

Title: Generalized Global Symmetries and Magnetohydrodynamics

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Abstract: I will discuss a global symmetry approach to constructing the most general effective field theory of magnetohydrodynamics.

# Generalized Global Symmetries and MHD

W / S. Grozdanov (Leiden)

N. Iqbal (Amsterdam)

What is EM?

# Generalized Global Symmetries and MHD

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What is EM?

$$1) \frac{1}{g_2} \partial_\mu F^{\mu\nu} = J_e^\nu$$

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$$F_{\rho\sigma} = \partial_{[\rho} A_{\sigma]}$$

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What is EM?

$$1) \frac{1}{g^2} \partial_\mu F^{\mu\nu} = J_e^\nu$$

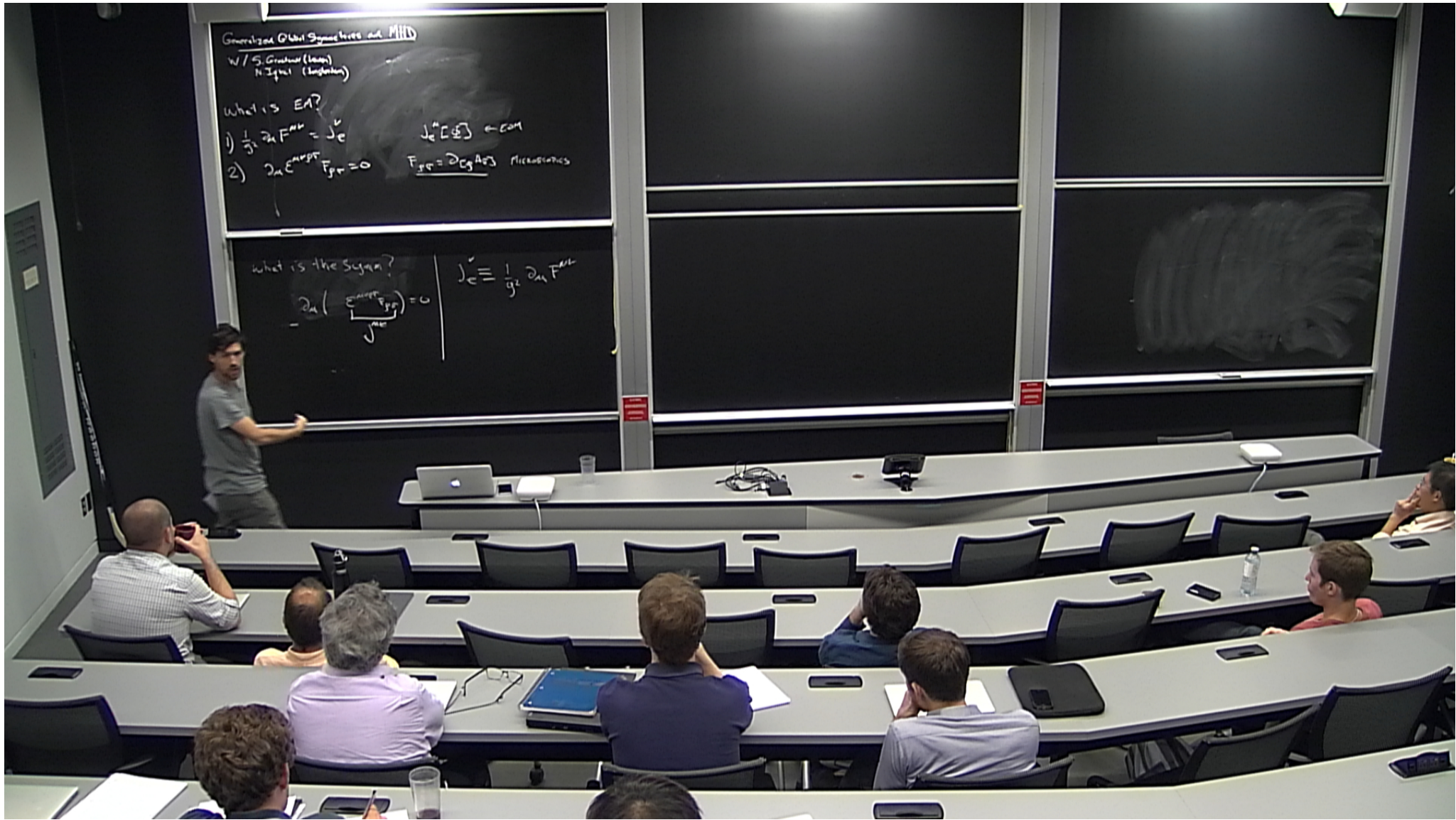
$$2) \partial_\mu \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} = 0$$

$$J_e^\mu [\Phi] \leftarrow \text{E2M}$$

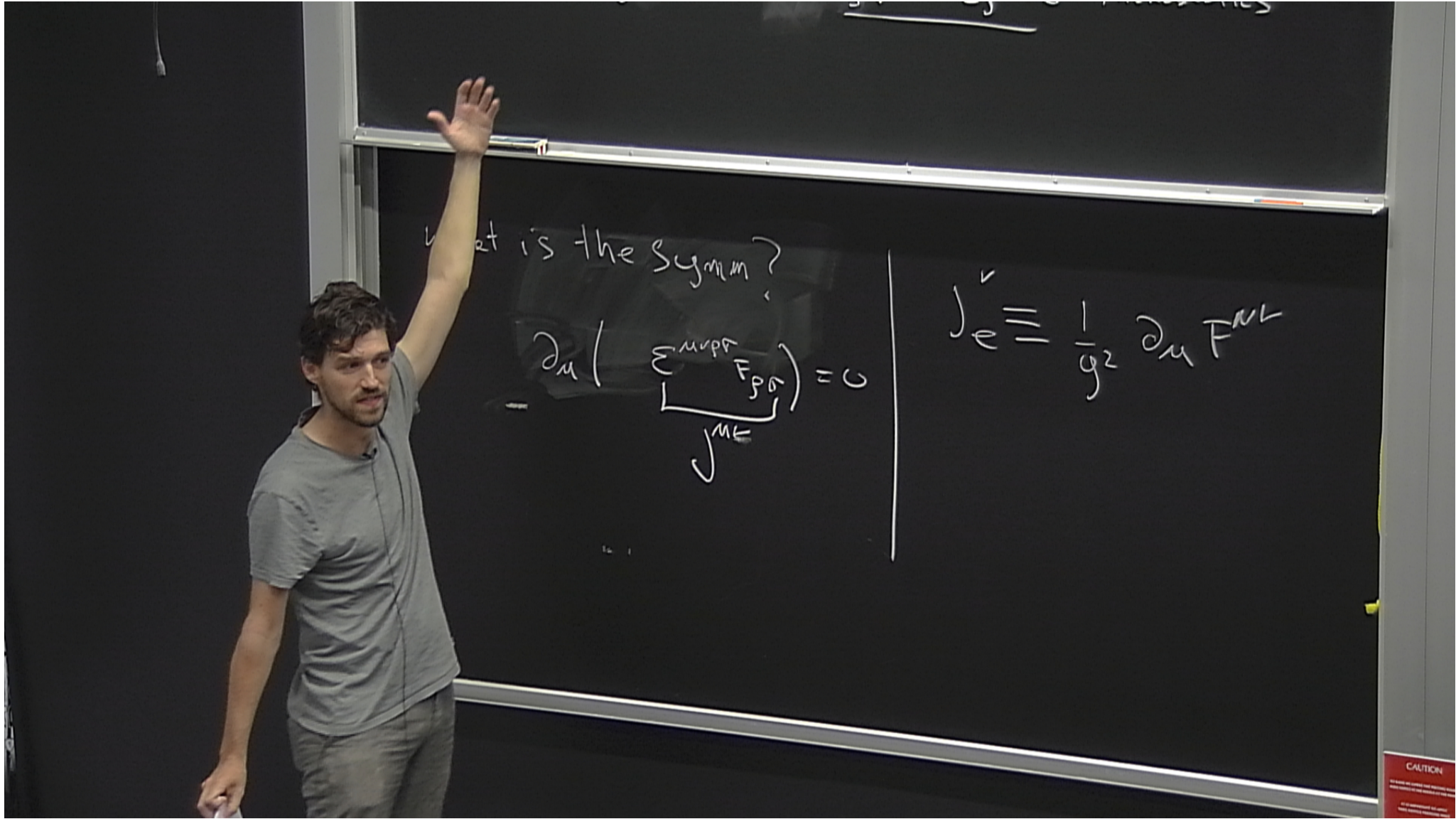
$$\underline{F_{\rho\sigma} = \partial_{[\rho} A_{\sigma]}} \quad \text{MICROSCOPICS}$$

what is the symm?

$$\partial_\mu \left( \underbrace{\sum_{\mu\nu\rho\sigma} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}}_{j_{\mu\nu}} \right) = 0$$







what is the Symm?

$$\partial_\mu \left( \underbrace{\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}}_{J^{\mu\nu}} \right) = 0$$

$$J_e^\nu \equiv \frac{1}{g^2} \partial_\mu F^{\mu\nu}$$

1)  $\frac{1}{g^2} d * F = * J_e$

2)  $dF = 0$

$$\left. \begin{array}{l} J = *F \\ \frac{1}{g^2} dJ = *J_e \\ d * J = 0 \end{array} \right\}$$

0

$$J_e = \frac{1}{g^2} \partial_\mu F^{\mu\nu}$$

- 1)  $\frac{1}{g^2} d * F = * J_e$
- 2)  $dF = 0$

$$J = * F \quad \left. \begin{array}{l} \frac{1}{g^2} dJ = * J_e \\ d * J = 0 \end{array} \right\}$$

0-form Sym

$$\partial_\mu J^\mu = 0$$

$$\Rightarrow d * J = 0$$

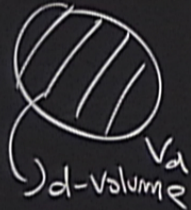
$\underbrace{\hspace{10em}}_{d-1 \text{ form}}$   
 $\underbrace{\hspace{10em}}_{d \text{ form}}$



0-form Sym

$$\partial_M J^M = 0$$

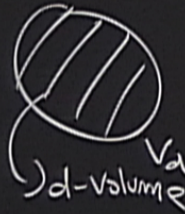
$$\Rightarrow d \underbrace{(*J)}_{\substack{\text{d-1 form} \\ \text{d-form}}} = 0$$


$$\int_{V_d} d(*J) = 0 \Rightarrow \int_{\partial V_d} *J = 0$$

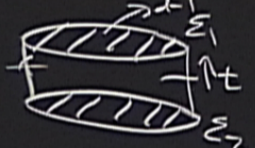
0-form Sym

$$\partial_M J^M = 0$$

$$\Rightarrow \underbrace{d \underbrace{*_J}_{d-1 \text{ form}}}_{d \text{ form}} = 0$$

  $\int_{V_d} d*_J = 0 \Rightarrow \int_{\partial V_d} *_J = 0$

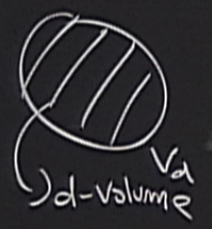
$\Rightarrow \int_{\Sigma_1} *_J - \int_{\Sigma_2} *_J = 0$



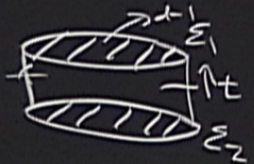
0-form sym

$$\partial_\mu J^\mu = 0$$

$$\Rightarrow \underbrace{d \underbrace{*\!J}_{d-1 \text{ form}}}_{d \text{ form}} = 0$$



$$\int_{V_d} d*\!J = 0 \Rightarrow \int_{\partial V_d} *\!J = 0$$



$$\Rightarrow \int_{S_1} *\!J - \int_{S_2} *\!J = 0$$

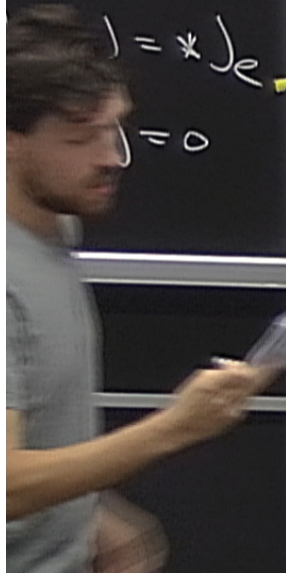
$$V_{d-1} = S_{d-2} \times [0, T]$$

1-form

$$= *\!J_e$$

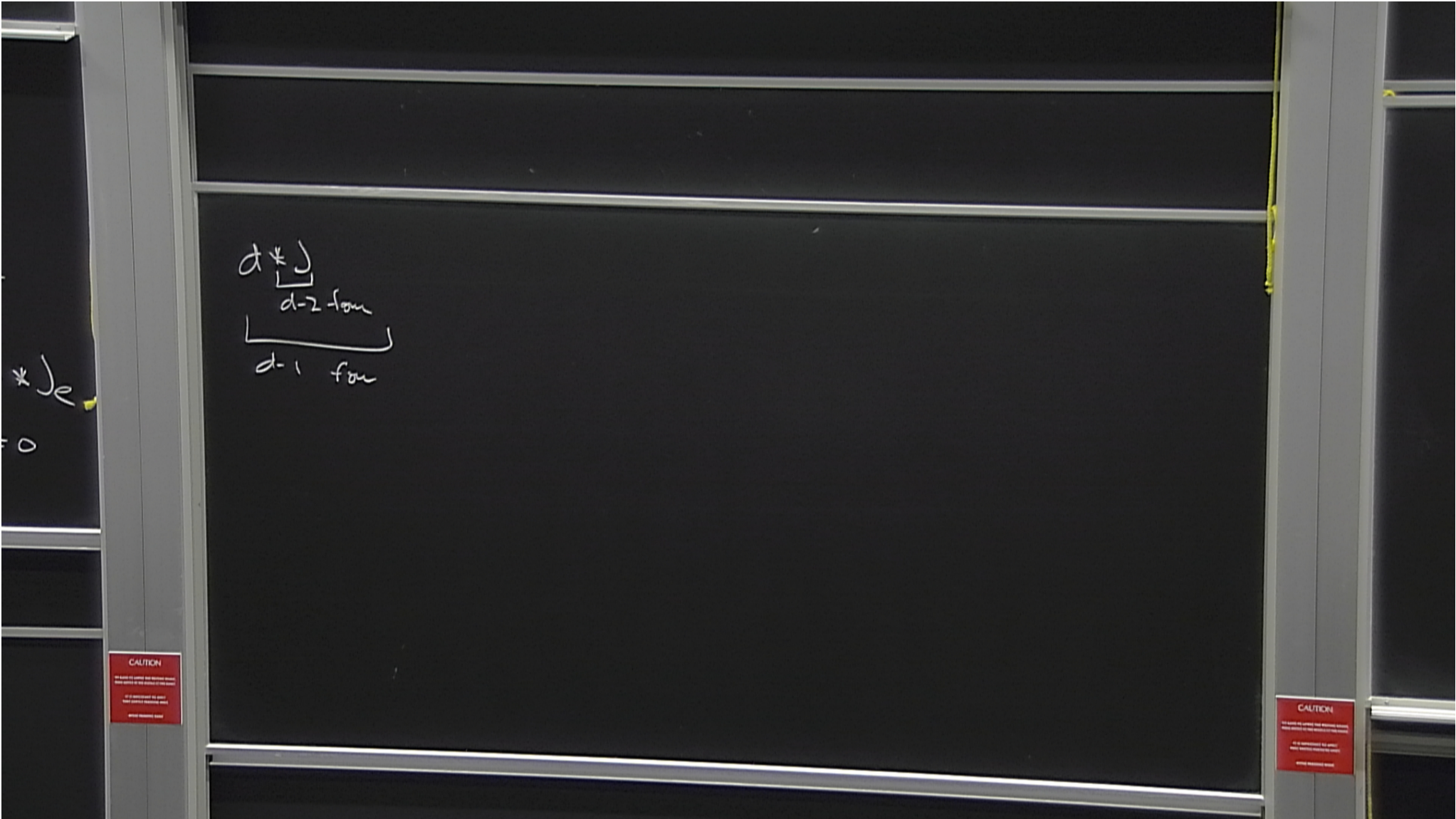
$$J = *\!J_e$$

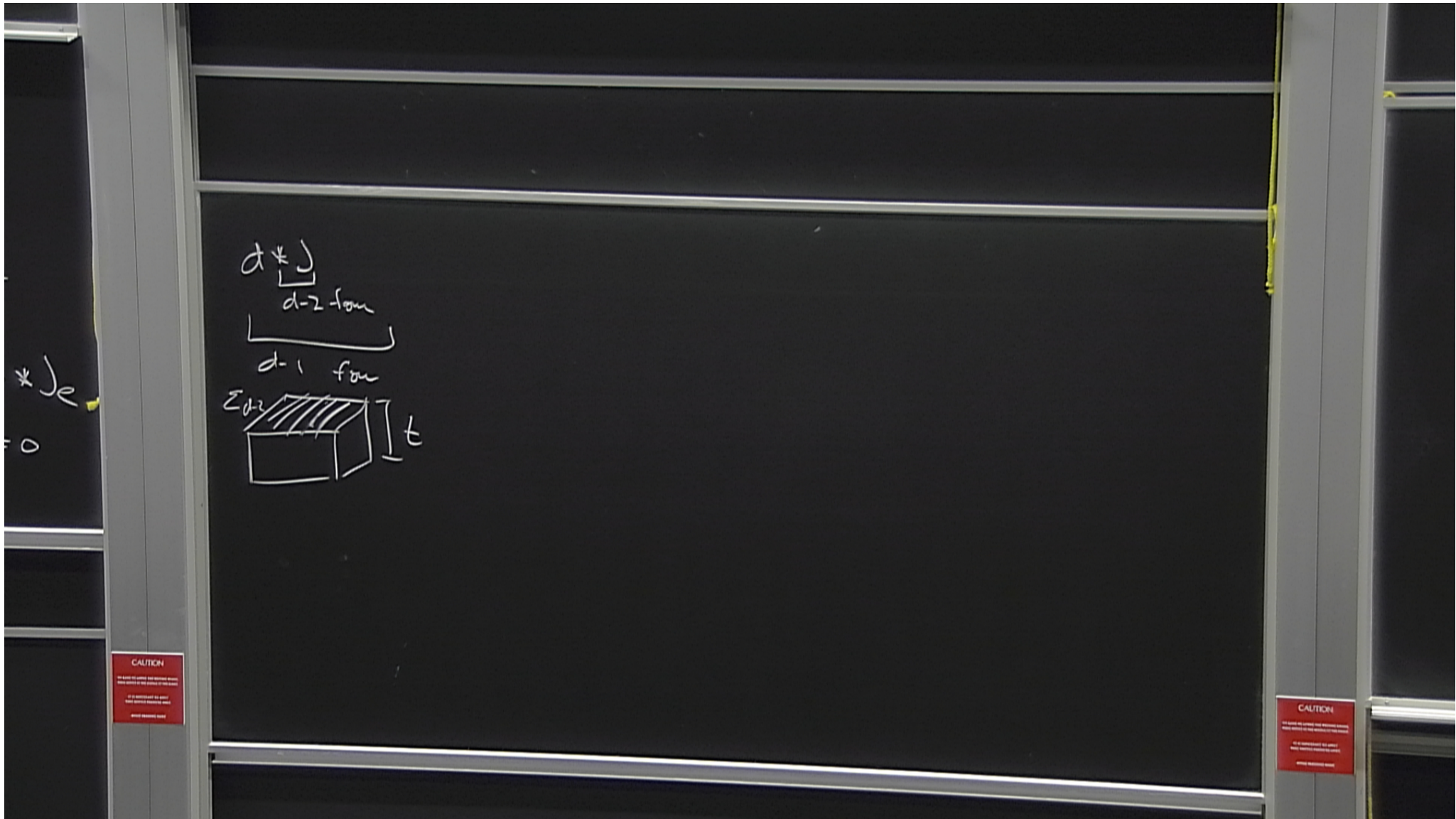
$$J = 0$$



CAUTION  
Do not touch the board when it is hot.  
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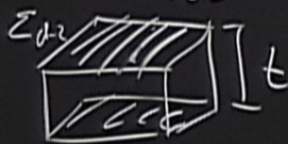
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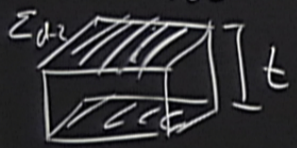
$$\underbrace{d^* \int_{\Sigma_{d-2}}}_{d-1 \text{ form}} \Rightarrow Q = \int_{\Sigma_{d-2}} *J = \int_{\Sigma_2} F$$



$d^*(\underbrace{\quad}_{d-2 \text{ form}}) \Rightarrow Q = \int_{\Sigma_{d-2}} *J = \int_{\Sigma_2} F$   
 $\underbrace{\quad}_{d-1 \text{ form}}$   
 $\Sigma_{d-2}$

$\underbrace{\quad}_{d-1 \text{ form}} \rightarrow 4\text{-form Sym.}$

$Q_{(10)} : G(x) \rightarrow e^{i q \Lambda} G(x)$



CAUTION  
 DO NOT TOUCH THE SURFACE OF THE BOARD  
 IF NECESSARY TO DO SO, PLEASE CONTACT THE STAFF

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$$\underbrace{d^*J}_{\substack{\text{d-2 form} \\ \text{d-1 form}}} \Rightarrow Q = \int_{\Sigma_{d-2}} *J = \int_{\Sigma_2} F$$

4-form Sym.

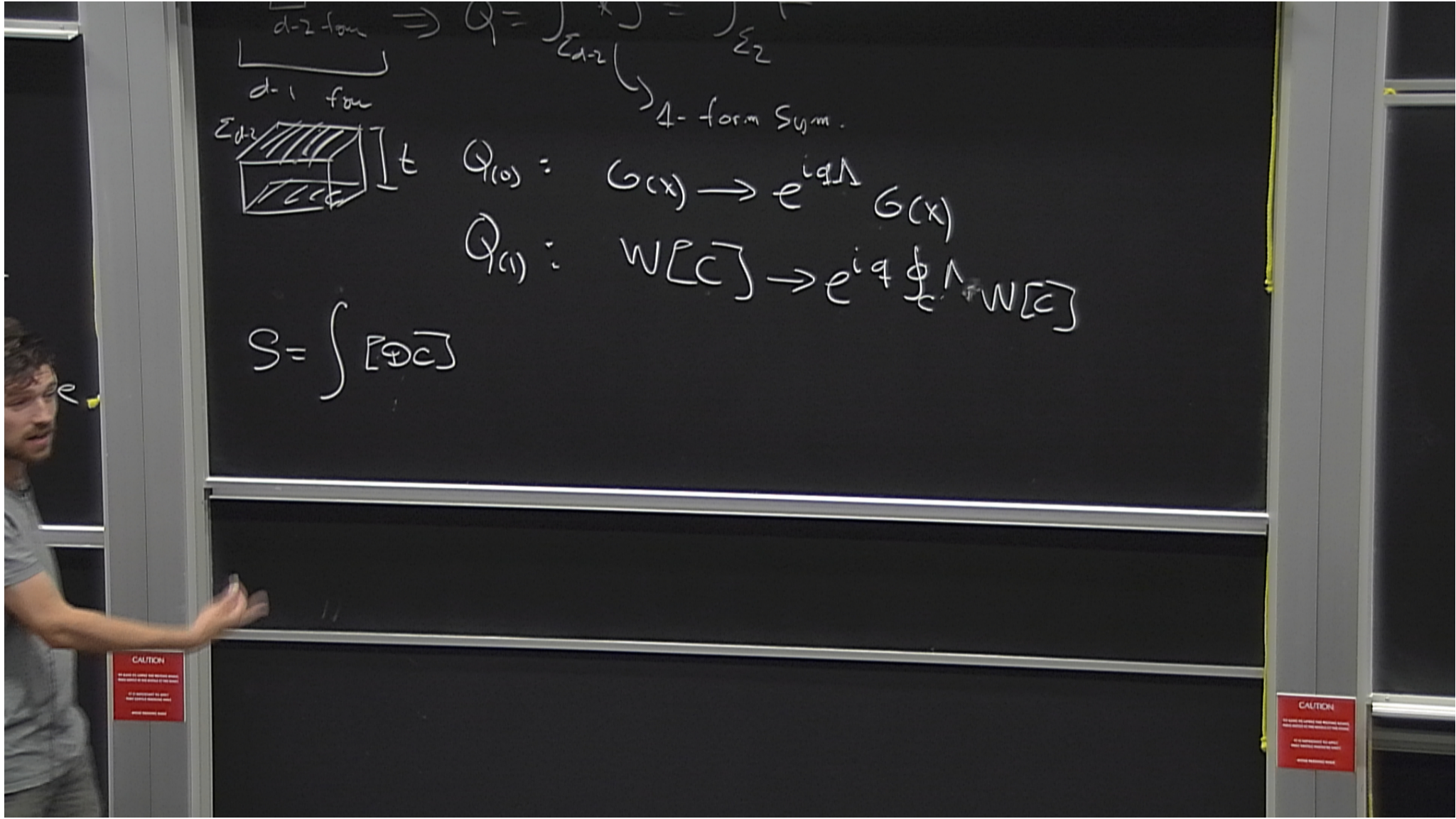


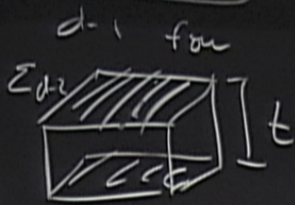
$$Q_{(0)} : G(x) \rightarrow e^{iq\Lambda} G(x)$$

$$Q_{(1)} : W[C] \rightarrow e^{iq \oint_C \Lambda} W[C]$$

CAUTION  
DO NOT TOUCH THE BOARD SURFACE  
OR THE BOARD OR THE BOARD SURFACE  
OR THE BOARD SURFACE

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4-form Sym.

$$Q_{(0)}: G(x) \rightarrow e^{iq\Lambda} G(x)$$

$$\Phi[\mathcal{C}] : \mathcal{C} \rightarrow \Phi \quad Q_{(1)}: W[\mathcal{C}] \rightarrow e^{iq\oint_{\mathcal{C}} \Lambda} W[\mathcal{C}]$$

$$S = \int [\Phi \bar{\mathcal{C}}] \left( \frac{\delta}{\delta C^{\mu\nu}} \Phi^* \right) \left( \frac{\delta}{\delta C^{\mu\nu}} \Phi \right) + V[\Phi^* \Phi]$$

$$\frac{\delta}{\delta C^{\mu\nu}} : \bigcirc \xrightarrow{\mu} \xrightarrow{\nu}$$

\*Je  
o

CAUTION  
DO NOT TOUCH THE BOARD WHEN  
IT IS BEING USED BY THE BOARD OR THE BOARD  
IT IS PROHIBITED TO TOUCH  
THE BOARD WHEN IT IS BEING USED  
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$$\underline{\Phi} \rightarrow e^{i\phi} \wedge \underline{\Phi}$$

$$\underline{\Phi} \rightarrow e^{i\phi} \underline{\Phi}$$
$$D_{\mu\nu} = \frac{\delta}{\delta c^{\mu\nu}} - B_{\mu\nu}$$

CAUTION  
DO NOT TOUCH THE BOARD WHEN  
IT IS BEING USED FOR  
TEACHING PURPOSES.

$$\underline{\Phi} \rightarrow e^{i\phi} \underline{\Phi}$$

$$D_{\mu\nu} = \int \frac{\delta_{\mathcal{L}}}{\delta C_{\mu\nu}} - i q B_{\mu\nu}$$

$$B_{\mu\nu} \rightarrow B_{\mu\nu} + \partial_{\mu} \Lambda_{\nu} - \partial_{\nu} \Lambda_{\mu}$$



$$D_{\mu\nu} = \frac{\partial}{\partial x^{\mu\nu}} - iq\gamma B_{\mu\nu}$$

$$B_{\mu\nu} \rightarrow B_{\mu\nu} + \partial_{\mu}\Lambda_{\nu} - \partial_{\nu}\Lambda_{\mu}$$

$$\Rightarrow \Phi \{E, C\} = \bar{\Phi}_0$$

$$\Lambda_{\mu} \rightarrow \tilde{A}_{\mu}$$

CAUTION  
DO NOT TOUCH THE BOARD WHEN  
IT IS BEING USED BY THE INSTRUCTOR  
OR VISITOR

$$\underline{\Phi} \rightarrow e^{i\phi\Lambda} \underline{\Phi}$$

$$D_{\mu\nu} = \frac{\delta}{\delta C^{\mu\nu}} - \int \mathcal{B}_{\mu\nu}$$

$$B_{\mu\nu} \rightarrow \tilde{B}_{\mu\nu} + \partial_{[\mu} \Lambda_{\nu]}$$

$$\Rightarrow \underline{\Phi} \{C\} = \underline{\Phi}_0$$

$$\Lambda_{\mu} \rightarrow \tilde{A}_{\mu}$$

$$J = d\tilde{A} - B$$

$$c^{\dagger} \\ J^{\mu} = d\theta - A$$

CAUTION

DO NOT TOUCH THE BOARD WHEN  
IT IS BEING USED BY THE LECTURER  
IF YOU HAVE ANY QUESTIONS  
PLEASE ASK THE LECTURER

$$\underline{\Phi} \rightarrow e^{i\phi} \underline{\Phi}$$

$$D_{\mu\nu} = \int \frac{\delta_{\mu\nu}}{\epsilon} - \int \frac{B_{\mu\nu}}{4}$$

$$B_{\mu\nu} \rightarrow B_{\mu\nu} + \partial_{\mu} \Lambda_{\nu} - \partial_{\nu} \Lambda_{\mu}$$

$$\Rightarrow \underline{\Phi} \{E\} = \underline{\Phi}_0$$

$$\Lambda_{\mu} \rightarrow \tilde{A}_{\mu}$$

$$J = d\tilde{A} - B$$

$B=0$  gauge

$$dF=0$$

$$\Rightarrow d * J = 0 = d * d\tilde{A} = 0$$

$$J = d\tilde{A}$$

$$\Rightarrow dJ=0 \Rightarrow *J \Rightarrow \text{Electric flux}$$

$$Q_e = \oint_{\Sigma_2} J = \text{winding number}$$

$c \neq$

$$J^{\mu} = d\theta - A$$

Free photon

## Generalized Global Symmetries and MHD

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What is MHD?

$$T_{\mu\nu} = T_{\mu\nu}^{\text{fluid}} + T_{\mu\nu}^{\text{EM}}$$

$$\partial_\mu T^{\mu\nu} = F^{\nu\rho} J_{\rho e}$$

$$\sigma(E + v \times B) = J_e$$

+ Maxwell

U(1) GGS

$$\nabla_\mu J^{\mu\nu} = 0$$

$$\nabla_\mu T^{\mu\nu} = H^{\nu\rho\sigma} J_{\rho\sigma}$$

$$H = d\vec{B}$$

$$S = S_0 + \int d^4x \sqrt{g} B_{\mu\nu} J^{\mu\nu}$$

$$d * J = 0 \Rightarrow * J = dA$$

$$fS = \int B_{\nu} \epsilon_{\nu\rho\sigma} \partial_{\rho} A_{\sigma}$$

$$f_a = - \int \partial_{\rho} (B_{\mu\nu} \epsilon_{\mu\nu\rho\sigma}) A_{\sigma}$$

NORMAL LIFE

$$J = \nabla \cdot E$$

SOME

$$E = \nabla \cdot J$$

*[Faded handwritten notes and scribbles on the chalkboard]*

CAUTION  
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 IT IS EQUIPOTENTIAL TO GROUND  
 PLEASE REPORT ANY DAMAGE TO THE BOARD

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$$\rho = \exp\left(\frac{1}{T(x)}(H - \mu Q)\right)$$

$$SO(3,1) \xrightarrow{T} SO(3)$$

$$u^\mu$$

$$T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu + p g^{\mu\nu}$$

$$J^M = \int \rho u^M$$

$$\Phi = U - TS - Q\mu$$

$$dP = SdT + p d\mu$$

$$d\epsilon = T ds + \mu dp$$

$$SO(3,1) \xrightarrow{\text{bzo}} SO(2) \times SO(1,1) \xrightarrow{\text{T}}$$



Normal

10

CAUTION  
DO NOT TOUCH THE BOARD  
WHEN IT IS HOT

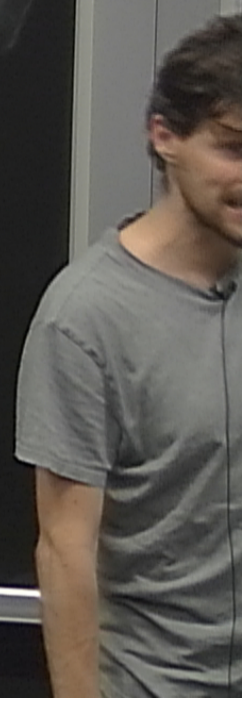
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WHEN IT IS HOT

$$Sd(3,1) \xrightarrow{\text{bzo}} SO(2) \times SO(1,1) \xrightarrow{\text{T}} SO(2)$$

Normal

*[The rest of the chalkboard is heavily scribbled over with dark chalk, obscuring any other text or diagrams.]*

CAUTION  
 THE BOARD IS HOTTER THAN THE SUN  
 AND COULD BE DAMAGED BY THE HEAT OF THE BOARD  
 IT IS RECOMMENDED TO WEAR  
 YOUR SAFETY GOGGLES  
 WHEN WEARING BOARD





$$SO(3,1) \xrightarrow{\text{bzt}} SO(2) \times SO(1,1) \quad \text{IT}$$

$$\Rightarrow u^\mu u_\mu = -1 \quad u^\mu h_{\mu\nu} = 0$$

$$h^\mu{}_\nu h_{\mu\sigma} = \delta^\mu{}_\sigma$$

$$T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu + p g^{\mu\nu} - \mu p h^{\mu\lambda} h^\nu{}_\lambda$$

$$J^{\mu\nu} = \rho u^{[\mu} h^{\nu]\lambda}$$

$\delta c$

$\xi_{\perp}, \xi_{\parallel}, \eta_{\perp}, \eta_{\parallel}, \Gamma_{\perp}, \Gamma_{\parallel}$

$\perp \cdot \parallel$

- Alfvén Waves

- Magnetosonic Waves

