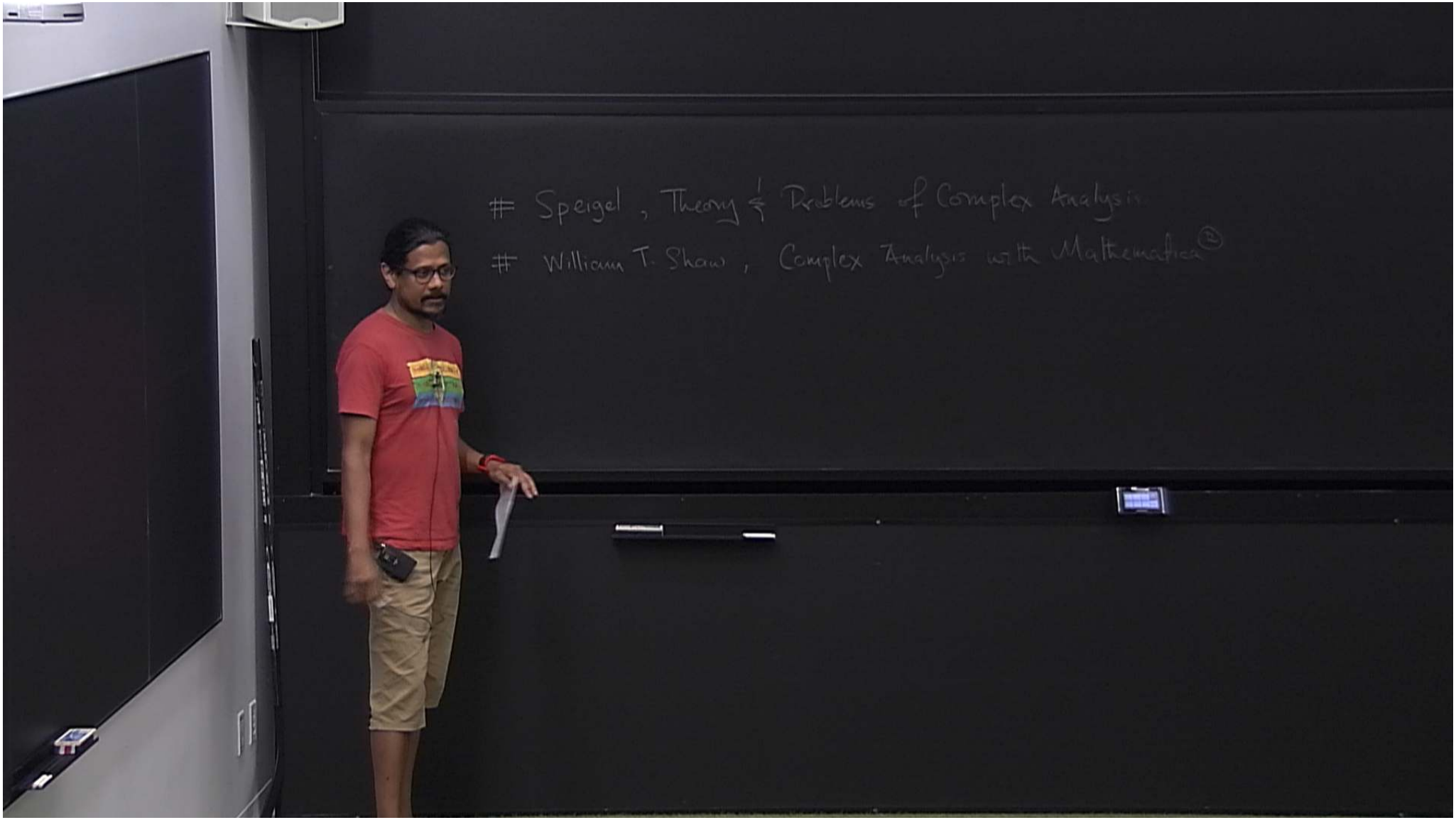
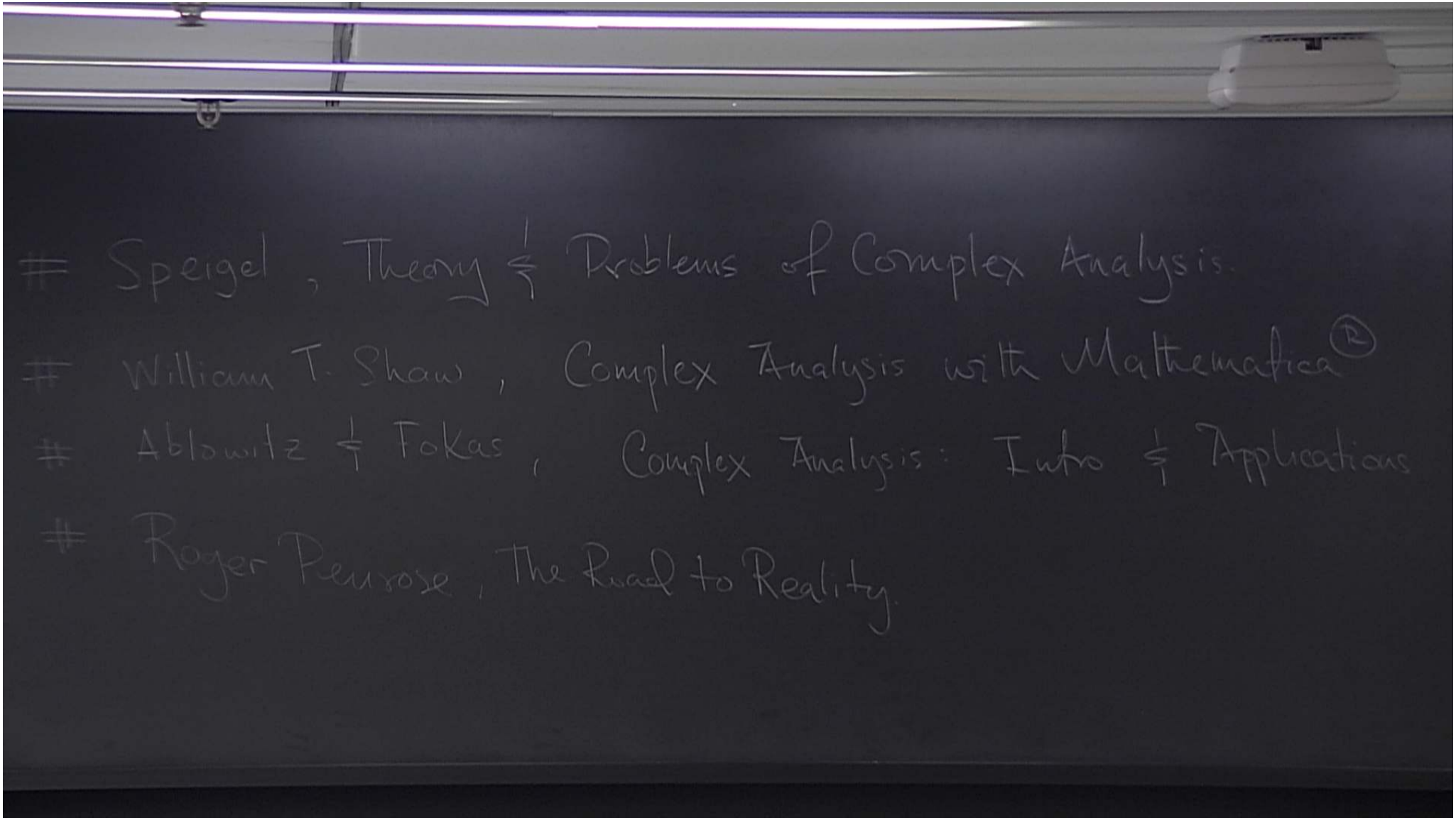
Title: Front End - Complex Analysis - 1

Date: Aug 09, 2016 09:00 AM

URL: http://pirsa.org/16080057

Abstract:


Spiegel, Theory & Problems of Complex Analysis
William T. Shaw, Complex Analysis with Mathematica


Spiegel, Theory & Problems of Complex Analysis.
William T. Shaw, Complex Analysis with Mathematica®
Ablowitz & Fokas, Complex Analysis: Intro & Applications
Roger Penrose, The Road to Reality.

History & Motivation

Girolamo Cardano (1545)

$x(10-x) = 40$

$ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

if $b^2 < 4ac$

$\sqrt{-1}$

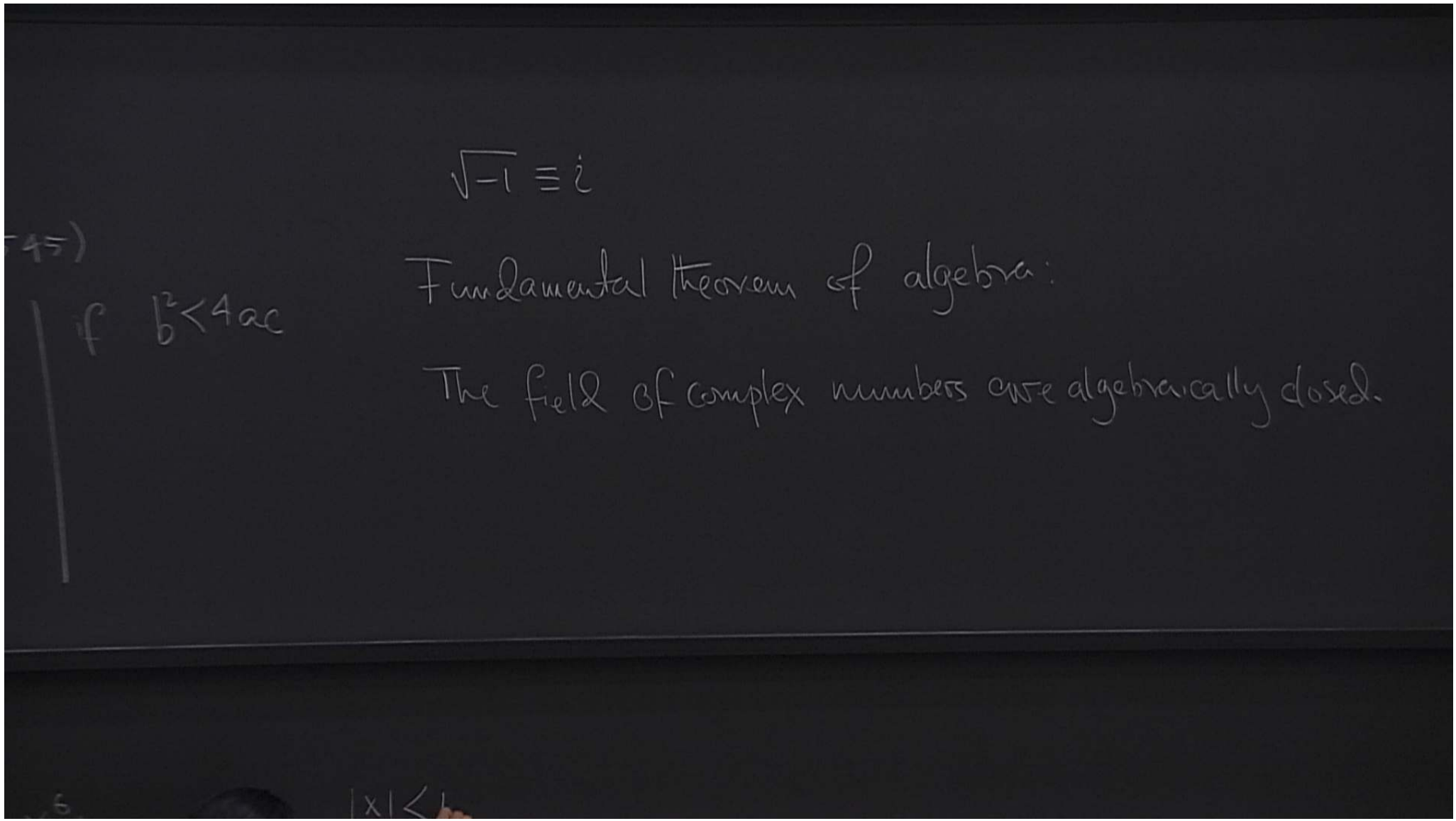
Fund

The

$\sqrt{-1} \equiv i$

Fundamental theorem of algebra:

The field of complex numbers are algebraically closed.



$$f(x) = \frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \cdots \quad , \quad |x| < 1$$

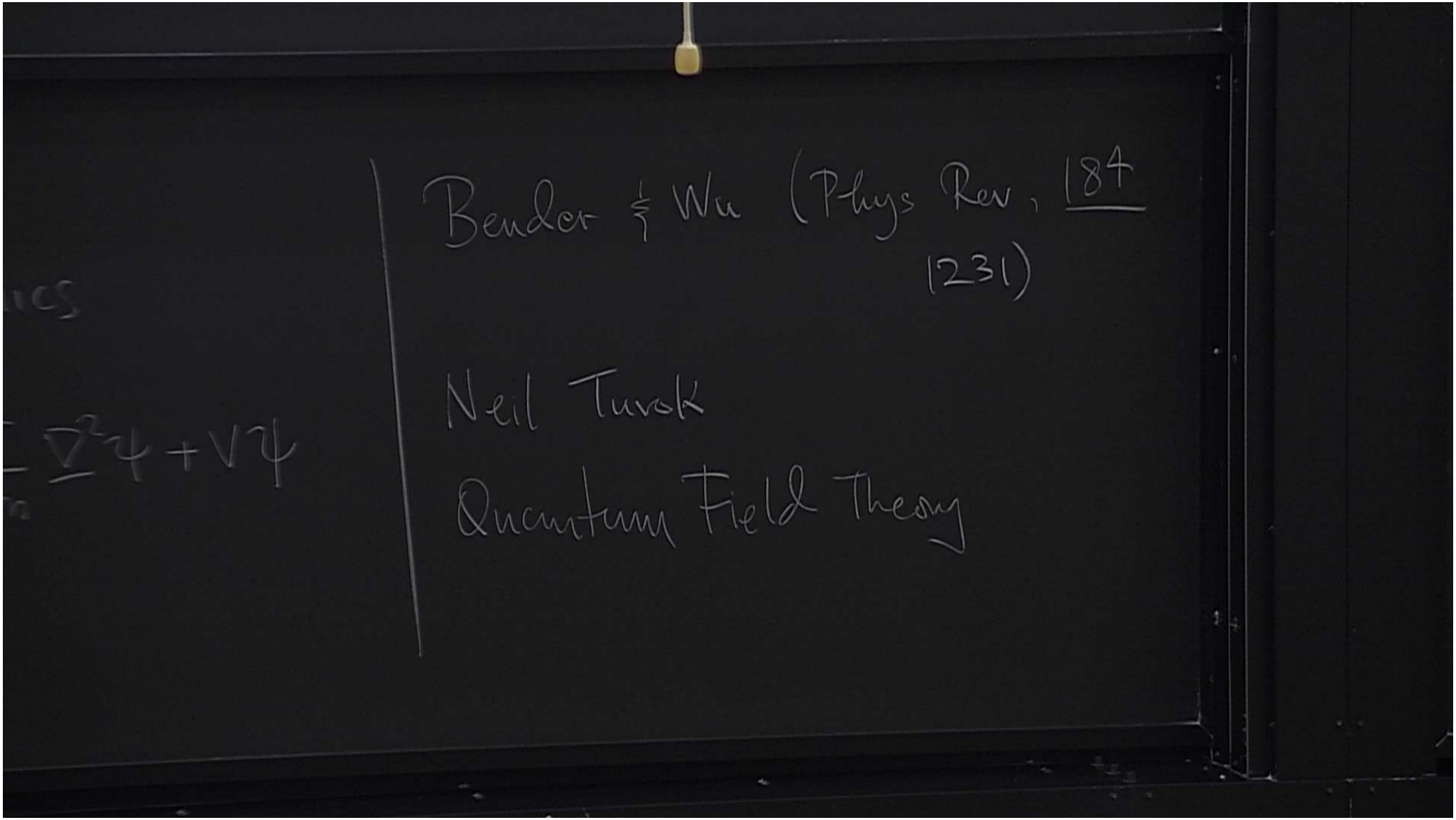
$x = \pm i$

$|x|<1$

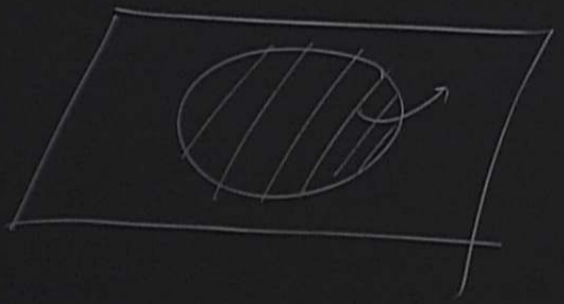
Physics

1. Quantum Mechanics

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi$$

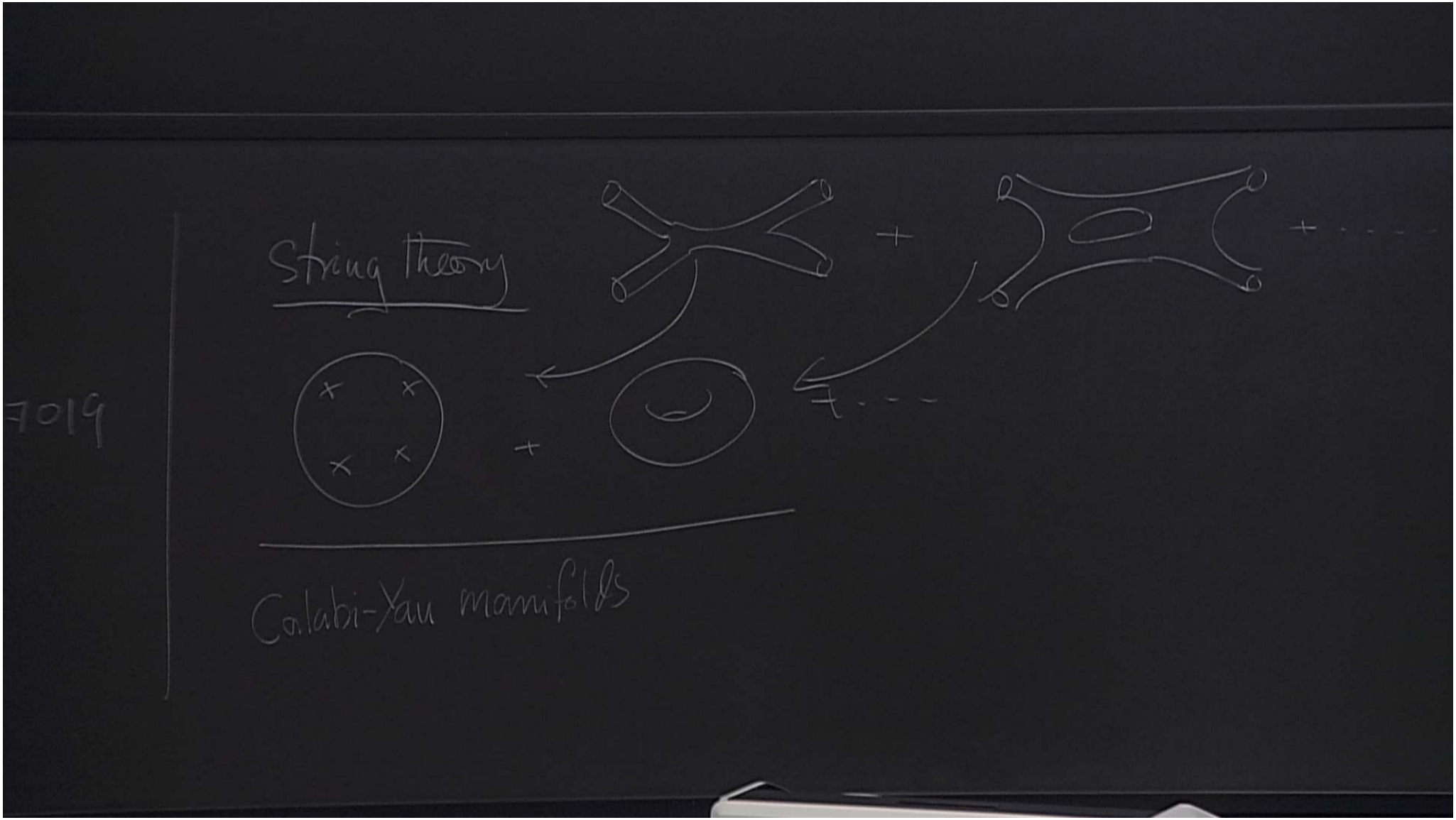

Bender & Wu (Phys Rev, 184
1231)
Neil Turok
Quantum Field Theory

GR ~ $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu}$



Bull, gr-qc/9507019

Twistor Program

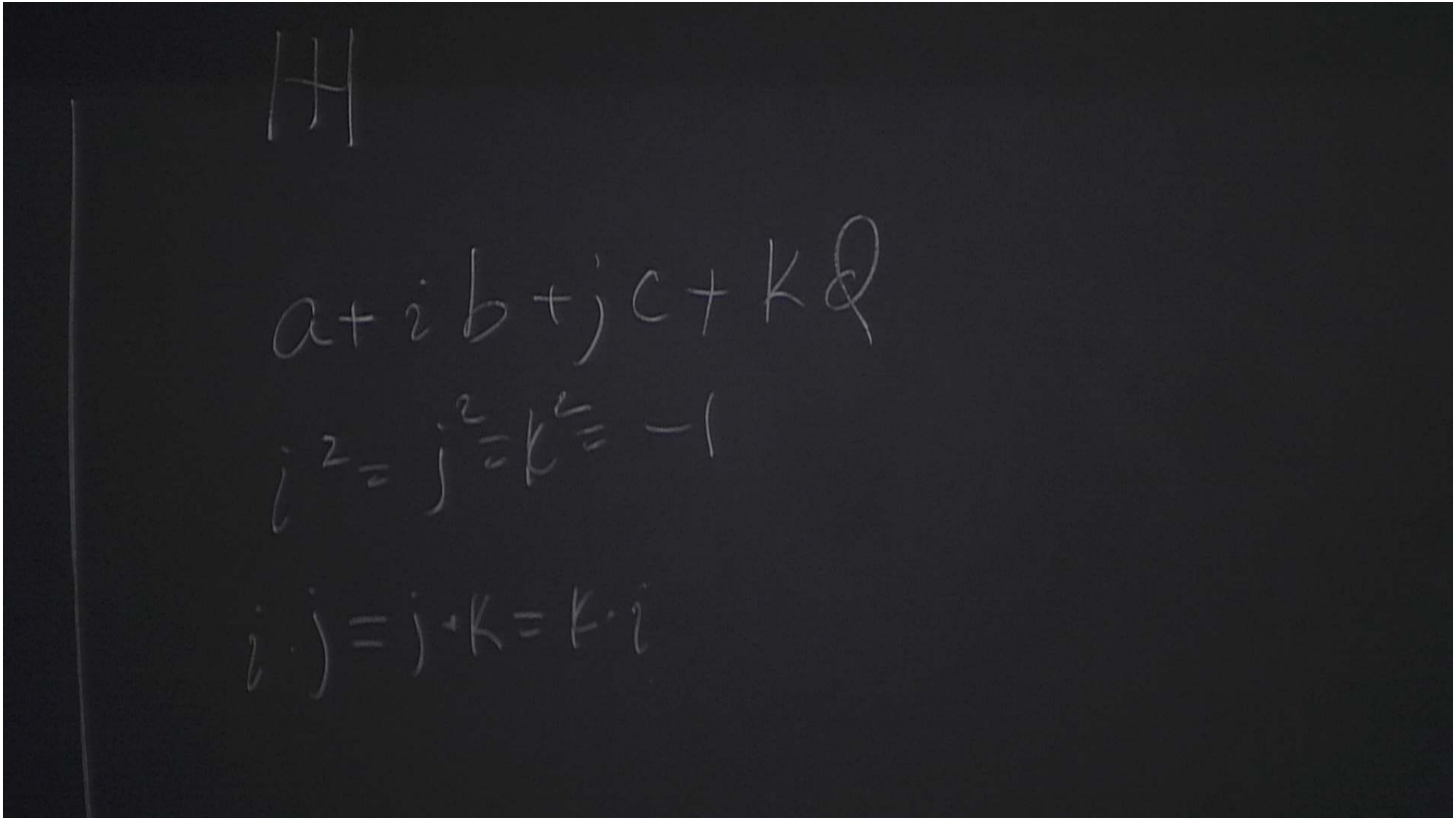

7019
String theory
+
+ . . . .
+
+ . . . .
Calabi-Yau manifolds

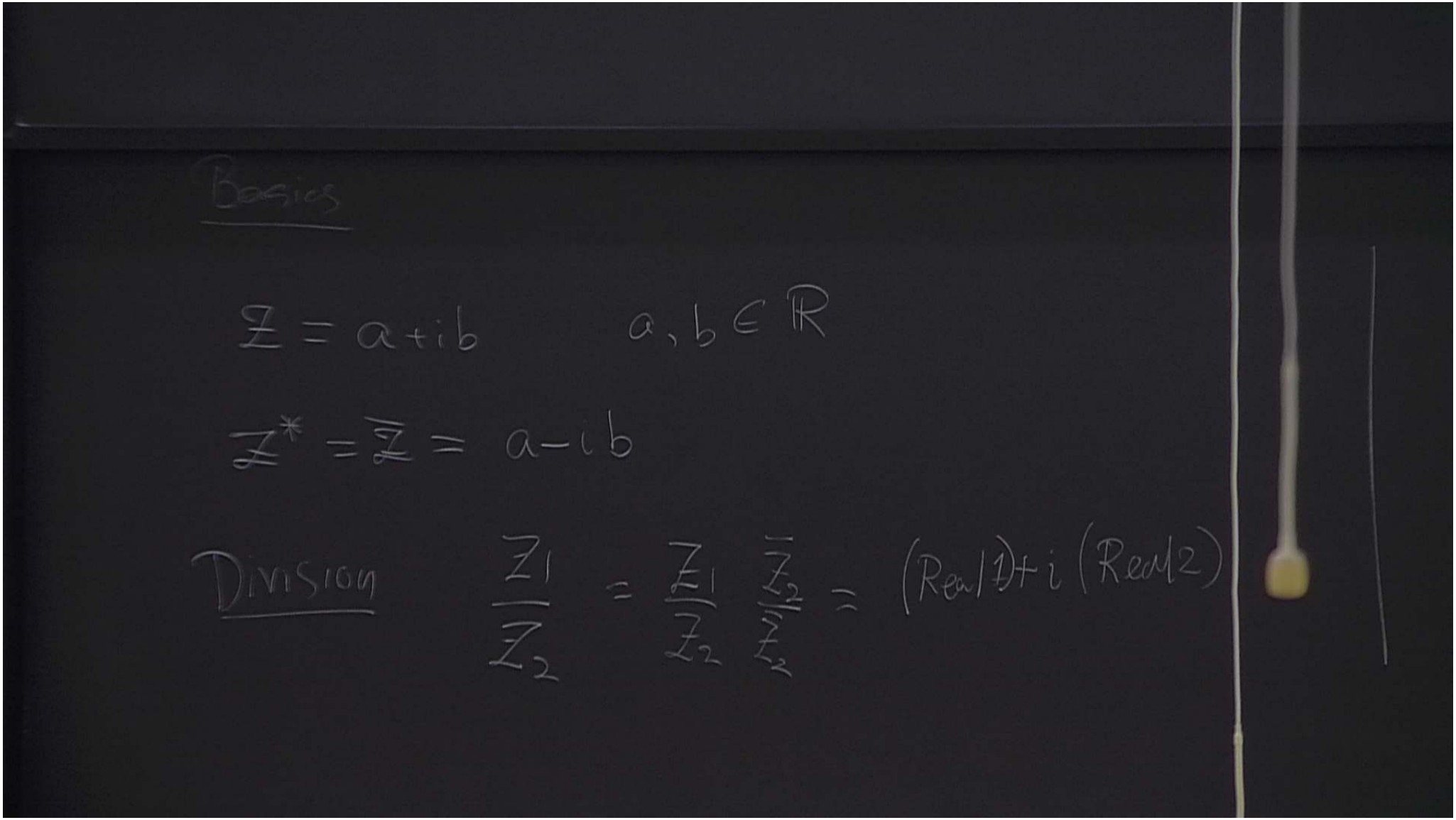
Basics

$z = a + ib \qquad a, b \in \mathbb{R}$

$z^* = \bar{z} = a - ib$

Division $\quad \dfrac{z_1}{z_2} = \dfrac{z_1}{z_2}\dfrac{\bar{z}_2}{\bar{z}_2} = (\mathrm{Real}\,1) + i\,(\mathrm{Real}\,2)$


H
a + ib + jc + kd
$i^2 = j^2 = k^2 = -1$
$ij = jk = ki$


Basics
$z = a + ib \qquad a, b \in \mathbb{R}$
$z^* = \bar{z} = a - ib$
Division
$\frac{z_1}{z_2} = \frac{z_1}{z_2}\frac{\bar{z}_2}{\bar{z}_2} = (\text{Real1}) + i\,(\text{Real2})$

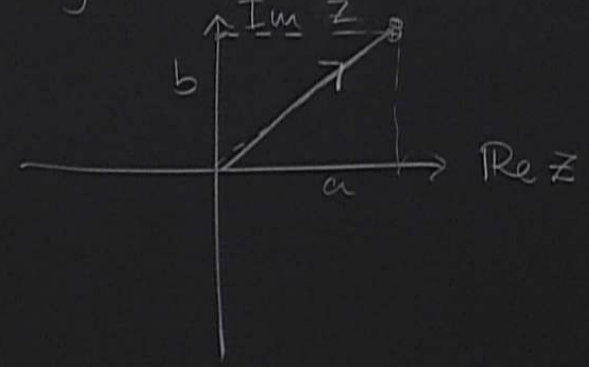
$\overline{\overline{z}} = z$

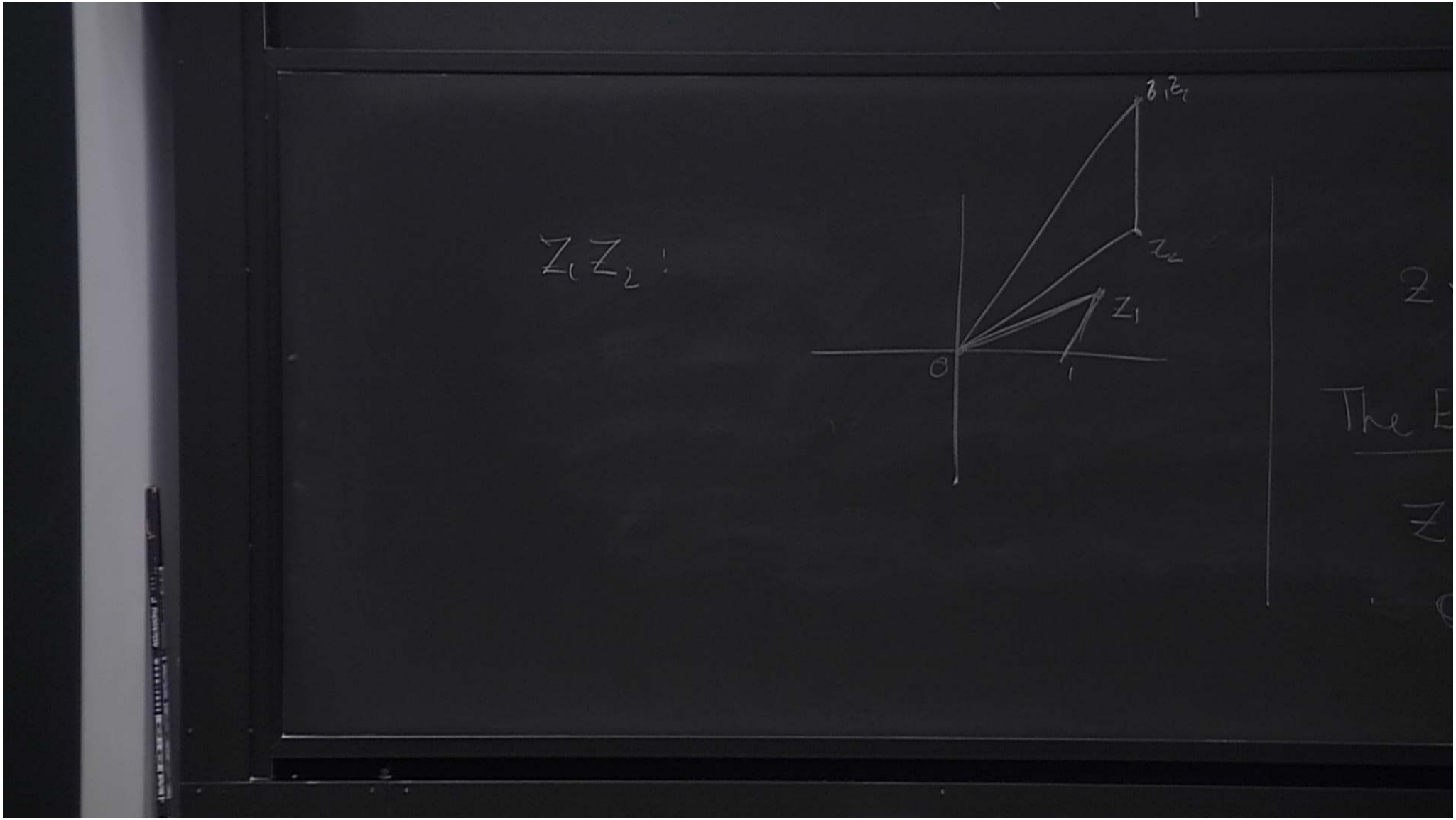
$\overline{z_1 z_2} = \overline{z}_1 \overline{z}_2$

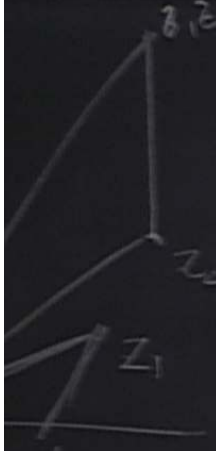
$\overline{z_1 + z_2} = \overline{z}_1 + \overline{z}_2$

$|z| = \sqrt{z\overline{z}} = \sqrt{a^2 + b^2}$

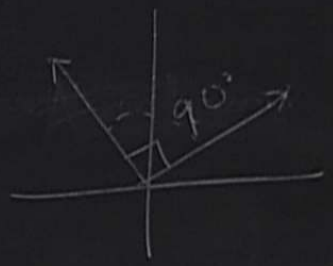
Argand-Wessel Plane




$z_1 z_2$!
$z_1 z_2$
z_2
z_1
0
1



$z \times i$



The Euler Formula:

$$z = r(\cos\theta + i\sin\theta), \quad r = |z|$$

$$\theta = \arg(z)$$

$\cos\theta$

$\sin$

$i^2 = -$

$z e^{i}$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!}$$

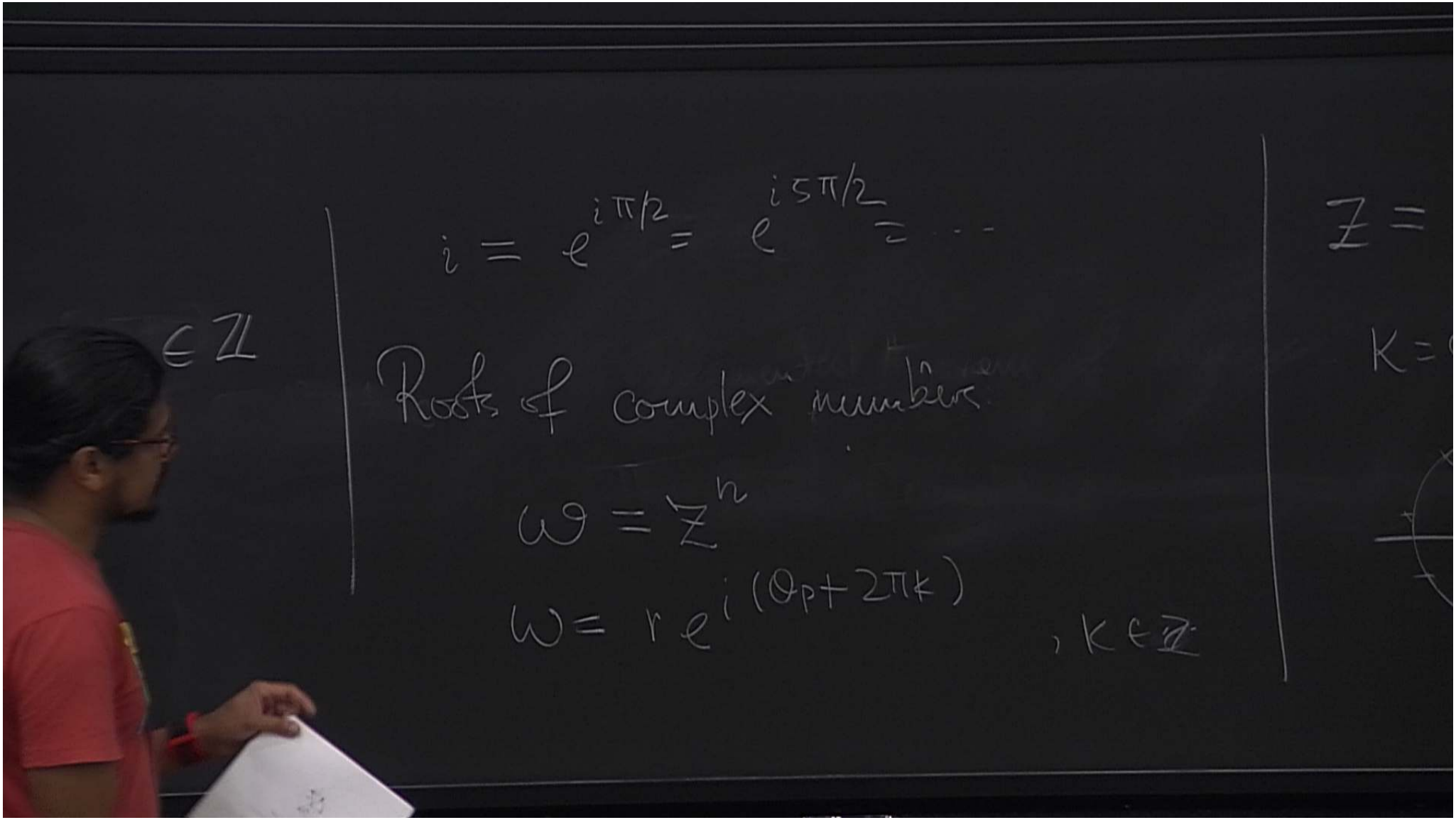
$$\sin\theta = \theta - \frac{\theta^3}{3!} + \cdots$$

$$i^2 = -1$$

$$r(\cos\theta + i\sin\theta) = re^{i\theta}$$

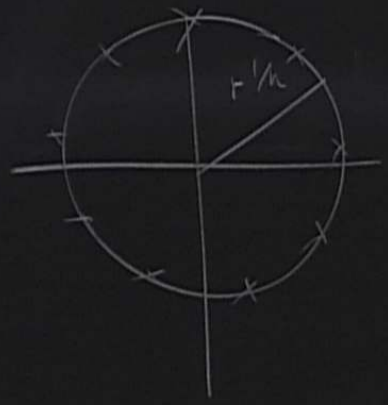
$r = |z|$

$$ze^{i\theta} = re^{i\varphi}e^{i\theta} = re^{i(\varphi+\theta)}$$


$i = e^{i\pi/2} = e^{i5\pi/2} = \cdots$
$\in \mathbb{Z}$
Roots of complex numbers
$\omega = z^n$
$\omega = re^{i(\theta_p + 2\pi k)}$
$, k \in \mathbb{Z}$
$z =$
$k =$

$$z = r^{\frac{1}{n}} e^{i\left(\frac{\theta_p}{n} + \frac{2\pi k}{n}\right)}$$

$k = 0, 1, \ldots, n-1$



$, k \in \mathbb{Z}$

$$z = re^{i\theta}$$

$$\arg(z) = \theta_p + 2\pi n, \quad n \in \mathbb{Z}$$

θ_p → Principle value

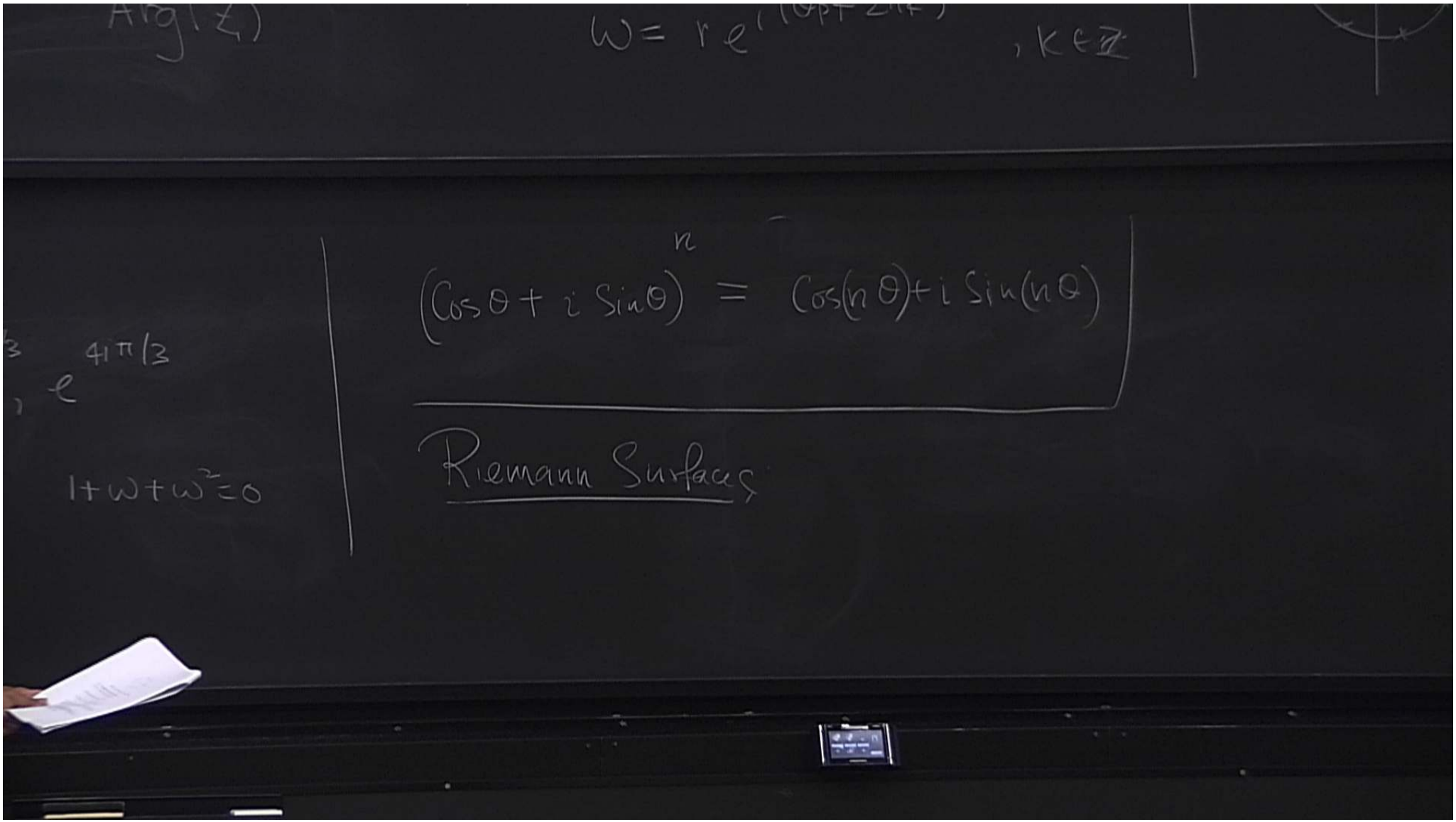
$\mathrm{Arg}(z)$

$z^3 = 1$

$z = 1, e^{2\pi i/3}, e^{4i\pi/3}$

$\hookrightarrow \omega$

$1 + \omega + \omega^2 = 0$


Arg(z)
$\omega = re^{i(\theta + 2k\pi)}$, $k \in \mathbb{Z}$
$(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$
Riemann Surfaces
$e^{4i\pi/3}$
$1 + \omega + \omega^2 = 0$

ltiple value

$\text{Arg}(z)$

$\omega = z^n$

$\omega = r e^{i(\theta_p + 2\pi k)}$, $k \in \mathbb{Z}$

$(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$

$z = |z| e^{i(\theta_p + 2\pi k)}$

$\log z = \log|z| + i(\theta_p + 2\pi k)$

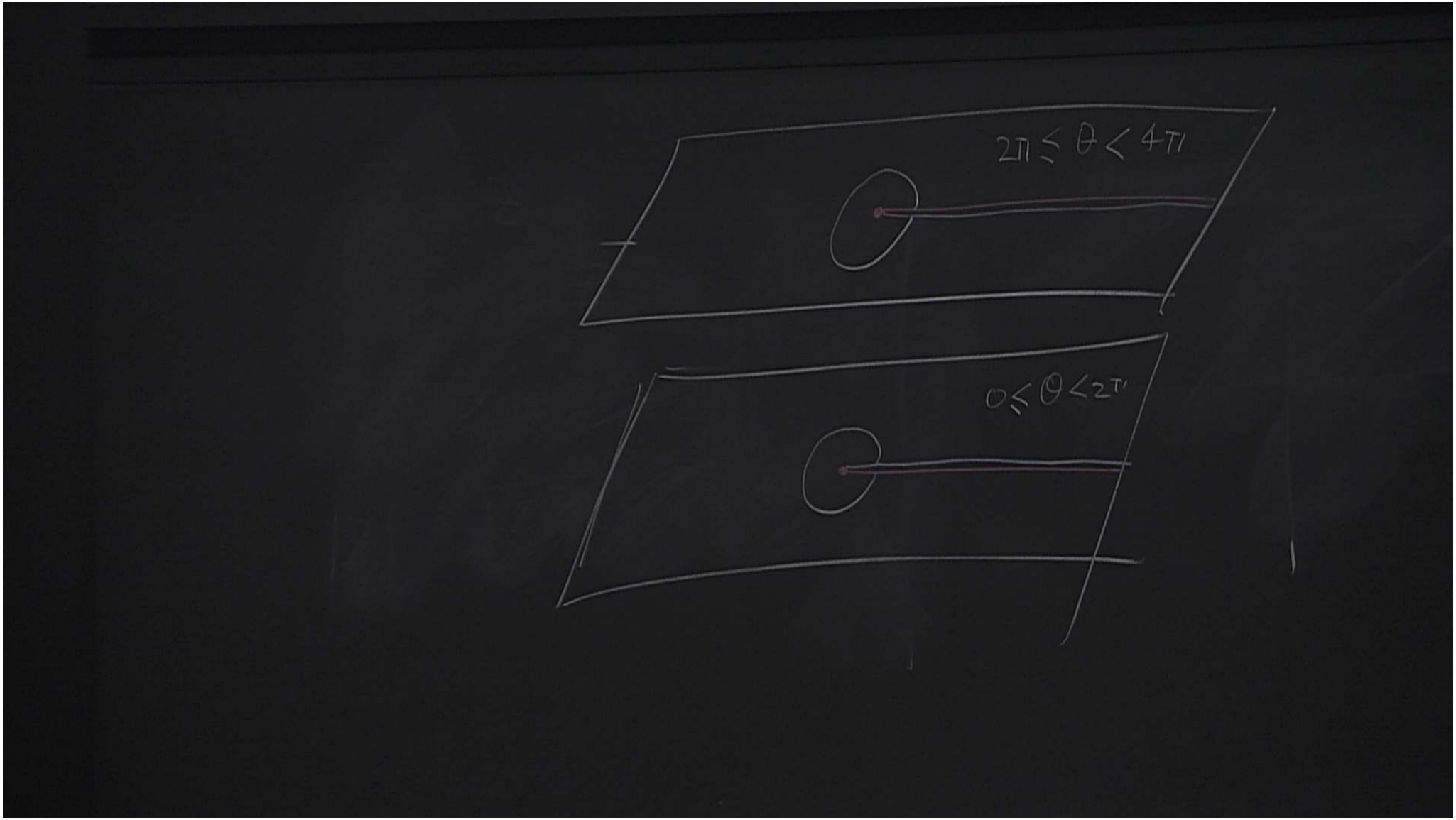
$\pi/3$

$\omega + \omega^2 = 0$

Riemann Surfaces

$\sqrt{z} = \sqrt{r}\, e^{i\theta/2}$

$= \sqrt{r}\, e^{i\theta_p/2}\, e^{in\pi}$


2π ≤ θ < 4π
0 ≤ θ < 2π