Title: Hydrodynamic theory of transport in Dirac and Weyl semimetals

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Abstract: I will discuss recent progress in understanding the consequences of hydrodynamic electron flow on measurable transport properties of metals, focusing on metals where the electrons behave as a charge neutral relativistic plasma. In graphene, I will connect our theoretical models with experimental data and show how we can explain features of transport in graphene that are inconsistent with quasiparticle transport. I will then discuss the extension of these results to Weyl semimetals, which are modeled by a system of multiple chiral fluids. Negative magnetoresistance can occur in both electric and thermal transport; the latter is a consequence of a distinct axial-gravitational anomaly. Future transport experiments on Weyl semimetals can discover this exotic type of anomaly in the lab.

Hydrodynamic theory of transport for Dirac and Weyl semimetals

Andrew Lucas

Stanford Physics

Low Energy Challenges for High Energy Physicists II; Perimeter Institute

August 24, 2016

Collaborators

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Harvard Physics & Perimeter Institute

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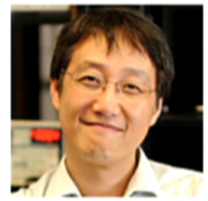
Harvard Physics



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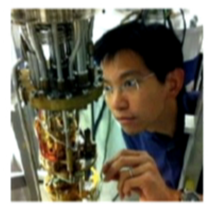
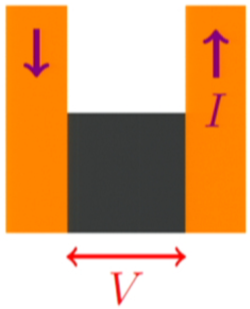


Kin Chung Fong
Raytheon BBN

- Ohm's law – the "simplest" experiment:



$$V = IR \qquad R \sim \frac{1}{\sigma}$$

Transport

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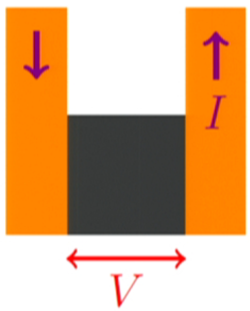


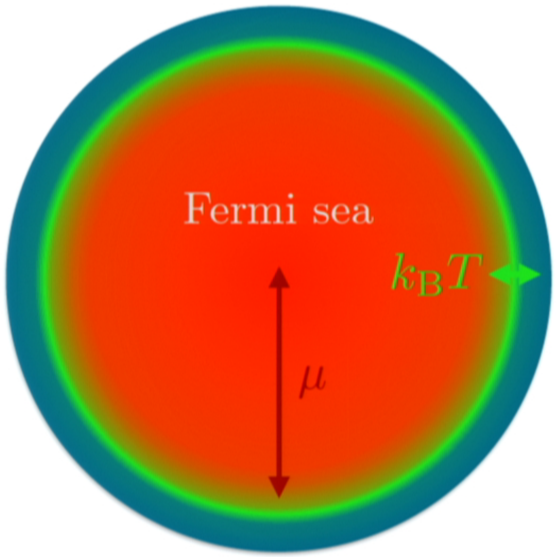
$$V = IR \qquad R \sim \frac{1}{\sigma}$$

- more generally, thermoelectric transport:

$$\begin{pmatrix} \mathbf{J} \\ \mathbf{Q} \end{pmatrix} = \begin{pmatrix} \sigma & \alpha \\ T\bar{\alpha} & \bar{\kappa} \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ -\nabla T \end{pmatrix}$$

Fermi "Liquids"

...are Analogous to a Classical **Gas**



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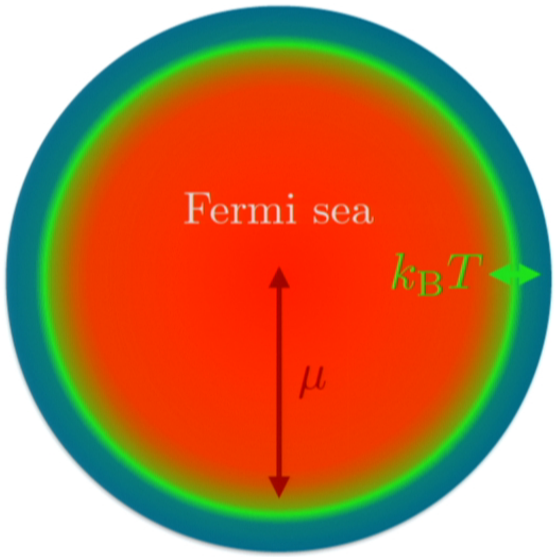

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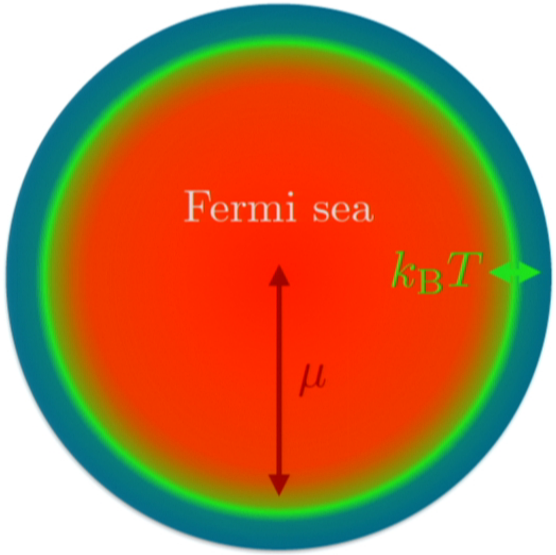

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$$t_{ee} \sim \frac{\hbar\mu}{(k_B T)^2} \sim 10^{-11}\ \mathrm{s}$$

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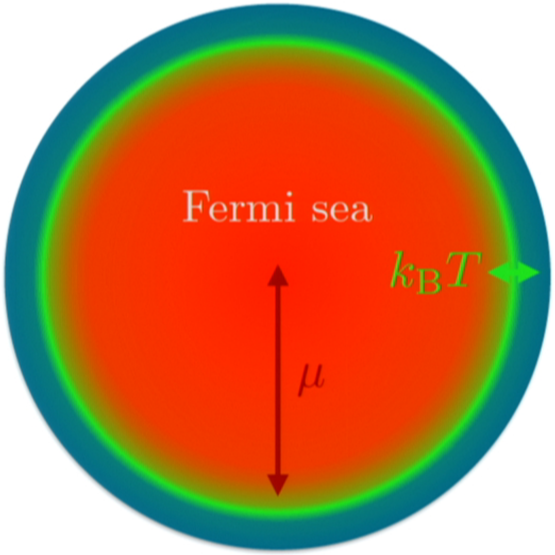

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- QP interacts with impurity or phonon $\sim$ every 10^{-14} s (exceptions are rare, e.g. GaAs, doped graphene)

Fermi "Liquids"

Wiedemann-Franz Law

- experimentally measured thermal conductivity:

$$\mathbf{Q}|_{\mathbf{J}=\mathbf{0}} \equiv -\kappa \nabla T, \quad \kappa = \bar{\kappa} - \frac{T\alpha^2}{\sigma}.$$

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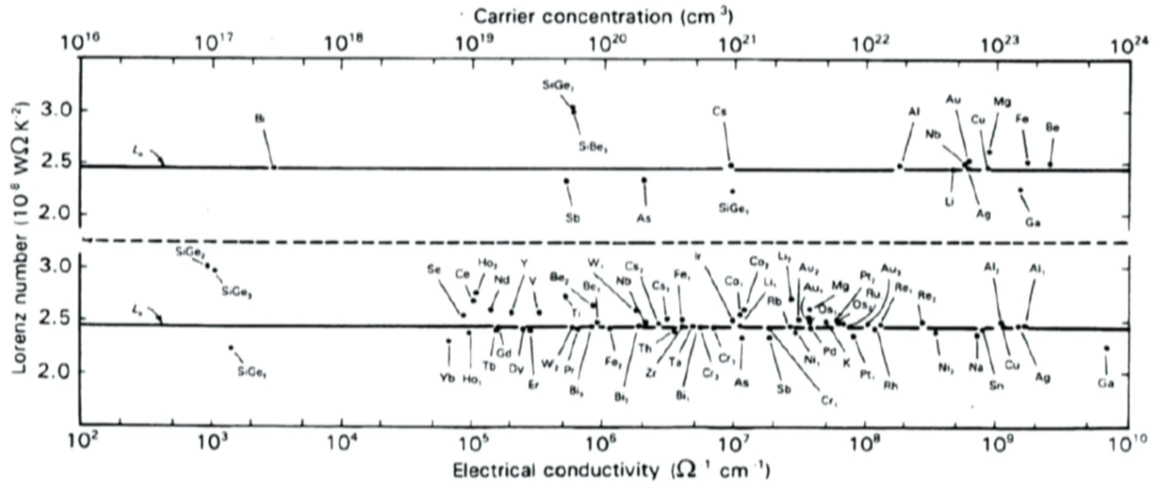
$$\mathbf{Q}\big|_{\mathbf{J}=\mathbf{0}} \equiv -\kappa \nabla T, \quad \kappa = \bar{\kappa} - \frac{T\alpha^2}{\sigma}.$$

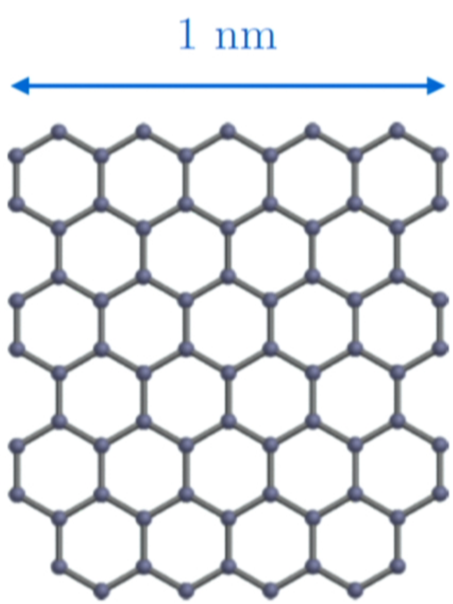
- Wiedemann-Franz law in a Fermi liquid:

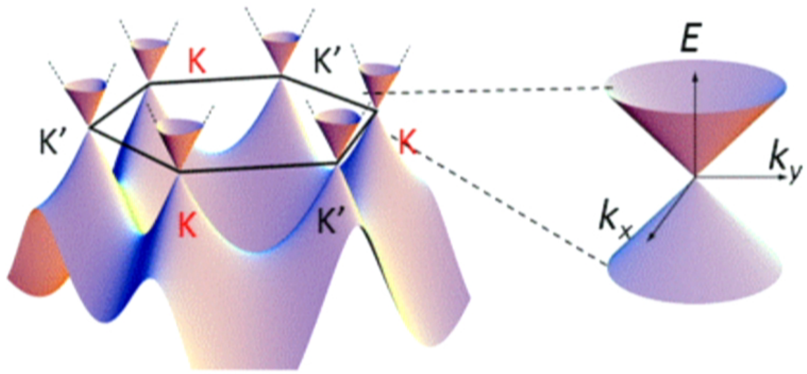
$$\mathcal{L} \equiv \frac{\kappa}{\sigma T} \approx \frac{\pi^2 k_{\mathrm{B}}^2}{3e^2} \approx 2.45 \times 10^{-8}\ \frac{\mathrm{W} \cdot \Omega}{\mathrm{K}^2}.$$



[Kumar, Prasad, Pohl (1993)]

Crash Course in Graphene



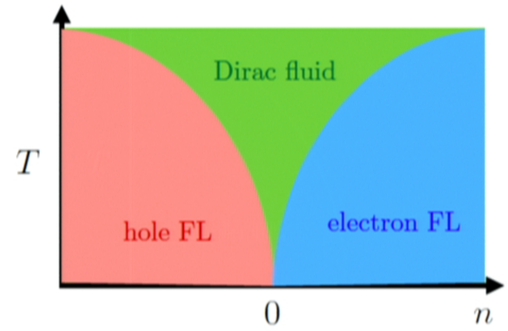


Crash Course in Graphene

$$\epsilon_{a\sigma} = \hbar v_{\mathrm{F}} k$$

$$+$$

$$V_{\mathrm{int}} = \frac{\alpha_{\mathrm{eff}}}{r}$$



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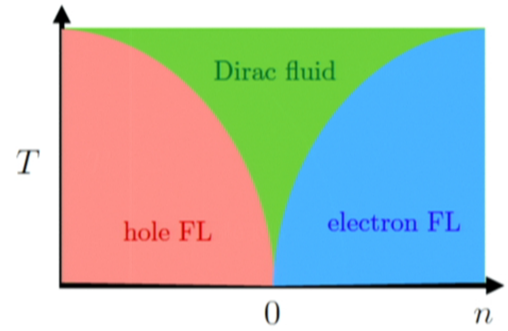

- marginally irrelevant $1/r$ Coulomb interactions:

$$\alpha_{\mathrm{eff}} = \frac{\alpha_0}{1 + (\alpha_0/4)\log((10^5\ \mathrm{K})/T)}, \quad \alpha_0 \approx \frac{1}{137}\frac{c}{v_{\mathrm{F}}\epsilon_{\mathrm{r}}} \sim 0.5.$$

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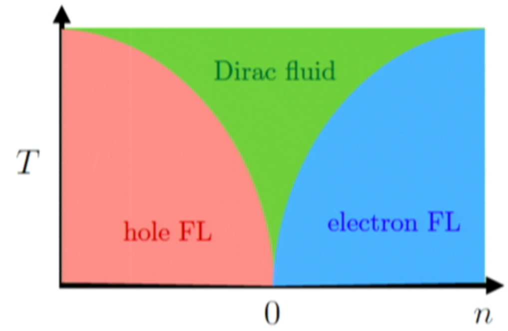

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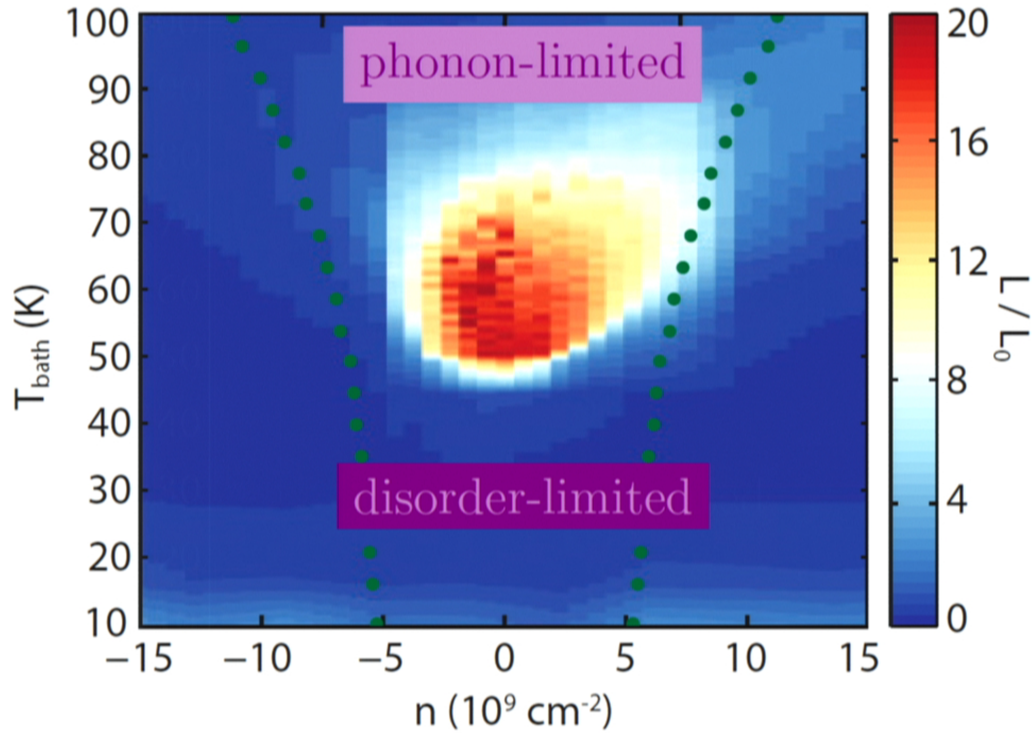
- not a Fermi liquid; (reasonably) strongly interacting at room temperature $\Longrightarrow$ hydrodynamics in clean samples

e.g. [Sheehy, Schmalian (2007); Müller, Fritz, Sachdev (2008)]

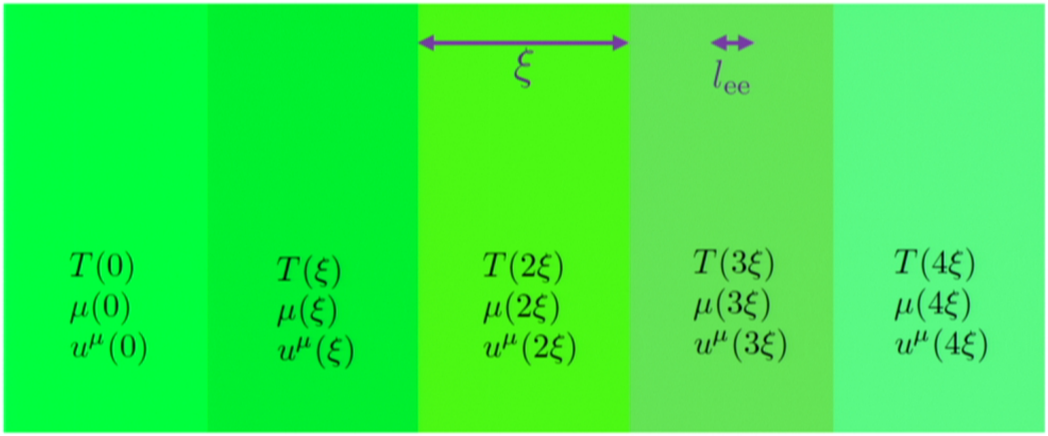
Transport in Graphene

Wiedemann-Franz Law Violations in Experiment



[Crossno *et al*, (2016)]

What is Hydrodynamics?



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- perturbative expansion in l_{ee}/ξ – opposite of usual QFT

Hydrodynamics

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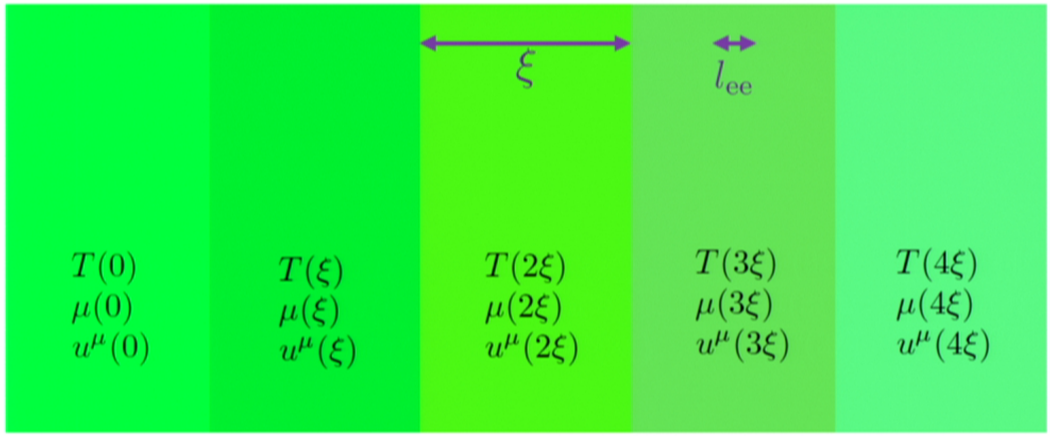

- effective description of relaxing to thermal equilibrium on **long length scales**
- perturbative expansion in l_{ee}/ξ – opposite of usual QFT
- **classical** equations of motion: **conservation laws**

$$\partial_\mu T^{\mu\nu} = F^{\mu\nu} J_\nu, \qquad \partial_\mu J^\mu = 0$$

The Gradient Expansion for a Lorentz-Invariant Fluid

- expand $T^{\mu\nu}$, J^{μ} in perturbative parameter $l_{\mathrm{ee}}\partial_\mu$:
  [Hartnoll, Kovtun, Müller, Sachdev (2007)]

$$T^{\mu\nu} = P\eta^{\mu\nu} + (\epsilon + P)u^\mu u^\nu - 2\mathcal{P}^{\mu\rho}\mathcal{P}^{\nu\sigma}\eta\partial_{(\rho}u_{\sigma)} - \mathcal{P}^{\mu\nu}\left(\zeta - \frac{2\eta}{d}\right)\partial_\rho u^\rho + \cdots,$$

$$J^\mu = nu^\mu - \sigma_{\mathrm{Q}}\mathcal{P}^{\mu\rho}\left(\partial_\rho\mu - \frac{\mu}{T}\partial_\rho T - u^\nu F_{\rho\nu}\right) + \cdots,$$

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- quantum physics → values of P, σ_{Q}, etc...
- fluid has *both electrons/holes*; *not* separately conserved. distinct from usual plasma physics

Hydrodynamic Transport

Conductivity of a Clean Metal

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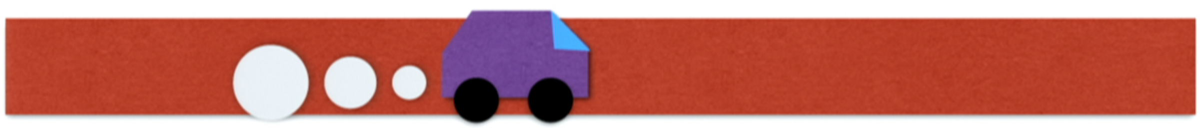
$J = 0$

$J = nv$ and $E = 0$



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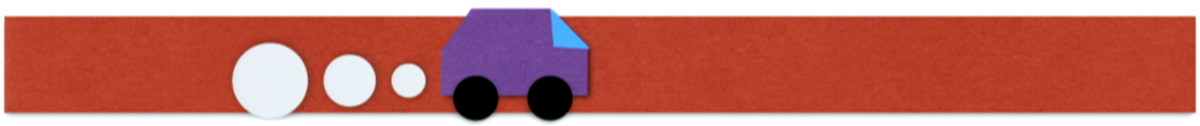

- σ sensitive to how translational symmetry broken

Mean Field: Drude Model

mean field treatment of translational symmetry breaking:

[Hartnoll, Kovtun, Müller, Sachdev (2007)]

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this formula is qualitatively OK; but quantitatively wrong...

Wiedemann-Franz Revisited

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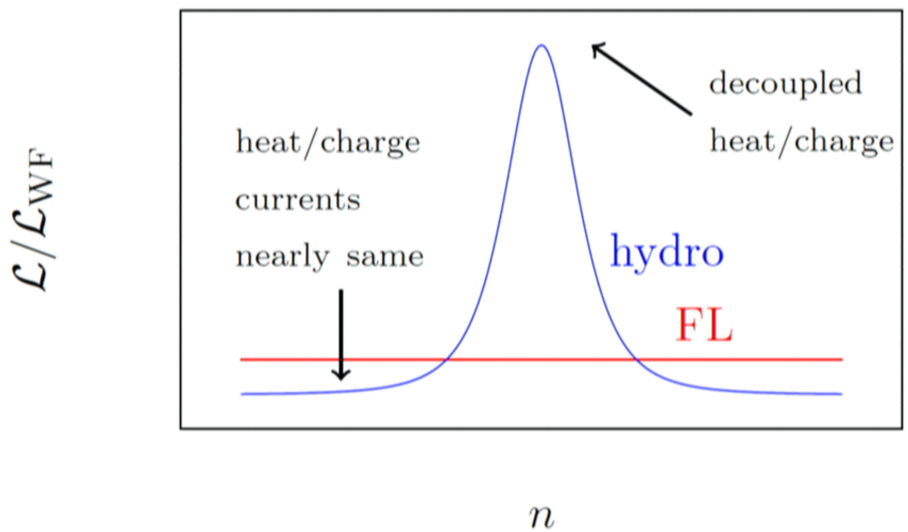

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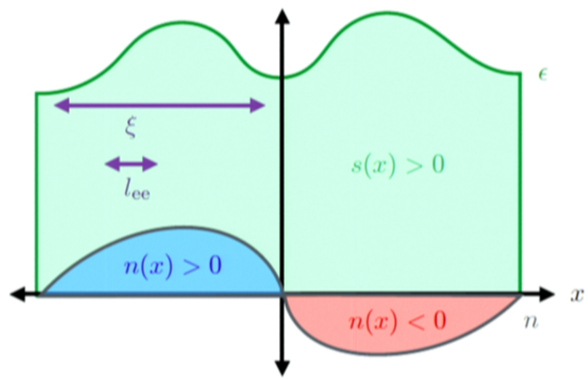
Non-Perturbative Approach



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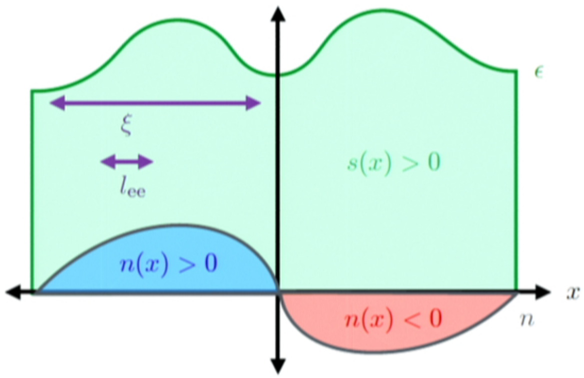

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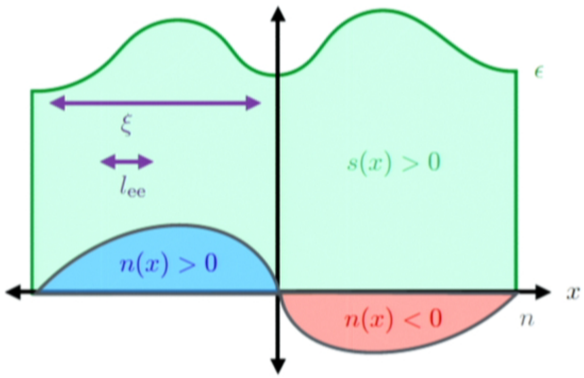

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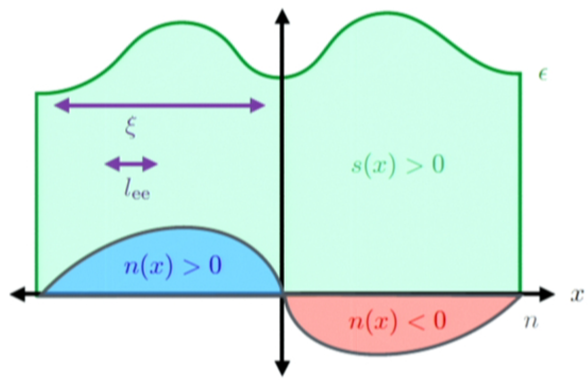

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- static fluid in an inhomogeneous chemical potential $\mu_0(\mathbf{x})$

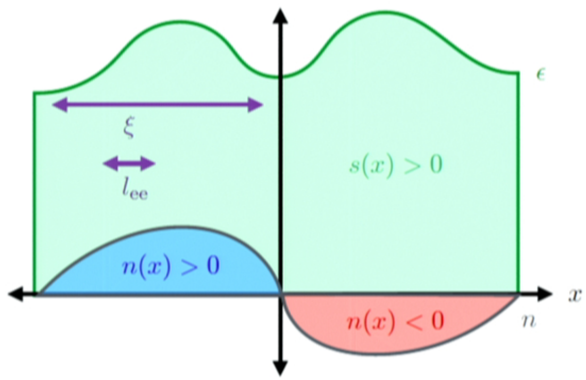
- transport from linearized hydrodynamic equations
- recover Drude model (without $\sigma_{\rm Q}$) as $\Delta\mu_0 \to 0$:

$$\sigma \approx \frac{n^2\tau}{\epsilon + P}, \quad \frac{1}{\tau} \sim (\Delta\mu_0)^2 \left(\frac{\partial n}{\partial \mu}\right)^2 \left[\frac{1}{\sigma_{\rm Q}(\epsilon+P)} + \frac{4\eta\mu^2}{\xi^2(\epsilon+P)^3}\right]$$

[Lucas (2015)]; [Lucas, Crossno, Fong, Kim, Sachdev (2016)]

similar ideas in FL: [Andreev, Kivelson, Spivak (2011)]

A Hydrodynamic Model for Graphene



$$s = C_0 T_0^2 + \frac{C_2}{2}\mu_0^2 - \frac{C_4 \mu_0^4}{4T_0^2} + \cdots,$$

$$n = C_2 \mu_0 T_0 + \frac{C_4}{T_0}\mu_0^3 + \cdots,$$

$$\eta = \eta_0 T_0^2, \quad \zeta = 0, \quad \sigma_{\mathrm{Q}} = \sigma_0$$

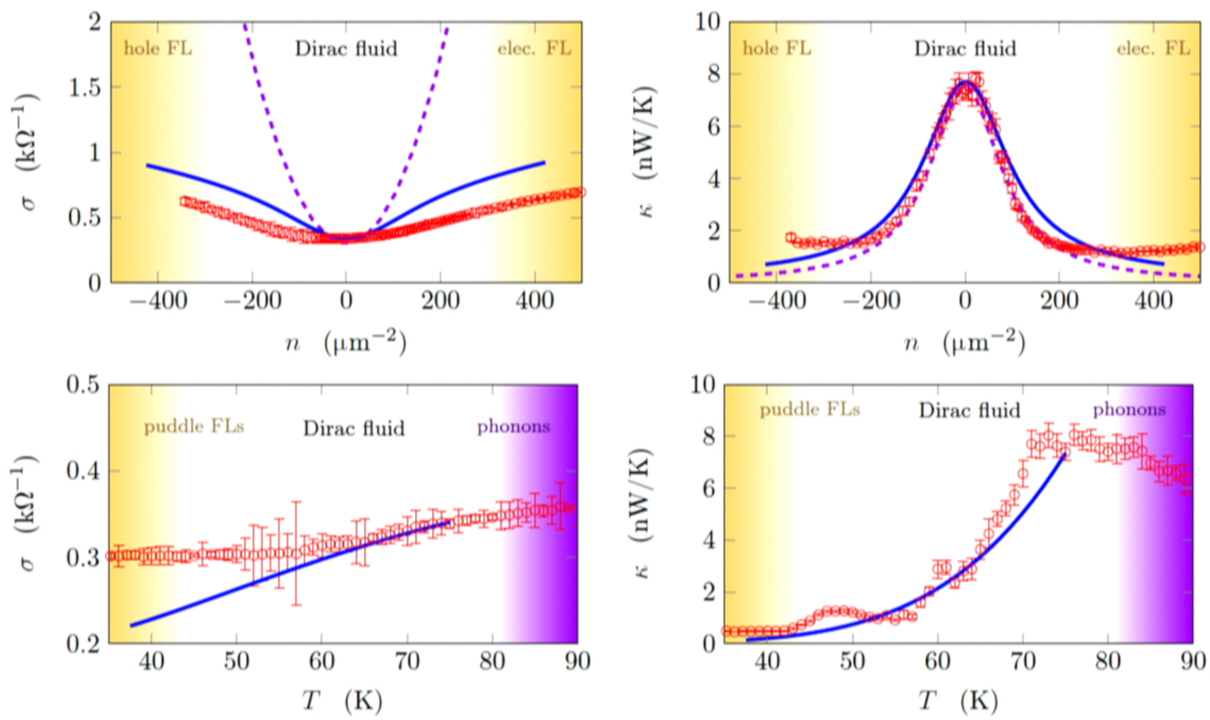
numerically solve linearized hydrodynamic equations:

$$0 = \nabla \cdot \left(n\mathbf{v} + \sigma_{\mathrm{Q}} \left(\mathbf{E} - \nabla\mu - \mu_0 \boldsymbol{\zeta} + \frac{\mu_0}{T_0}\nabla T \right)\right),$$

$$0 = \nabla \cdot \left(T_0 s\mathbf{v} - \mu_0 \sigma_{\mathrm{Q}} \left(\mathbf{E} - \nabla\mu - \mu_0 \boldsymbol{\zeta} + \frac{\mu_0}{T_0}\nabla T \right)\right),$$

$$0 = n(\nabla\mu - \mathbf{E}) + s\left(\nabla T - T_0 \boldsymbol{\zeta}\right) - \nabla \cdot \left(\eta \left(\nabla \mathbf{v} + \nabla \mathbf{v}^{\mathsf{T}} \right)\right) + \nabla(\eta \nabla \cdot \mathbf{v})$$

Comparing Theory to Experiment



[Crossno *et al*, (2016); Lucas, Crossno, Fong, Kim, Sachdev (2016)]

Weyl Semimetals

Band Theory

- Weyl Hamiltonian:

$$H = \pm\hbar v_{\mathrm{F}}(\sigma_x q_x + \sigma_y q_y + \sigma_z q_z).$$

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$$\mathcal{A}_i = \mathrm{i}\langle q|\frac{\partial}{\partial q_i}|q\rangle, \qquad \frac{1}{2\pi}\int \mathrm{d}^3\mathbf{q}\, \epsilon_{ijk}\mathcal{A}_i\partial_j\mathcal{A}_k = \pm 1$$

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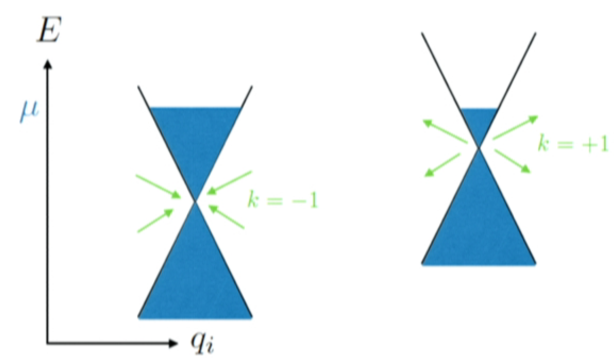
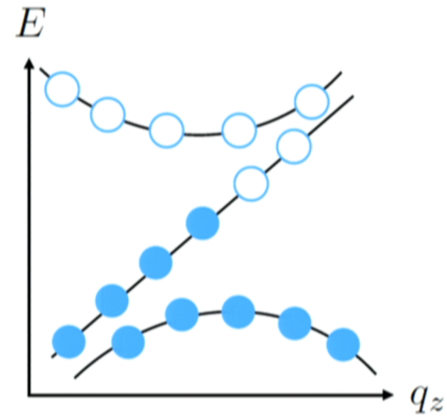
- theorem: net Berry flux must vanish on a lattice (BZ is compact) [Nielsen, Ninomiya (1983)]



Axial Anomaly

- consider applying $\mathbf{B} = B\hat{\mathbf{z}}$ to $k = 1$ Weyl fermion:



Weyl Semimetals

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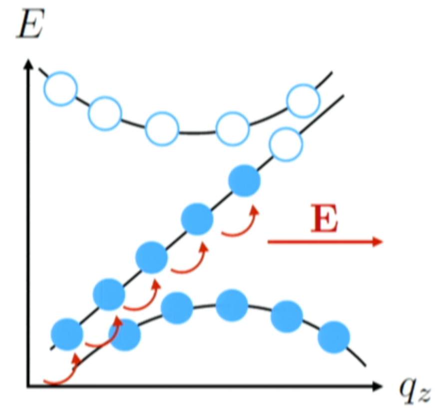

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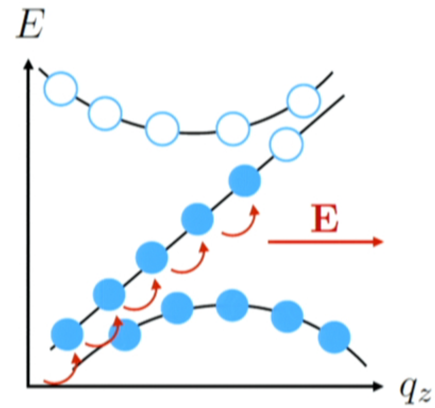

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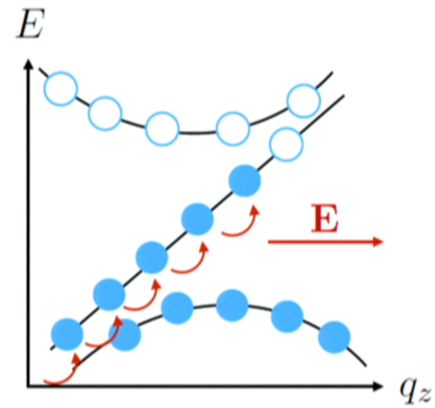

- quantum mechanical effect spoils current conservation: electromagnetic anomaly
- effect on *classical* hydrodynamics: [Son, Surówka (2009)]

$$\partial_\mu \langle J^\mu \rangle = -\frac{C}{8}\varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} = C\mathbf{E}\cdot\mathbf{B}, \quad C = \frac{k}{4\pi^2}.$$

Negative Magnetoresistance

- no static solution to equations for $k = \pm 1$ Weyl fermion:

$$\int \mathrm{d}^3\mathbf{x}\, \partial_i J^i = 0 \neq \int \mathrm{d}^3\mathbf{x}\, C\mathbf{E}\cdot\mathbf{B}.$$

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- but on the lattice:

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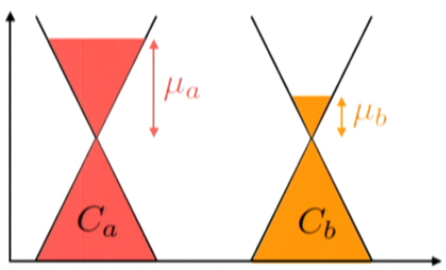

Negative Magnetoresistance

- no static solution to equations for $k = \pm 1$ Weyl fermion:

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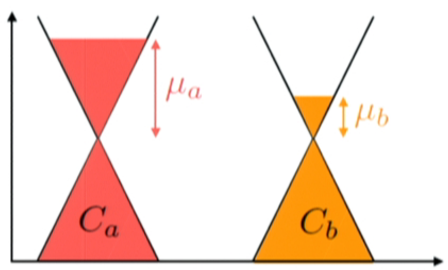

- finite $\sigma \implies$ exchange of charge between "valleys"

[Son, Spivak (2012)]

Axial-Gravitational Anomaly

- generation of *heat*: [Lucas, Davison, Sachdev (2016)]

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- there's no gravity in thermal transport – consistency on conical spaces $\Longrightarrow$ G_a alters thermo/hydro on flat space [Jensen, Loganayagam, Yarom (2012)]

Hydrodynamic Formalism

- $\mathbf{J}_a$ contains chiral components (fluid rest frame):

$$\mathbf{J}_a = (\mathbf{J}_a)_{\text{non-chiral}} + \mathcal{D}_1 \nabla \times \mathbf{v} + \frac{\mathcal{D}_2}{2}\mathbf{B}$$

$$\mathcal{D}_1 = \frac{C\mu^2}{2}\left(1 - \frac{2}{3}\frac{n\mu}{\epsilon + P}\right) - \frac{4G\mu n T^2}{\epsilon + P}, \quad \mathcal{D}_2 = C\mu\left(1 - \frac{1}{2}\frac{n\mu}{\epsilon + P}\right) - \frac{GnT^2}{\epsilon + P}.$$

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- hydrodynamics w/ intervalley scattering (rest frame):

$$\partial_\mu J_a^\mu = -\sum_b [\mathcal{R}_{ab}\nu_b + \mathcal{S}_{ab}\beta_b] \qquad \text{(charge)}$$

$$\partial_\mu T_a^{\mu t} = \sum_b [\mathcal{U}_{ab}\nu_b + \mathcal{V}_{ab}\beta_b] \qquad \text{(energy)}$$

with $\nu_a = \mu_a / T_a$ and $\beta_a = 1/T_a$, and

$$\sum_b \mathcal{R}_{ab} = \sum_b \mathcal{S}_{ab} = \sum_b \mathcal{U}_{ab} = \sum_b \mathcal{V}_{ab} = 0.$$

Second Law of Thermodynamics

- second law of thermodynamics:

$$\partial_\mu \left(\sum_a s_a u_a^\mu \right) = \sum_{ab} \begin{pmatrix} 1/T \\ \mu/T \end{pmatrix}_a^{\mathsf{T}} \begin{pmatrix} \mathcal{R}_{ab} & \mathcal{S}_{ab} \\ \mathcal{U}_{ab} & \mathcal{V}_{ab} \end{pmatrix}^{-1} \begin{pmatrix} 1/T \\ \mu/T \end{pmatrix}_b \geq 0$$

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Negative Magnetoresistance

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- dissipative $\mathcal{R}_{ab}$, etc., can be approximated as

$$\mathcal{R}_{ab} = \lim_{\omega \to 0} \frac{1}{\omega} \mathrm{Im} \left(G^{\mathrm{R}}_{\partial_t J_a^t, \partial_t J_b^t}(\omega) \right).$$

Negative Magnetoresistance

Perturbative Theory of Electrical Negative Magnetoresistance

$\mathbf{B} = B\hat{\mathbf{z}}$; disorder is long wavelength; solve transport equations:

$$\sigma_{ij} = \begin{pmatrix} \sum_a \frac{n_a^2 \Gamma_a}{\Gamma_a^2 + (n_a B)^2} & \sum_a \frac{n_a^3 B}{\Gamma_a^2 + (n_a B)^2} & 0 \\ \sum_a \frac{-n_a^3 B}{\Gamma_a^2 + (n_a B)^2} & \sum_a \frac{n_a^2 \Gamma_a}{\Gamma_a^2 + (n_a B)^2} & 0 \\ 0 & 0 & \sum_a \frac{n_a^2}{\Gamma_a} + \mathfrak{s} B^2 \end{pmatrix}$$

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Perturbative Theory of Thermoelectric Negative Magnetoresistance

- we find more generally

$$\alpha_{ij} = \alpha_{ij}^{\text{Drude}} + \mathfrak{a} B_i B_j, \quad \bar{\kappa}_{ij} = \bar{\kappa}_{ij}^{\text{Drude}} + \mathfrak{h} B_i B_j,$$

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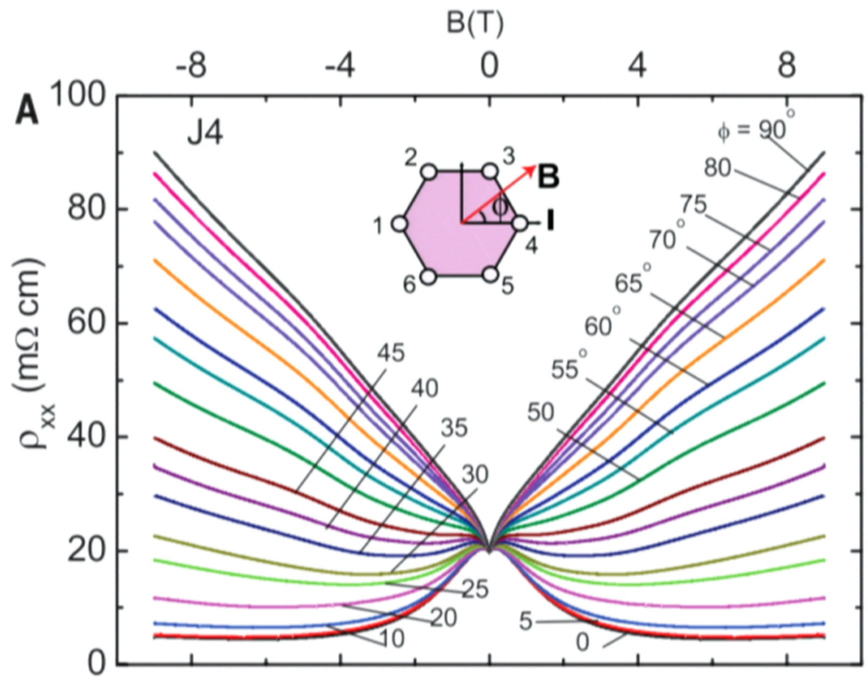
- thermoelectric conductivity matrix is Onsager reciprocal / positive definite.
- basic relaxation time approach does not obey these properties [Landsteiner, Liu, Sun (2015)]
- (N)MR in α, $\bar{\kappa}$ only possible if $G \neq 0$

Negative Magnetoresistance

Experimental Outlook

- NMR in σ has been measured (many times) [Xiong *et al* (2015)]



Wiedemann-Franz Law in Weyl Systems

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Negative Magnetoresistance

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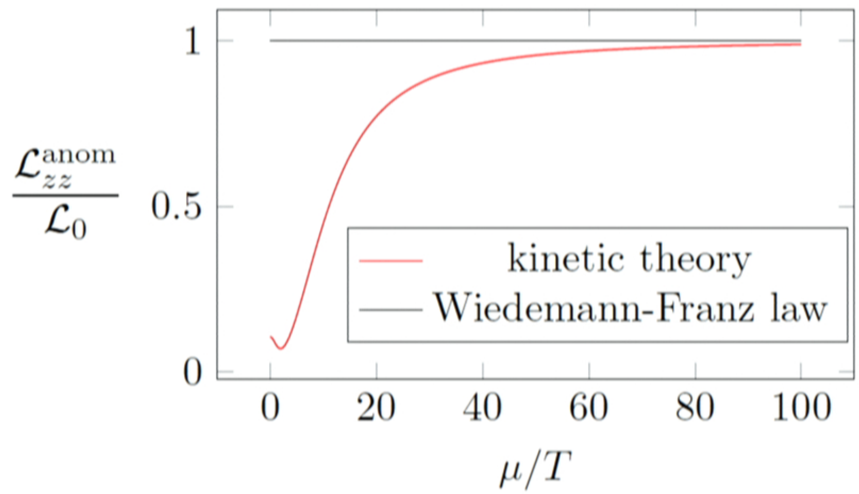
- using kinetic theory approximations to $\mathcal{R}$ (valid if $t_{\text{intervalley}} \gg \mu/T^2$):



Outlook

- hydrodynamics: tractable, non-perturbative approach to transport in very clean interacting metals
- experimental evidence emerging for hydrodynamic transport in graphene
- axial-gravitational anomaly plays a crucial role in thermal transport in Weyl materials