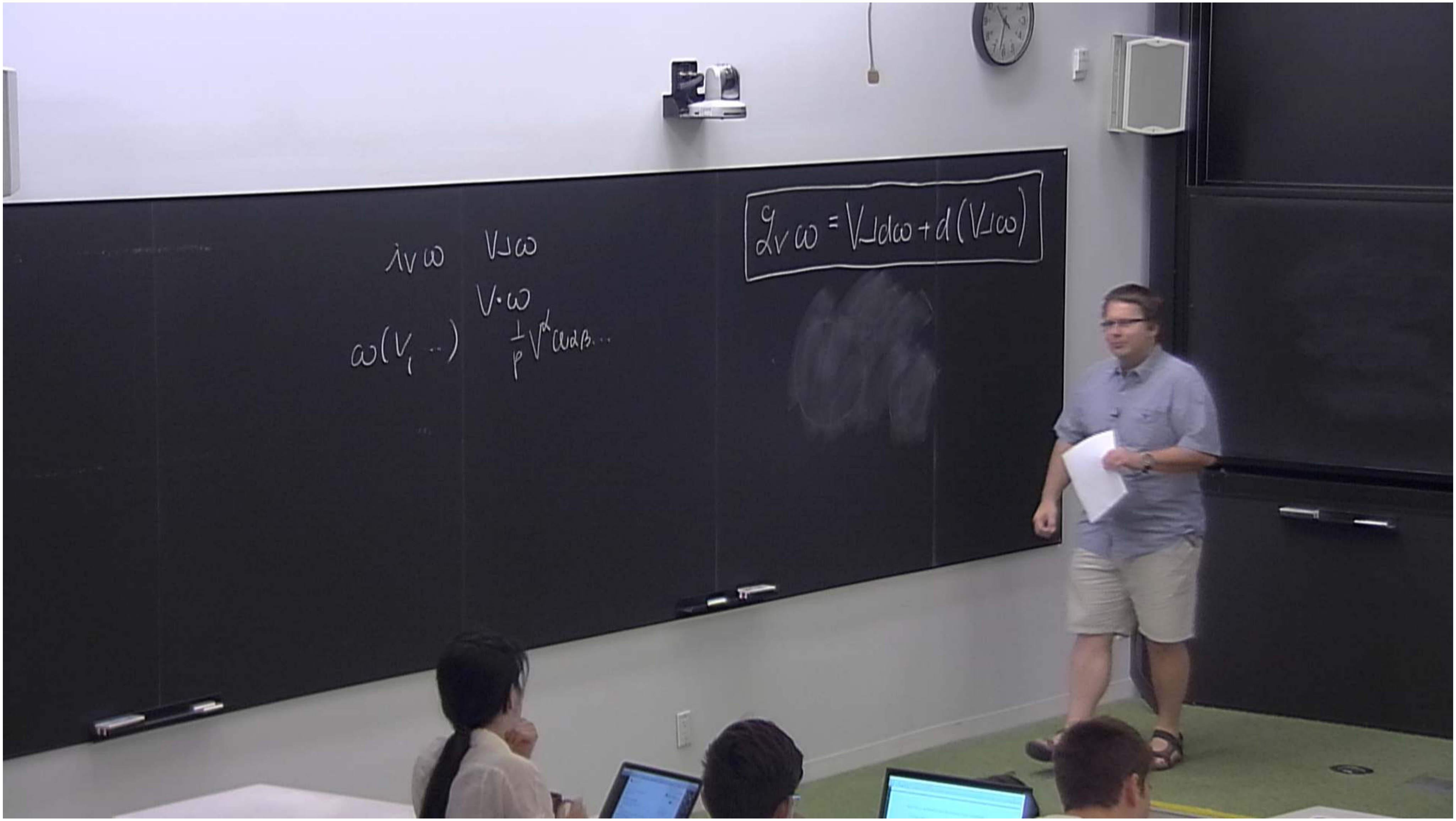
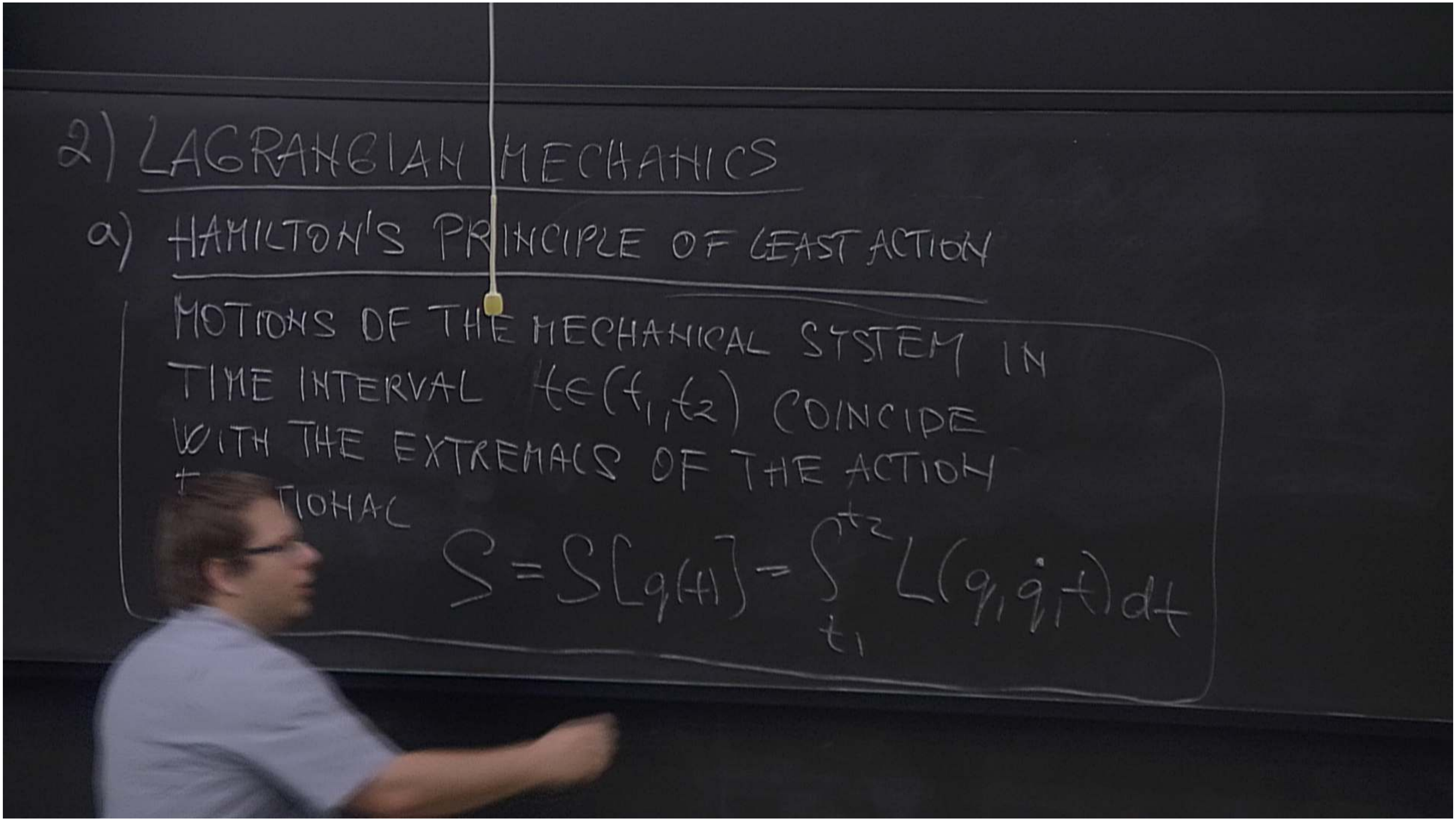
Title: Front End - Classical Mechanics 2

Date: Aug 11, 2016 10:30 AM

URL: http://pirsa.org/16080063

Abstract:


$\mathcal{L}_V \omega = V \lrcorner d\omega + d(V \lrcorner \omega)$
$i_V \omega$
$V \lrcorner \omega$
$V \cdot \omega$
$\omega(V_1 \dots)$
$\frac{1}{p} V^\alpha \omega_{\alpha\beta\dots}$


2) LAGRANGIAN MECHANICS
a) HAMILTON'S PRINCIPLE OF LEAST ACTION
MOTIONS OF THE MECHANICAL SYSTEM IN
TIME INTERVAL $t \in (t_1, t_2)$ COINCIDE
WITH THE EXTREMALS OF THE ACTION
TIONAL
$S = S[q(t)] = \int_{t_1}^{t_2} L(q, \dot{q}, t)\, dt$

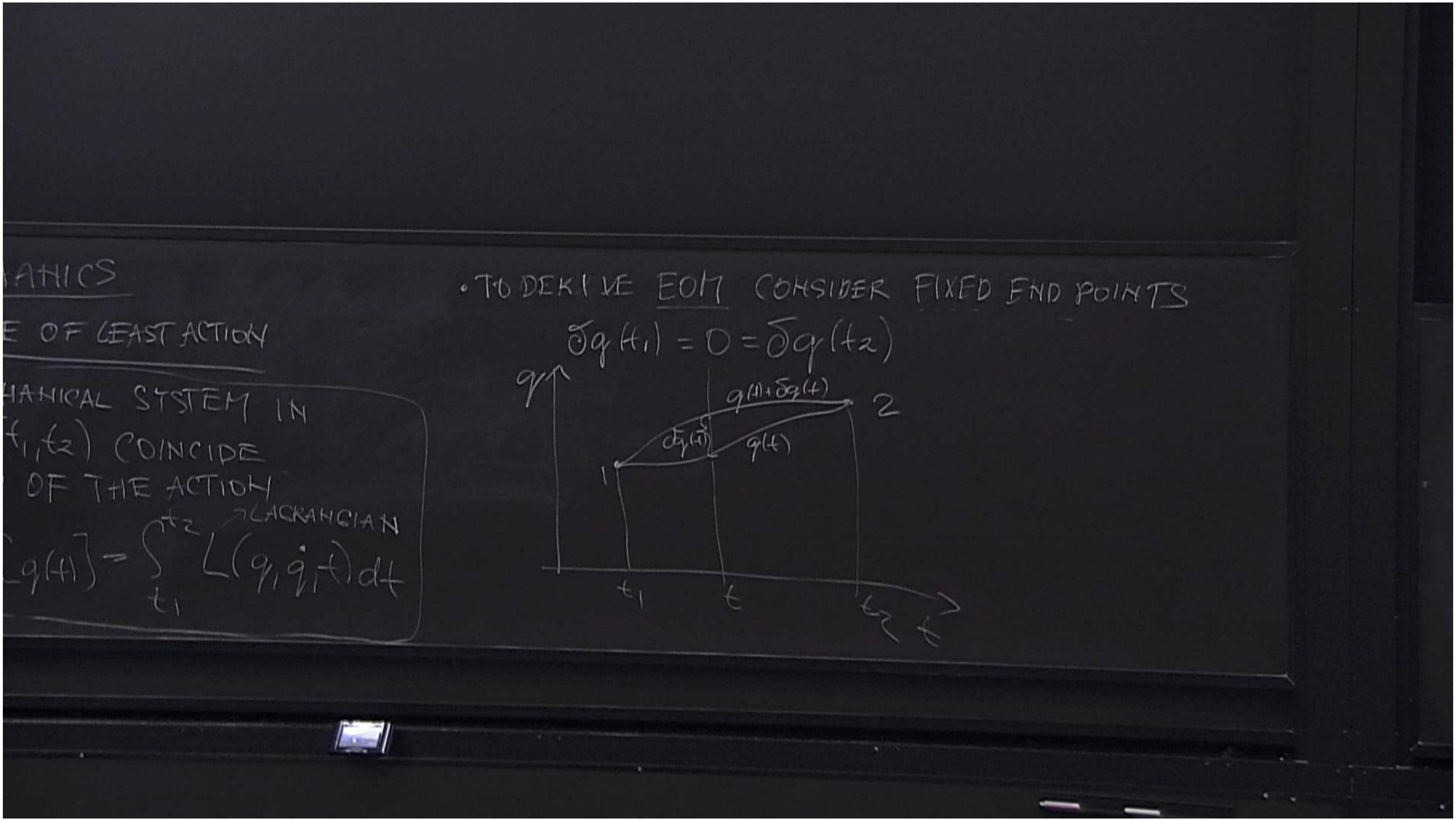
2) LAGRANGIAN MECHANICS

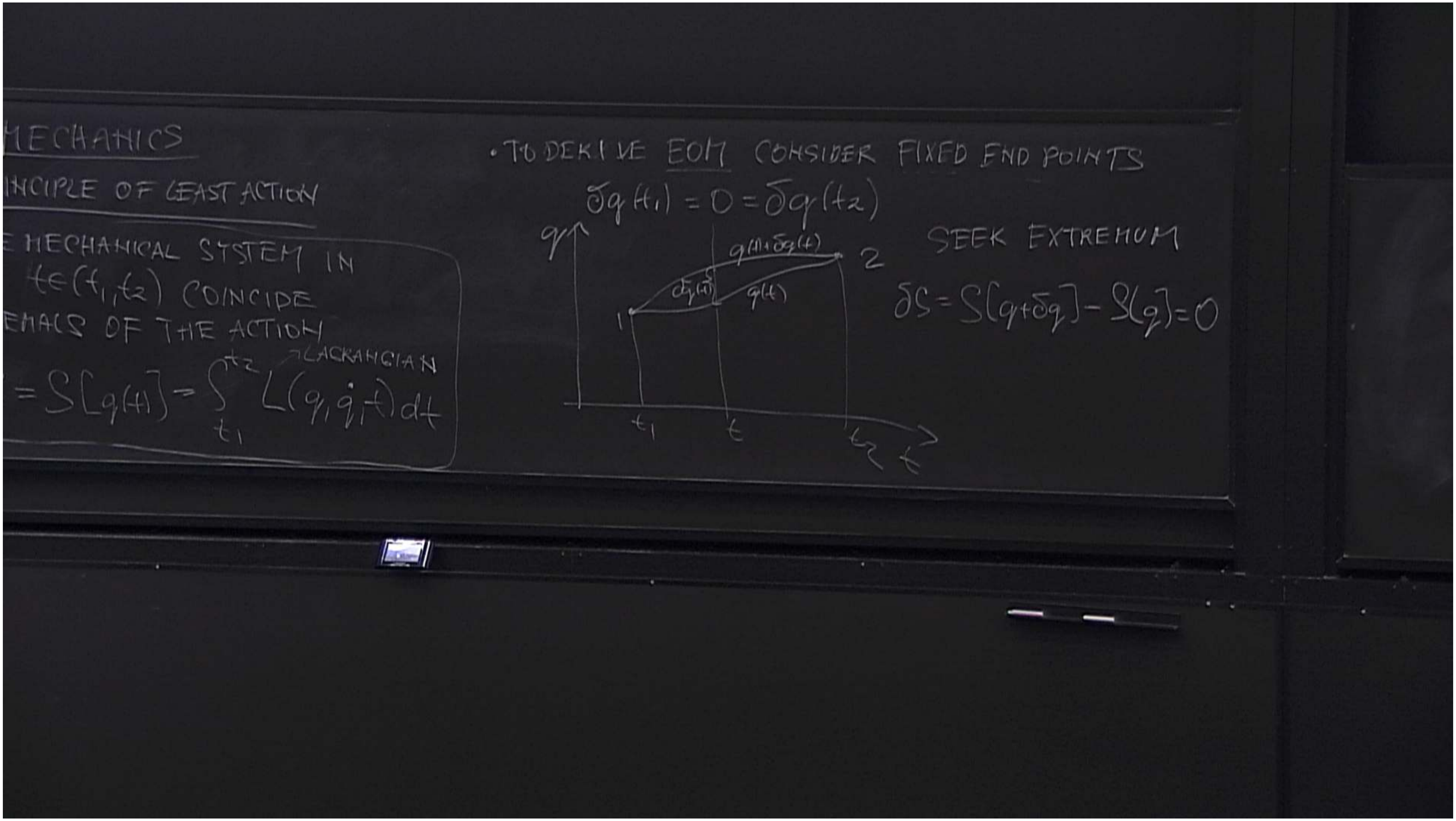
a) HAMILTON'S PRINCIPLE OF LEAST ACTION

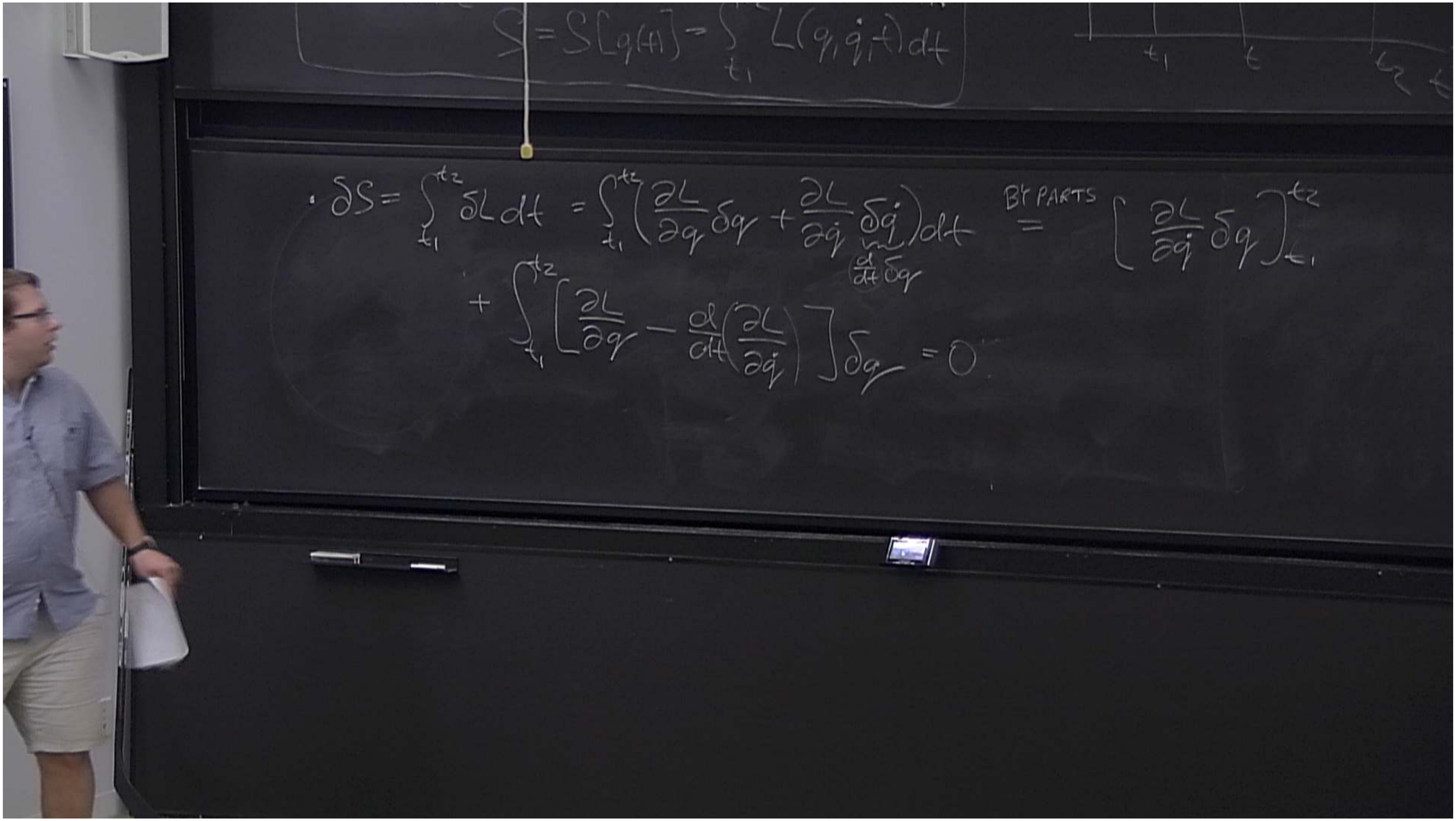
MOTIONS OF THE MECHANICAL SYSTEM IN TIME INTERVAL $t \in (t_1, t_2)$ COINCIDE WITH THE EXTREMALS OF THE ACTION FUNCTIONAL

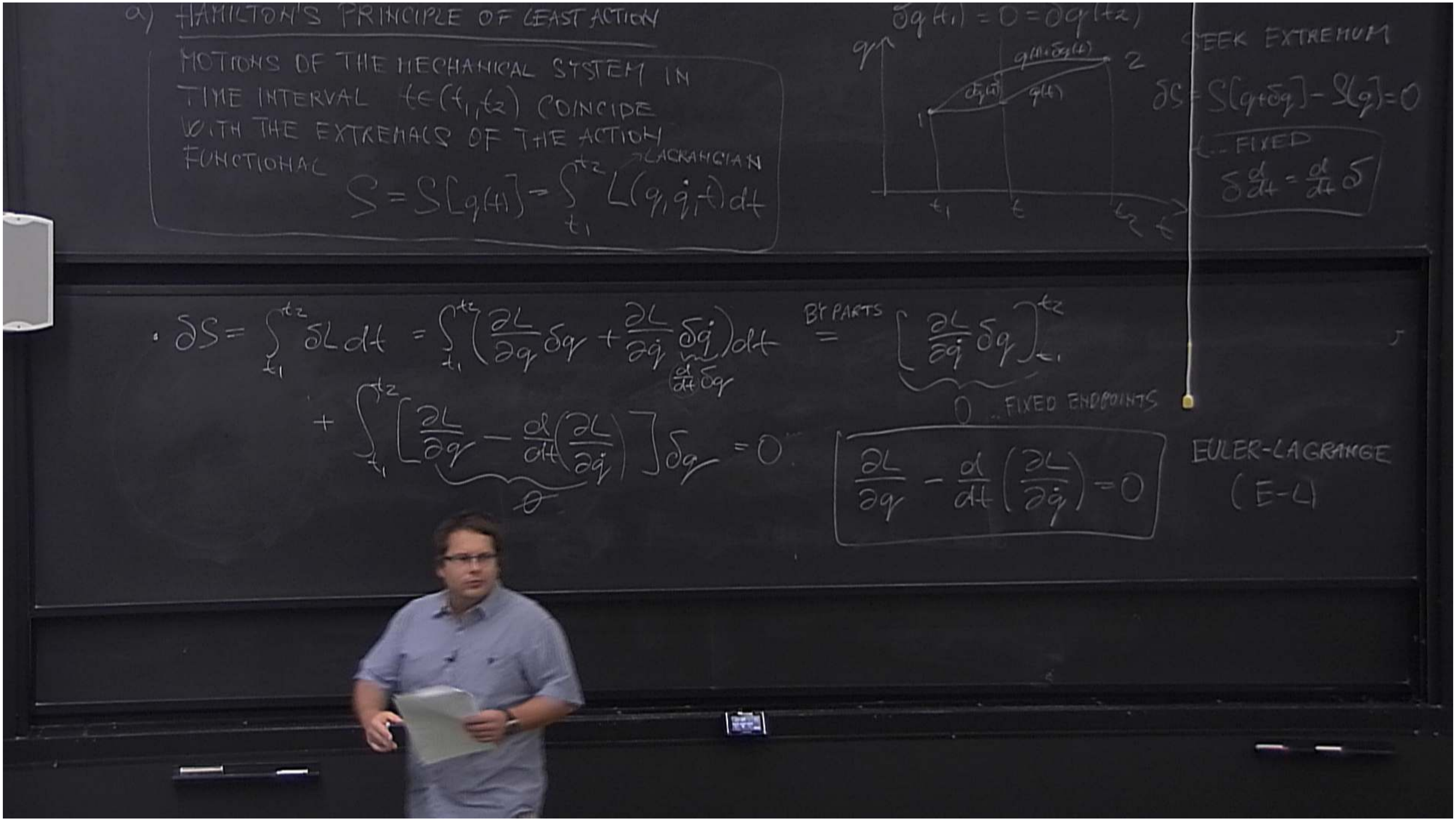
LAGRANGIAN

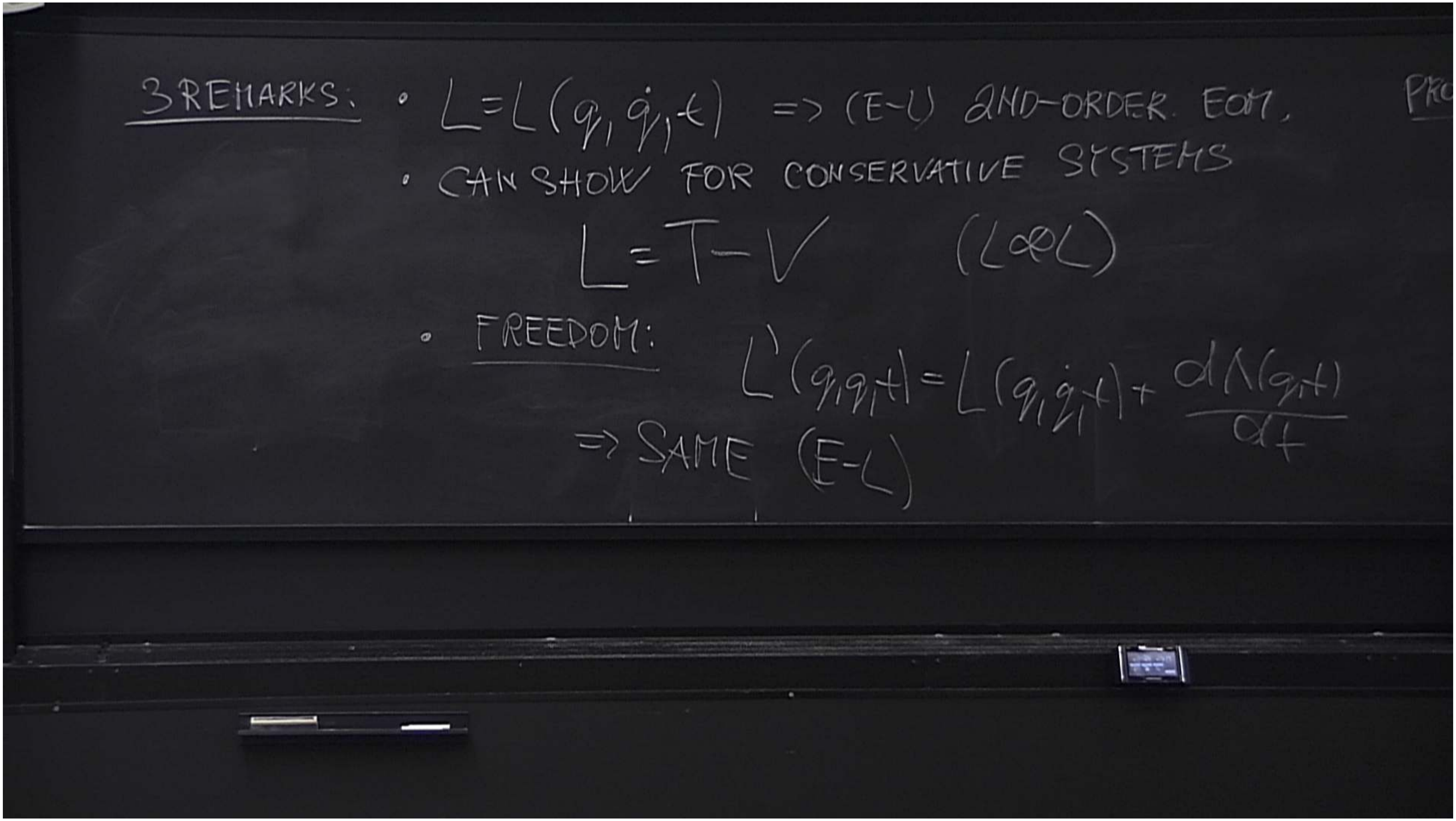
$$S = S[q(t)] = \int_{t_1}^{t_2} L(q, \dot{q}, t)\, dt$$


ANICS
E OF LEAST ACTION
HANICAL SYSTEM IN
t1, t2) COINCIDE
OF THE ACTION
LAGRANGIAN
q(t)] = ∫ t1 t2 L(q, q̇, t) dt
• TO DERIVE EOM CONSIDER FIXED END POINTS
δq(t1) = 0 = δq(t2)
q
q(t)+δq(t)
2
δq(t)
q(t)
1
t1
t
t2
t


MECHANICS
INCIPLE OF LEAST ACTION
E MECHANICAL SYSTEM IN
$t \in (t_1, t_2)$ COINCIDE
EMALS OF THE ACTION
LAGRANGIAN
$= S[q(t)] = \int_{t_1}^{t_2} L(q, \dot{q}, t)\,dt$
• TO DERIVE EOM CONSIDER FIXED END POINTS
$\delta q(t_1) = 0 = \delta q(t_2)$
q
$q(t)+\delta q(t)$
2
$\delta q(t)$
$q(t)$
1
t_1
t
t_2
t
SEEK EXTREMUM
$\delta S = S[q+\delta q] - S[q] = 0$


$S = S[q(t)] = \int_{t_1} L(q,\dot{q},t)\,dt$
t_1 t t_2
$\delta S = \int_{t_1}^{t_2} \delta L\, dt = \int_{t_1}^{t_2}\left(\frac{\partial L}{\partial q}\delta q + \frac{\partial L}{\partial \dot{q}}\underbrace{\delta\dot{q}}_{\frac{d}{dt}\delta q}\right)dt$
BY PARTS
$= \left[\frac{\partial L}{\partial \dot{q}}\delta q\right]_{t_1}^{t_2} + \int_{t_1}^{t_2}\left[\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)\right]\delta q = 0$


a) HAMILTON'S PRINCIPLE OF LEAST ACTION
MOTIONS OF THE MECHANICAL SYSTEM IN TIME INTERVAL $t\in(t_1,t_2)$ COINCIDE WITH THE EXTREMALS OF THE ACTION FUNCTIONAL
LAGRANGIAN
$S = S[q(t)] = \int_{t_1}^{t_2} L(q,\dot q,t)\,dt$
$\delta q(t_1) = 0 = \delta q(t_2)$
$q(t)+\delta q(t)$
$\delta q(t)$
$q(t)$
SEEK EXTREMUM
$\delta S = S[q+\delta q] - S[q] = 0$
t_1 FIXED
$\delta \frac{d}{dt} = \frac{d}{dt}\delta$
$\delta S = \int_{t_1}^{t_2} \delta L\, dt = \int_{t_1}^{t_2}\left(\frac{\partial L}{\partial q}\delta q + \frac{\partial L}{\partial \dot q}\delta\dot q\right)dt$
$\delta \dot q = \frac{d}{dt}\delta q$
BY PARTS
$= \underbrace{\left[\frac{\partial L}{\partial \dot q}\delta q\right]_{t_1}^{t_2}}_{0}$ FIXED ENDPOINTS
$+ \int_{t_1}^{t_2}\underbrace{\left[\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot q}\right)\right]}_{0}\delta q = 0$
$\boxed{\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot q}\right) = 0}$
EULER-LAGRANGE
(E-L)


3 REMARKS:
• $L = L(q, \dot{q}, t)$ => (E-L) 2ND-ORDER EOM.
PRO
• CAN SHOW FOR CONSERVATIVE SYSTEMS
$L = T - V$ (L&L)
• FREEDOM:
$L'(q, \dot{q}, t) = L(q, \dot{q}, t) + \frac{d\Lambda(q,t)}{dt}$
=> SAME (E-L)

=> (E-L) 2ND-ORDER EOM.

R CONSERVATIVE SYSTEMS

$-V \quad (L \not\propto L)$

$$L'(q,\dot{q},t) = L(q,\dot{q},t) + \frac{d\Lambda(q,t)}{dt}$$

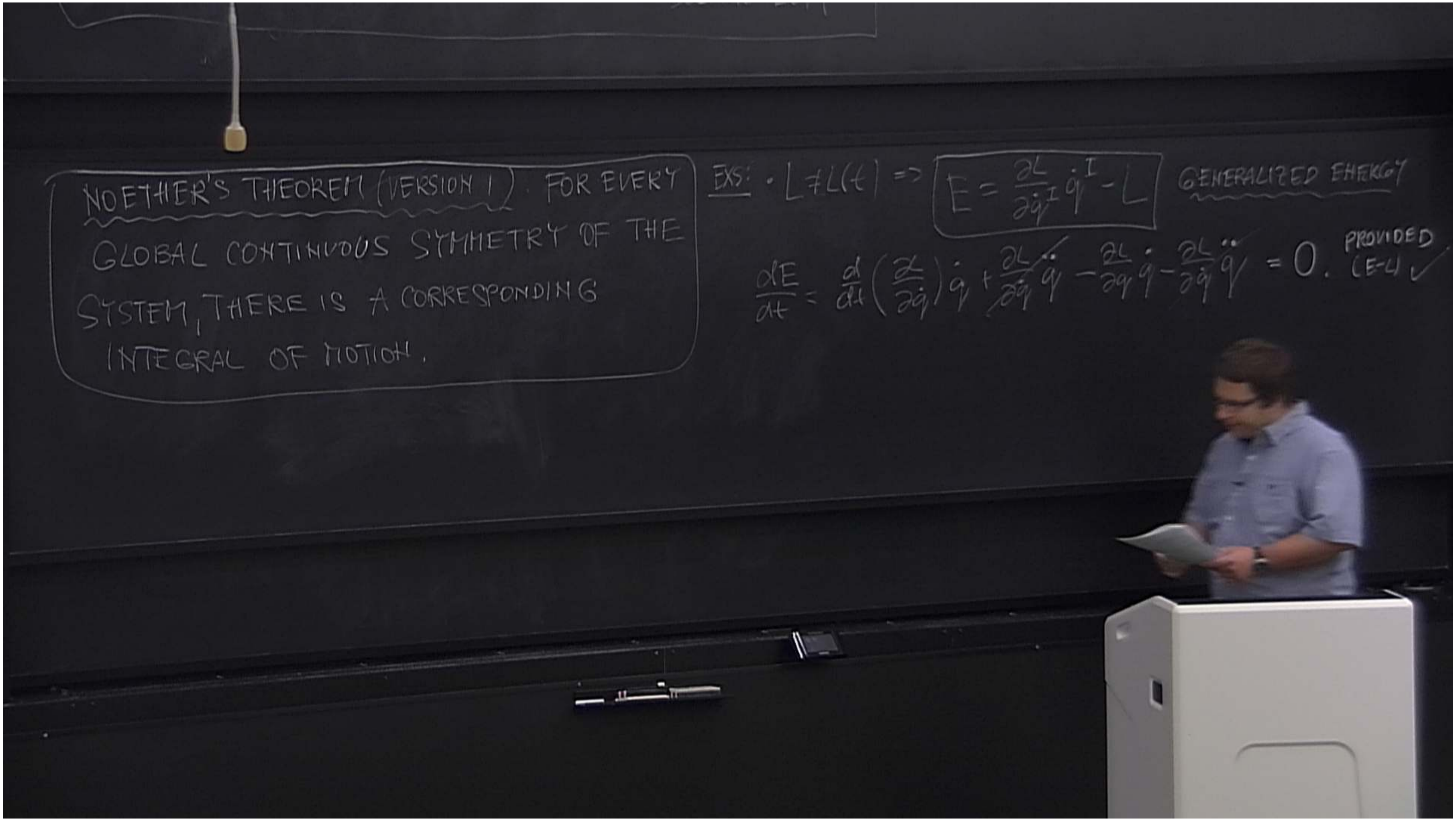
E (E-L)

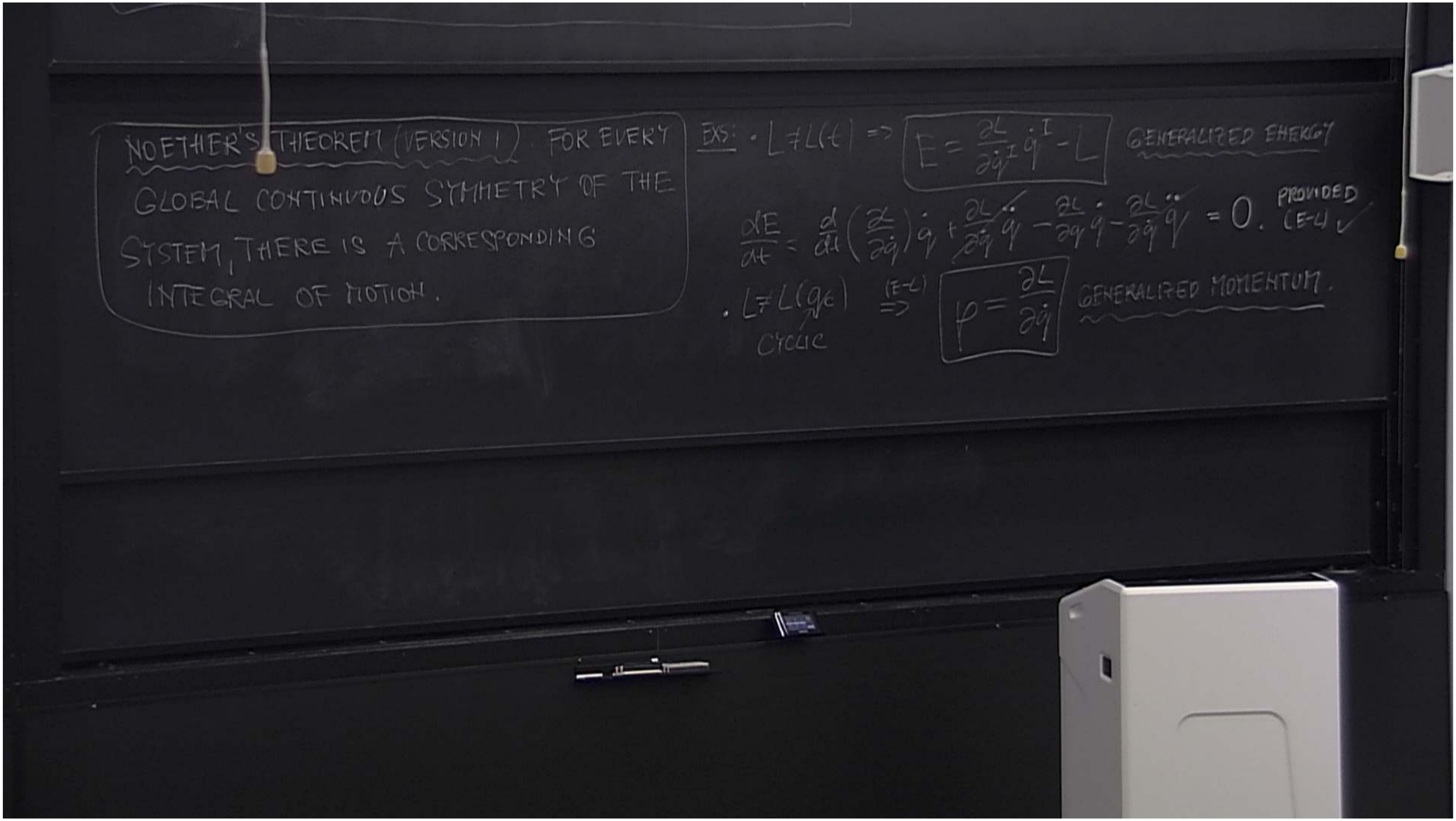
PROOF: $S' = \int L'\,dt = \int L\,dt + \int_{t_1}^{t_2} \frac{d\Lambda}{dt}\,dt$

$= S + [\Lambda]_{t_1}^{t_2}$

$$\delta S' = \delta S + \underbrace{\left[\frac{\partial\Lambda}{\partial q}\delta q\right]_{t_1}^{t_2}}_{0}$$

NOETHER'S THEOREM (VERSION I). FOR EVERY GLOBAL CONTINUOUS SYMMETRY OF THE SYSTEM, THERE IS A CORRESPONDING INTEGRAL OF MOTION.


NOETHER'S THEOREM (VERSION I). FOR EVERY GLOBAL CONTINUOUS SYMMETRY OF THE SYSTEM, THERE IS A CORRESPONDING INTEGRAL OF MOTION.
EXS: • L ≠ L(t) => E = ∂L/∂q̇^I q̇^I − L
GENERALIZED ENERGY
dE/dt = d/dt (∂L/∂q̇) q̇ + ∂L/∂q̇ q̈ − ∂L/∂q q̇ − ∂L/∂q̇ q̈ = 0.
PROVIDED E-L ✓

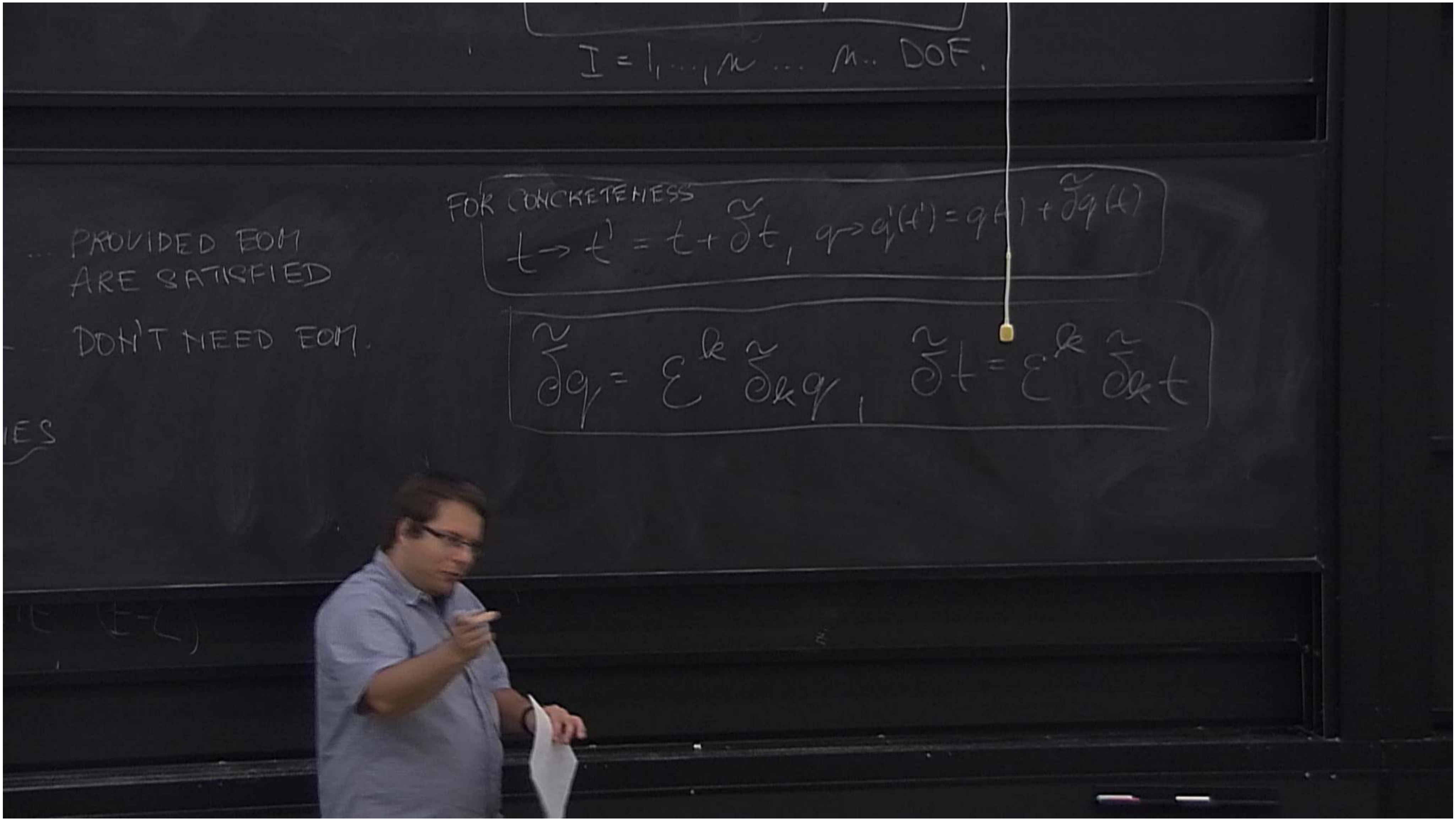

NOETHER'S THEOREM (VERSION 1): FOR EVERY GLOBAL CONTINUOUS SYMMETRY OF THE SYSTEM, THERE IS A CORRESPONDING INTEGRAL OF MOTION.
EXS: • $L \neq L(t) \Rightarrow \boxed{E = \frac{\partial L}{\partial \dot{q}^I}\dot{q}^I - L}$ GENERALIZED ENERGY
$\frac{dE}{dt} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)\dot{q} + \cancel{\frac{\partial L}{\partial \dot{q}}\ddot{q}} - \frac{\partial L}{\partial q}\dot{q} - \cancel{\frac{\partial L}{\partial \dot{q}}\ddot{q}} = 0.$ PROVIDED (E-L) ✓
• $L \neq L(q_c) \overset{(E-L)}{\Rightarrow} \boxed{p = \frac{\partial L}{\partial \dot{q}}}$ GENERALIZED MOMENTUM.
CYCLIC

2 REMARKS ON TERMINOLOGY

- VALID ON SHELL ... PROVIDED EOM ARE SATISFIED

  OFF SHELL ... DON'T NEED EOM.

- TYPES OF SYMMETRIES



$I = 1, \ldots, n \ldots n$.. DOF.
FOR CONCRETENESS
$t \to t' = t + \tilde{\delta} t, \quad q \to q'(t') = q(t) + \tilde{\delta} q(t)$
... PROVIDED EOM
ARE SATISFIED
DON'T NEED EOM.
$\tilde{\delta} q = \varepsilon^k \tilde{\delta}_k q, \quad \tilde{\delta} t = \varepsilon^k \tilde{\delta}_k t$

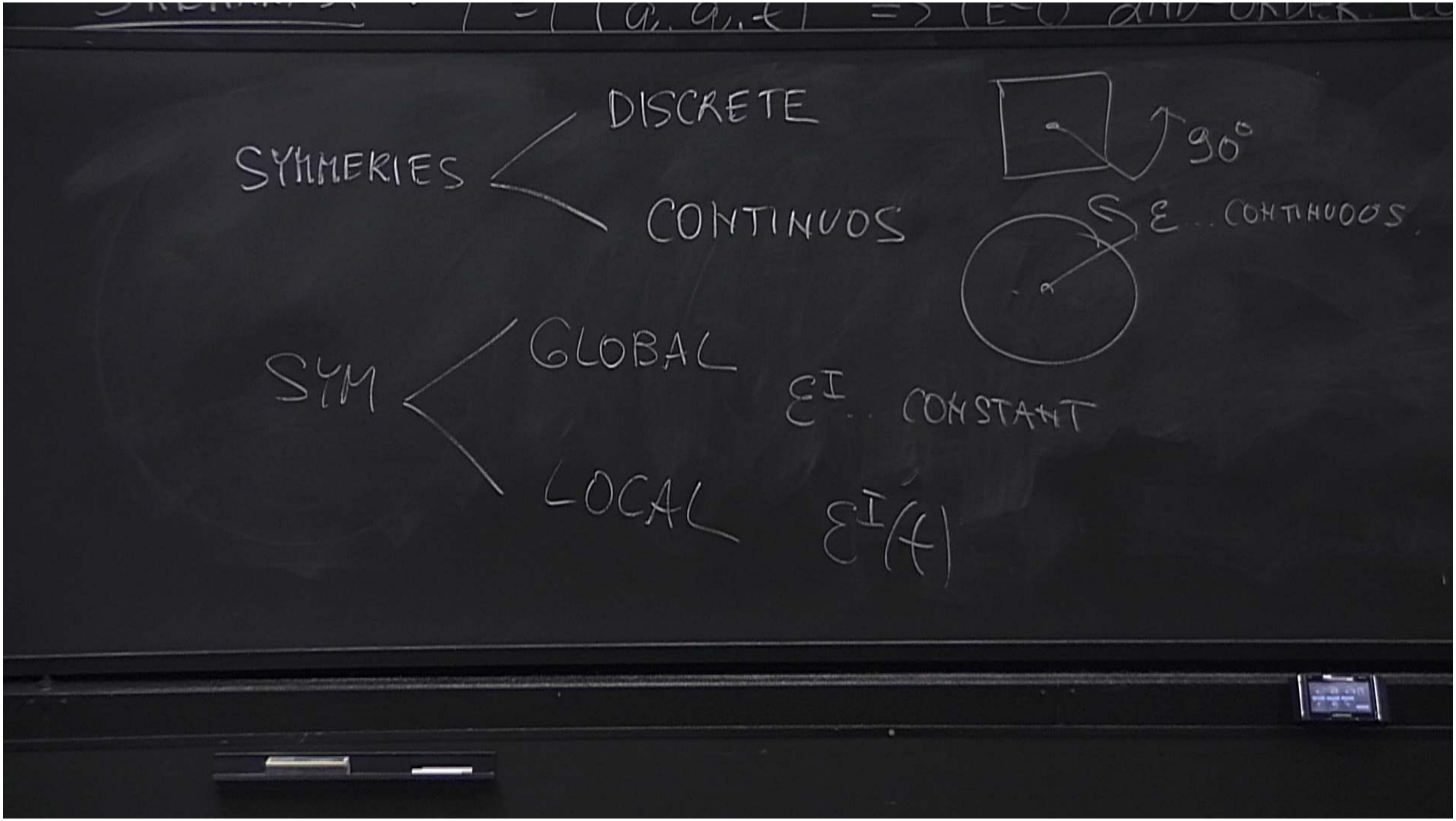
FOR CONCRETENESS

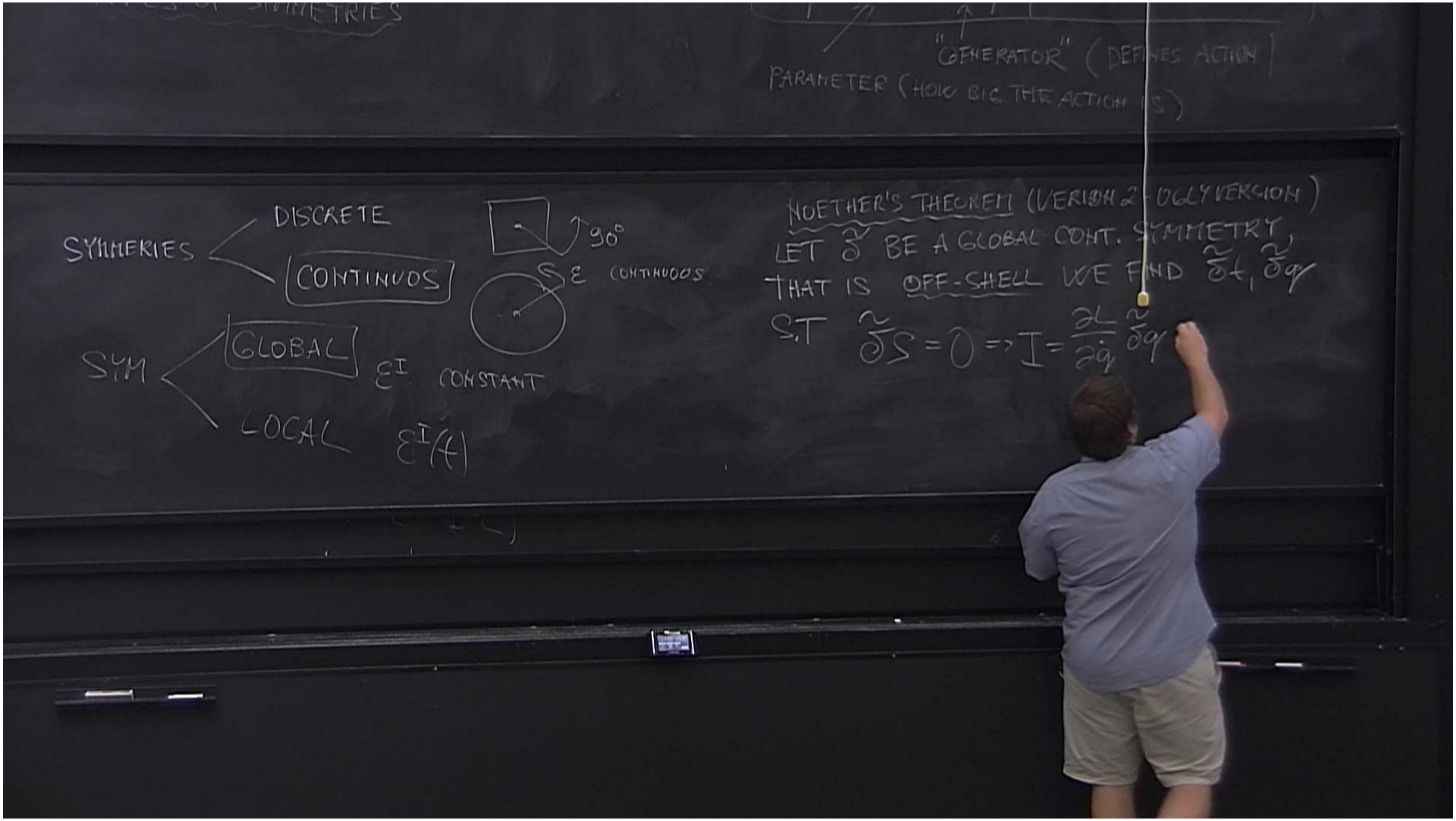
$$\boxed{t \to t' = t + \tilde{\delta} t, \quad q \to q'(t') = q(t) + \tilde{\delta} q(t)}$$

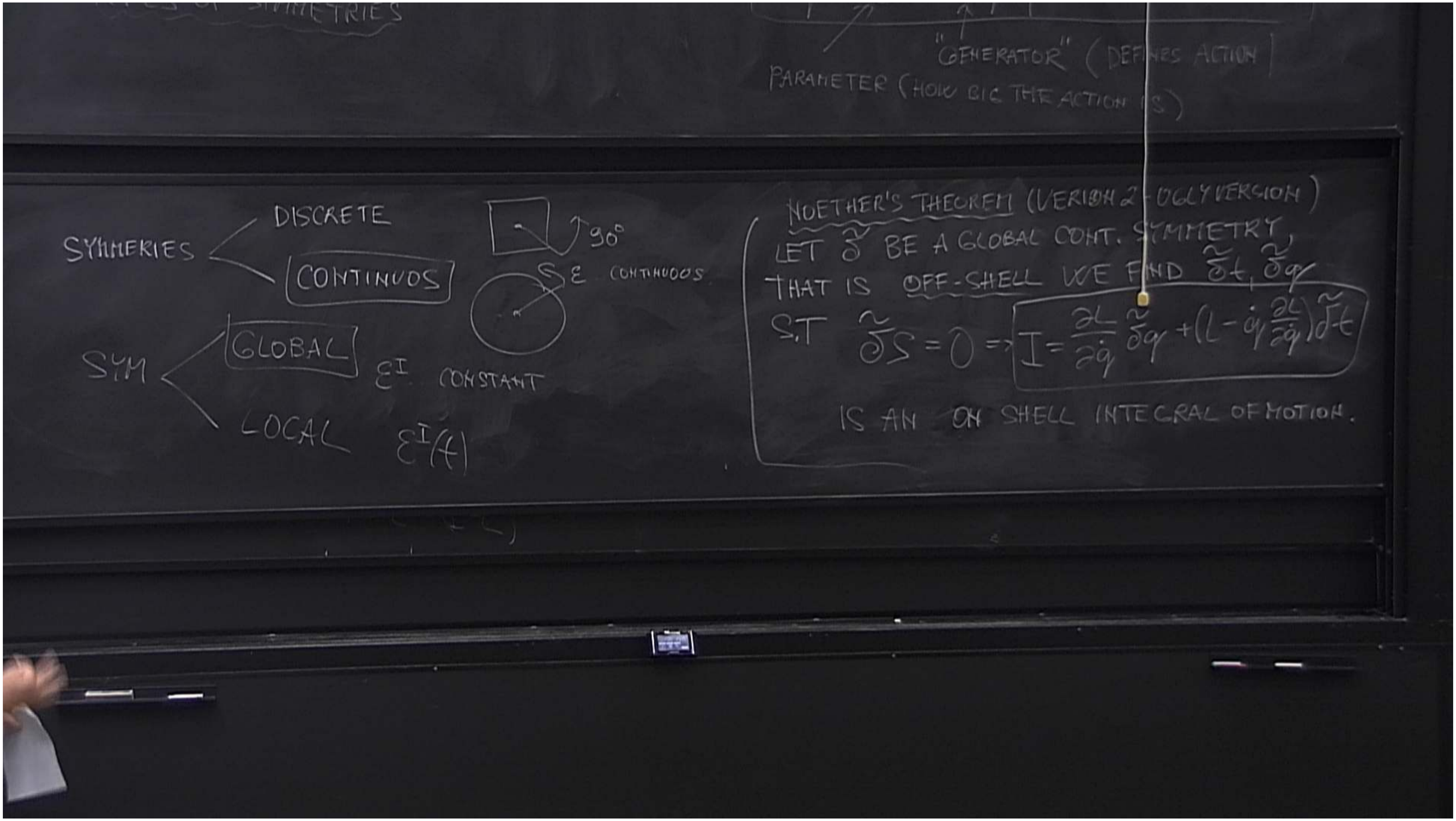
$$\boxed{\tilde{\delta} q = \varepsilon^{k} \tilde{\delta}_{k} q, \quad \tilde{\delta} t = \varepsilon^{k} \tilde{\delta}_{k} t}$$

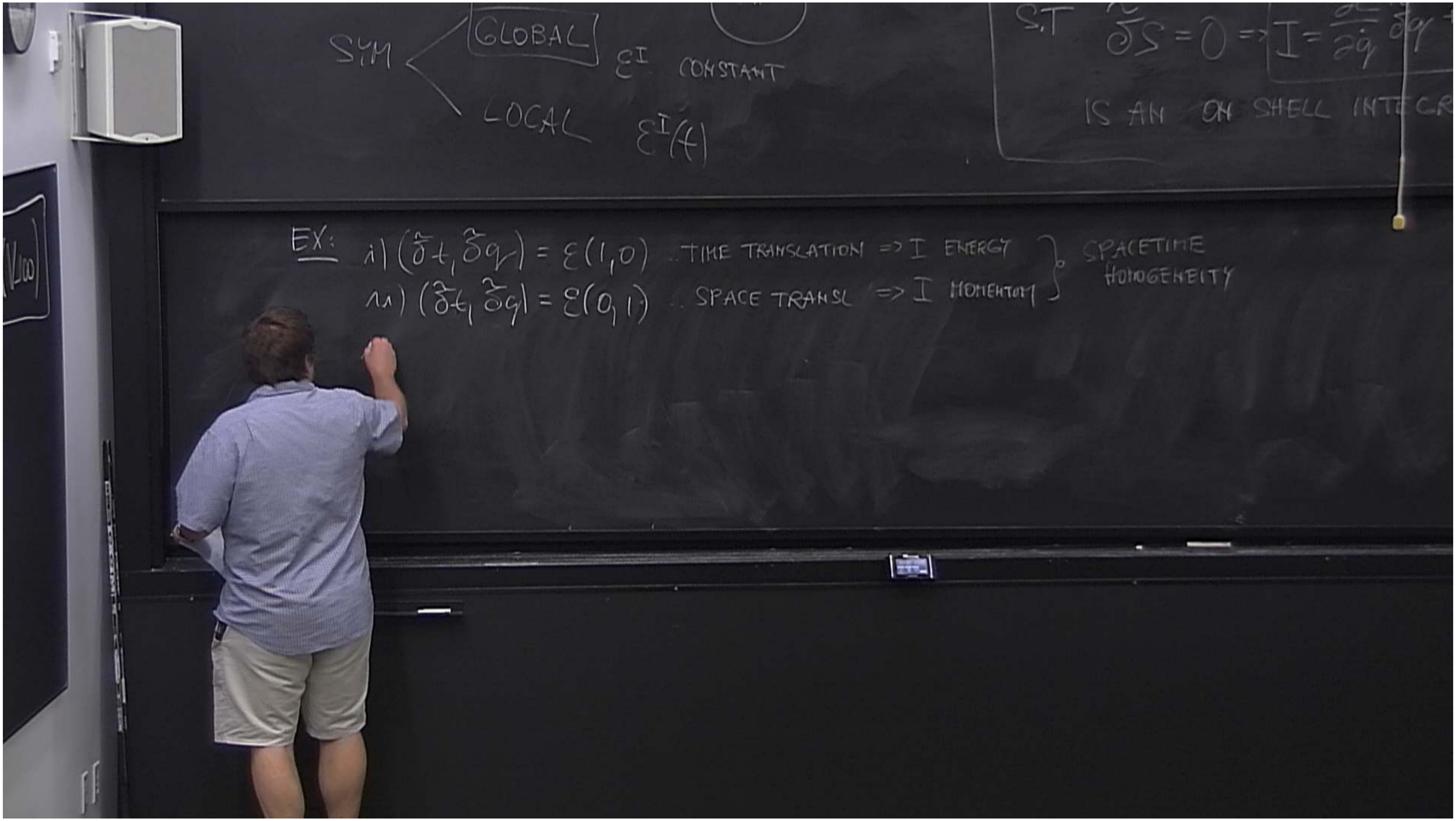
"GENERATOR" (DEFINES ACTION)

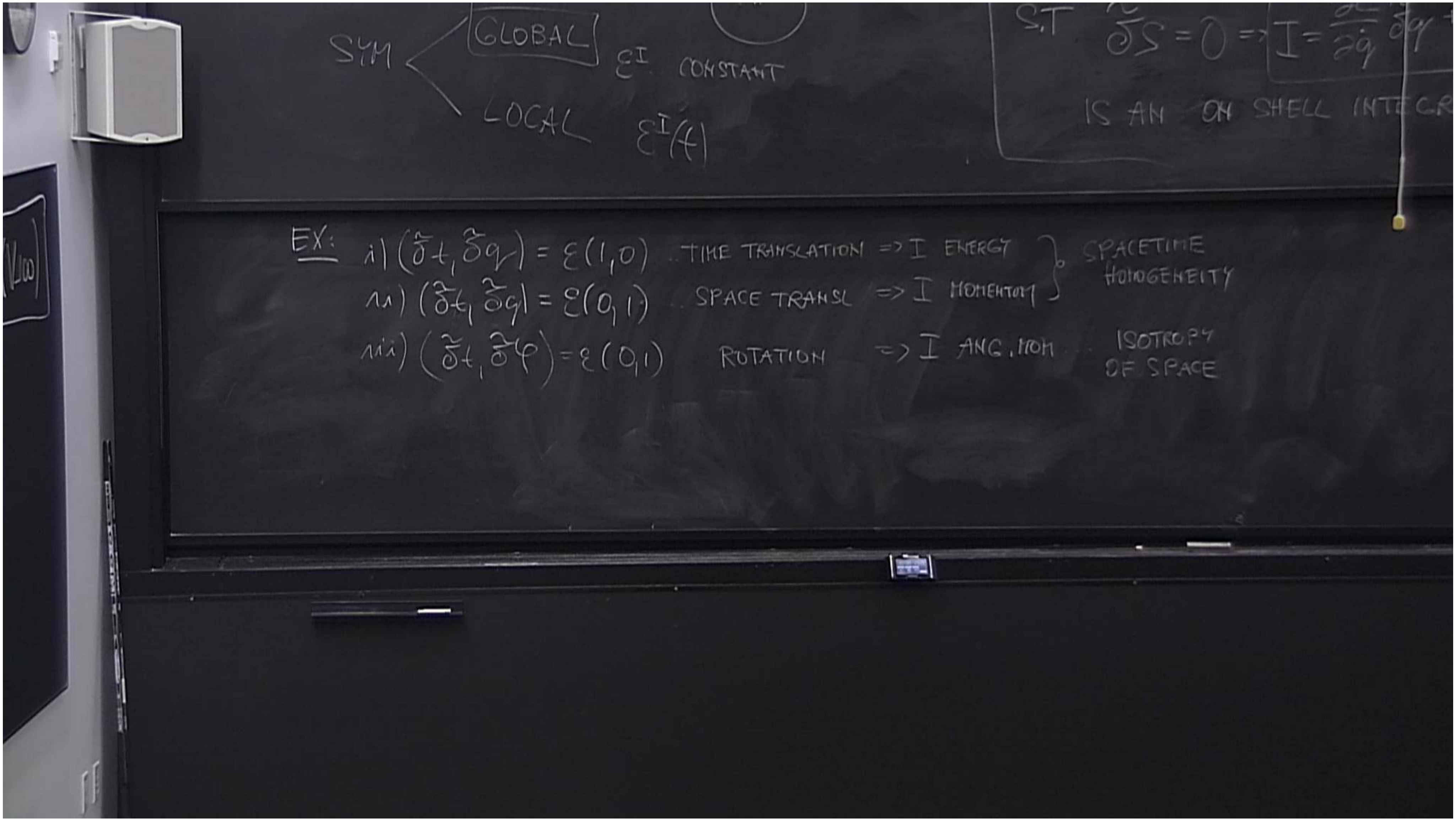
PARAMETER (HOW BIG THE ACTION IS)

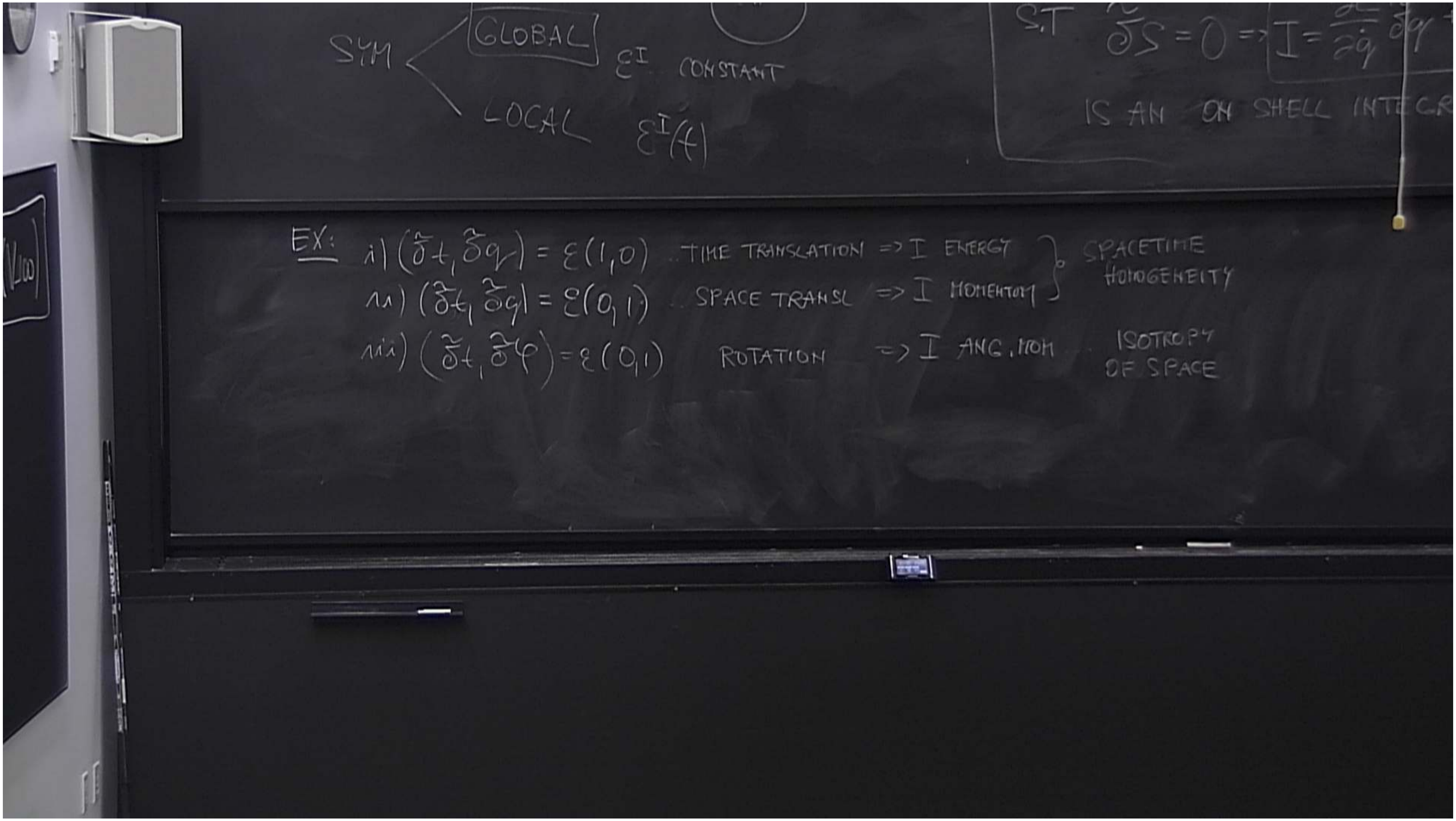

SYMMERIES
DISCRETE
CONTINUOS
90°
ε ... CONTINUOOS
SYM
GLOBAL
LOCAL
ε^I ... CONSTANT
$\varepsilon^I(t)$


"GENERATOR" (DEFINES ACTION)
PARAMETER (HOW BIG THE ACTION IS)
SYMMERIES
DISCRETE
CONTINUOS
90°
CONTINUOUS
SYM
GLOBAL
CONSTANT
LOCAL
NOETHER'S THEOREM (VERSION 2 - UGLY VERSION)
LET $\tilde{\delta}$ BE A GLOBAL CONT. SYMMETRY,
THAT IS OFF-SHELL WE FIND $\tilde{\delta}t$, $\tilde{\delta}q$
S.T $\tilde{\delta}S = 0 \Rightarrow I = \frac{\partial L}{\partial \dot{q}} \tilde{\delta}q$

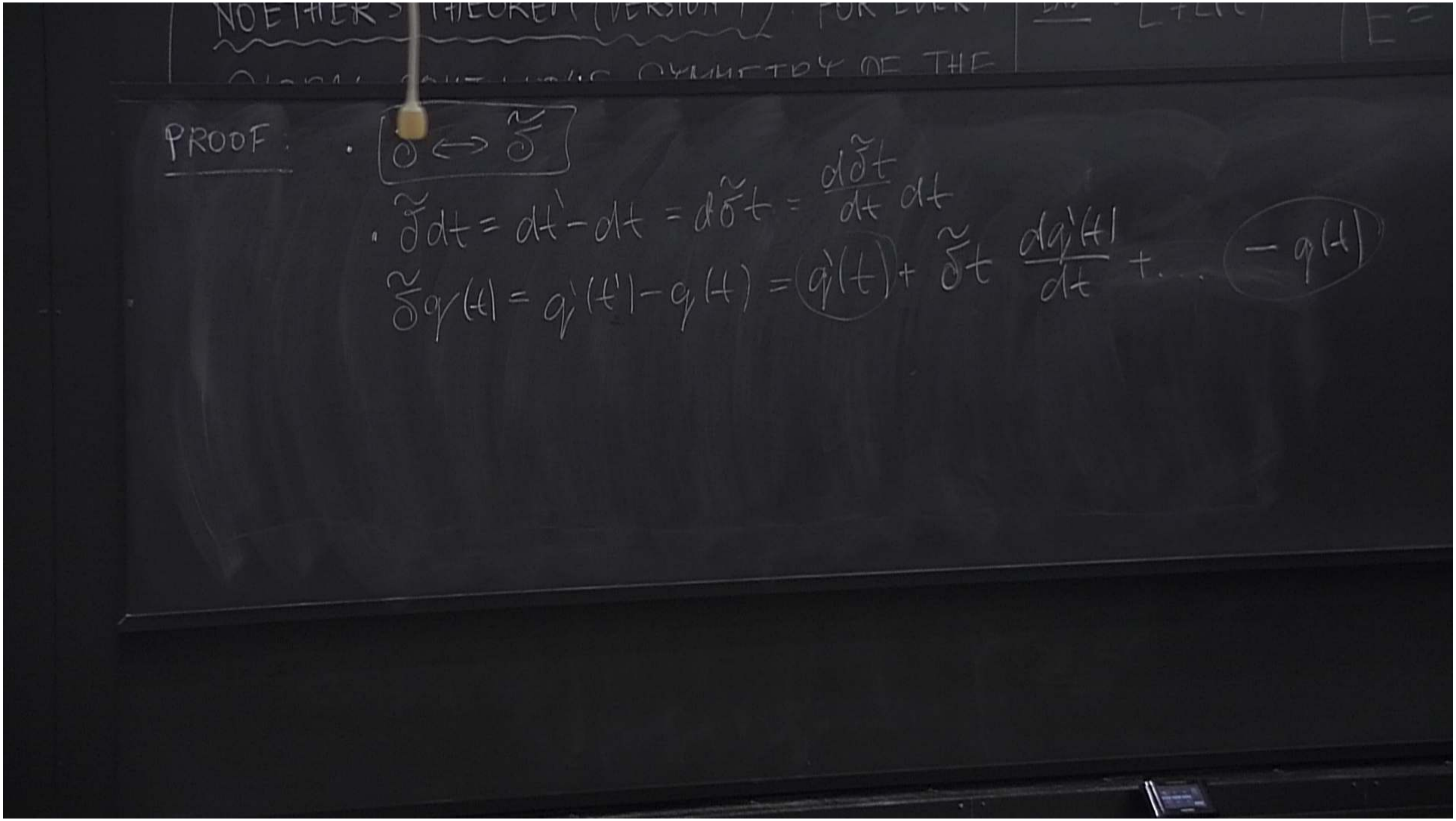

"GENERATOR" (DEFINES ACTION)
PARAMETER (HOW BIG THE ACTION IS)
SYMMERIES
DISCRETE
CONTINUOS
90°
$\delta\varepsilon$ CONTINUOUS
SYM
GLOBAL
ε^I CONSTANT
LOCAL
$\varepsilon^I(t)$
NOETHER'S THEOREM (VERSION 2 - UGLY VERSION)
LET δ BE A GLOBAL CONT. SYMMETRY,
THAT IS OFF-SHELL WE FIND $\tilde{\delta}t$, $\tilde{\delta}q$
S.T $\tilde{\delta}S = 0 \Rightarrow \boxed{I = \frac{\partial L}{\partial \dot{q}}\tilde{\delta}q + \left(L - \dot{q}\frac{\partial L}{\partial \dot{q}}\right)\tilde{\delta}t}$
IS AN ON SHELL INTEGRAL OF MOTION.

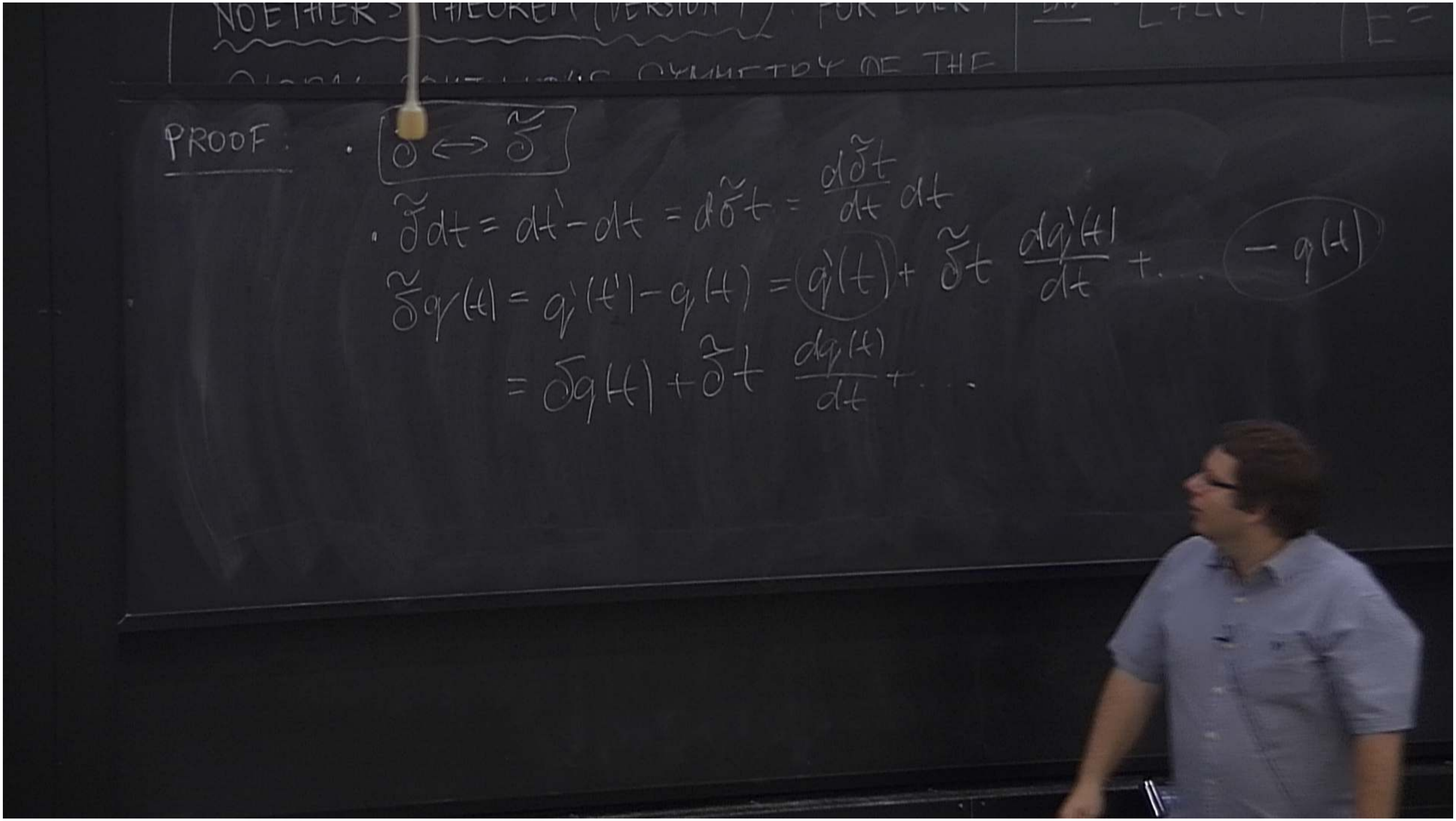

SYM
GLOBAL
ε^I CONSTANT
LOCAL
ε^I(t)
S.T
IS AN ON SHELL INTEGR
EX:
i) (δ̃t, δ̃q) = ε(1,0) TIME TRANSLATION => I ENERGY
ii) (δ̃t, δ̃q) = ε(0,1) SPACE TRANSL => I MOMENTUM
SPACETIME HOMOGENEITY

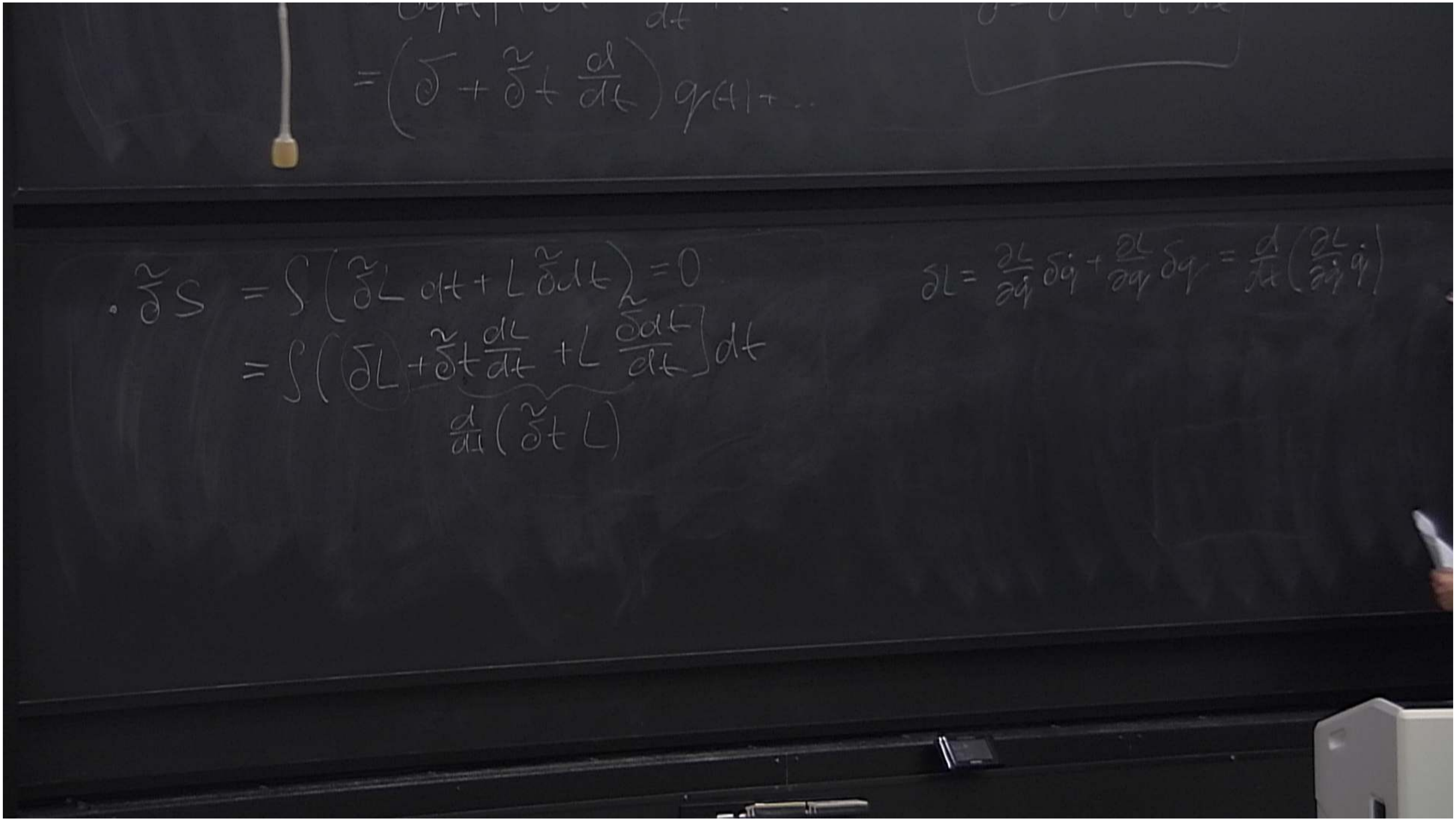

SYM
GLOBAL ε^I CONSTANT
LOCAL ε^I(t)
S.T δS = 0 => I =
IS AN ON SHELL INTEGR
EX:
i) (δ̃t, δ̃q) = ε(1,0) TIME TRANSLATION => I ENERGY
ii) (δ̃t, δ̃q) = ε(0,1) SPACE TRANSL => I MOMENTUM
SPACETIME HOMOGENEITY
iii) (δ̃t, δ̃φ) = ε(0,1) ROTATION => I ANG. MOM
ISOTROPY OF SPACE



SYM
GLOBAL
ε^I CONSTANT
LOCAL
$\varepsilon^I(t)$
S.T. $\delta S = 0 \Rightarrow I = \frac{\partial L}{\partial \dot{q}} \delta q$
IS AN ON SHELL INTEGR
EX: i) $(\tilde{\delta} t, \tilde{\delta} q) = \varepsilon(1,0)$ TIME TRANSLATION => I ENERGY
ii) $(\tilde{\delta} t, \tilde{\delta} q) = \varepsilon(0,1)$ SPACE TRANSL => I MOMENTUM
SPACETIME HOMOGENEITY
iii) $(\tilde{\delta} t, \tilde{\delta} \varphi) = \varepsilon(0,1)$ ROTATION => I ANG. MOM
ISOTROPY OF SPACE


PROOF:
$\delta \longleftrightarrow \tilde{\delta}$
$\tilde{\delta} dt = dt' - dt = d\tilde{\delta}t = \frac{d\tilde{\delta}t}{dt} dt$
$\tilde{\delta} q(t) = q'(t') - q(t) = q'(t) + \tilde{\delta}t \frac{dq'(t)}{dt} + \ldots - q(t)$


PROOF:
$\delta \leftrightarrow \tilde{\delta}$
$\tilde{\delta} dt = dt' - dt = d\tilde{\delta}t = \frac{d\tilde{\delta}t}{dt} dt$
$\tilde{\delta} q(t) = q'(t') - q(t) = q'(t) + \tilde{\delta}t \frac{dq'(t)}{dt} + \dots - q(t)$
$= \delta q(t) + \tilde{\delta}t \frac{dq(t)}{dt} + \dots$



$= \left(\delta + \tilde{\delta} t \frac{d}{dt}\right) q(t) + \dots$
$\tilde{\delta} S = \int \left(\tilde{\delta} L \, dt + L \tilde{\delta} dt\right) = 0$
$= \int \Big(\delta L + \underbrace{\tilde{\delta} t \frac{dL}{dt} + L \frac{\tilde{\delta} dt}{dt}}_{\frac{d}{dt}(\tilde{\delta} t L)} \Big] dt$
$\delta L = \frac{\partial L}{\partial \dot{q}} \delta \dot{q} + \frac{\partial L}{\partial q} \delta q = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}} \dot{q}\right)$

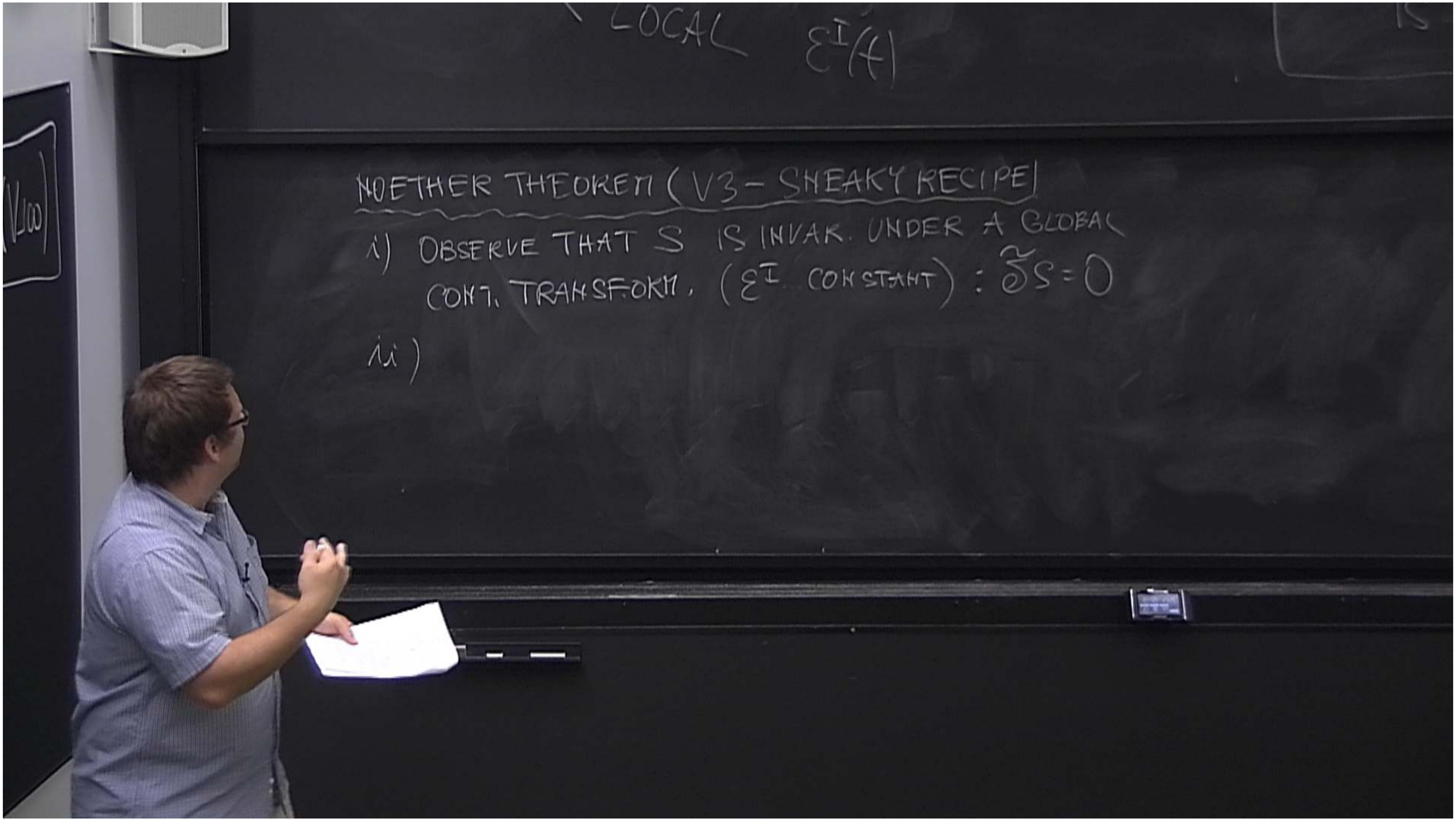
$$\tilde\delta S = \int\left(\tilde\delta L\, dt + L\,\tilde\delta dt\right) = 0$$

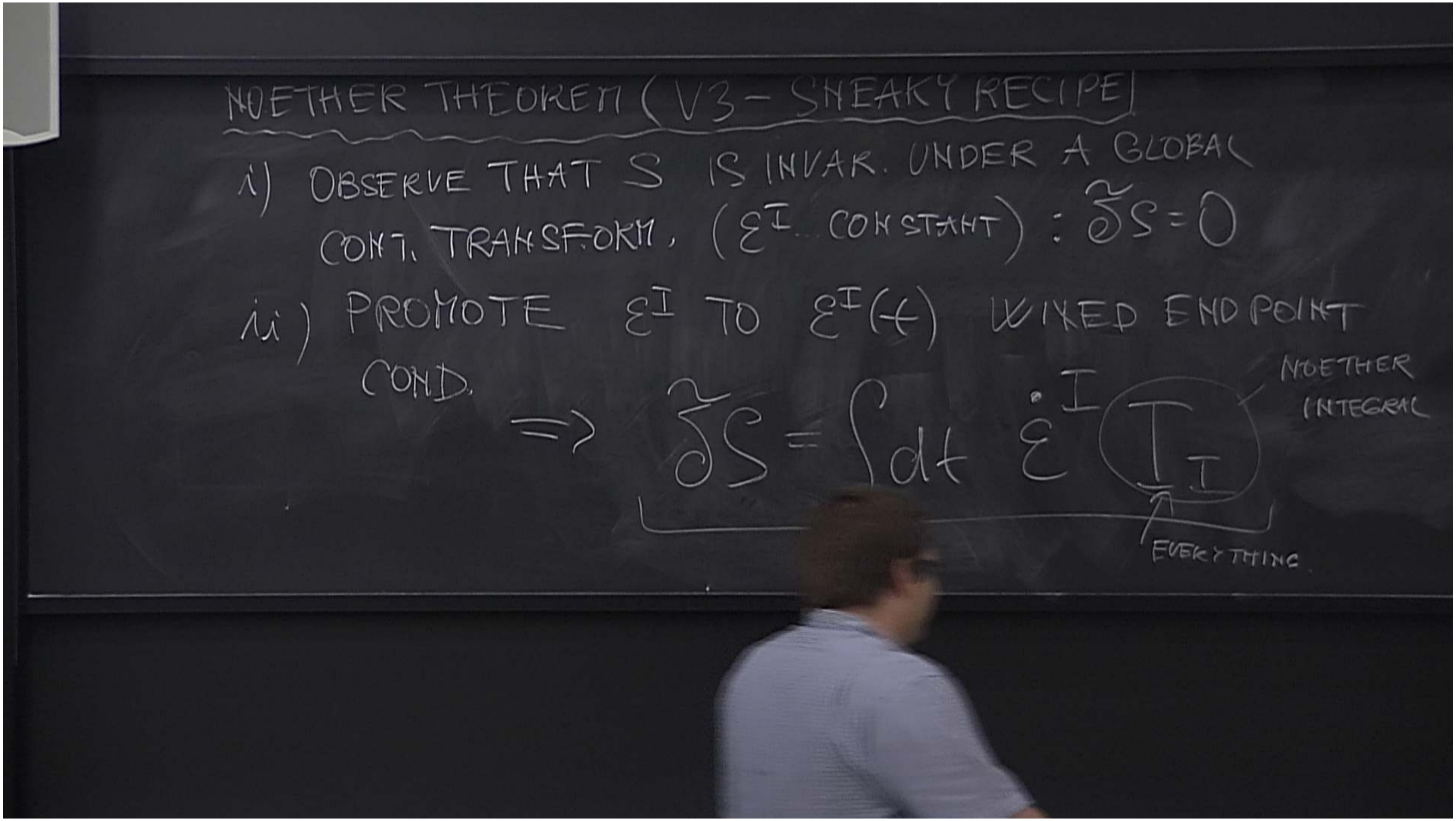
$$\delta L = \frac{\partial L}{\partial q}\delta q + \frac{\partial L}{\partial \dot q}\delta \dot q = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot q}\delta q\right) - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot q}\right)\delta q + \frac{\partial L}{\partial q}\delta q$$

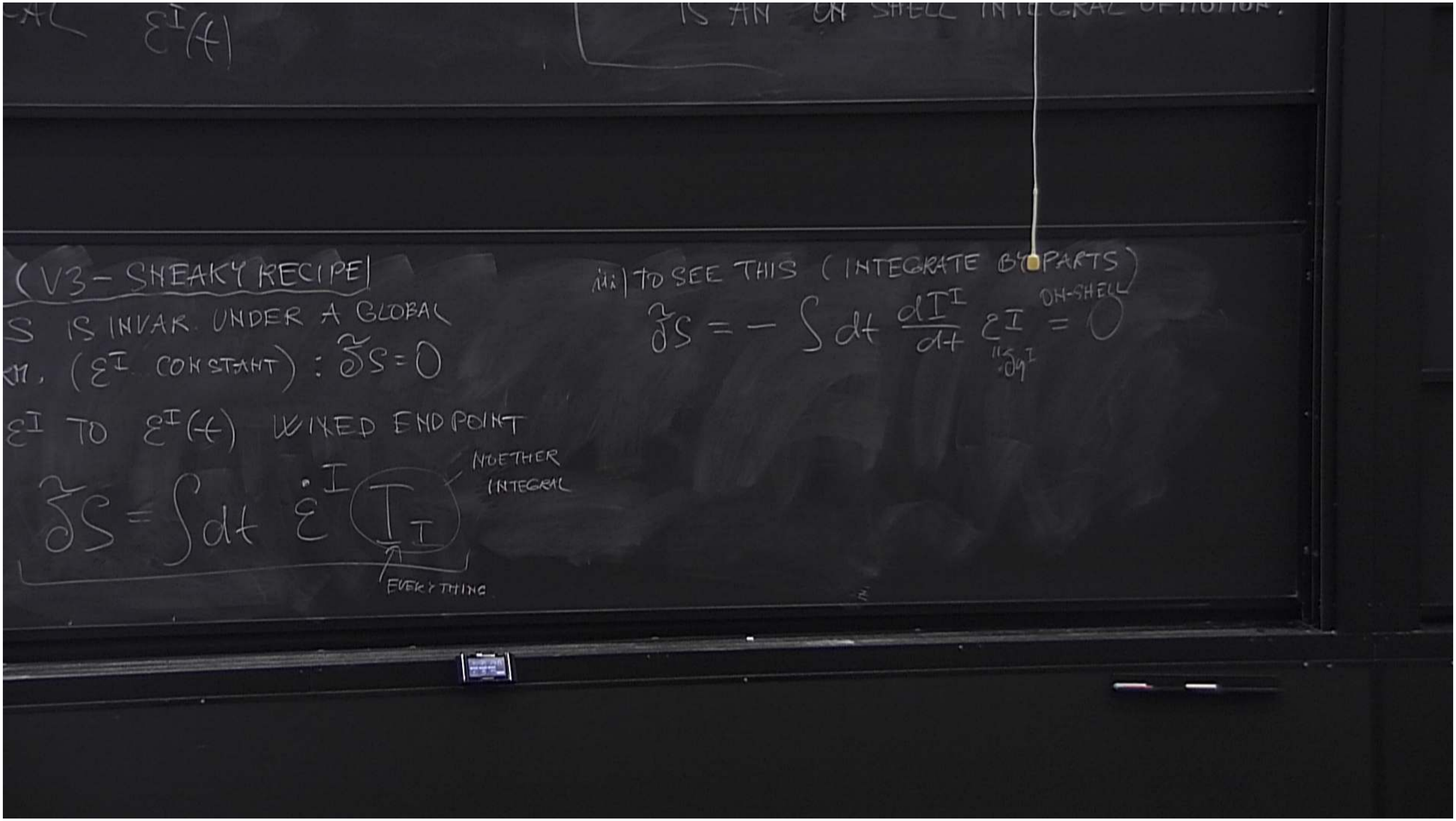
$$= \int\left(\delta L + \underbrace{\tilde\delta t\,\frac{dL}{dt} + L\,\frac{\tilde\delta dt}{dt}}_{\frac{d}{dt}(\tilde\delta t\, L)}\right) dt$$

$$= \int\left(\frac{d}{dt}\left(\underbrace{\frac{\partial L}{\partial \dot q}\delta q + L\tilde\delta t}_{I}\right) + \left[\underbrace{\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot q}\right)}_{0 \text{ ON SHELL}}\right]\delta q\right) dt$$

$$I(t_1) = I(t_2)$$


LOCAL
$\varepsilon^I(t)$
NOETHER THEOREM (V3 - SNEAKY RECIPE)
i) OBSERVE THAT S IS INVAR. UNDER A GLOBAL
CONT. TRANSFORM. (ε^I CONSTANT) : $\tilde{\delta} S = 0$
ii)


NOETHER THEOREM (V3 – SNEAKY RECIPE)
i) OBSERVE THAT S IS INVAR. UNDER A GLOBAL CONT. TRANSFORM. (ε^I CONSTANT) : $\tilde{\delta} S = 0$
ii) PROMOTE ε^I TO $\varepsilon^I(t)$ FIXED END POINT COND.
$\Longrightarrow \underbrace{\tilde{\delta} S = \int dt\, \dot{\varepsilon}^I\, T_I}$
NOETHER INTEGRAL
EVERYTHING



IS AN ON SHELL INTEGRAL OF MOTION.
ε^I(t)
(V3 - SNEAKY RECIPE)
S IS INVAR. UNDER A GLOBAL
(ε^I CONSTANT) : δ̃S = 0
ε^I TO ε^I(t) W FIXED END POINT
δ̃S = ∫dt ε̇^I T_I
NOETHER INTEGRAL
EVERYTHING
iii) TO SEE THIS (INTEGRATE BY PARTS)
δ̃S = − ∫dt dT^I/dt ε^I = 0
"δq^I"
ON-SHELL