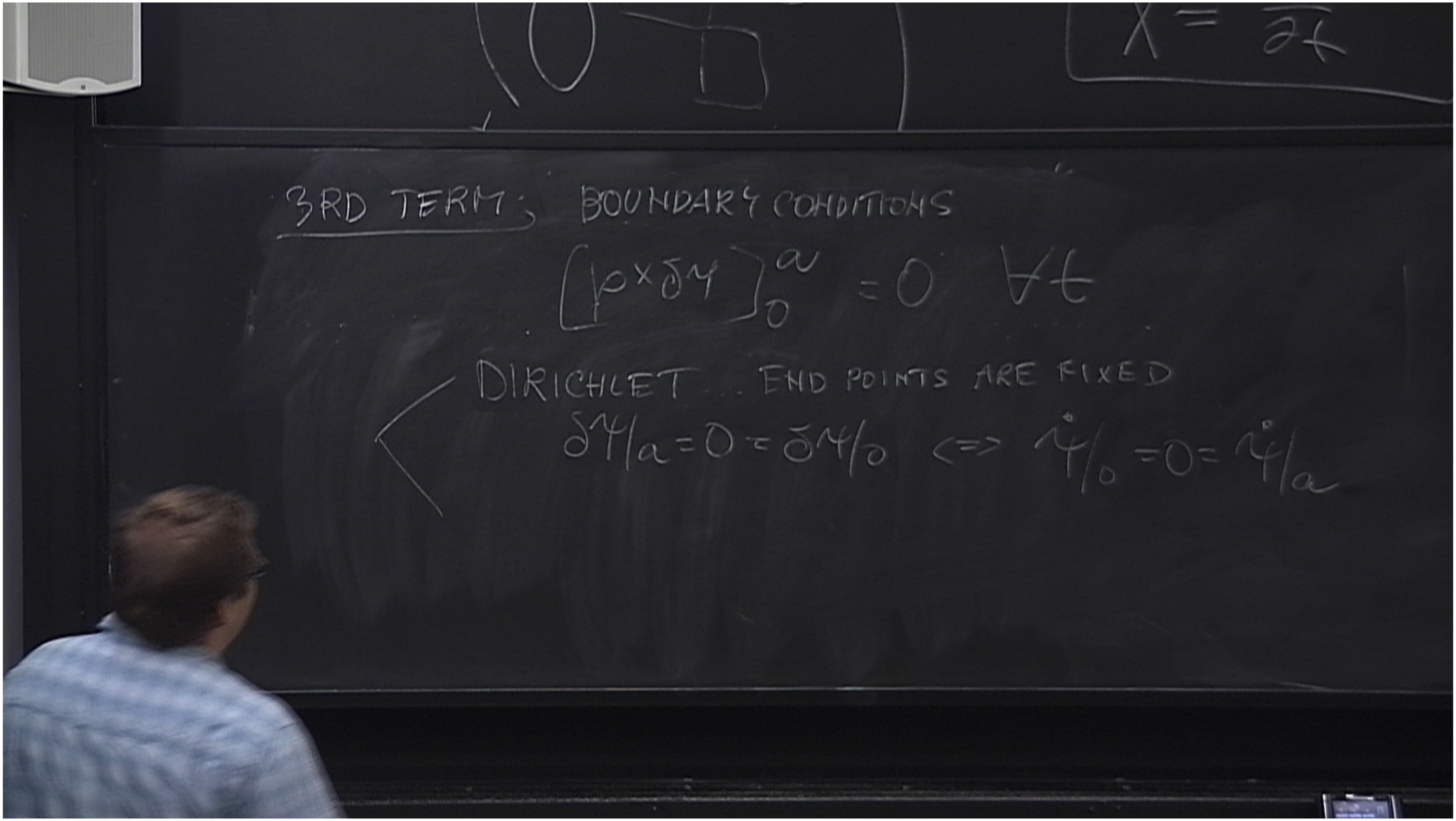
$$= -\int_{t_1}^{t_2} dt \int_0^a dx \Big[\underbrace{\dot{p}^t + \partial_x p^x}_{0} \Big]\delta\psi + \int_0^a dx \left[p^t \ldots \right.$$

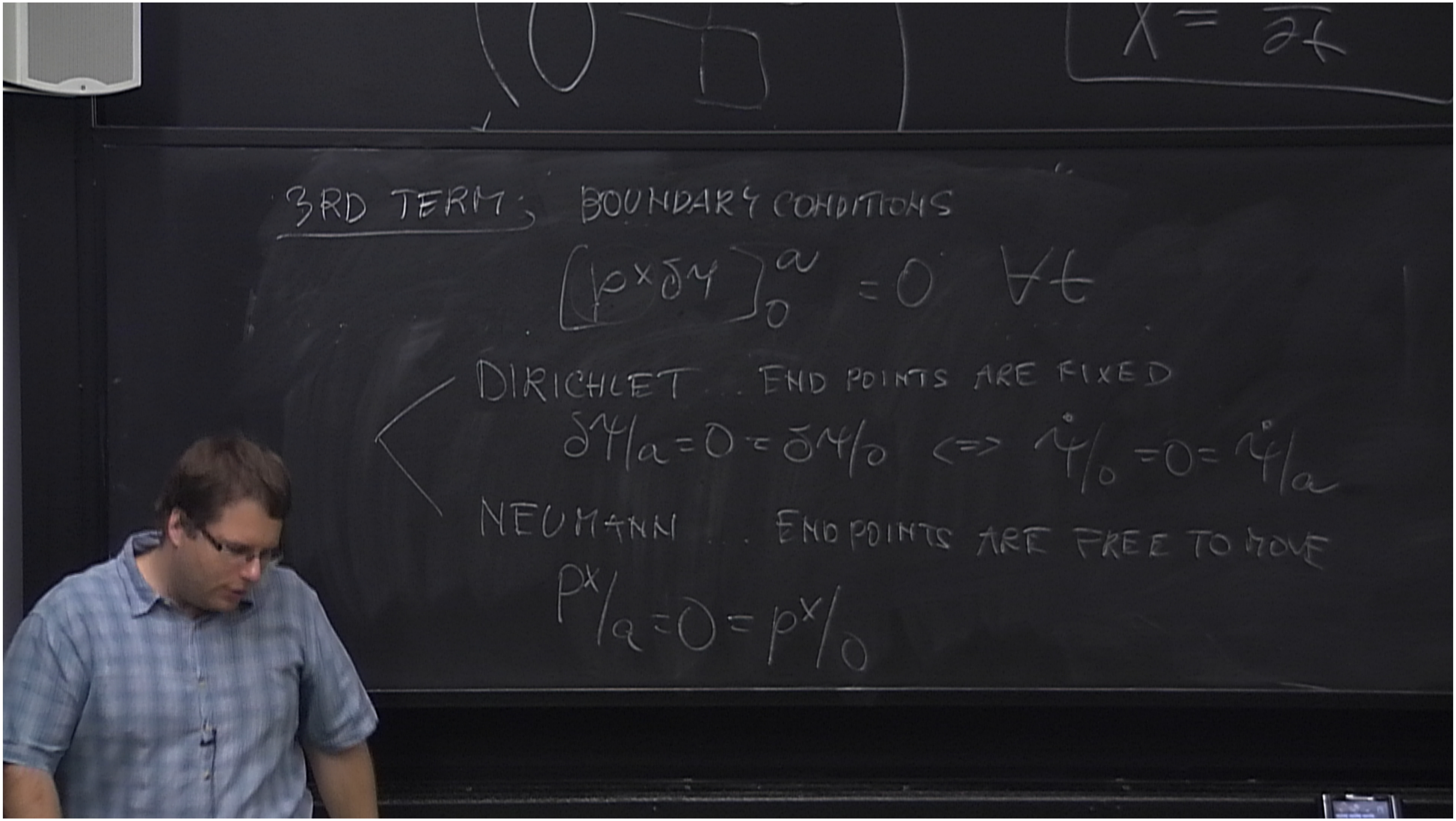
$$\boxed{\psi_{,xx} - \frac{1}{v^2}\psi_{,tt} = 0} \quad \boxed{v = \sqrt{T/\mu}}$$


$p^t \quad \frac{\partial}{\partial t}\delta\psi \quad p^x \quad \partial_x\delta\psi$
$\int_0^a dx\,[p^t\delta\psi]_{t_1}^{t_2} + \int_{t_1}^{t_2}[p^x\delta\psi]_0^a\,dt$
$\emptyset$
$v = \sqrt{T/\mu}$
INITIAL & FINAL COND.
$\delta\psi(t_1,x)=0 \quad \delta\psi(t_0,x)=0$


3RD TERM: BOUNDARY CONDITIONS
[p^x δy]_0^a = 0 ∀t
DIRICHLET .. END POINTS ARE FIXED


3RD TERM: BOUNDARY CONDITIONS
$[p^x \delta y]_0^a = 0 \quad \forall t$
DIRICHLET .. END POINTS ARE FIXED
$\delta y|_a = 0 = \delta y|_0$


3RD TERM: BOUNDARY CONDITIONS
$\left[p_x \delta x\right]_0^a = 0 \quad \forall t$
DIRICHLET ... END POINTS ARE FIXED
$\delta x|_a = 0 = \delta x|_0 \iff \dot{x}|_0 = 0 = \dot{x}|_a$


3RD TERM: BOUNDARY CONDITIONS
$[p^x \delta \varphi]_0^a = 0 \quad \forall t$
DIRICHLET ... END POINTS ARE FIXED
$\delta\varphi|_a = 0 = \delta\varphi|_0 \iff \dot{\varphi}|_0 = 0 = \dot{\varphi}|_a$
NEUMANN ... END POINTS ARE FREE TO MOVE
$p^x|_a = 0 = p^x|_0$

3RD TERM: BOUNDARY CONDITIONS

$$\left[p^x \delta\psi\right]_0^a = 0 \quad \forall t$$

DIRICHLET ... END POINTS ARE FIXED

$$\delta\psi|_a = 0 = \delta\psi|_0 \iff \dot{\psi}|_0 = 0 = \dot{\psi}|_a$$

NEUMANN ... END POINTS ARE FREE TO MOVE

$$p^x|_a = 0 = p^x|_0 \iff \partial_x\psi|_0 = \partial_x\psi|_a = 0$$

$= \dot{\varphi}/a$ ... MOMENTUM EXCHANGES WITH THE WALL .... D-BRANE

TO MOVE

REL. FIELD THEORY: $S = \int \underbrace{dt\, d^3x}_{d^4x} \, \mathcal{L}\left(\varphi, \frac{\partial \varphi}{\partial x^\mu}, x^\mu\right)$

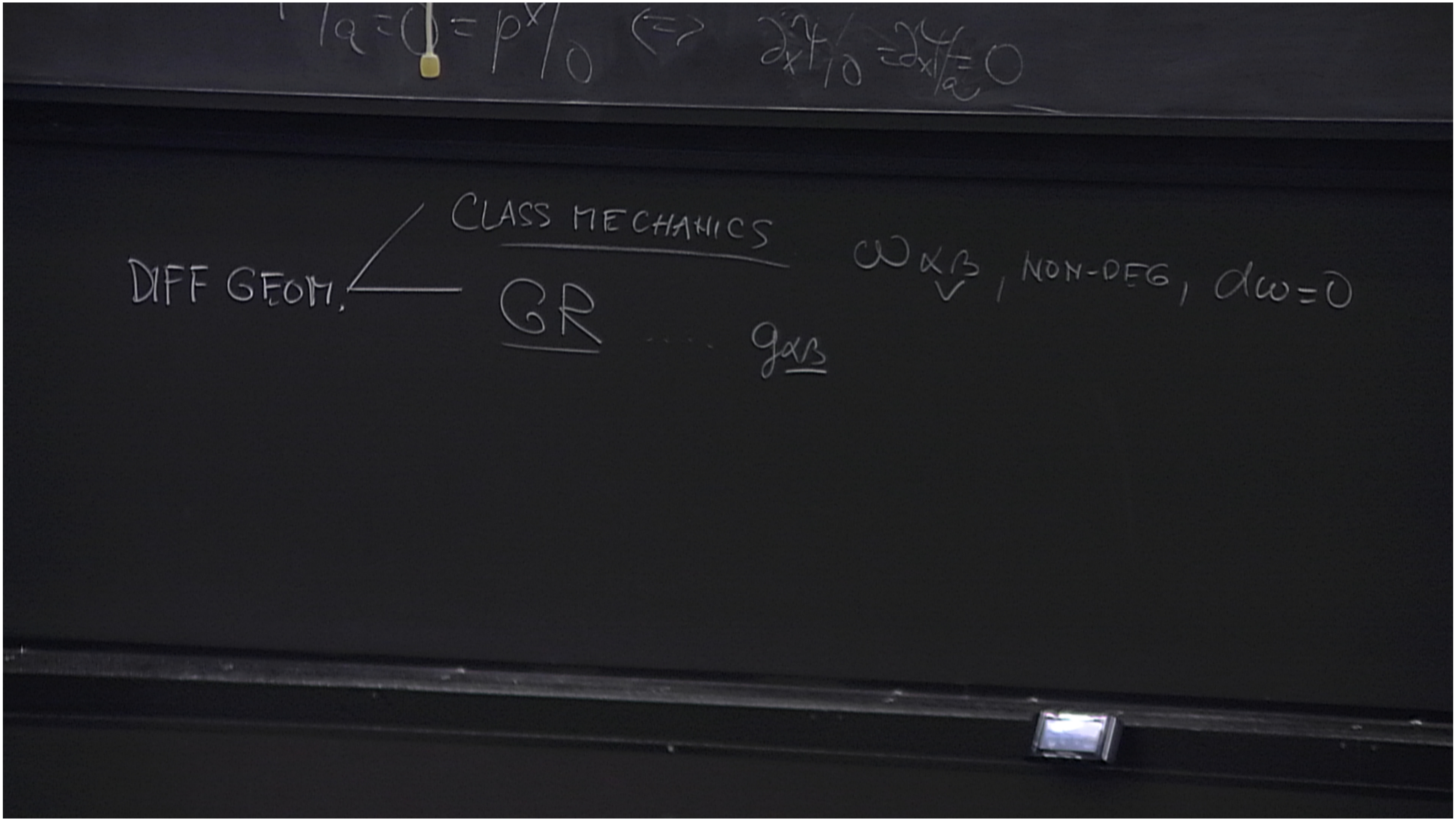
$\dot{x}/a = 0$

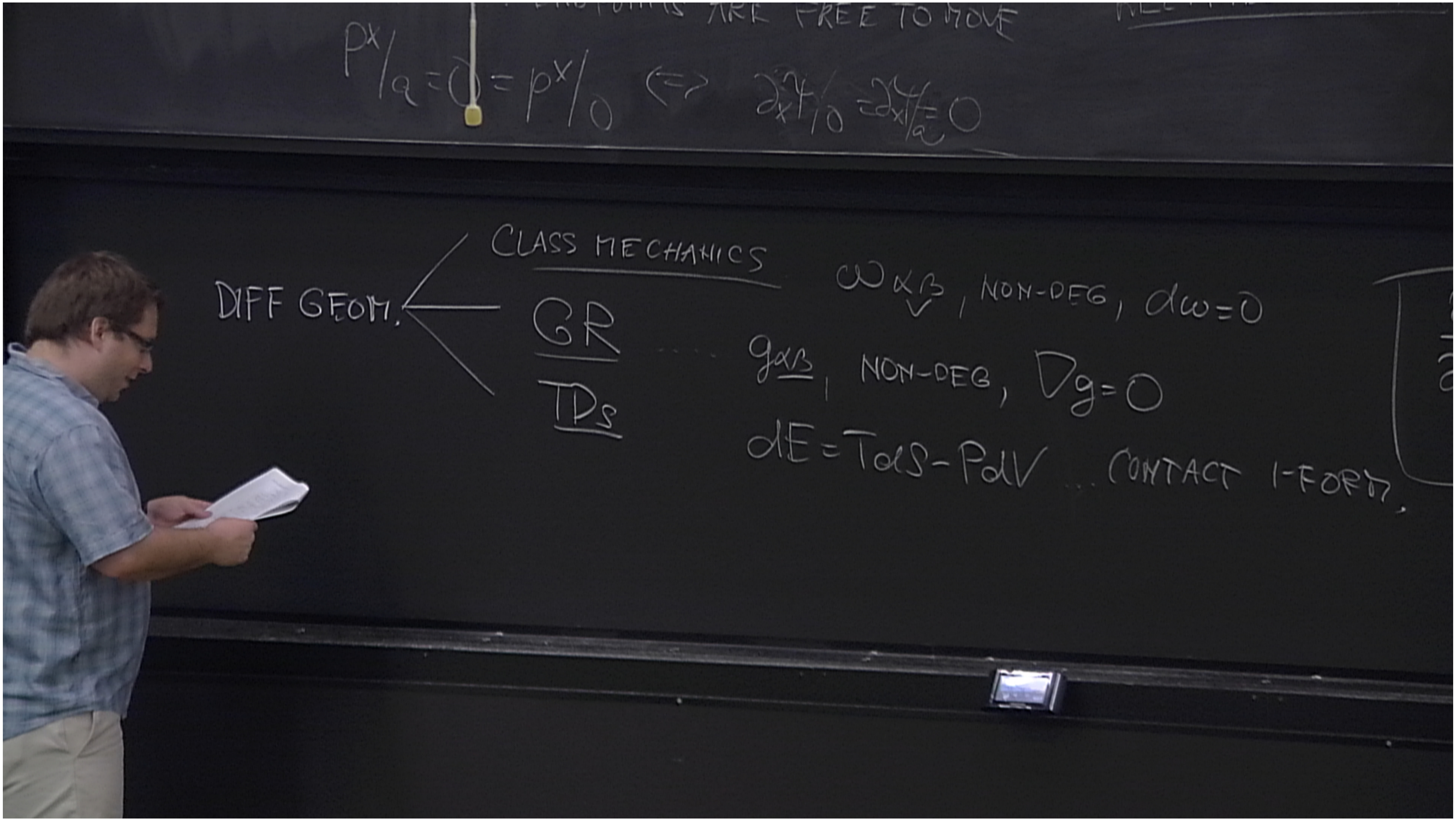
MOMENTUM EXCHANGES WITH THE WALL

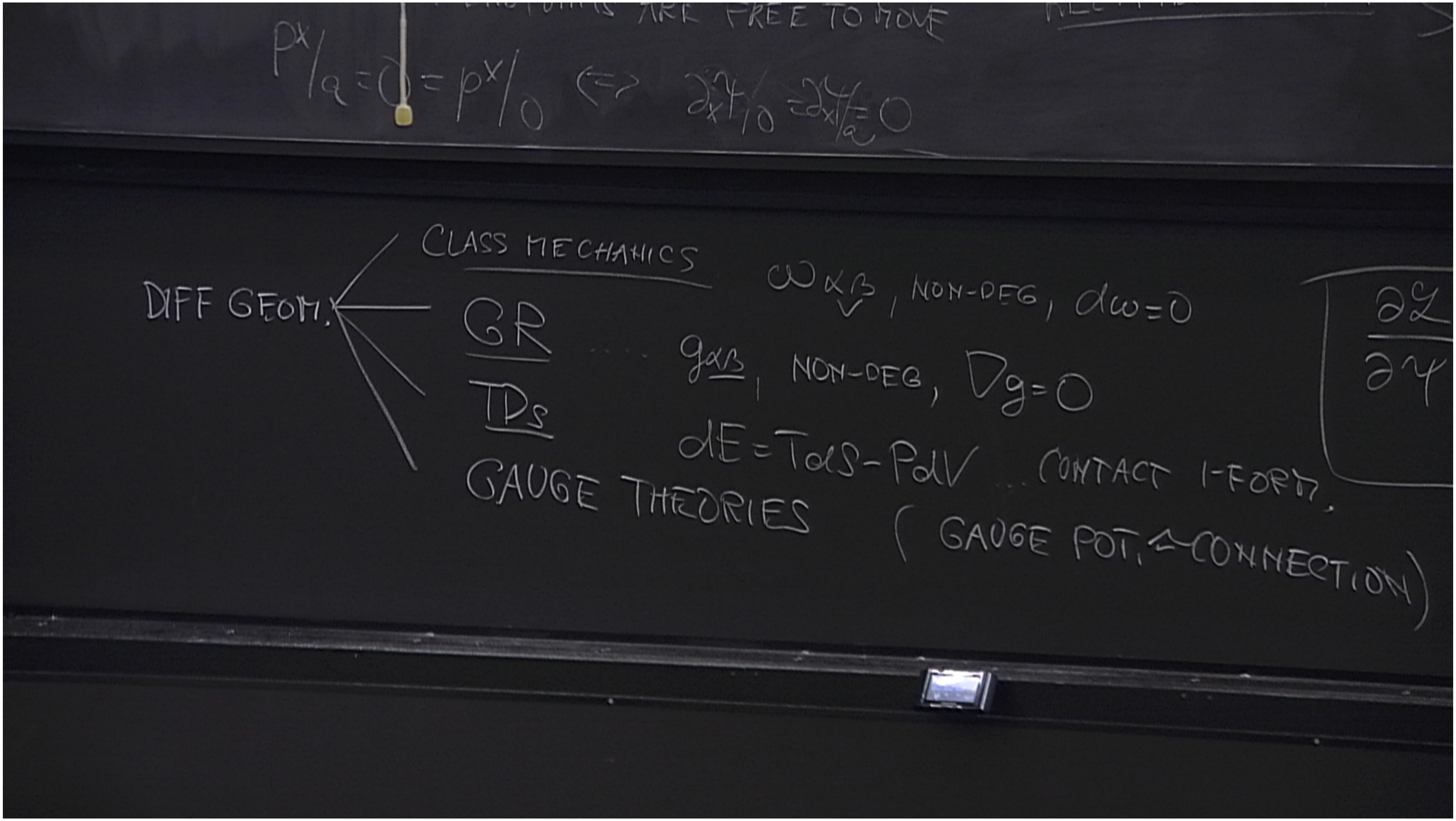
TO MOVE

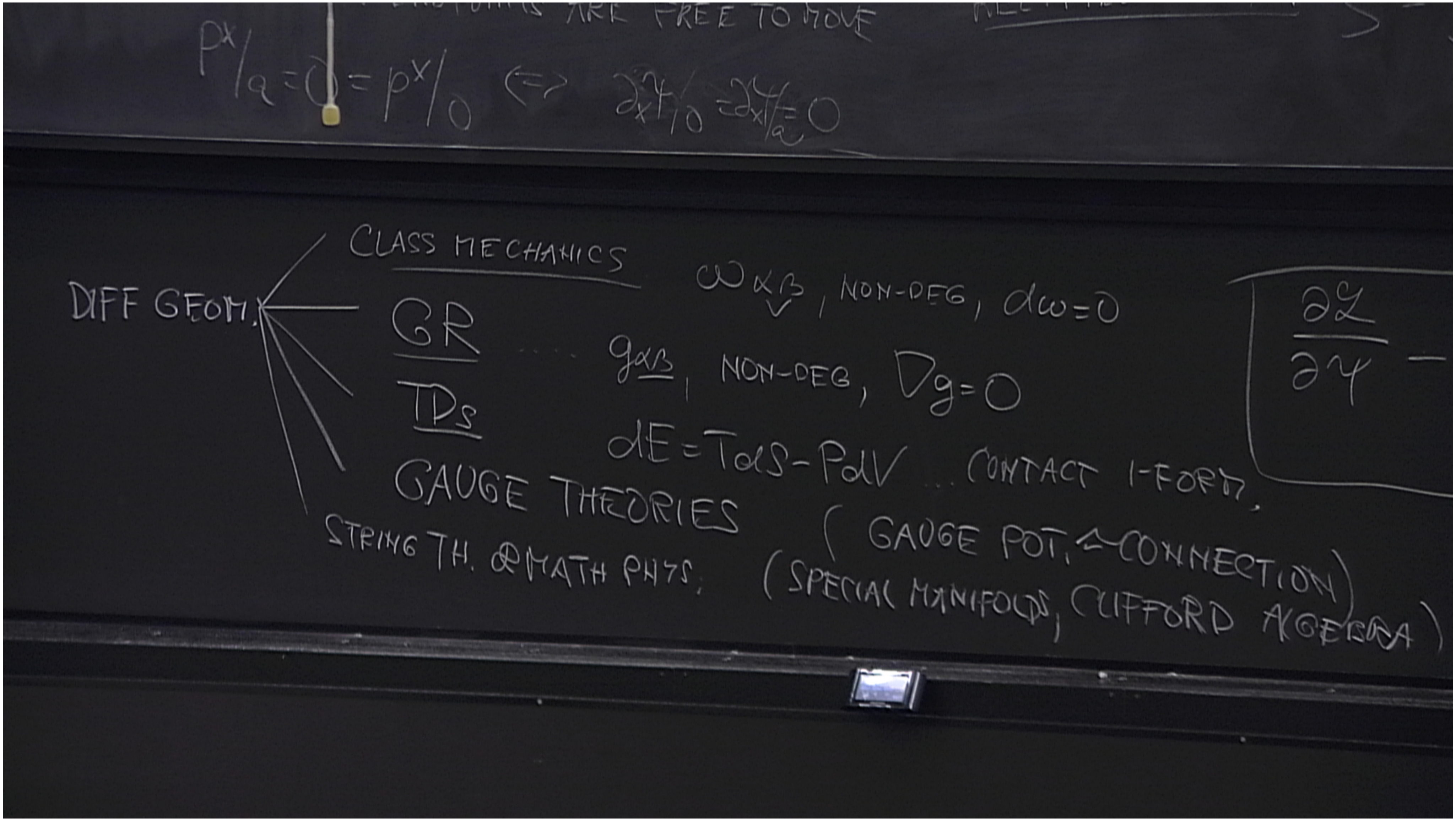
REL. FIELD THEORY: $S = \int \underbrace{dt\, d^3x}_{d^4x} \, \mathcal{L}\left(\psi, \frac{\partial \psi}{\partial x^\mu}, x^\mu\right)$

$$\boxed{\frac{\partial \mathcal{L}}{\partial \psi} - \frac{\partial}{\partial x^\mu}\left(\frac{\partial \mathcal{L}}{\partial\left(\frac{\partial \psi}{\partial x^\mu}\right)}\right) = 0}$$


CLASS MECHANICS
DIFF GEOM.
GR
ωαβ , NON-DEG, dω=0
gαβ


ARE FREE TO MOVE
DIFF GEOM.
CLASS MECHANICS
GR
TDs
ωαβ, NON-DEG, dω=0
gαβ, NON-DEG, Dg=0
dE=TdS-PdV ... CONTACT 1-FORM.


FREE TO MOVE
DIFF GEOM.
CLASS MECHANICS
ωαβ, NON-DEG, dω=0
GR
gαβ, NON-DEG, ∇g=0
TDs
dE=TdS-PdV ... CONTACT 1-FORM.
GAUGE THEORIES
(GAUGE POT. ⇐ CONNECTION)


ARE FREE TO MOVE
p^x/a = 0 = p^x/0 ⟺ ∂xψ/0 = ∂xψ/a = 0
DIFF GEOM.
CLASS MECHANICS
ωαβ, NON-DEG, dω=0
GR
gαβ, NON-DEG, ∇g=0
TDs
dE=TdS−PdV ... CONTACT 1-FORM.
GAUGE THEORIES
(GAUGE POT. ↔ CONNECTION)
STRING TH. & MATH PHYS.
(SPECIAL MANIFOLDS, CLIFFORD ALGEBRA)
∂L/∂ψ −