Title: Hydrodynamic theory of quantum fluctuating superconductivity

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Abstract: A hydrodynamic theory of transport in quantum mechanically phase-disordered superconductors is possible when supercurrent relaxation can be treated as a slow process. We obtain general results for the frequency-dependent conductivity of such a regime. With time-reversal invariance, the conductivity is characterized by a Drude-like peak, with width given by the supercurrent relaxation rate. Using the memory matrix formalism, we obtain a formula for this width (and hence also the dc resistivity) when the supercurrent is relaxed by short range Coulomb interactions. This leads to a new -- effective field theoretic and fully quantum -- derivation of a classic result on flux flow resistance. With strong breaking of time-reversal invariance, the optical conductivity exhibits what we call a `hydrodynamic supercyclotron' resonance. We obtain the frequency and decay rate of this resonance for the case of supercurrent relaxation due to an emergent Chern-Simons gauge field. The supercurrent decay rate in this `topologically ordered superfluid vortex liquid' is determined by the conductivities of the normal component of the liquid. Our work gives a controlled framework for low temperature metallic phases arising from phase-disordered superconductivity.

Hydrodynamic theory of quantum fluctuating superconductivity

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Friday Aug 26, 2016

Low energy challenges for high energy physicists
Perimeter Institute, Waterloo, Canada

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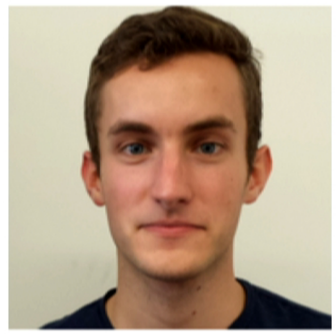
Acknowledgments

- Based on
  'Hydrodynamics theory of quantum fluctuating superconductivity',
  arXiv:1602.08171, Phys. Rev. B 94, 054502 (2016)
  together with Richard Davison, Luca Delacrétaz and Sean Hartnoll

- My research is supported by a Marie Curie International Outgoing Fellowship, Seventh European Community Framework Programme.

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Infinite vs finite DC conductivities

- If a conserved quantity (momentum) overlaps with the electric current, the electric conductivity at zero frequency diverges. For instance, in relativistic hydrodynamics:

$$\sigma(\omega) = \sigma_Q + \frac{i\rho^2}{(\epsilon + p)\omega}, \quad J^\mu = \rho u^\mu - T\sigma_Q \partial_\mu \left(\frac{\mu}{T}\right)$$

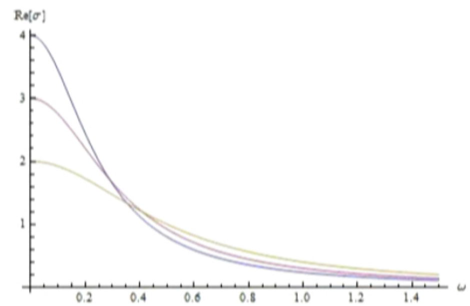
  The divergence is due to momentum conservation.

  σ_Q is the conductivity of the diffusive dofs of the system (no overlap with momentum) [Davison, B.G. & Hartnoll'15].

- If momentum is slowly relaxed $\Gamma \ll T$:

$$\sigma(\omega) = \sigma_1 + \frac{\rho^2}{(\epsilon + p)(\Gamma - i\omega)}.$$

  A lot of work recently in gauge/gravity duality and hydrodynamics to model this [Hartnoll et al'07, Davison & B.G'15, Lucas'15].



Superfluidity

- In a superfluid, a U(1) symmetry is spontaneously broken: a complex order parameter condenses.

- Its phase is a Goldstone boson and by gauge invariance

$$\dot{\phi} = -\mu$$

Taking a spatial derivative

$$\dot{u}_\phi = -\frac{1}{m}\nabla\mu \,, \qquad u_\phi = \frac{1}{m}\nabla\phi$$

A phase gradient sources an electric field! The (conserved) superfluid current couples to the electric current.

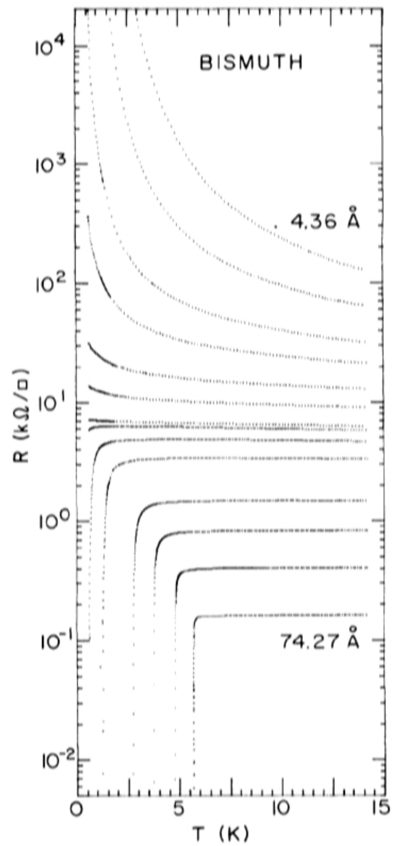
- The electric conductivity contains a superfluid delta function

$$\sigma(\omega) = \sigma_0 + \frac{i\rho_s}{m^2\omega}$$

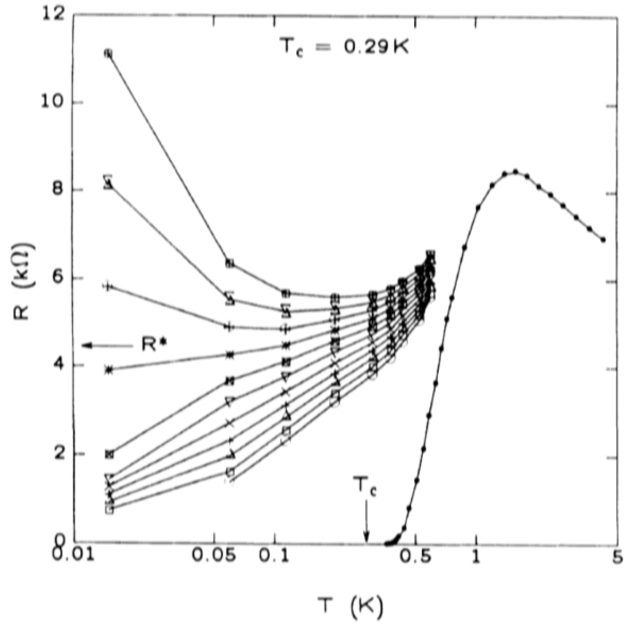
- Is there a sense in which this delta function can be resolved?

Superfluid/insulator transitions in thin superfluid films

2D superfluid films exhibit two different kinds of (quantum) superfluid/insulator phase transition, disorder or magnetic field-driven.

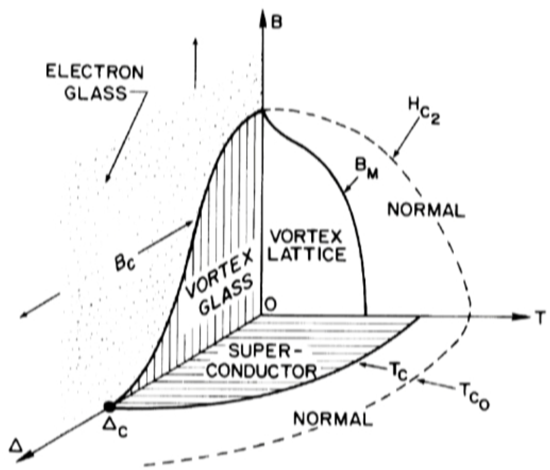


[HAVILAND ET AL'89]

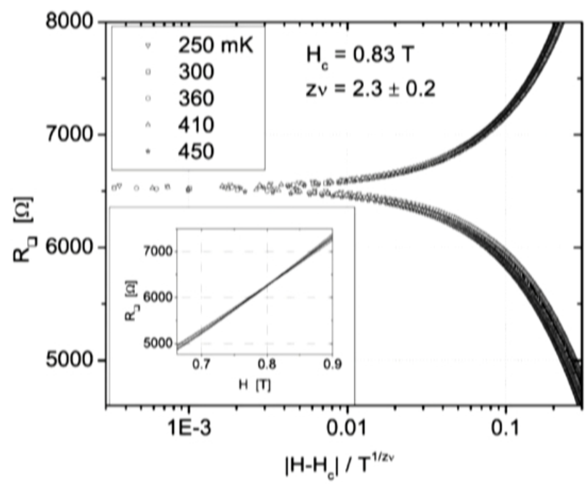


[HEBARD & PAALANEN'90]

Phase diagram of SIT – early days



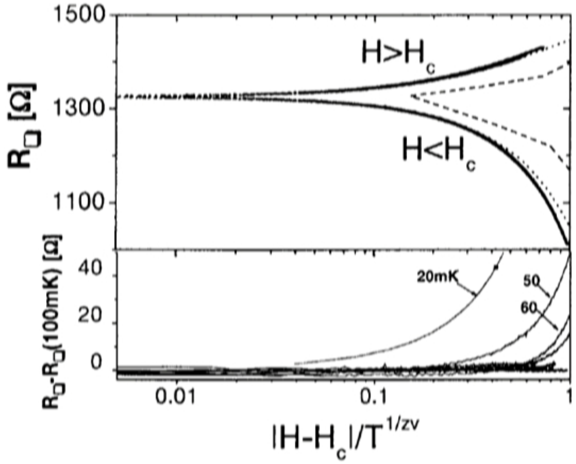
[MPA Fisher'90]



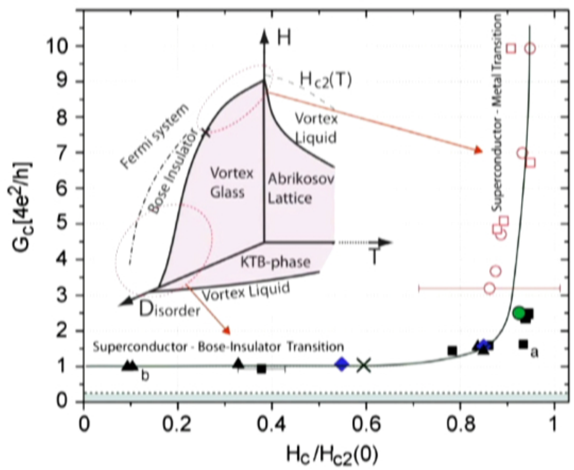
[Steiner-Breznay-Kapitulnik'08]

A quantum critical point sits at the SIT. At this quantum critical point, the resistivity takes a universal value $R_Q = h/4e^2$ and a scaling theory of transport applies [MPA Fisher'90].

A surprise: intermediate metallic phases



[Mason & Kapitulnik'99]



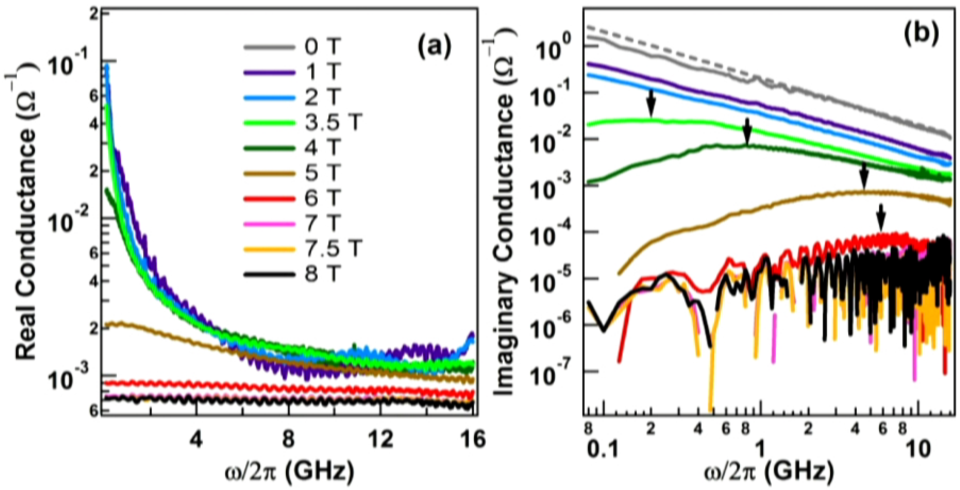
[Steiner-Breznay-Kapitulnik'08]

For weakly disordered films, the quantum critical resistivity differs from R_Q and scaling does not apply at low temperatures.

The resistivity becomes temperature independent over a range of fields: intermediate metallic phase between superfluid and insulator.

AC measurements: peaks in the conductivity

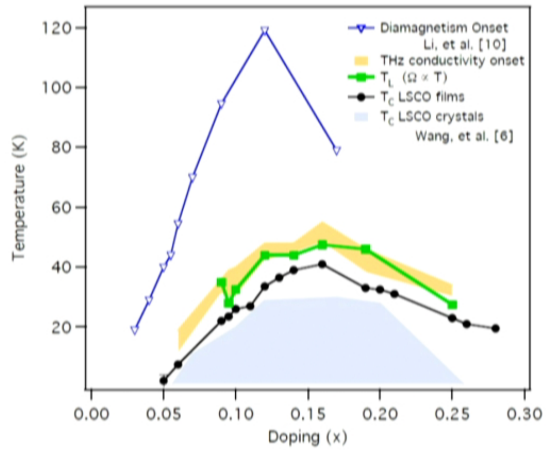
Sharp Drude-like peaks appear in the real part of the conductivity.
The superfluid $1/\omega$ pole in the imaginary part is resolved.



[Liu-Pan-Wen-Kim-Sambandamurthy-Armitage'13]

Cuprates

Superfluid phase fluctuations are also important in the phase diagram of the underdoped cuprates (here LSCO).



[Bilbro et al, Nature Physics 7, 2011]

Plan of the talk

1. Construct a theory of superfluid hydrodynamics which includes weak phase relaxation.
2. Match it to a memory matrix treatment (exact, no approximation) and use this to calculate the width of the peak and the DC conductivity when the phase fluctuations are weak.
3. First restrict to parity invariant setups (no magnetic field), then move on to parity-violating systems.

Incoherent superfluid hydrodynamics

- Hydrodynamics is the universal long wavelength, small frequencies description of the dynamics.
  dofs: conserved quantities and broken symmetry Goldstone modes.

- Work in the incoherent limit [HARTNOLL'14]: due to fast momentum relaxation by disorder, momentum is short lived and not part of the hydro.

- We do include the Goldstone mode of the spontaneously broken U(1) symmetry (eg phase of the complex scalar order parameter). By gauge invariance, it couples to the chemical potential as

$$\dot{\phi} = -\mu$$

  This Josephson equation gets corrected at higher order in derivatives.

Incoherent superfluid hydrodynamics

- Our set of hydrodynamic variables are the energy and charge density as well as superfluid velocity $(\delta\epsilon, \delta\rho, u_\phi = \nabla\phi/m)$.

- The equations of motion are the conservation of energy and charge

$$\partial_t \delta\epsilon + \nabla \cdot j_E = 0\,, \qquad \partial_t \delta\rho + \nabla \cdot j = 0$$

- Supplemented by constitutive relations for the charge and heat currents + Josephson relation for the superfluid velocity

$$j = \frac{\rho_s}{m} u_\phi - \sigma_0 \nabla\mu - \alpha_0 \nabla T$$

$$\frac{1}{T} j_Q = \alpha_0 \nabla\mu - \bar{\kappa}_0 \frac{\nabla T}{T}$$

$$\partial_t u_\phi = -\frac{1}{m}\nabla\mu + \frac{\xi}{m}\rho_s \nabla(\nabla \cdot u_\phi)$$

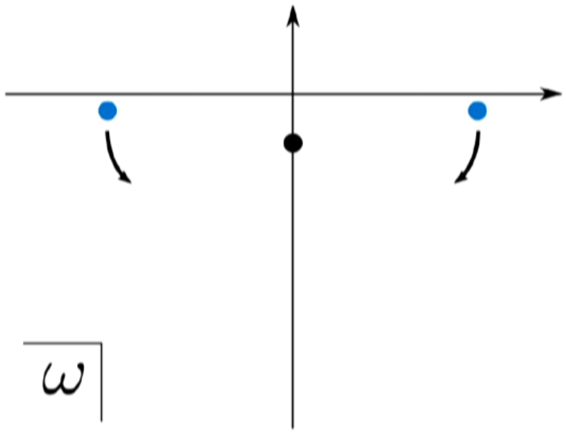
Incoherent superfluid conductivities

- The conductivities are (by Kadanoff and Martin)

$$\sigma = \sigma_0 + \frac{i\rho_s}{m^2\omega}, \quad \alpha = \bar{\alpha} = \alpha_0 , \quad \bar{\kappa} = \bar{\kappa}_0$$

No phase relaxation yet, so infinite DC electric conductivity: **superfluidity**.

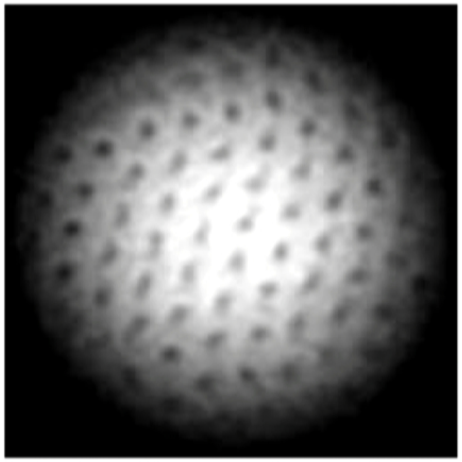
- The collective excitations are a thermal diffusion mode and two superfluid second sound modes.



[Credit: Luca V. Delacrétaz (Stanford)]

Vortices in two dimensions

- At finite temperature, vortices can proliferate due to thermal fluctuations and destroy quasi long range order (BKT transition).
- At a vortex, the amplitude of the order parameter vanishes.



[CREDIT: ANDRE SCHIROTZEK (MIT)]

- This requires that the circulation of the superfluid velocity is quantized

$$\oint_{\text{vortex}} u_\phi = \frac{2\pi n}{m}$$

- At a vortex location, the superfluid velocity is no longer a pure gradient

$$u_\phi = \frac{1}{m}\left(\nabla\phi + \epsilon \times \nabla\psi\right)$$

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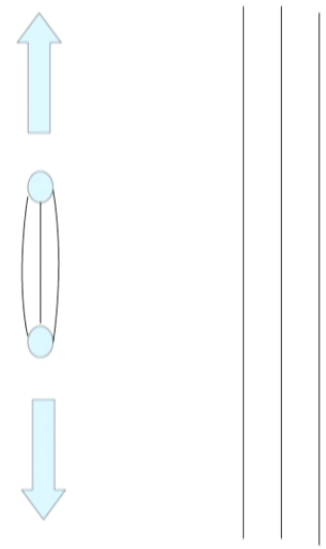
Vortices in two dimensions

- Vortices are nucleated in pairs of vortices anti-vortices: pinned vortices do not relax the supercurrent.
- Mobile vortices will relax the supercurrent, $\partial_t u_\phi \neq 0$, by (un)winding the phase.



- As vortex cores are not superconducting, expect that mobile vortices produce dissipation and regulate the conductivity

$$\sigma = \sigma_0 + \frac{\rho_s}{m^2} \frac{1}{-i\omega + \Omega}$$

Classically [BARDEEN & STEPHEN'65]

$$\Omega \sim \frac{n_f}{\sigma_n}$$

The vortex current

- Introduce the vortex density

$$n_v = \frac{m}{2\pi}\epsilon^{ij}\nabla^i u_\phi^j$$

- There exists a conserved vortex current (locally n_v is conserved)

$$\partial_t n_v + \nabla \cdot j_v = 0$$

which is consistent with the modified Josephson relation

$$m\partial_t u_\phi = -\nabla\mu + 2\pi\epsilon^{ij} j_v^j + \xi\rho_s \nabla\nabla \cdot u_\phi$$

Interpretation: a vortex flow induces a transverse voltage [Huebener-Seher'68].

Vortex constitutive relation and phase relaxation

- We can now write the constitutive relation for the vortex current

$$j_v = \frac{m}{2\pi}\Omega\epsilon^{ij}u^j_\phi - \gamma\nabla^i n_v$$

- γ is the intrinsic vortex diffusivity.
- Ω encodes the Magnus force felt by the vortices in the presence of a supercurrent [de Gennes-Matricon'64].

  Positivity of divergence of entropy current: $\Omega > 0$, generates dissipation.
- It breaks the conservation of the phase

$$\partial_t\phi = -\mu - \Omega\phi$$

  and relaxes the delta function in the electric conductivity

$$\sigma(\omega) = \sigma_0 + \frac{\rho_s}{m^2}\frac{1}{\Omega - i\omega}\,, \quad \sigma_{DC} = \sigma_0 + \frac{\rho_s}{m^2\Omega}$$

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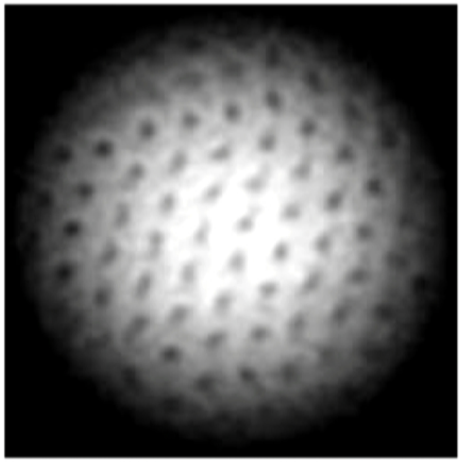

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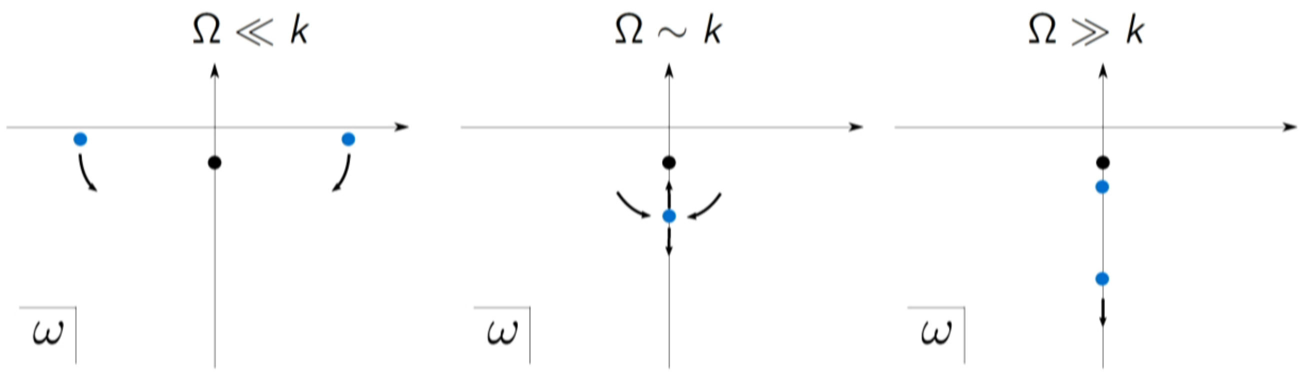
Relaxed spectrum



[CREDIT: LUCA V. DELACRÉTAZ (STANFORD)]

- At $\Omega \ll k$, the spectrum is qualitatively unchanged: the superfluid sound modes acquire a small mass $O(\Omega)$.
- At $\Omega \sim k$, the sound modes collide on the imaginary axis.
- At $\Omega \gg k$, the spectrum qualitatively changes: the superfluid sound modes now split into a gapped mode (Drude) and a gapless diffusive mode with $D_\omega \sim \sigma_{DC} \sim 1/\Omega$ (Einstein relation).

Decay rate from the memory matrix formalism

- The memory matrix [Forster'75] is a powerful formalism to calculate the decay rate of a slowly relaxing operator:

$$H = H_0 + \varepsilon \Delta H , \qquad \varepsilon \ll 1$$

$$\partial_t J_\phi = \varepsilon i \left[\Delta H, J_\phi \right] , \qquad J_\phi = \frac{1}{m} \int_{T^2 \setminus \{\text{vortex cores}\}} d^2x \nabla \phi$$

- It has proven very useful for momentum relaxation [Hartnoll-Kovtun-Mueller-Sachdev'07, Hartnoll-Herzog'07, Hartnoll-Hofman'12, Lucas-Sachdev'15].
- In our case, the slow operators are J, J_ϕ and J_Q.
- The conductivities take the same functional form as in hydro. Importantly, we also get a formula for Ω

$$\Omega = \varepsilon^2 \rho_s \lim_{\omega \to 0} \frac{\mathrm{Im}\, G^R_{i[\Delta H, J_\phi] i[\Delta H, J_\phi]}}{\omega} \ll k_B T$$

Onsite Coulomb interaction

- A natural starting point is the commutation relation between the charge density and the phase, which are canonical variables

$$[\phi(x), \rho(y)] = i\delta(x - y)$$

- This relation encodes a Heisenberg uncertainty relation:

$$\Delta\phi\Delta\rho \gtrsim \hbar$$

- Coulomb interactions penalize charge density fluctuations [Efetov'80, Doniach'81... Emery-Kivelson'95]. Consequently phase fluctuations are enhanced.

- A simple choice is an on-site density-density interaction [Doniach'84, Sachdev-Starykh'00]

$$\Delta H = \frac{1}{2\chi_{\rho\rho}} \int d^2x \, \rho(x)^2$$

Decay rate due to onsite Coulomb interaction

- The calculation of the decay rate

$$\Omega = \varepsilon^2 \frac{1}{\chi_{J_\phi J_\phi}} \lim_{\omega \to 0} \frac{\mathrm{Im}\, G^R_{i[\Delta H, J_\phi] i[\Delta H, J_\phi]}}{\omega}$$

now involves knowledge of the density-density retarded Green's function **of the normal fluid inside the vortex cores**, for which we assume the form (neglecting thermoelectric effects)

$$G^R_{\rho\rho} = \frac{k^2 D \chi_{\rho\rho}}{-i\omega + Dk^2}, \qquad D = \frac{\sigma_n}{\chi_{\rho\rho}}$$

- We can compute the decay rate due to this interaction as

$$\Omega = \frac{\rho_s}{m^2} \frac{n_f \pi r_v^2}{2\sigma_n}$$

where r_v is the typical radius of a vortex core, n_f the vortex density and σ_n the conductivity of the normal state.

- This is exactly [Bardeen-Stephen'65], in a fully quantum treatment.

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Summary so far

- Objective: construct a theory of slowly decaying supercurrent due to quantum fluctuations. We have done this in two ways.
- First, we constructed incoherent superfluid hydrodynamics with mobile vortices, showing that the conductivity takes the form

$$\sigma(\omega) = \sigma_0 + \frac{\rho_s}{m^2} \frac{1}{\Omega - i\omega}$$

  where Ω is the 'Magnus' force felt by vortices.

- Second, we recovered it using the memory matrix approach. Turning on an on-site Coulomb interaction, we computed the decay rate as

$$\Omega = \frac{\rho_s}{m^2} \frac{n_f \pi r_v^2}{2\sigma_n}$$

  which shows that phase relaxation is intimately connected with dissipation in the normal cores of the vortices for this interaction.

Why parity violation is important

- Magnetic fields play a central role in the study of superconductivity.
- Meissner effect, depairing of Cooper pairs above the critical field $H_{c2}\ldots$
- More importantly for this talk, the phase diagram of superfluid thin films displays a field-tuned superfluid-insulator phase transition, possibly with an intermediate metallic phase.
- In the incoherent limit, since terms like $F^{ij}u_j$ have been integrated out (no normal velocity), the magnetic field only appears implicitly through terms which violate parity.

In a nutshell

- We now authorize terms in the constitutive relations like

$$j^i - \frac{\rho_s}{m}\left(\delta^{ij} - \rho_v \epsilon^{ij}\right) u^j_\phi = -\hat{\alpha}^{ij}_1 \nabla^j s - \hat{\alpha}^{ij}_2 \nabla^j \rho + \cdots ,$$

$$\tfrac{1}{T} j^{Q\,i} + \frac{\rho_s}{m} s_v \epsilon^{ij} u^j_\phi = -\hat{\beta}^{ij}_1 \nabla^j s - \hat{\beta}^{ij}_2 \nabla^j \rho + \cdots$$

with

$$\hat{\alpha}^{ij}_a = \alpha_a \delta^{ij} + \alpha^H_a \epsilon^{ij} , \qquad \hat{\beta}^{ij}_a = \beta_a \delta^{ij} + \beta^H_a \epsilon^{ij} ,$$

where the $\hat{\alpha}$ and $\hat{\beta}$ are related to the incoherent conductivities.

- More importantly, there are new contributions to the vortex current

$$j^i_v - \frac{m}{2\pi}\Omega \epsilon^{ij} u^j_\phi - \frac{m}{2\pi}\Omega^H u^i_\phi = -\frac{s_v}{2\pi}\nabla^i T - \frac{\rho_v}{2\pi}\nabla^i \mu - \gamma \nabla^i n_v + \cdots .$$

There is a new force term Ω_H.

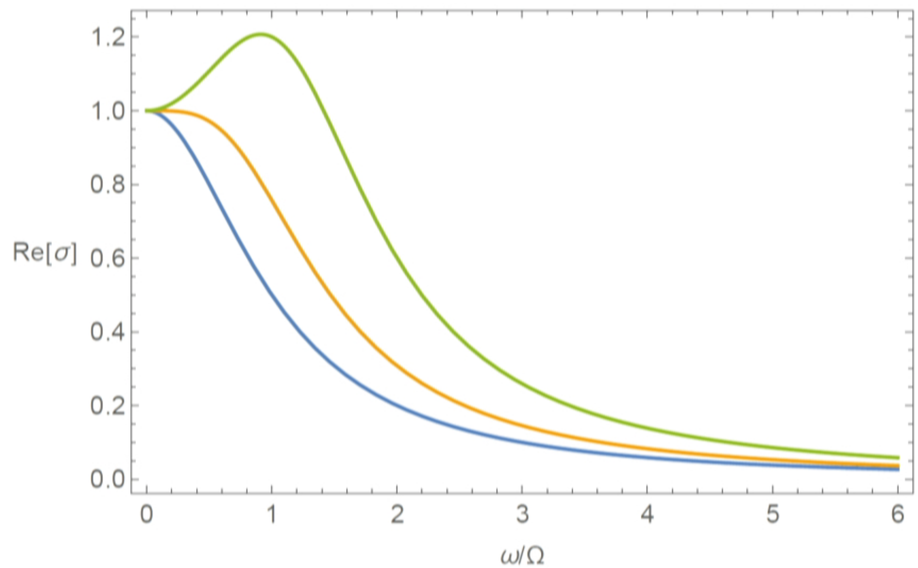
- The equations of motion and Josephson relation are unchanged.

Supercyclotron pole and optical conductivity

- Computing the conductivities as before reveals the appearance of a 'supercyclotron' pole in the retarded Green's functions. For instance,

$$\sigma(\omega) = \frac{\rho_s\left(1+\rho_v^2\right)}{m^2}\frac{-i\omega+\Omega}{\left(-i\omega+\Omega\right)^2+\left(\Omega^H\right)^2}+\sigma_0$$

- The peak only moves off the vertical axis once $\Omega_H > \Omega/\sqrt{3}$ (large enough parity violation).



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There is a new force term Ω_H.

- The equations of motion and Josephson relation are unchanged.

Nernst coefficient et Lorentz ratios

- The Nernst signal (transverse electric field generated by a temperature gradient in the presence of a magnetic field) is related to vortex physics, as expected from previous work [Huebener-Seher'70]

$$e_N \equiv -(\hat{\rho}\hat{\alpha}_-)_{yx} = \frac{s_v}{1+\rho_v^2}$$

- For small Ω and Ω_H, we can also calculate

$$L \equiv \frac{\kappa}{\sigma T} \ll 1\,, \qquad L^H \equiv \frac{\kappa^H}{\sigma^H T} \ll 1$$

where κ is the heat conductivity with open circuit boundary conditions. This is reminiscent of the Wiedemann-Franz law in strongly-coupled theories with weak momentum relaxation [Mahajan-Barkeshli-Hartnoll'13].

Memory matrix and parity violation

- Here as well we can recover these results from the memory matrix.
- Hence we can also compute both Ω and Ω_H by turning on the appropriate parity-violating Hamiltonian deformation.

$$\Omega = \rho_s M_{J^x_\phi J^y_\phi} \,, \qquad \Omega_H = -\rho_s \left(M_{J^x_\phi J^y_\phi} + N_{J^x_\phi J^y_\phi} \right) , \quad N_{J^x_\phi J^y_\phi} = \chi_{J^x_\phi \dot{J}^y_\phi}$$

- For the flux-flow resistance

$$\Omega_H = \frac{\rho_s n_f \pi r_v^2}{\chi_{\rho\rho} m} \chi_\rho \nabla \times j_\phi$$

Chern-Simons deformation

- A rather universal possibility is turning on a Chern-Simons term for an emergent gauge field, which leads to

$$\Delta H = \frac{\lambda'}{2} \int \frac{d^2k}{(2\pi)^2} \frac{\rho_{-k} (\nabla \times j)^z_k}{k^2} + \text{h.c.}\,.$$

and

$$i[\Delta H, J^i_\phi] = -\frac{\lambda'}{m} \lim_{k\to 0} \epsilon^{ij} j^{T\,j}\,.$$

- This leads to decay rates Ω, Ω_H set by the conductivity of the normal component σ_0, σ_0^H – no BKT input needed:

$$\Omega = \frac{\lambda'^2 \rho_s}{m^2} \sigma_0 + O(\lambda'^3)\,, \qquad \Omega_H = \frac{\lambda'^2 \rho_s}{m^2}(1 - \sigma_0^H) + O(\lambda'^2)$$

Ω is parametrically suppressed compared to Ω_H.

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$$\Delta H = \frac{\lambda'}{2} \int \frac{d^2k}{(2\pi)^2} \frac{\rho_{-k} (\nabla \times j)^z_k}{k^2} + \text{h.c.}\,.$$

and

$$i[\Delta H, J^i_\phi] = -\frac{\lambda'}{m} \lim_{k \to 0} \epsilon^{ij} j^{T\,j}\,.$$

- This leads to decay rates Ω, Ω_H set by the conductivity of the normal component σ_0, σ_0^H – no BKT input needed:

$$\Omega = \frac{\lambda'^2 \rho_s}{m^2} \sigma_0 + O(\lambda'^3)\,, \qquad \Omega_H = \frac{\lambda'^2 \rho_s}{m^2} (1 - \sigma_0^H) + O(\lambda'^2)$$

Ω is parametrically suppressed compared to Ω_H.

Summary and Outlook

- We have seen how to construct a theory a weak quantum phase fluctuations, both when parity is violated or not.
- These phase fluctuations give rise to sharp peaks in the optical conductivities.
- With parity, vortices play a prominent role in relaxing the phase, and we recover previous results but in a fully quantum treatment.
- When parity is violated, a supercyclotron pole appears.

Summary and Outlook

- While most of the discussion has been phrased in terms of hydrodynamics for physical transparency, the memory matrix formalism is a more systematic tool to capture the effects of long-lived operators, as it does not need extra assumptions.
- We have given two examples of Hamiltonian deformations, but others surely exist and our framework would still apply.
- What can we say about $T = 0$ metallic phases? This depends on whether the parameters $(n_f, \sigma_0, \sigma_0^H)$ which control the dissipation survive at $T = 0$.
- Experiments: some version of particle-hole symmetry $(\chi_{JP} = \rho = 0)$ seems to be at play $\Omega_H \ll \Omega$. How do we understand this in our formalism $(\chi_{JJ_\phi} \neq 0)$?