

Title: Hydrodynamic electron transport in a graphene field effect transistor

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Abstract:

Hydrodynamic transport in a graphene FET



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ISTITUTO ITALIANO
DI TECNOLOGIA

Perimeter Institute
Waterloo (CA), August 25, 2016

www.graphene-flagship.eu



Collaborators

Theory:

I. Torre, F. Pellegrino, A. Tomadin, A. Principi



Experiments:

D. Bandurin, R.K. Kumar, M.B. Shalom,
G.H. Auton, L.A. Ponomarenko, and A.K. Geim



- a) D. Bandurin, I. Torre, R.K. Kumar, M. Ben Shalom, A. Tomadin, A. Principi, G.H. Auton, E. Khestanova, K.S. Novoselov, I.V. Grigorieva, L.A. Ponomarenko, A.K. Geim, and M. Polini, *Science* **351**, 1055 (2016)
- b) I. Torre, A. Tomadin, A.K. Geim, and M. Polini, *Phys. Rev. B* **92**, 165433 (2015)
- c) A. Principi, G. Vignale, M. Carrega, and M. Polini, *Phys. Rev. B* **93**, 125410 (2016)
- d) F.M.D. Pellegrino, I. Torre, A.K. Geim, and M. Polini, arXiv:1607.03726 (*Phys. Rev. B*, to appear)

In the same Science issue

REPORT

Observation of the Dirac fluid and the breakdown of the Wiedemann-Franz law in graphene

Jesse Crossno^{1,2}, Jing K. Shi¹, Ke Wang¹, Xiaomeng Liu¹, Achim Harzheim¹, Andrew Lucas¹, Subir Sachdev^{1,3}, Philip Kim^{1,2,*}, Takashi Taniguchi⁴, Kenji Watanabe⁴, Thomas A. Ohki⁵, Kin Chung Fong^{5,*}

+ Author Affiliations

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Science 04 Mar 2016;
Vol. 351, Issue 6277, pp. 1058-1061
DOI: 10.1126/science.aad0343

REPORT

Evidence for hydrodynamic electron flow in PdCoO₂

Philip J. W. Moll^{1,2,3}, Pallavi Kushwaha³, Nabhanila Nandi³, Burkhard Schmidt³, Andrew P. Mackenzie^{3,4,*}

+ Author Affiliations

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Science 04 Mar 2016;
Vol. 351, Issue 6277, pp. 1061-1064
DOI: 10.1126/science.aac8385

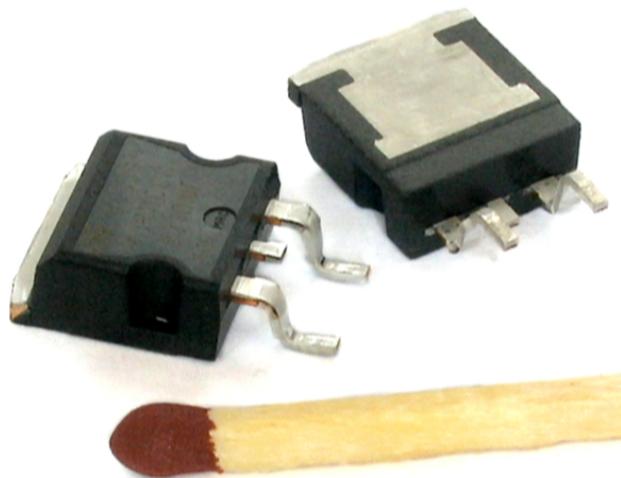
Outline

- ⌚ Brief introduction to electric field effect, graphene transport, and hydrodynamic transport in the solid state
- ⌚ Theory of hydrodynamic nonlocal transport in doped graphene sheets (Phys. Rev. B 2015)
- ⌚ Analytical solution: negative nonlocal resistance and current whirlpools (Phys. Rev. B 2015, 2016)
- ⌚ Experimental data and *rheological* properties of graphene electron liquids (Science 2016: please read 29 pages of SOM!)
- ⌚ Future perspectives

Closely related theoretical results
by Levitov and Falkovich (Nature Phys. 2016)



2D systems are very appealing because you can change density over a broad range by local control electrodes (called “*gates*”)



Integer and fractional quantum Hall effects

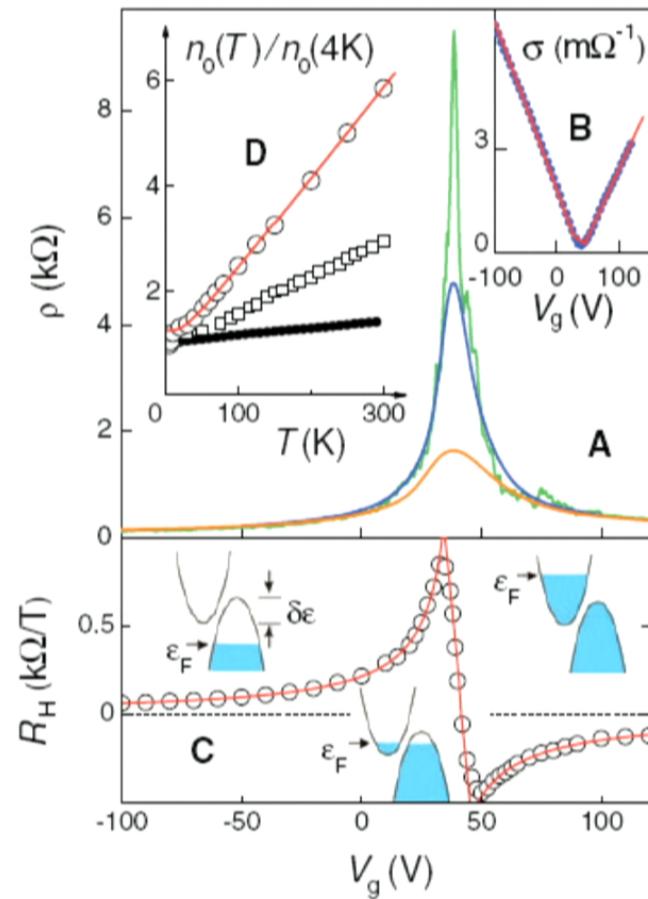
Metal-insulator transitions

Multiple quantum wells (Coulomb drag, interlayer coherence, charge-transfer instability, etc)

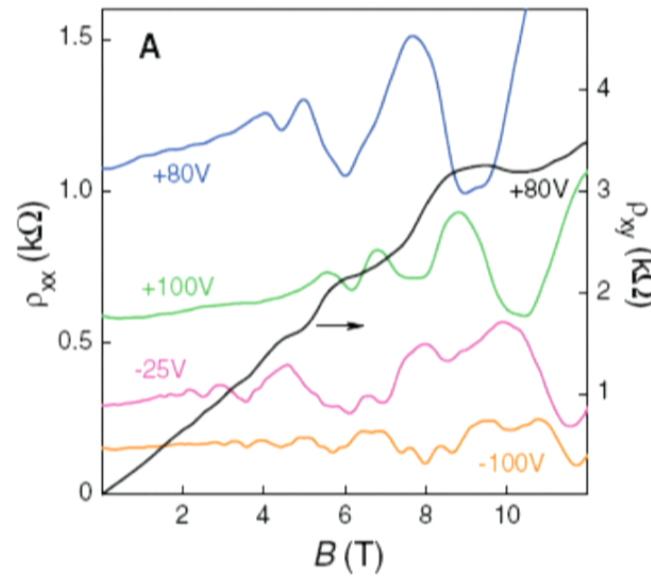
...

Electric Field Effect in Atomically Thin Carbon Films

K. S. Novoselov,¹ A. K. Geim,^{1,*} S. V. Morozov,² D. Jiang,¹
Y. Zhang,¹ S. V. Dubonos,² I. V. Grigorieva,¹ A. A. Firsov²

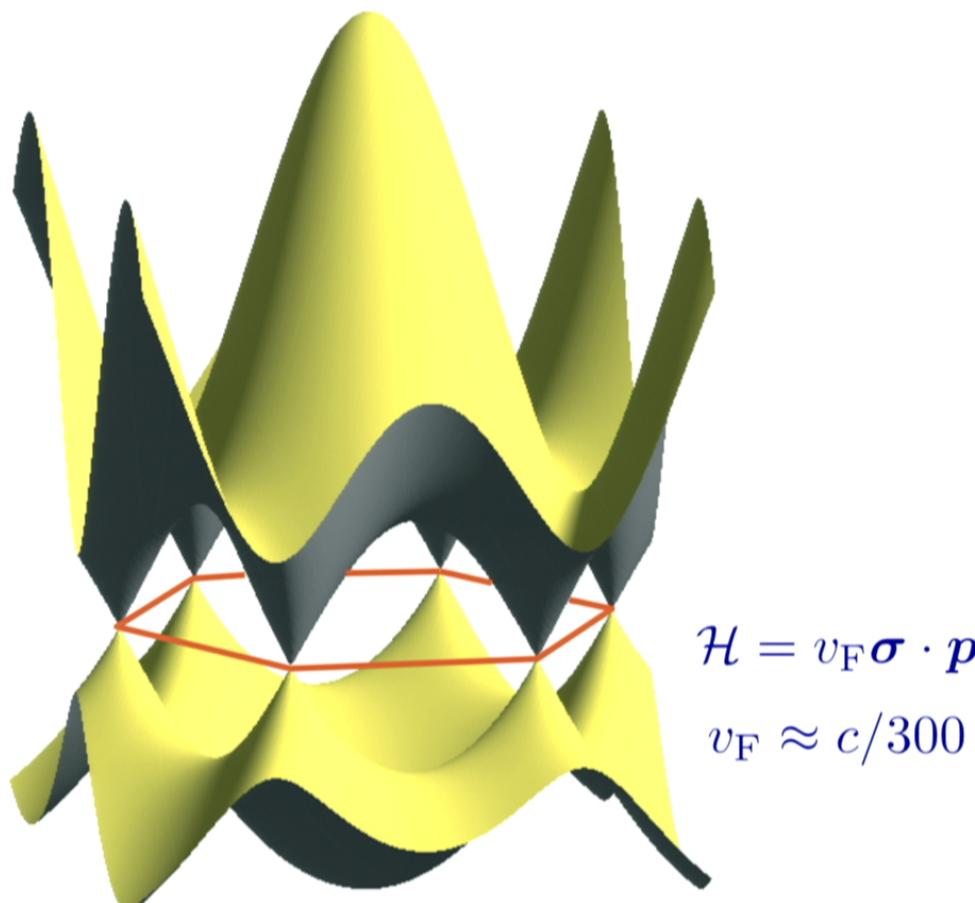


weak-field magneto-resistance (SdH oscillations)



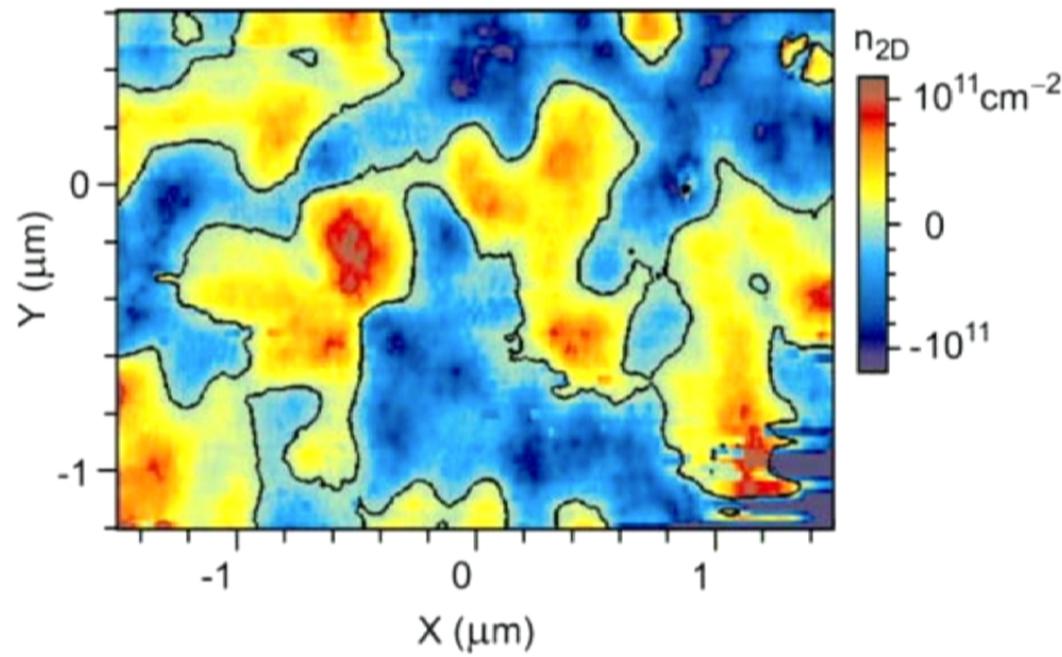
K.S. Novoselov *et al.*, Science **306**, 666 (2004)

2D Dirac-Weyl fermions



P.R. Wallace, Phys. Rev. **71**, 622 (1947)
J.C. Slonczewski and P.R. Weiss, Phys. Rev. **109**, 272 (1958)

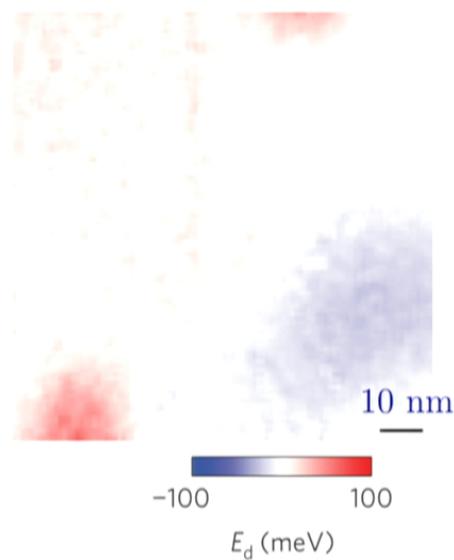
Physics near CNP is dominated by e-h puddles



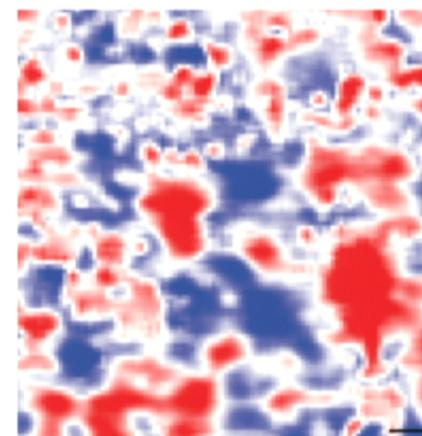
J. Martin *et al.*, Nature Phys. 4, 144 (2008)

Mitigating the role of disorder

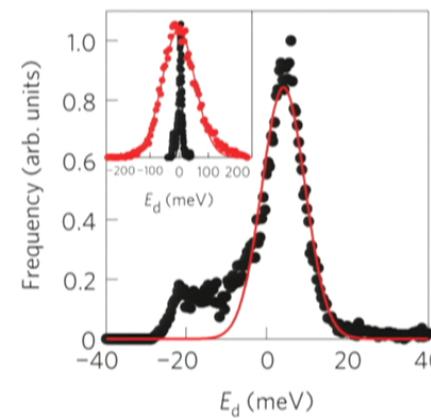
Graphene on BN



Graphene on SiO_2

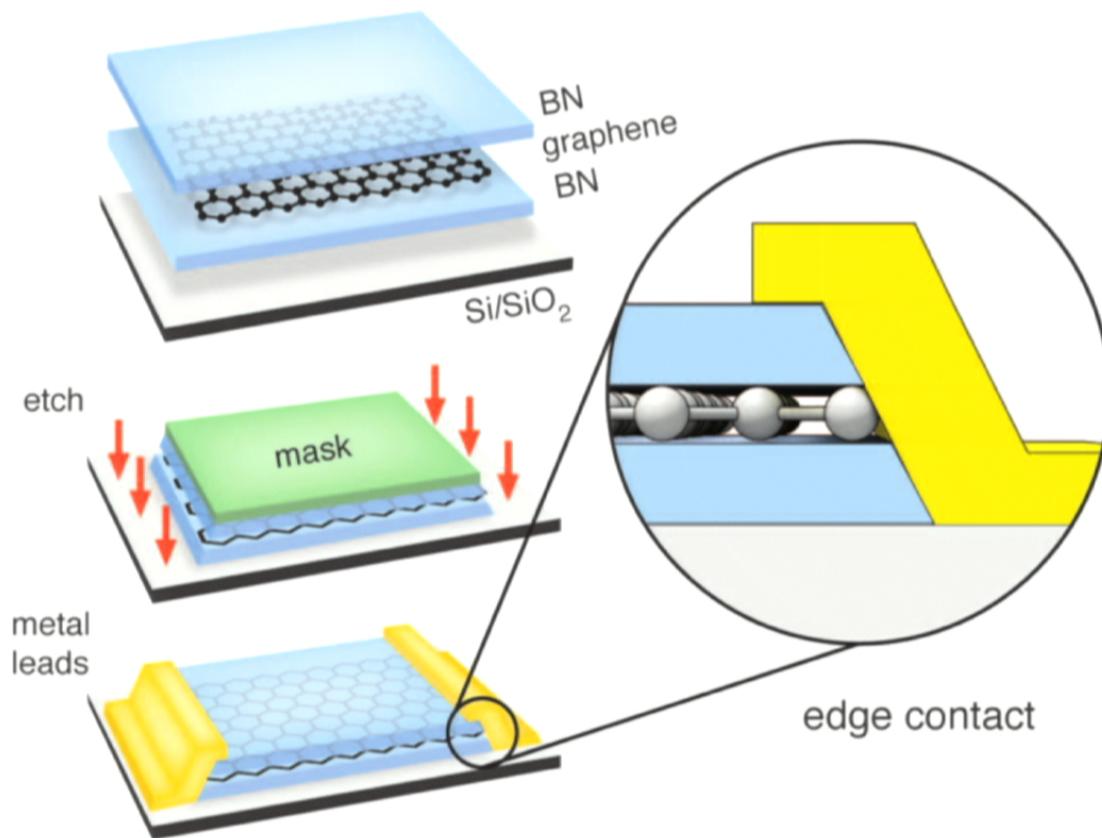


By depositing graphene on BN (rather than SiO_2): **one** order of magnitude reduction in the amplitude of height fluctuations and **two** orders of magnitude reduction in the amplitude of carrier density fluctuations



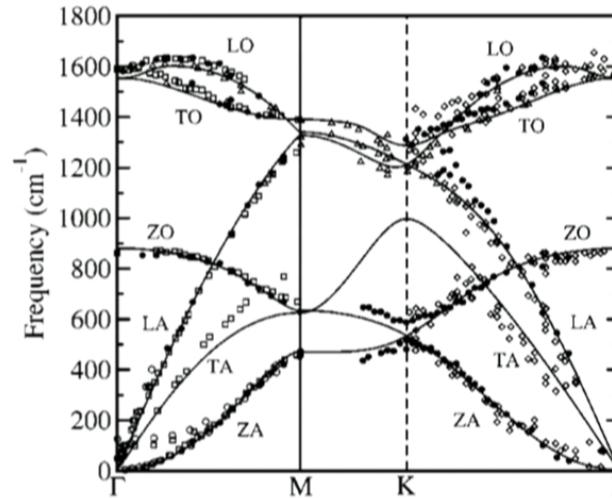
J. Xue *et al.*, Nature Mater. **10**, 282 (2011); R. Decker *et al.*, Nano Lett. **11**, 2291 (2011)

Encapsulation and “1D” edge contacts



L. Wang *et al.*, Science 342, 614 (2013)
See also works published by Manchester, MIT, Harvard, Aachen, etc

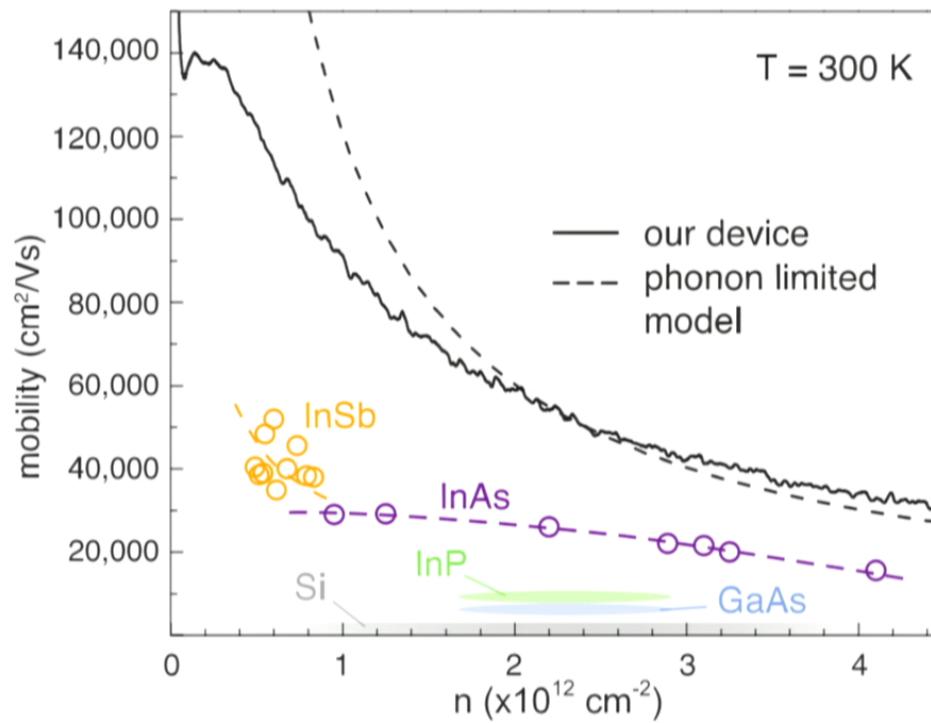
Graphene phonons



- The lattice is exceptionally stiff. Its highest energy phonons have an energy exceeding 100 meV
- There are 6 branches of phonon modes – corresponding to the 3 directions in which each of the atoms in the unit cell can move. There are 3 modes whose energy vanishes in the long-wavelength limit (asymptotic relaxation of hot carriers, etc)
- Very soft phonon modes: flexural modes (Lifshitz, 1952), crucial for transport in suspended devices: E.V. Castro *et al.*, Phys. Rev. Lett. **105**, 266601 (2010)

N. Mounet and N. Marzari, Phys. Rev. B **71**, 205214 (2005)

Ideal transport properties



L. Wang *et al.*, Science 342, 614 (2013)

Hydrodynamic flow in doped graphene

$$\ell_{\text{ee}} \ll \ell, W, v_F/\omega$$

Dimensionless coupling constant

In graphene (or any other Dirac material), the strength of electron-electron interactions is controlled by the following dimensionless parameter, which is usually called “fine structure constant” (because of its analogy with the QED fine structure constant):

$$\alpha_{ee} = \frac{e^2 / (\epsilon L)}{\hbar v_F / L} = \frac{e^2}{\epsilon \hbar v_F}$$

This dimensionless number is:

- 1) **not** small, i.e. it is of order unity
- 2) **not** gate tunable (Fermi wave number drops out)
- 3) **sensitive** to dielectric environment (the “epsilon factor”)

V. N. Kotov *et al.*, Rev. Mod. Phys. **84**, 1067 (2012)

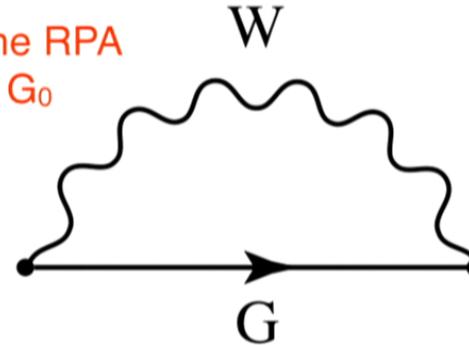
Microscopic theory of the electron-electron collision time

For a weakly correlated electron system the collision time is usually calculated from the on-shell quasiparticle self-energy (evaluated on the Fermi surface):

$$\frac{1}{\tau_{\mathbf{k},\lambda}} = 2 \Im m[\Sigma_\lambda(\mathbf{k}, \xi_{\mathbf{k},\lambda})]$$

The quasiparticle self-energy is usually calculated within the famous “GW approximation”:

- Screened potential W evaluated at the level of the RPA
- Green's function G replaced by bare propagator G_0
- Vertex corrections are neglected



M. Polini and G. Vignale, arXiv:1404.5728

No-nonsense physicist: an overview of Gabriele Giuliani's work and life (Edizioni della Normale, Pisa, 2016)

Qualitative discussion and asymptotics

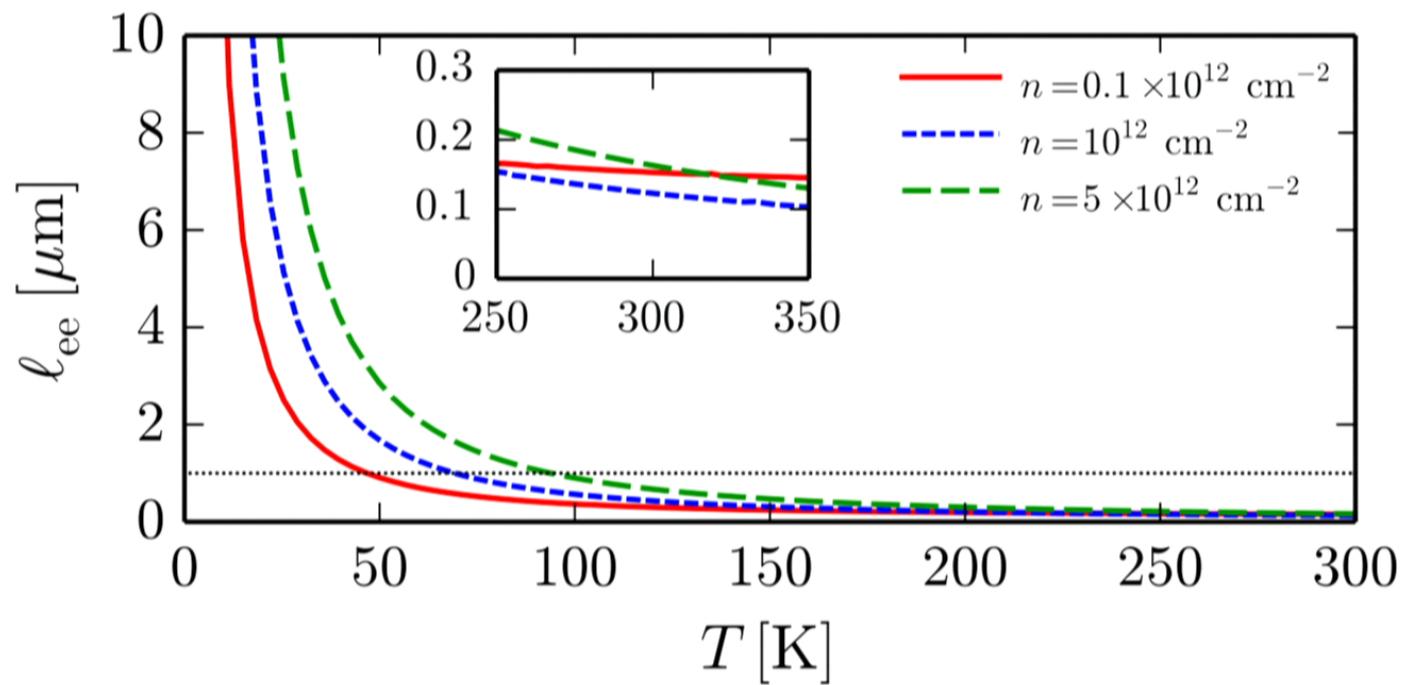
- A simple Fermi golden rule approach with statically screened Coulomb interactions is **not** viable in graphene as it yields logarithmically-divergent intra-band scattering rates due to the **collinear scattering singularity**
- The leading-order contribution to the quasiparticle decay rate in the low-energy and low-temperature limits is completely controlled by collinear scattering events with **small momentum transfer**: $2k_F$ contributions are **suppressed** by the chiral nature of massless Dirac fermions
- The leading order contribution to the quasiparticle decay rate is **completely independent** on the fine-structure constant: the result is therefore **universal** in that it **does not depend on the surrounding dielectrics**

$$\frac{1}{\tau_{\mathbf{k},+}} \Big|_{\text{FS}} = \frac{\varepsilon_F}{\hbar} \frac{\pi}{N_f} \left(\frac{k_B T}{\varepsilon_F} \right)^2 \ln \left(\frac{\Lambda}{k_B T} \right)$$

M. Polini and G. Vignale, arXiv:1404.5728

No-nonsense physicist: an overview of Gabriele Giuliani's work and life (Edizioni della Normale, Pisa, 2016)

Numerical results



In 2D electron systems, electron-acoustic phonon mean free path decays in a much slower fashion (like $1/T$): hydrodynamic window must exist above liquid nitrogen temperatures

A. Principi, G. Vignale, M. Carrega, and M. Polini, Phys. Rev. B **93**, 125410 (2016)

Hydrodynamic equations in the solid state

Continuity equation:

$$\partial_t n(\mathbf{r}, t) + \nabla_{\mathbf{r}} \cdot [n(\mathbf{r}, t) \mathbf{v}(\mathbf{r}, t)] = 0$$

Navier-Stokes equation:

$$mn(\mathbf{r}, t) \{ \partial_t + [\mathbf{v}(\mathbf{r}, t) \cdot \nabla] \mathbf{v}(\mathbf{r}, t) \} = -\mathbf{E}(\mathbf{r}, t)n(\mathbf{r}, t) - \nabla P(\mathbf{r}, t) + \eta \nabla^2 \mathbf{v}(\mathbf{r}, t) - \frac{1}{\tau} mn(\mathbf{r}, t) \mathbf{v}(\mathbf{r}, t)$$

Effective mass Convective derivative

External and internal electric field forces Pressure Shear viscosity Phenomenological treatment of momentum-non-conserving collisions

```
graph TD; A[Effective mass] --> B["mn(r,t){\partial_t + [v(r,t) · ∇]v(r,t)}"]; C[Convective derivative] --> B; D[External and internal electric field forces] --> E["-E(r,t)n(r,t)"]; F[Pressure] --> G["-∇P(r,t)"]; H[Shear viscosity] --> I["η∇²v(r,t)"]; J[Phenomenological treatment of momentum-non-conserving collisions] --> K["-1/τ mn(r,t)v(r,t)"];
```

Boundary conditions

No-slip boundary conditions: velocity tangential to the walls of the container must vanish. Good for water in a pipe, when the interactions between the molecules of the fluid and the walls of the container are of the same nature as of those between two molecules of the fluid

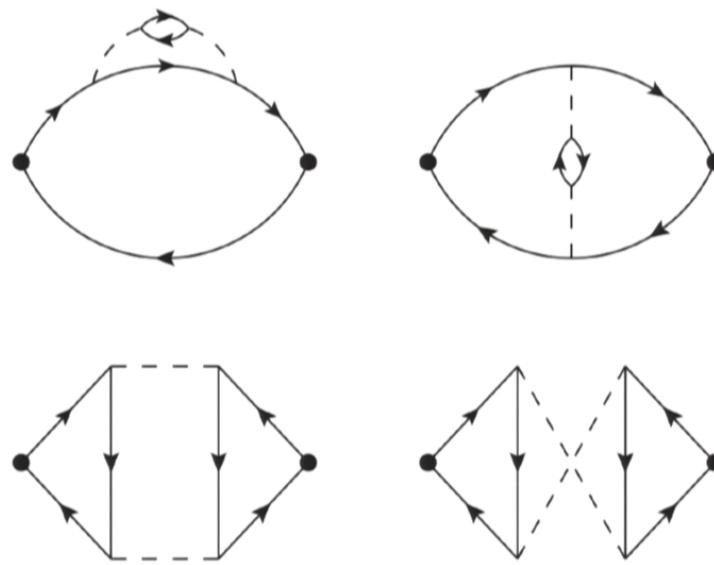
Free-surface boundary conditions: tangential force applied from the boundary to the fluid element has to vanish at the boundary

Boundary conditions at the contacts: total current at each contact is set to zero (except at the contact where current is injected); contacts are equipotential; one contact is grounded to fix arbitrary zero of the potential

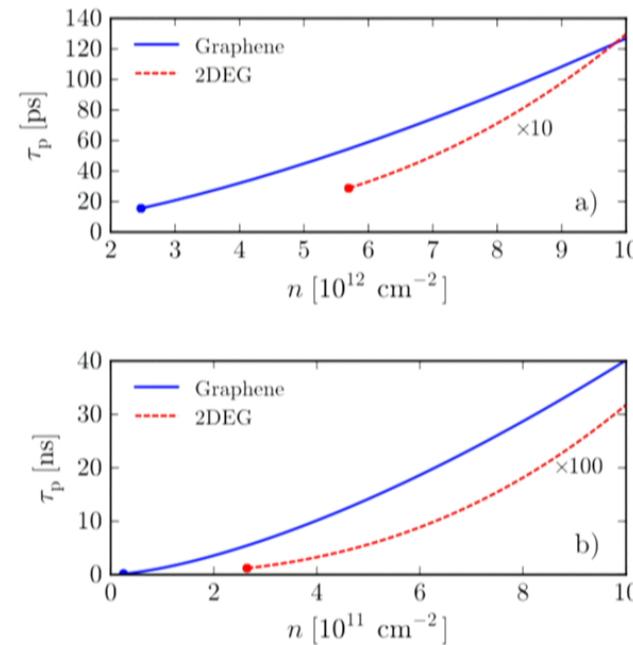
A. Tomadin, G. Vignale, and M. Polini, Phys. Rev. Lett. **113**, 235901 (2014)

How do you probe hydrodynamic flow
in solid-state devices?

Viscosity and the intrinsic plasmon lifetime

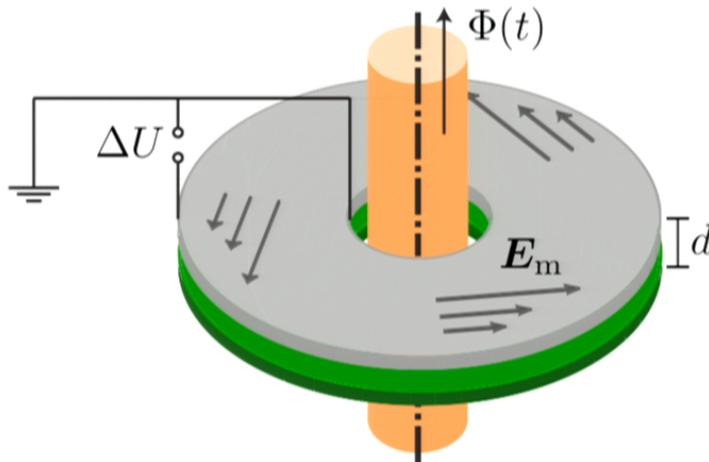


$$\Gamma_p(q) = \frac{\varepsilon_F}{\hbar} \mathcal{A}_{N_f} (\alpha_{ee}) \left(\frac{q}{k_F} \right)^2$$



A. Principi, G. Vignale, M. Carrega, and M. Polini, Phys. Rev. B **88**, 195405 (2013)

Corbino disk viscometers



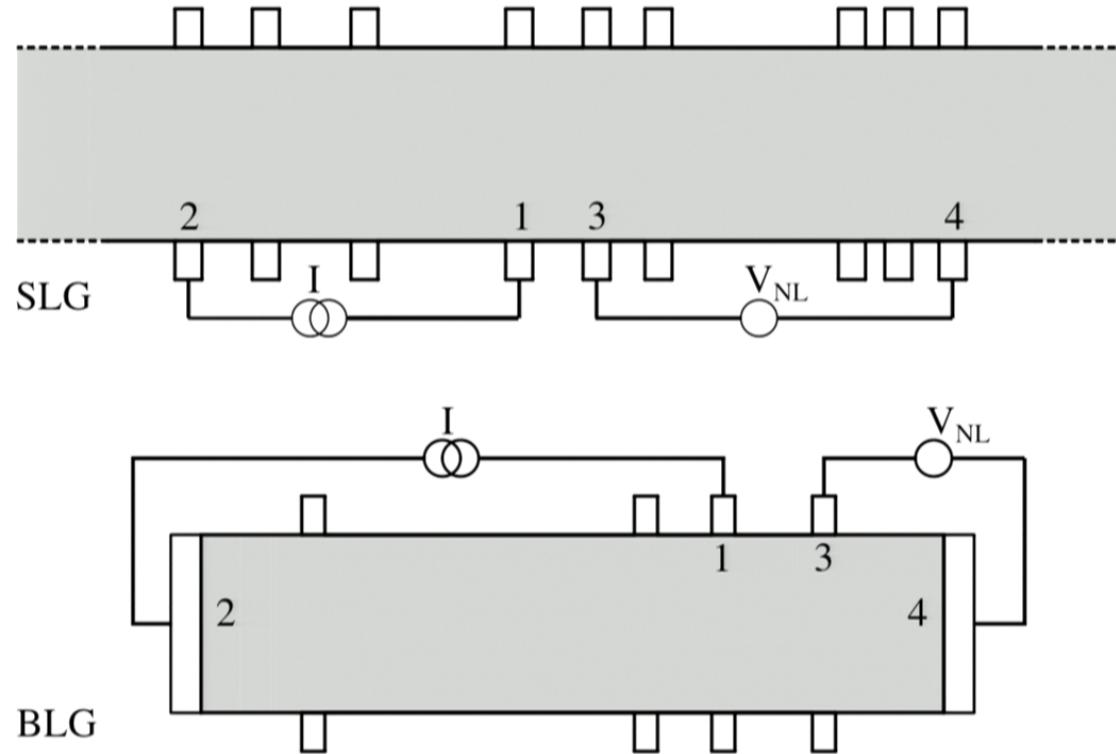
A time-dependent magnetic flux induces by Faraday's law an azimuthal electric field (with a non-zero gradient along the radial direction):

$$\mathbf{E}_{\hat{\theta}}(r, t) = -\frac{1}{2\pi cr} \partial_t \Phi(t) \hat{\theta}$$

A **dc** potential energy difference ΔU appears between inner and outer edges: this a rectified signal stemming from the non-linear nature of hydrodynamic equations

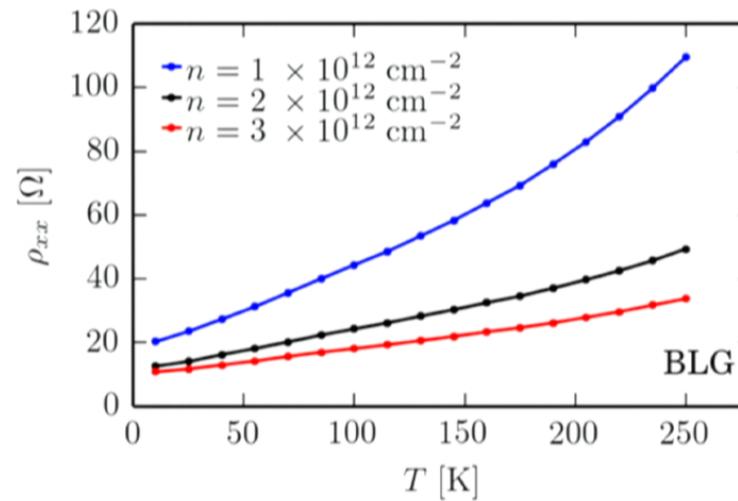
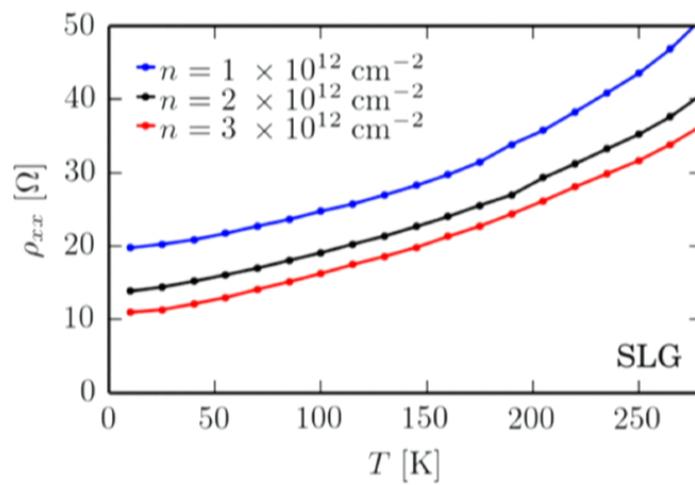
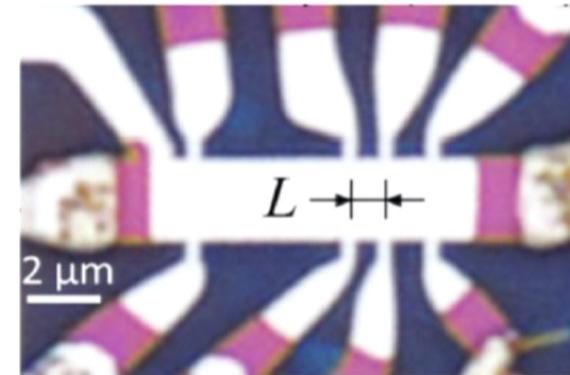
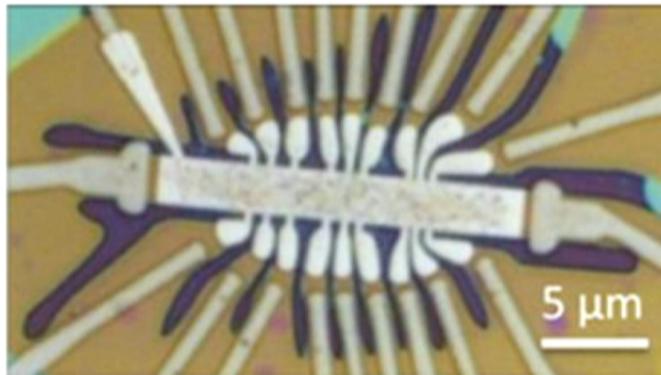
A. Tomadin, G. Vignale, and M. Polini, Phys. Rev. Lett. **113**, 235901 (2014)

Nonlocal transport and the “vicinity” resistance



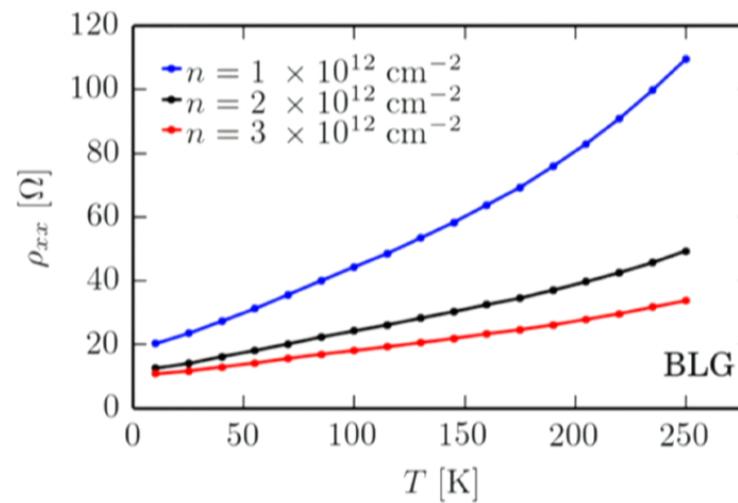
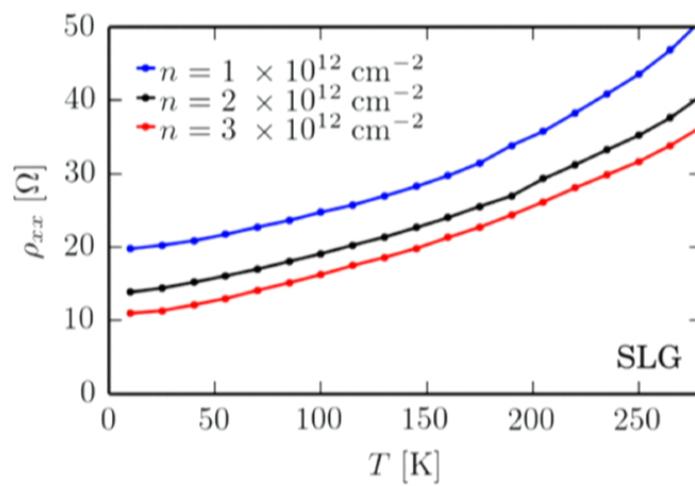
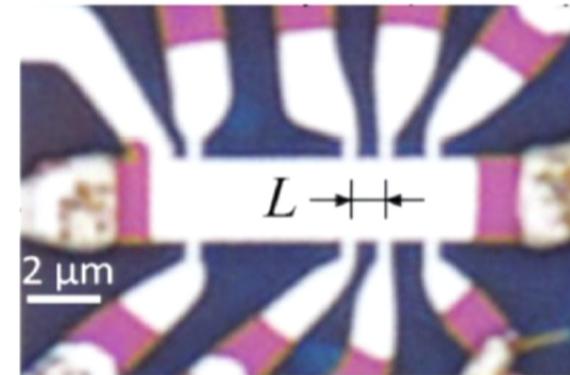
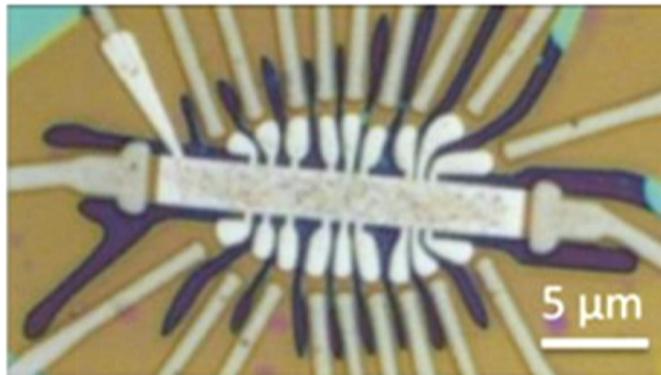
D. Bandurin, I. Torre, R.K. Kumar, M. Ben Shalom, A. Tomadin, A. Principi, G.H. Auton, E. Khestanova, K.S. Novoselov, I.V. Grigorieva, L.A. Ponomarenko, A.K. Geim, and M. Polini, Science **351**, 1055 (2016)

“Local” longitudinal transport



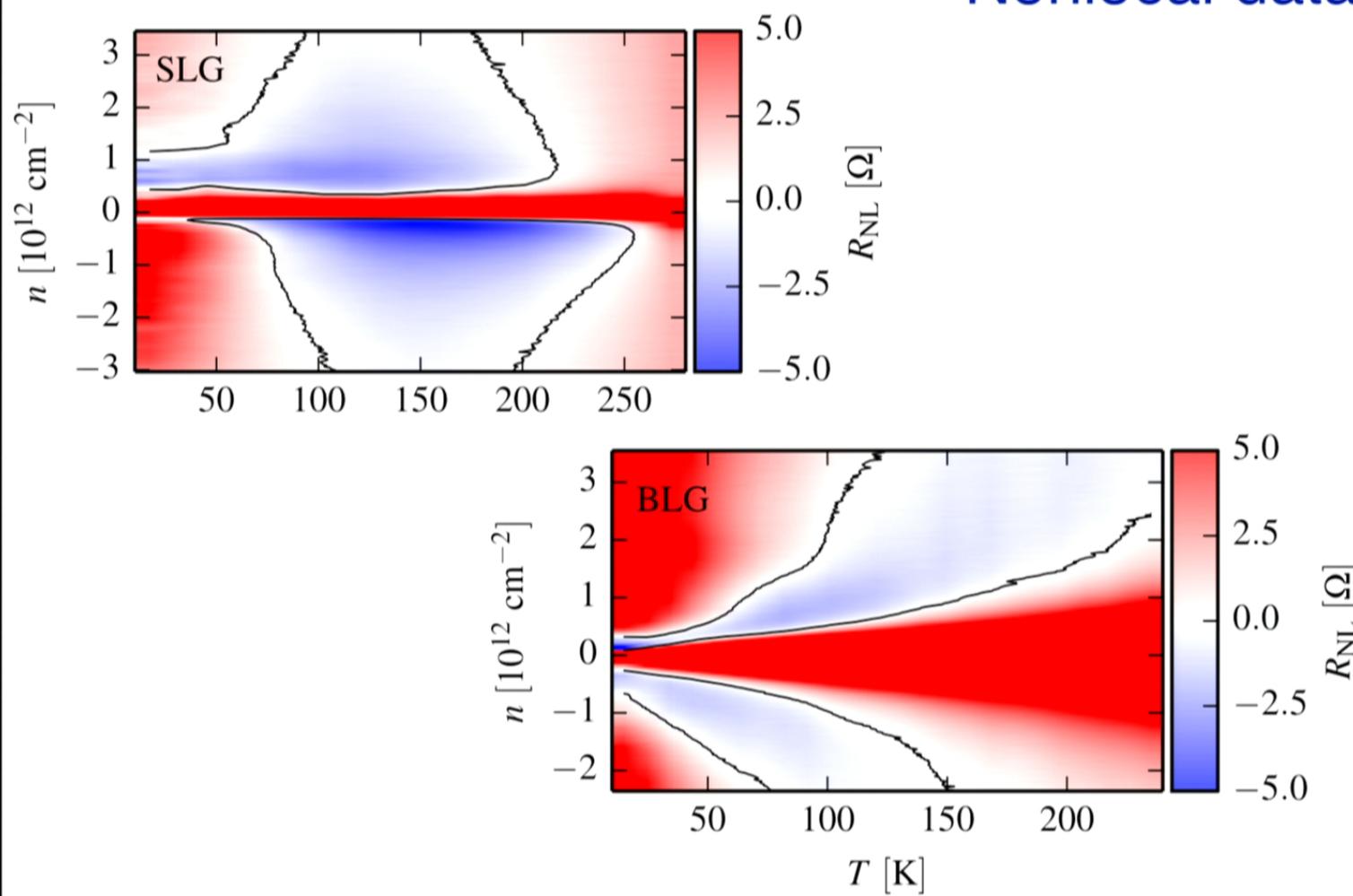
D. Bandurin, I. Torre, R.K. Kumar, M. Ben Shalom, A. Tomadin, A. Principi, G.H. Auton, E. Khestanova, K.S. Novoselov, I.V. Grigorieva, L.A. Ponomarenko, A.K. Geim, and M. Polini, Science **351**, 1055 (2016)

“Local” longitudinal transport



D. Bandurin, I. Torre, R.K. Kumar, M. Ben Shalom, A. Tomadin, A. Principi, G.H. Auton, E. Khestanova, K.S. Novoselov, I.V. Grigorieva, L.A. Ponomarenko, A.K. Geim, and M. Polini, Science **351**, 1055 (2016)

Nonlocal data



D. Bandurin, I. Torre, R.K. Kumar, M. Ben Shalom, A. Tomadin, A. Principi, G.H. Auton, E. Khestanova, K.S. Novoselov, I.V. Grigorieva, L.A. Ponomarenko, A.K. Geim, and M. Polini, Science **351**, 1055 (2016)



Minimal theory

Viscous linear-response theory

Linearized steady-state equations (continuity + Navier-Stokes):

$$\nabla \cdot \mathbf{J}(\mathbf{r}) = 0$$
$$\frac{en}{m} \nabla \phi(\mathbf{r}) + \nu \nabla^2 \mathbf{J}(\mathbf{r}) - \frac{1}{\tau} \mathbf{J}(\mathbf{r}) = 0$$

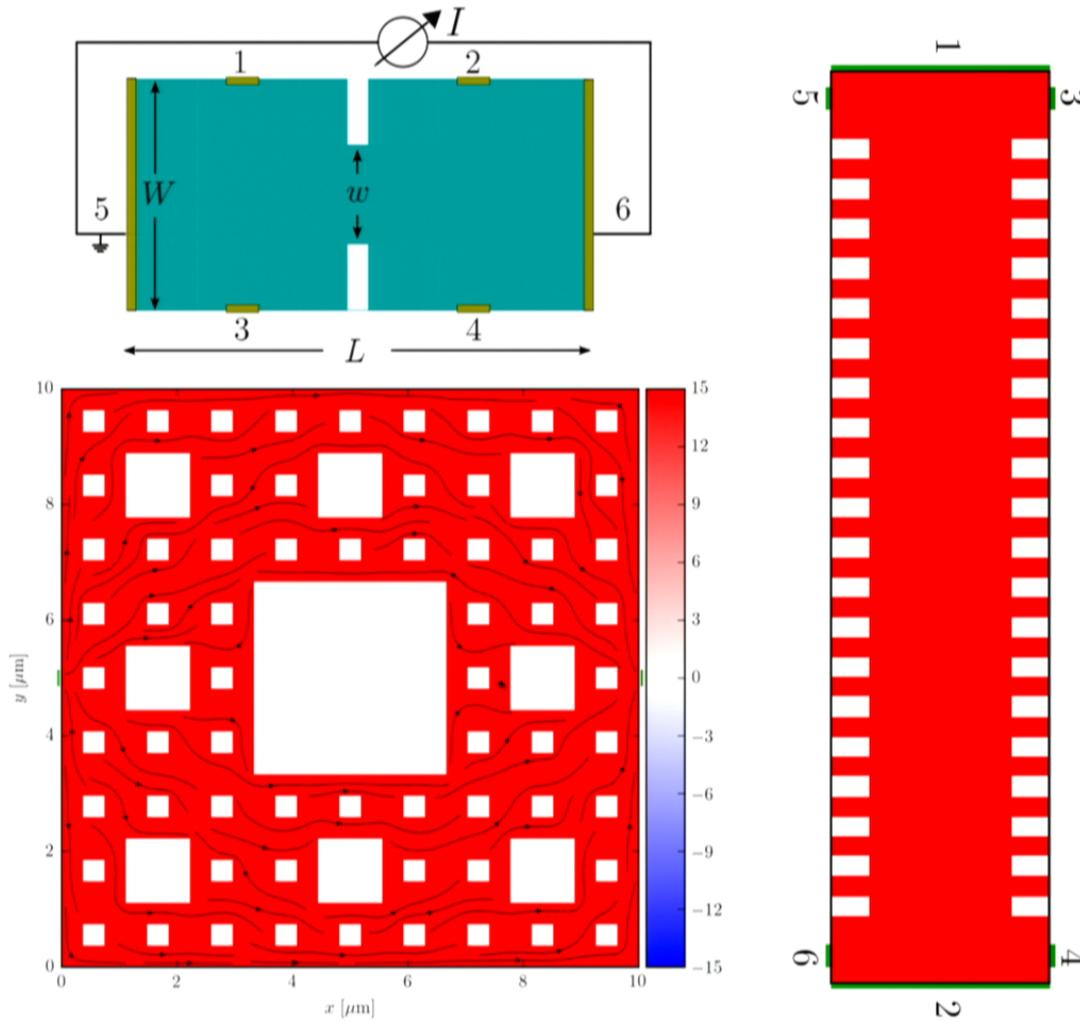
where the **kinematic viscosity** is $\nu = \frac{\eta}{mn}$

Crucial length scale: $D_\nu = \sqrt{\nu \tau}$

Reynolds number: $\text{Re} \simeq \frac{|\mathbf{v}| W}{\nu} \sim \frac{I}{env}$

I. Torre, A. Tomadin, A.K. Geim, and M. Polini, Phys. Rev. B **92**, 165433 (2015)

Going wild

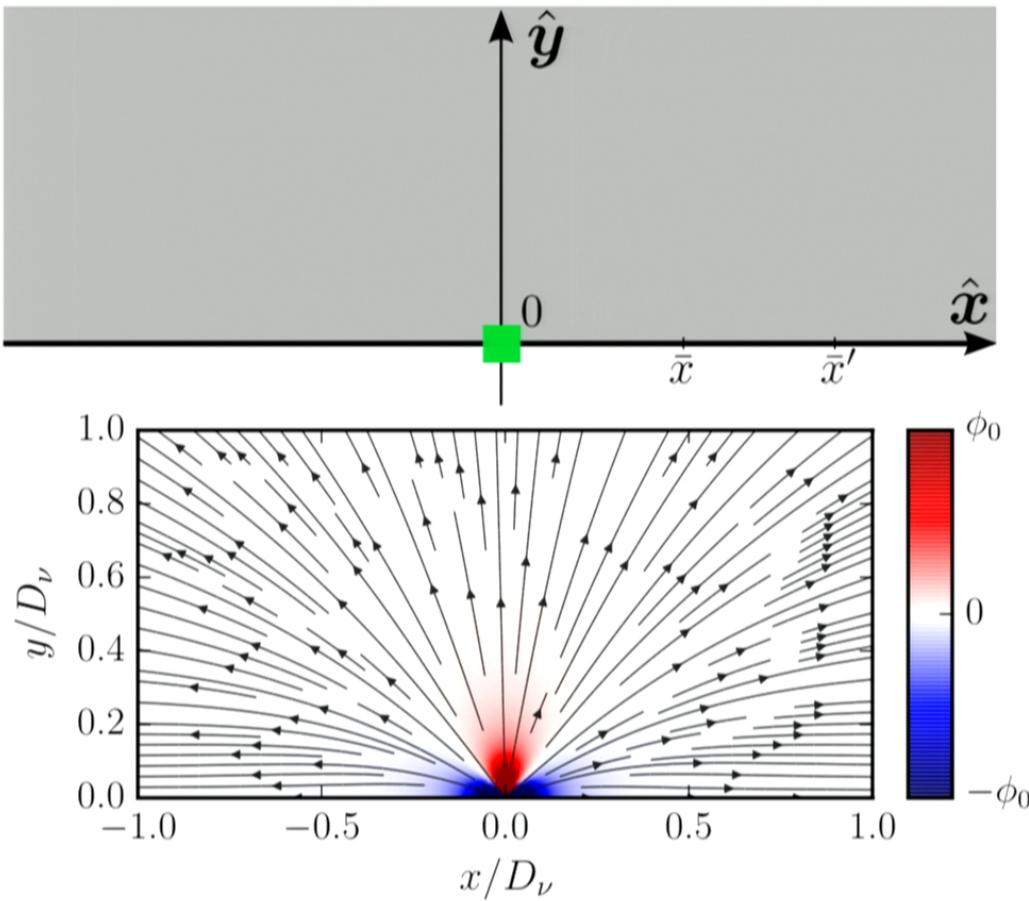


Physical facts

- a. In the linear-response regime, no-slip BCs imply non-monotonic T-dependence (Gurzhi effect) of the ordinary longitudinal local resistance (not observed experimentally). We use free-surface BCs
- b. Independent of BCs, we find spatial sign changes in the 2D electrostatic potential, which are due to viscosity and are responsible for the measured negative nonlocal resistance
- c. In the Hall bar geometry and with our arrangement of contacts, current whirlpools (and backflow at the injector) do exist for arbitrarily small values of the electron viscosity. The latter simply determines the spatial extent of the vortex away from the injector
- d. Our calculations indicate that current whirlpools can be detected by scanning magnetometry (e.g. Yacoby), with tens of micro Tesla for currents of 200 microamps

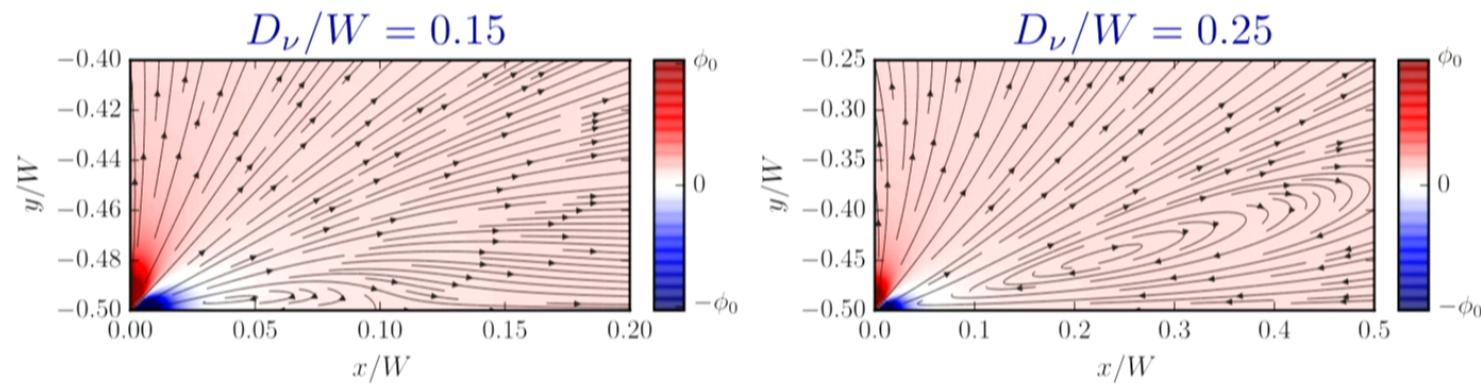
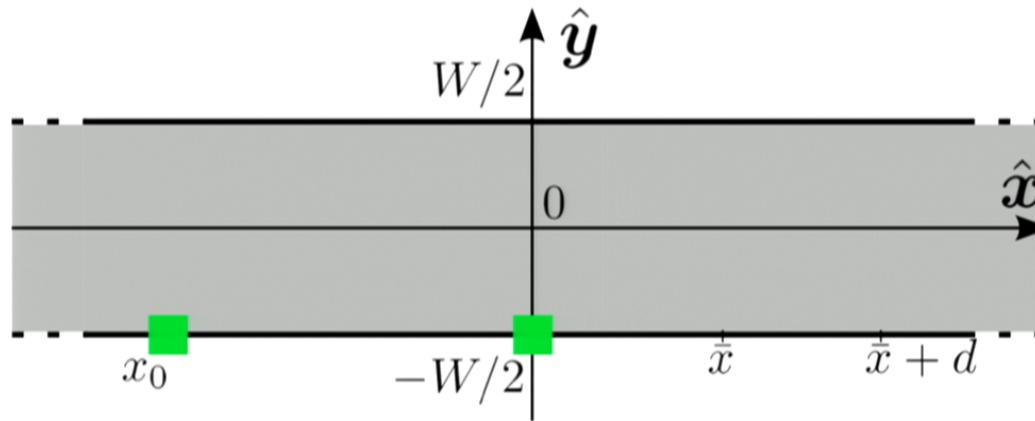
I. Torre, A. Tomadin, A.K. Geim, and M. Polini, Phys. Rev. B **92**, 165433 (2015)
F.M.D. Pellegrino, I. Torre, A.K. Geim, and M. Polini, arXiv:1607.03726 (Phys. Rev. B, 2016)

The hydro dilemma: whirlpools or no whirlpools?



G. Falkovich and L. Levitov, arXiv:1607.00986 (Falkovich's talk at Perimeter some time ago, Levitov's talk)
F.M.D. Pellegrino, I. Torre, A.K. Geim, and M. Polini, arXiv:1607.03726 (Phys. Rev. B, 2016)

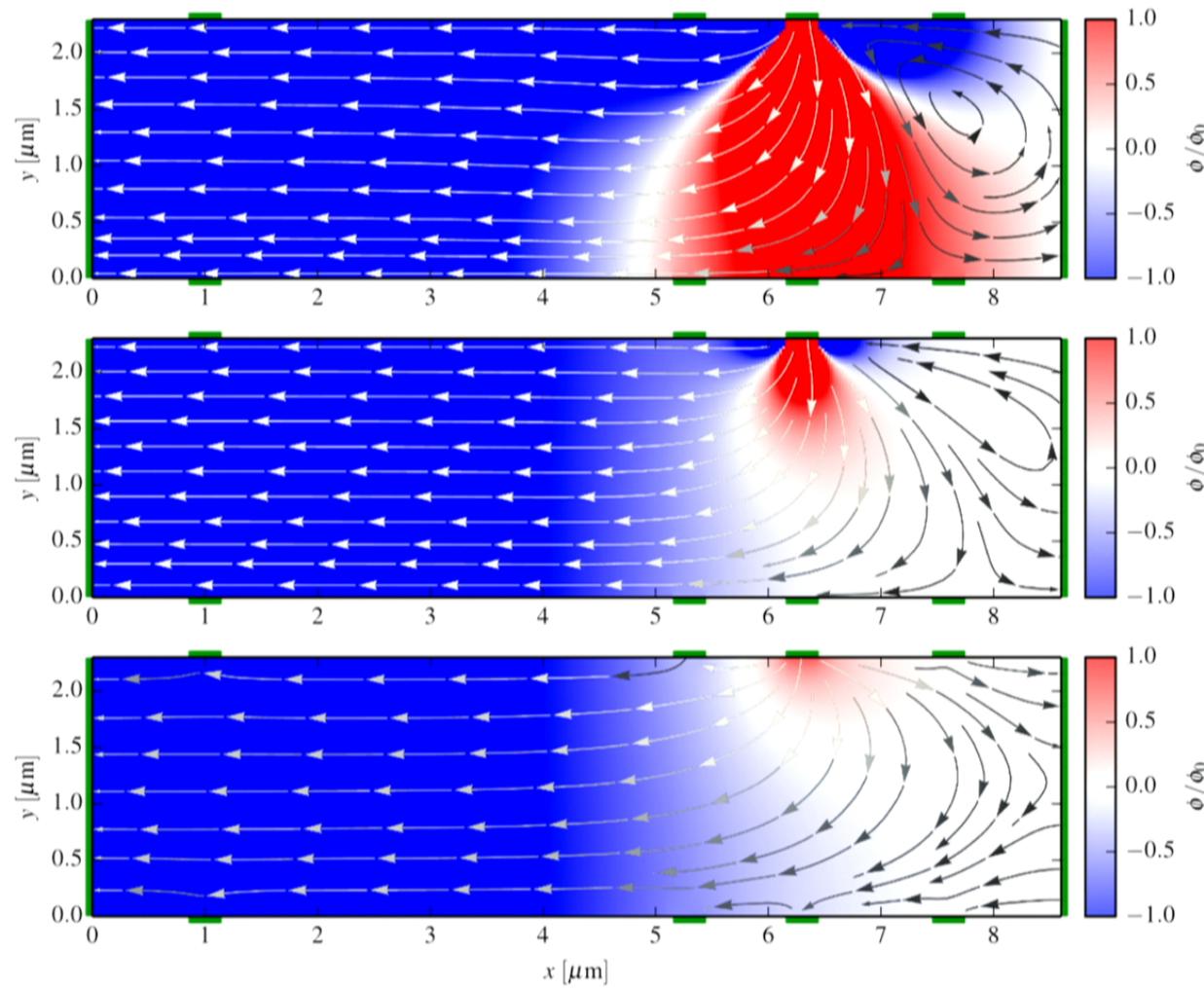
The hydro dilemma: whirlpools or no whirlpools?



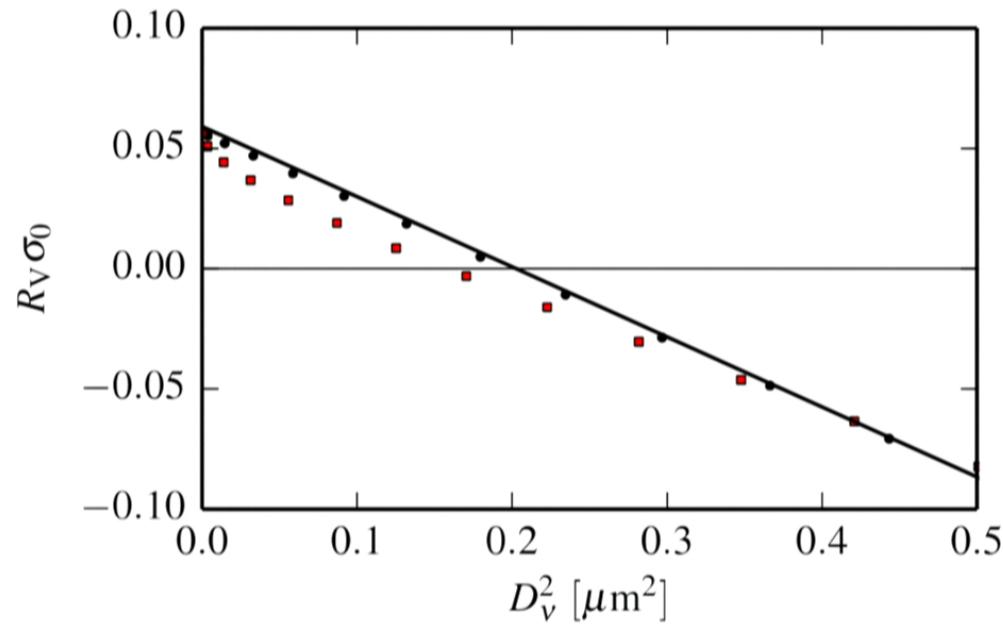
$$D_\nu/W = \frac{1}{\sqrt{2}\pi} \simeq 0.225$$

F.M.D. Pellegrino, I. Torre, A.K. Geim, and M. Polini, arXiv:1607.03726 (Phys. Rev. B, 2016)

Negative nonlocal resistance and current whirlpools



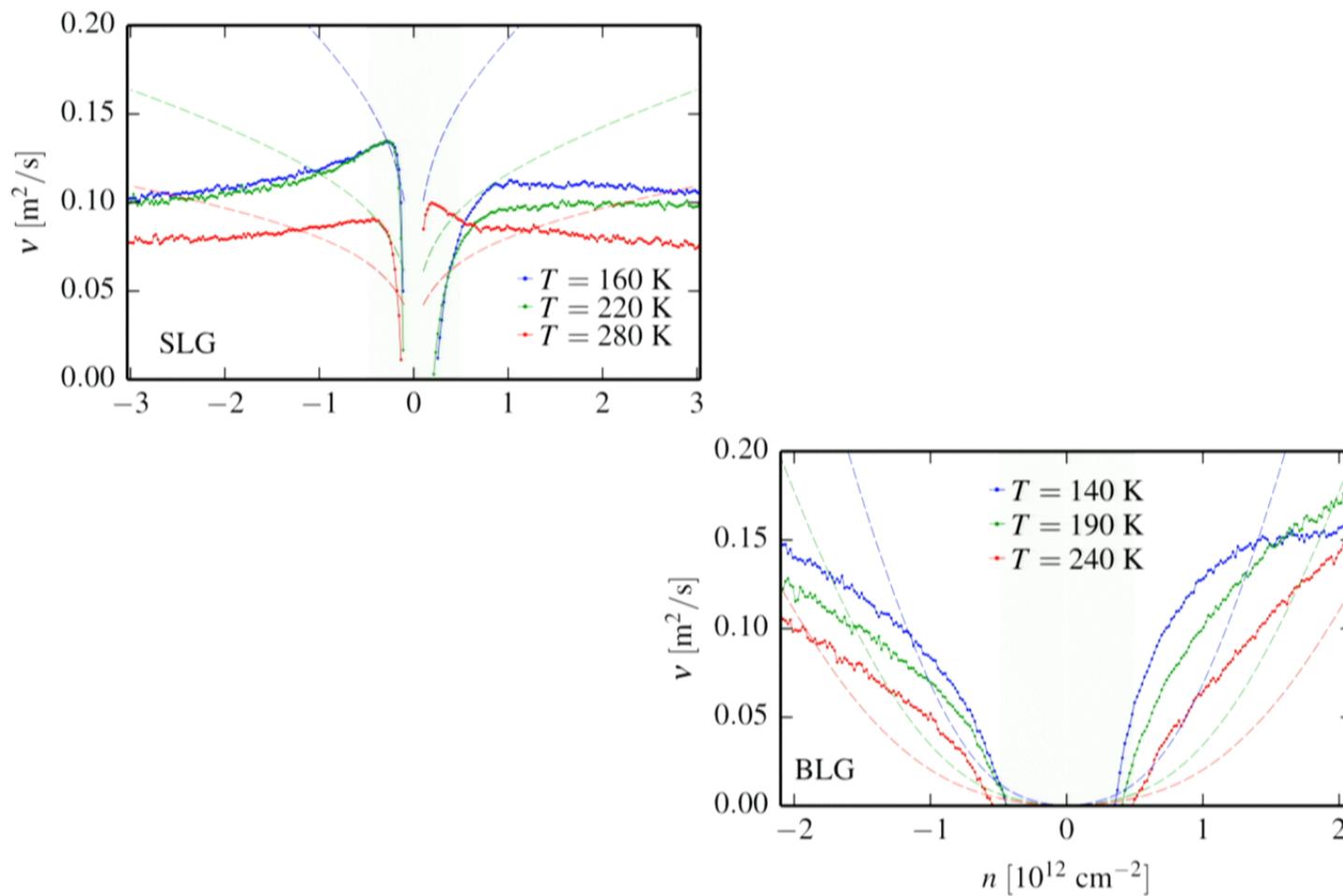
All-electrical rheometry of 2D Dirac-Weyl liquids



$$R_V = (b + aD_v^2)\sigma_0^{-1}$$

- a. Black filled circles: numerical results for the actual sample (BLG)
- b. Black solid line: analytical result for an infinitely long sample
- c. Red squares: numerical results for no-slip boundary conditions

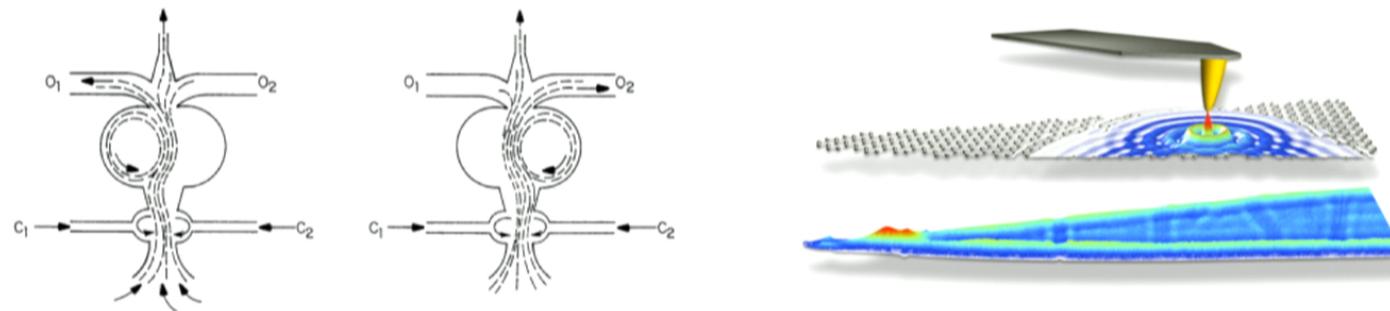
Kinematic viscosity



D. Bandurin, I. Torre, R.K. Kumar, M. Ben Shalom, A. Tomadin, A. Principi, G.H. Auton, E. Khestanova, K.S. Novoselov, I.V. Grigorieva, L.A. Ponomarenko, A.K. Geim, and M. Polini, Science **351**, 1055 (2016)

Conclusions

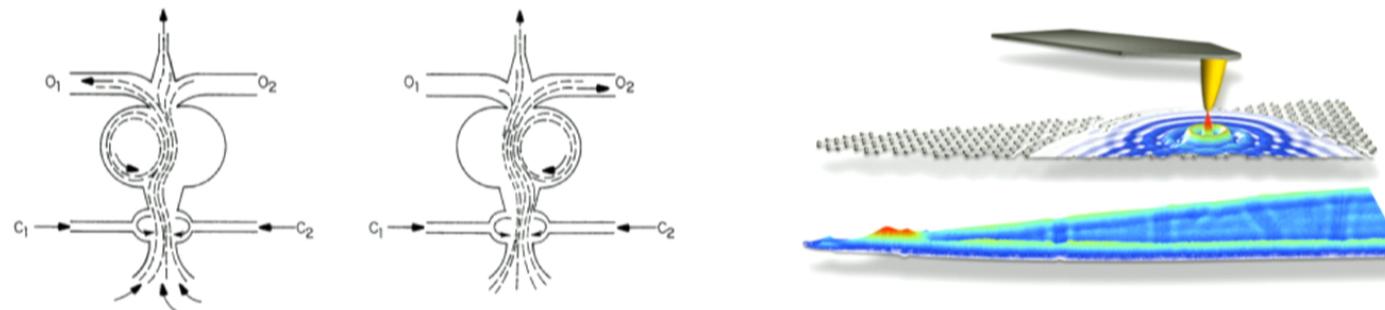
- 1) Encapsulated (doped) graphene samples display **hydrodynamic flow and viscosity-induced negative nonlocal resistances**
- 2) Spatial structures in the current and electrical potential distribution can be measured by existing **scanning probe methods**
- 3) **All-electrical rheometry** of electron liquids in solid-state devices is now possible



- Electron fluidic logic (non-linear interactions at $Re \sim 1$)
- **Pre-turbulence** at $Re \sim 10$
- **Magneto-hydrodynamics** of electron liquids (beyond-NS equation)
- Hydrodynamic effects, shear viscosities, and thermal conductivities by **THz near-field optics** (Koppens, Hillenbrand, Basov, etc)

Conclusions

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