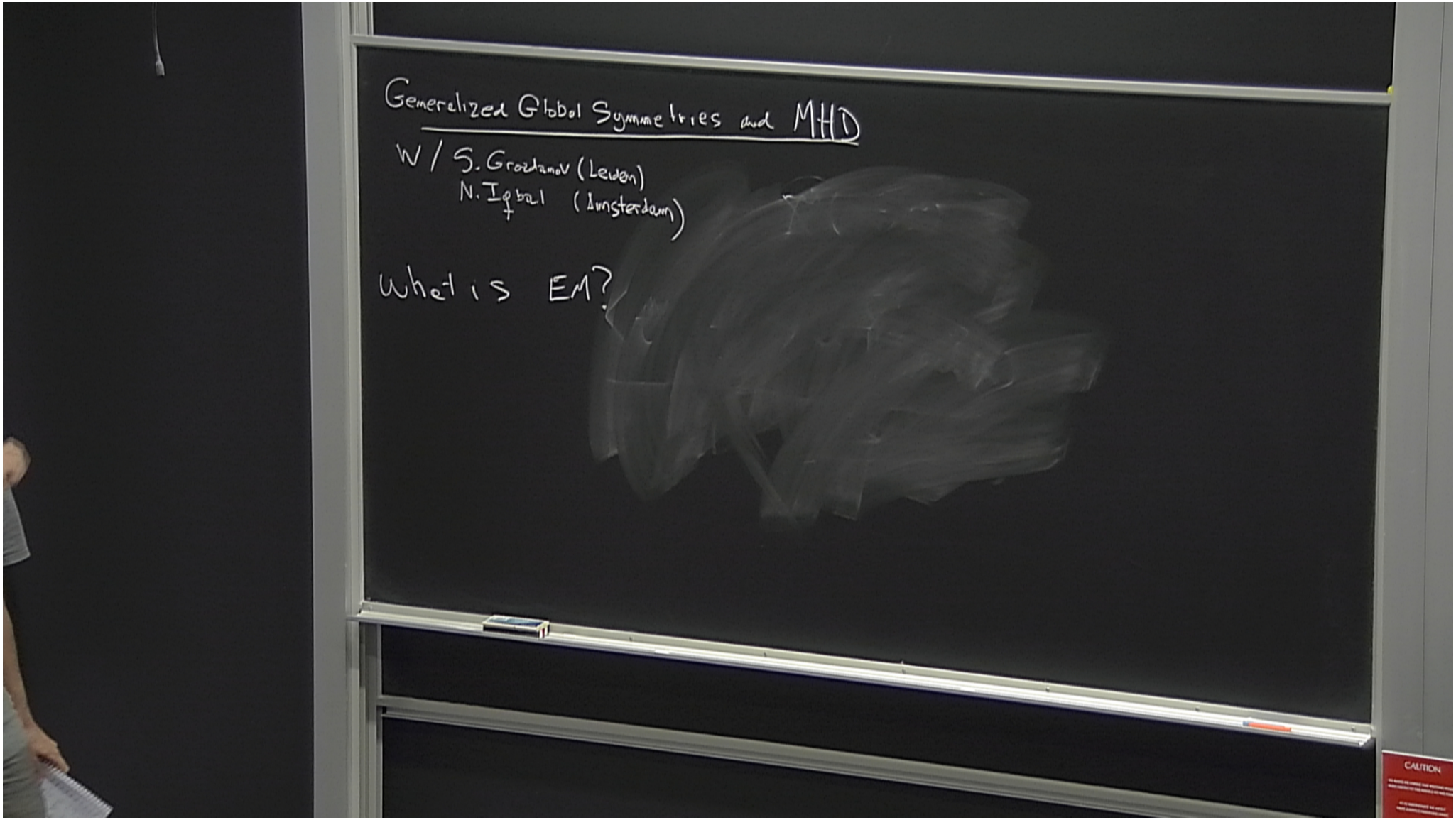
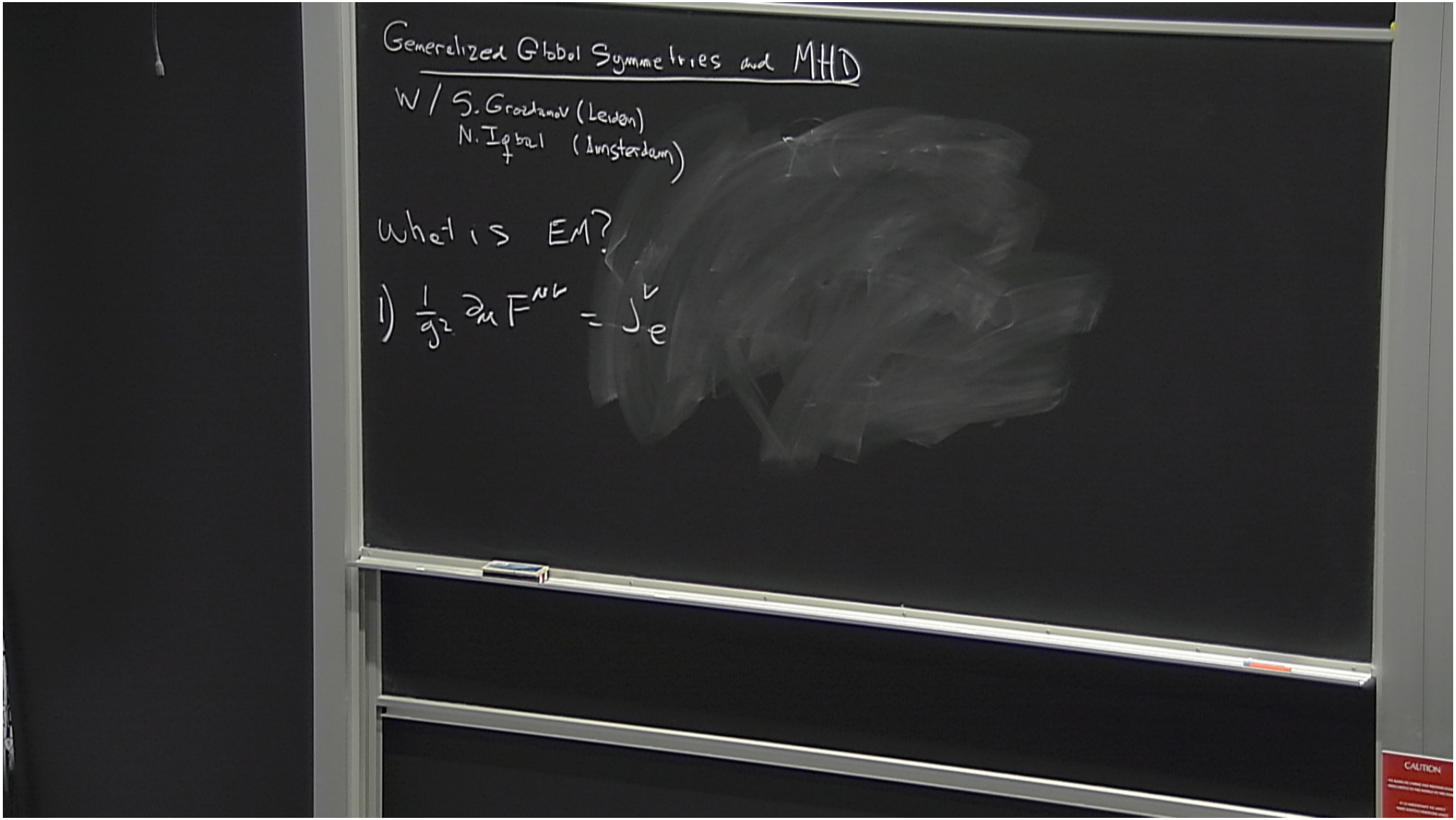
Title: Generalized Global Symmetries and Magnetohydrodynamics

Date: Aug 25, 2016 04:00 PM

URL: http://pirsa.org/16080050

Abstract: I will discuss a global symmetry approach to constructing the most general effective field theory of magnetohydrodynamics.


Generalized Global Symmetries and MHD
W/ S. Grozdanov (Leiden)
N. Iqbal (Amsterdam)
What is EM?


Generalized Global Symmetries and MHD
W/ S. Grozdanov (Leiden)
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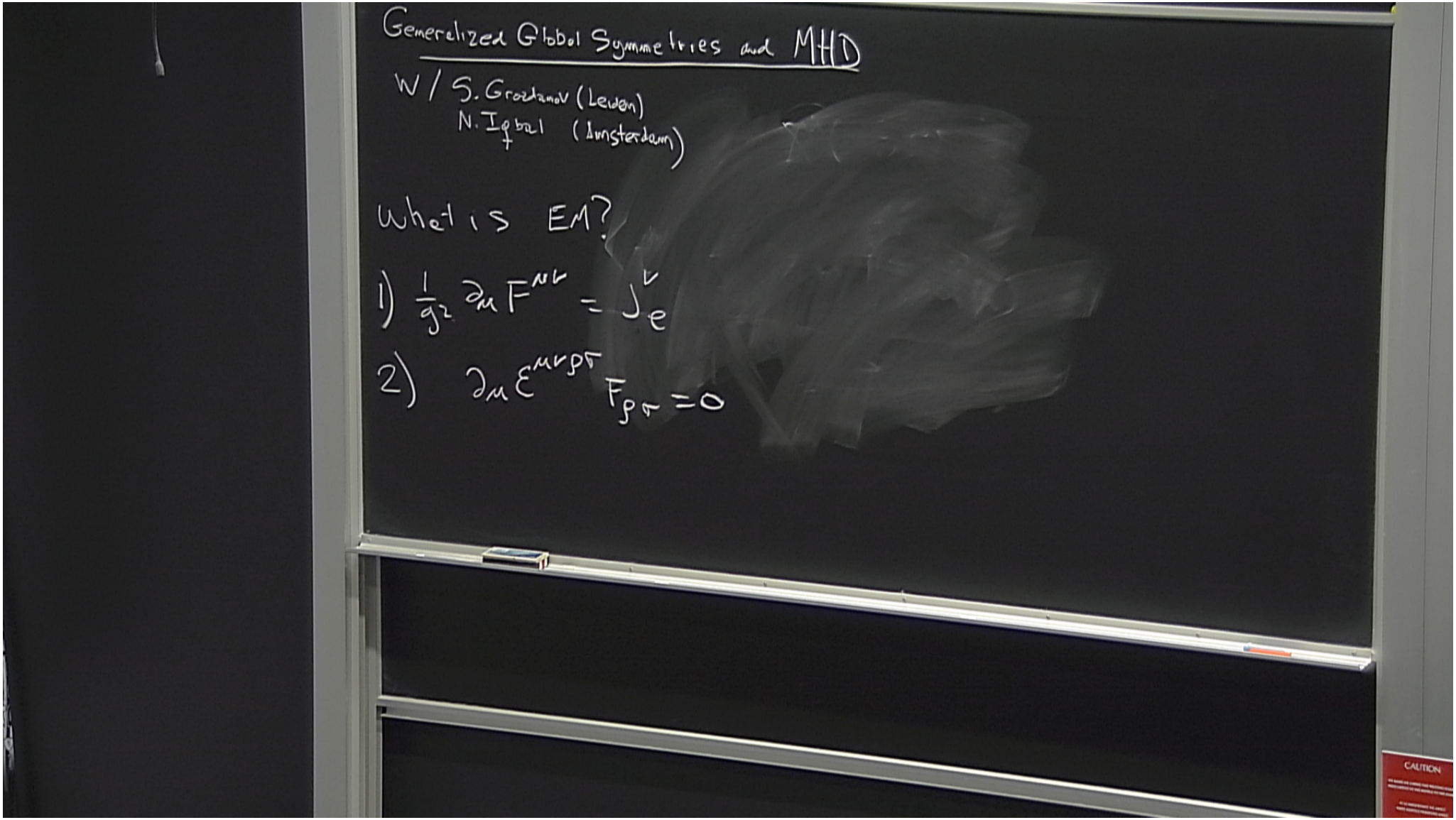
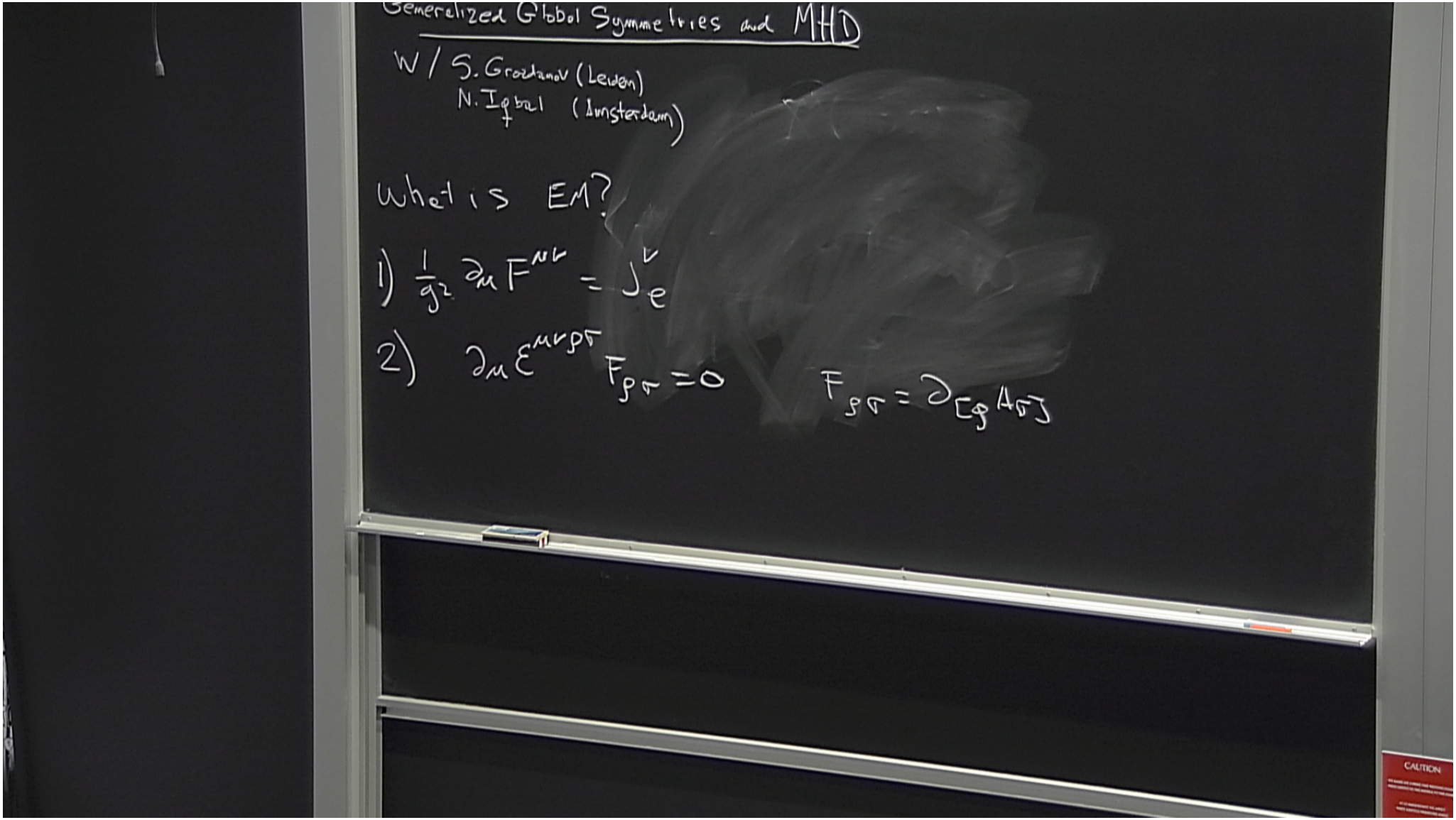
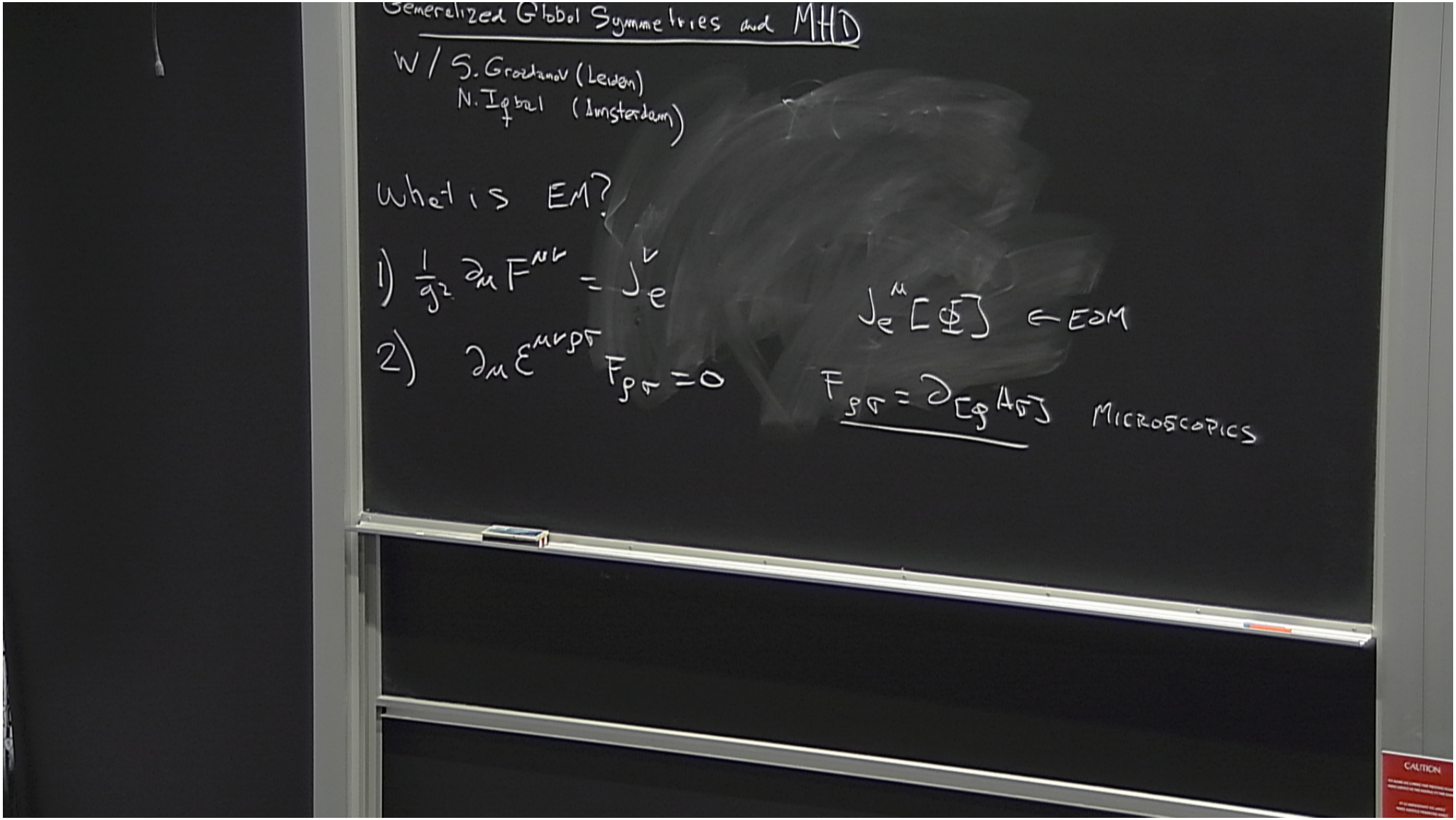
What is EM?
1) $\frac{1}{g^2}\partial_\mu F^{\mu\nu} = J_e^\nu$
CAUTION


Generalized Global Symmetries and MHD
W/ S. Grozdanov (Leiden)
N. Iqbal (Amsterdam)
What is EM?
1) $\frac{1}{g^2}\partial_\mu F^{\mu\nu} = J_e^\nu$
2) $\partial_\mu \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} = 0$
CAUTION


Generalized Global Symmetries and MHD
W/ S. Grozdanov (Leiden)
N. Iqbal (Amsterdam)
What is EM?
1) $\frac{1}{g^2}\partial_\mu F^{\mu\nu} = J_e^\nu$
2) $\partial_\mu \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} = 0$
$F_{\rho\sigma} = \partial_{[\rho} A_{\sigma]}$
CAUTION


Generalized Global Symmetries and MHD
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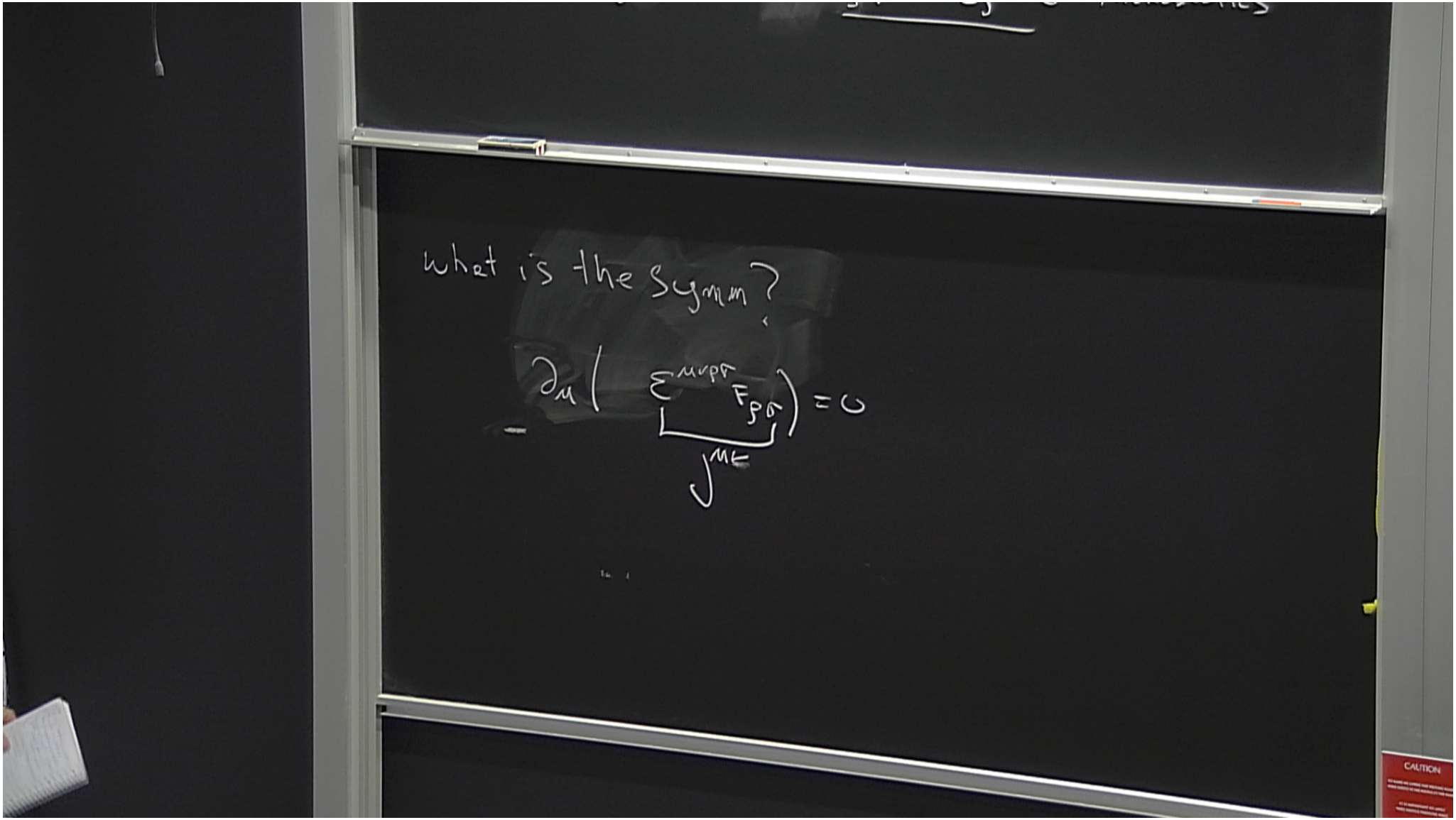
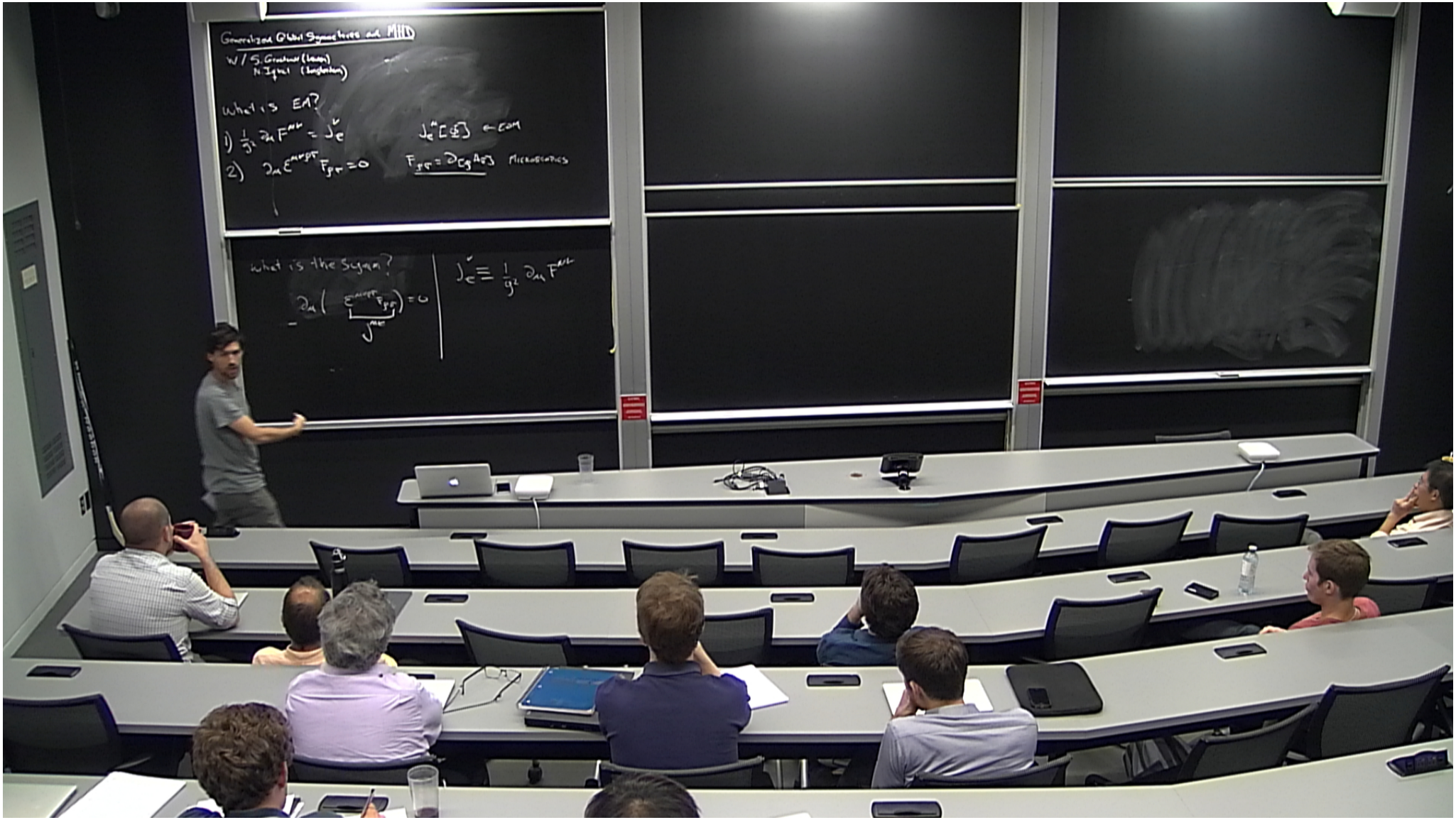
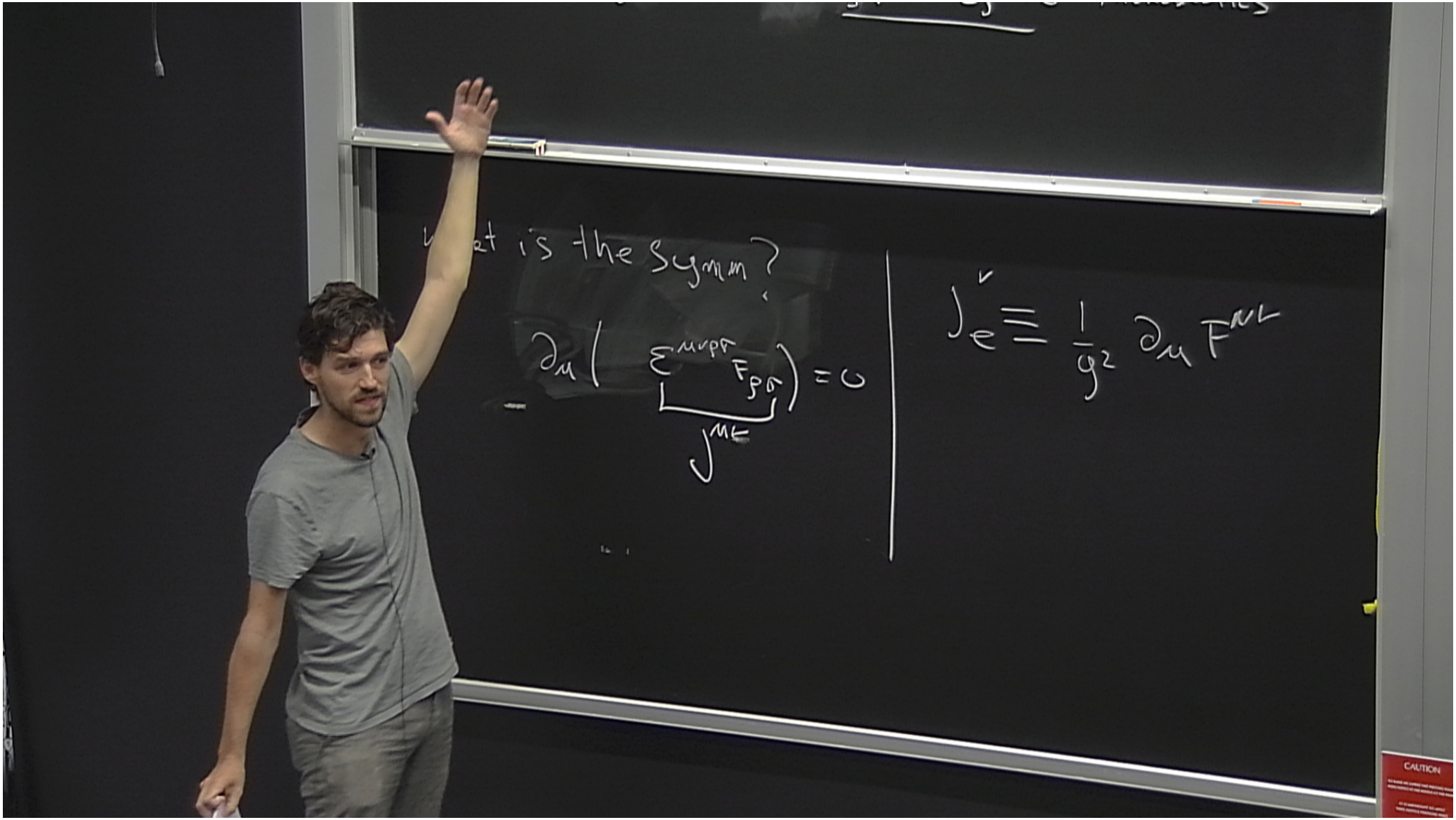
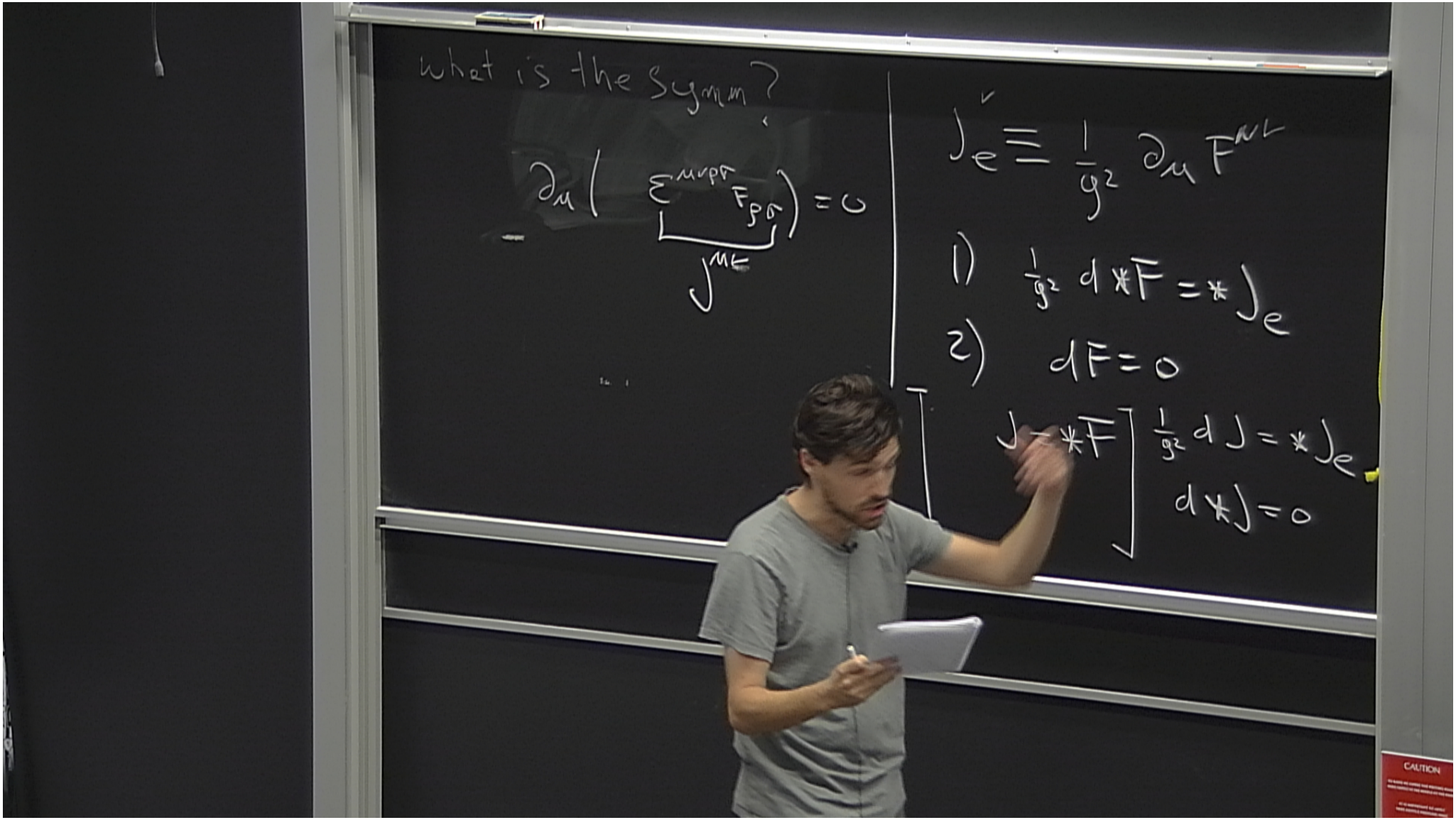
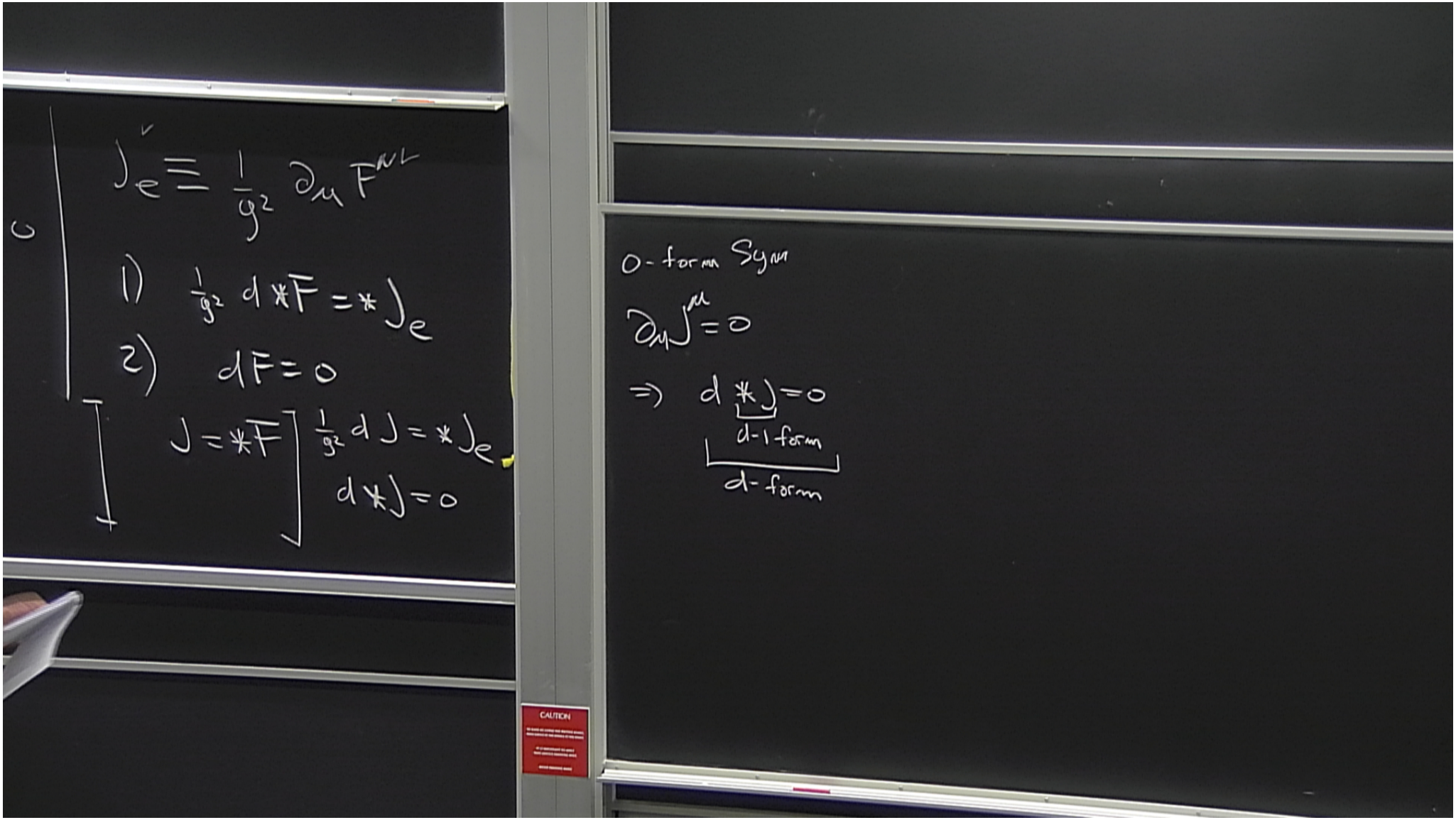
What is EM?
1) $\frac{1}{g^2}\partial_\mu F^{\mu\nu} = J_e^\nu$
2) $\partial_\mu \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} = 0$
$J_e^\mu[\Phi] \in$ EOM
$F_{\rho\sigma} = \partial_{[\rho} A_{\sigma]}$ MICROSCOPICS
CAUTION


what is the symm?
$\partial_\mu(\varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}) = 0$
$J^{\mu\nu}$
CAUTION


Generalized Global Symmetries and MHD
W/ S. Gukov (Caltech)
N. Iqbal (Amsterdam)
What is EM?
1) $\frac{1}{g^2}\partial_\mu F^{\mu\nu} = J_e^\nu$
$J_e^\mu[\Sigma]$ ← Wilson
2) $\partial_\mu \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} = 0$
$F_{\rho\sigma} = \partial_{[\rho} A_{\sigma]}$ Microscopics
What is the Sugra?
$-\partial_\mu(\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}) = 0$
$J^{\mu\nu}$
$J_e^\nu \equiv \frac{1}{g^2}\partial_\mu F^{\mu\nu}$


t is the symm?
$\partial_\mu(\varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}) = 0$
$J^{\mu\nu}$
$J_e^\nu \equiv \frac{1}{g^2}\partial_\mu F^{\mu\nu}$
CAUTION


what is the symm?
$\partial_\mu(\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma})=0$
$J^{\mu\nu}$
$J_e^\nu \equiv \frac{1}{g^2}\partial_\mu F^{\mu\nu}$
1) $\frac{1}{g^2} d*F = *J_e$
2) $dF=0$
$J - *F$
$\frac{1}{g^2} dJ = *J_e$
$d*J=0$
CAUTION

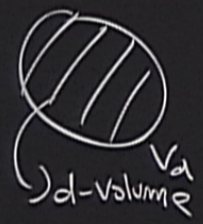

$J_e \equiv \frac{1}{g^2} \partial_\mu F^{\mu\nu}$
1) $\frac{1}{g^2} d*F = *J_e$
2) $dF = 0$
$J = *F$
$\frac{1}{g^2} dJ = *J_e$
$d*J = 0$
0-form Sym
$\partial_\mu J^\mu = 0$
$\Rightarrow d*J = 0$
d-1 form
d-form
CAUTION

0-form Sym

$\partial_\mu J^\mu = 0$

$\Rightarrow \; d\,\underbrace{*J}_{d-1\text{-form}} = 0$

d-form



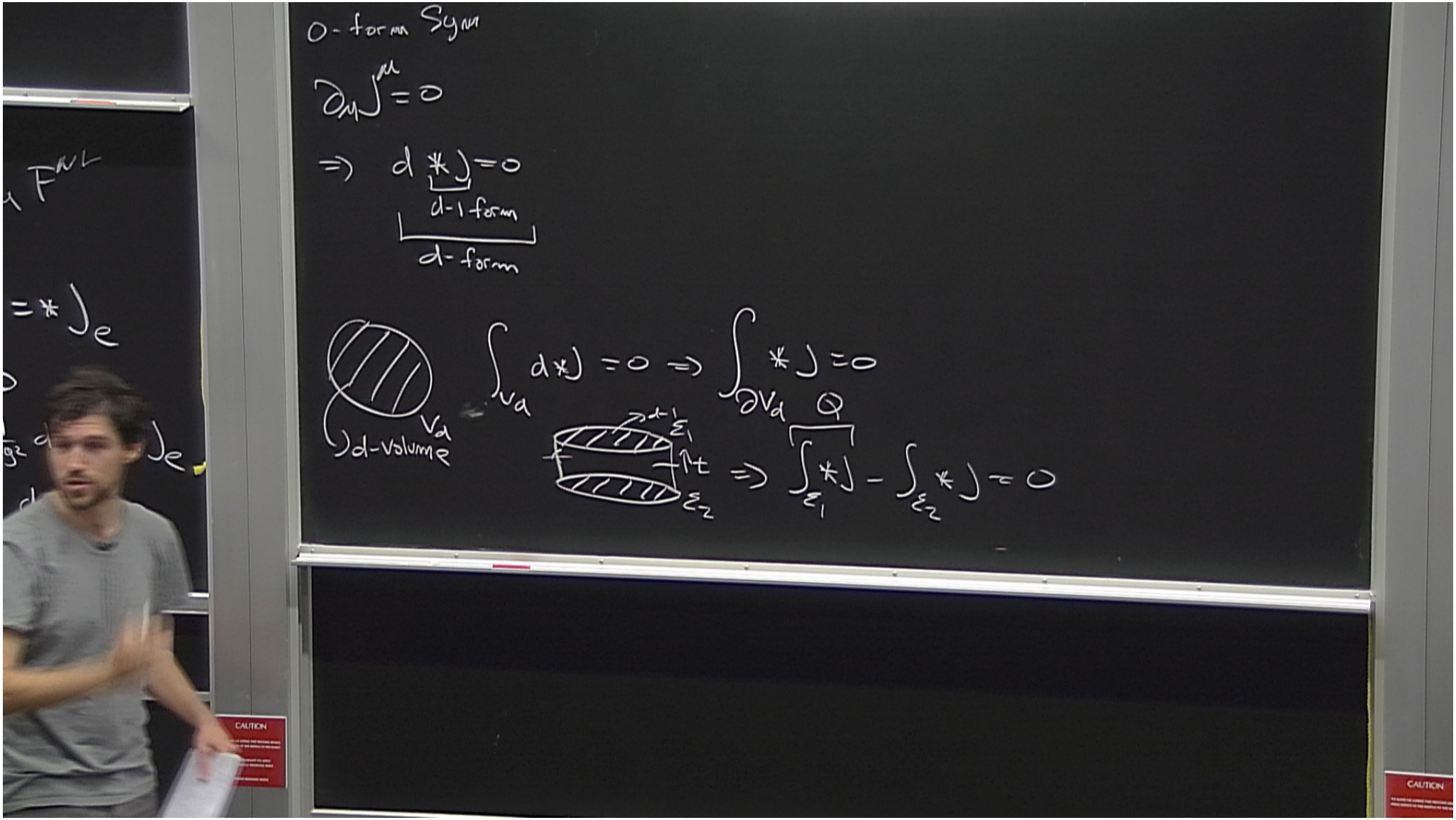
$\int_{V_d} d*J = 0 \Rightarrow \int_{\partial V_d} *J = 0$

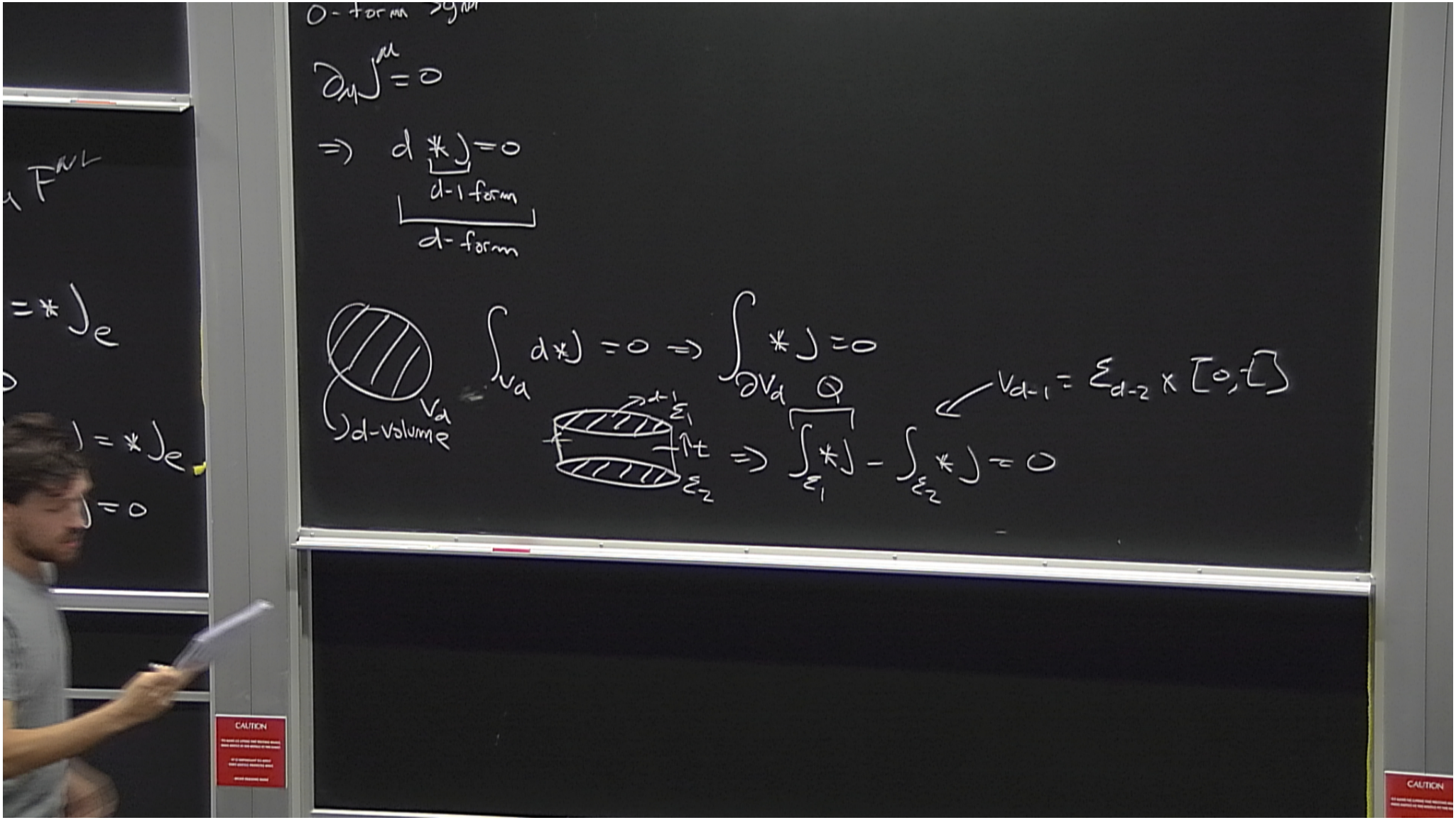
$F^{\mu\nu}$

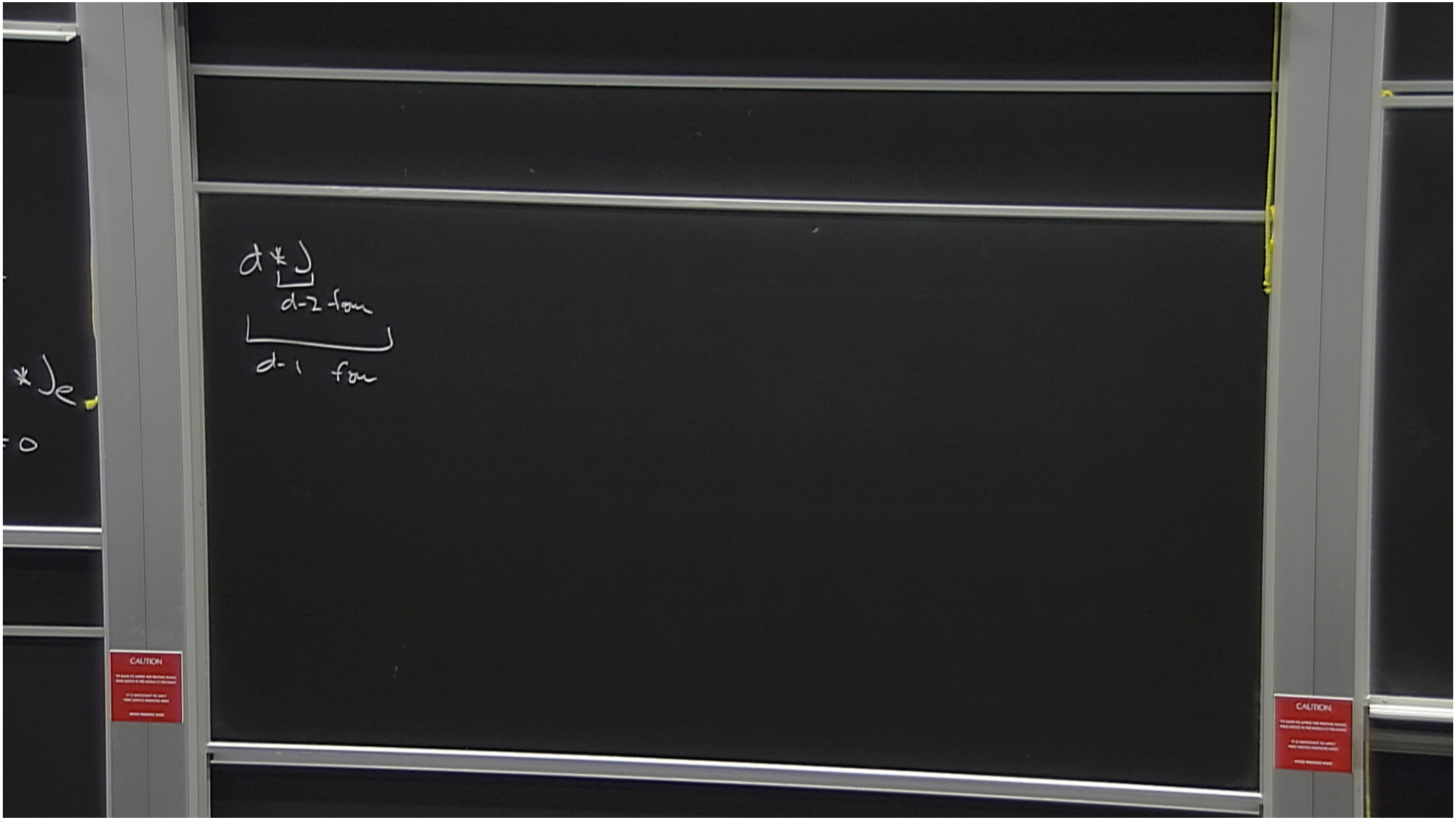
$= *J_e$

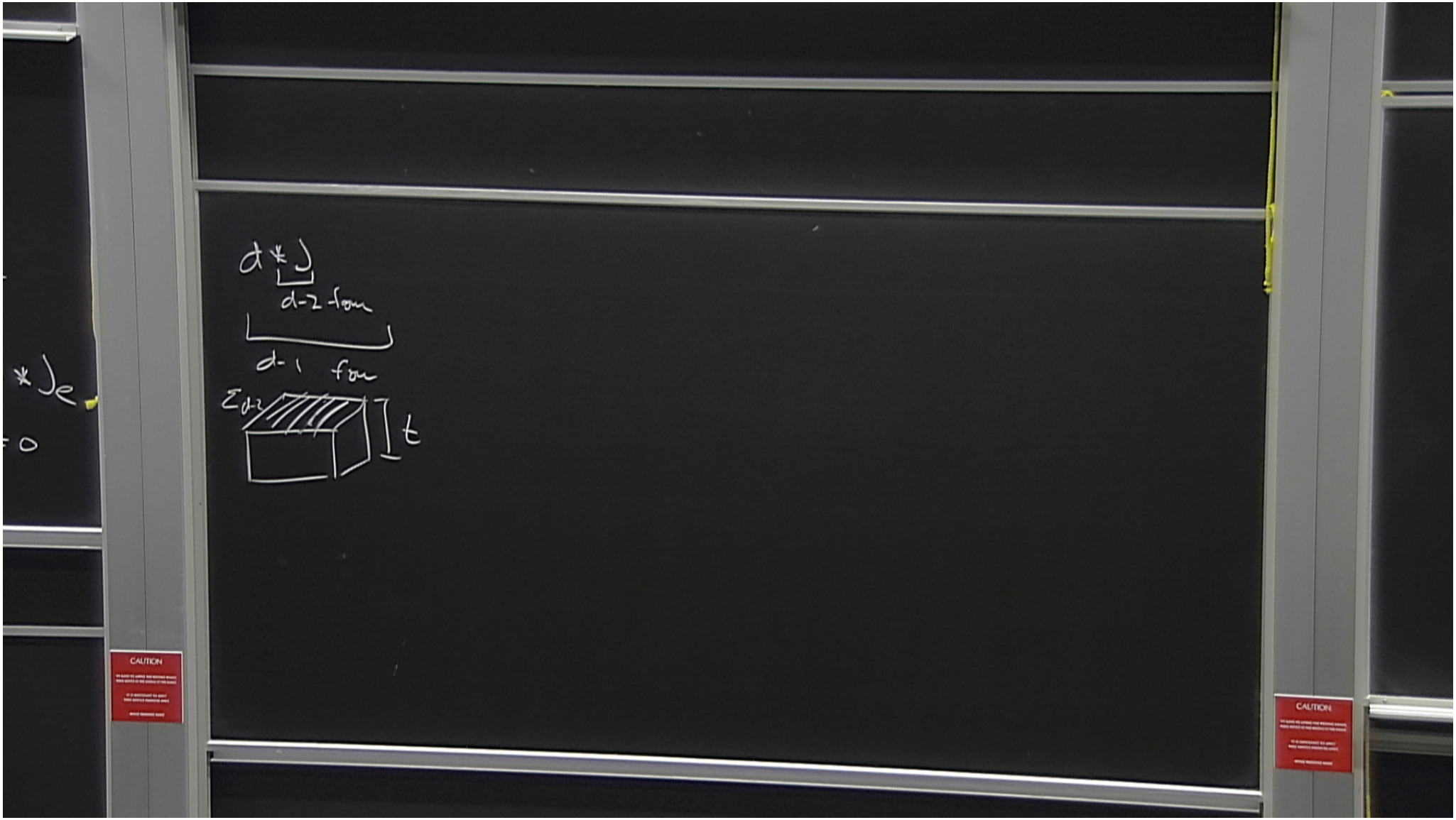
$\frac{1}{g^2} dJ = *J_e$

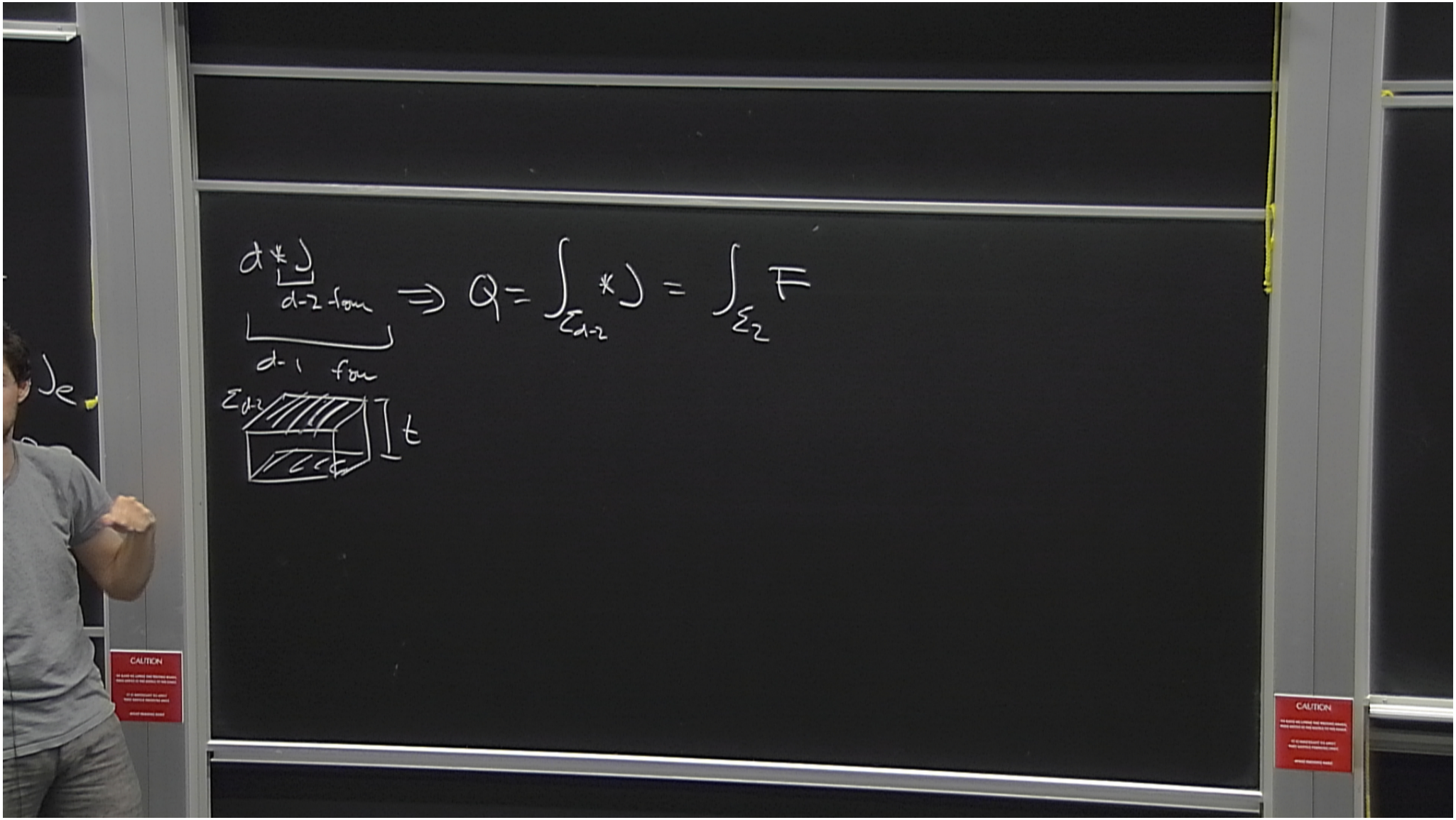
$d*J = 0$


0-form Sym
$\partial_\mu J^\mu = 0$
$\Rightarrow d * J = 0$
d-1 form
d-form
$F^{\mu\nu}$
$= * J_e$
J_e
d-volume
V_d
$\int_{V_d} d*J = 0 \Rightarrow \int_{\partial V_d} *J = 0$
$\int_{\Sigma_1} *J - \int_{\Sigma_2} *J = 0$
CAUTION


0-form
$\partial_\mu J^\mu = 0$
$\Rightarrow d\,{*J} = 0$
d-1 form
d-form
$F^{\mu\nu}$
$= *J_e$
$= *J_e$
$= 0$
V_d
d-volume
$\int_{V_d} d{*J} = 0 \Rightarrow \int_{\partial V_d} {*J} = 0$
∂V_d
Σ_1
Σ_2
t
$\Rightarrow \int_{\Sigma_1} {*J} - \int_{\Sigma_2} {*J} = 0$
$V_{d-1} = \Sigma_{d-2} \times [0, t]$
CAUTION
CAUTION

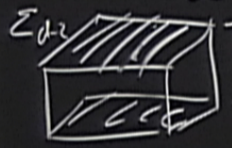

d * J
d-2 forms
d-1 forms


d * J
d-2 form
d-1 form
t


d * J
d-2 form
d-1 form
Σ_{d-2}
t
$\Rightarrow Q = \int_{\Sigma_{d-2}} *J = \int_{\Sigma_2} F$
CAUTION
CAUTION

$d * J$ ($d-2$ form) $\Rightarrow Q = \int_{\Sigma_{d-2}} * J = \int_{\Sigma_2} F$

($d-1$ form)

Σ_{d-2}  t

Σ_{d-2} ↪ 1-form Sym.

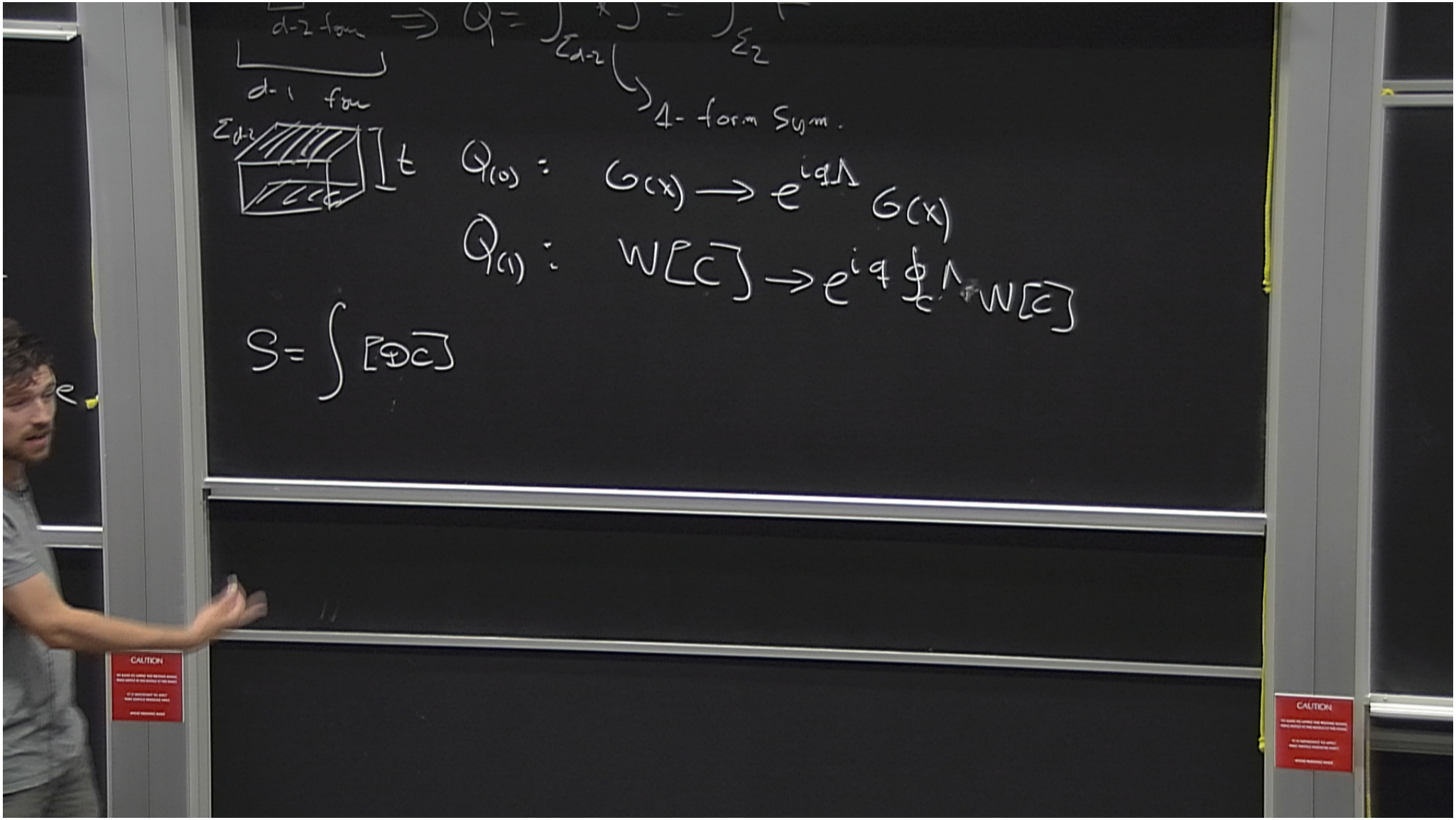
$Q_{(0)}$: $G(x) \longrightarrow e^{iq\Lambda} G(x)$

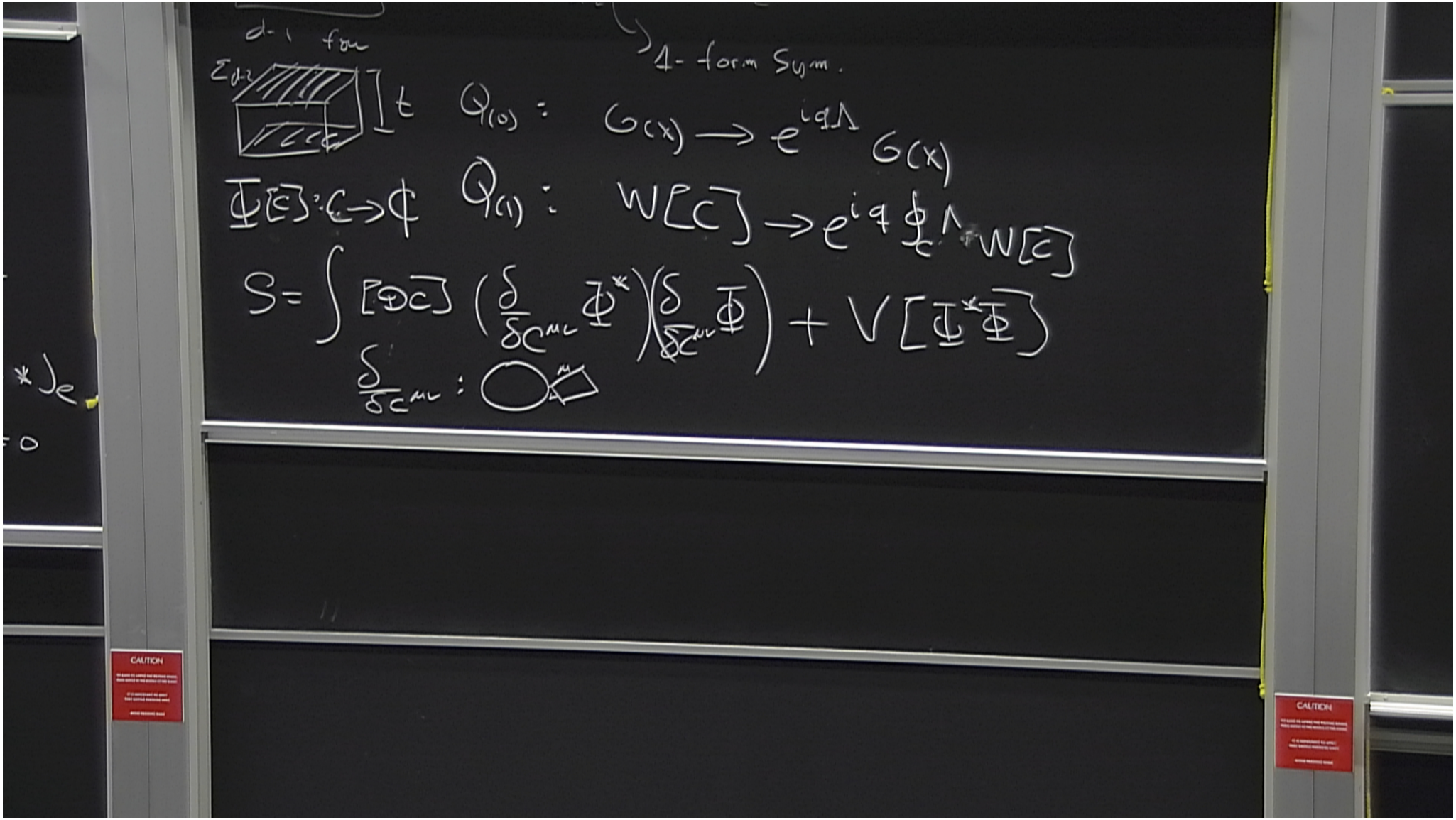
$d * J$ ($d-2$ form) $\Rightarrow Q = \int_{\Sigma_{d-2}} * J = \int_{\Sigma_2} F$

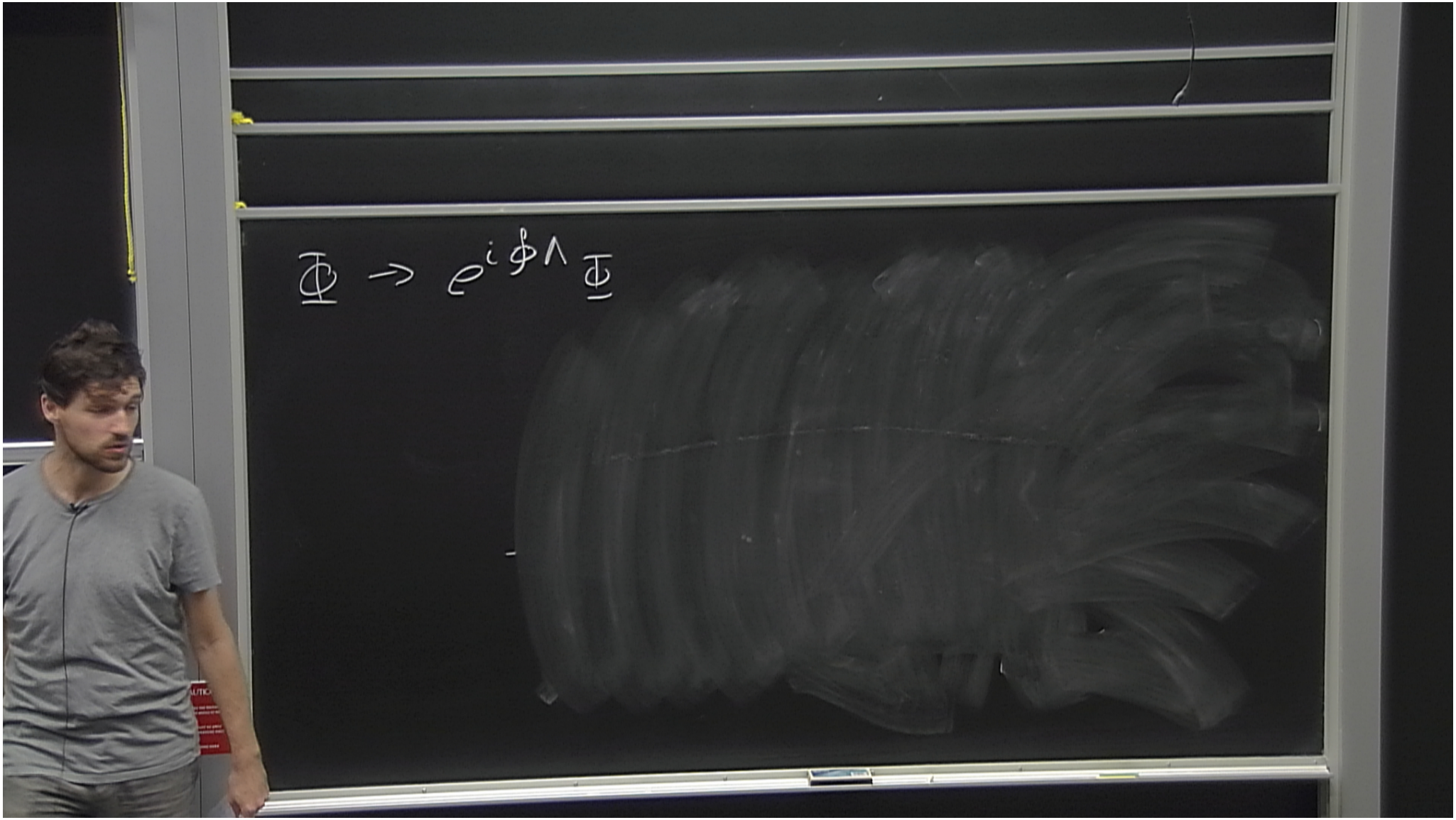
($d-1$ form; F: 1-form Sym.)

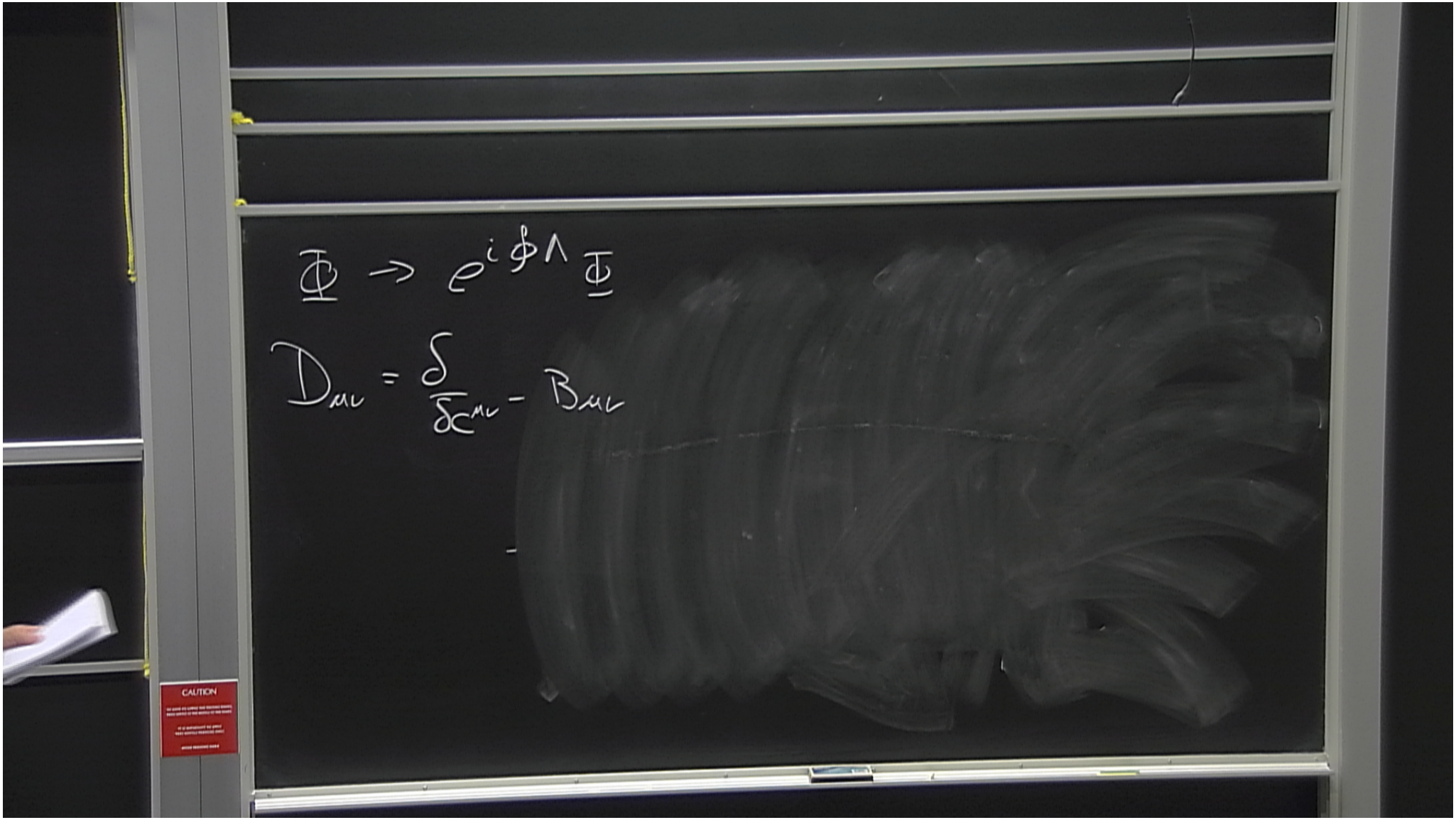
$Q_{(0)}$: $G(x) \to e^{iq\Lambda} G(x)$

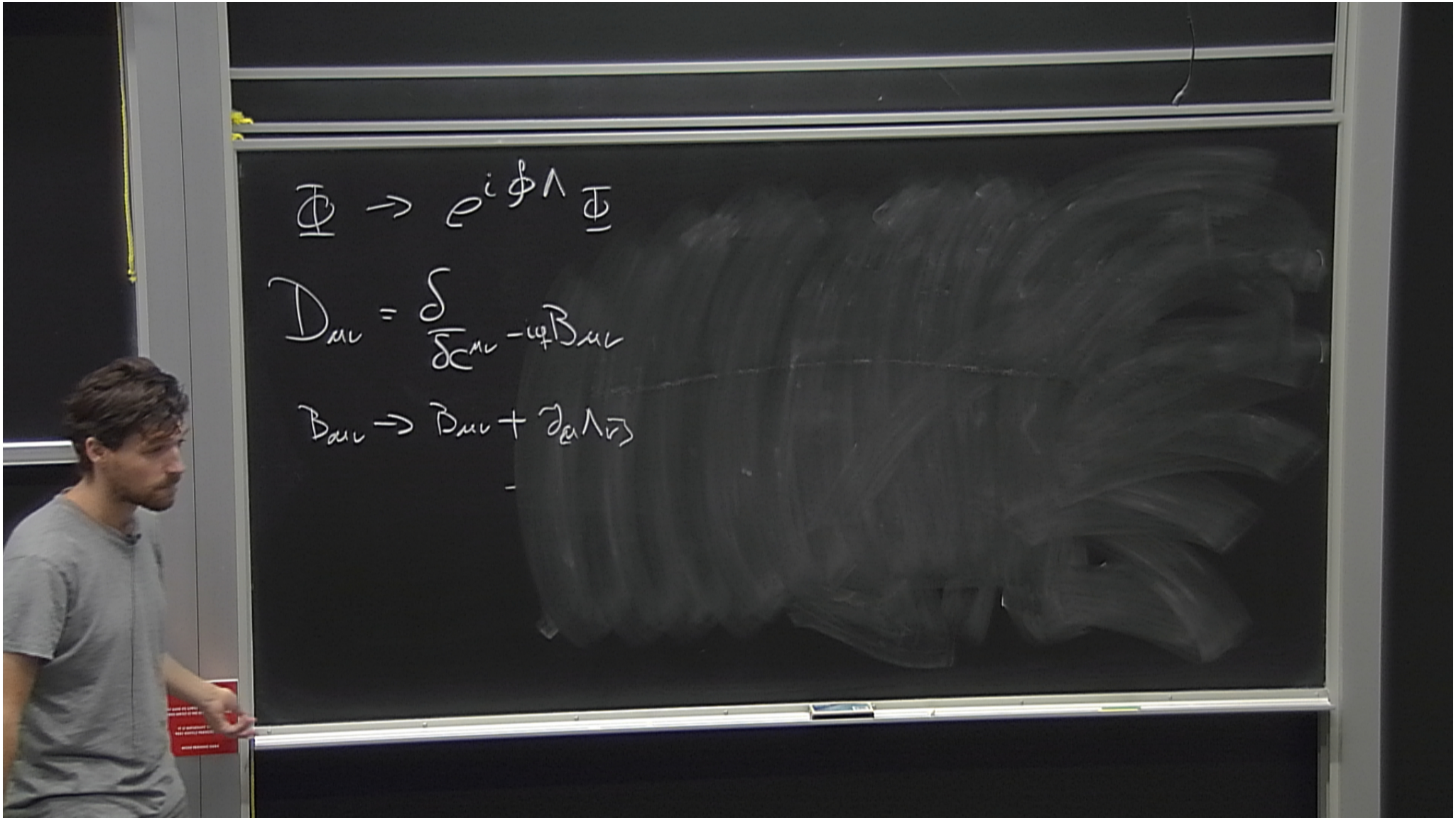
$Q_{(1)}$: $W[C] \to e^{iq \oint_C \Lambda} W[C]$

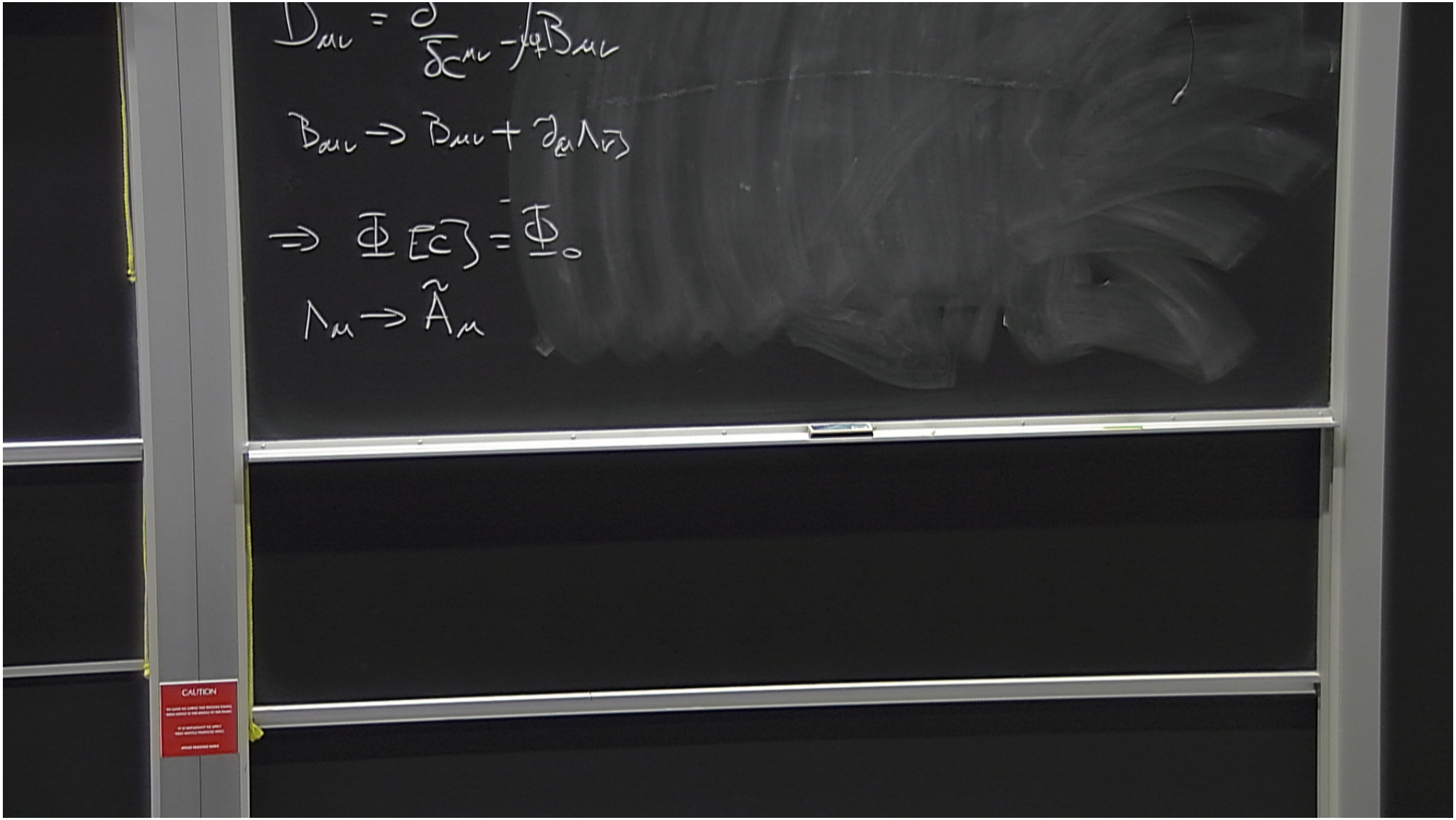

d-2 form
d-1 form
Σ_{d-2}
t
Σ_{d-2}
Σ_2
1- form Sym.
$Q_{(0)}$: $G(x) \to e^{iq\Lambda} G(x)$
$Q_{(1)}$: $W[C] \to e^{iq\oint_C \Lambda} W[C]$
$S = \int [\mathcal{D}C]$
CAUTION
CAUTION

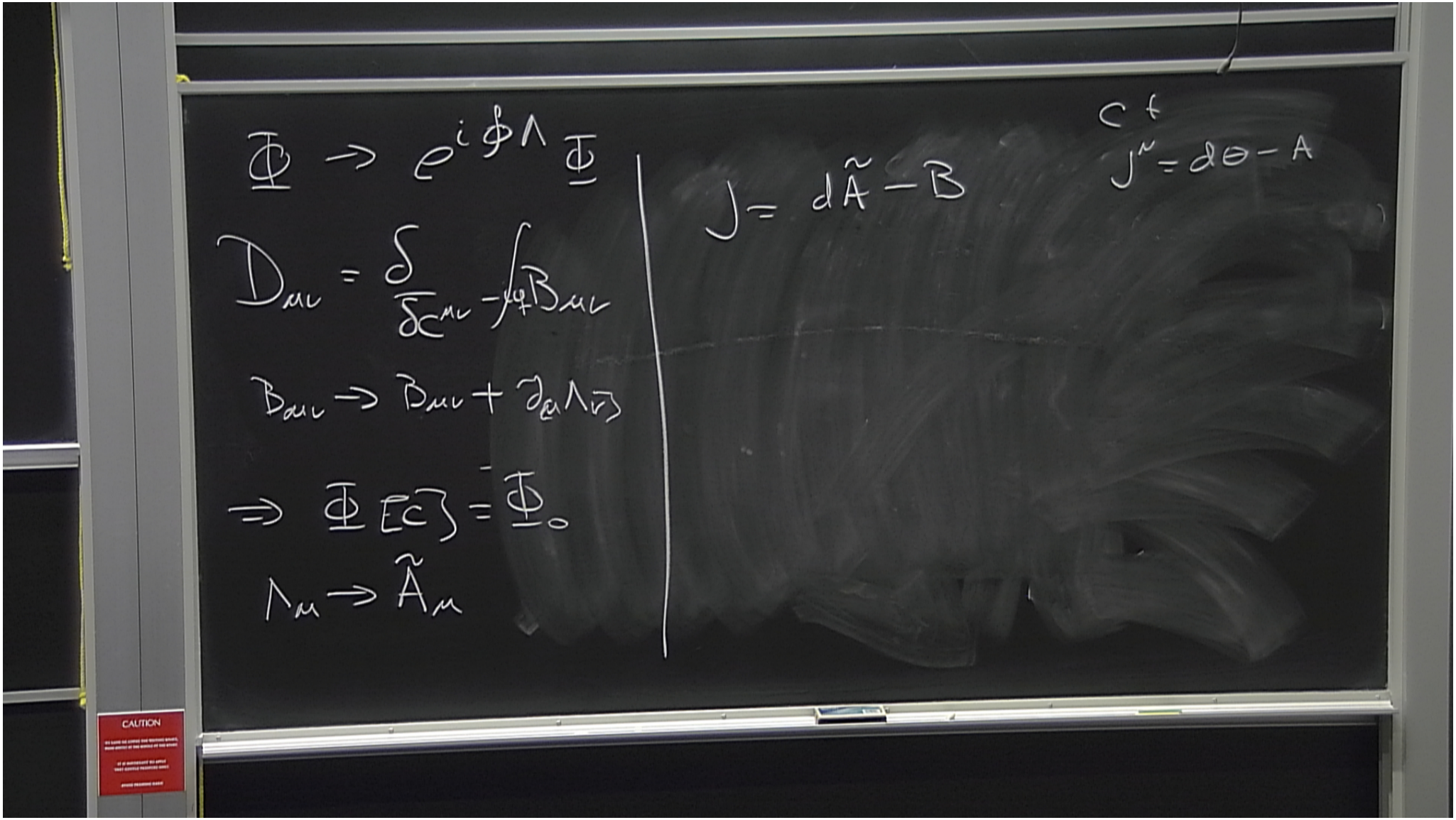

d-1 form
1-form Sym.
Σ_{d-2}
$\int_t$
$Q_{(0)}: \quad G(x) \to e^{iq\Lambda} G(x)$
$\Phi[C]: C \to \mathbb{C}$
$Q_{(1)}: \quad W[C] \to e^{iq \oint_C \Lambda} W[C]$
$S = \int [\mathcal{D}C] \left(\frac{\delta}{\delta C^{\mu\nu}} \Phi^*\right)\left(\frac{\delta}{\delta C^{\mu\nu}} \Phi\right) + V[\Phi^* \Phi]$
$\frac{\delta}{\delta C^{\mu\nu}}$:
M


$\Psi \to e^{i\phi\gamma_5}\Psi$


$\underline{\Phi} \to e^{i\phi\Lambda}\underline{\Phi}$
$D_\mu = \frac{\delta}{\delta C^\mu} - B_\mu$
CAUTION


$\Phi \to e^{i\phi\Lambda}\Phi$
$D_{\mu\nu} = \frac{\delta}{\delta C^{\mu\nu}} - iqB_{\mu\nu}$
$B_{\mu\nu} \to B_{\mu\nu} + \partial_{[\mu}\Lambda_{\nu]}$


$D_{\mu\nu} = \frac{\partial}{\partial C^{\mu\nu}} - \frac{1}{4} B_{\mu\nu}$
$B_{\mu\nu} \to B_{\mu\nu} + \partial_{[\mu} \Lambda_{\nu]}$
$\Rightarrow \Phi[C] = \Phi_0$
$\Lambda_\mu \to \tilde{A}_\mu$
CAUTION


$\Phi \to e^{i\phi\Lambda}\Phi$
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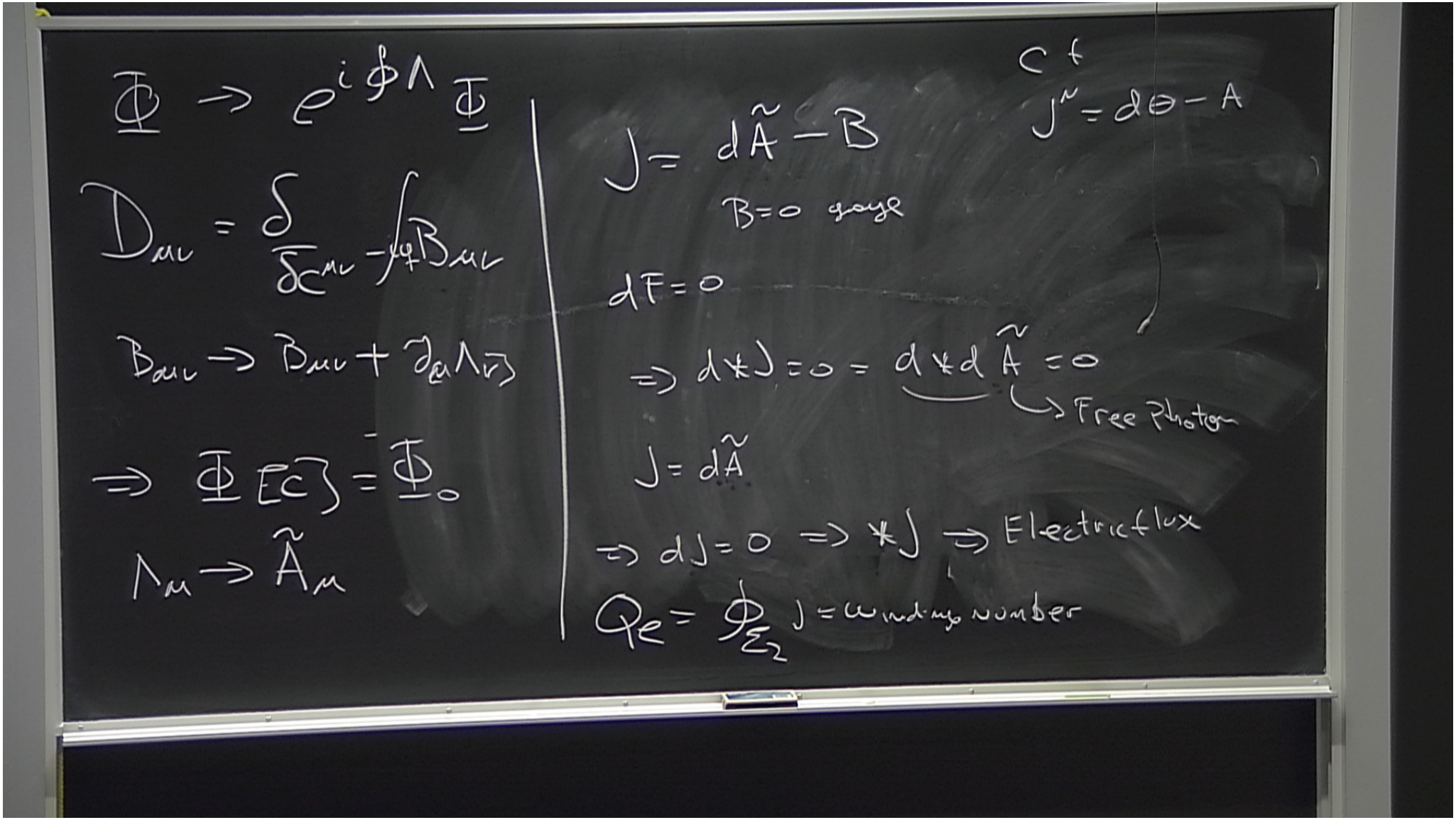
$\Rightarrow \Phi[C] = \Phi_0$
$\Lambda_\mu \to \tilde{A}_\mu$
$J = d\tilde{A} - B$
c.f.
$J^\mu = d\theta - A$
CAUTION


$\Phi \to e^{i\phi\Lambda}\Phi$
$D_{\mu\nu} = \frac{\delta}{\delta C^{\mu\nu}} - \frac{1}{4}B_{\mu\nu}$
$B_{\mu\nu} \to B_{\mu\nu} + \partial_{[\mu}\Lambda_{\nu]}$
$\Rightarrow \Phi[C] = \Phi_0$
$\Lambda_\mu \to \tilde{A}_\mu$
$J = d\tilde{A} - B$
$B=0$ gauge
$dF = 0$
$\Rightarrow d*J = 0 = d*d\tilde{A} = 0$
Free Photon
$J = d\tilde{A}$
$\Rightarrow dJ = 0 \Rightarrow *J \Rightarrow$ Electric flux
$Q_e = \oint_{\Sigma_2} J$ = winding number
cf
$J^\mu = d\Theta - A$

Generalized Global Symmetries and MHD

W / S. Grozdanov (Leiden)
N. Iqbal (Amsterdam)

What is MHD?

$$T_{\mu\nu} = T^{\text{fluid}}_{\mu\nu} + T^{EM}_{\mu\nu}$$

$$\partial_\mu T^{\mu\nu} = F^{\nu\rho} J_{\rho e}$$

$$\sigma(E + v \times B) = J_e$$

+ Maxwell

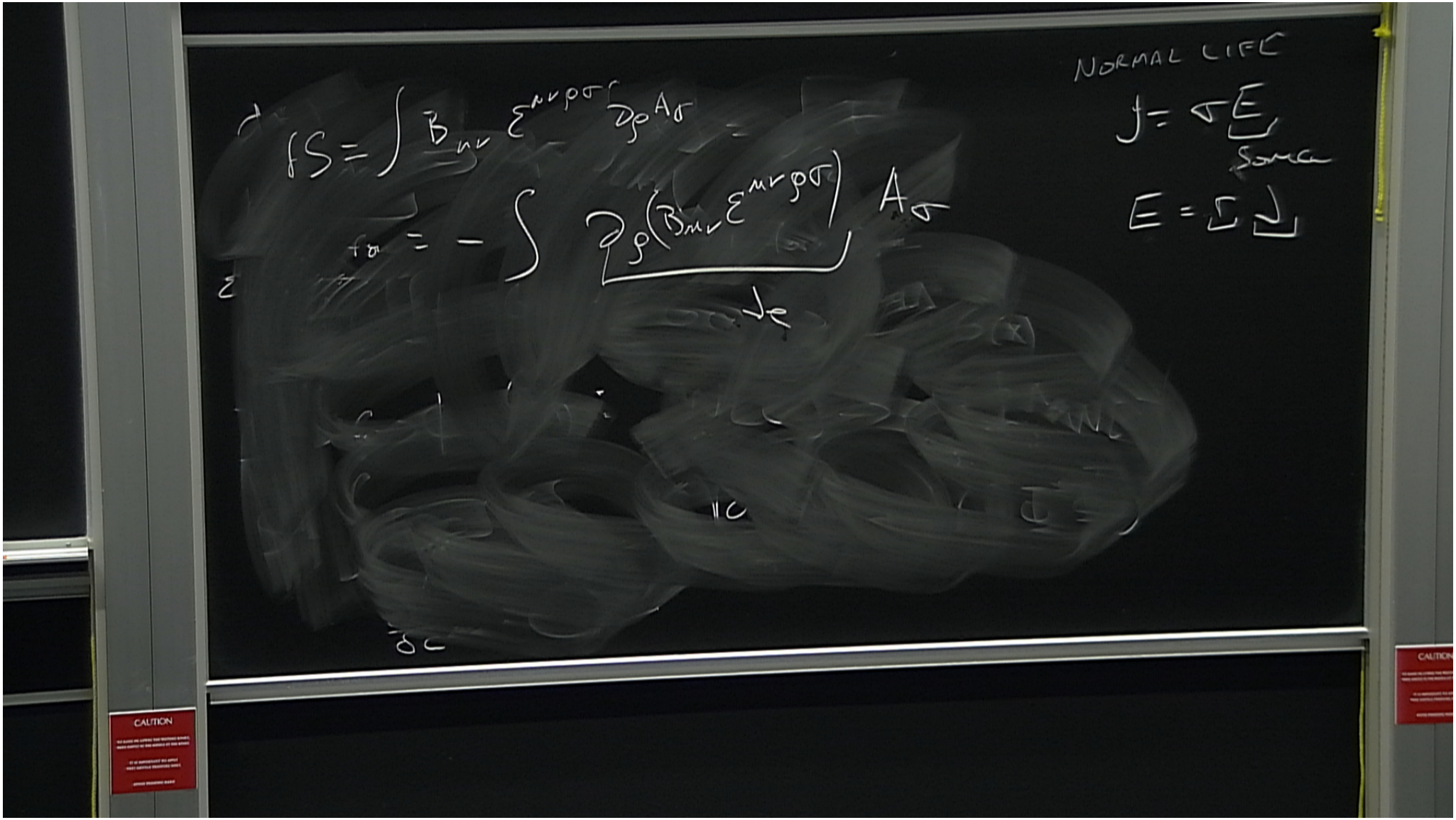
U(1) GGS

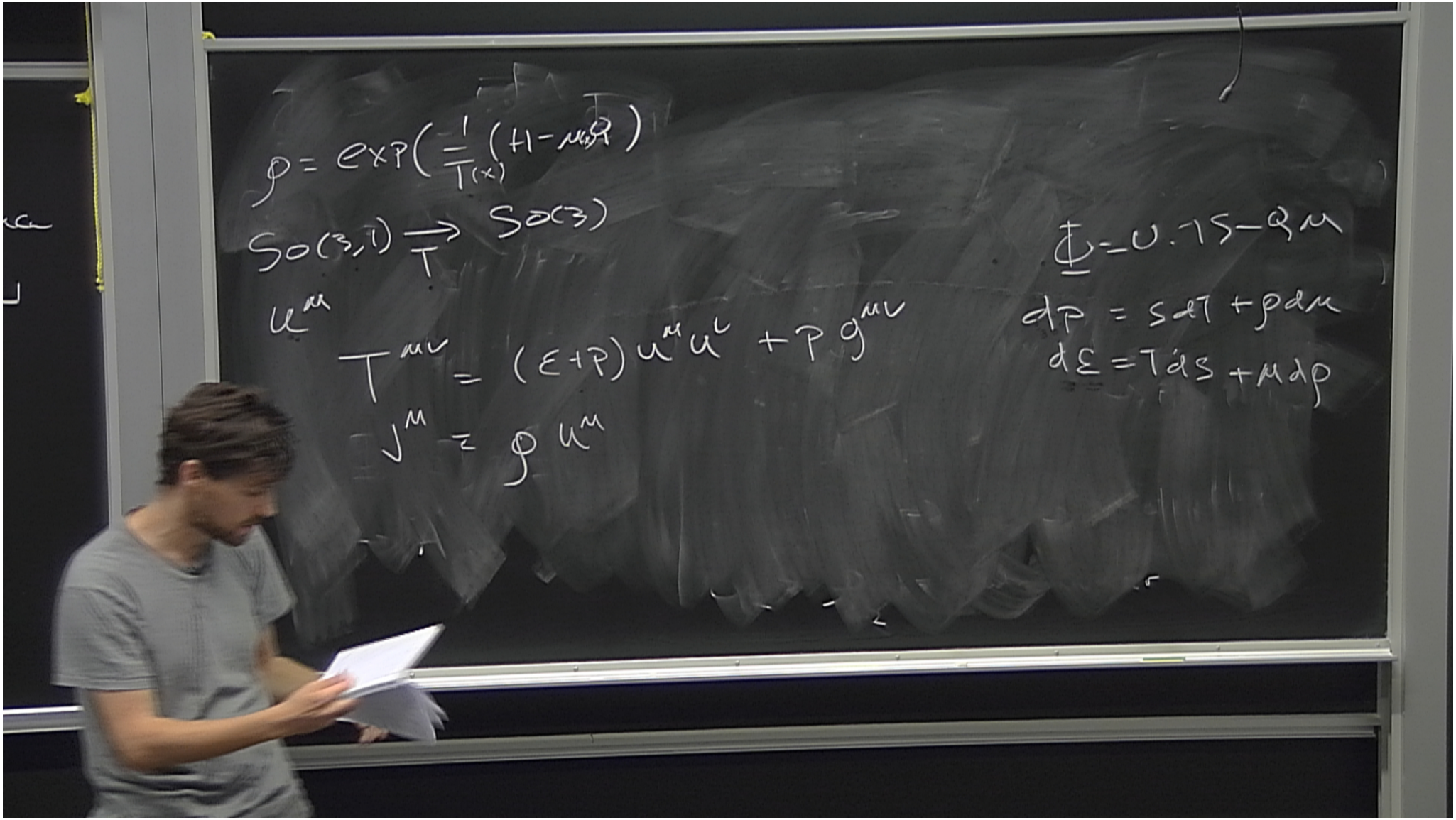
$$\nabla_\mu J^{\mu\nu} = 0 \qquad H = dB$$

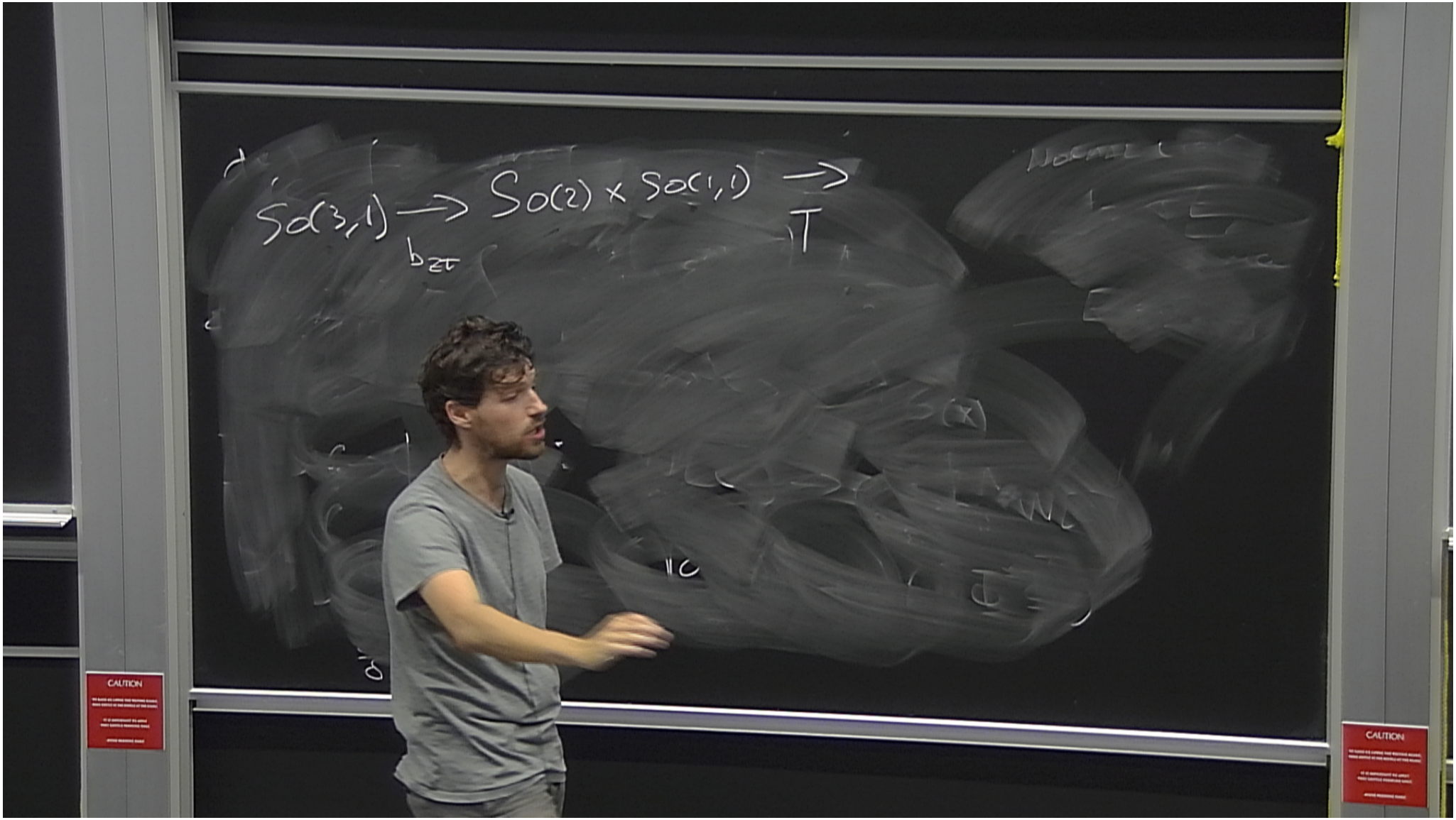
$$\nabla_\mu T^{\mu\nu} = H^{\nu}{}_{\rho\sigma} J^{\rho\sigma}$$

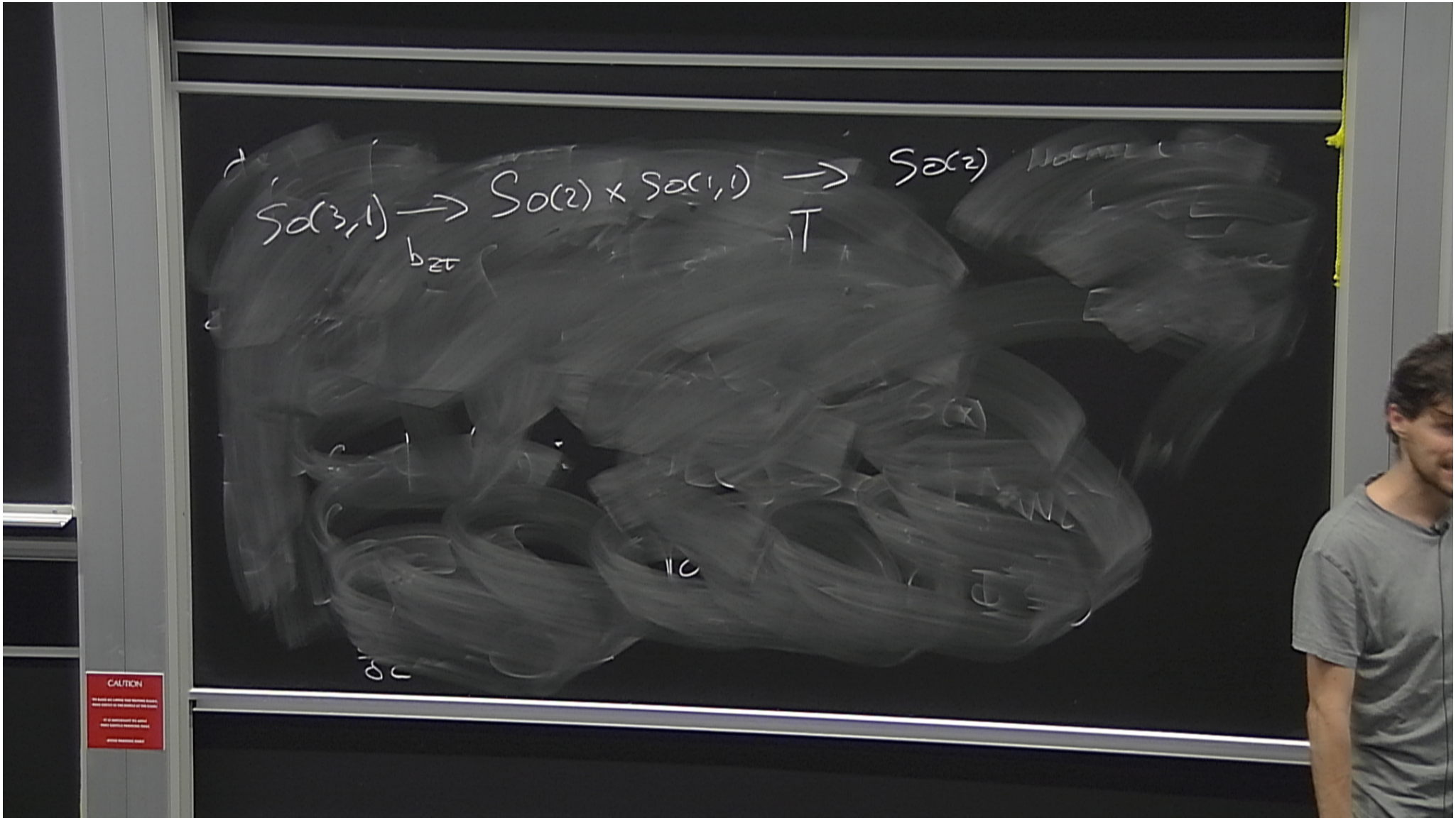
$$S = S_0 + \int d^4x \sqrt{g}\, B_{\mu\nu} J^{\mu\nu}$$

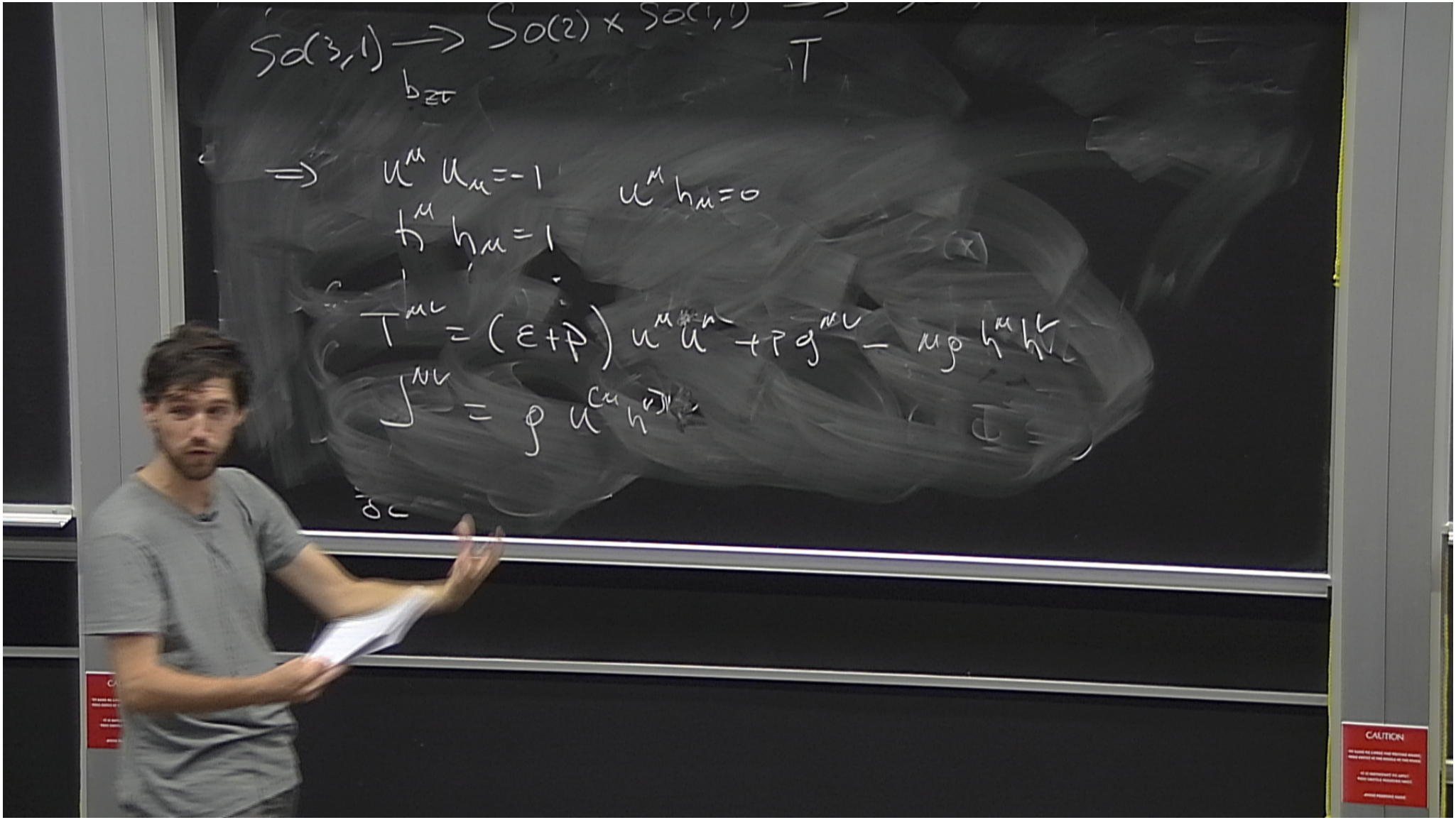
$$d{*}J = 0 \Rightarrow {*}J = dA$$

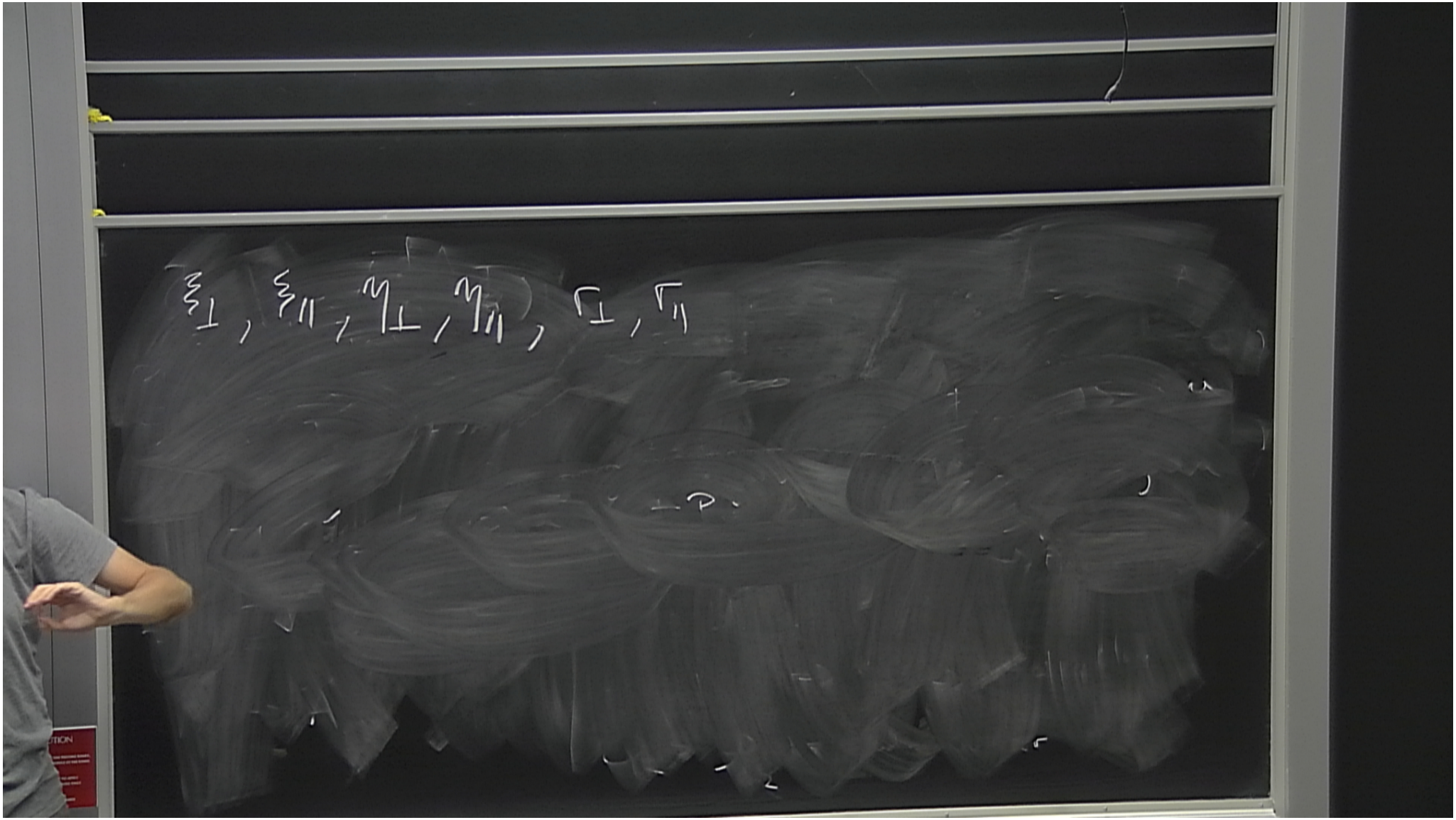

NORMAL LIFE
$J = \sigma E$
Some
$E = \Sigma J$
$\delta S = \int B_{\mu\nu} \varepsilon^{\mu\nu\rho\sigma} \partial_\rho A_\sigma$
$= -\int \partial_\rho (B_{\mu\nu} \varepsilon^{\mu\nu\rho\sigma}) A_\sigma$
J_σ
∂_σ
CAUTION

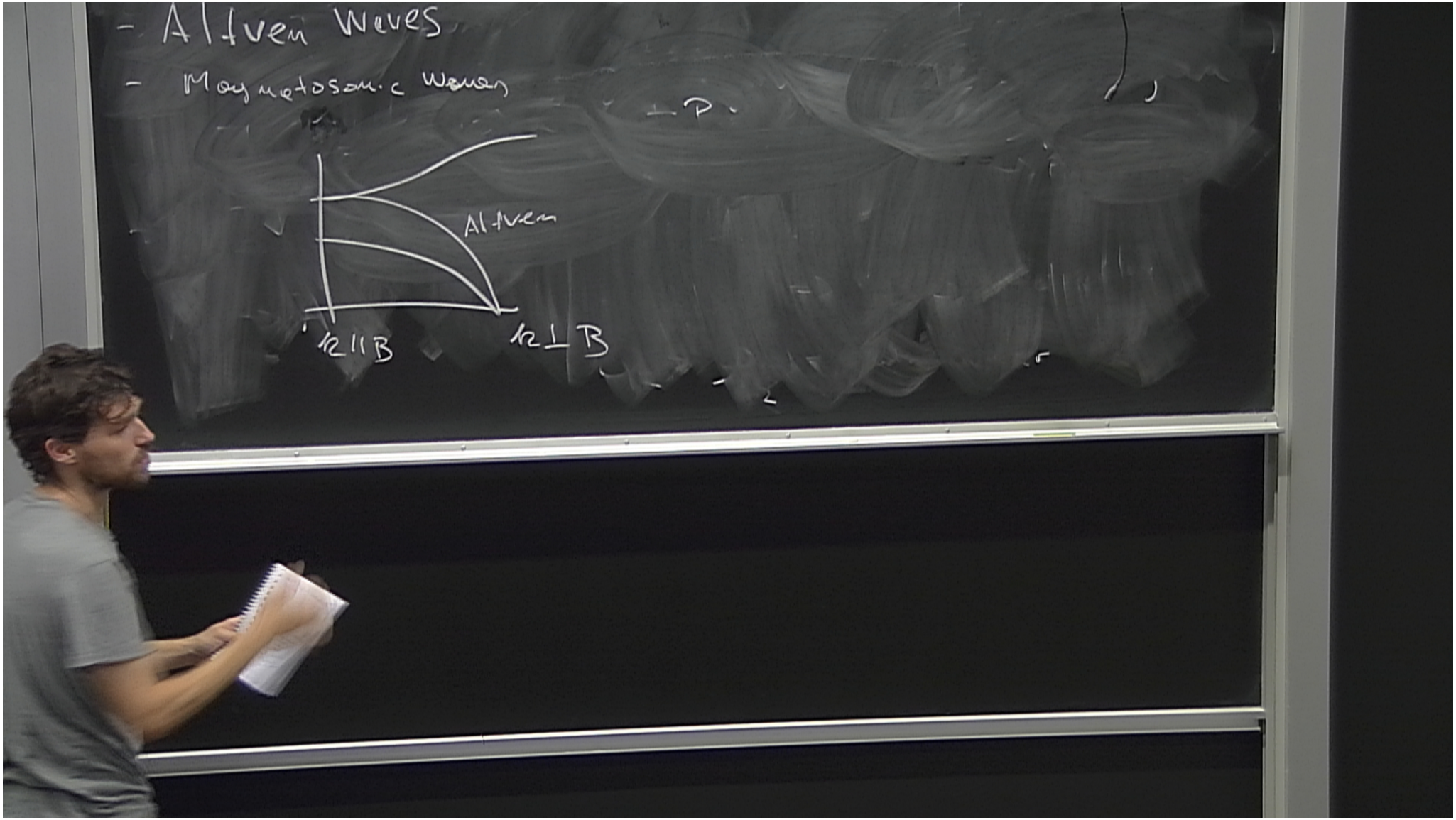

$\rho = \exp\left(\frac{1}{T(x)}(H - \mu_i Q^i)\right)$
$SO(3,1) \xrightarrow{T} SO(3)$
u^μ
$T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu + p g^{\mu\nu}$
$J^\mu = \rho u^\mu$
$\Phi = U - TS - Q\mu$
$dp = s dT + \rho d\mu$
$d\varepsilon = T ds + \mu d\rho$


So(3,1) → So(2) × So(1,1) →
boost
T


SO(3,1) → SO(2) × SO(1,1) → SO(2)
boost
T
CAUTION


$SO(3,1) \to SO(2) \times SO(1,1)$
boost
T
$\Rightarrow$ $u^\mu u_\mu = -1$
$u^\mu h_\mu = 0$
$h^\mu h_\mu = 1$
$T^{\mu\nu} = (\varepsilon + p)\, u^\mu u^\nu + p\, g^{\mu\nu} - \mu\rho\, h^\mu h^\nu$
$J^\nu = \rho\, u^\mu h^\nu$
CAUTION


$\xi_\perp, \xi_\parallel, \eta_\perp, \eta_\parallel, \Gamma_\perp, \Gamma_\parallel$


- Alfven Waves
- Magnetosonic Waves
Alfven
k∥B
k⊥B