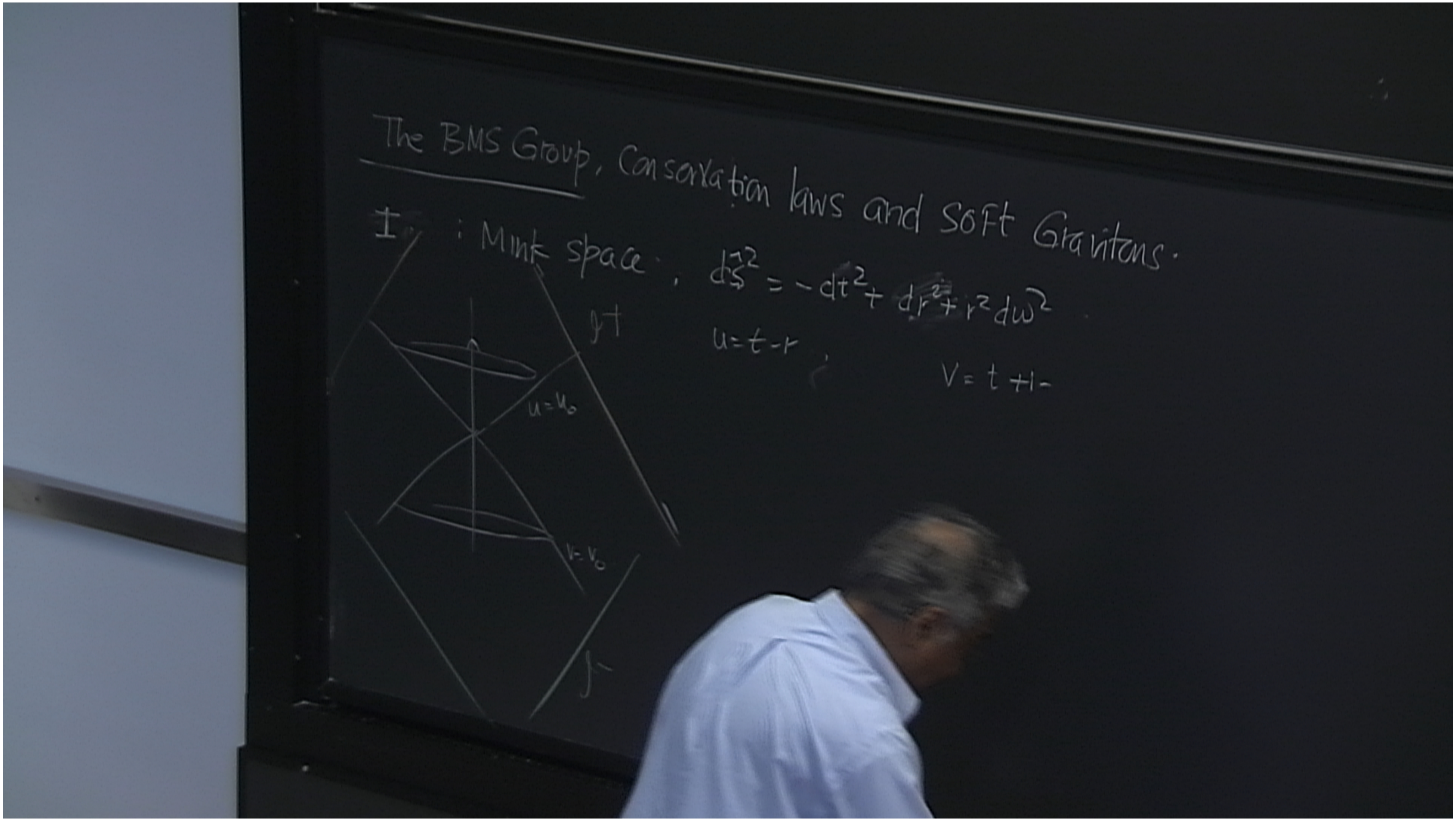
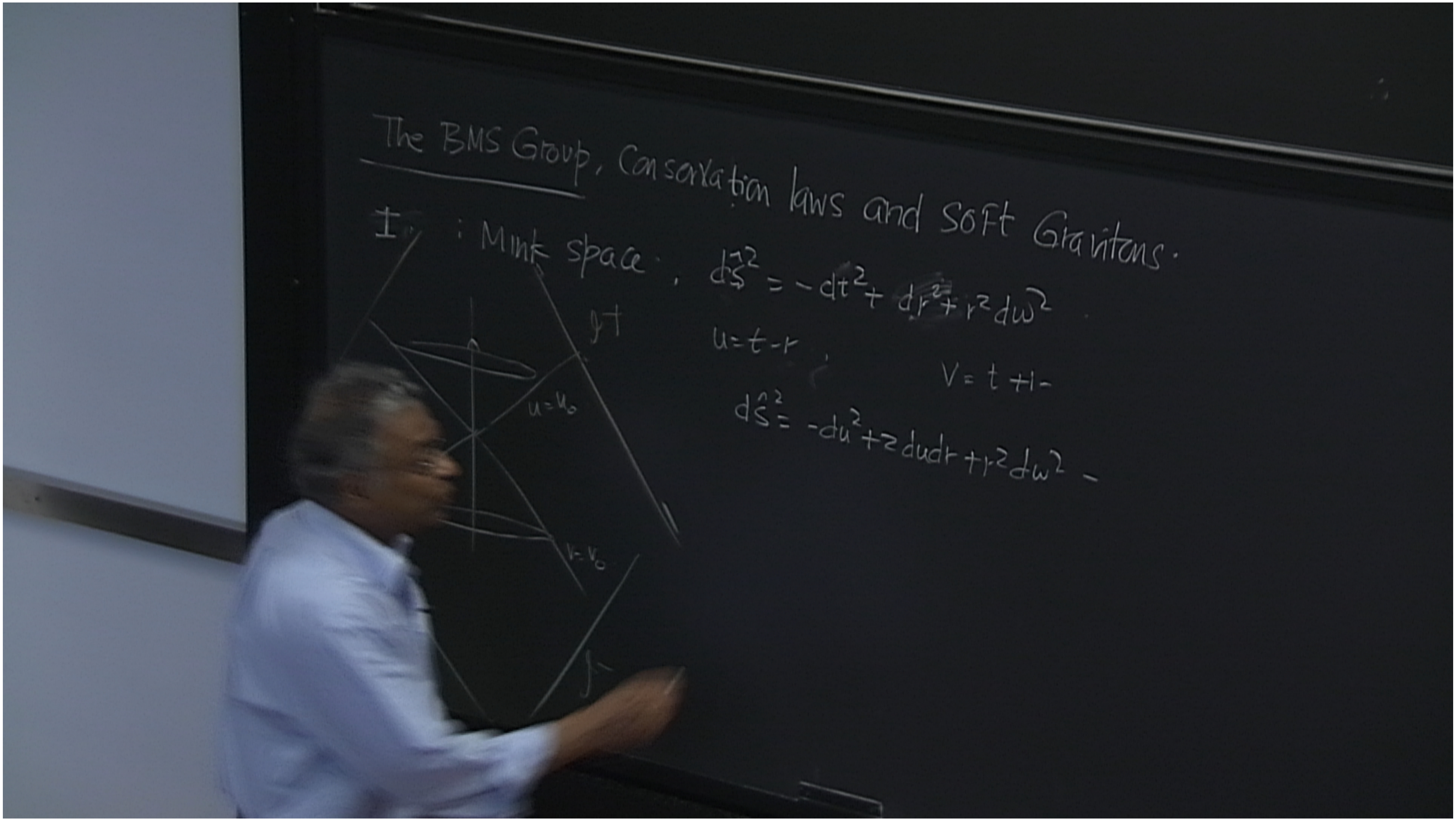
Title: The BMS group, conservation laws and soft gravitons

Date: Aug 11, 2016 02:30 PM

URL: http://pirsa.org/16080055

Abstract: <p>I will give a broad overview of this old subject which has come back to light recently in connection with the issue of black hole evaporation. It will be an informal, black board talk.</p>


The BMS Group, Conservation laws and Soft Gravitons.
I : Mink space : ds^2 = -dt^2 + dr^2 + r^2 dω^2
u = t - r
V = t + r
u = u_0
v = v_0
I^+
I^-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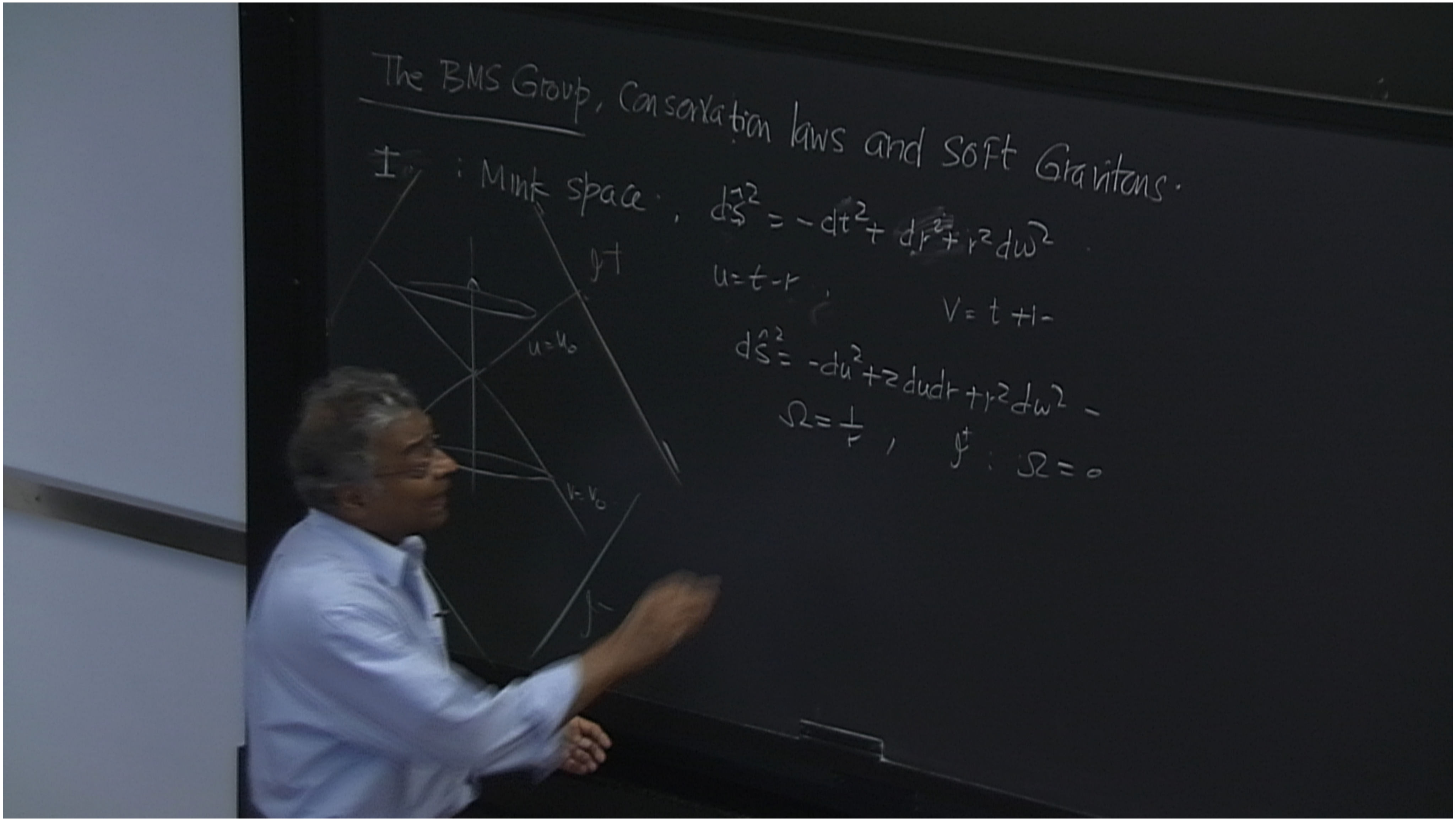
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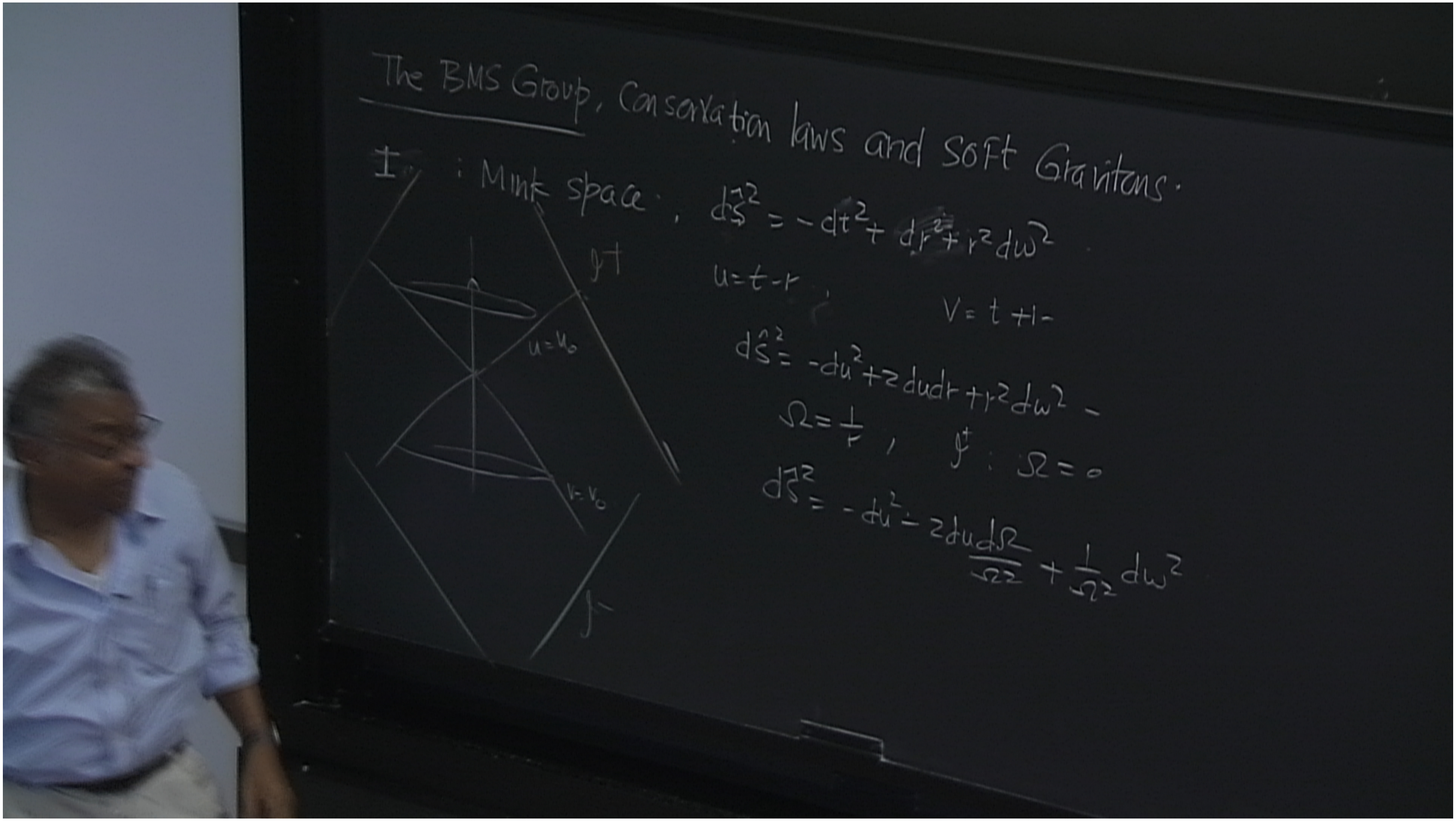
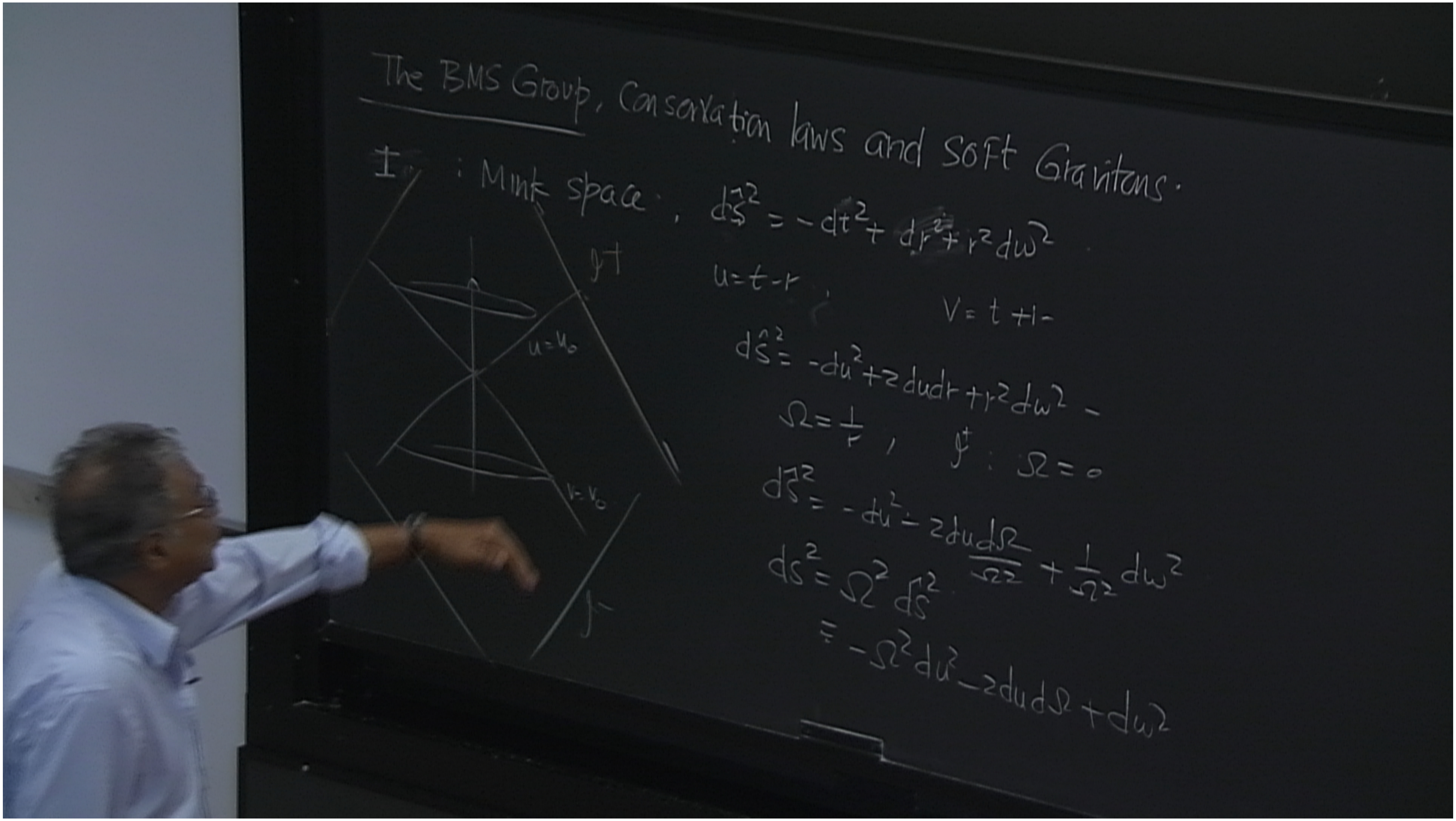
Ω = 1/r , 𝒥+ : Ω = 0
𝒥+
u = u0
v = v0
𝒥-


The BMS Group, Conservation laws and Soft Gravitons.
: Mink space
u = t - r
V = t + r
u = u₀
v = v₀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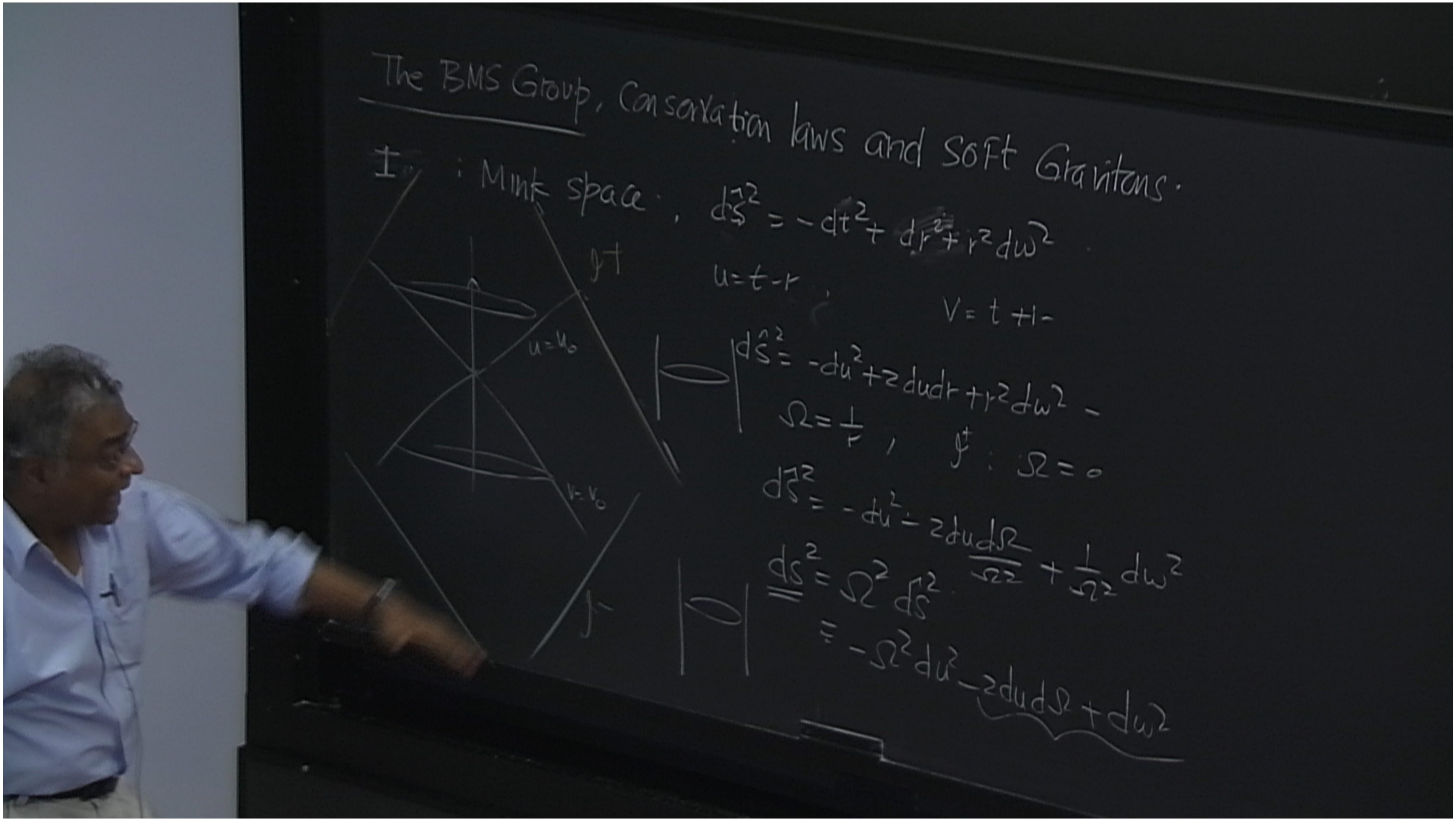
$u = t - r$
$v = t + r$
$d\hat{s}^2 = -du^2 + 2dudr + r^2 d\omega^2$
$\Omega = \frac{1}{r}$, $\mathcal{J}^+ : \Omega = 0$
$d\hat{s}^2 = -du^2 - 2du\frac{d\Omega}{\Omega^2} + \frac{1}{\Omega^2}d\omega^2$
$ds^2 = \Omega^2 d\hat{s}^2$
$= -\Omega^2 du^2 - 2dud\Omega + d\omega^2$
$\mathcal{J}^+$
$u = u_0$
$v = v_0$
$\mathcal{J}^-$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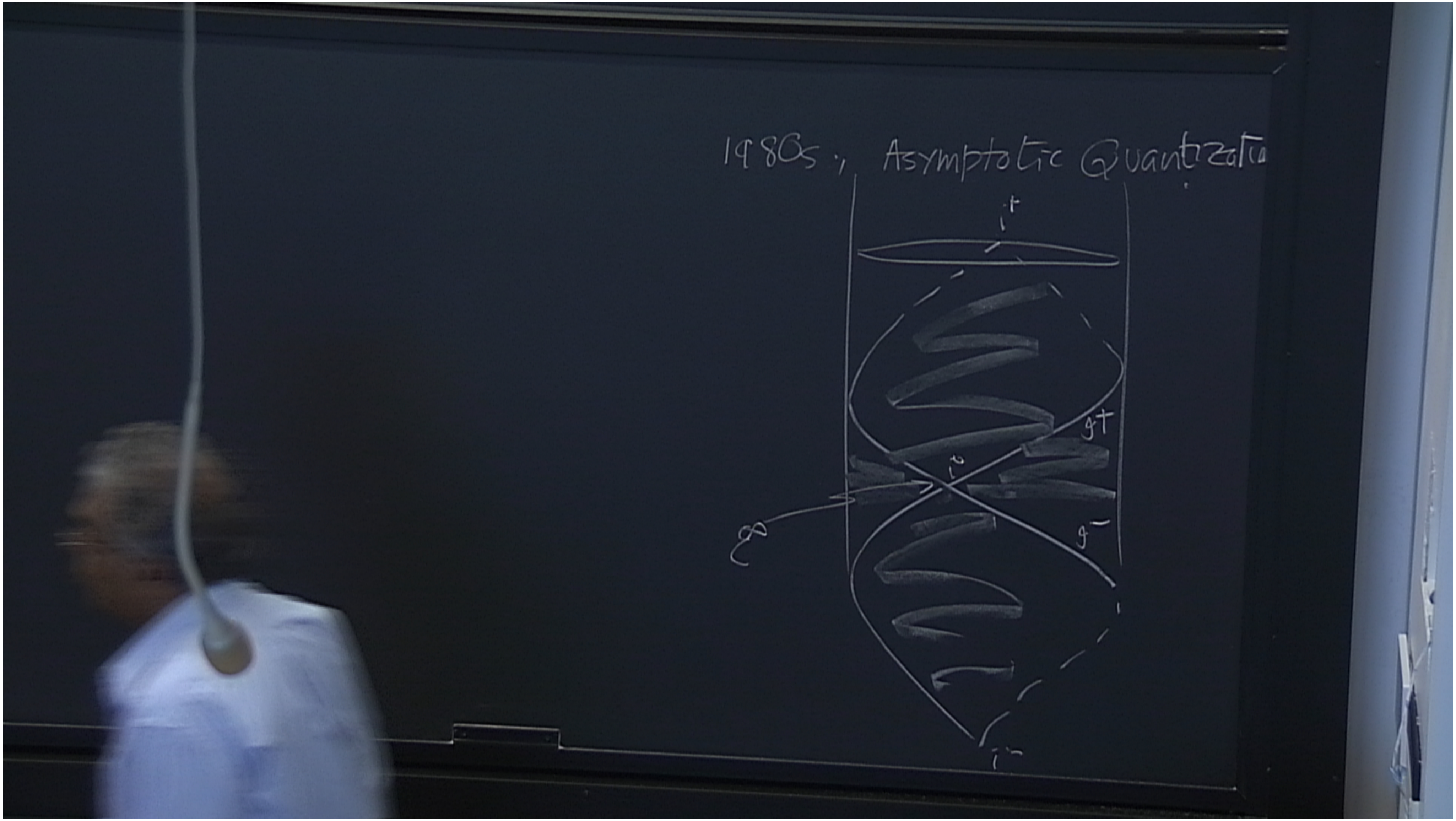
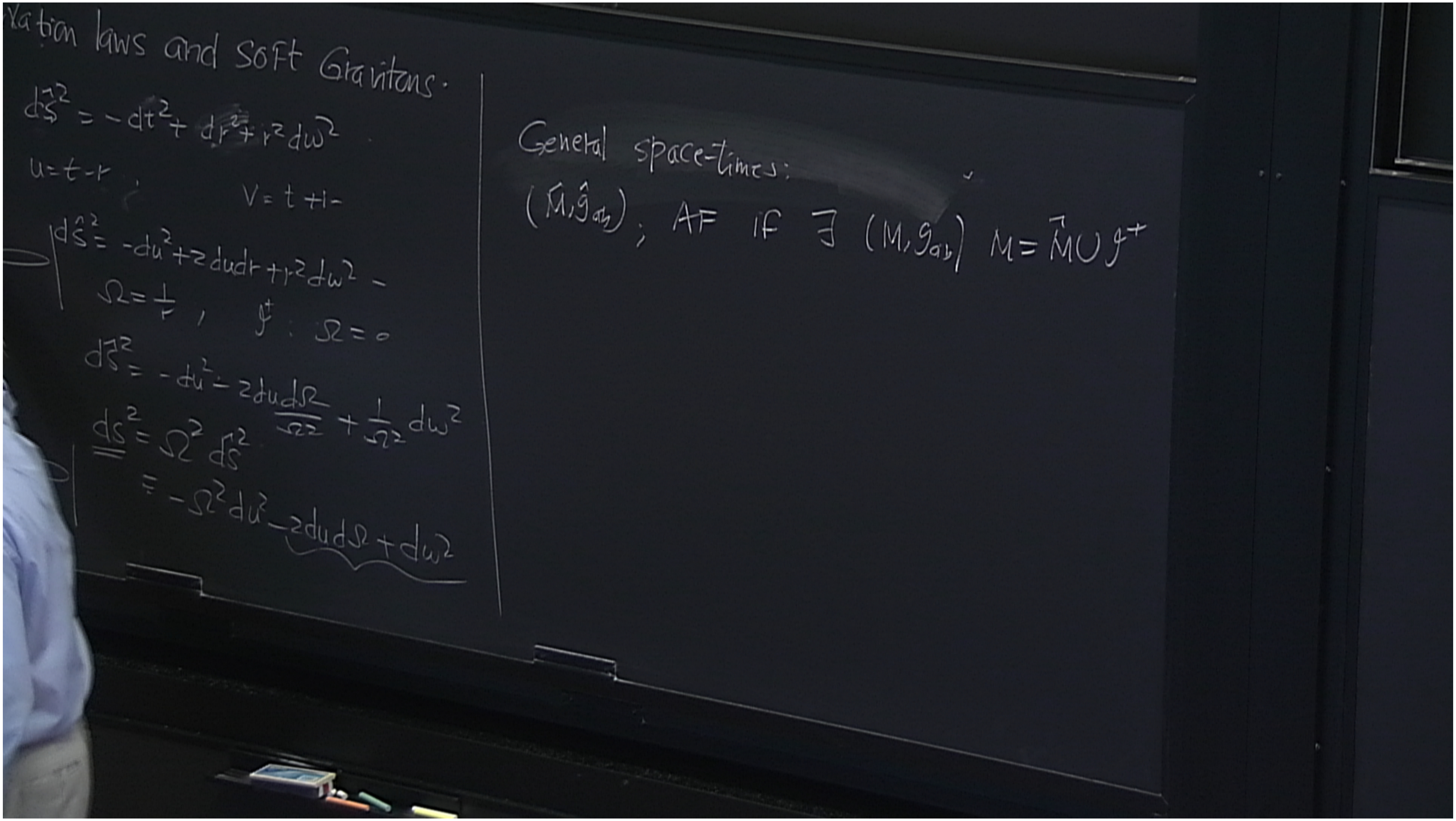
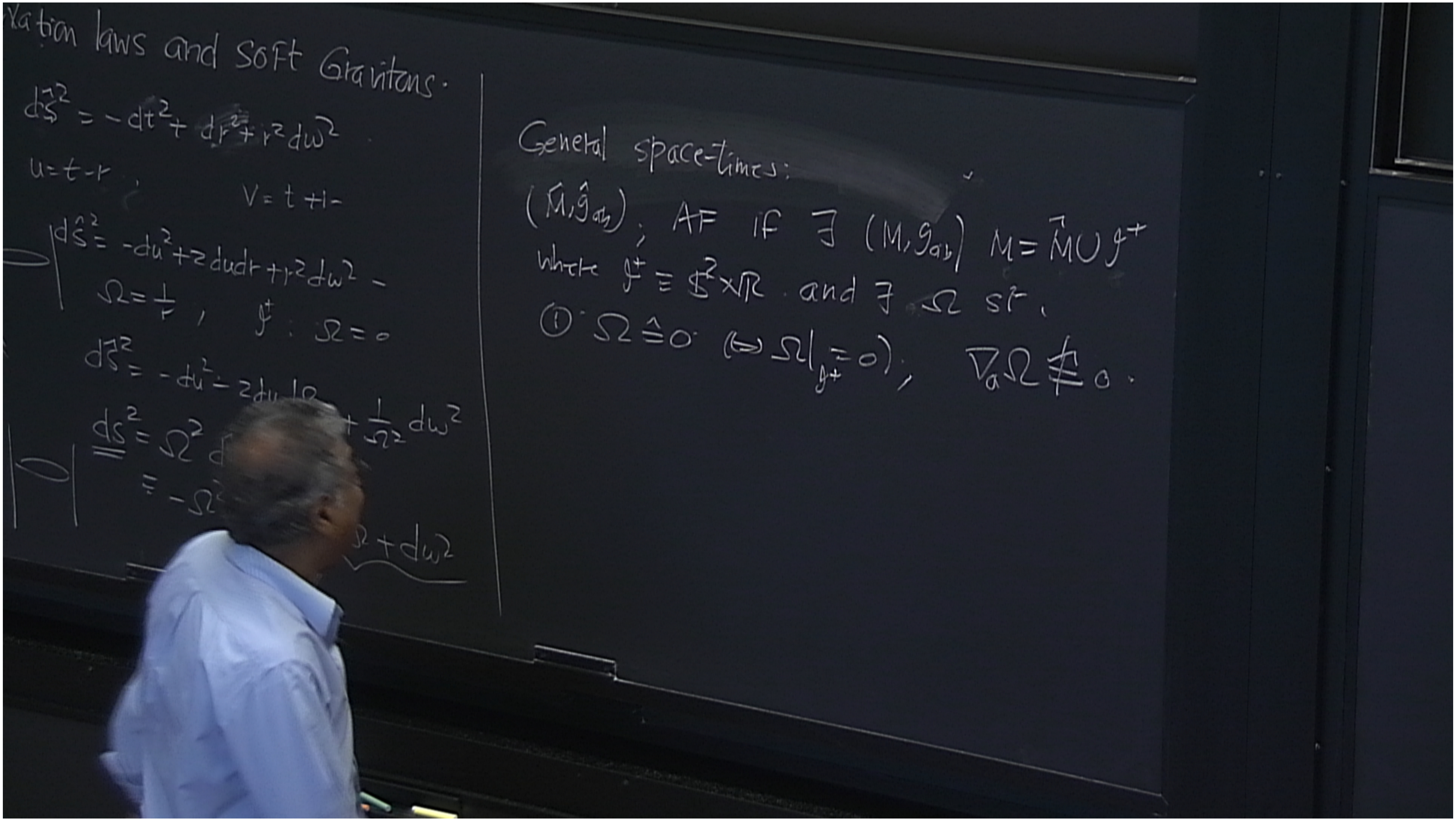
$ds^2 = \Omega^2 d\tilde{s}^2$
$= -\Omega^2 du^2 - 2 du d\Omega + d\omega^2$


1980s :, Asymptotic Quantization
i+
ℐ+
i0
ℐ−
i−


...vation laws and soft Gravitons.
$d\tilde{s}^2 = -dt^2 + dr^2 + r^2 d\omega^2$
$u = t - r$; $v = t + r$
$d\hat{s}^2 = -du^2 + 2du\,dr + r^2 d\omega^2$
$\Omega = \frac{1}{r}$, $\mathcal{I}^+ : \Omega = 0$
$d\tilde{s}^2 = -du^2 - 2du\frac{d\Omega}{\Omega^2} + \frac{1}{\Omega^2} d\omega^2$
$ds^2 = \Omega^2 d\tilde{s}^2$
$= -\Omega^2 du^2 - 2du\,d\Omega + d\omega^2$
General space-times:
$(\hat{M}, \hat{g}_{ab})$; AF if $\exists$ (M, g_{ab}) $M = \hat{M} \cup \mathcal{I}^+$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u = t - r
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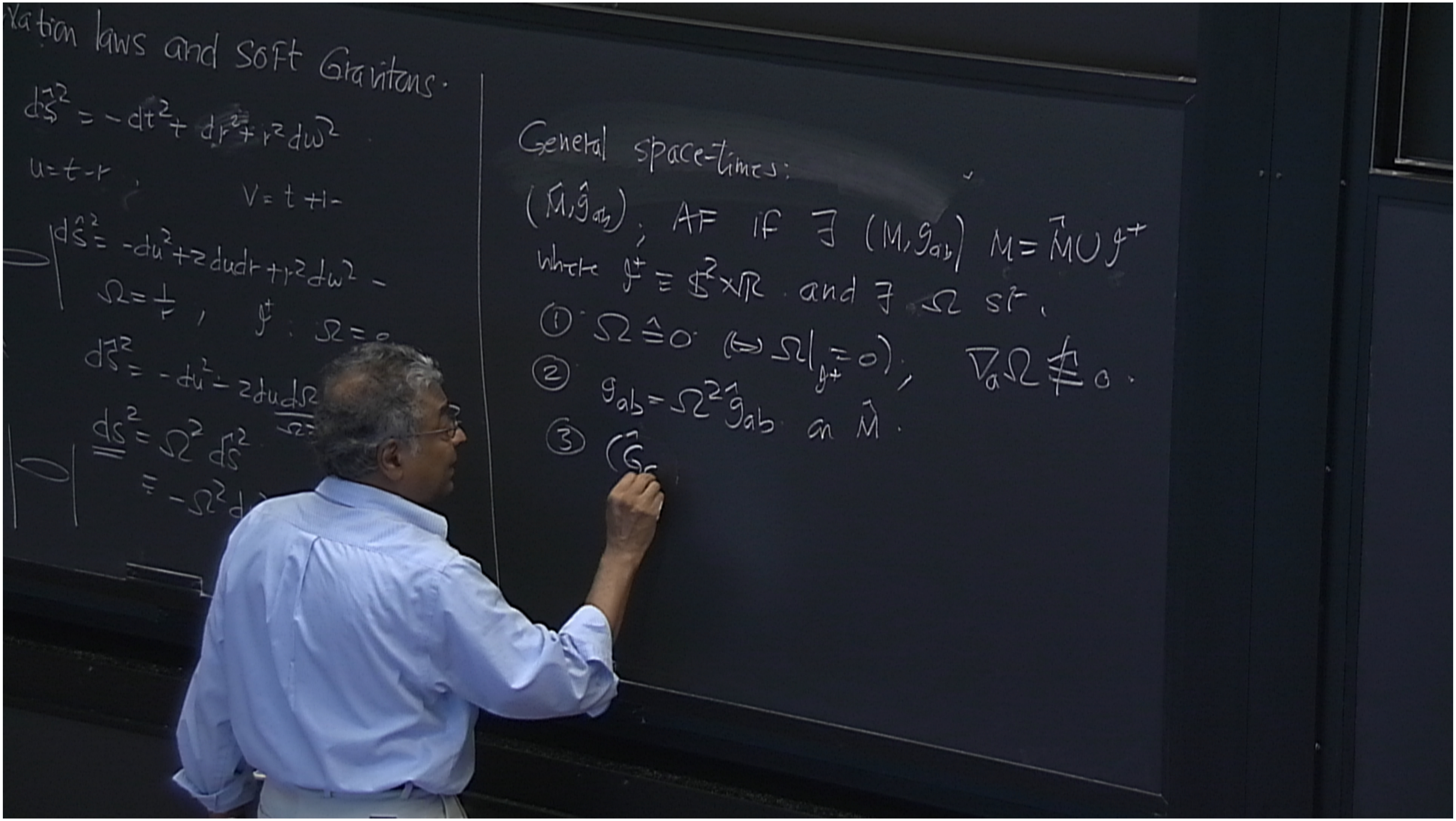
General space-times:
AF if
where
and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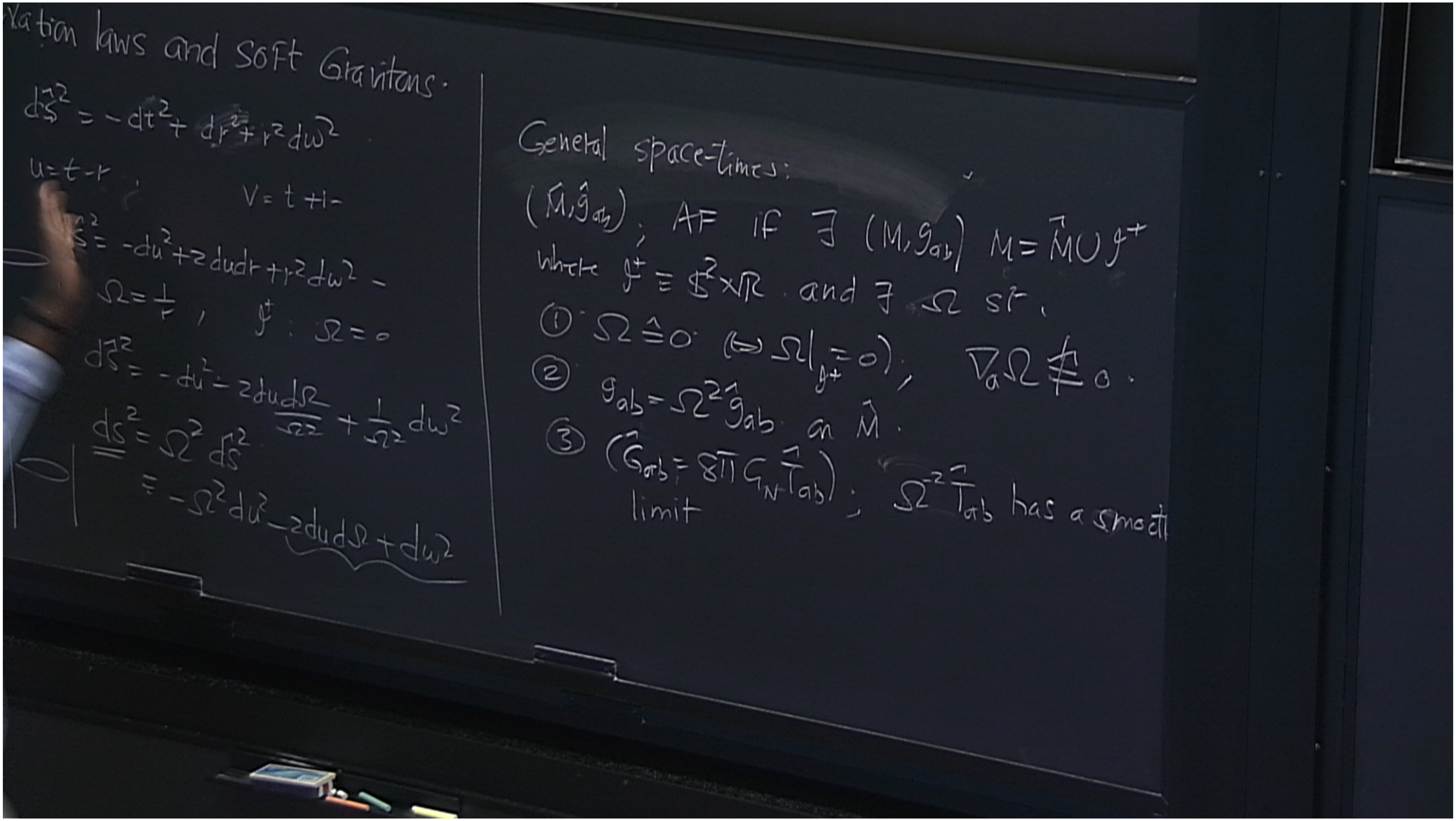
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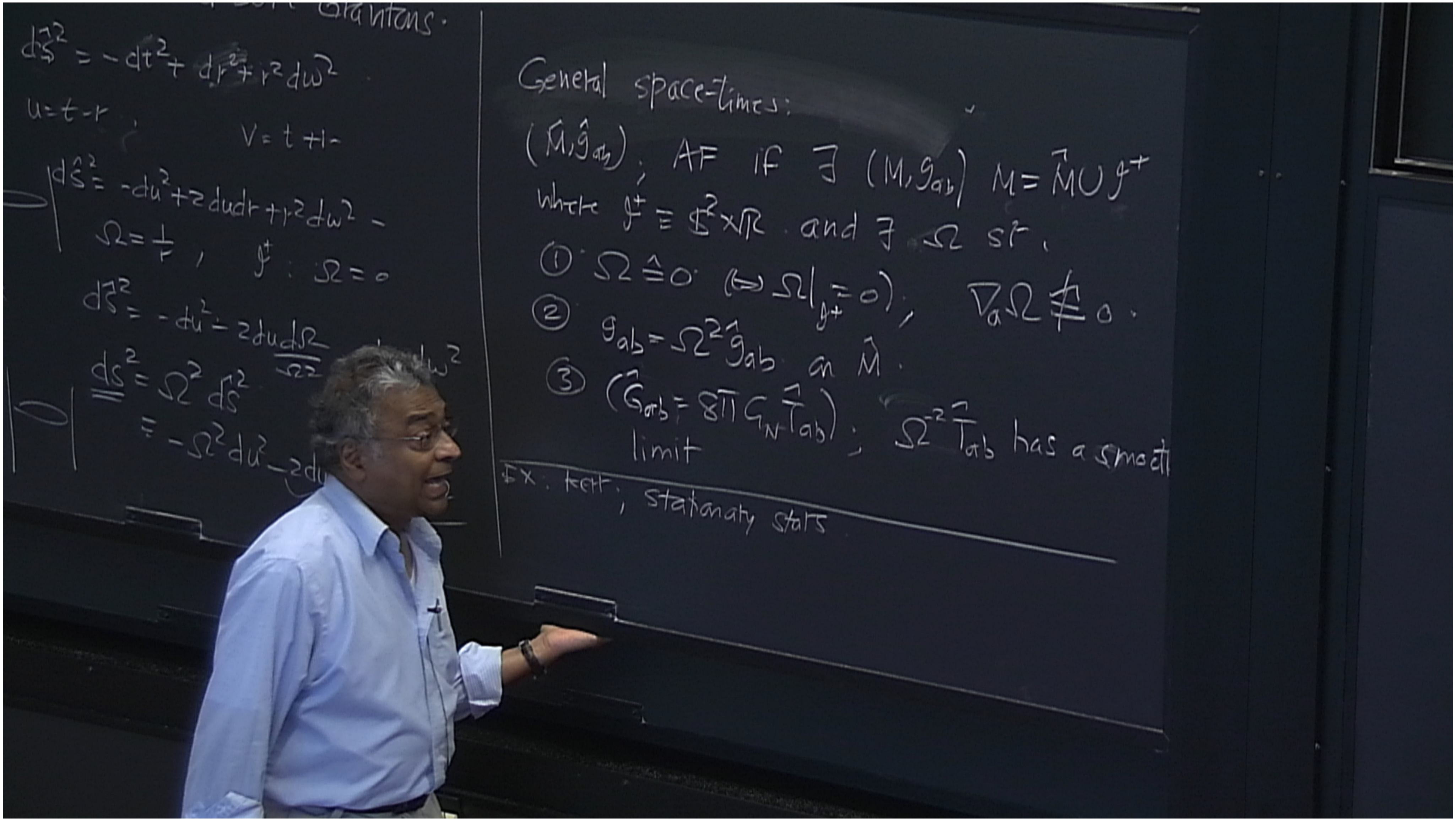
u = t - r
v = t + r
General space-times:
AF if
where
and
limit
has a smooth


General space-times:
AF if
where
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limit
has a
Ex: Kerr, stationary stars

$v = t + r$

$d\hat{s}^2 = -du^2 + 2\,du\,dr + r^2 d\omega^2$

$\Omega = \frac{1}{r}$, $\mathcal{I}^+ : \Omega = 0$

$d\tilde{s}^2 = -du^2 - 2\,du\frac{d\Omega}{\Omega^2} + \frac{1}{\Omega^2}d\omega^2$

$ds^2 = \Omega^2 d\tilde{s}^2$

$= -\Omega^2 du^2 - 2\,du\,d\Omega + d\omega^2$

$(\tilde{M}, \tilde{g}_{ab})$; AF if $\exists$ (M, g_{ab}) $M = \tilde{M} \cup \mathcal{I}^+$

where $\mathcal{I}^+ \equiv S^2 \times \mathbb{R}$ and $\exists$ Ω st.

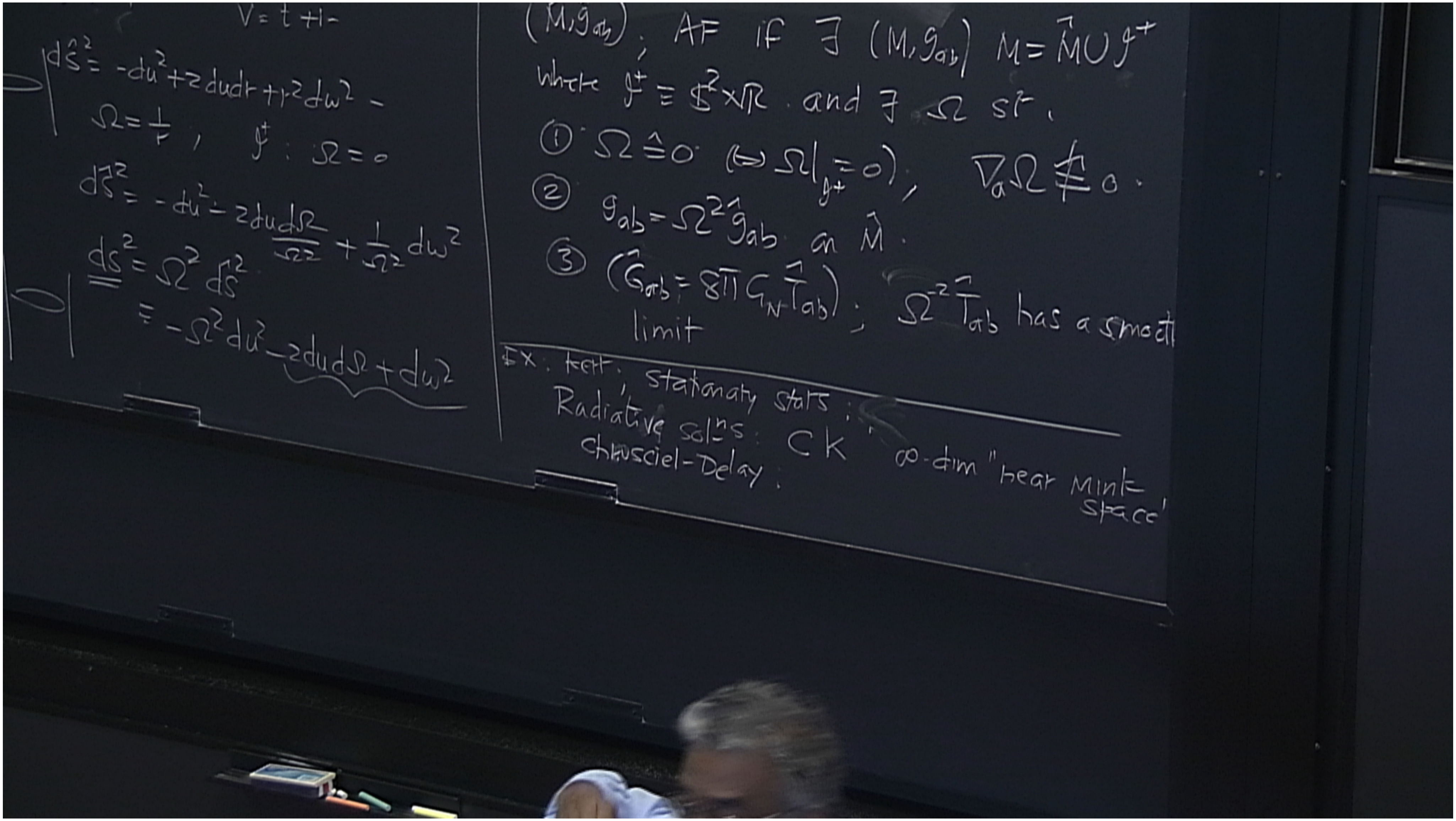
① $\Omega \hat{=} 0$ ($\Leftrightarrow \Omega|_{\mathcal{I}^+} = 0$), $\nabla_a \Omega \hat{\neq} 0$.

② $g_{ab} = \Omega^2 \tilde{g}_{ab}$ on $\tilde{M}$.

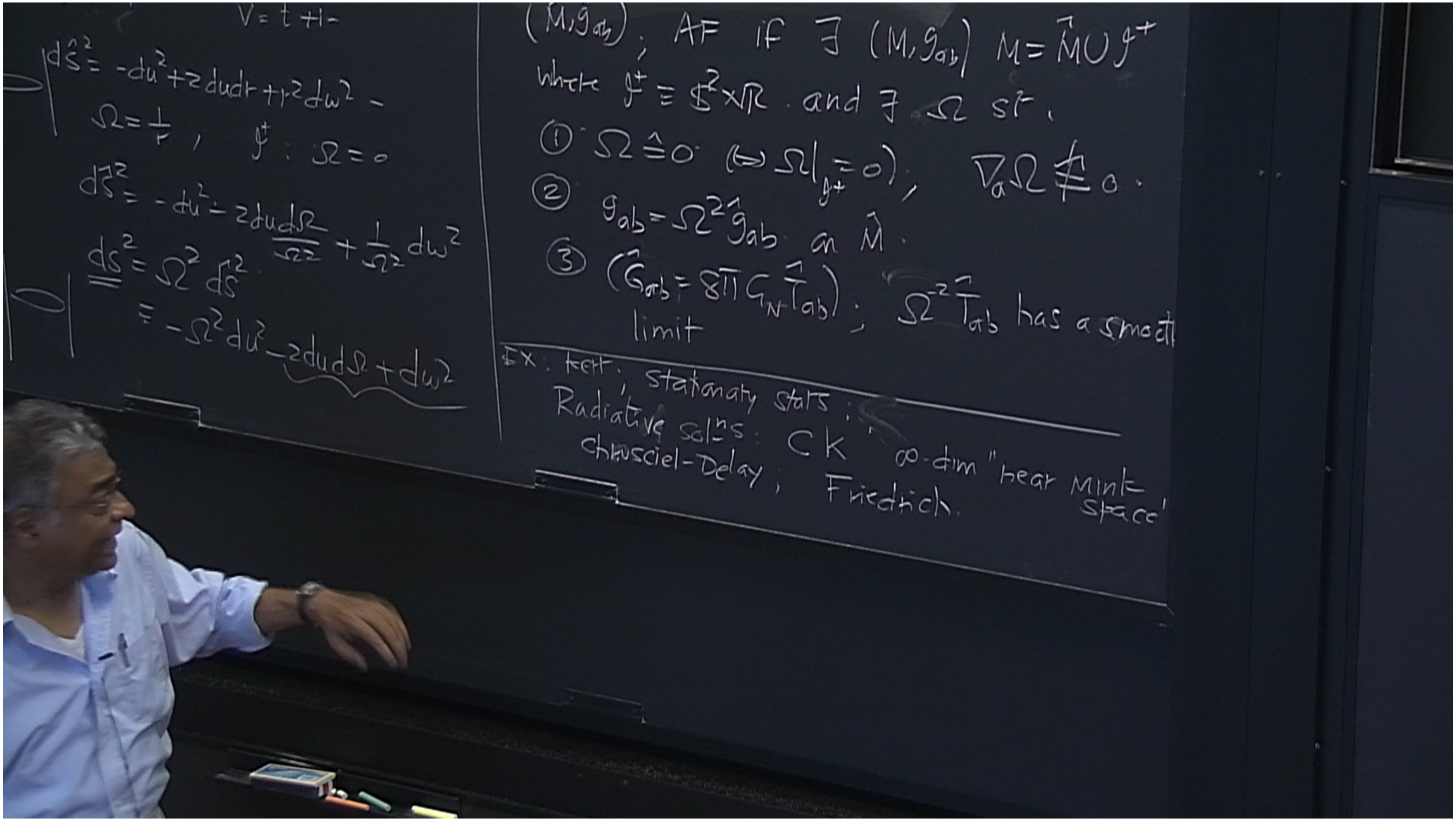
③ $(\tilde{G}_{ab} = 8\pi G_N \tilde{T}_{ab})$; $\Omega^{-2}\tilde{T}_{ab}$ has a smooth limit

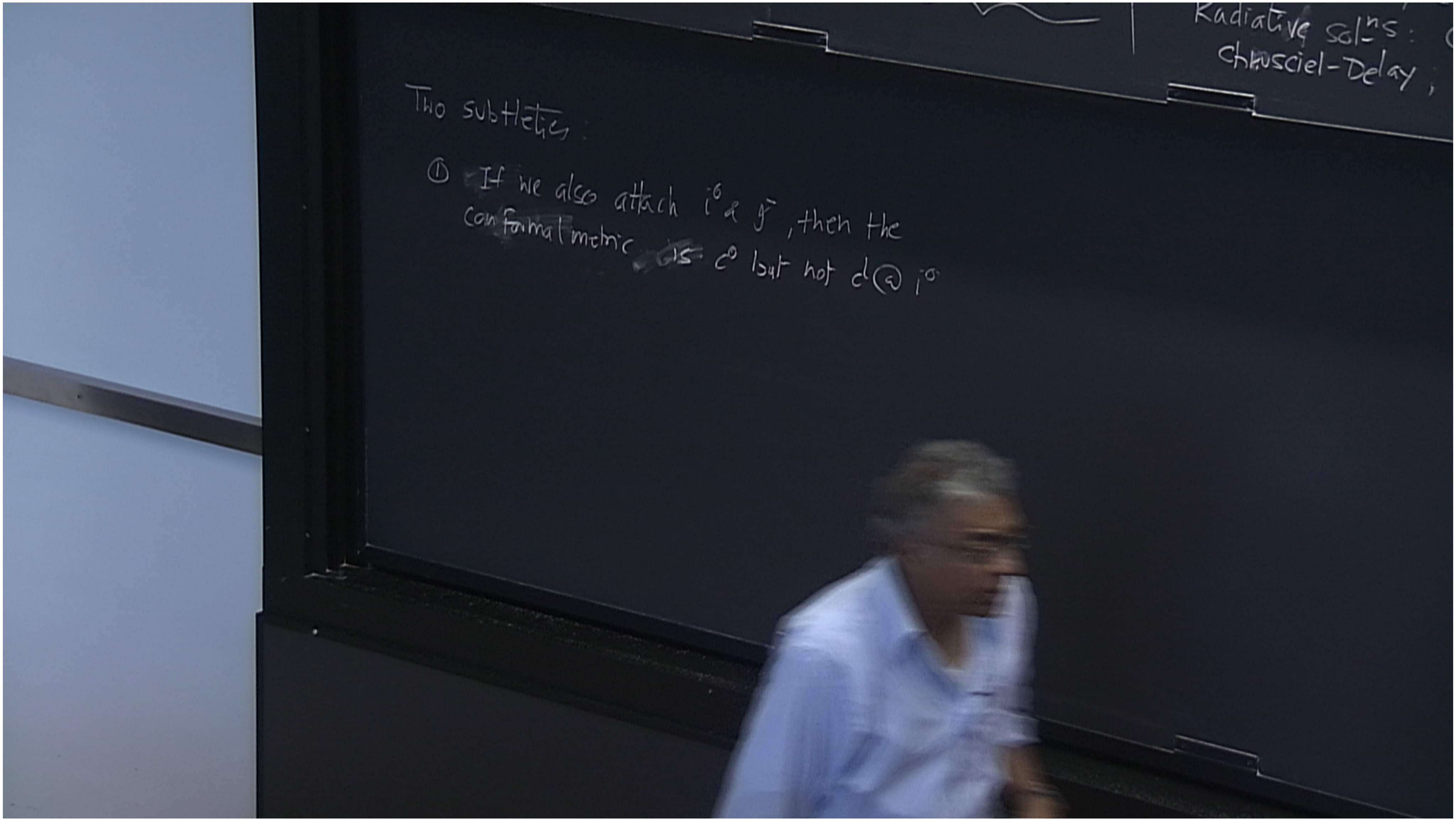
Ex: Kerr, stationary stars;

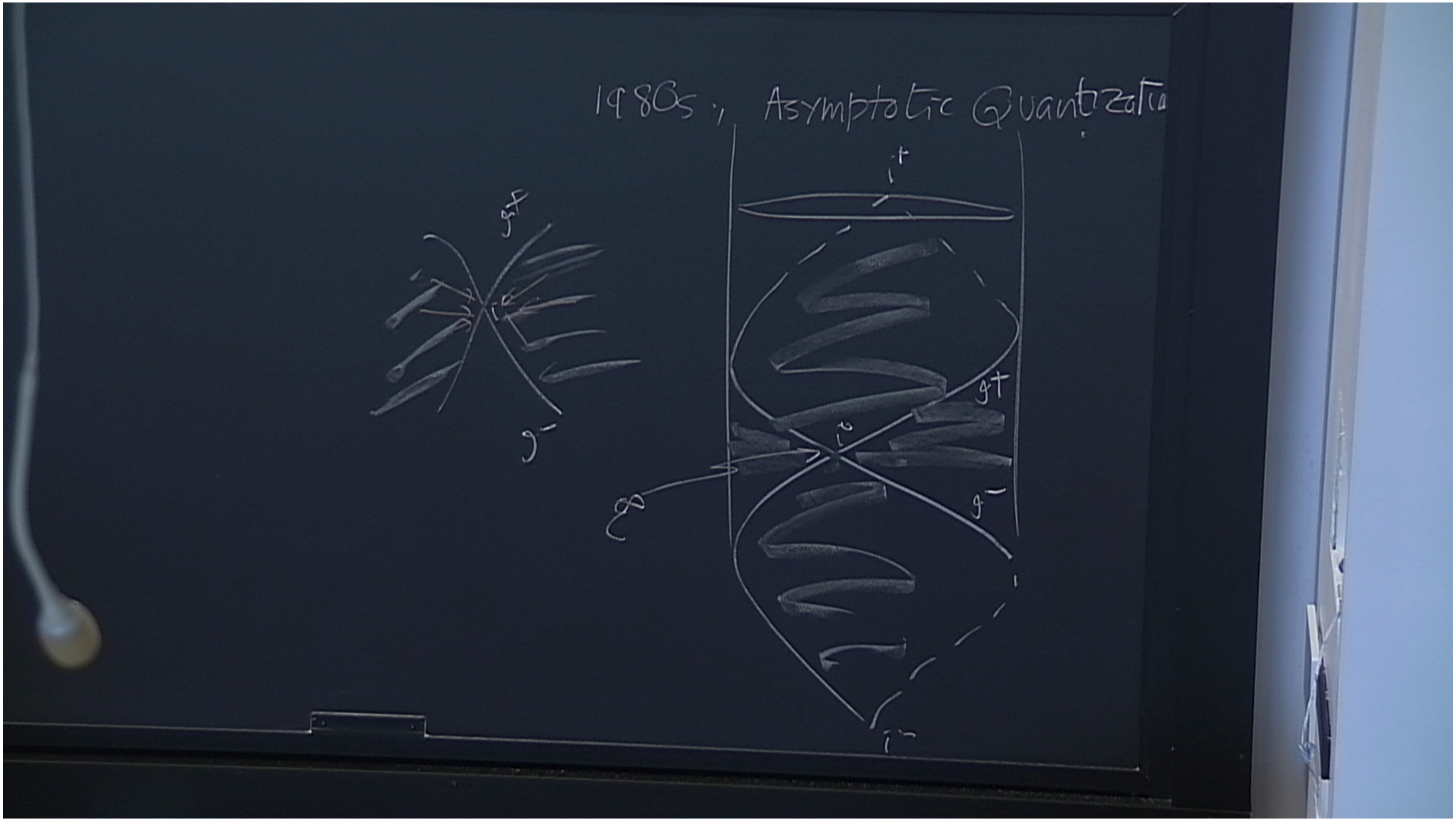
Radiative sol$^{\underline{n}}$s: CK "∞-dim "near Mink space"

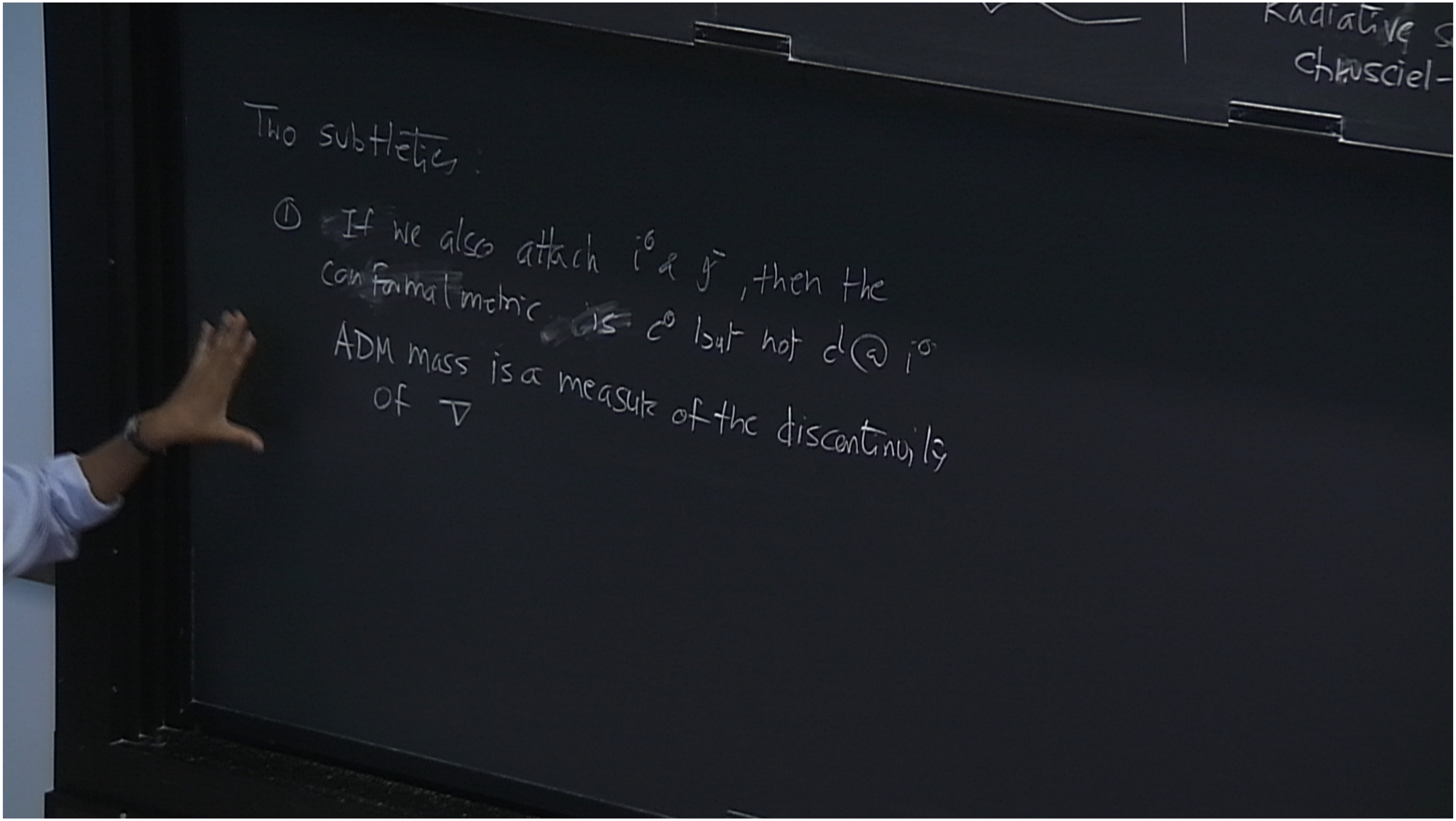


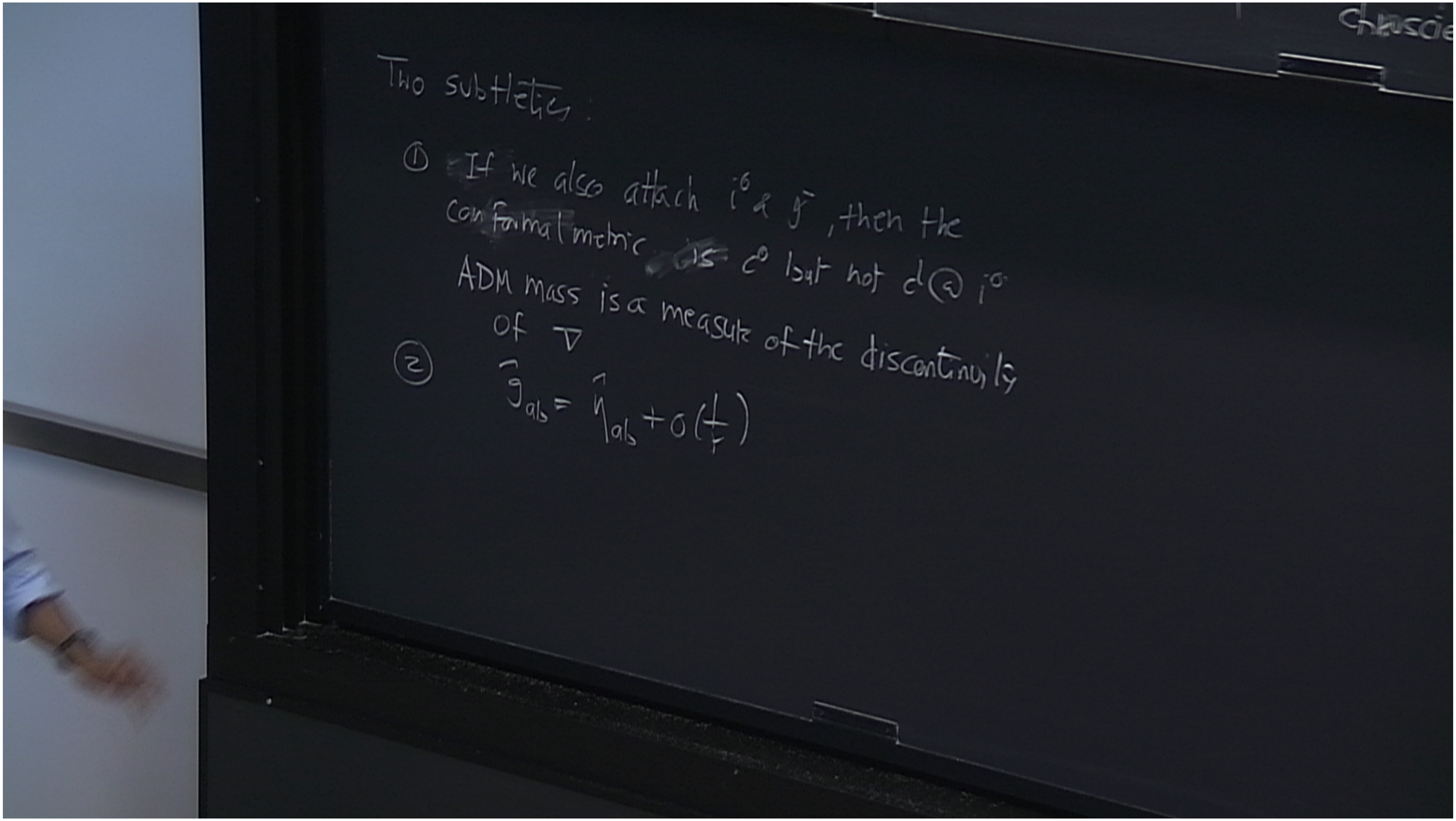
AF if ∃ (M, g_ab) M = M̃ ∪ ℐ+
where ℐ+ ≡ S² × R and ∃ Ω st.
① Ω ≙ 0 (⟺ Ω|ℐ+ = 0), ∇_a Ω ≙̸ 0.
② g_ab = Ω² g̃_ab on M̃.
③ (G̃_ab = 8π G_N T̃_ab); Ω⁻² T̃_ab has a smooth limit
Ex: Kerr, stationary stars;
Radiative solns: CK ∞-dim "near Mink space"
Chrusciel-Delay;
ds² = -du² + 2 du dr + r² dω²
Ω = 1/r, ℐ+: Ω = 0
ds² = Ω² dŝ²
= -Ω² du² - 2 du dΩ + dω²


AF if ∃ (M, g_ab) M = M ∪ ℐ+
where ℐ+ ≡ S2 × R and ∃ Ω st.
Ω ≙ 0
gab = Ω2 gab on M
has a smooth limit
Ex: Kerr, Stationary stars ;
Radiative solns : CK ∞-dim "near Mink space"
Chrusciel-Delay , Friedrich


Radiative sol^ns :
Chrusciel-Delay ;
Two subtleties:
① If we also attach i° & ℐ⁻, then the
conformal metric is C° but not C¹ @ i°


1980s : Asymptotic Quantization
i+
i0
i-
∞


Two subtleties:
① If we also attach i^0 & $\mathscr{J}^-$, then the conformal metric is C^0 but not C^1 @ i^0
ADM mass is a measure of the discontinuity of ∇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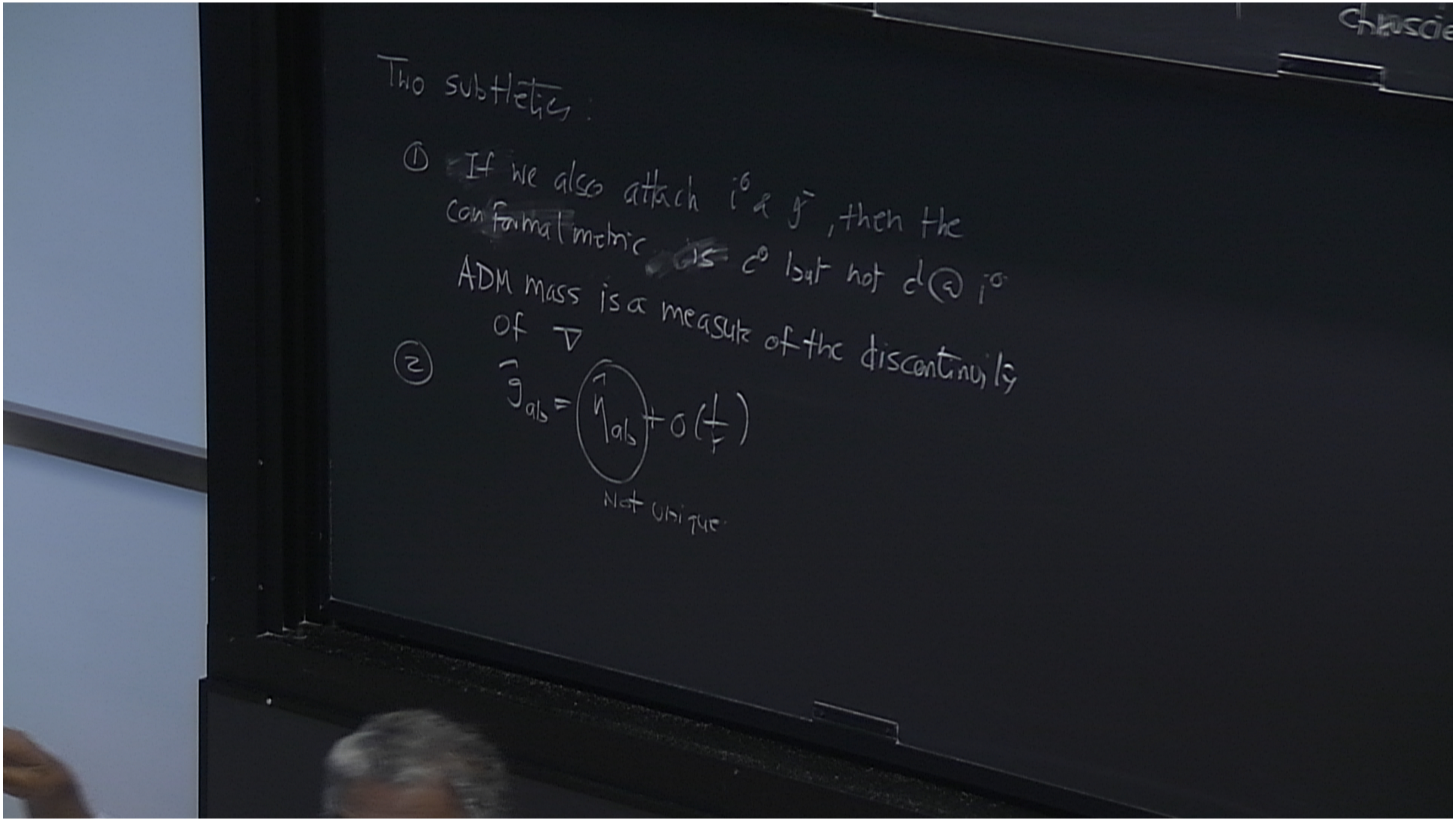
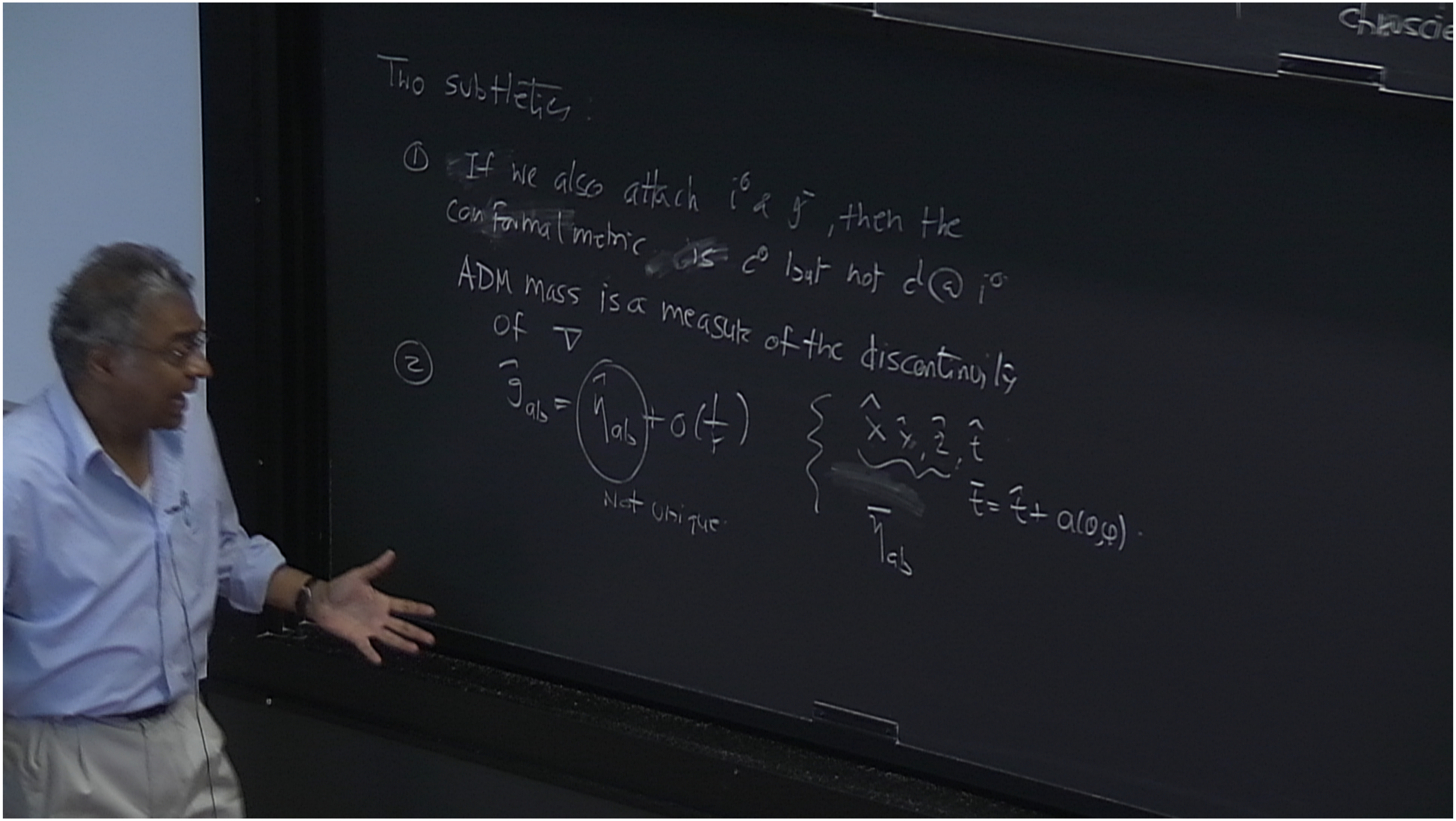
① If we also attach i^0 & $\mathscr{J}^-$, then the conformal metric is C^0 but not C^1 @ i^0
ADM mass is a measure of the discontinuity of ∇
② $\bar{g}_{ab} = \hat{\eta}_{ab} + O(\frac{1}{r})$


Two subtleties:
① If we also attach i⁰ & ℐ⁻, then the conformal metric is C⁰ but not C¹ @ i⁰
ADM mass is a measure of the discontinuity of ∇
② ḡ_ab = (η̄_ab) + O(1/r)
Not unique.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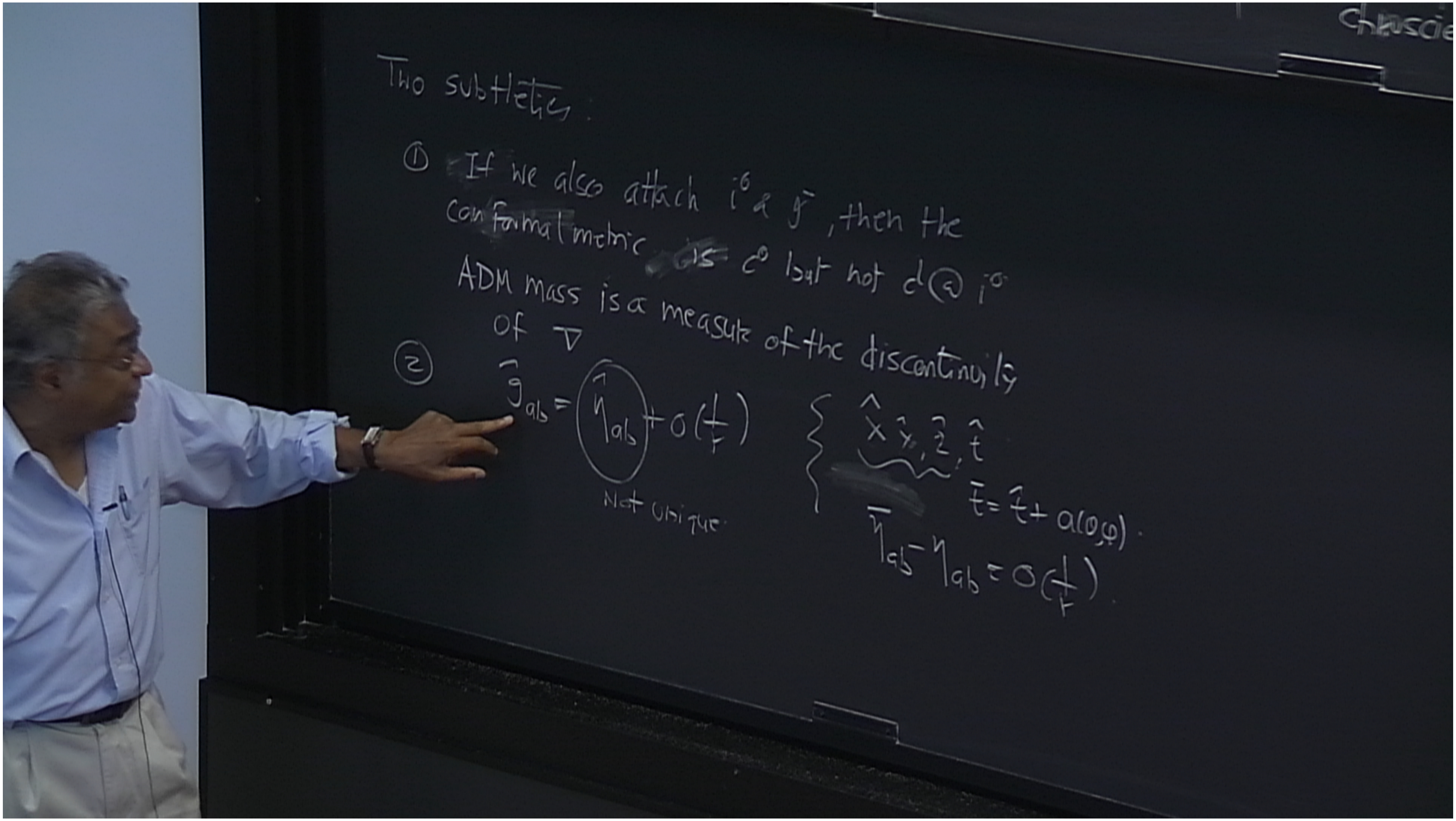
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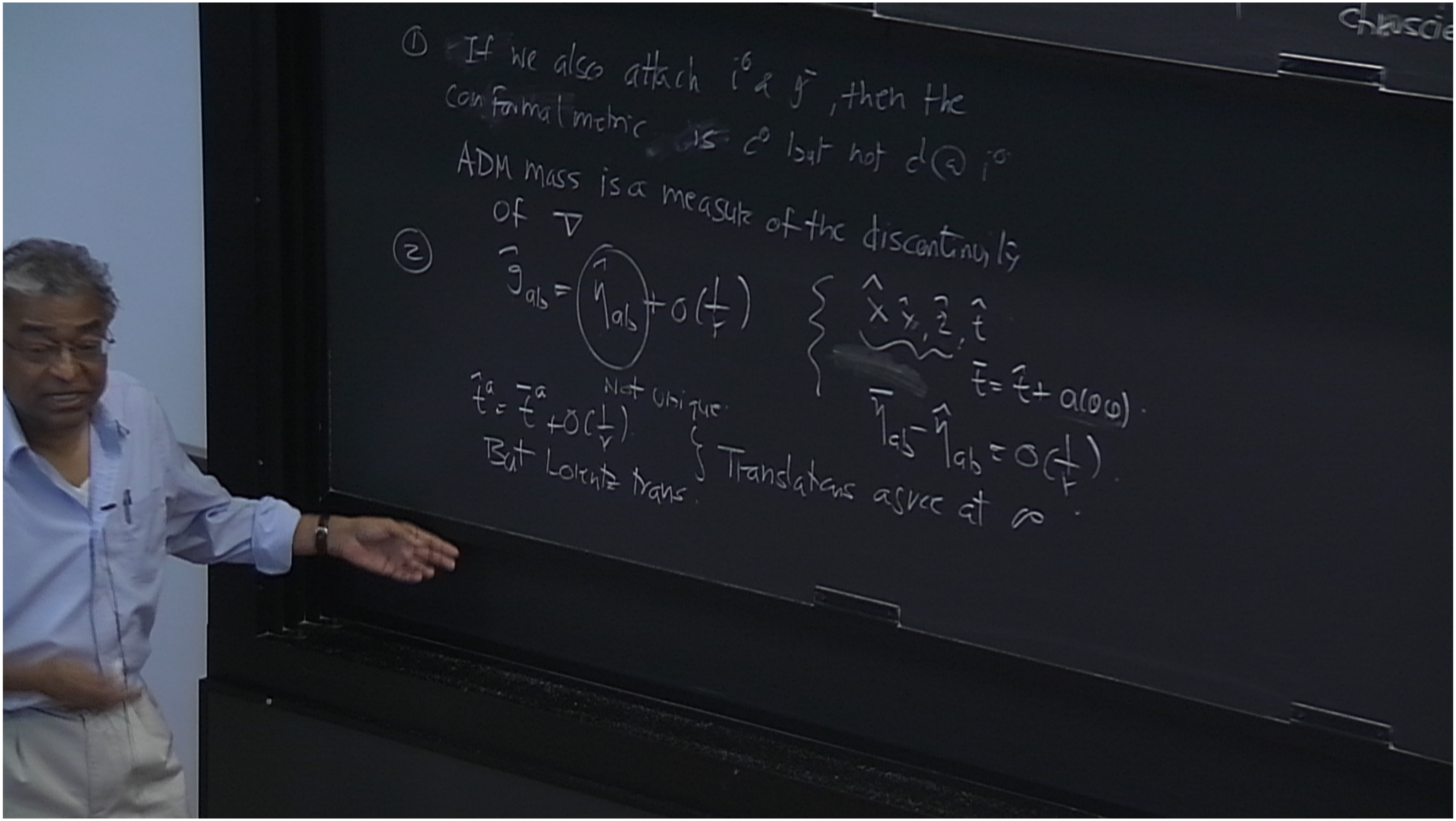
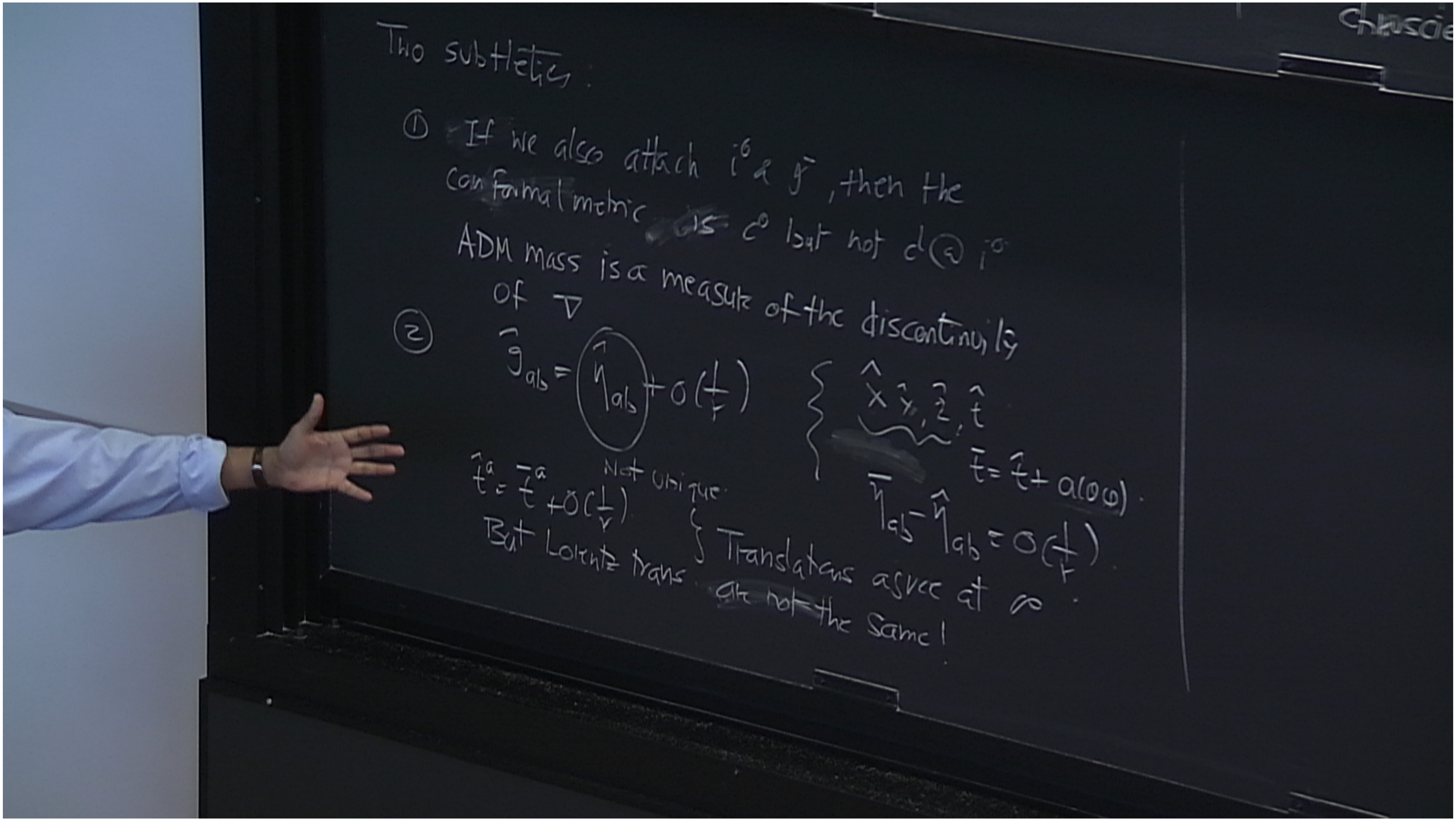
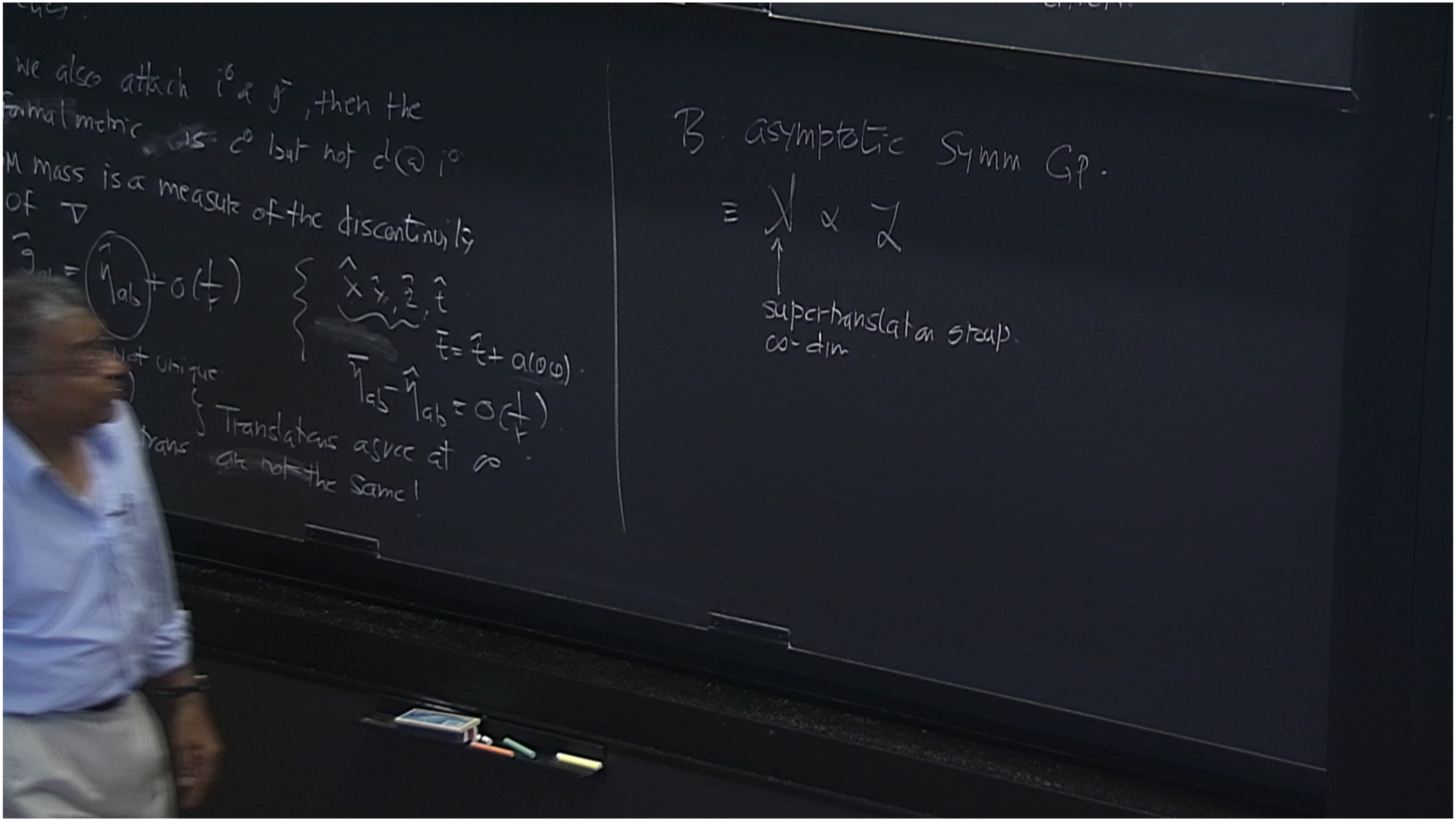
② $\bar{g}_{ab} = \bar{\eta}_{ab} + O(\frac{1}{r})$
Not unique.
$\hat{x}, \hat{y}, \hat{z}, \hat{t}$
$\bar{t} = \hat{t} + a(\theta,\varphi)$.
$\bar{\eta}_{ab} - \eta_{ab} = O(\frac{1}{r})$.


① If we also attach i° & J⁻, then the conformal metric is C° but not C¹ @ i°
ADM mass is a measure of the discontinuity of ∇
②
Not unique
But Lorentz trans.
Translations agree at ∞


Two subtleties:
① If we also attach
then the
conformal metric
but not
ADM mass is a measure of the discontinuity
of ∇
②
Not unique.
Translations agree at ∞.
But Lorentz trans
the same!


we also attach i° & J⁻, then the
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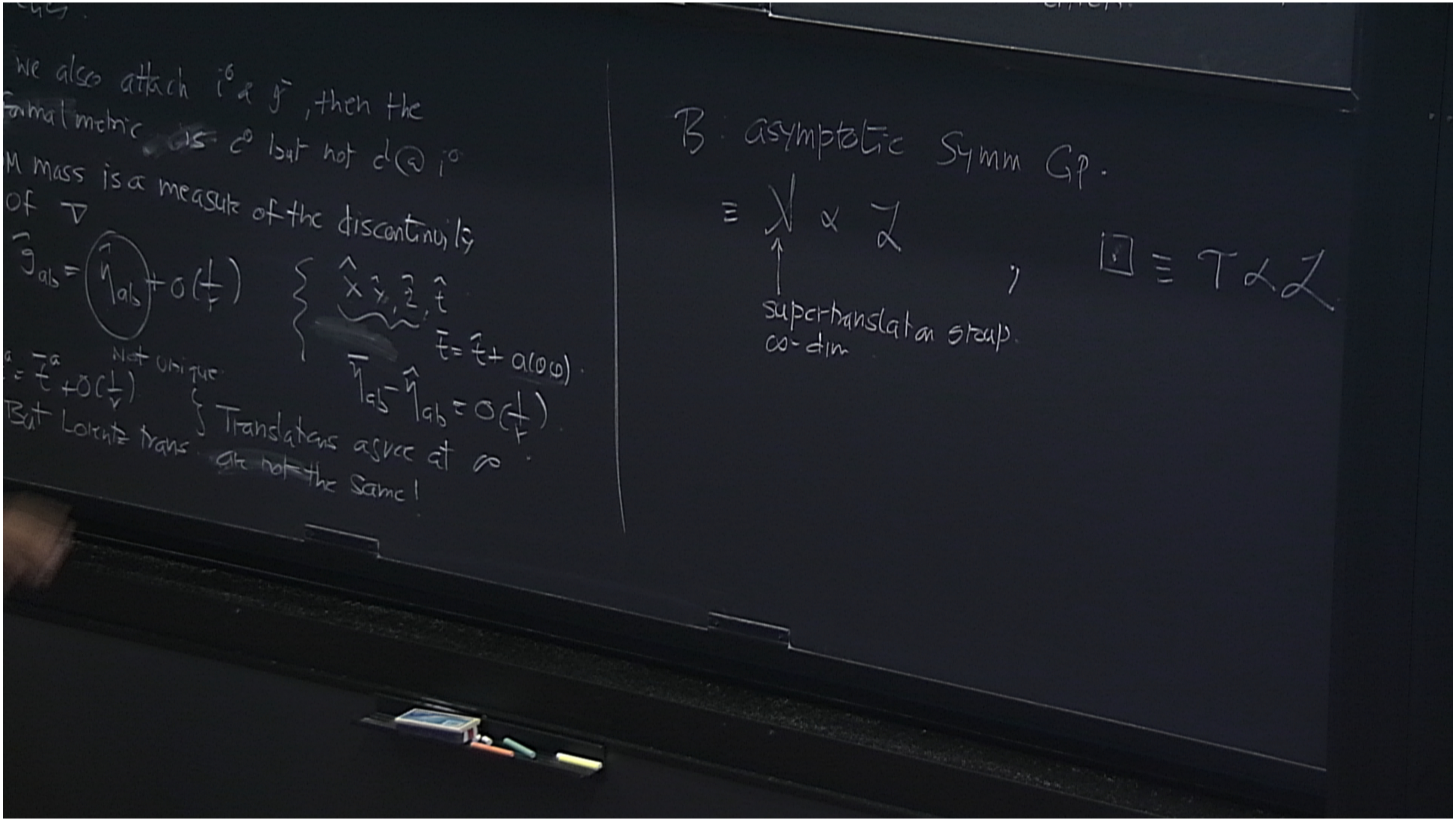
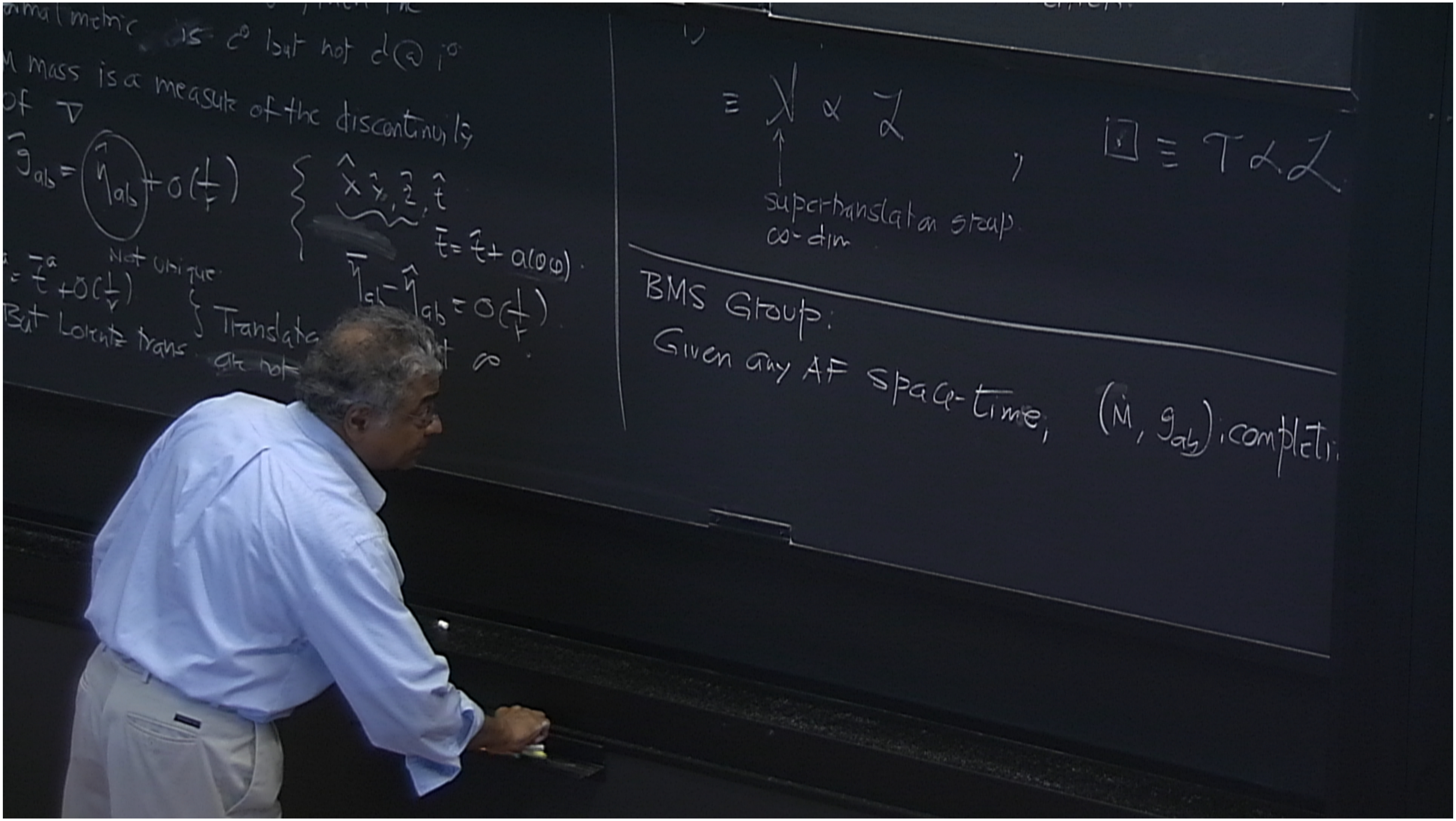
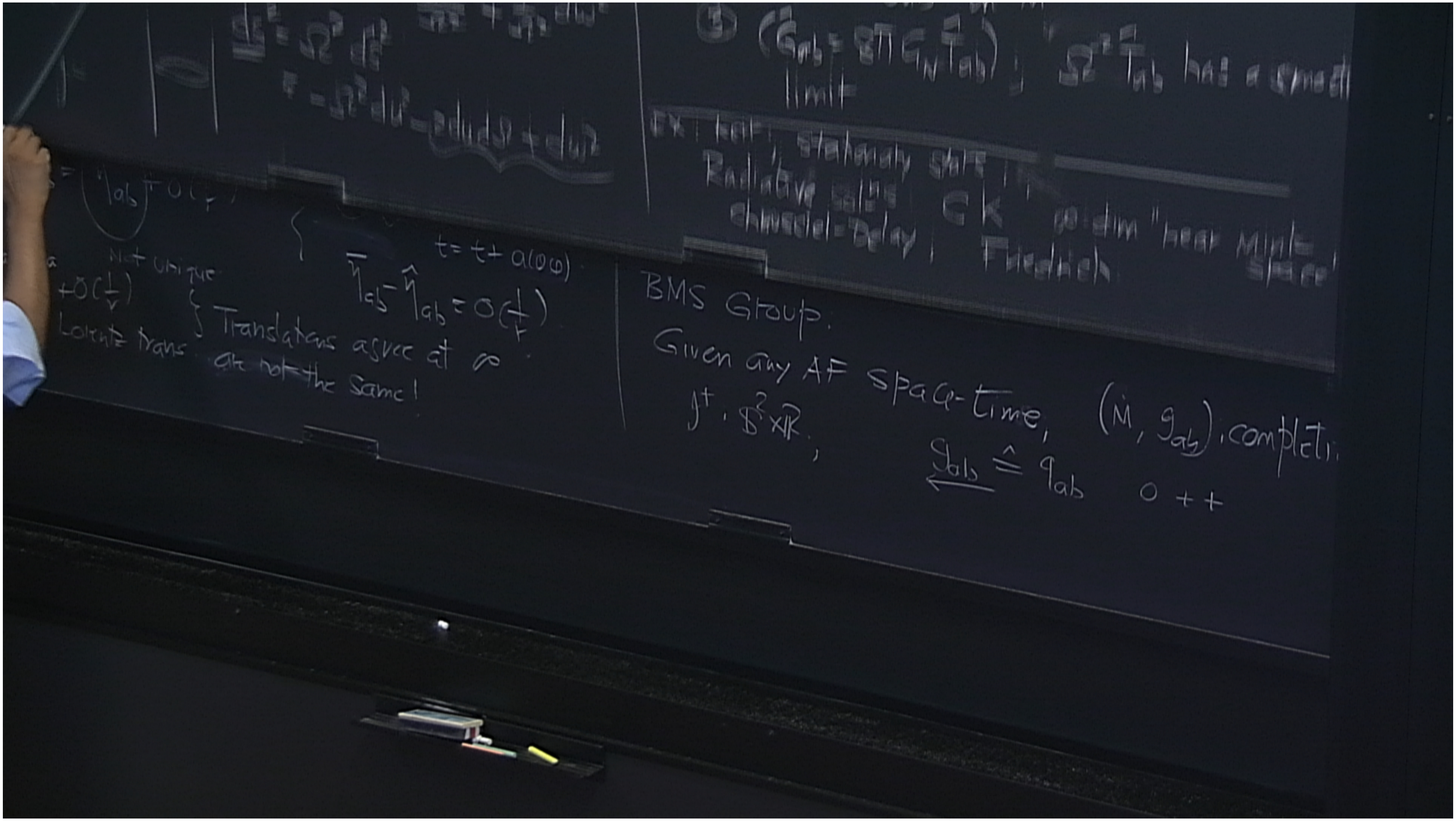
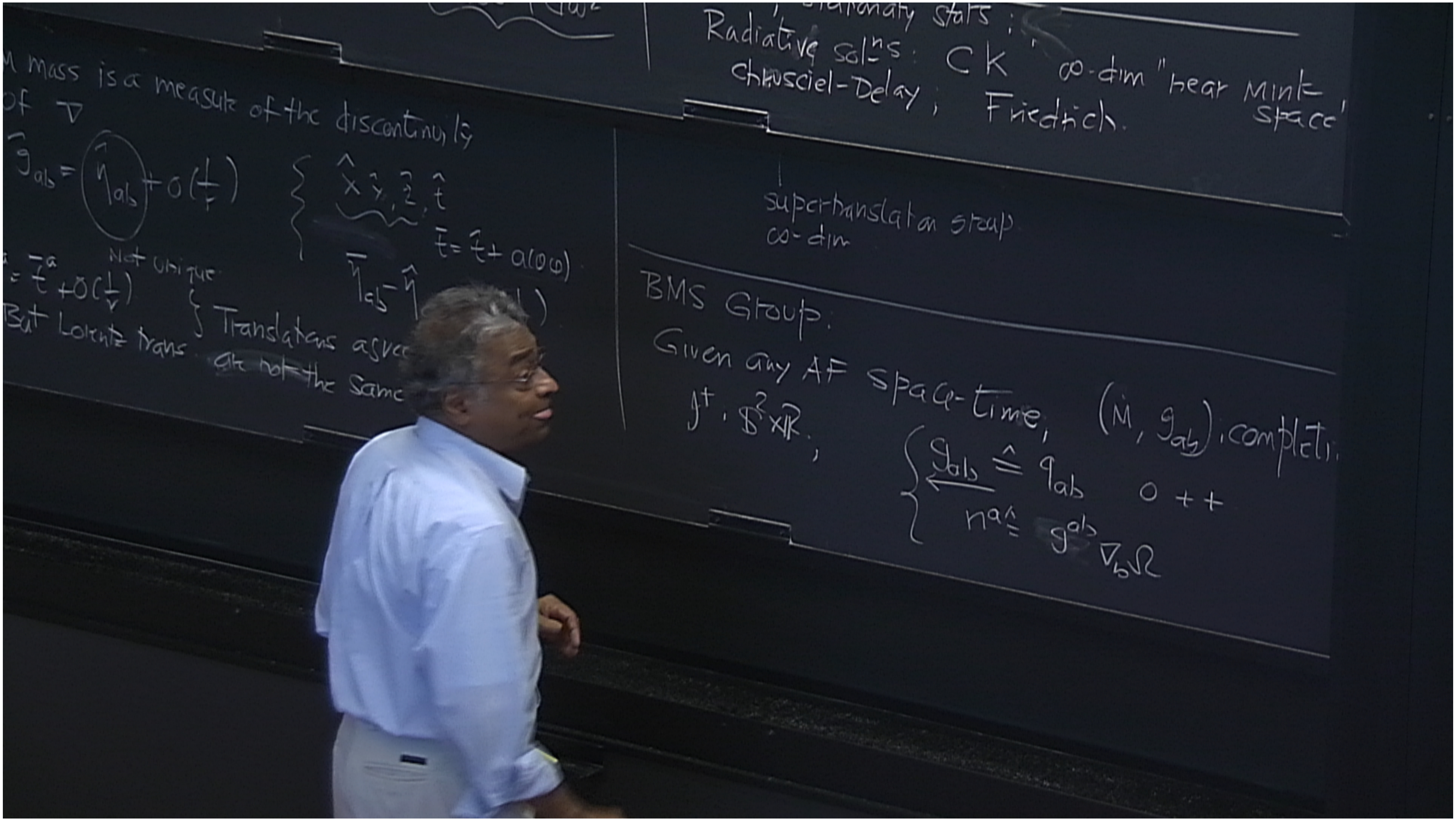
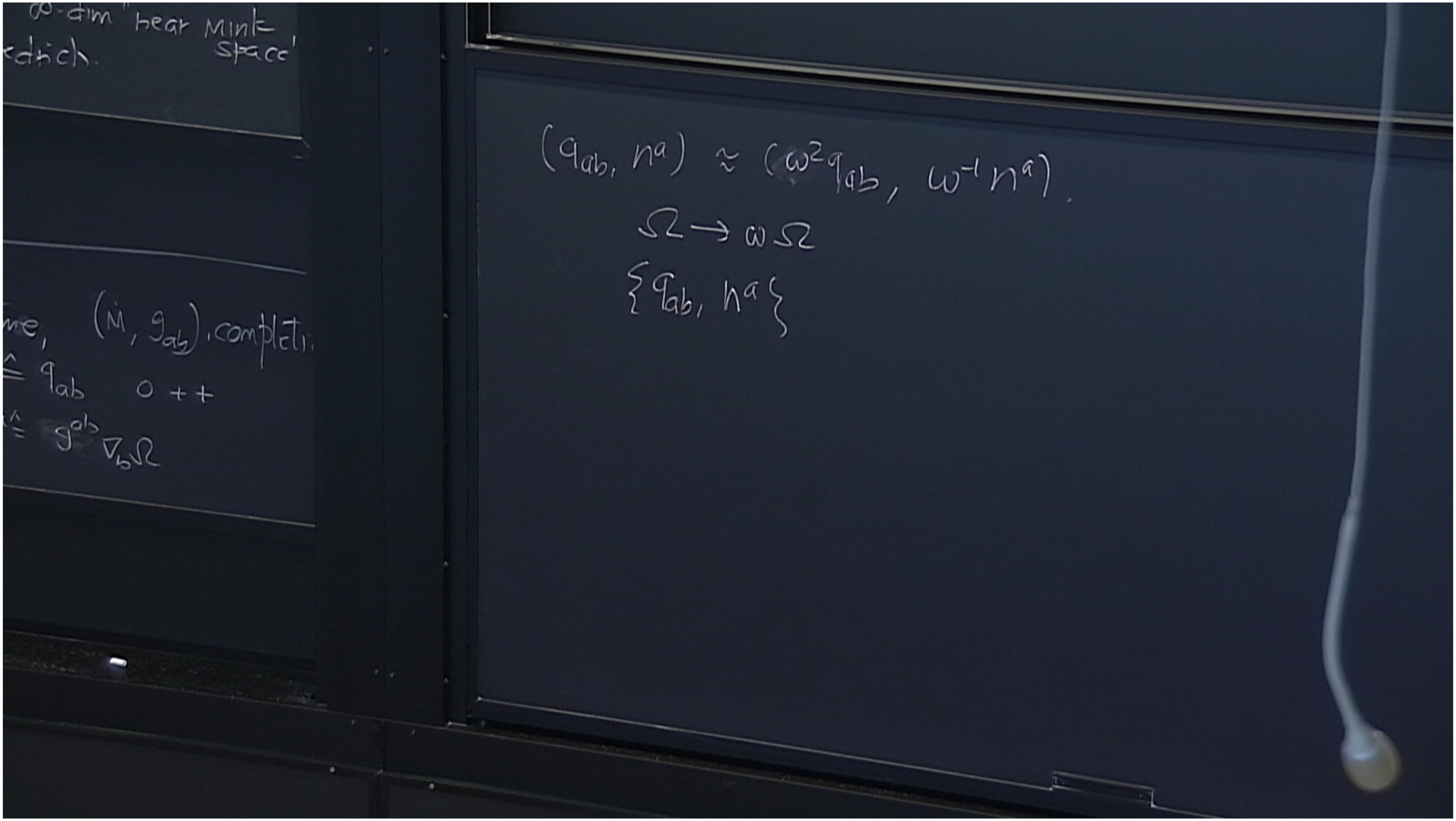
Not unique
Translations agree at ∞
trans are not the same!
B. asymptotic Symm Gp.
supertranslation group
∞-dim

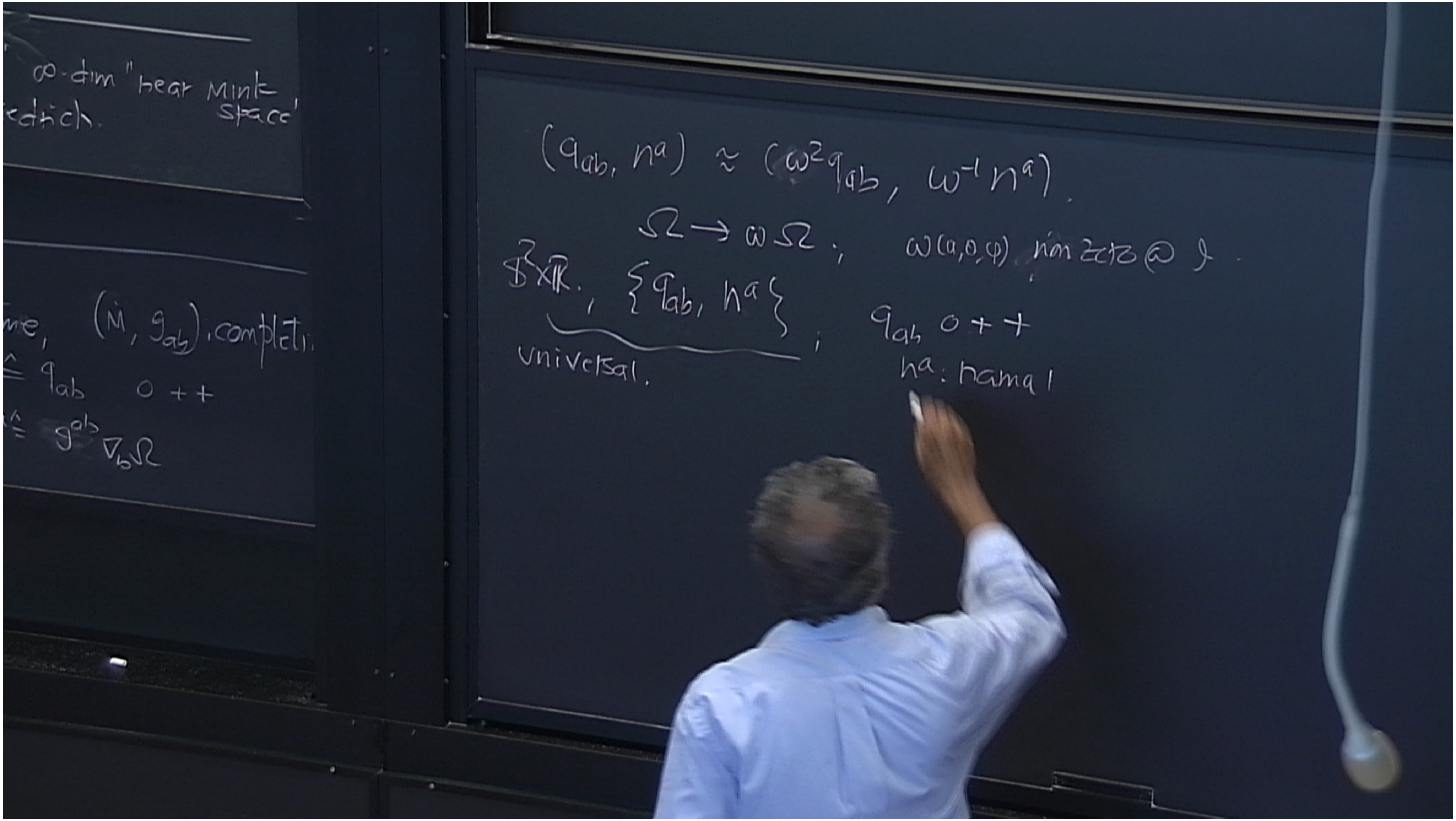

we also attach i^0 & J^- , then the
metric is C^0 but not C^1 @ i^0
mass is a measure of the discontinuity
Not unique
But Lorentz trans are not the same!
Translations agree at ∞
B: asymptotic Symm Gp.
super-translation group
∞-dim

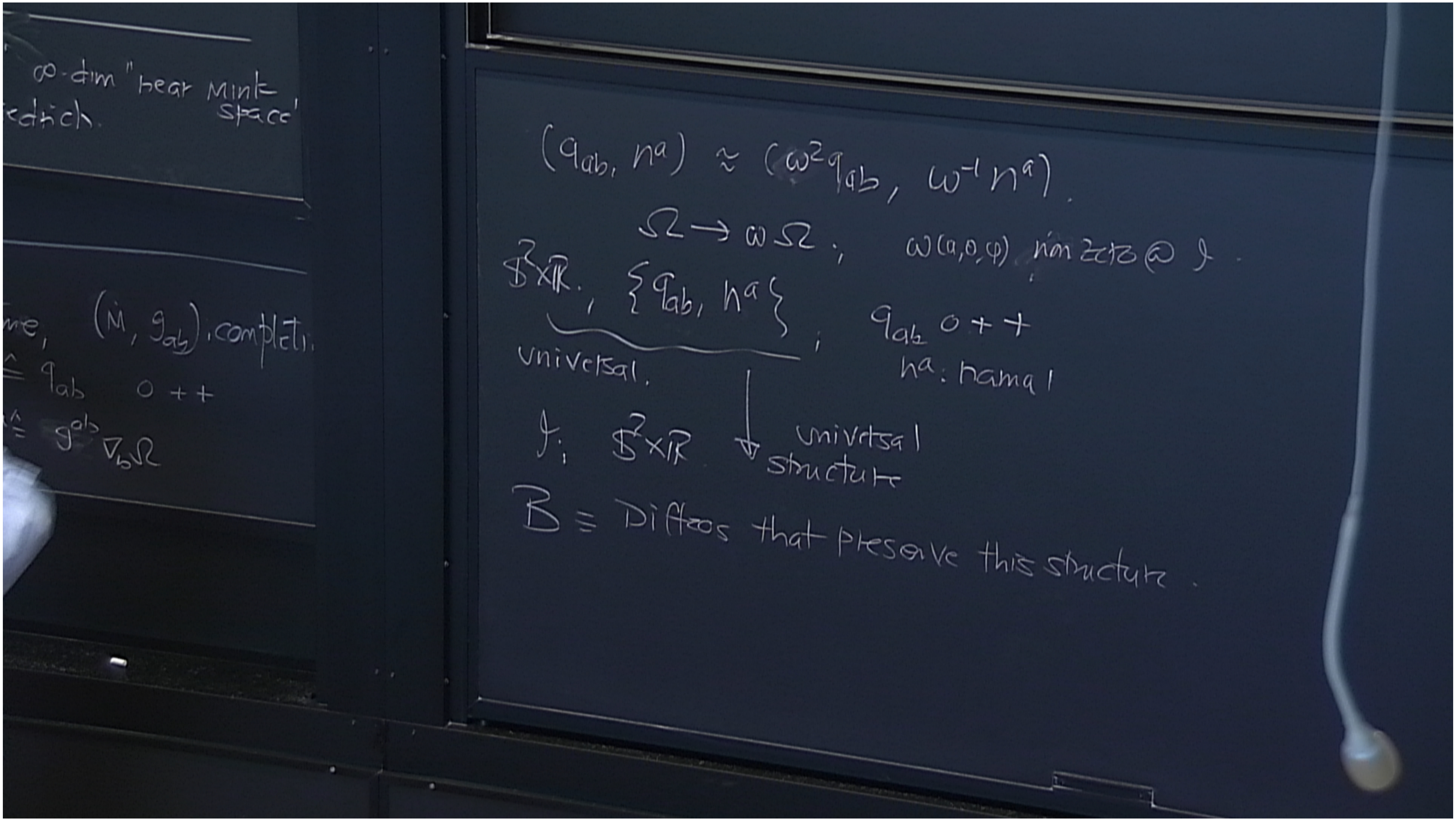

BMS Group:
Given any AF space-time,
supertranslation group
Not unique
But Lorentz trans
Translations

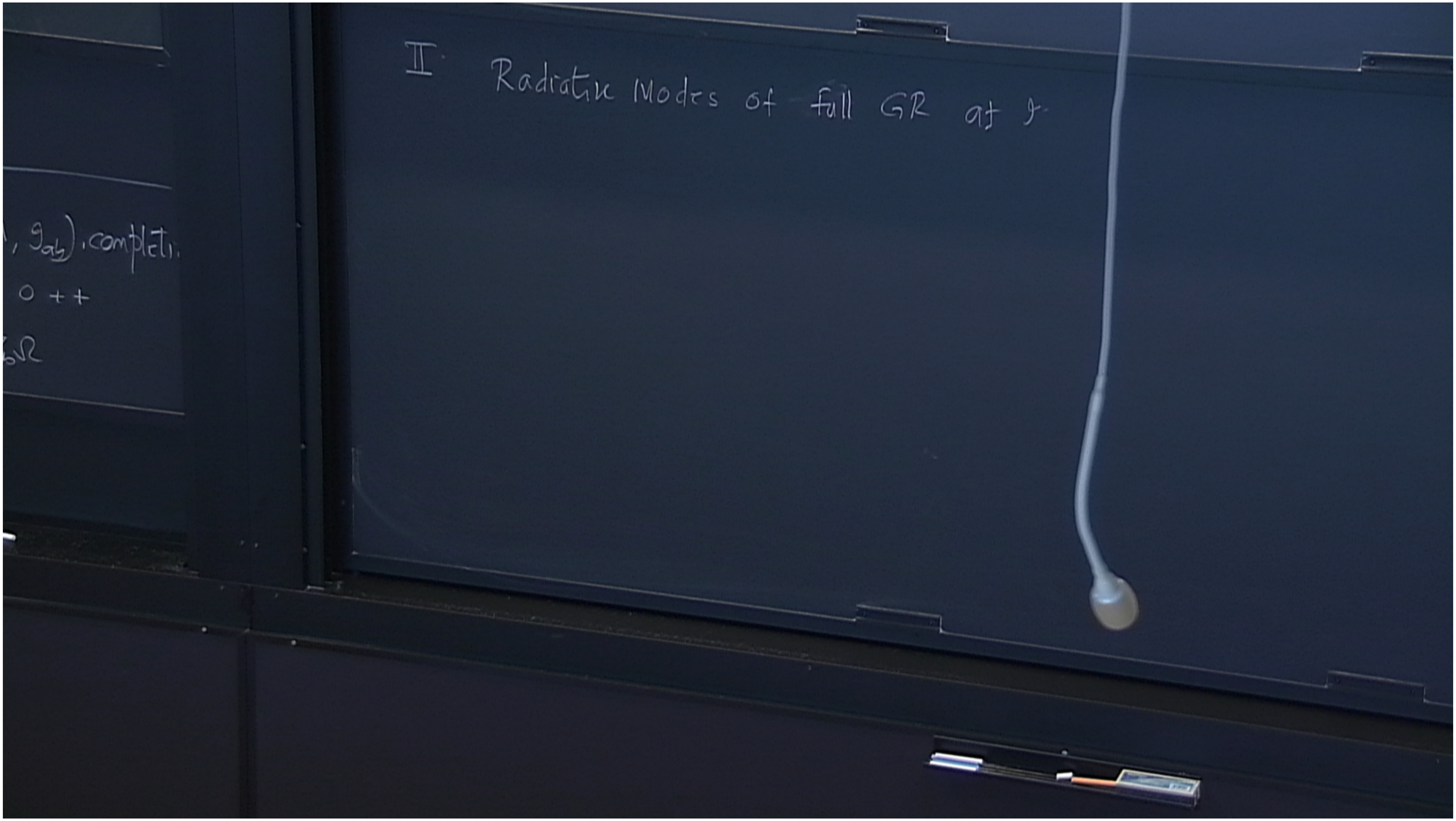

BMS Group.
Given any AF space-time,
Translations agree at ∞
are not the same!
Not unique
Lorentz trans
Radiative sol'ns
limit

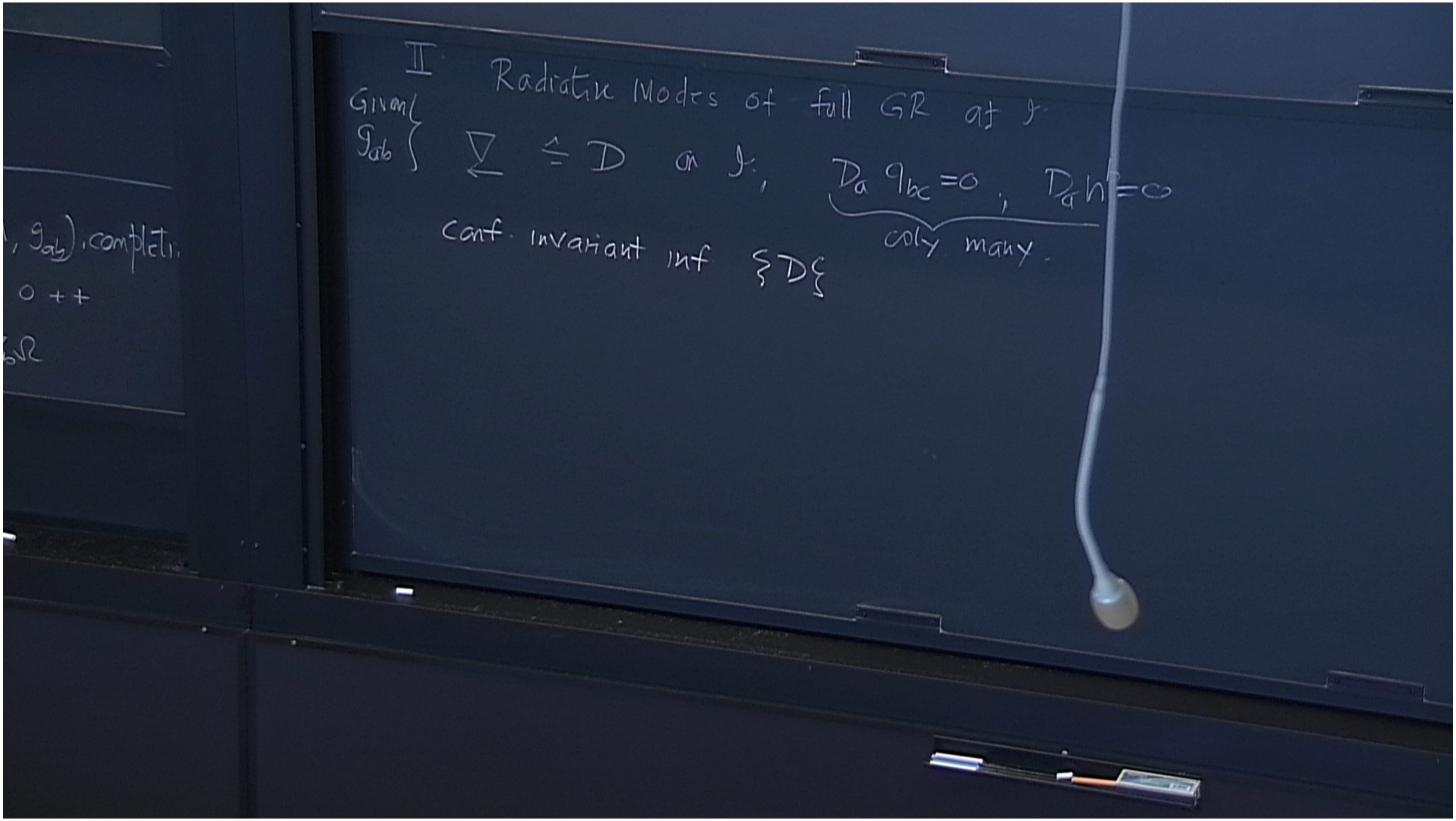

Radiative solns: CK ∞-dim "near Mink space"
Chrusciel-Delay; Friedrich.
mass is a measure of the discontinuity of ∇
Not unique
But Lorentz trans are not the same
Translations agree
supertranslation group
∞-dim
BMS Group:
Given any AF space-time, (M, gab), completion
ℐ+ : S2×R;

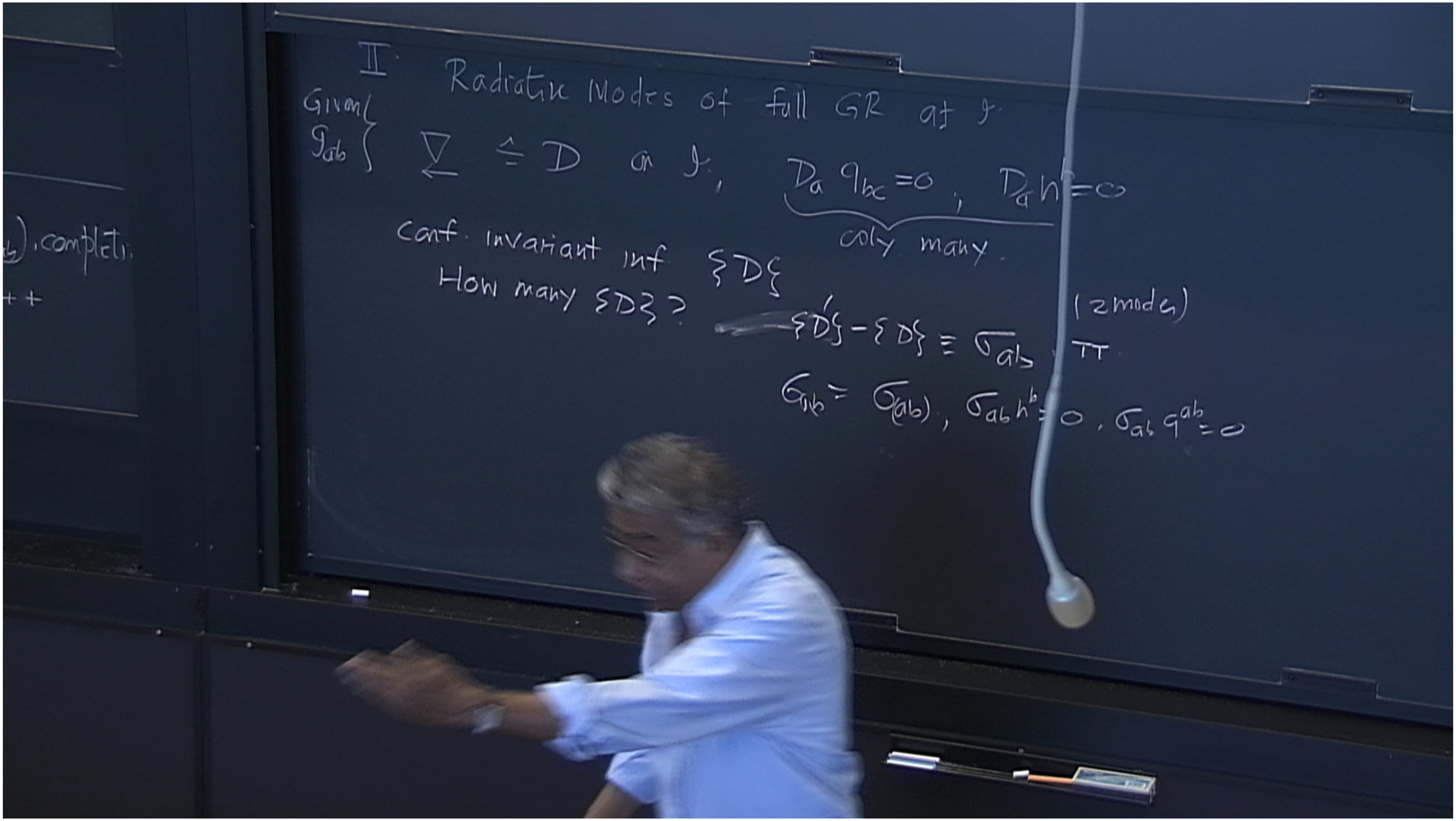

∞-dim "near Mink
space'
$(\dot{M}, g_{ab})$, complet
q_{ab} 0 + +
$g^{ab} \nabla_b \Omega$
$(q_{ab}, n^a) \approx (\omega^2 q_{ab}, \omega^{-1} n^a)$.
$\Omega \to \omega \Omega$
$\{q_{ab}, n^a\}$

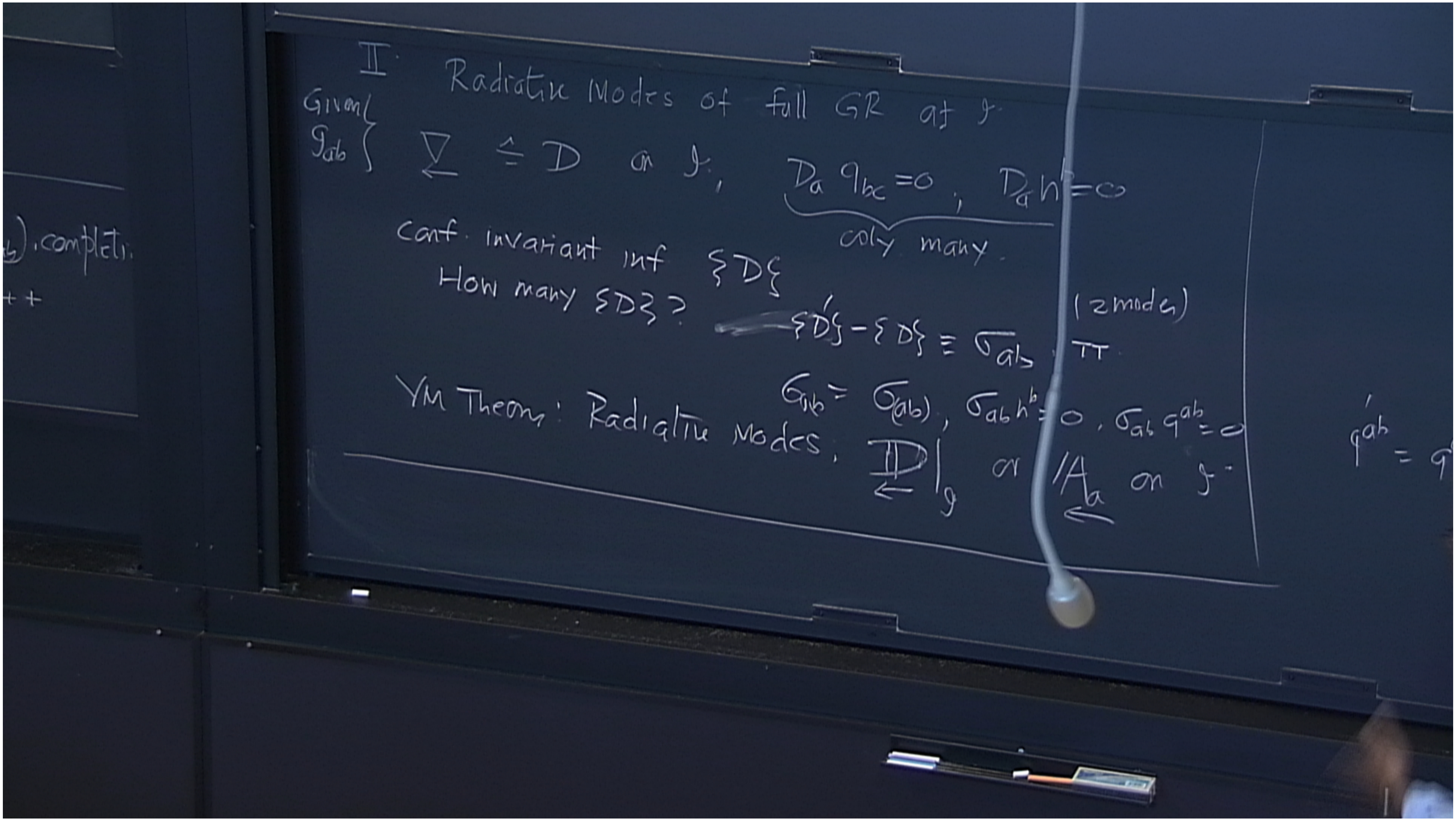

∞-dim "near Mink space'
(q_ab, n^a) ≈ (ω^2 q_ab, ω^-1 n^a).
Ω → ω Ω ; ω(θ,φ) non zero @ ℐ.
S^2×R , {q_ab, n^a} ; q_ab 0++
universal.
n^a : null
(M, g_ab) complet
q_ab 0++
g^ab ∇_b Ω

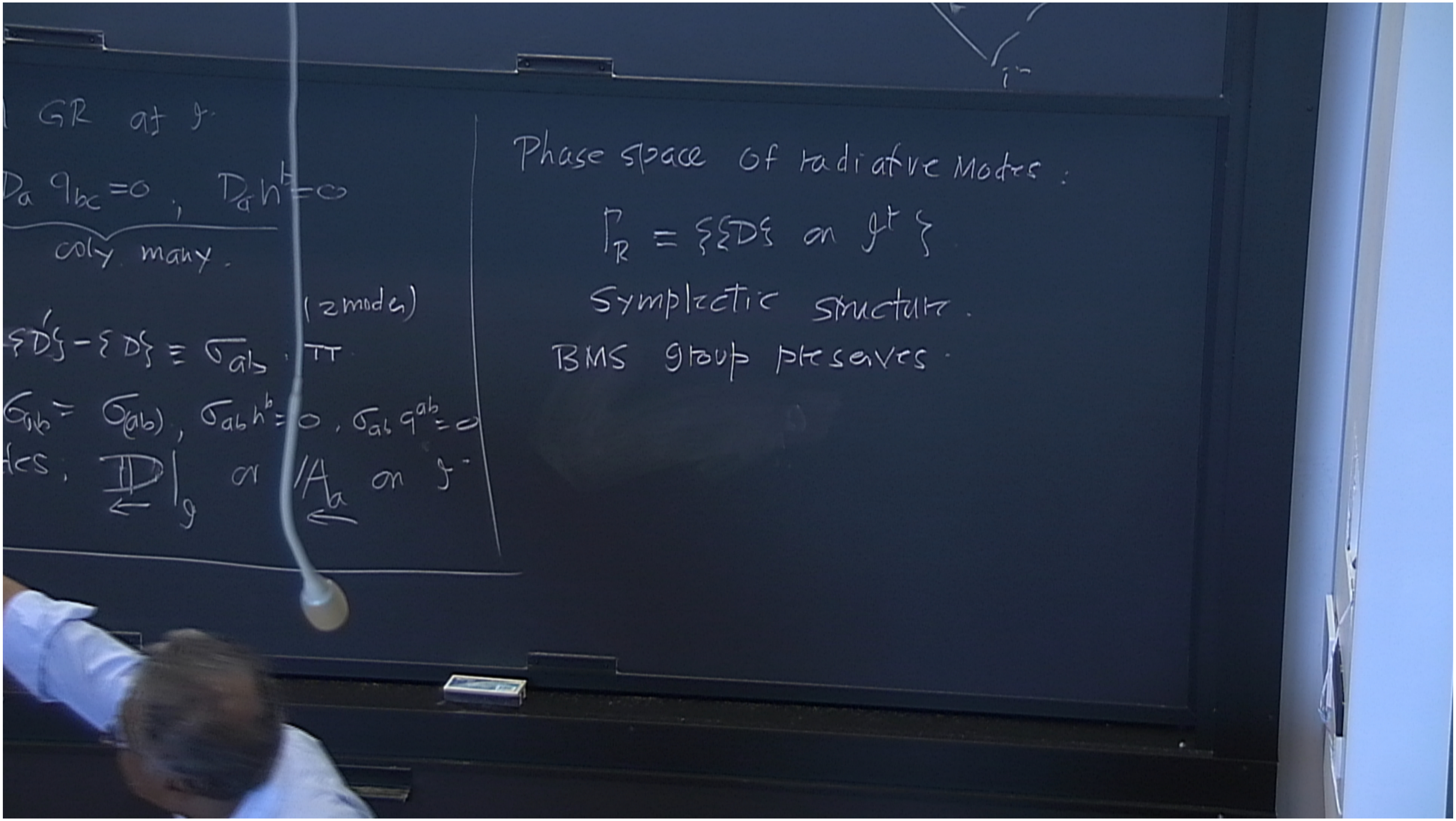

∞-dim "near Mink space"
(M, g_ab), complet
q_ab 0 + +
g^ab ∇_b Ω
(q_ab, n^a) ≈ (ω² q_ab, ω⁻¹ n^a).
Ω → ω Ω ; ω(u,θ,φ) non zero @ ℐ.
S² × ℝ, {q_ab, n^a} ; q_ab 0 + +
universal. n^a : normal
ℐ ; S² × ℝ universal structure
B ≡ Diffeos that preserve this structure.

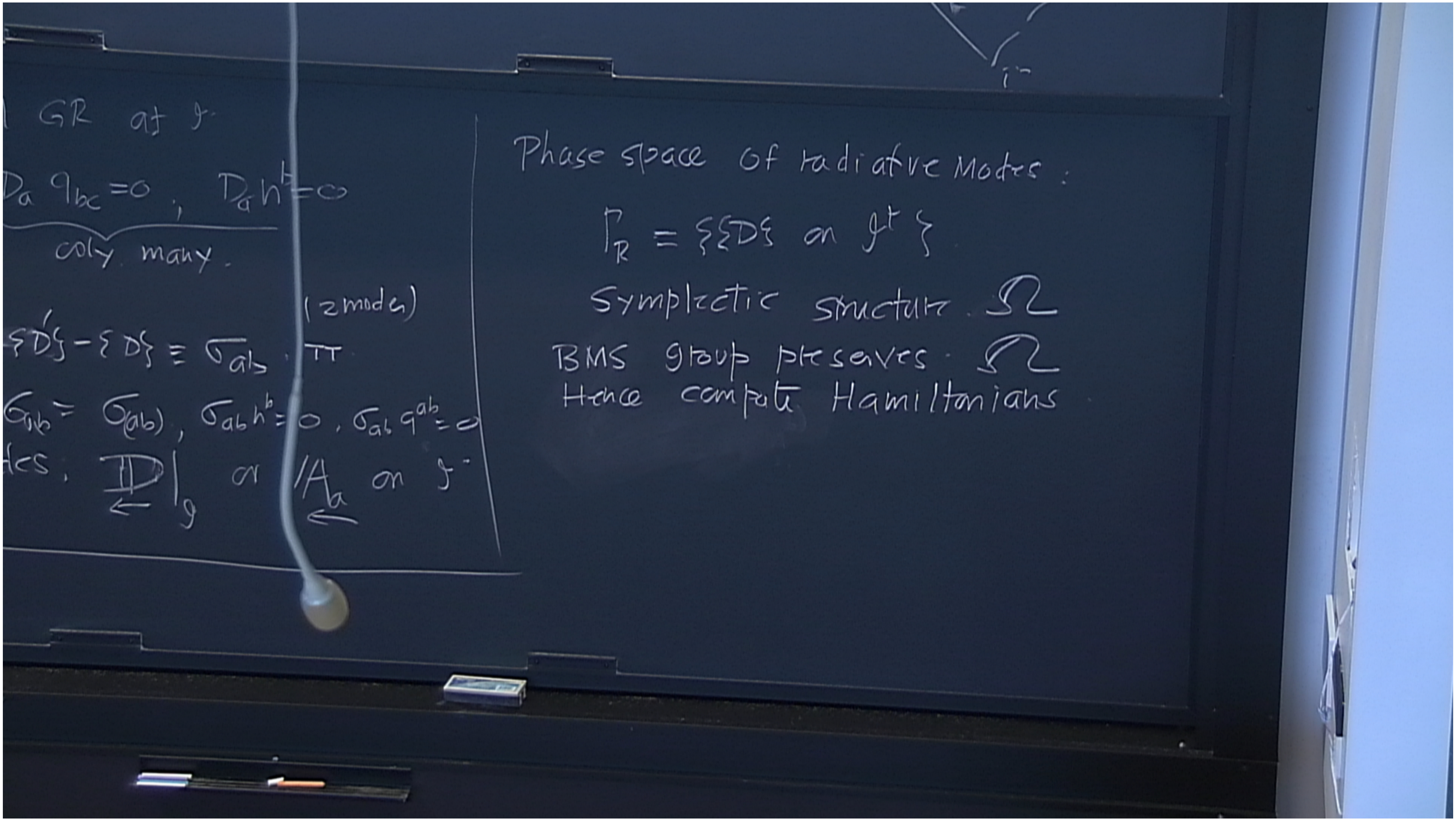

II. Radiative Modes of full GR at ℐ.

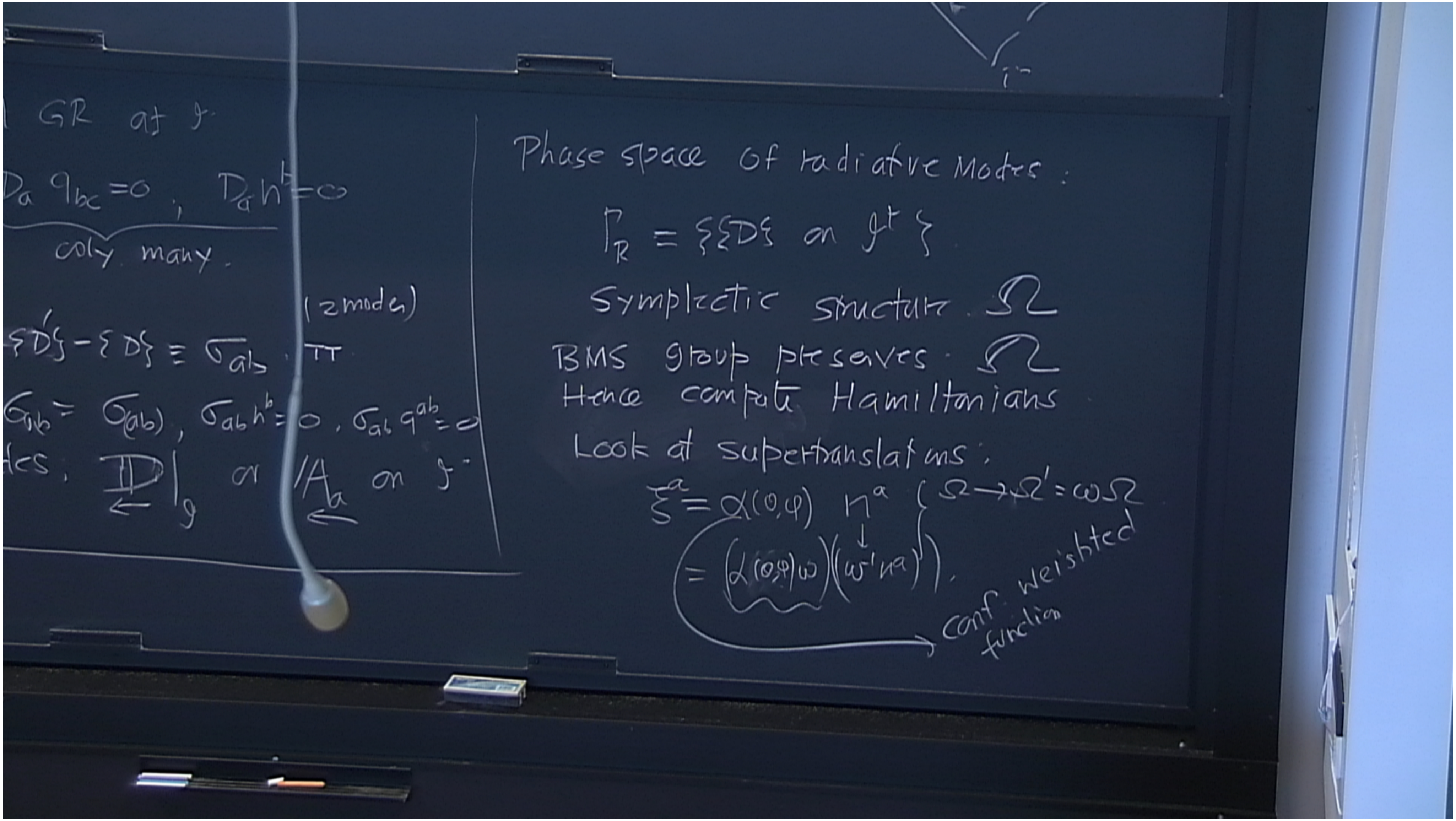

II. Radiative Modes of full GR at ℐ.
Given g_ab
∇ ≙ D on ℐ, D_a q_bc = 0, D_a n^b = 0
cntly many.
conf. invariant inf {D}

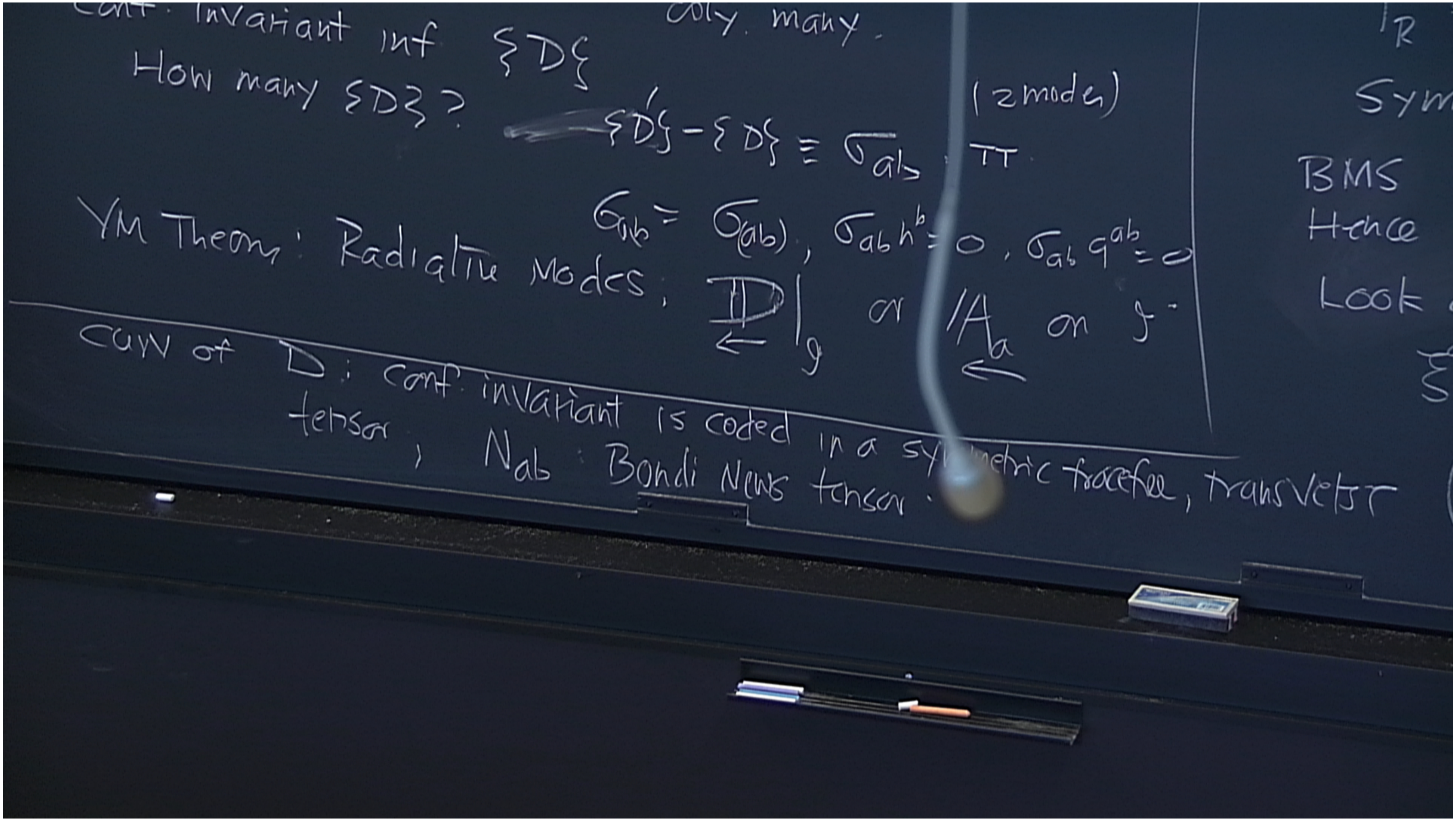

II. Radiative Modes of full GR at ℐ.
Given g_{ab}
$\nabla \hat{=} D$ on ℐ, $D_a q_{bc} = 0$, $D_a n^b = 0$
only many.
conf. invariant inf {D}
How many {D}?
(2 modes)
{D'} − {D} ≡ σ_{ab} TT
$\sigma_{ab} = \sigma_{(ab)}$, $\sigma_{ab} n^b = 0$, $\sigma_{ab} q^{ab} = 0$

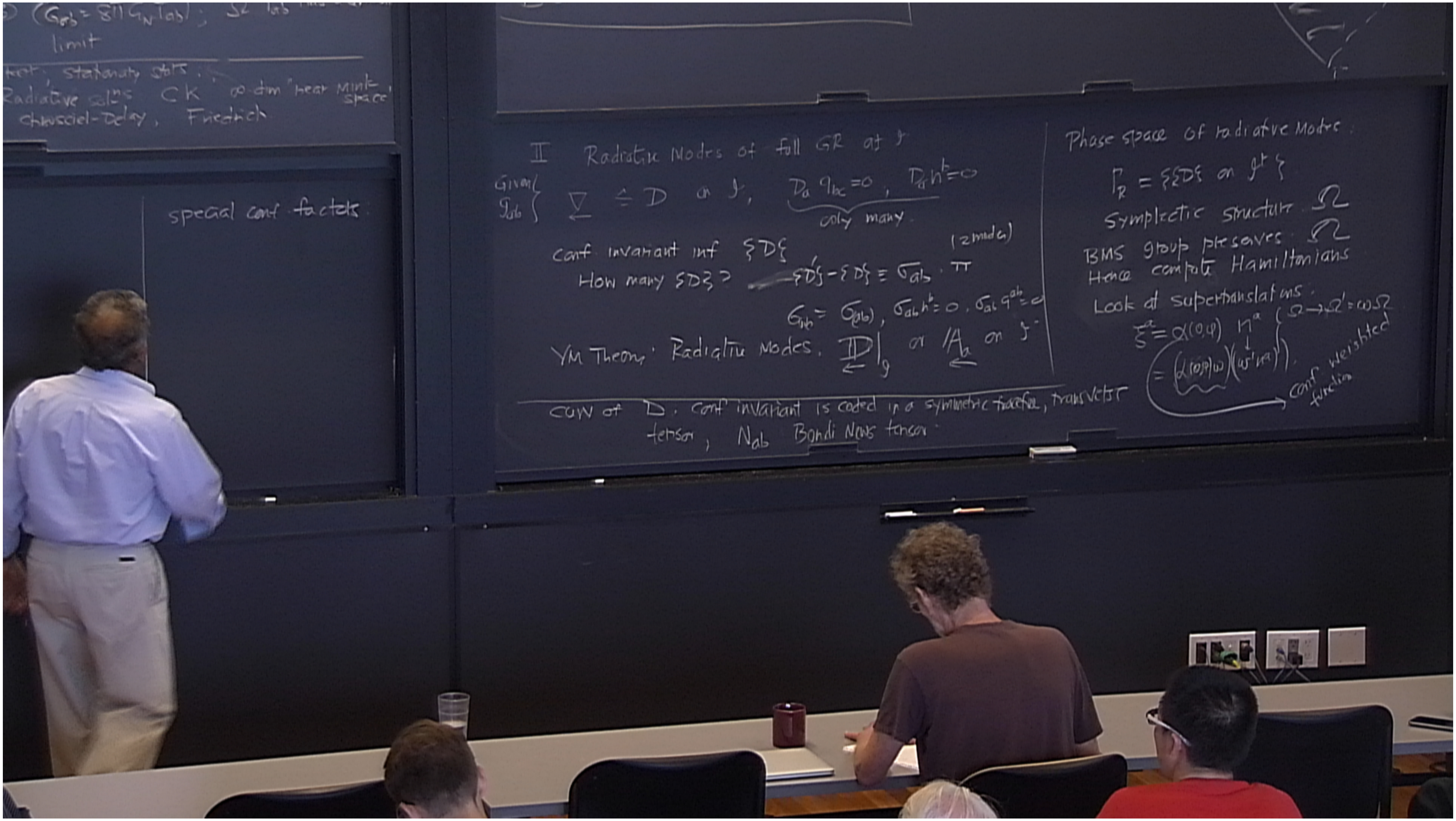

II. Radiative Modes of full GR at ℐ.
Given g_ab
∇ ≙ D on ℐ, D_a q_bc = 0, D_a n^b = 0
∞ly many
conf. invariant inf {D}
How many {D}?
{D′} − {D} ≡ σ_ab TT
(2 modes)
σ_ab = σ_(ab), σ_ab n^b = 0, σ_ab q^ab = 0
YM Theory: Radiative Modes. D|_ℐ or A_a on ℐ.

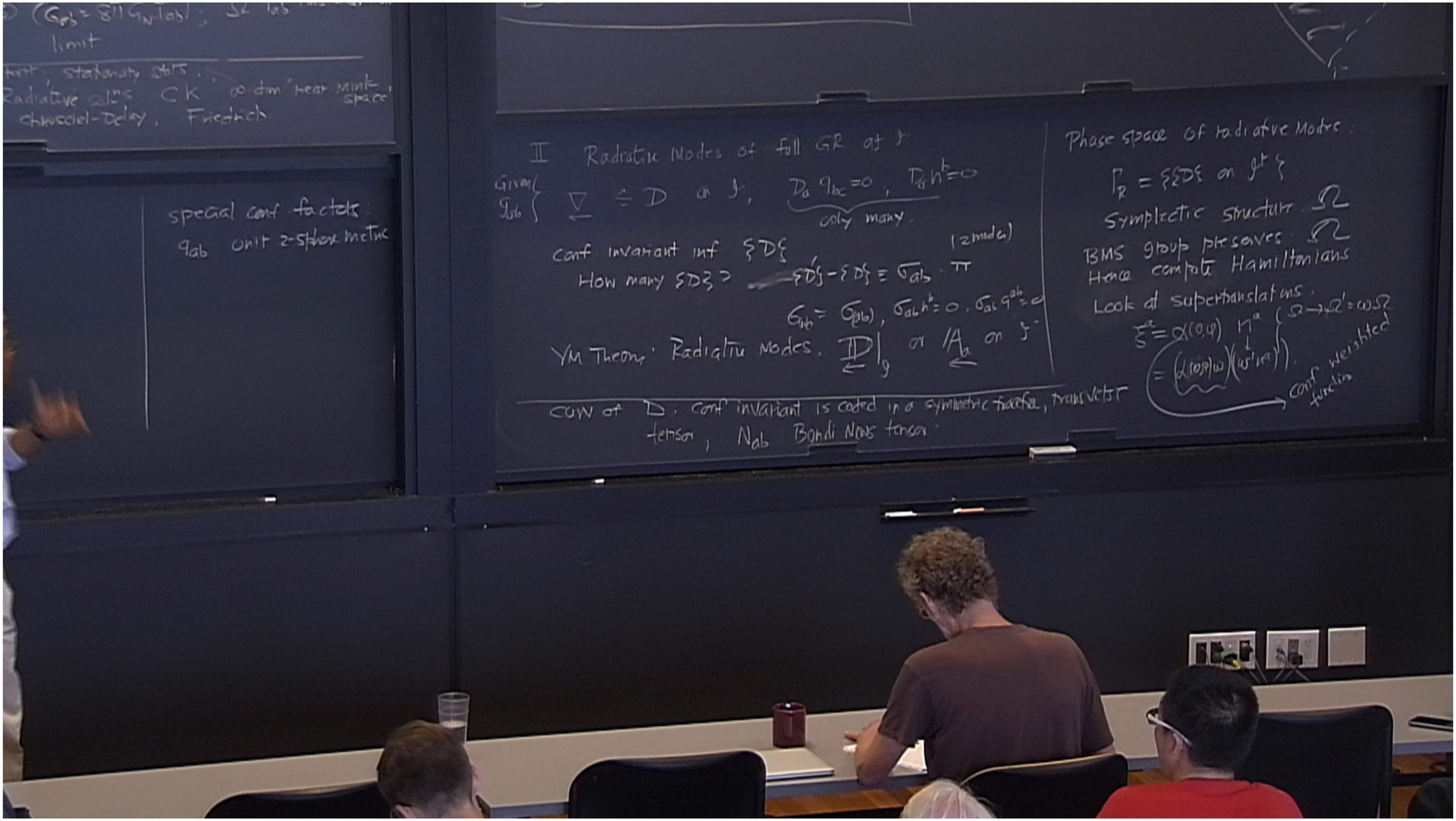

GR at
Phase space of radiative modes :
Symplectic structure.
BMS group preserves.

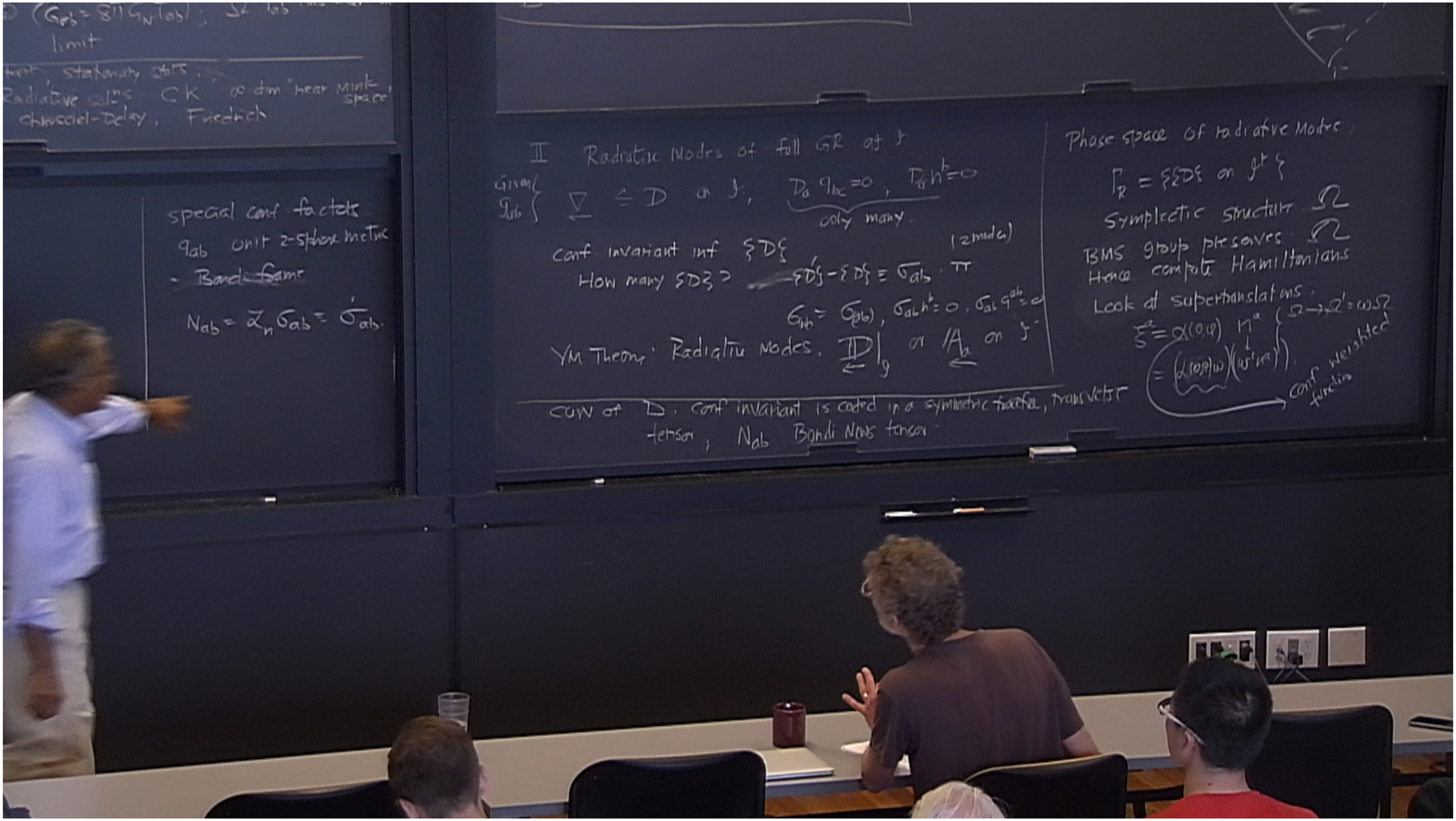

GR at $\mathcal{I}$
$D_a q_{bc} = 0$, $D_a n^b = 0$
∞ly many
(2 modes)
$\{D'\} - \{D\} \equiv \sigma_{ab}$ TT
$\sigma_{ab} = \sigma_{(ab)}$, $\sigma_{ab} n^b = 0$, $\sigma_{ab} q^{ab} = 0$
$\mathbb{D}|_{\mathcal{I}}$ or A_a on $\mathcal{I}$
Phase space of radiative Modes :
$\Gamma_R = \{\{D\}$ on $\mathcal{I}^+\}$
Symplectic structure Ω
BMS group preserves Ω
Hence compute Hamiltonians

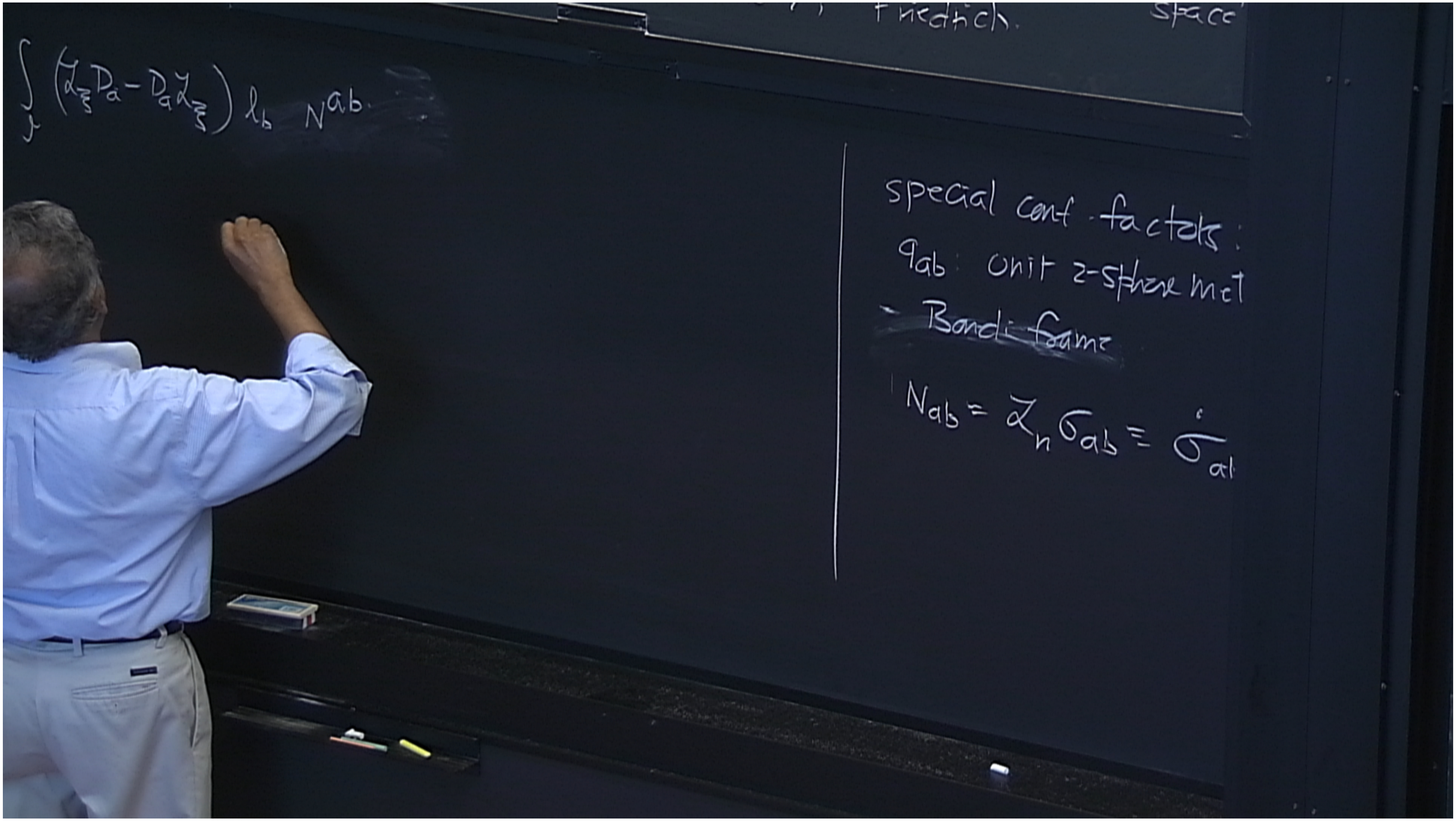

GR at
$D_a q_{bc} = 0$, $D_a n^b = 0$
coly many.
(2 modes)
$\{D'\} - \{D\} \equiv \sigma_{ab}$ TT
$\sigma_{ab} = \sigma_{(ab)}$, $\sigma_{ab} n^b = 0$, $\sigma_{ab} q^{ab} = 0$
Phase space of radiative Modes:
$\Gamma_R = \{\{D\}$ on $\mathcal{I}^+\}$
Symplectic structure Ω
BMS group preserves Ω
Hence compute Hamiltonians
Look at supertranslations:
$\xi^a = \alpha(\theta,\varphi)\, n^a$
$\Omega \to \Omega' = \omega\Omega$
$= (\alpha(\theta,\varphi)\omega)(\omega^{-1} n^a)$
conf. weighted function

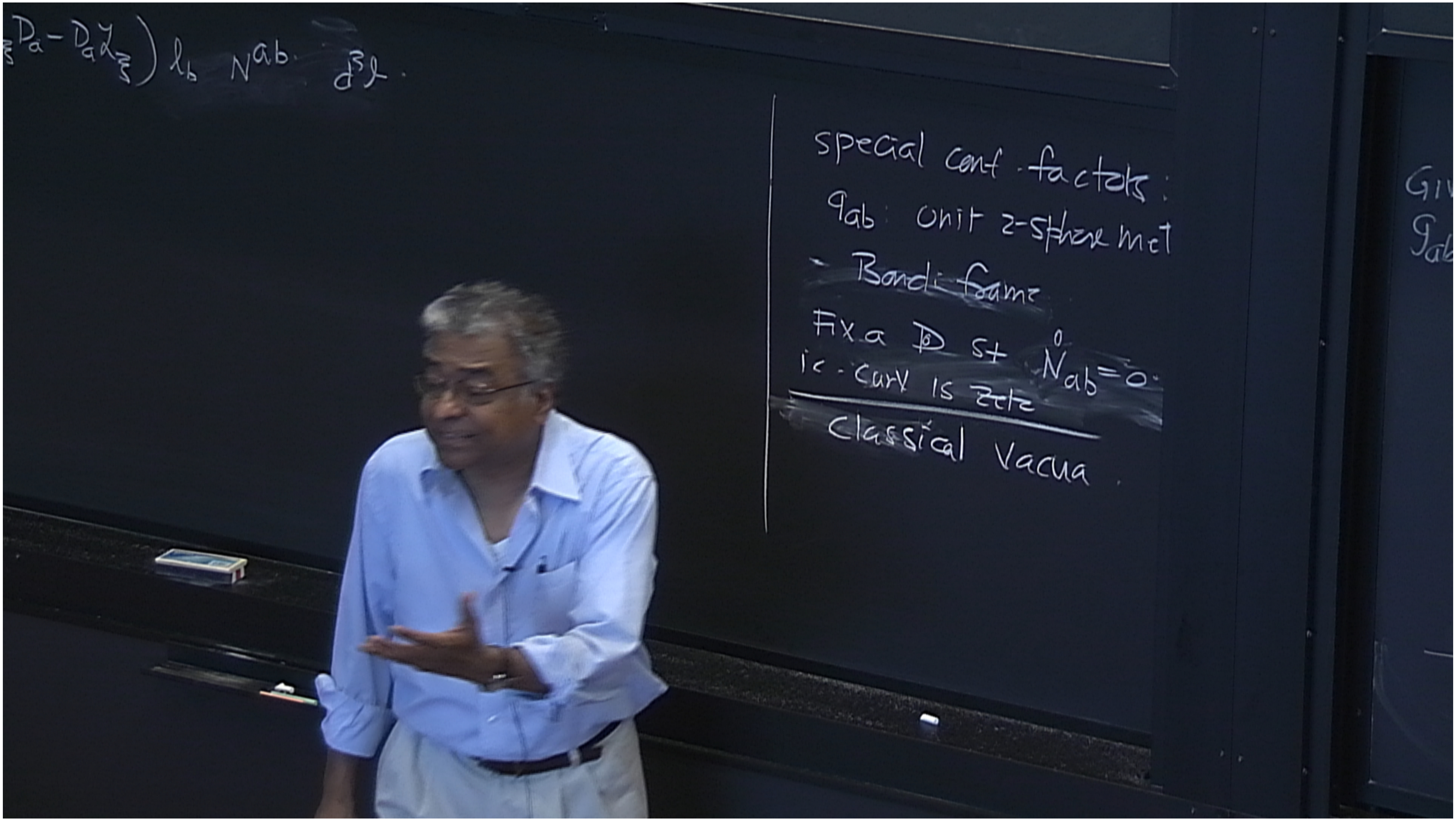

conf. invariant inf {D}
only many.
How many {D}?
(2 modes)
{D'} − {D} ≡ σ_ab TT
σ_ab = σ_(ab), σ_ab n^b = 0, σ_ab q^ab = 0
YM Theory: Radiative Modes: D| or A_a on
Curv of D: conf. invariant is coded in a symmetric tracefree, transverse
tensor, N_ab: Bondi News tensor.
Sym
BMS
Hence
Look

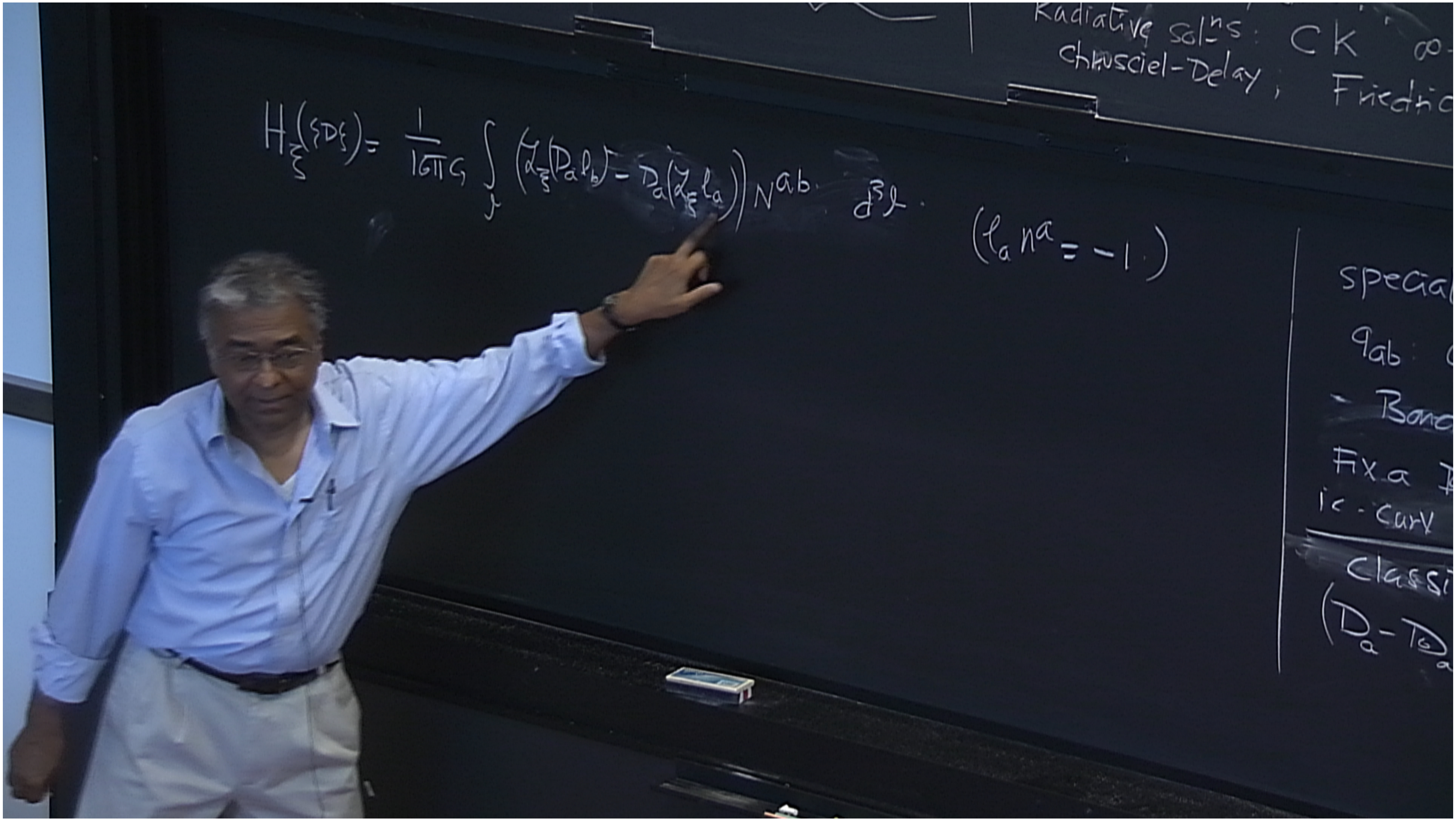

limit
stationary sol^ns
Radiative sol^ns CK ∞-dim "near Mink space"
Chrusciel-Delay, Friedrich
special conf factors
II Radiative Modes of full GR at ℐ
Given q_{ab}
∇ ≙ D on ℐ, $D_a q_{bc} = 0$, $D_a n^b = 0$
only many
conf invariant inf {D}
How many {D}?
(2 modes)
$G_{ab} = G_{(ab)}$, $\sigma_{ab} n^b = 0$, $\sigma_{ab} q^{ab} = 0$
YM Theory: Radiative Modes. D| or A on ℐ
Curv of D. conf invariant is coded in a symmetric tracefree, transverse tensor, N_{ab} Bondi News tensor
Phase space of radiative Modes:
Γ_R = {{D} on ℐ⁺}
Symplectic structure Ω
BMS group preserves Ω
Hence compute Hamiltonians
Look at supertranslations
conf weighted function

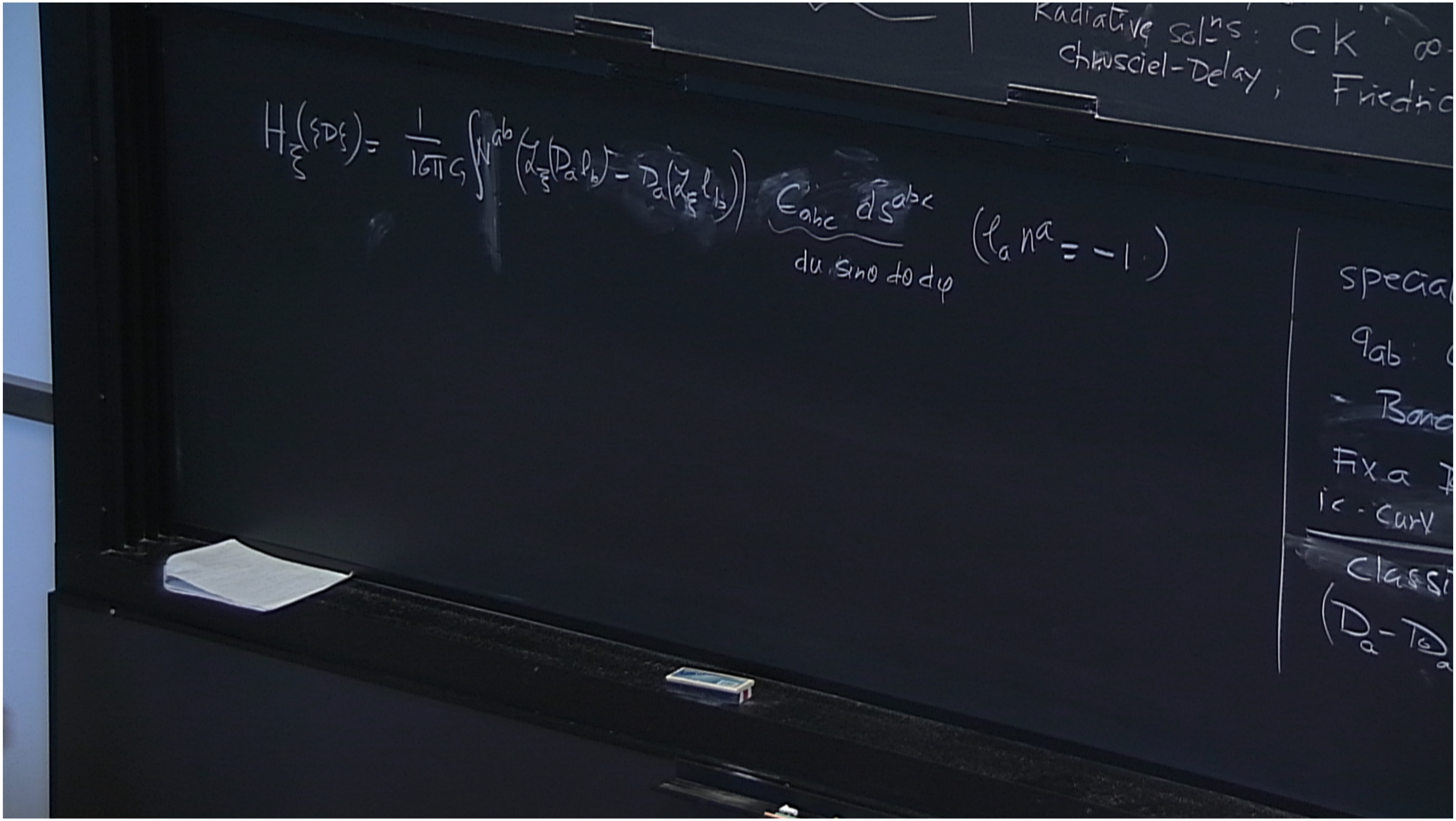

limit
stationary sol^ns
Radiative sol^ns CK ∞-dim "near Mink space"
Chruściel-Delay, Friedrich
special conf factors:
q_ab unit 2-sphere metric
II Radiative Modes of full GR at ℐ
Given q_ab: ∇ ≙ D on ℐ, D_a q_bc = 0, D_a n^b = 0
only many
conf invariant inf {D}
How many {D}? {D'} − {D} ≡ σ_ab
(2 modes)
YM Theory: Radiative Modes. D|_ℐ or A_a on ℐ
curv of D. conf invariant is coded in a symmetric tracefree, transverse tensor, N_ab Bondi News tensor
Phase space of radiative Modes:
Γ_R = {{D} on ℐ^+}
Symplectic structure Ω
BMS group preserves Ω
Hence compute Hamiltonians
Look at supertranslations.
conf weighted function

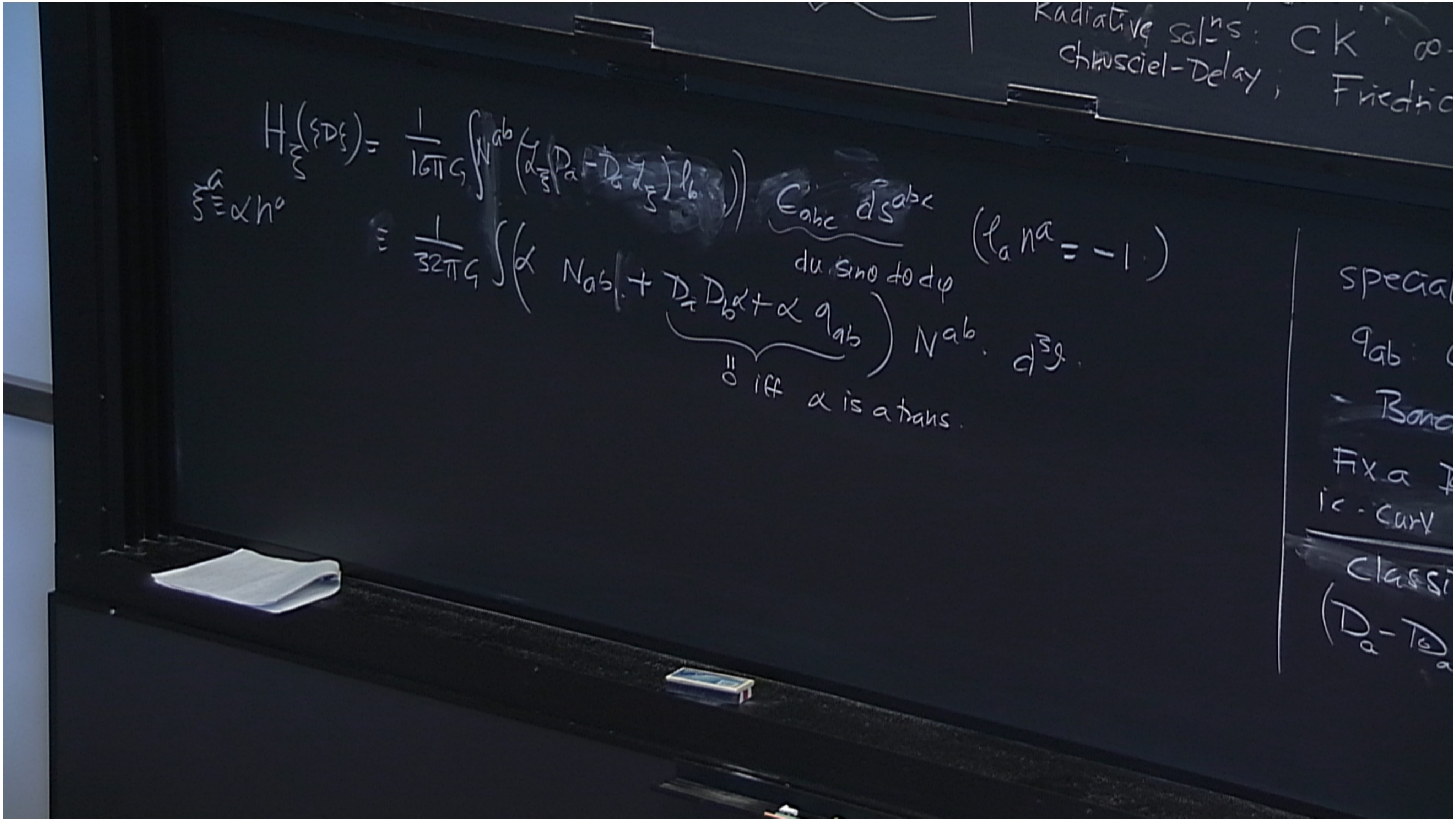

limit
stationary sol^ns
Radiative sol^ns CK ∞ dim "near Mink space"
Chrusciel-Delay, Friedrich
special conf factors
q_ab unit 2-sphere metric
- Bondi frame
N_ab = L_n σ_ab = σ̇_ab
II Radiative Modes of full GR at 𝒥
Given q_ab
∇ ≙ D on 𝒥, D_a q_bc = 0, D_a n^b = 0
∞ly many
conf invariant inf {D}
How many {D}? {D'} − {D} ≡ σ_ab
(2 modes)
σ_ab = σ_(ab), σ_ab n^b = 0, σ_ab q^ab = 0
YM Theory: Radiative Modes. 𝔻|_𝒥 or A_a on 𝒥.
CoW of D. conf invariant is coded in a symmetric tracefree, transverse tensor, N_ab Bondi News tensor
Phase space of radiative Modes:
Γ_R = {{D} on 𝒥^+}
Symplectic structure Ω
BMS group preserves Ω
Hence compute Hamiltonians
Look at supertranslations.
ξ^a = α(θ,φ) n^a (Ω → Ω' = ωΩ)
conf weighted function

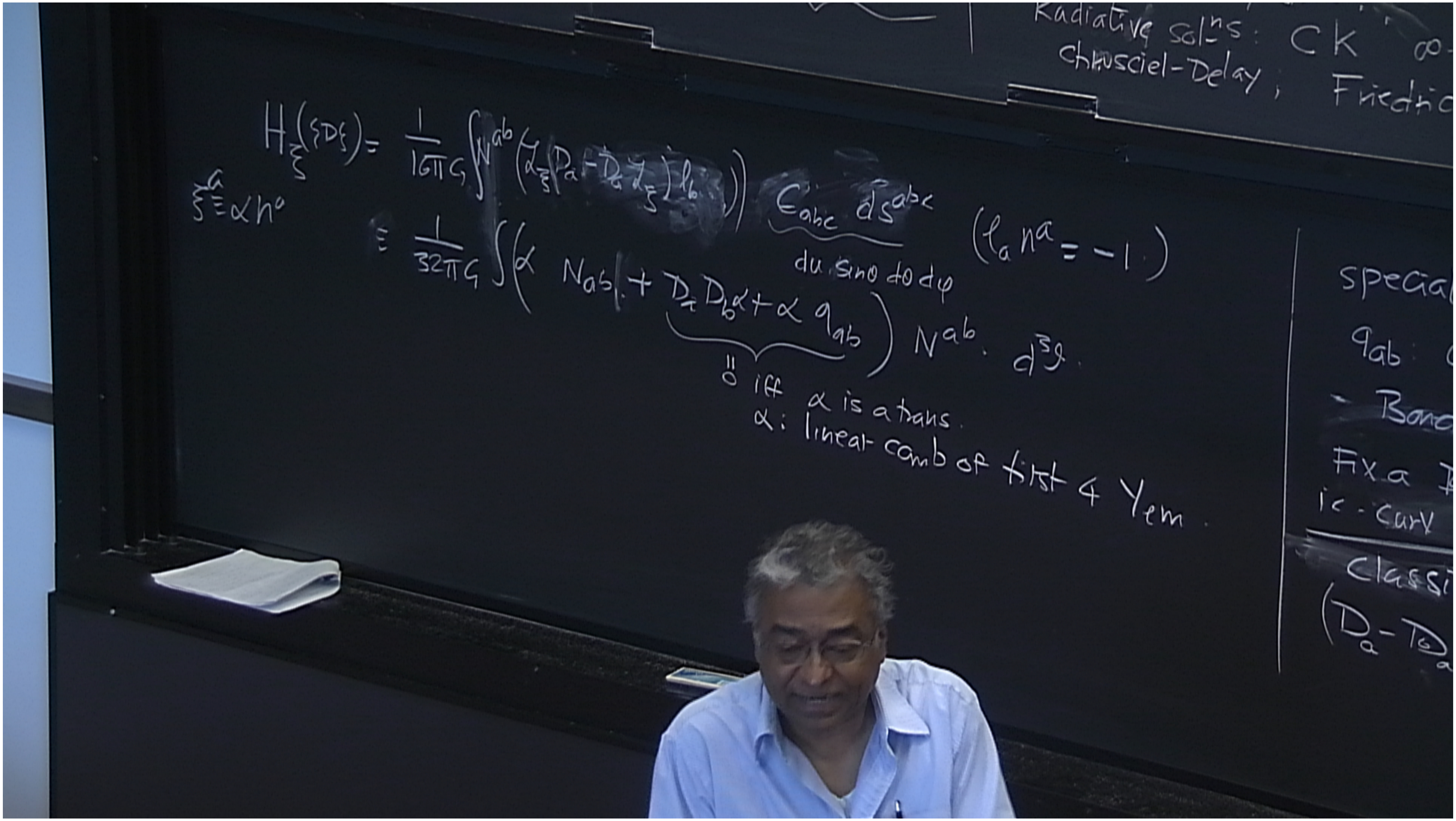

Friedrich.
space
$\int (\mathcal{L}_\xi D_a - D_a \mathcal{L}_\xi)\, l_b \; N^{ab}$
special conf. factors:
q_{ab}: unit 2-sphere met
Bondi frame
$N_{ab} = \mathcal{L}_n \sigma_{ab} = \dot{\sigma}_{ab}$

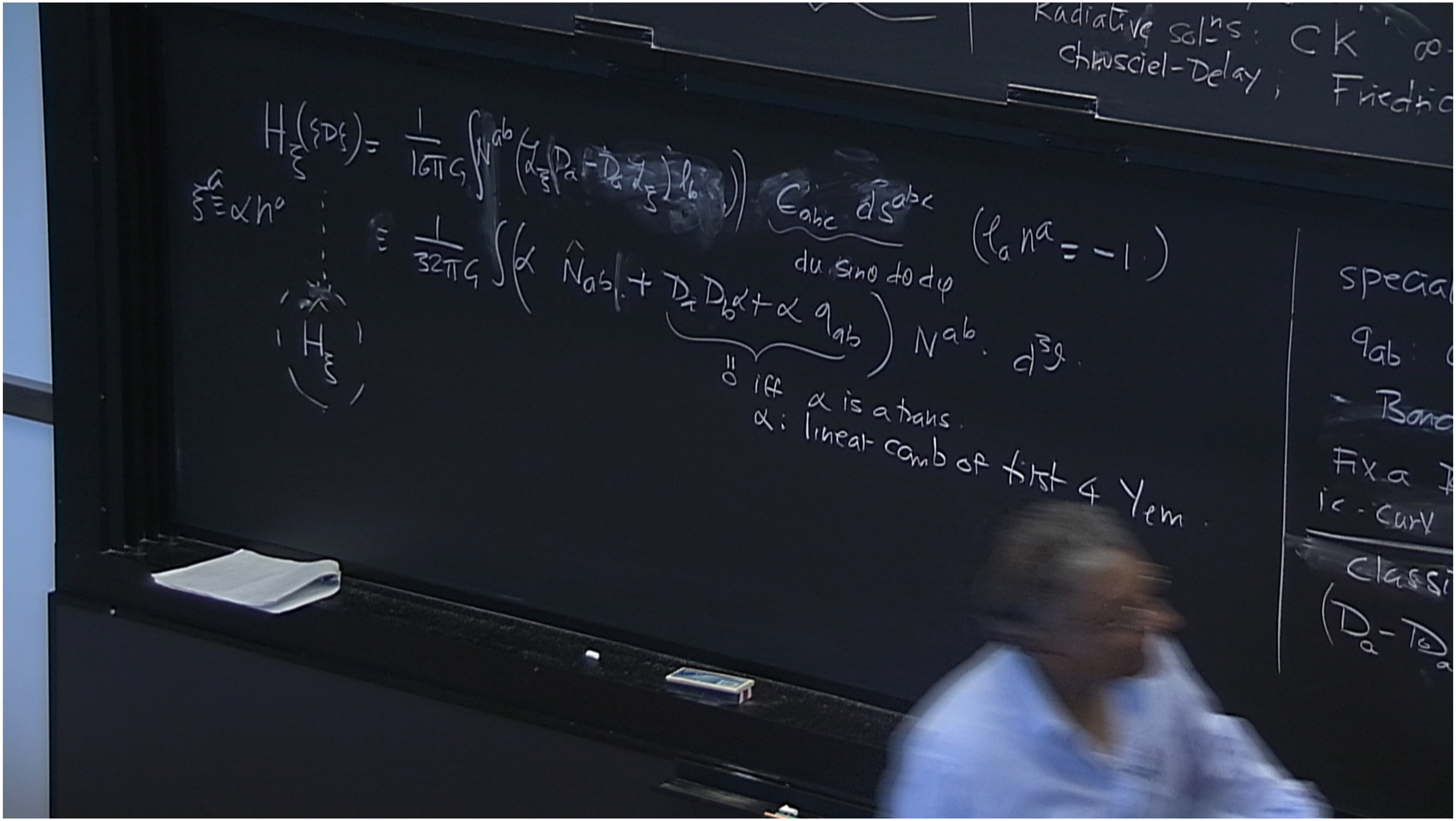

special conf. factors:
q_{ab}: unit 2-sphere met
Bondi frame
Fix a D st $\overset{0}{N}_{ab} = 0$.
i.e. curv is zero
Classical vacua.

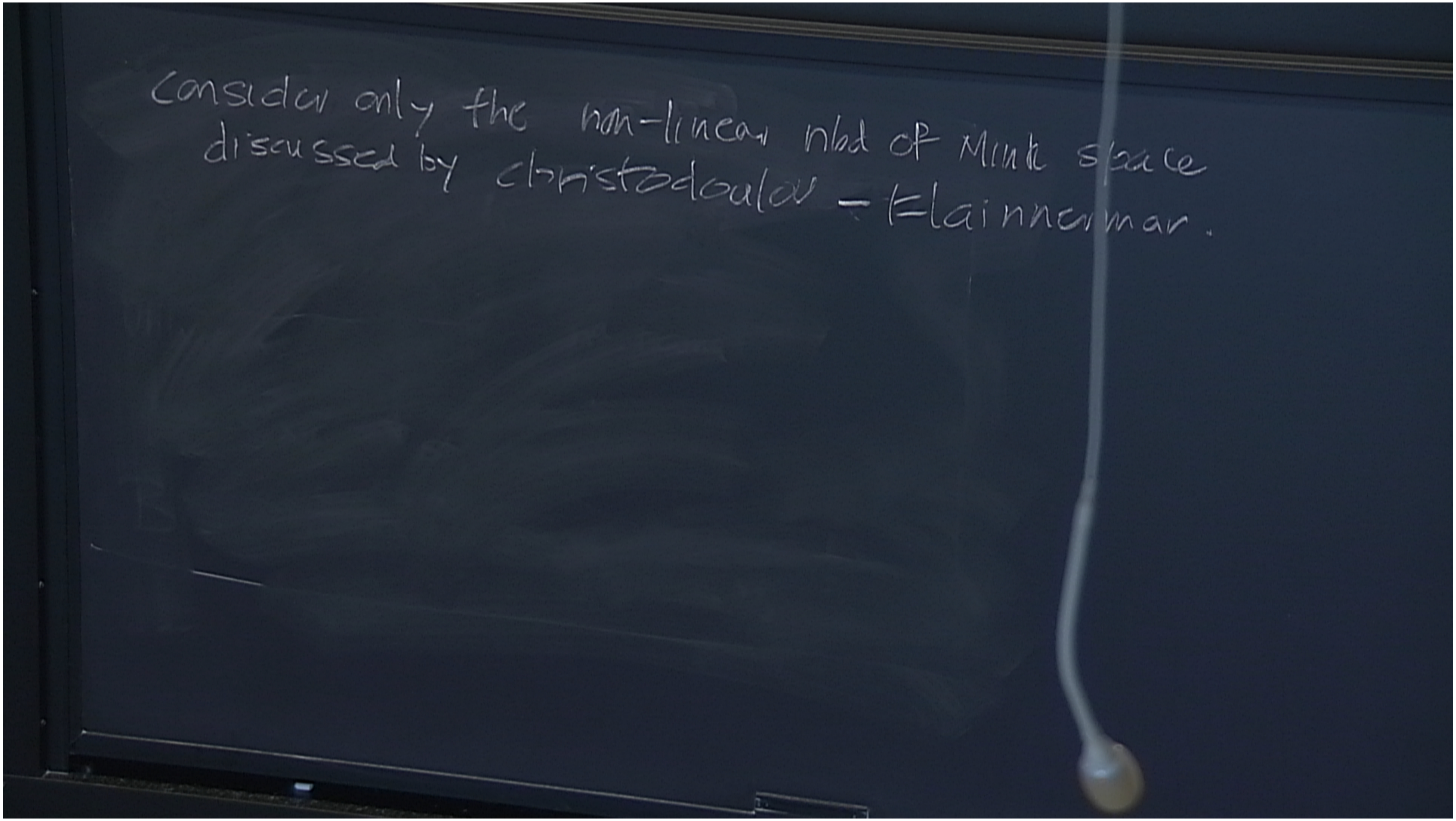


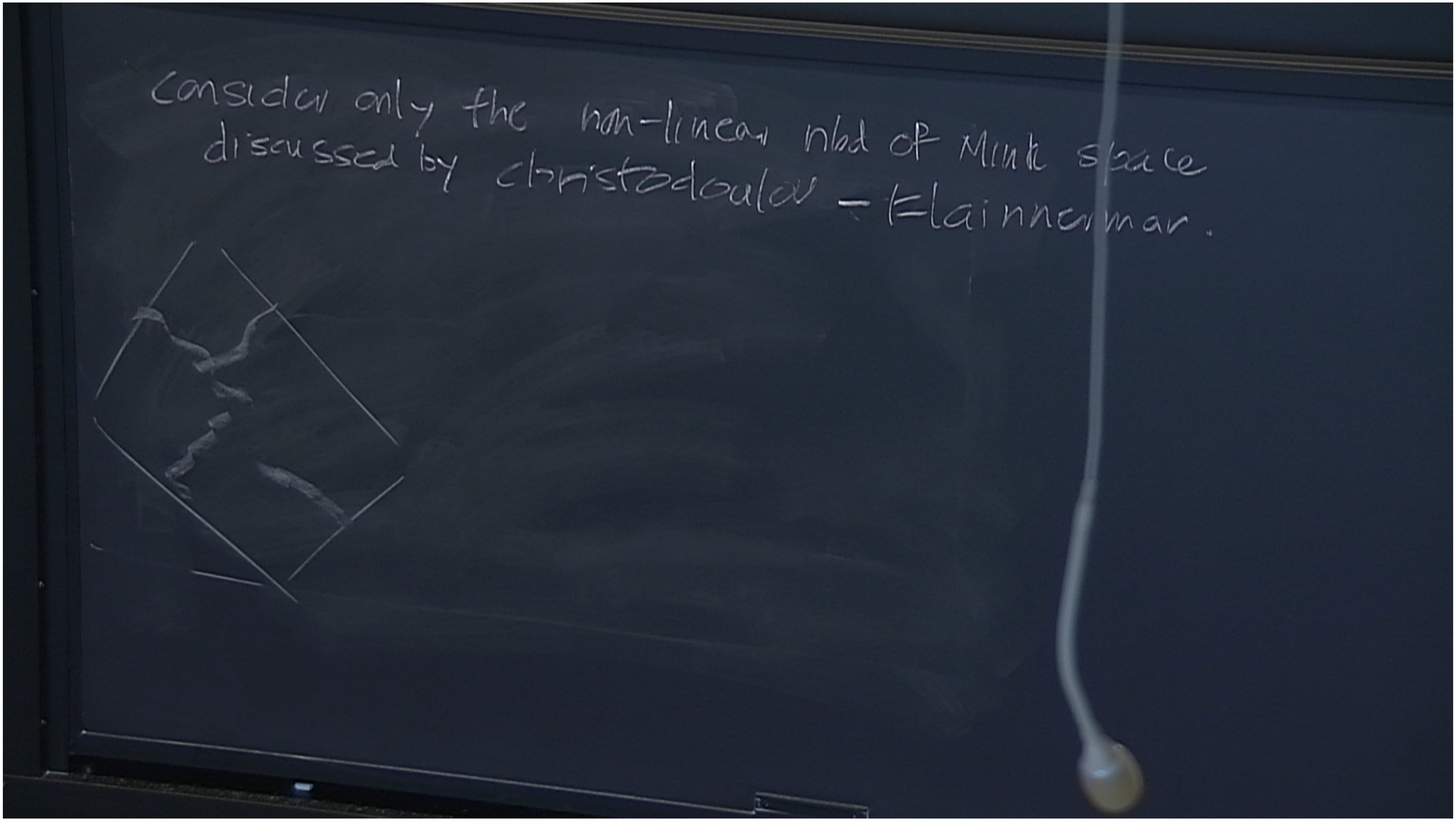

Radiative solns : CK
Chrusciel-Delay ; Friedric
H_ξ(ξDξ) = 1/16πG ∫ N^ab (α_ξ(D_a l_b) − D_a(α_ξ l_b)) ε_abc dS^abc
du sinθ dθ dφ
(l_a n^a = −1)
special
q_ab
Fix a
Classi


Radiative sol^ns: CK
Chrusciel-Delay; Friedric
$H_\xi(\xi D\xi) = \frac{1}{16\pi G}\int N^{ab}(\mathcal{L}_\xi D_a - D_a \mathcal{L}_\xi)\, l_b\, \epsilon_{abc}\, dS^{abc}$
$\xi^a = \alpha n^a$
du sinθ dθ dφ
$(l_a n^a = -1)$
$= \frac{1}{32\pi G}\int (\alpha N_{ab} + D_a D_b \alpha + \alpha q_{ab}) N^{ab} d^3 S$
0 iff α is a trans.
special
q_{ab}:
Bond
Fix a
i.e. curv
Classi
$(D_a - D_a$


Radiative sol^ns : CK
Chrusciel-Delay ; Friedric
$H_\xi(\xi D\xi) = \frac{1}{16\pi G}\int N^{ab}(\mathcal{L}_\xi D_a - D_a \mathcal{L}_\xi) l_b)\ \epsilon_{abc}\, dS^{abc}$
$\xi^a = \alpha n^a$
du sinθ dθ dφ
$(l_a n^a = -1)$
$= \frac{1}{32\pi G}\int (\alpha\, N_{ab} + D_a D_b \alpha + \alpha\, q_{ab})\, N^{ab}\, d^3S$
0 iff α is a trans.
α : linear comb of first 4 $Y_{\ell m}$
special
q_{ab}
Fix a
Classi
$(D_a - D_a$


Radiative solns : CK
Chrusciel-Delay ; Friedric
$(l_a n^a = -1)$
du sinθ dθ dφ
iff α is a trans.
α : linear comb of first 4 $Y_{\ell m}$.
special
Fix a
Classi


Consider only the non-linear nbd of Mink space
discussed by Christodoulou - Klainerman.


Consider only the non-linear nbd of Mink space
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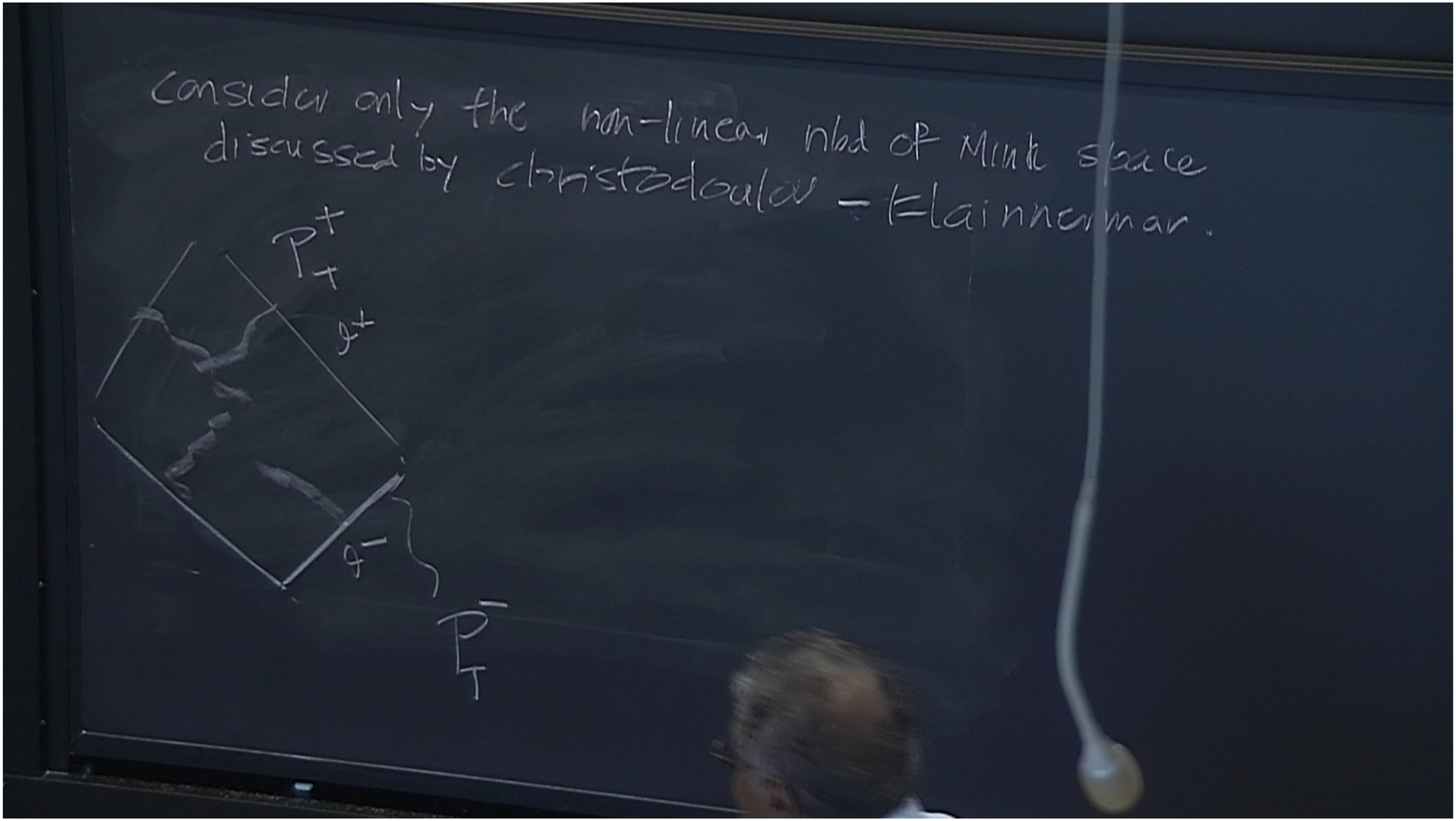
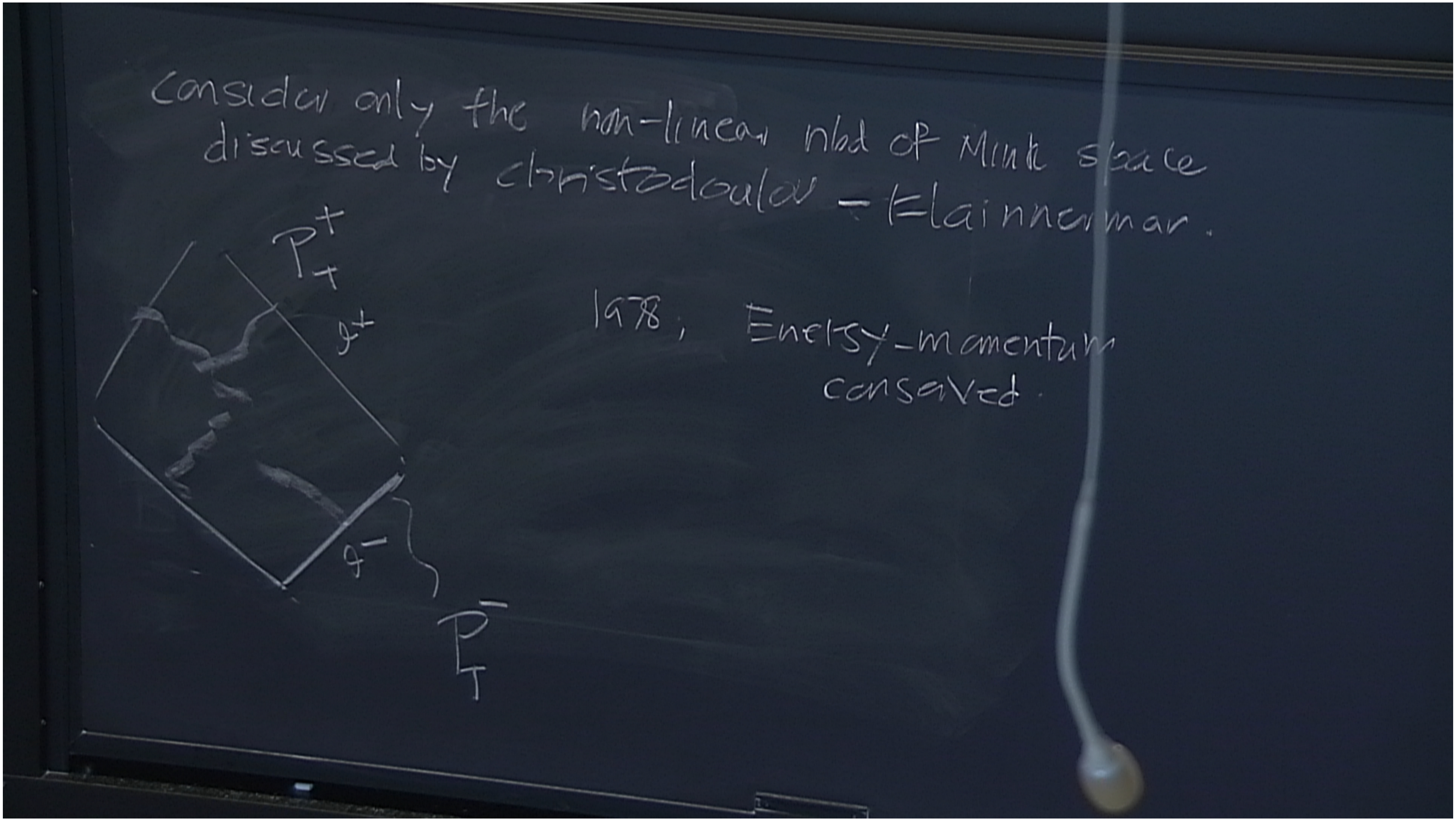
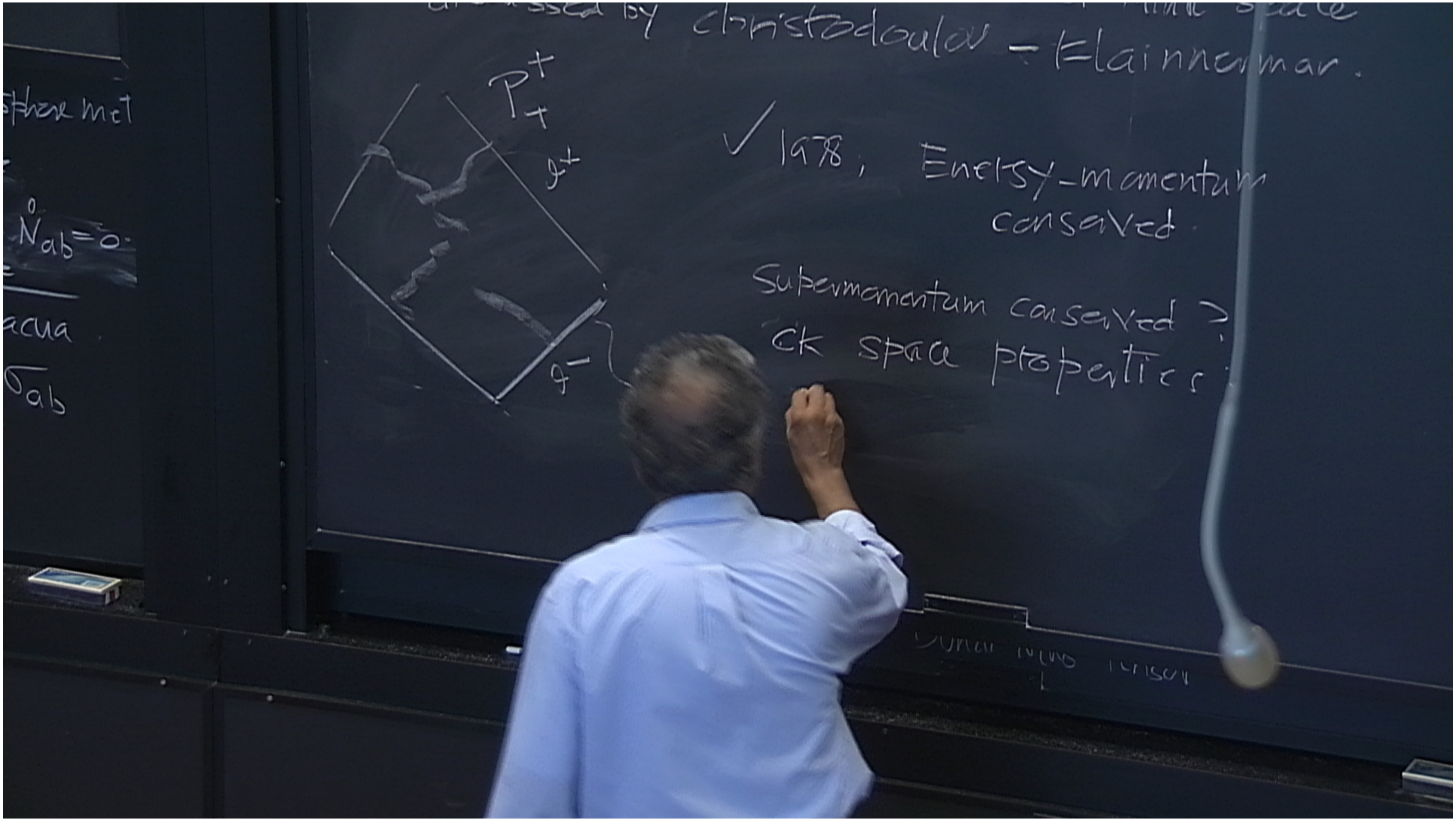
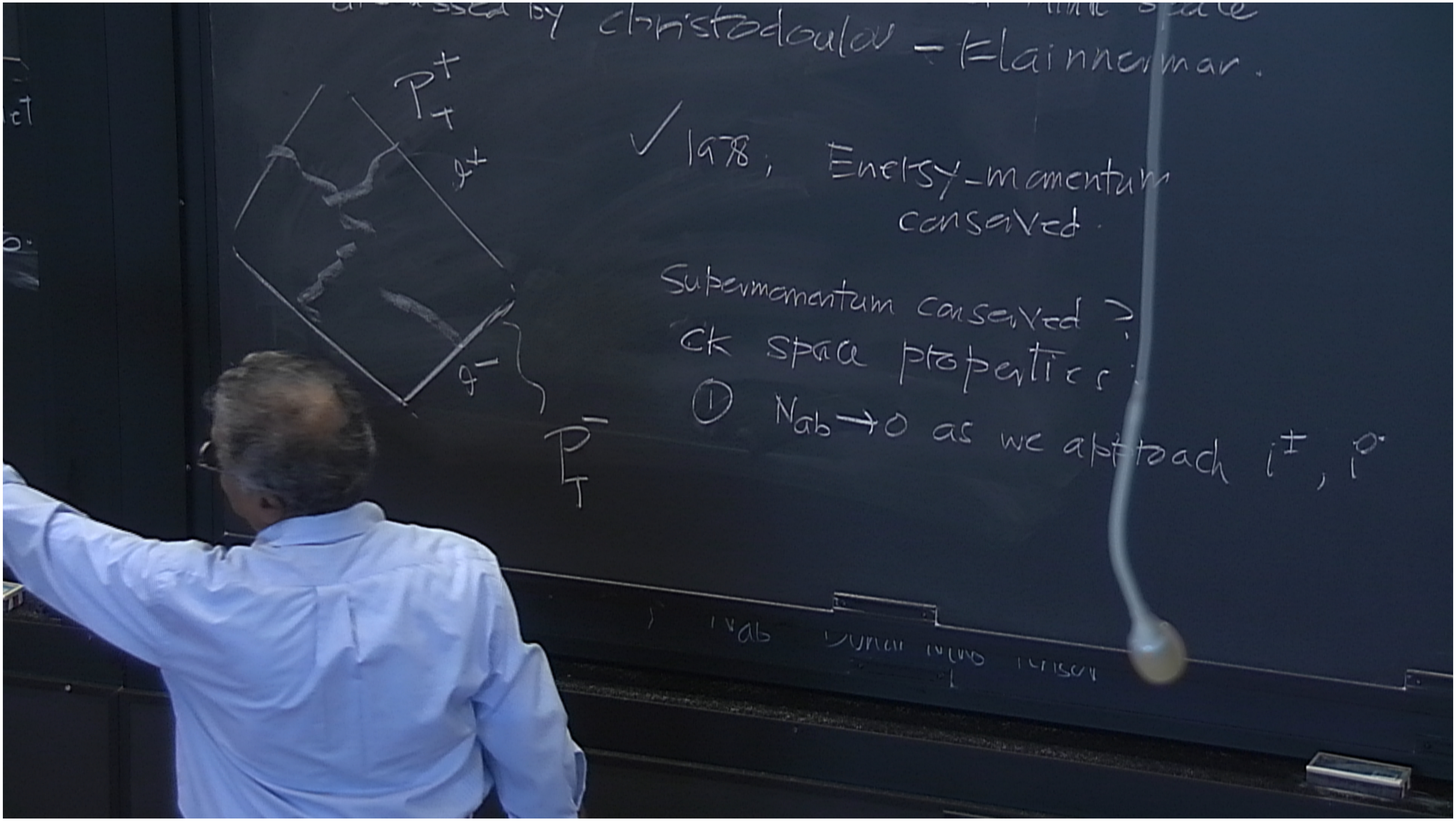

Consider only the non-linear nbd of Mink space
discussed by Christodoulou - Klainerman.
P_T^+
$\mathcal{I}^+$
$\mathcal{I}^-$
P_T^-

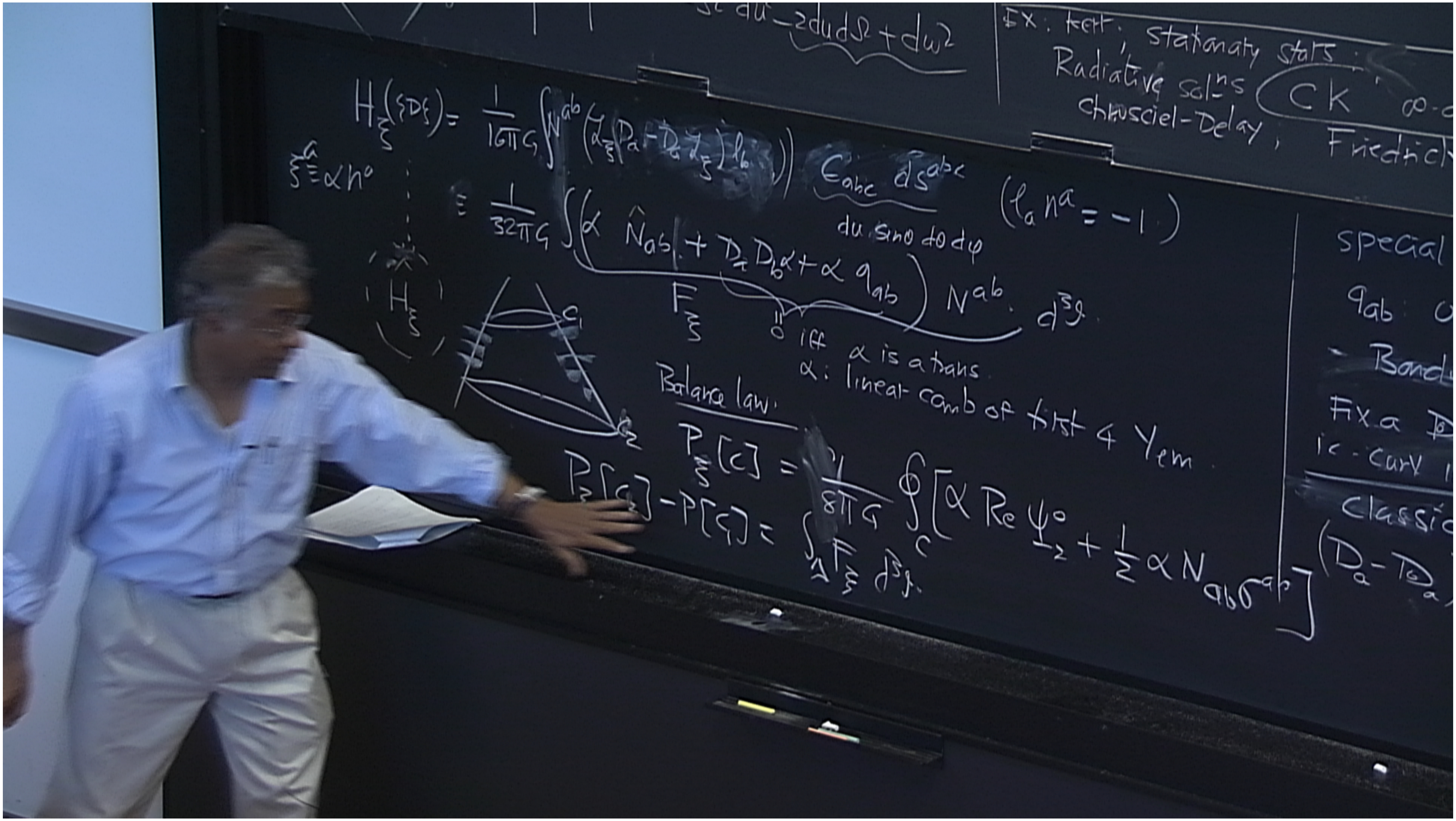

consider only the non-linear nbd of Mink space
discussed by Christodoulou - Klainerman.
1978; Energy-momentum
conserved.

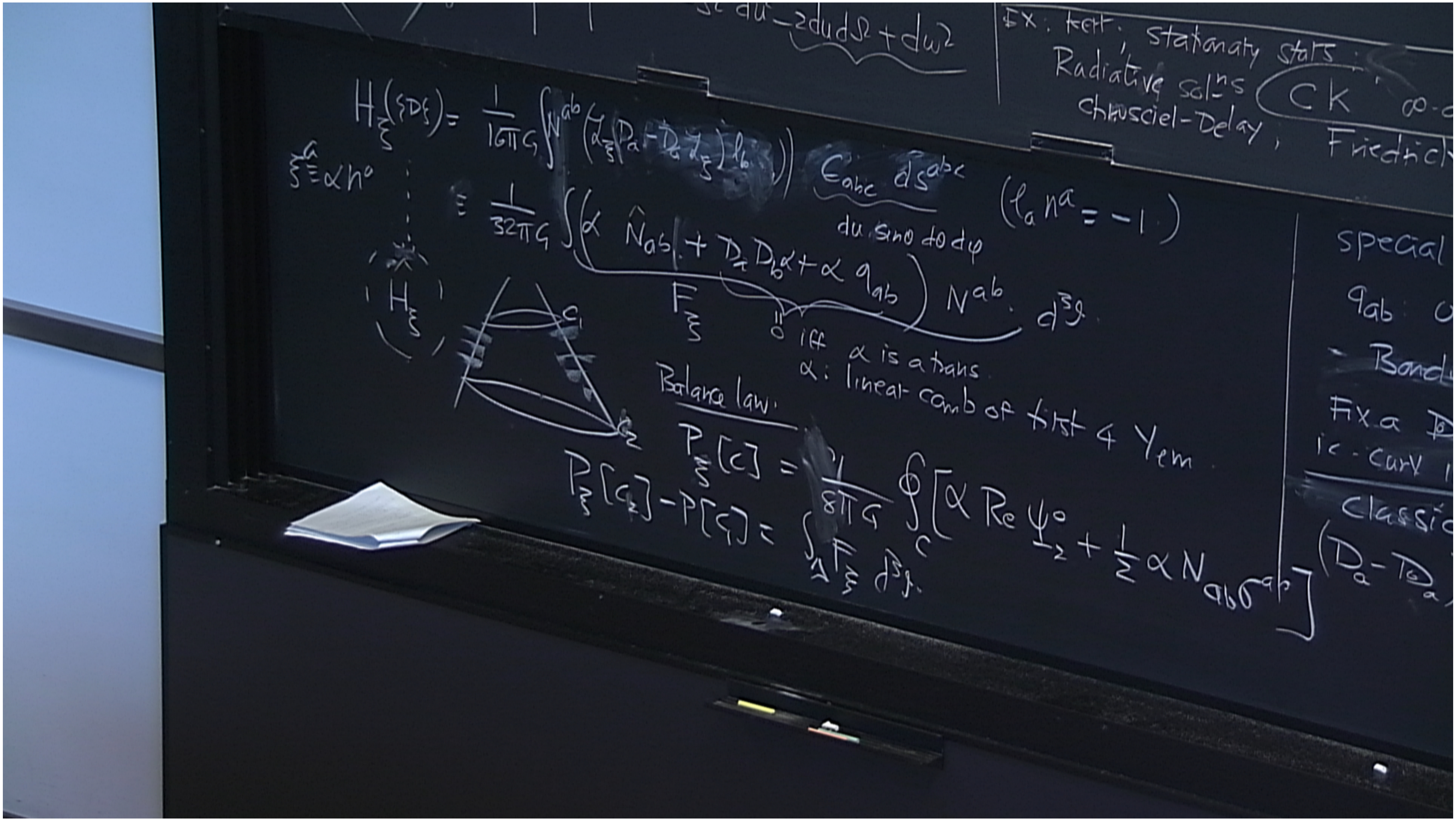


Christodoulou - Klainerman.
1978, Energy-momentum conserved.
Supermomentum conserved ?
space properties

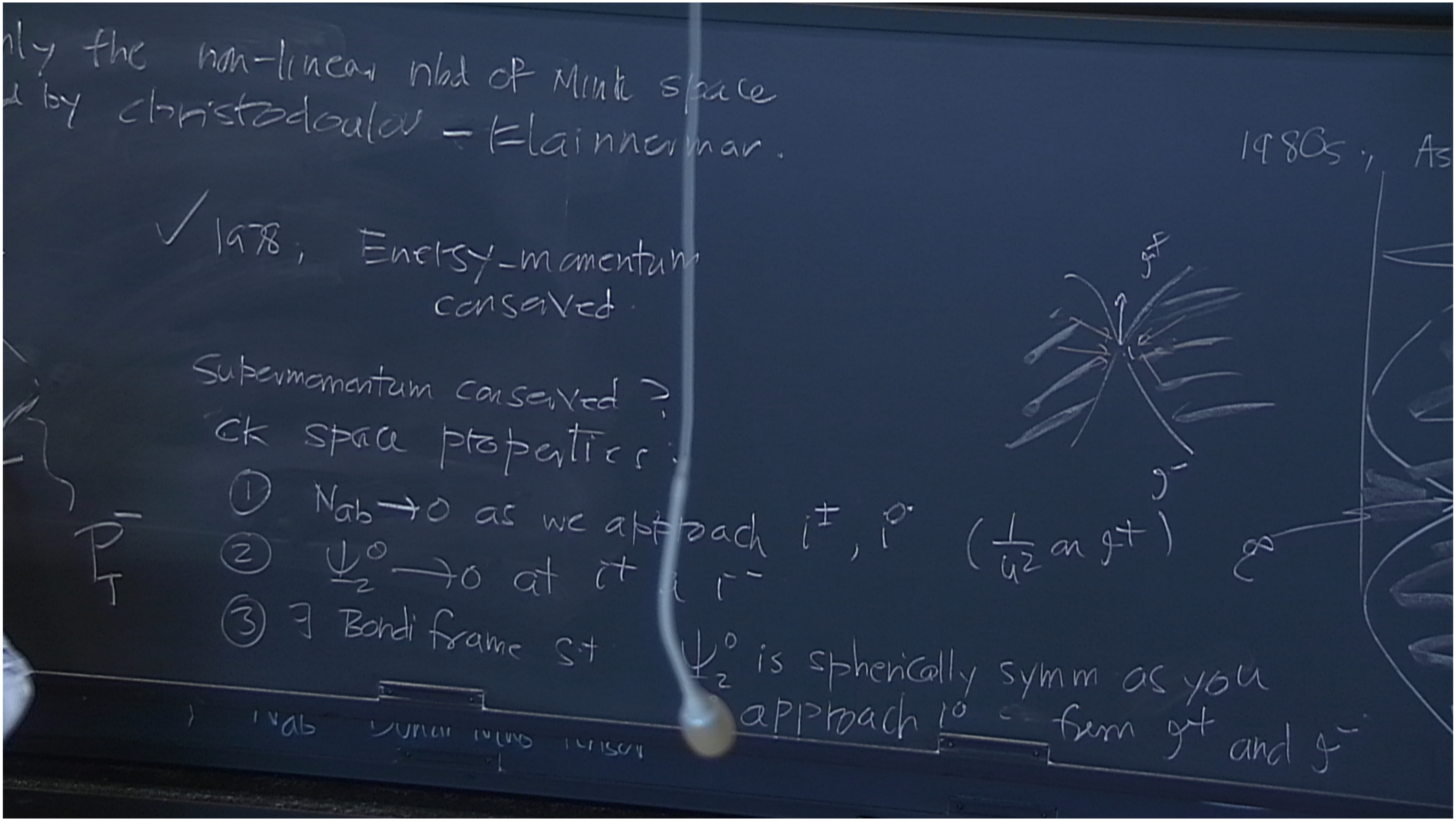


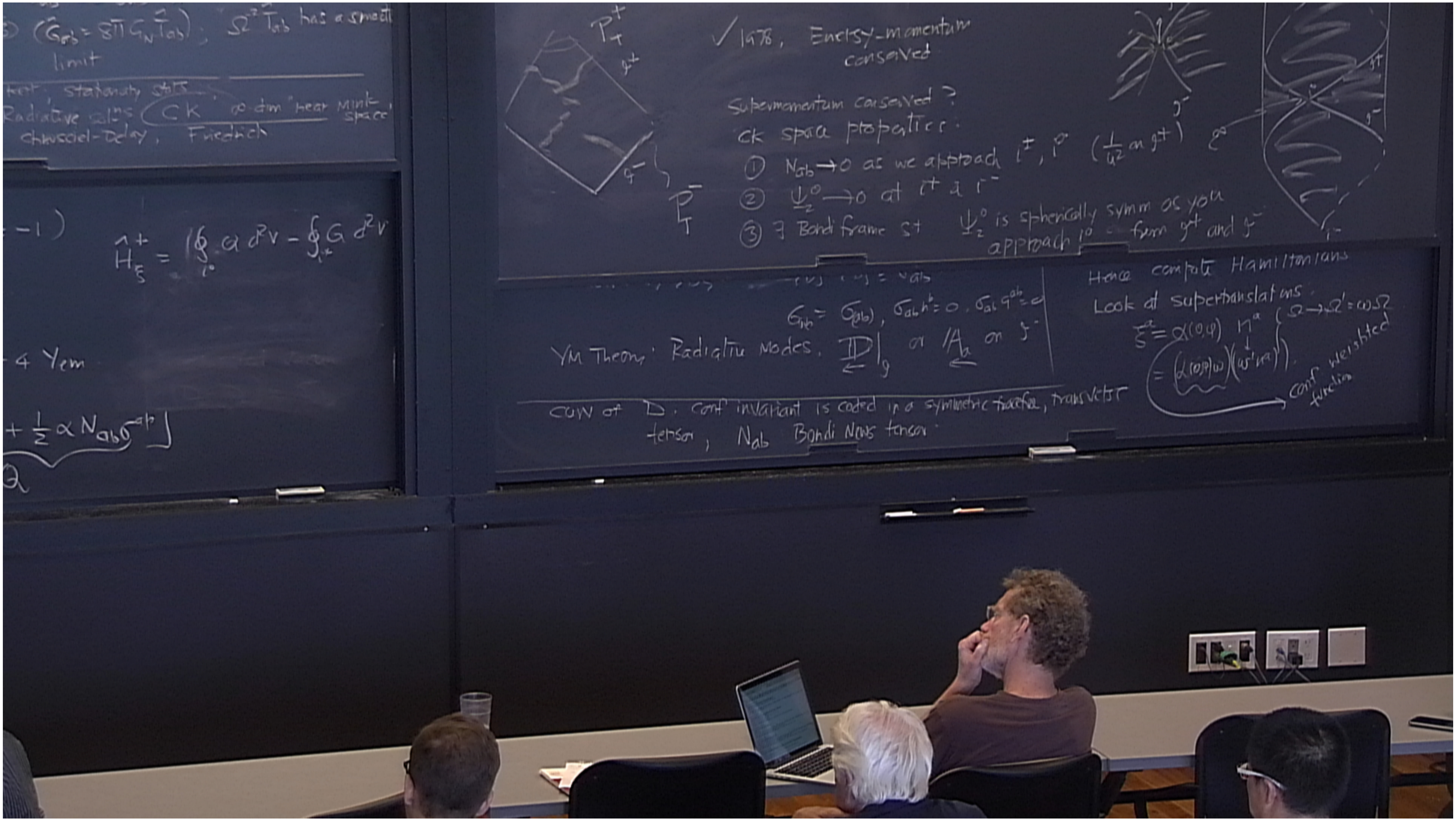
Christodoulou - Klainnerman.
P+
√ 1978; Energy-momentum conserved.
Supermomentum conserved?
ck space properties:
① Nab → 0 as we approach i±, i°
P−

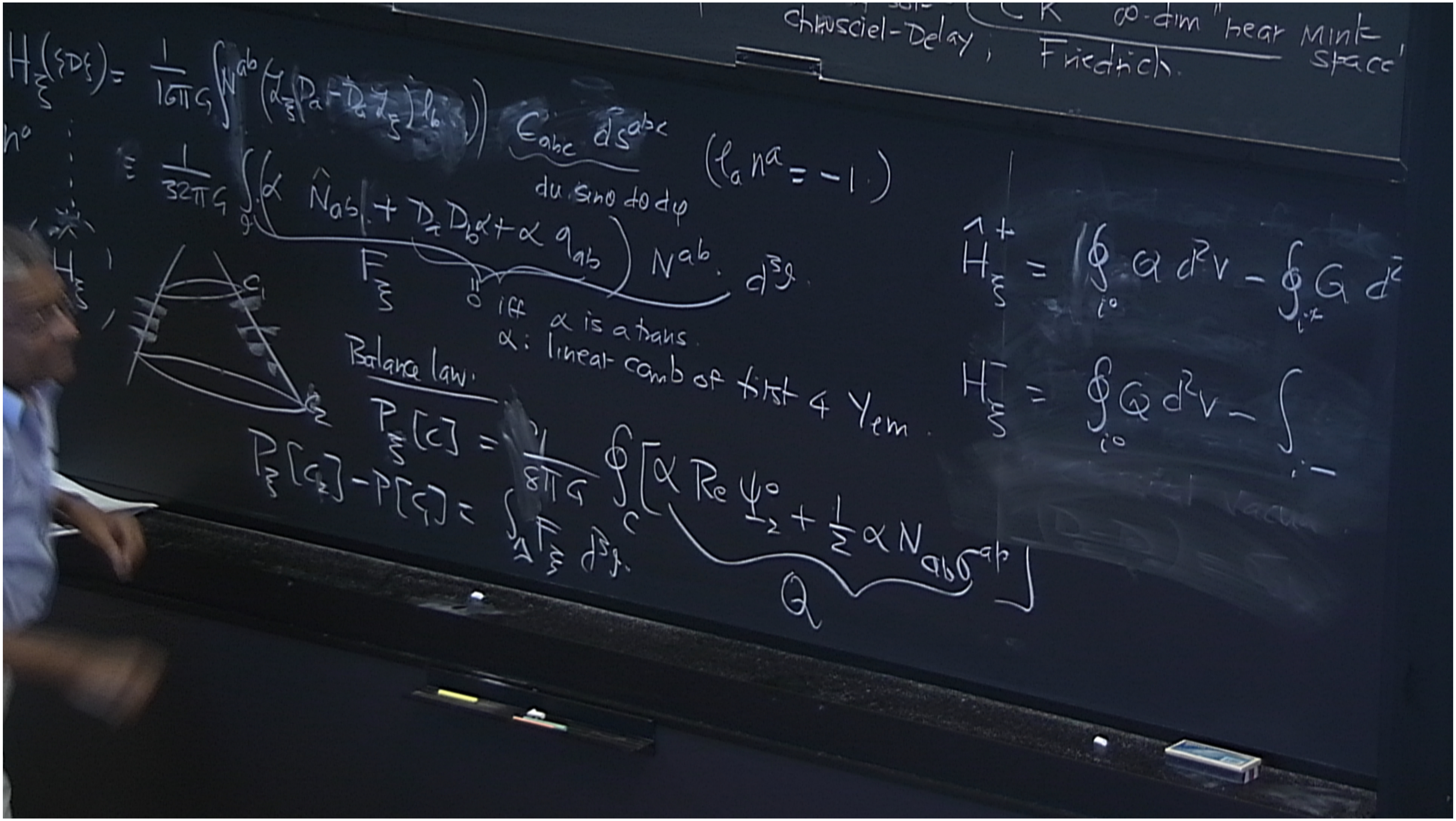

Radiative sol-ns
Balance law
special
Classic

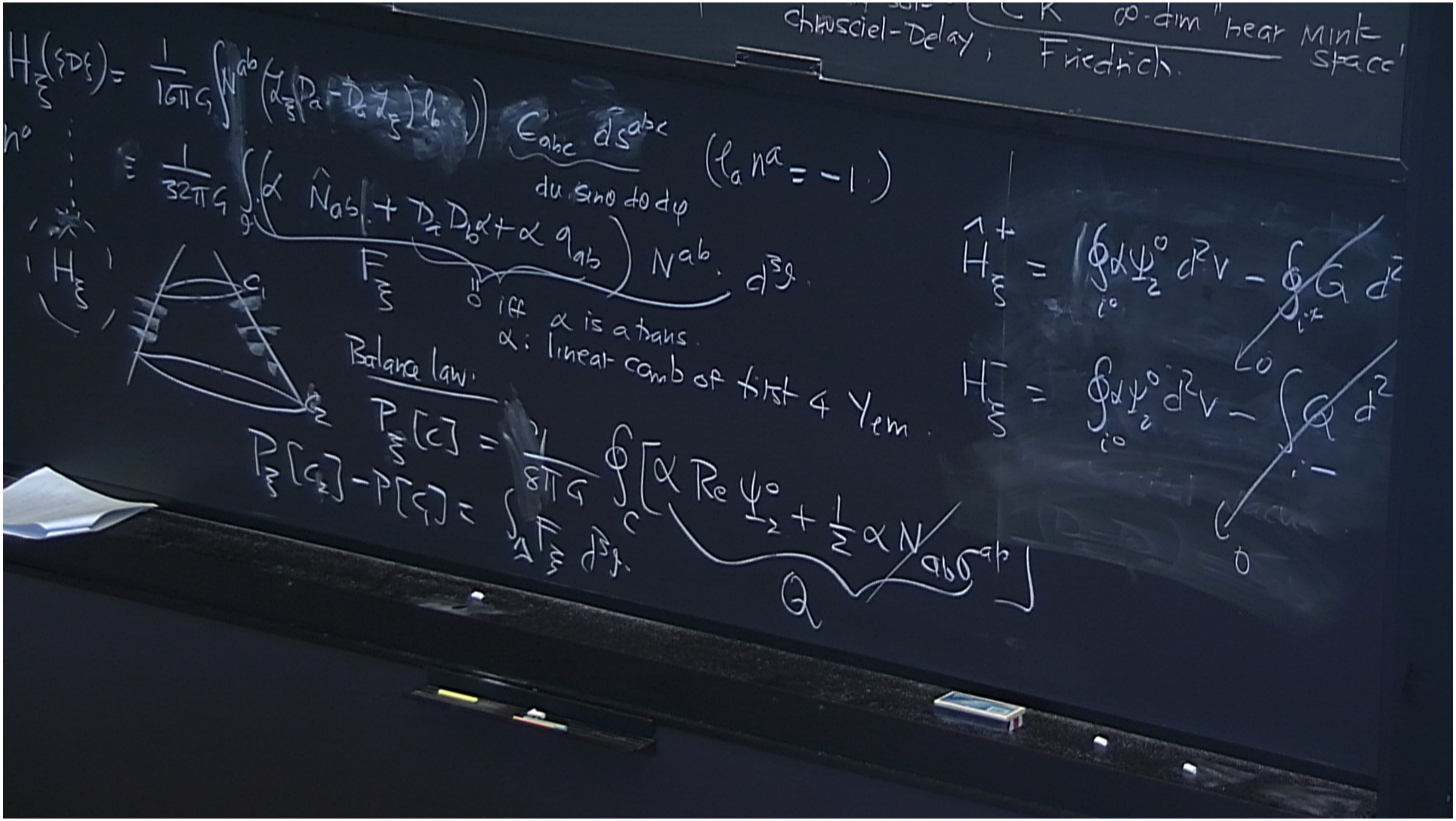


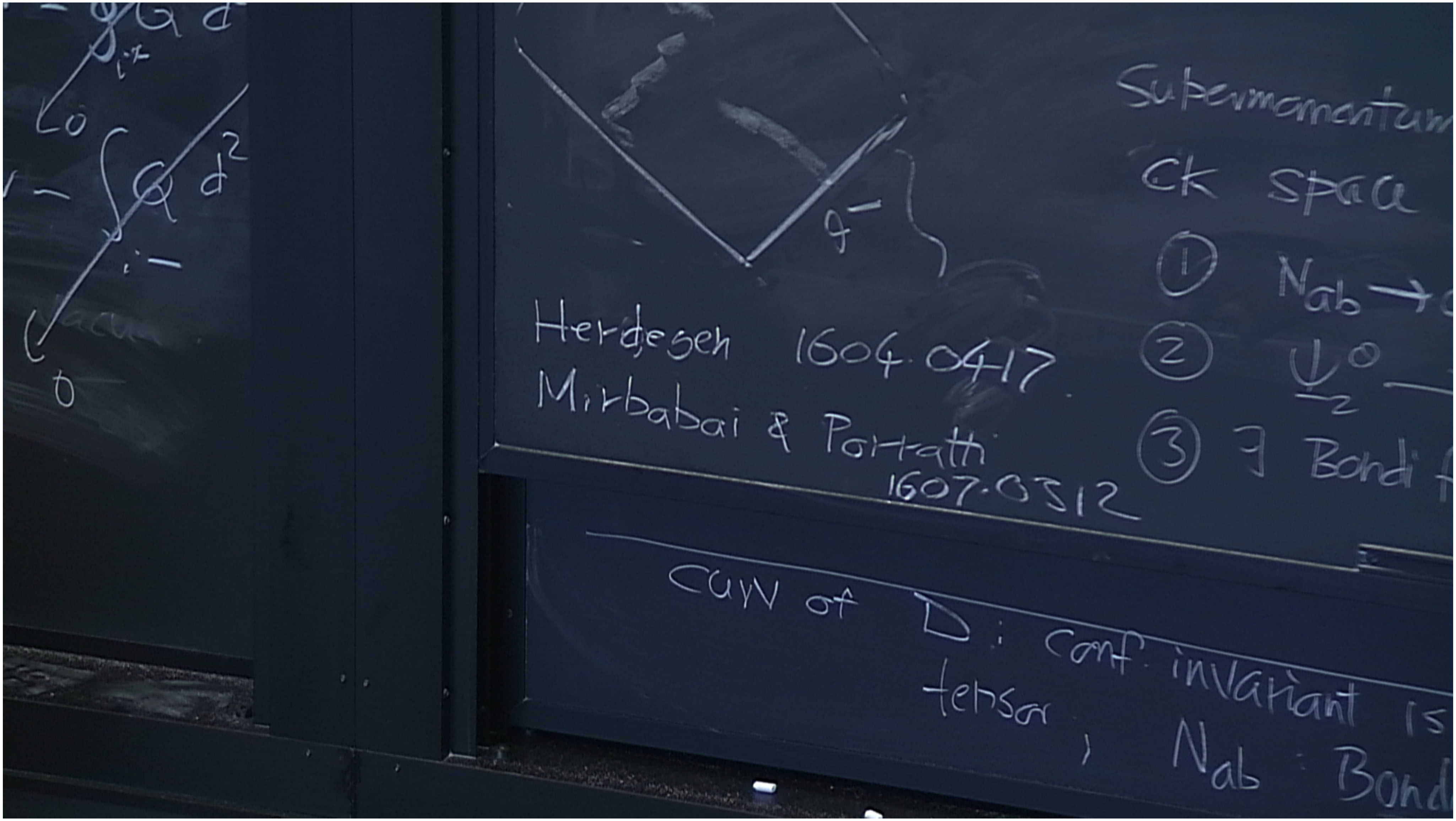
Ex: Kerr, stationary stars
Radiative sol^ns CK
Chrusciel-Delay, Friedrich
Balance law
iff α is a trans
α: linear comb of first 4 Yem
special
Fix a
Classic


the non-linear nbd of Mink space
by Christodoulou - Klainnerman.
1980s ; As
1978 ; Energy-momentum conserved.
Supermomentum conserved ?
ck space properties :
① $N_{ab} \to 0$ as we approach $i^{\pm}, i^{0}$ $(\frac{1}{u^2}$ on $\mathscr{I}^{+})$
② $\Psi_2^0 \to 0$ at i^{+} & i^{-}
③ ∃ Bondi frame s.t. Ψ_2^0 is spherically symm as you approach i^0 from $\mathscr{I}^{+}$ and $\mathscr{I}^{-}$
$\mathscr{I}^{+}$
$\mathscr{I}^{-}$
P_T^-




Chrusciel-Delay, Friedrich
∞-dim "near Mink space"
Balance law.
iff α is a trans
α: linear comb of first 4 Yem


Chrusciel-Delay; Friedrich.
∞-dim "near Mink space"
(l_a n^a = -1)
du sinθ dθ dφ
iff α is a trans
α: linear comb of first 4 Y_em
Balance law.
Q


Supermomentum
ck space
Herdegen 1604.0417
Mirbabai & Porrati
1607.0312
curl of D: conf. invariant
tensor, Nab : Bondi