

Title: The BMS group, conservation laws and soft gravitons

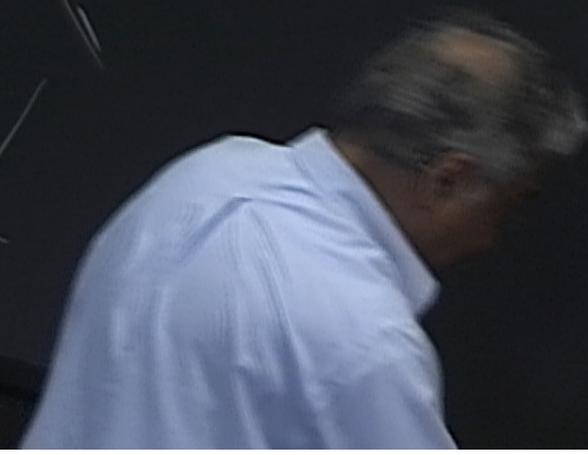
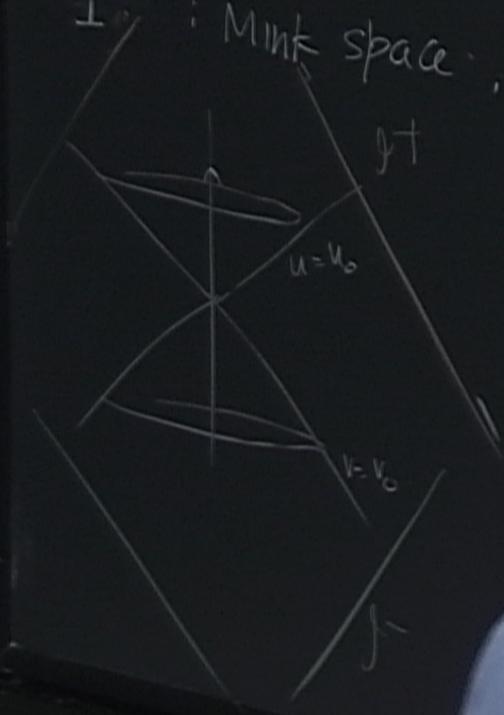
Date: Aug 11, 2016 02:30 PM

URL: <http://pirsa.org/16080055>

Abstract: <p>I will give a broad overview of this old subject which has come back to light recently in connection with the issue of black hole evaporation. It will be an informal, black board talk.</p>

The BMS Group, Conservation laws and Soft Gravitons.

I: Mink space,  $ds^2 = -dt^2 + dr^2 + r^2 d\omega^2$   
 $u = t - r$ ,  $v = t + r$



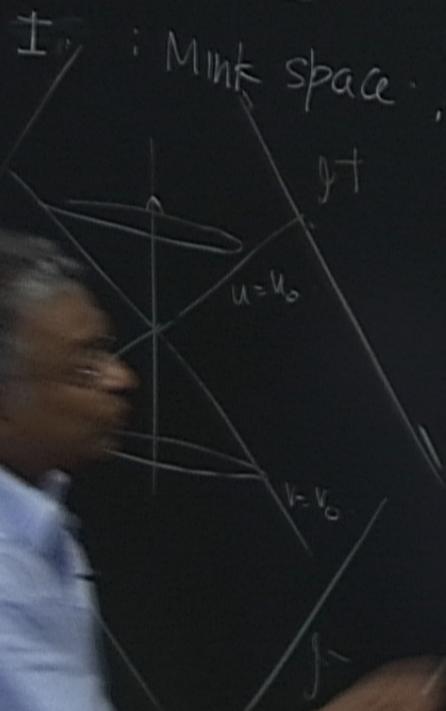
# The BMS Group, Conservation laws and Soft Gravitons.

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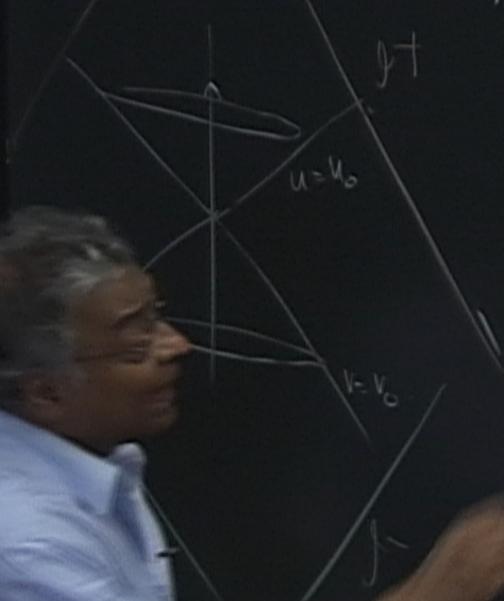
$$v = t + r$$

$$ds^2 = -du^2 + 2 du dr + r^2 d\omega^2$$



# The BMS Group, Conservation laws and Soft Gravitons.

I. Mink space



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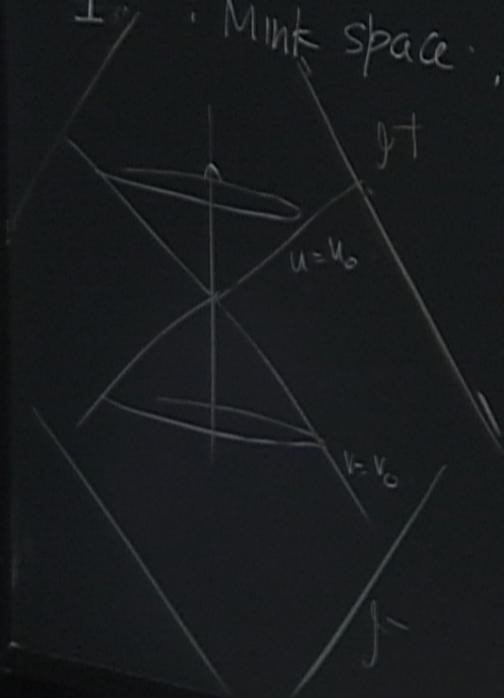
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$$\Omega = \frac{1}{r}, \quad \mathcal{I}^+ : \Omega = 0$$

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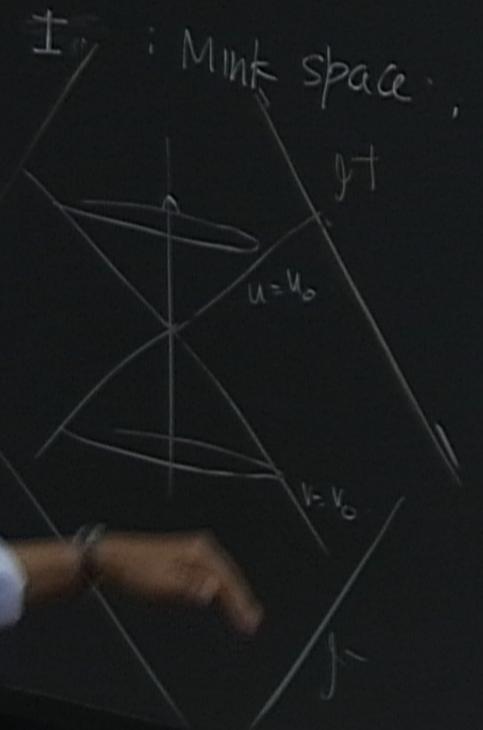
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$$\Omega = \frac{1}{r}, \quad J^+ : \Omega = 0$$

$$ds^2 = -du^2 + 2du d\left(\frac{1}{\Omega}\right) + \frac{1}{\Omega^2} d\omega^2$$

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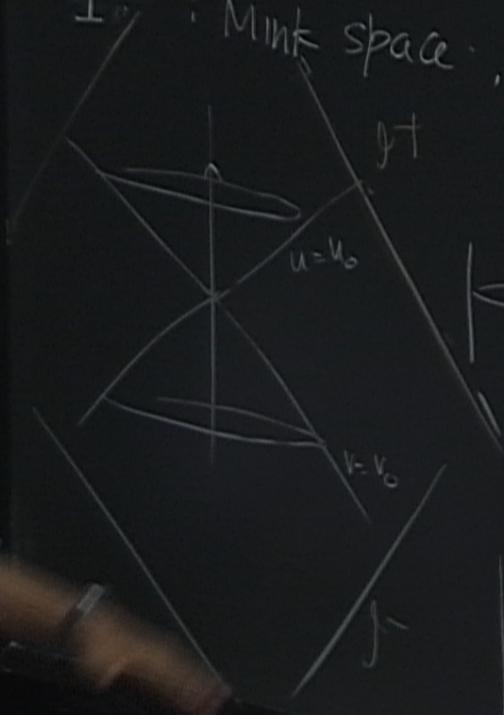
$$\Omega = \frac{1}{r}, \quad \mathcal{I}^+ : \Omega = 0$$

$$ds^2 = -du^2 + 2du d\Omega + \frac{1}{\Omega^2} d\omega^2$$

$$ds^2 = \Omega^2 ds^2_{\text{sp}} - 2du d\Omega + d\omega^2$$

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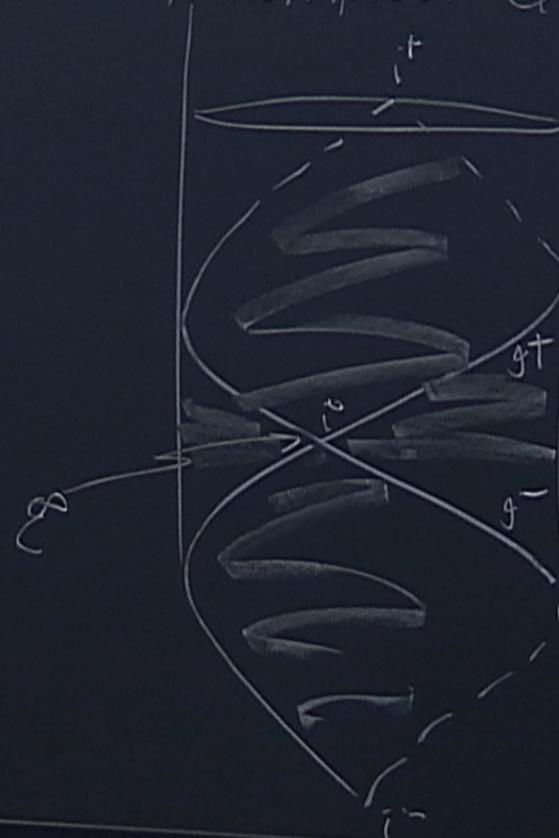
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1980s, Asymptotic Quantization



Variation laws and soft gravitons.

$$ds^2 = -dt^2 + dr^2 + r^2 d\omega^2$$

$$u = t - r, \quad v = t + r$$

$$ds^2 = -du^2 + 2du dr + r^2 d\omega^2 -$$

$$\Omega = \frac{1}{r}, \quad \mathcal{I}^+ : \Omega = 0$$

$$ds^2 = -du^2 - 2du dR + \frac{1}{\Omega^2} d\omega^2$$

$$\underline{ds^2} = \Omega^2 ds^2 = -\Omega^2 du^2 - 2du dR + d\omega^2$$

General space-times:

$$(M, g_{ab}); \text{ AF if } \exists (M, g_{ab}) \text{ } M = \bar{M} \cup \mathcal{I}^+$$

Variation laws and Soft Gravitons.

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where  $\mathcal{I}^+ \cong \mathbb{S}^2 \times \mathbb{R}$  and  $\exists \Omega$  s.t.

①  $\Omega \stackrel{\Delta}{=} 0 \iff \Omega|_{\mathcal{I}^+} = 0, \quad \nabla_a \Omega \neq 0.$

Evolution laws and Soft Gravitons.

$$ds^2 = -dt^2 + dr^2 + r^2 d\omega^2$$

$$u = t - r \quad v = t + r$$

$$ds^2 = -du^2 + 2du dr + r^2 d\omega^2 -$$

$$\Omega = \frac{1}{r}, \quad \mathcal{I}^+ : \Omega = 0$$

$$ds^2 = -du^2 - 2du \frac{dr}{\Omega^2}$$

$$\underline{ds^2} = \Omega^2 ds^2 = -\Omega^2 d\left(\frac{r}{\Omega}\right)^2$$

General space-times:

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③  $(\tilde{g})$

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Gravitons.

$$ds^2 = -dt^2 + dr^2 + r^2 d\omega^2$$

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$$\Omega = \frac{1}{r}, \quad \mathcal{I}^+ : \Omega = 0$$

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Ex. Kerr; stationary stars

$$V = t + i -$$

$$ds^2 = -du^2 + 2dudr + r^2 dw^2 -$$

$$\Omega = \frac{1}{r}, \quad \mathcal{I}^+ : \Omega = 0$$

$$d\tilde{s}^2 = -du^2 - 2dudr \frac{1}{\Omega^2} + \frac{1}{\Omega^2} dw^2$$

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$(M, g_{ab})$ ; AF if  $\exists (M, g_{ab}) M = \tilde{M} \cup \mathcal{I}^+$

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Ex. Kerr, stationary stars;  
Radiative solns: CK "∞-dim" near Mink space

$$V = t + 1$$

$$ds^2 = -du^2 + 2du dr + r^2 dw^2 -$$

$$\Omega = \frac{1}{r}, \quad \mathcal{I}^+ : \Omega = 0$$

$$d\tilde{s}^2 = -du^2 - 2du d\rho + \frac{1}{\Omega^2} + \frac{1}{\Omega^2} dw^2$$

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Ex. Kerr, stationary stars;

Radiative solns. CK

Christiel-Delort

" $\infty$ -dim" near Mink space

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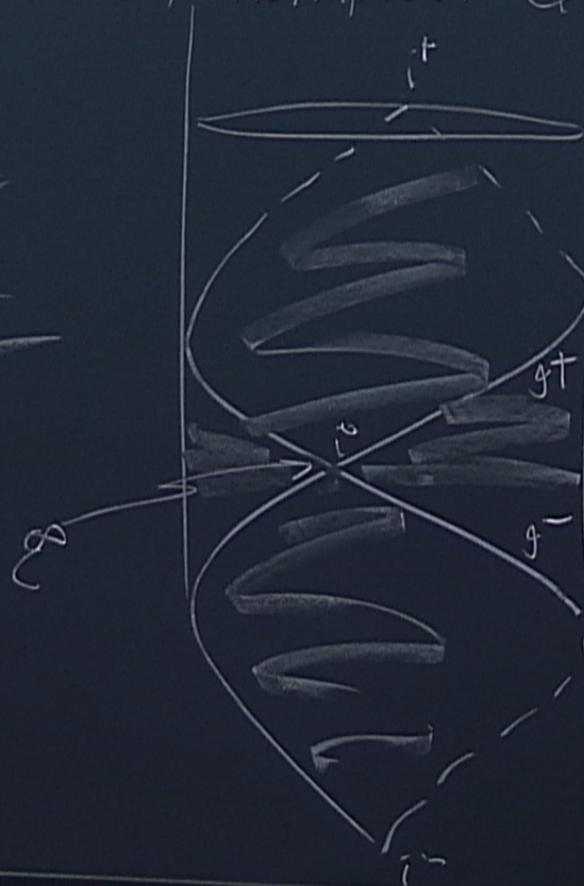
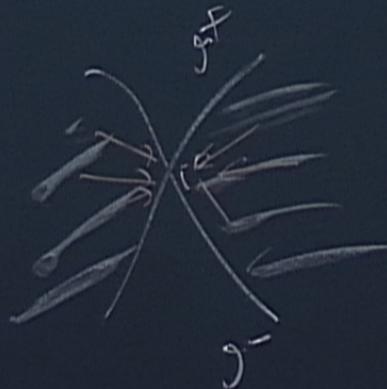
Ex. Kerr, stationary stars;  
Radiative solns. CK "oo-dim" near Mink space  
Christiel-Delort, Friedrich.

Radiative solns:  
Chrusciel-Delay

Two subtleties:

- ① If we also attach  $i^0$  &  $j^0$ , then the conformal metric is  $c^0$  but not  $d^0$  at  $i^0$

1980s, Asymptotic Quantization



Radiative S  
Chrusciel-

Two subtleties:

- ① If we also attach  $i^0$  &  $\bar{y}^-$ , then the conformal metric is  $C^0$  but not  $C^1$  at  $i^0$ .  
ADM mass is a measure of the discontinuity of  $\nabla$

Two subtleties:

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② 
$$\bar{g}_{ab} = \bar{\eta}_{ab} + O\left(\frac{1}{r}\right)$$

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②  $\bar{g}_{ab} = \left( \bar{\gamma}_{ab} + O\left(\frac{1}{r}\right) \right)$   
Not Unique

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②  $\bar{g}_{ab} = \underbrace{\hat{\gamma}_{ab}}_{\text{Not Unique}} + O\left(\frac{1}{r}\right)$

$\underbrace{\hat{x}, \hat{y}, \hat{z}, \hat{t}}_{\bar{\gamma}_{ab}} \quad \bar{t} = \hat{t} + a(\theta, \varphi)$

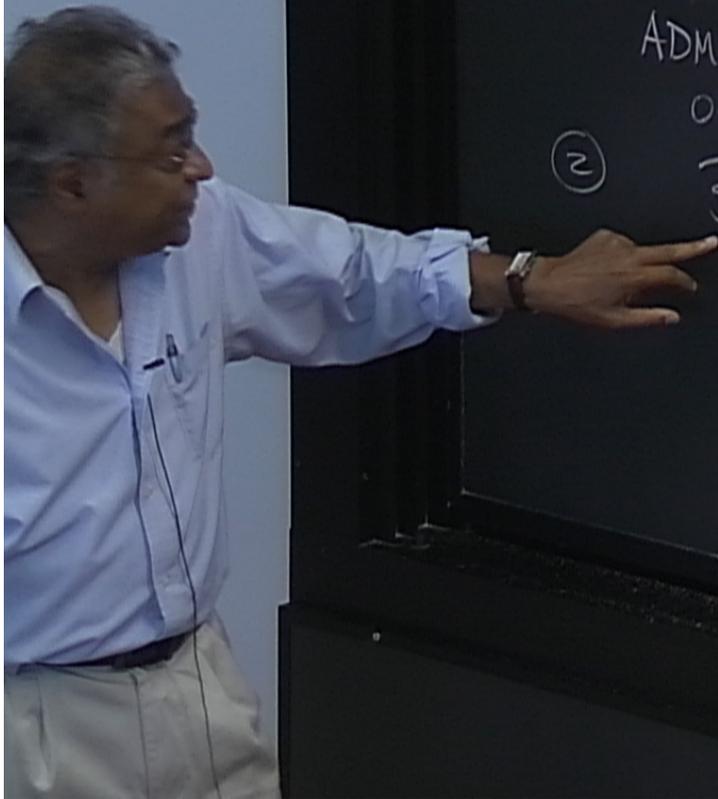
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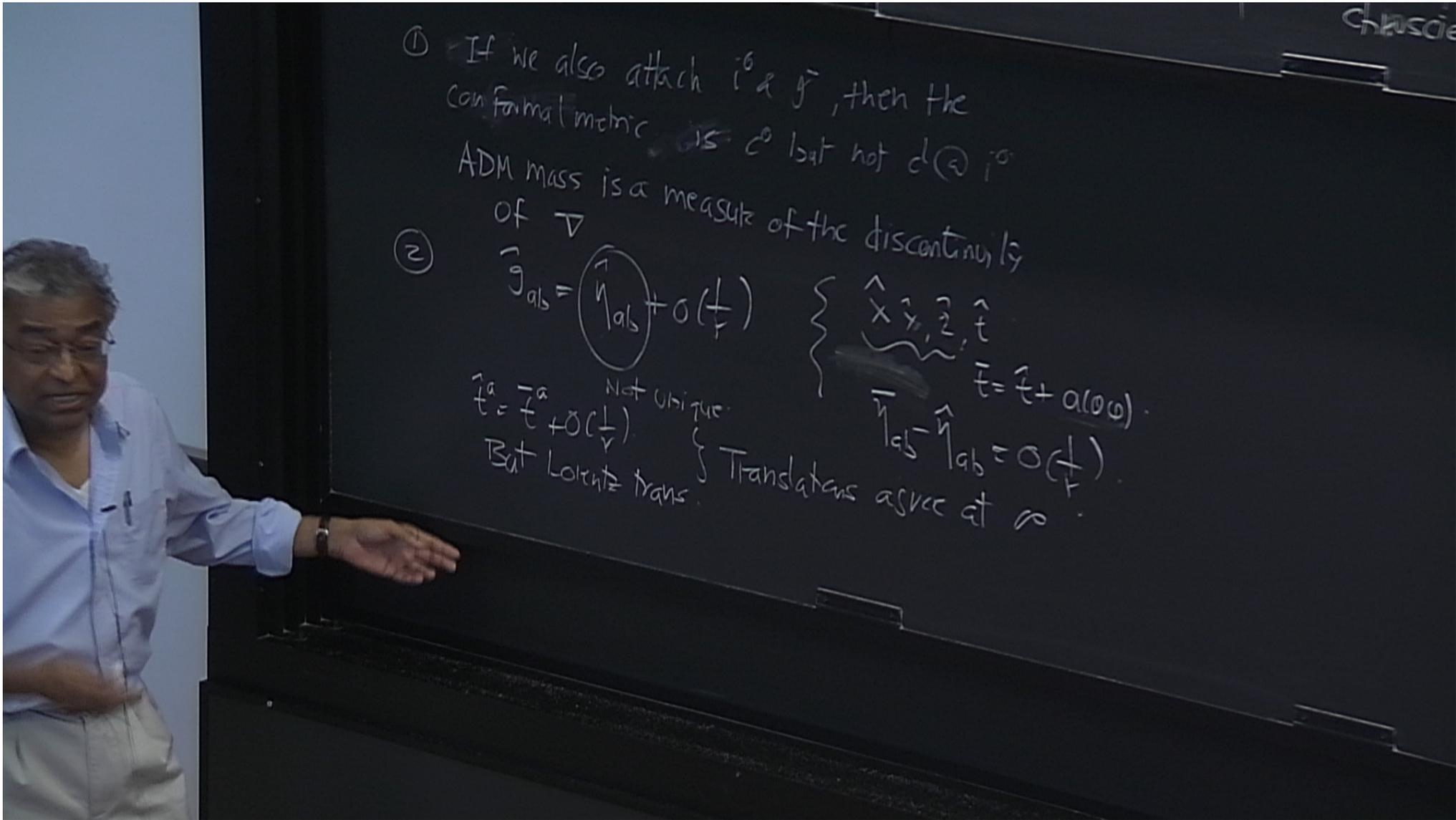
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$\underbrace{\hat{x}, \hat{y}, \hat{z}, \hat{t}}_{\text{Coordinates}}$   
 $\bar{t} = \hat{t} + a(\theta, \varphi)$   
 $\bar{\eta}_{ab} - \eta_{ab} = O\left(\frac{1}{r}\right)$





① If we also attach  $i^0$  &  $\bar{g}$ , then the conformal metric is  $c^0$  but not  $d^0$   $i^0$

ADM mass is a measure of the discontinuity of  $\nabla$

②  $\bar{g}_{ab} = \hat{\gamma}_{ab} + O(1/r)$  }  $\hat{x}^1, \hat{x}^2, \hat{t}$   
Not unique }  $\bar{t} = \hat{t} + a(\theta)$   
 $\bar{t}^a = \hat{t}^a + O(1/r)$  }  $\bar{\gamma}_{ab} - \hat{\gamma}_{ab} = O(1/r)$   
But Lorentz trans. } Translations asymp at  $\infty$

Two subtleties:

① If we also attach  $i^0$  &  $\bar{g}$ , then the conformal metric is  $c^0$  but not  $d @ i^0$

ADM mass is a measure of the discontinuity of  $\nabla$

②

$$\bar{g}_{ab} = \hat{\gamma}_{ab} + O\left(\frac{1}{r}\right)$$

Not unique

$$\hat{t}^a = \bar{t}^a + O\left(\frac{1}{r}\right)$$

But Lorentz trans

Translations agree at  $\infty$  are not the same!

$$\left\{ \begin{array}{l} \hat{x}, \hat{y}, \hat{z}, \hat{t} \\ \bar{t} = \hat{t} + a(\omega) \end{array} \right.$$

$$\bar{\gamma}_{ab} - \hat{\gamma}_{ab} = O\left(\frac{1}{r}\right)$$

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 M mass is a measure of the discontinuity  
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$\hat{x}, \hat{y}, \hat{z}, \hat{t}$   
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 $\hat{\gamma}_{ab} - \hat{\gamma}'_{ab} = O(\frac{1}{r})$

Not unique  
 Translations asymp at  $\infty$   
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$\mathcal{B}$  asymptotic Symm Gp.

$\cong \mathcal{N} \times \mathcal{Z}$   
 ↑  
 supertranslation group  
 co-dim

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M mass is a measure of the discontinuity of  $\nabla$

$$\hat{g}_{ab} = \hat{\eta}_{ab} + O\left(\frac{1}{r}\right) \quad \left\{ \begin{array}{l} \hat{x}, \hat{y}, \hat{z}, \hat{t} \\ \hat{t} = \hat{t} + a(\omega) \end{array} \right.$$

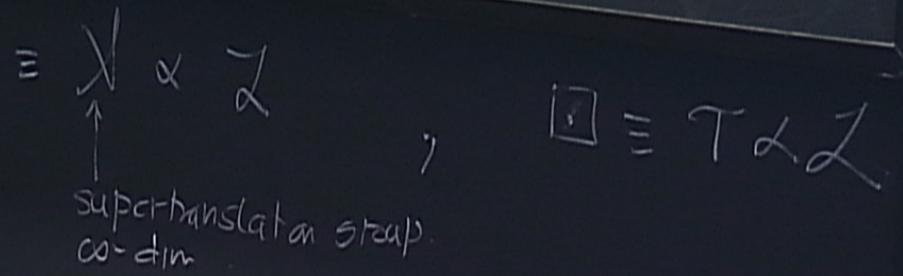
Not unique  
 $\bar{g}_{ab} = \hat{g}_{ab} + O\left(\frac{1}{r}\right)$   
 But Lorentz trans. etc not the same!  
 Translations agree at  $\infty$

$\mathcal{B}$ : asymptotic Symm Gp.

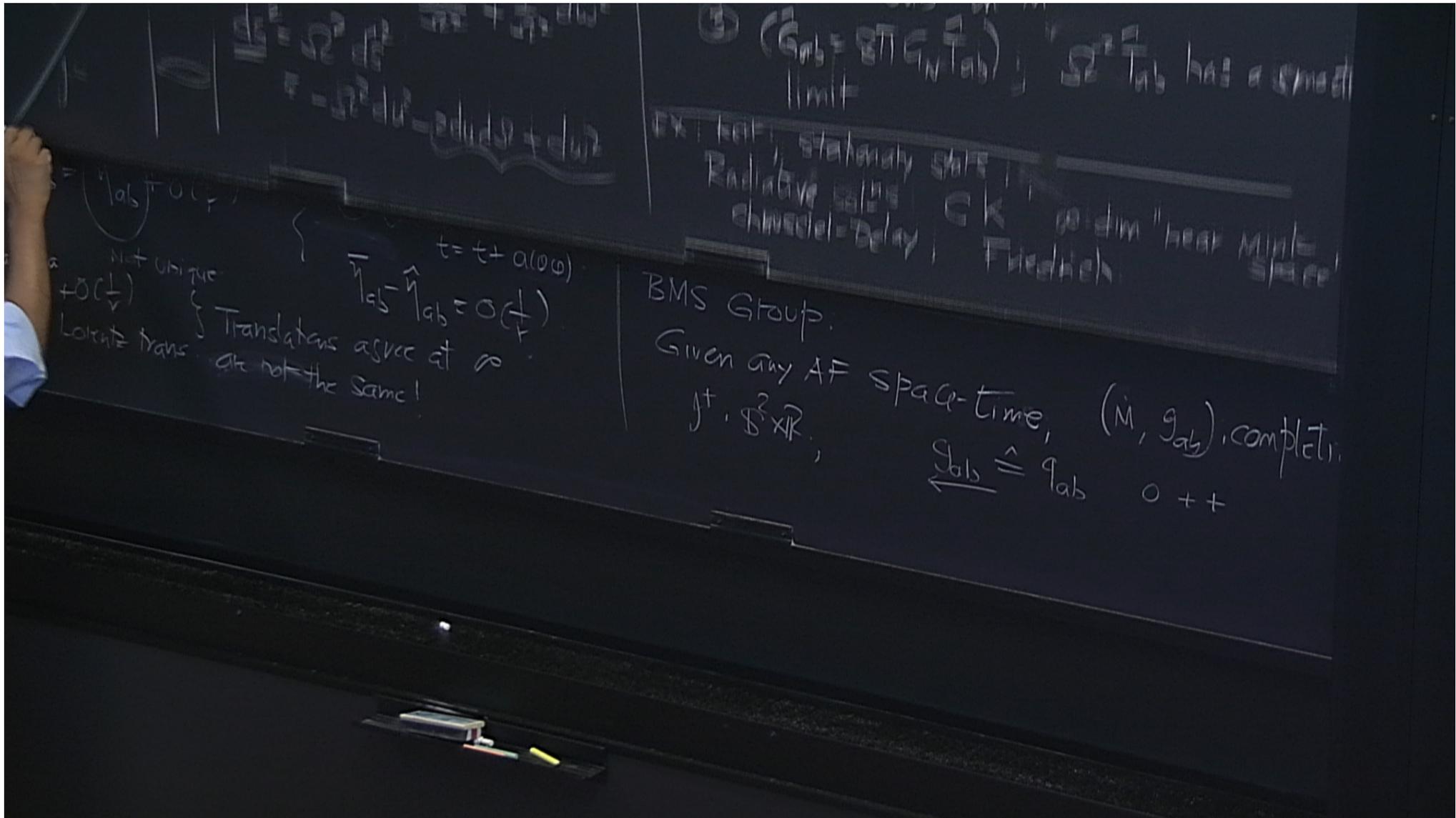
$$\equiv \mathcal{N} \ltimes \mathcal{L} \quad ; \quad \boxed{\mathcal{B}} \equiv \mathcal{T} \ltimes \mathcal{L}$$

↑  
supertranslation group  
co-dim

normal metric is  $d^2$  but not  $d(\omega)^2$   
 A mass is a measure of the discontinuity  
 of  $\nabla$   
 $\hat{g}_{ab} = \hat{\eta}_{ab} + O(\frac{1}{r})$   
 Not unique  
 $\hat{t} = \hat{t} + a(\omega)$   
 $\hat{\eta}_{ab} - \hat{\eta}'_{ab} = O(\frac{1}{r})$   
 But Lorentz trans. are not



BMS Group:  
 Given any AF space-time,  $(M, g_{ab})$ , complete.



BMS Group:

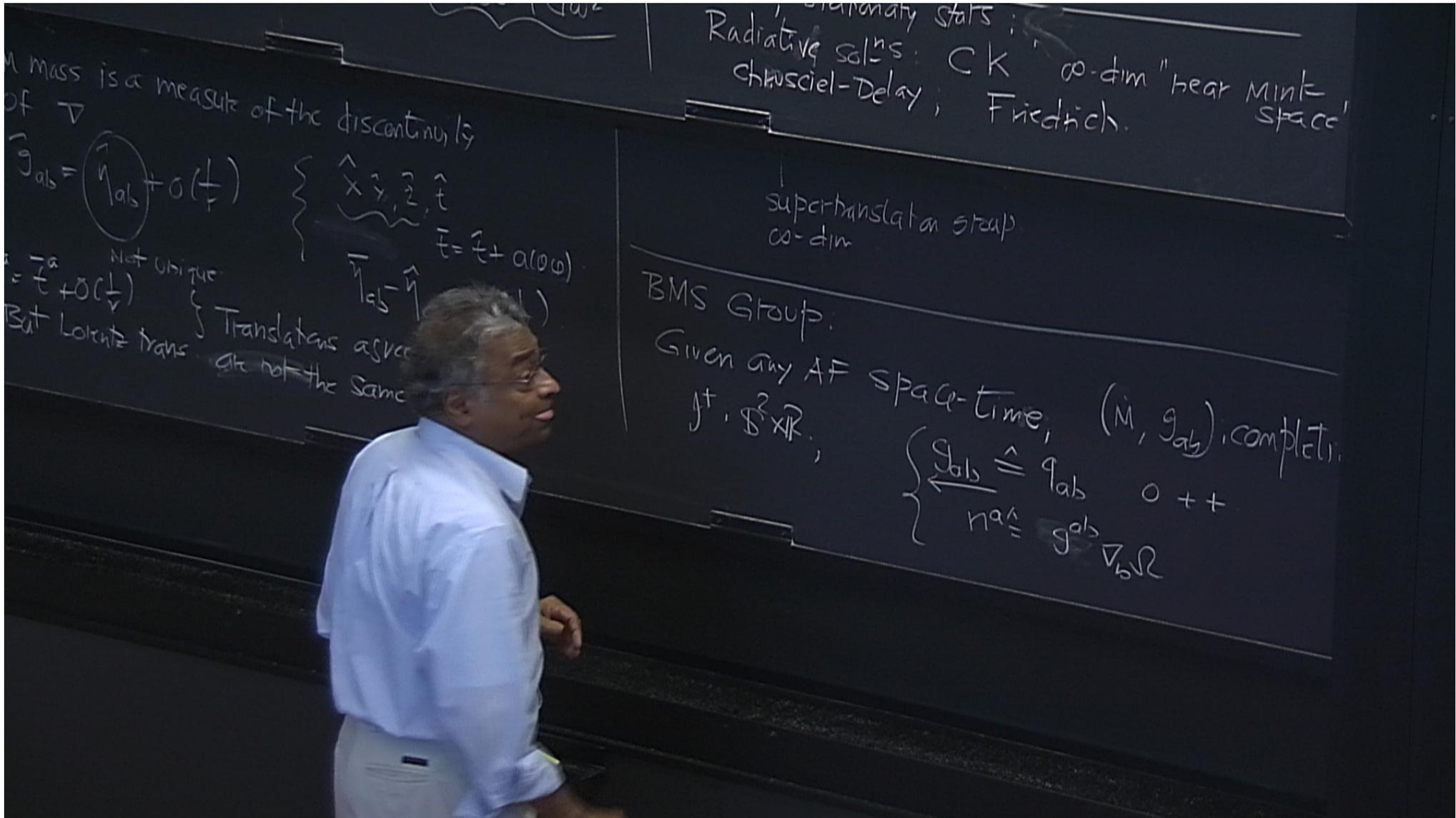
Given any AF space-time,  $(M, g_{ab})$ , complete,  
 $J^+, S^2 \times \mathbb{R}$ ,  
 $\overleftarrow{g}_{ab} \hat{=} g_{ab} \quad 0++$

$\hat{=} g_{ab} + O(\frac{1}{r})$

Not unique  
 $+O(\frac{1}{r})$   
Lorentz trans.

Translations asymp at  $\infty$   
are not the same!

$t = t + a(\theta, \phi)$   
 $\hat{=} g_{ab} - \hat{=} g_{ab} = O(\frac{1}{r})$



stationary states  
 Radiative solns: CK "near Mink space"  
 Chruściel-Delays, Friedrich.

Mass is a measure of the discontinuity  
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$$\hat{g}_{ab} = \hat{\eta}_{ab} + O\left(\frac{1}{r}\right)$$

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Not unique  
 $\hat{t} = \hat{t} + O\left(\frac{1}{r}\right)$   
 But Lorentz trans. are not the same  
 Translations as well

supertranslation group  
 co-dim

BMS Group:

Given any AF space-time,  $(M, g_{ab})$ , complete,  
 $M^+ \cong \mathbb{S}^2 \times \mathbb{R}$ ,

$$\left\{ \begin{array}{l} g_{ab} \hat{=} \eta_{ab} \quad 0^{++} \\ \text{near } \mathbb{S}^2 \times \mathbb{R} \end{array} \right.$$

$\omega$ -dim "near Mink  
space".

me,  $(M, g_{ab})$  complete  
 $\hat{=} g_{ab} \quad 0 \quad ++$   
 $\hat{=} g^{ab} \nabla_b \mathcal{R}$

$$(g_{ab}, n^a) \approx (\omega^2 g_{ab}, \omega^{-1} n^a)$$

$$\Omega \rightarrow \omega \Omega$$

$$\{g_{ab}, n^a\}$$

$\infty$ -dim "near Mink-  
space"

$(M, g_{ab})$  complete  
 $\hat{=} g_{ab} \quad 0++$   
 $\hat{=} g^{ab} \nabla_b \mathbb{R}$

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$$\Omega \rightarrow \omega \Omega; \quad \omega(a, 0, \varphi) \text{ in } \mathbb{Z} \times \mathbb{Z} @ \mathcal{I}$$

$\mathbb{R}^2 \times \mathbb{R}$ ,  $\{g_{ab}, h^a\}$   
universal.

$g_{ab} \quad 0++$   
 $h^a: \text{hama!}$

$\infty$ -dim "near Mink space"  
 edlich.

$(M, g_{ab})$  complete  
 $\hat{=} g_{ab} \quad 0++$   
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$\mathbb{S}^2 \times \mathbb{R}, \{g_{ab}, h^a\}$   
 universal,  $g_{ab} \quad 0++$   
 $h^a: \text{hama!}$

$\mathcal{J}: \mathbb{S}^2 \times \mathbb{R} \downarrow$  universal structure

$\mathcal{B} = \text{Diffeos that preserve this structure}$

II. Radiative Modes of full GR at  $\mathcal{I}^+$

$(g_{ab})$ , complete.

$0++$

GR

II. Radiative Modes of full GR at  $\mathcal{I}$

Given  $g_{ab}$   $\left\{ \begin{array}{l} \nabla \cdot D \text{ on } \mathcal{I}, \\ D_a g_{bc} = 0, \quad D_a h^a_b = 0 \end{array} \right.$

conf. invariant inf  $\xi D \xi$  poly. many.

$g_{ab}$  complete  
 $0++$   
 $\mathcal{I}^R$

II. Radiative Modes of full GR at  $\mathcal{I}$

Given  $\left. \begin{array}{l} \mathcal{I} \\ \mathcal{I}^{ab} \end{array} \right\} \nabla \leq D \text{ on } \mathcal{I}, \underbrace{D_a g_{bc} = 0, D_a h^b = 0}_{\text{only many}}$

conf. invariant inf  $\{D\}$

How many  $\{D\}$ ?

~~$\{D\} - \{D\} = \sigma_{ab} \quad \text{TT}$~~  (2 modes)

$G_{ab} = \sigma_{ab}, \sigma_{ab} h^b = 0, \sigma_{ab} g^{ab} = 0$

II. Radiative Modes of full GR at  $\mathcal{I}^-$

Given  $g_{ab}$   $\nabla \leq D$  on  $\mathcal{I}^-$ ,  $D_a g_{bc} = 0$ ,  $D_a h^b = 0$

(poly many)

conf. invariant inf  $\xi D \xi$

How many  $\xi D \xi$ ?

$$\xi D \xi - \xi D \xi = \sigma_{ab} \quad \text{(2 modes)}$$

YM Theorem: Radiative Modes,  $\mathbb{D} \Big|_g$  or  $\mathbb{A}_a$  on  $\mathcal{I}^-$

$G_{ab} = \sigma_{ab}$ ,  $\sigma_{ab} h^b = 0$ ,  $\sigma_{ab} g^{ab} = 0$

$$g^{ab} = g$$

GR at  $\mathcal{I}^-$   
 $D_a q_{bc} = 0, D_a h^b = 0$   
 only many.  
 (2 modes)  
 $\{D\} - \{D\} = \sigma_{ab} \cdot \pi$   
 $\sigma_{ab} = \sigma_{(ab)}, \sigma_{ab} h^b = 0, \sigma_{ab} q^{ab} = 0$   
 fcs:  $\mathbb{D}|_g$  or  $A_a$  on  $\mathcal{I}^-$

Phase space of radiative modes:  
 $\Gamma_R = \{ \{D\} \text{ on } \mathcal{I}^+ \}$   
 Symplectic structure.  
 BMS group preserves.

GR at  $\mathcal{I}^-$

$$D_a g_{bc} = 0, \quad D_a h^b = 0$$

only many.

$$\{D\} - \{D\} = \sigma_{ab} \quad \text{(2 modes)}$$

$$G_{ab} = \sigma_{(ab)}, \quad \sigma_{ab} h^b = 0, \quad \sigma_{ab} g^{ab} = 0$$

fcs,  $\mathbb{D}|_g$  or  $A_a$  on  $\mathcal{I}^-$

Phase space of radiative modes:

$$\mathcal{P}_R = \{ \{D\} \text{ on } \mathcal{I}^+ \}$$

Symplectic structure  $\Omega$

BMS group preserves  $\Omega$   
Hence compute Hamiltonians

GR at  $\mathcal{I}^-$

$$D_a q_{bc} = 0, \quad D_a h^b = 0$$

only many.

$$\{D\} - \{D\} = \sigma_{ab} \quad \text{(2 modes)}$$

$$G_{ab} = \sigma_{(ab)}, \quad \sigma_{ab} h^b = 0, \quad \sigma_{ab} q^{ab} = 0$$

fcs,  $\mathbb{D}|_g$  or  $A_a$  on  $\mathcal{I}^-$

Phase space of radiative modes:

$$\mathcal{P}_R = \{ \{D\} \text{ on } \mathcal{I}^+ \}$$

Symplectic structure  $\Omega$

BMS group preserves  $\Omega$   
Hence compute Hamiltonians

Look at supertranslations:

$$\xi^a = \alpha(\theta, \varphi) n^a \quad \left\{ \begin{array}{l} \Omega \rightarrow \Omega' = \omega \Omega \\ = (\alpha(\theta, \varphi) \omega) (\omega^{-1} n^a) \end{array} \right.$$

conf. weighted function

cont. invariant inf  $\xi D \xi$  only many.

How many  $\xi D \xi$ ?

~~$\xi D \xi - \xi D \xi \equiv \sigma_{ab}$~~   $\sigma_{ab}$  TT (2 modes)

YM Theory: Radiative Modes,  $G_{ab} = \sigma_{ab}$ ,  $\sigma_{ab} h^b = 0$ ,  $\sigma_{ab} \eta^{ab} = 0$   
 $\mathbb{D} \downarrow g$  or  $A_a$  on  $\mathcal{I}^-$

curv of  $\mathbb{D}$ : cont. invariant is coded in a symmetric tracefree, transverse tensor,  $N_{ab}$  Bondi News tensor.

IR  
Sym  
BMS  
Hence  
Look  
S

$(G_{ab} = 8\pi G_N \rho_{ab})$ ;  $\Sigma$  has  $3$  dim  
 limit  
 stat, stationary state  
 Radiative solns CK  $\infty$  dim "near" Mink space  
 Chruściel-Delays, Friedrich

special conf factors

II Radiative Modes of full GR at  $\mathcal{I}^-$   
 Given  $\left\{ \begin{array}{l} \Sigma \cong \mathcal{D} \text{ on } \mathcal{I}^- \\ \rho_{ab} \end{array} \right\}$ ,  $\rho_{ab} \rho_{bc} = 0$ ,  $\rho_{ab} h^b = 0$   
 only many  
 Conf invariant inf  $\{SD\}$  (2 mod 6)  
 How many  $SD\}$ ?  $\int \rho_{ab} \rho^{ab} = \pi$   
 $G_{ab} = G_{ab}$ ,  $G_{ab} h^b = 0$ ,  $G_{ab} \rho^{ab} = 0$   
 YM Theory: Radiative Modes,  $\mathbb{D}|_g$  or  $A|_g$  on  $\mathcal{I}^-$

COW of  $\mathcal{D}$ . conf invariant is coded in a symmetric traceful, transverse tensor,  $N_{ab}$  Bondi News tensor

Phase space of radiative Modes  
 $\Gamma_R = \{SD\}$  on  $\mathcal{I}^-$   
 Symplectic structure  $\Omega$   
 BMS group preserves  $\Omega$   
 Hence compute Hamiltonians  
 Look at supertranslations  
 $\xi^a = \alpha(0, \varphi) n^a$  ( $\Omega \rightarrow \Omega' = \omega \Omega$ )  
 $= (\alpha(\theta, \varphi) \omega) (\omega' n^a)$   
 Conf weighted function

$(G_{ab} = 8\pi G_N \rho_{ab})$ ;  $\mathcal{I}^+$   $\rho_{ab}$  limit  
 flat, stationary stars  
 Radiative solutions CK  $\infty$  dim "near" Mink space  
 Christodoulou-Delays, Friedrich

special conf factors:  
 $g_{ab}$  unit 2-sphere metric

II Radiative Modes of full GR at  $\mathcal{I}^+$   
 Given  $\left\{ \begin{array}{l} \nabla \cdot D \text{ on } \mathcal{I}^+, \\ \rho_{ab} \text{ on } \mathcal{I}^+ \end{array} \right.$ ,  $D_a \rho_{bc} = 0$ ,  $D_a h^b = 0$   
 only many.  
 Conf invariant inf  $\{SD\}$  (2 mod 6)  
 How many SDs?  $\#SDs - \#D\bar{S} = \sigma_{ab} \cdot \pi$   
 $G_{ab} = G_{ab}$ ,  $G_{ab} h^b = 0$ ,  $G_{ab} \rho^{ab} = 0$   
 YM Theory: Radiative Modes,  $\mathbb{D}|_g$  or  $A_a$  on  $\mathcal{I}^+$

Phase space of radiative Modes:  
 $\Gamma_R = \{SDs \text{ on } \mathcal{I}^+\}$   
 Symplectic structure  $\Omega$   
 BMS group preserves  $\Omega$   
 Hence compute Hamiltonians  
 Look at supertranslations.  
 $\xi^a = \alpha(\theta, \varphi) n^a$  ( $\Omega \rightarrow \Omega' = \omega \Omega$ )  
 $= (\alpha(\theta, \varphi) \omega) \left( \omega^{-1} n^a \right)$   
 Conf weighted function

COW of  $D$ . Conf invariant is coded in a symmetric tracefree, transverse tensor,  $N_{ab}$  Bondi News tensor.

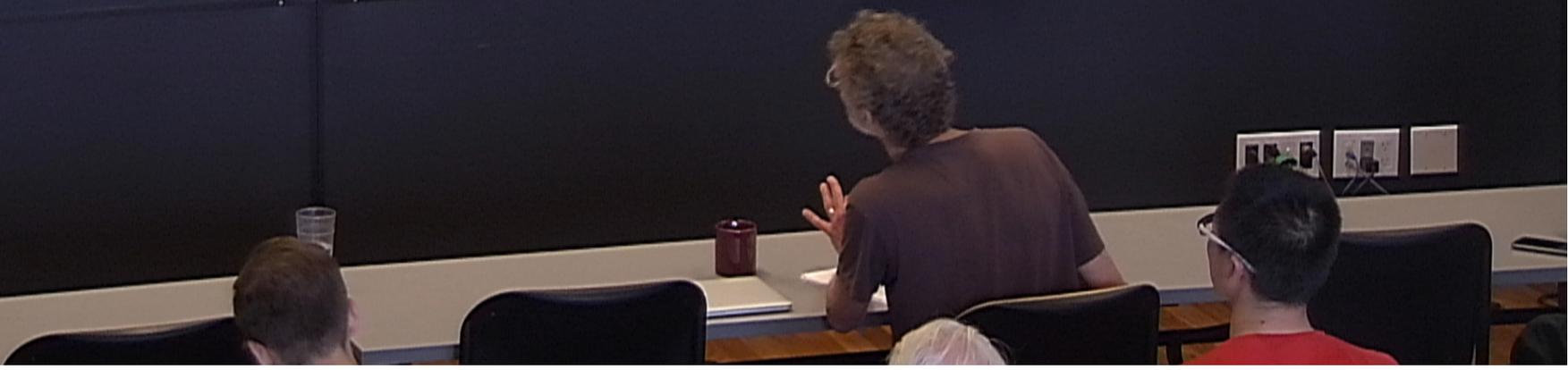
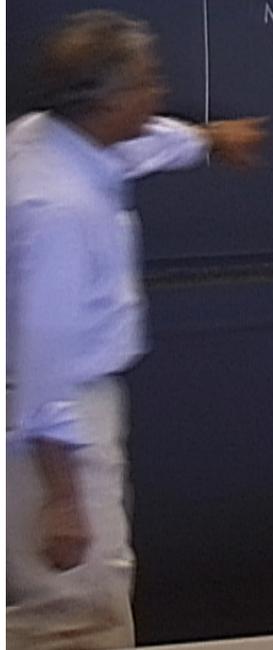
$(G_{ab} = 8\pi G_N T_{ab})$ ;  $\Sigma$  is  $3+1$  limit  
 flat, stationary stars  
 Radiative solutions: CK  $\infty$  dim "near Mink space"  
 Chruściel-Delays, Friedrich

special conf factors  
 $g_{ab}$  unit 2-sphere metric  
 - Bondi frame  
 $N_{ab} = \bar{\alpha}_n G_{ab} = \bar{\sigma}_{ab}$

II Radiative Modes of full GR at  $\mathcal{I}^+$   
 Given  $\{g_{ab}\}$   $\nabla \cdot D$  on  $\mathcal{I}^+$ ,  $D_a g_{bc} = 0$ ,  $D_a h^b = 0$   
 only many  
 Conf invariant inf  $\{SD\}$  (2 mod 6)  
 How many SDs?  $SDs - \{SDs\} = \bar{\sigma}_{ab} \cdot \pi$   
 $G_{ab} = G_{ab}$ ,  $\bar{\sigma}_{ab} h^b = 0$ ,  $\bar{\sigma}_{ab} g^{ab} = 0$   
 YM Theory: Radiative Modes,  $\mathbb{D}|_g$  or  $A_n$  on  $\mathcal{I}^+$

Phase space of radiative Modes  
 $\Gamma_R = \{SDs \text{ on } \mathcal{I}^+\}$   
 Symplectic structure  $\Omega$   
 BMS group preserves  $\Omega$   
 Hence compute Hamiltonians  
 Look at supertranslations  
 $\xi^a = \alpha(0, \varphi) n^a$  ( $\Omega \rightarrow \Omega' = \omega \Omega$ )  
 $= (\alpha(0, \varphi) \omega) (\omega^{-1} n^a)$   
 Conf. weighted function

CON of  $D$ : conf invariant is coded in a symmetric traceful, transverse tensor,  $N_{ab}$  Bondi News tensor



$$\int_V (\tilde{D}_\xi D_a - D_a \tilde{D}_\xi) \lambda_b N^{ab}$$

Friedrich.

space

special cont. factors:

$q_{ab}$ : unit 2-sphere met

- Bondi frame

$$N_{ab} = \tilde{\alpha}_n \sigma_{ab} = \overset{\circ}{\sigma}_{ab}$$

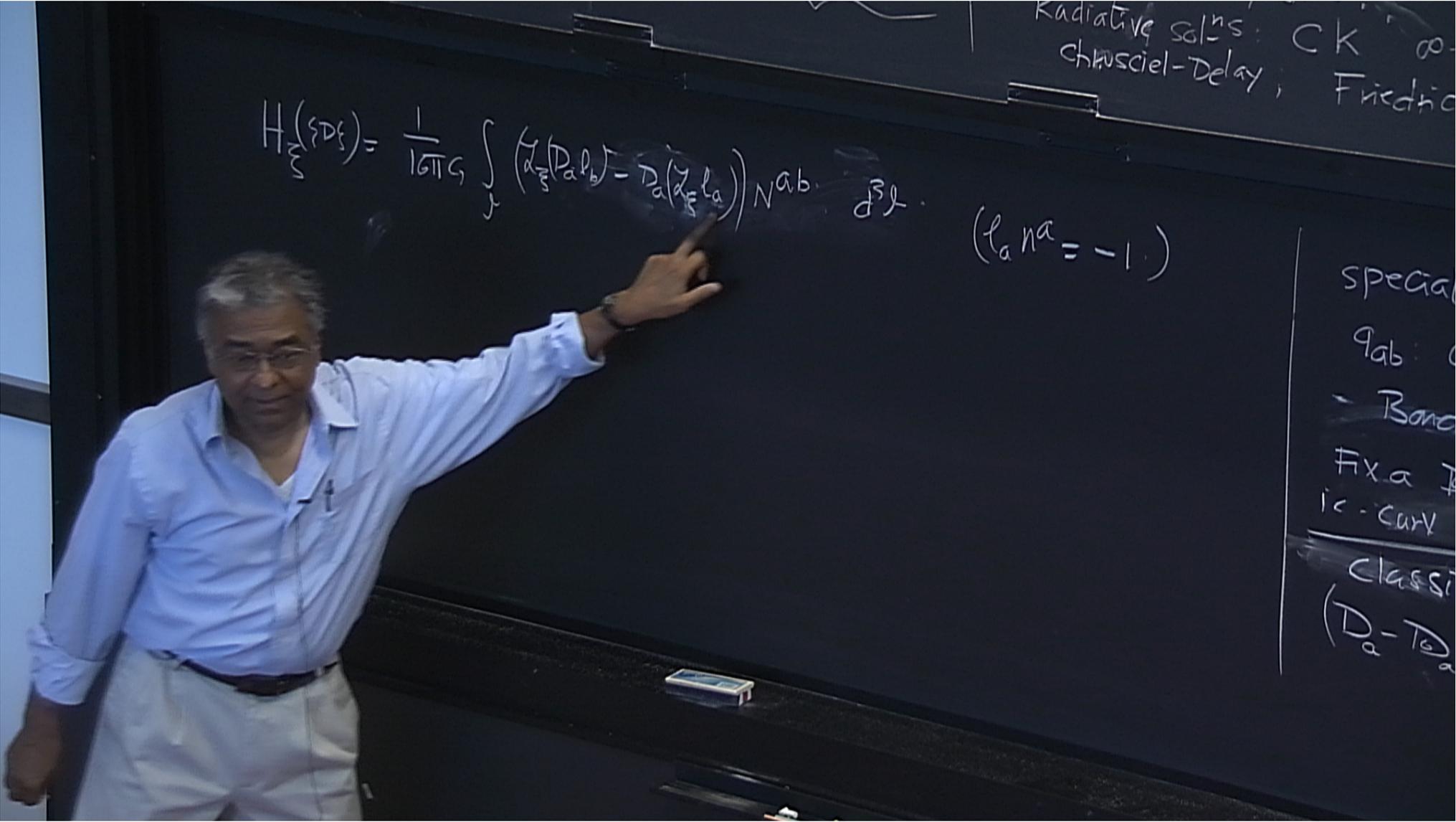
$$\left( D_a - D_a \gamma \right) l_b N^{ab} \quad d^3x$$

special cont. factors:  
 $g_{ab}$ : unit 2-sphere met  
- Bondi frame  
Fix a  $\mathbb{P}^1$  st  $N^{ab} = 0$   
ic. curv is zero  

---

Classical vacua

GIV  
Gab



$$H_{\xi}(\xi D\xi) = \frac{1}{16\pi G} \int_V (\xi^a D_a b^b - D_a(\xi^a b^a)) N^{ab} d^3x$$

$$(l_a n^a = -1)$$

Radiative solns: CK,  $\infty$   
Christoffel-Delay, Friedrich

special  
 $g_{ab}$   
- Bondi  
Fix a  
ic-curve  
CLASSI  
( $D_a - T_a$ )

Radiative solns: CK  
 Chrusciel-Delay, Friedrich

$$H_{\xi}(\xi D\xi) = \frac{1}{16\pi G} \int W^{ab} \left( \gamma_{\xi}^{(Dab)} - D_a \gamma_{\xi}^{(l_b)} \right) \epsilon_{abc} ds^{abc} \quad (l_a n^a = -1)$$

du sino do dy

special  
 $q_{ab}$   
 - Bondi  
 Fix a  
 ic-curve  


---

 CLASSI  
 ( $D_a - T_a$ )

Radiative solns: CK  
 Chrusciel-Delay, Friedrich

$$\begin{aligned}
 H_{\xi}(\xi D\xi) &= \frac{1}{16\pi G} \int N^{ab} \left( \alpha \frac{D_a D_b \xi}{\xi} + \dots \right) \epsilon_{abc} d^3s_{abc} \\
 \xi &= \alpha h^0
 \end{aligned}$$

$\int \epsilon_{abc} d^3s_{abc} = \int du \sin\theta d\theta d\phi$

$$\equiv \frac{1}{32\pi G} \int \left( \alpha N_{ab} + \underbrace{D_a D_b \xi + \alpha q_{ab}}_{=0 \text{ iff } \alpha \text{ is a trans.}} \right) N_{ab} d^3s$$

$(\epsilon_a n^a = -1)$

special  
 $q_{ab}$   
 - Bondi  
 Fix a  
 ic-curve  
 CLASSI  
 $(D_a - T_a)$

Radiative solns: CK, Chandrasekhar-Delany, Friedmann

$$H_{\xi}(\xi D\xi) = \frac{1}{16\pi G} \int N^{ab} \left( \alpha \frac{D_a - D_b}{\xi} \right) \epsilon_{abc} d^3s \quad (l_a n^a = -1)$$

$$\xi \equiv \alpha n^0$$

$$\equiv \frac{1}{32\pi G} \int \left( \alpha N_{ab} + \underbrace{D_a D_b}_{\text{du. sym. to } d\mu} + \alpha q_{ab} \right) N_{ab} d^3s$$

iff  $\alpha$  is a trans.  
 $\alpha$ : linear comb of first 4 Yem

special  
 $q_{ab}$   
 - Bondi  
 Fix a  
 ic-curve  
 CLASSI  
 $(D_a - T_a)$

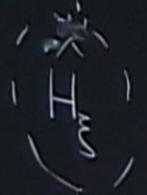


Radiative solns: CK  
 Chrusciel-Delay, Friedrich

$$H_{\xi}(\text{SDF}) = \frac{1}{16\pi G} \int N^{ab} \left( \alpha_{\xi} (D_a - D_b) \gamma_{\xi} \right) \epsilon_{abc} d^3s_{abc} \quad (l_a n^a = -1)$$

$$\equiv \frac{1}{32\pi G} \int \left( \alpha \hat{N}_{ab} + \underbrace{D_a D_b \alpha}_{\text{du sino do do}} + \alpha q_{ab} \right) N^{ab} d^3s$$

$\xi \equiv \alpha n^0$



$\hat{0}$  iff  $\alpha$  is a trans.  
 $\alpha$ : linear comb of first 4 Yem

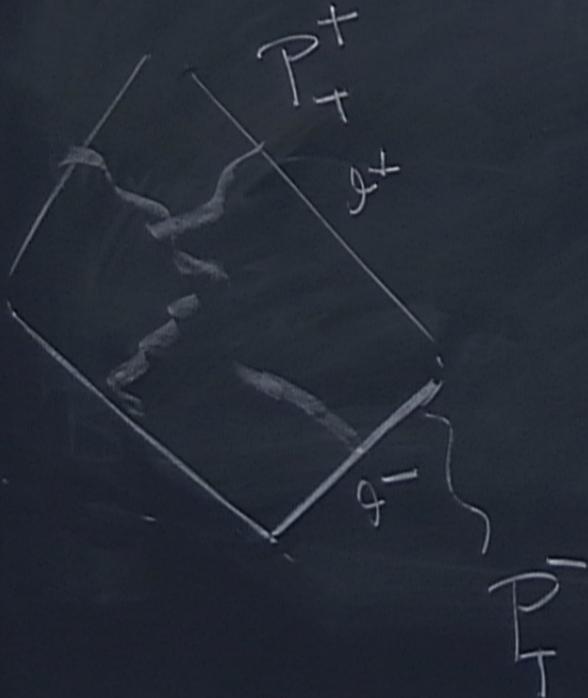
special  
 $q_{ab}$   
 - Bondi  
 Fix a J  
 ic-curl  
 CLASSI  
 $(D_a - T_a)$

Consider only the non-linear nbd of Mink space  
discussed by Christodoulou - Klainerman.

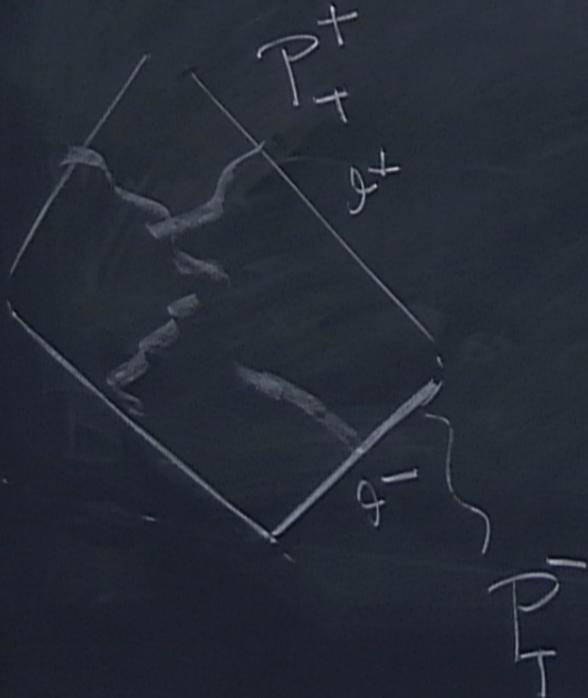
consider only the non-linear nbd of Mink space  
discussed by christodoulou - flannerman.



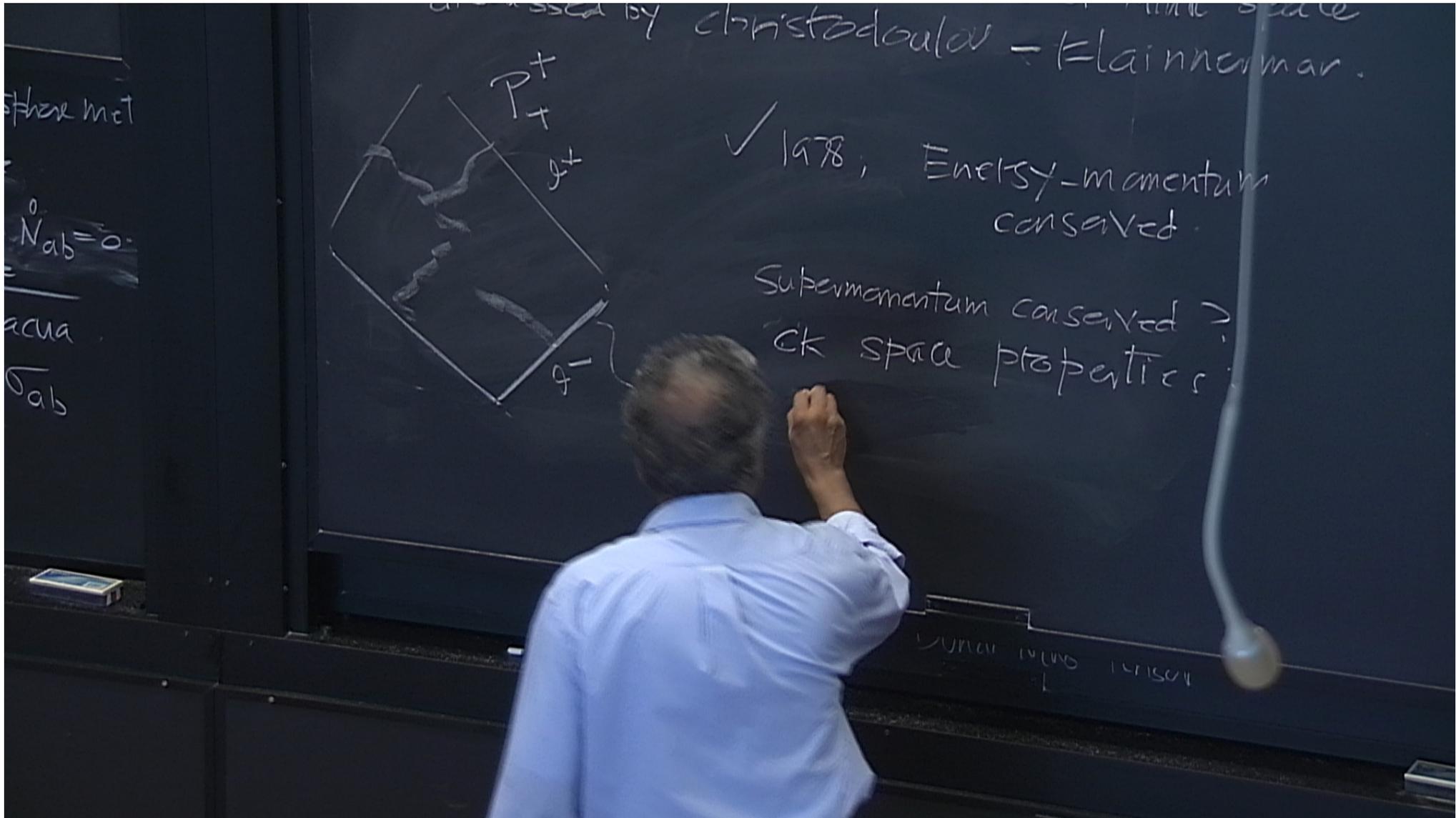
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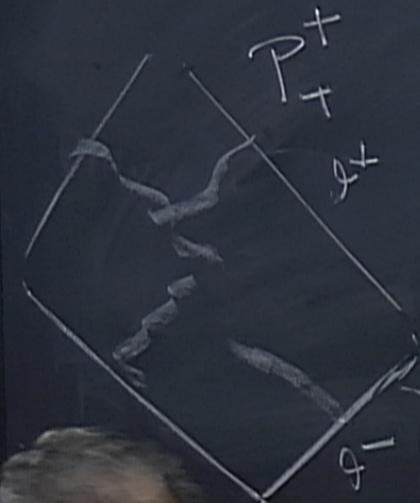
consider only the non-linear nbd of Mink space  
discussed by christodoulou - Klainnerman.



1978, Energy-momentum  
conserved.



discussed by christodoulou - klainnorman.



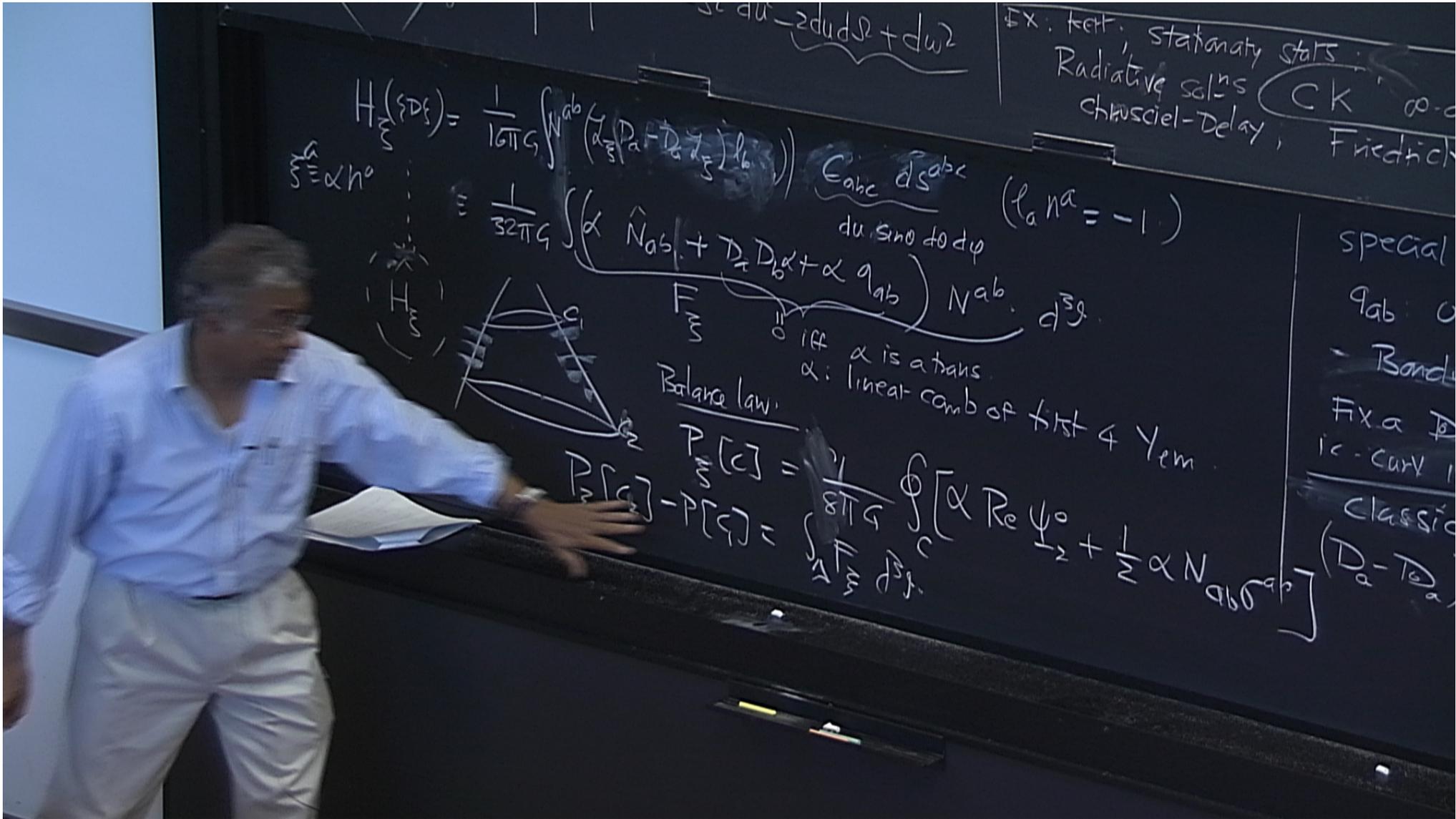
✓ 1978, Energy-momentum conserved.

Supermomentum conserved? CK space properties:

①  $N_{ab} \rightarrow 0$  as we approach  $i^\pm, i^0$ .

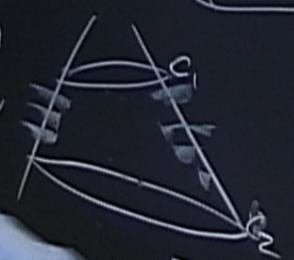
$P_T^-$

$N_{ab}$   $N_{ab}$   $N_{ab}$



$$H_S(SDE) = \frac{1}{16\pi G} \int N_{ab} \left( \frac{1}{\alpha} (D_a D_b - D_b D_a) \right) \epsilon_{abc} d^3s_{abc} \quad (l_a n^a = -1)$$

$$\equiv \frac{1}{32\pi G} \int \left( \alpha N_{ab} + D_a D_b + \alpha q_{ab} \right) N_{ab} d^3s$$



Balance law:  
 $F_S = 0$  iff  $\alpha$  is a trans  
 $\alpha$ : linear comb of first 4 Yem

$$P_S[c] = \frac{1}{8\pi G} \int_C \left[ \alpha \text{Re} \psi_0 + \frac{1}{2} \alpha N_{ab} q_{ab} \right]$$

$$P_S[c] - P_S[c'] = \int_{\Delta} F_S d^3s$$

Ex: Kerr, stationary stars  
 Radiative solns CK  
 Chrusciel-Delay, Friedrich

special  
 $q_{ab}$   
 - Bondi  
 Fix a D  
 ic-cov  
 classical  
 $(D_a - T_a)$

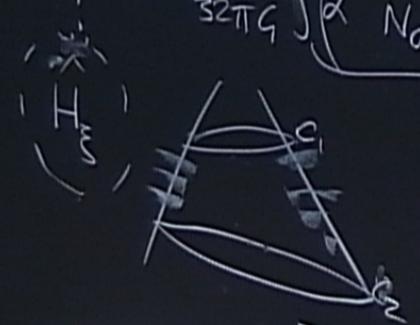
$$\int du - 2du d\Omega + dw^2$$

Ex: Kerr, stationary stars  
 Radiative solns CK  $\infty$ -d  
 Chruściel-Delays, Friedrich

$$H_{\Sigma}(\text{SDE}) = \frac{1}{16\pi G} \int N_{ab} \left( \frac{1}{3} (D_a D_b - D_b D_a) \right) \epsilon_{abc} d^3s_{abc}$$

$$\stackrel{\xi^a \equiv \alpha n^a}{=} \frac{1}{32\pi G} \int \left( \alpha N_{ab} + D_a D_b \alpha + \alpha q_{ab} \right) N_{ab} d^3s$$

$(\epsilon_a n^a = -1)$



Balance law:  
 $F_{\Sigma} = 0$  iff  $\alpha$  is a trans  
 $\alpha$ : linear comb of first 4 Yem

$$P_{\Sigma}[c_2] - P_{\Sigma}[c_1] = \int_{\Sigma} \frac{1}{8\pi G} \left[ \alpha \text{Re} \psi_0 + \frac{1}{2} \alpha N_{ab} q_{ab} \right] d^3s$$

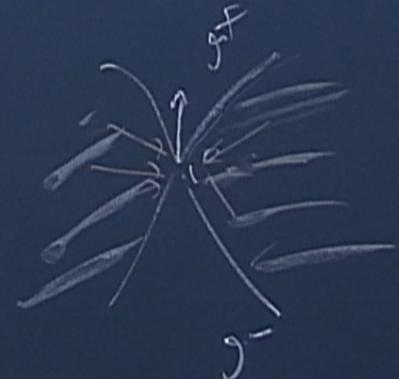
special  
 $q_{ab}$   
 - Bondi  
 Fix a  $\mathcal{I}^+$   
 ic-curve  
 classic  
 $(D_a - T_a)$

only the non-linear nbd of Mink space  
 by Christodoulou - Klainerman.

1980s, As

✓ 1978, Energy-momentum  
 conserved.

Supermomentum conserved?  
 ck space properties:



$\frac{P}{T}$

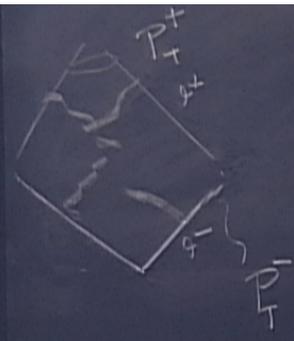
- ①  $N_{ab} \rightarrow 0$  as we approach  $i^\pm, i^0$  ( $\frac{1}{u^2}$  on  $g^\pm$ )
- ②  $\psi_2^0 \rightarrow 0$  at  $i^+$  &  $i^-$
- ③  $\exists$  Bondi frame s.t.  $\psi_2^0$  is spherically symm as you approach  $i^0$  - from  $g^+$  and  $g^-$

$(G_{ab} = 8\pi G_N T_{ab})$ ;  $\Omega^2 T_{ab}$  has a smooth limit  
 near stationary state  
 Radiative fields CK  $\infty$ -dim near Mink space  
 Chruściel-Delays, Friedrich

-1)  $H_E = \int_{i^0} \alpha d^3V - \int_{i^+} G d^3V$

4 Yem

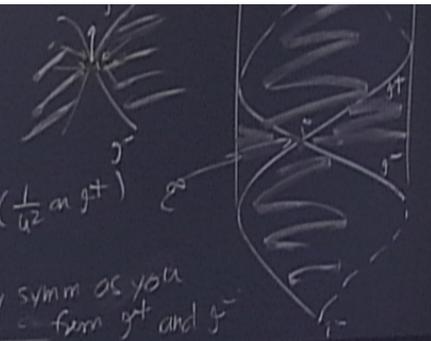
$+ \frac{1}{2} \alpha N_{ab} \omega^{ab}$



✓ 1978, Energy-momentum conserved

Supermomentum conserved?  
 CK space properties:

- ①  $N_{ab} \rightarrow 0$  as we approach  $I^\pm, i^0$  ( $\frac{1}{42}$  on  $g^{\pm}$ )
- ②  $\psi_z^0 \rightarrow 0$  at  $i^+$  &  $i^-$
- ③  $\exists$  Bondi frame s.t.  $\psi_z^0$  is spherically symm as you approach  $i^0$  - form  $g^+$  and  $g^-$



$G_{ab} = G_{(ab)}, G_{ab} h^a = 0, G_{ab} \omega^{ab} = 0$   
 YM Theory: Radiative Modes,  $\mathbb{D}|_g$  or  $A_a$  on  $S^2$

Hence compute Hamiltonians  
 Look at supertranslations  
 $\xi^a = \alpha(\theta, \varphi) \eta^a$  ( $\Omega \rightarrow \Omega' = \Omega + \xi$ )  
 $= (\alpha(\theta, \varphi) \omega^a)$   
 Conf. Weyl tensor function

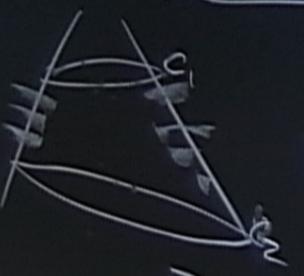
CON of  $\mathbb{D}$ . conf invariant is coded in a symmetric tracefree, transverse tensor,  $N_{ab}$  Bondi News tensor

Christoffel-Delay, Friedrich.  $\infty$ -dim "near Mink-space"

$$H_{\Sigma}^{\pm}(\mathcal{SDF}_{\Sigma}) = \frac{1}{16\pi G} \int_{\Sigma} N^{ab} (\partial_{\Sigma}^a \partial_{\Sigma}^b - \partial_{\Sigma}^b \partial_{\Sigma}^a) \epsilon_{abc} ds^{abc}$$

$$= \frac{1}{32\pi G} \int_{\Sigma} (\alpha \hat{N}_{ab} + D_{\Sigma}^a D_{\Sigma}^b \alpha + \alpha q_{ab}) N^{ab} d^3\Sigma$$

(if  $\alpha$  is a trans.  $\alpha$ : linear comb of first 4 Yem)



Balance law:

$$P_{\Sigma}^{\pm}[c] = \frac{1}{8\pi G} \int_{\Sigma} [\alpha \text{Re } \psi_0 + \frac{1}{2} \alpha N_{ab} \sigma^{ab}]$$

$$P_{\Sigma}^{\pm}[c_2] - P_{\Sigma}^{\pm}[c_1] = \int_{\Sigma} F_{\Sigma}^a d^3\Sigma$$

$$H_{\Sigma}^+ = \int_{i^0} \alpha d^2V - \int_{i^{\infty}} G d^2\bar{e}$$

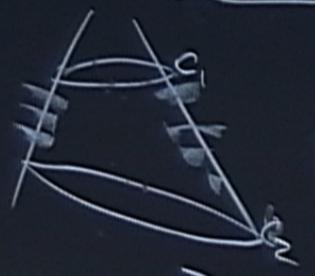
$$H_{\Sigma}^- = \int_{i^0} Q d^2V - \int_{i^{\infty}} \dots$$

Chrusciel-Delay, Friedrich.  $\infty$ -dim "near Mink-space"

$$H_{\Sigma}(\text{SDF}) = \frac{1}{16\pi G} \int N^{ab} (\dots) \epsilon_{abc} ds^{abc} \quad (\epsilon_{ab} n^a = -1)$$

$$= \frac{1}{32\pi G} \int_{\Sigma} (\alpha N^{ab} + D_i D_b \alpha + \alpha q_{ab}) N^{ab} d^3x$$

(I<sub>Σ</sub>)



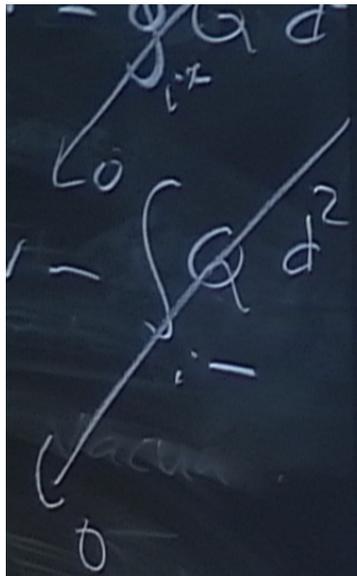
Balance law:  
 iff  $\alpha$  is a trans.  
 $\alpha$ : linear comb of first 4 Yem

$$P_{\Sigma}[C] = \frac{1}{8\pi G} \int_C [\alpha \text{Re } \Psi_0 + \frac{1}{2} \alpha N^{ab} q_{ab}]$$

$$P_{\Sigma}[C_2] - P_{\Sigma}[C_1] = \int_{\Sigma} F_{\Sigma} d^3x$$

$$H_{\Sigma}^+ = \int_{i^0} \alpha \Psi_2^0 d^2V - \int_{i^{\infty}} G d^2$$

$$H_{\Sigma}^- = \int_{i^0} \alpha \Psi_0^0 d^2V - \int_{i^0} Q d^2$$



Supermomentum  
CK space

- ①  $N_{ab} \rightarrow$
- ②  $\downarrow \circ$
- ③  $\exists$  Bondi

Herdegen 1604-0417

Mirbabai & Pottathi  
1607-0312

CUV of  $D$ : conf. invariant is  
tensor,  $N_{ab}$  Bondi