

Title: Superconducting quantum criticality of Dirac fermions

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Abstract: The semimetal-superconductor quantum phase transition of 2D Dirac fermions, such as found on the surface of a topological insulator, is conjectured to exhibit an emergent $N=2$ supersymmetry, based on a one-loop renormalization group analysis. In this talk I will present further evidence for this conjecture based on a three-loop analysis. Assuming the conjecture is true, I will present exact results for certain critical properties including the optical conductivity, shear viscosity, and entanglement entropy at zero temperature, as well as the finite-temperature optical conductivity.



Collaborators



Nikolai Zerf
(Heidelberg)



Chien-Hung Lin
(Alberta)

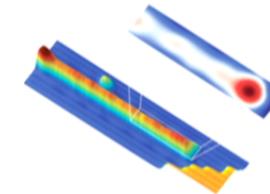
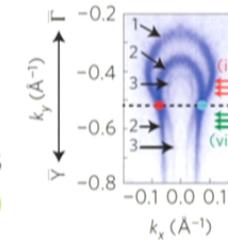
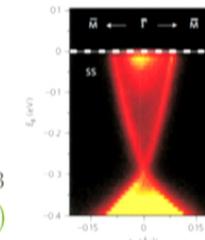


William Witczak-Krempa
(Montréal)

N. Zerf, C.-H. Lin, and JM, arXiv:1605.09423
W. Witczak-Krempa and JM, PRL 116, 100402 (2016) [arXiv:1510.06397]

Topological surface states

- ❖ Topological insulators: Dirac fermions
 Bi_2Se_3
(Xia et al., Nat. Phys. '09)
- ❖ Weyl semimetals: Fermi arcs
 TaAs
(Yang et al., Science '15)
- ❖ Topological superconductors: Majorana fermions
 Fe/Pb
(Nadj-Perge et al., Science '14)



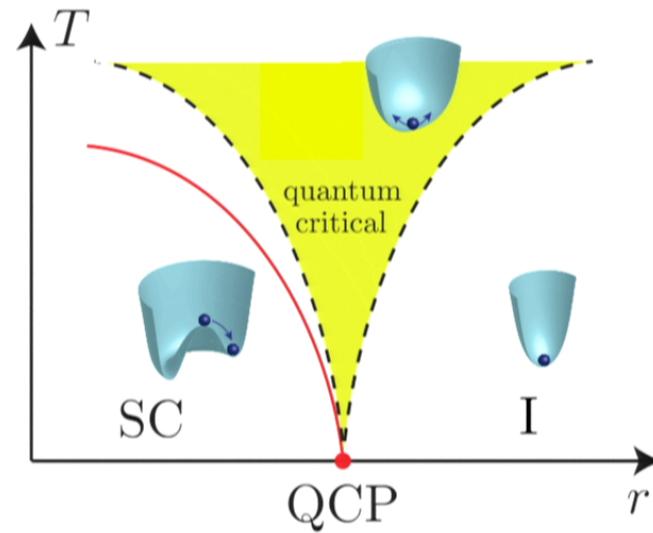
Topological surface states: novel gapless fermionic vacua

- ❖ New platforms for fermionic quantum criticality
- ❖ "Anomalous" character of surface state: phase transitions
not possible in lattice model of same dimensionality?
- ❖ Focus on semimetal-superconductor transition on surface
of 3D TI: single 2D Dirac fermion

Outline

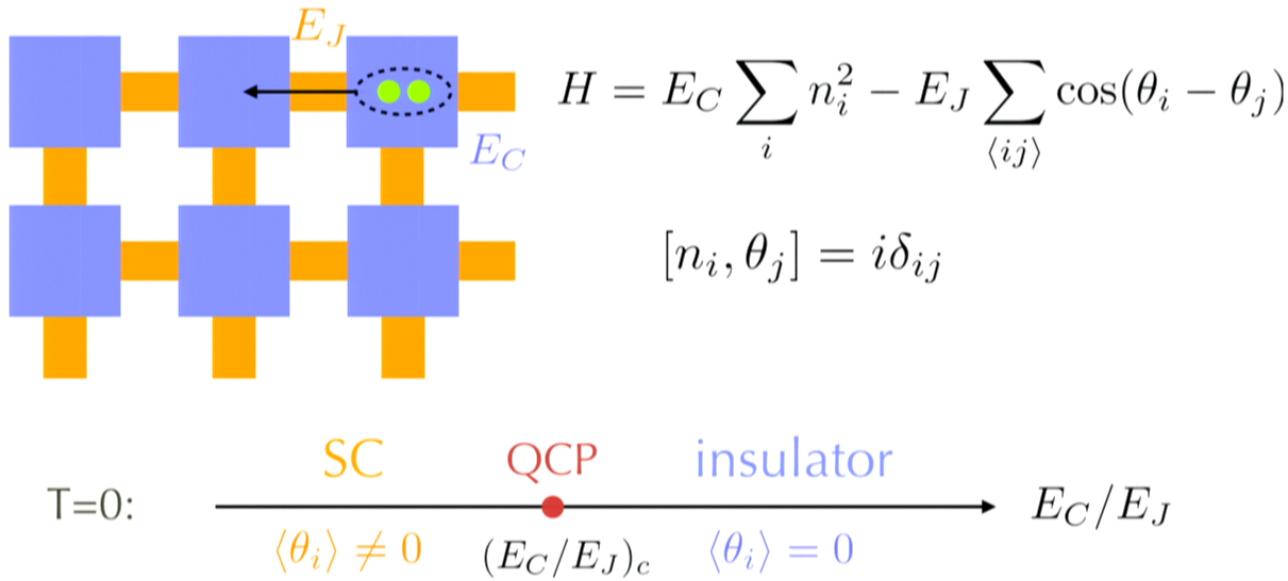
- ❖ Warm-up: superconductor-insulator transition of bosons
- ❖ Semimetal-superconductor transition of 2D Dirac fermions: emergent $N=2$ supersymmetry
 - ❖ Critical exponents (3-loop)
 - ❖ Response properties (exact)

Superconductor-insulator transition of bosons

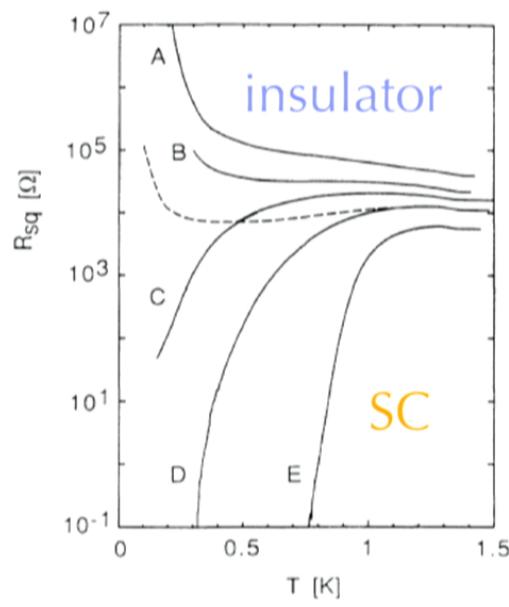


Route #1: Josephson junction array

- ❖ SC islands coupled via Cooper pair tunneling
- ❖ Assume $E_C, E_J \ll \Delta$: no low-energy fermions

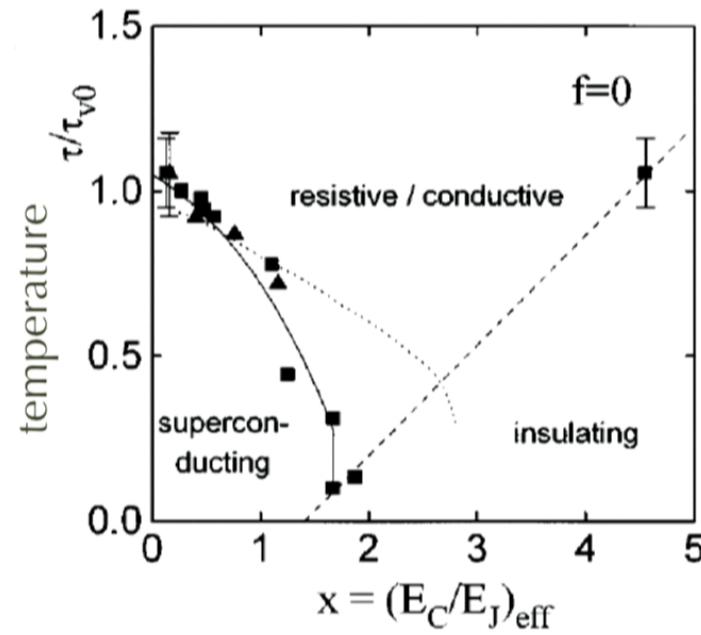


(Al junctions)



Geerligs et al., PRL '89

van der Zant et al., PRB '96

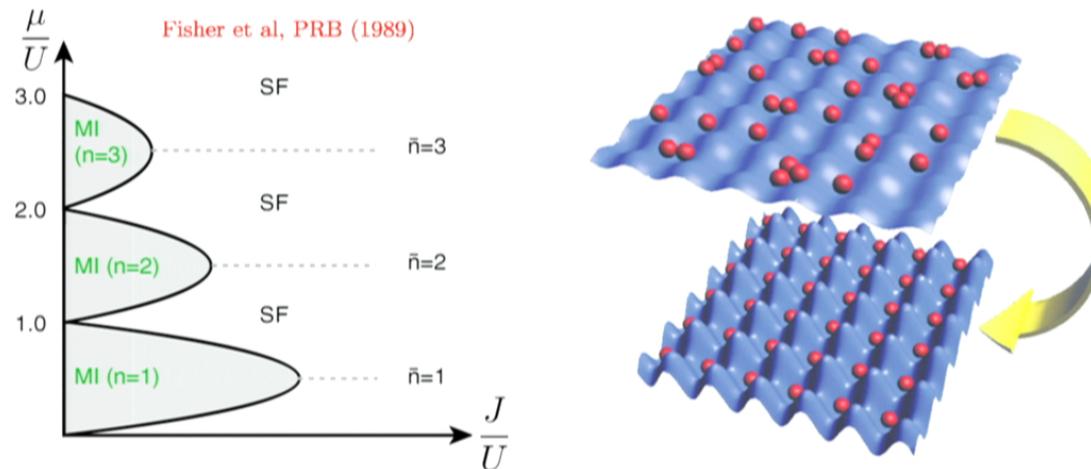


$$(E_C/E_J)_c \approx 1.7$$

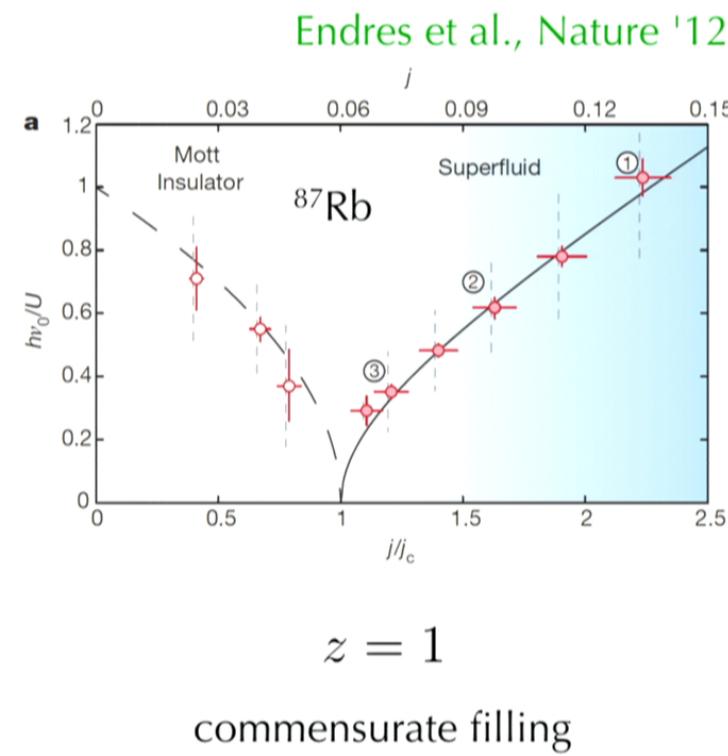
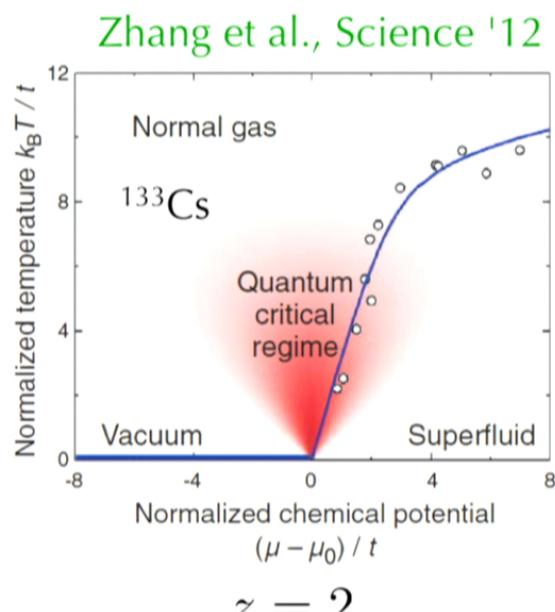
Route #2: BEC in optical lattice

- ❖ Near-ideal realization of 2D Bose-Hubbard model:

$$H = -J \sum_{\langle ij \rangle} b_i^\dagger b_j - \mu \sum_i n_i + \frac{U}{2} \sum_i n_i(n_i - 1)$$



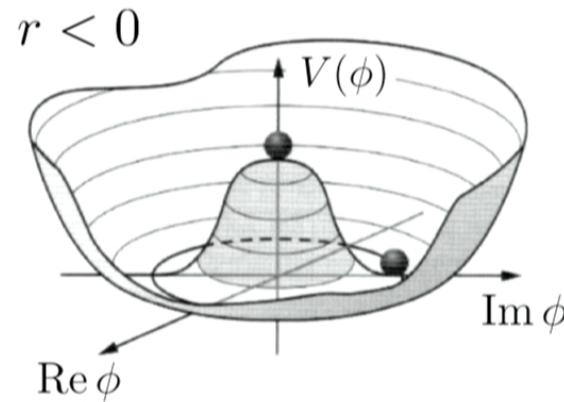
- ❖ Experimental observation of 2D superfluid-Mott insulator transition (Spielman et al., PRL '07)



Landau-Ginzburg theory

- ❖ Coarse-grained description: order parameter = bosonic Cooper pair field $\phi(\mathbf{r}, \tau)$

$$\mathcal{L} = |\partial_\tau \phi|^2 + c_b^2 |\nabla \phi|^2 + V(\phi)$$

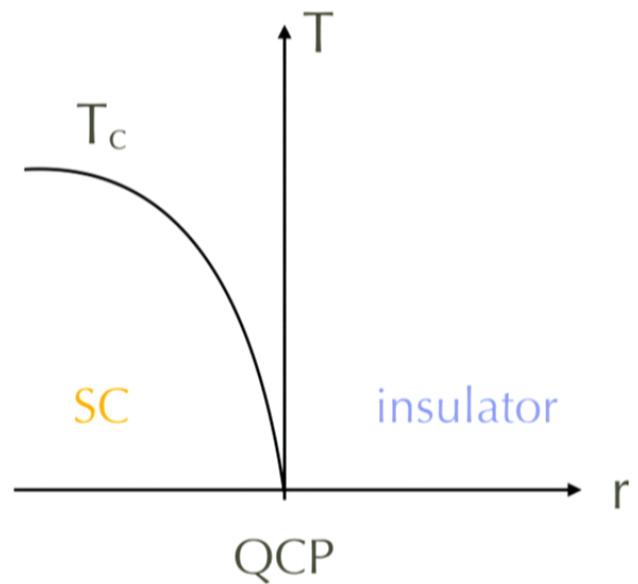


$$V(\phi) = r|\phi|^2 + u|\phi|^4$$

$$r \sim g - g_c,$$
$$g = \frac{E_C}{E_J}, \frac{J}{U}$$

- ❖ Spontaneous breaking of U(1) symmetry

Quantum critical point



- ❖ u is relevant in (2+1)D
- ❖ QCP is strongly coupled: O(2) Wilson-Fisher fixed point (3DXY)
- ❖ Emergent Lorentz invariance ($z=1$)

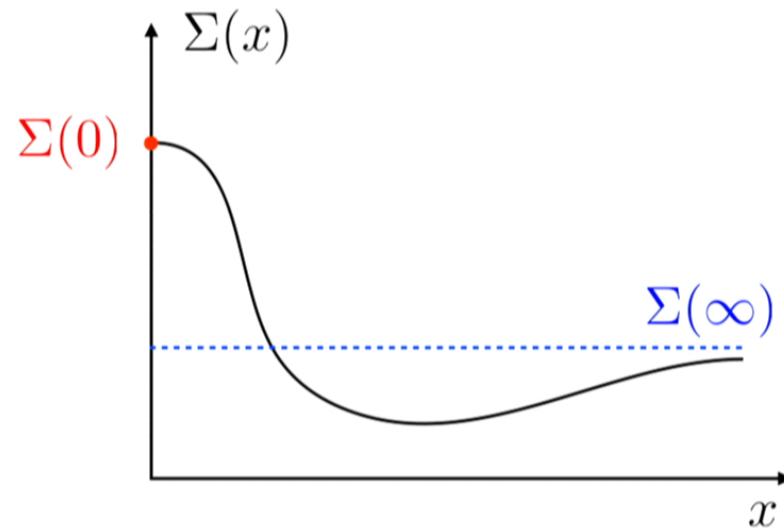
Optical conductivity

$$\sigma(\omega, T) = \frac{1}{i\omega} \langle J_x(\mathbf{q} = 0, \omega) J_x(\mathbf{q} = 0, -\omega) \rangle$$

- ❖ J_x : conserved current of the U(1) symmetry (boson current)

Optical conductivity

$$\sigma(\omega, T) = \frac{e^2}{\hbar} \left(\frac{k_B T}{\hbar c} \right)^{(d-2)/z} \Sigma \left(\frac{\hbar \omega}{k_B T} \right)$$



Damle & Sachdev, PRB '97

Optical conductivity: (2+1)D

$$\sigma(\omega, T) = \frac{e^2}{\hbar} \Sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

- ❖ T=0 optical conductivity = universal constant

$$\sigma(\omega, 0) = \frac{e^2}{\hbar} \Sigma(\infty) = \frac{e^2}{\hbar} \sigma_\infty$$

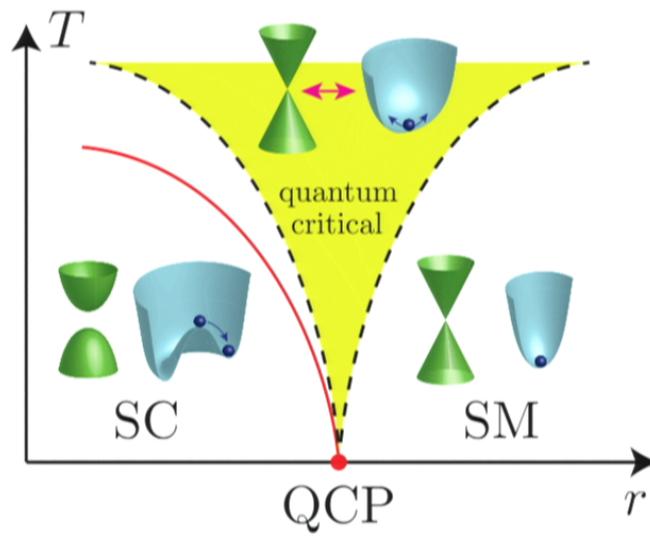
- ❖ σ_∞ related to T=0 JJ correlation function

Boson S-I transition

- ❖ Universal conductivity σ_∞ : no exact result, long history
(Fisher, Grinstein, Girvin, PRL '90; Cha et al., PRB '91; Fazio & Zappalà, PRB '96; Šmakov & Sørensen, PRL '05; ...)
- ❖ QMC + holography + conformal bootstrap (Katz et al., PRB '14; Gazit et al., PRB '13, PRL '14; Chen et al., PRL '14; Witczak-Krempa et al., Nat. Phys. '14; Kos et al., JHEP '15)

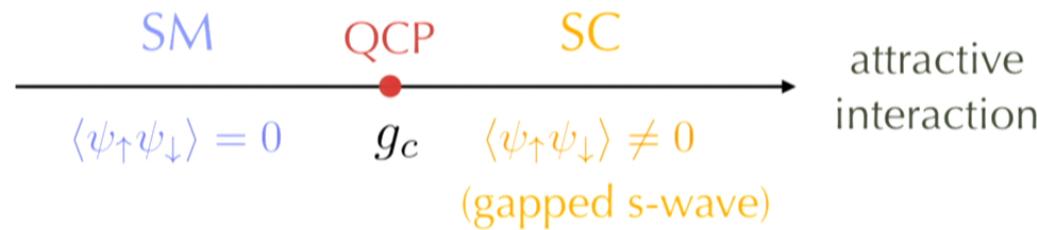
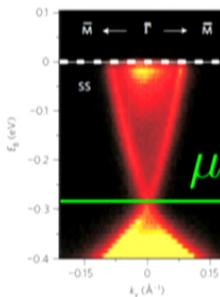
$$\sigma_\infty \simeq 0.226$$

Semimetal-superconductor transition of Dirac fermions



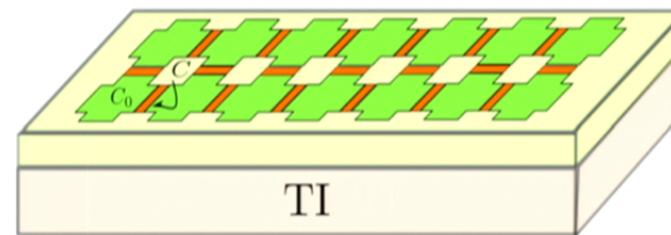
SM-SC transition of 2D Dirac fermions

- ❖ Pairing instability of **single** 2D Dirac fermion: 3D TI surface $\begin{pmatrix} \psi_{\uparrow} \\ \psi_{\downarrow} \end{pmatrix}$
- ❖ Consider μ at Dirac point: vanishing DOS implies QCP



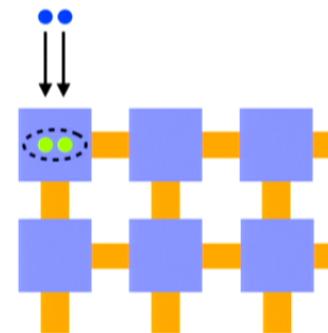
Route #1: Josephson engineering

- ❖ JJA on surface of TI



Ponte & Lee, NJP '14

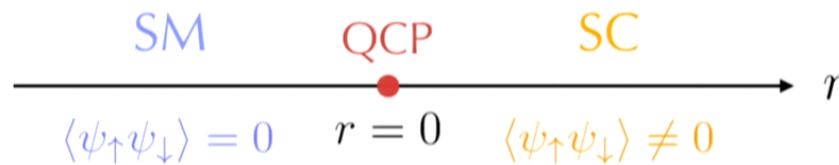
- ❖ Pairs of Dirac electrons tunnel to SC island and vice-versa



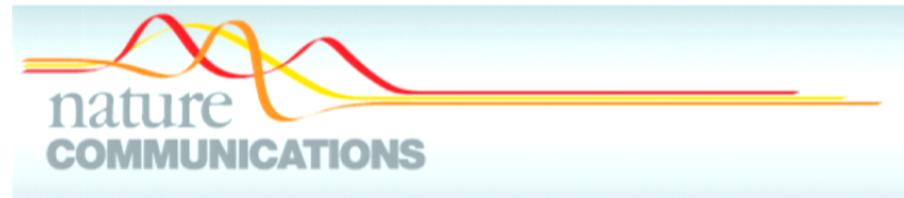
Landau-Ginzburg theory

- ❖ Low-energy theory has bosons **and** fermions (pair-breaking effects)

$$\begin{aligned}\mathcal{L} = & i\bar{\psi}(\gamma_0\psi_0 + c_f\gamma_i\partial_i)\psi && \text{Dirac} \\ & + |\partial_0\phi|^2 + c_b^2|\partial_i\phi|^2 + r|\phi|^2 + \lambda^2|\phi|^4 && \text{JJA} \\ & + 2h(\phi^*\psi_\uparrow\psi_\downarrow + \text{c.c.}) && \text{Dirac-JJA tunneling}\end{aligned}$$



Route #2: Intrinsic SC?



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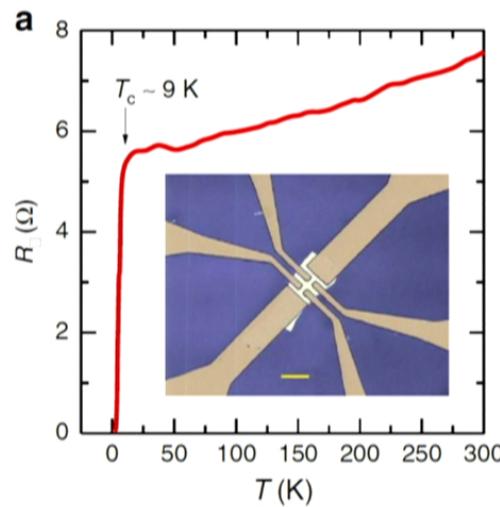
Received 31 Aug 2014 | Accepted 6 Aug 2015 | Published 11 Sep 2015

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Emergent surface superconductivity in the topological insulator Sb_2Te_3

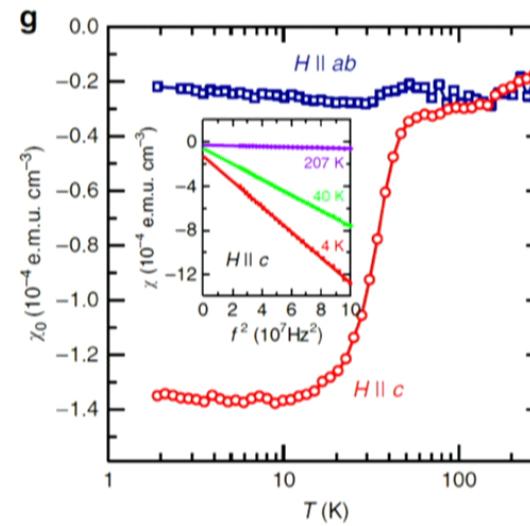
Lukas Zhao¹, Haiming Deng¹, Inna Korzhovska¹, Milan Begliarbekov¹, Zhiyi Chen¹, Erick Andrade², Ethan Rosenthal², Abhay Pasupathy², Vadim Oganesyan^{3,4} & Lia Krusin-Elbaum^{1,4}

Route #2: Intrinsic SC?



- ❖ Resistive transition at $T_c = 8.6\text{ K}$
- ❖ Anisotropic (2D) diamagnetic screening below $T \sim 50\text{ K}$ ($\sim 2\%$ of Meissner value)

- ❖ Tune residual Coulomb interaction with metallic gate to reach QCP (Ponte & Lee, NJP '14)



Quantum critical point

- ❖ 1-loop RG: At low energies, $c_f - c_b \rightarrow 0$: emergent Lorentz invariance (Lee, PRB '07)

$$\begin{aligned}\mathcal{L} = i\bar{\psi}\gamma_\mu\partial_\mu\psi + |\partial_\mu\phi|^2 + r|\phi|^2 \\ + \lambda^2|\phi|^4 + h(\phi^*\psi^T i\sigma_2\psi + \text{h.c.})\end{aligned}$$

- ❖ At the QCP ($r=0$), emergent SUSY! (Thomas '05; Lee, PRB '07; Grover, Sheng, Vishwanath, Science '14; Ponte, Lee, NJP '14)
- ❖ Strongly coupled (2+1)D SCFT: N=2 Wess-Zumino model (Aharony et al., NPB '97)
- ❖ Finite $\lambda = h$ at the QCP: strongly interacting; universality class neither Gaussian nor 3D XY

SUSY QCP

- ❖ SUSY fixes exact anomalous dimensions of ψ, ϕ

$$\eta_\phi = \eta_\psi = \frac{1}{3}$$

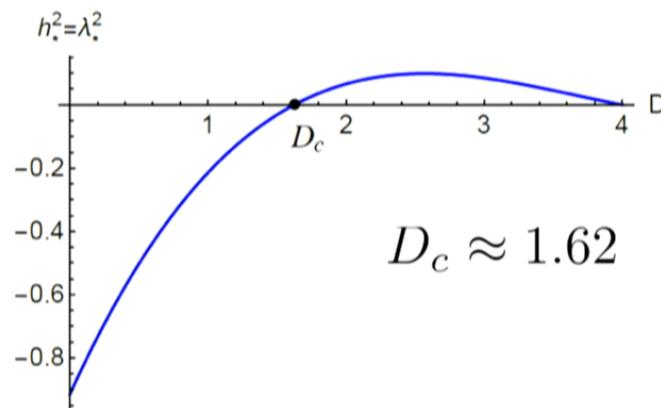
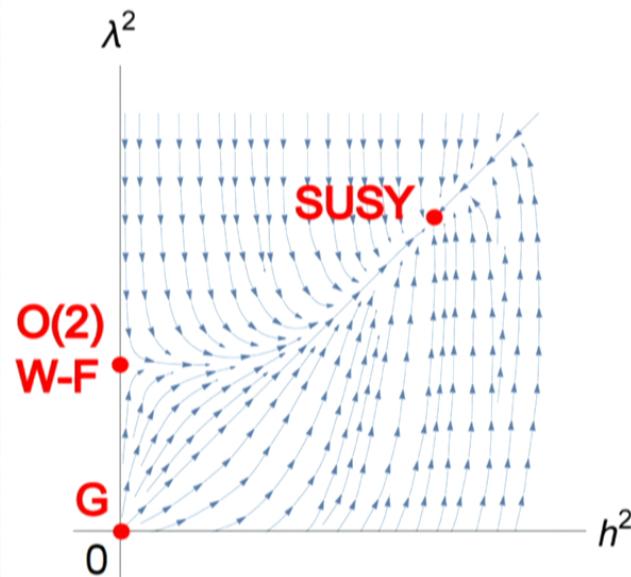
- ❖ Correlation length exponent not fixed by SUSY:
use ϵ -expansion

$$\nu = \frac{1}{2} + \frac{\epsilon}{4} + \mathcal{O}(\epsilon^2) \approx 0.75$$

1-loop RG (Thomas, '05;
Lee, PRB '07)

Higher loops

- ❖ Does the SUSY fixed point survive at higher loops?
- ❖ Up to 3 loops, yes (Zerf, Lin, JM, arXiv '16)



Correlation length exponent

$$\nu = \frac{1}{2} + \frac{\epsilon}{4} + \mathcal{O}(\epsilon^2) \approx 0.75$$

1-loop RG (Thomas, '05;
Lee, PRB '07)

$$\nu = \frac{1}{2} + \frac{\epsilon}{4} + \frac{\epsilon^2}{24} + \left(\frac{\zeta(3)}{6} - \frac{1}{144} \right) \epsilon^3 + \mathcal{O}(\epsilon^4) \approx 0.985$$

3-loop RG (Zerf, Lin, JM, arXiv '16)

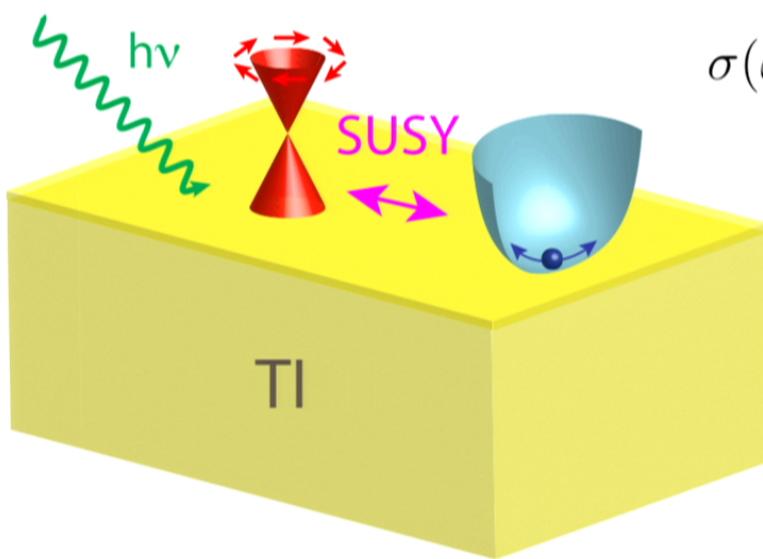
$$\nu \approx 0.9174$$

Padé extrapolation of 3-loop result (Fei et al.,
arXiv '16)

$$\nu \approx 0.9173$$

conformal bootstrap (Bobev et al., PRL '15)

Optical conductivity?



$$\sigma(\omega, 0) = \frac{e^2}{\hbar} \sigma_\infty$$

Measurement of the Optical Conductivity of Graphene

Kin Fai Mak,¹ Matthew Y. Sfeir,² Yang Wu,¹ Chun Hung Lui,¹ James A. Misewich,² and Tony F. Heinz^{1,*}

¹*Departments of Physics and Electrical Engineering, Columbia University, 538 West 120th Street, New York, New York 10027, USA*

²*Brookhaven National Laboratory, Upton, New York 11973, USA*

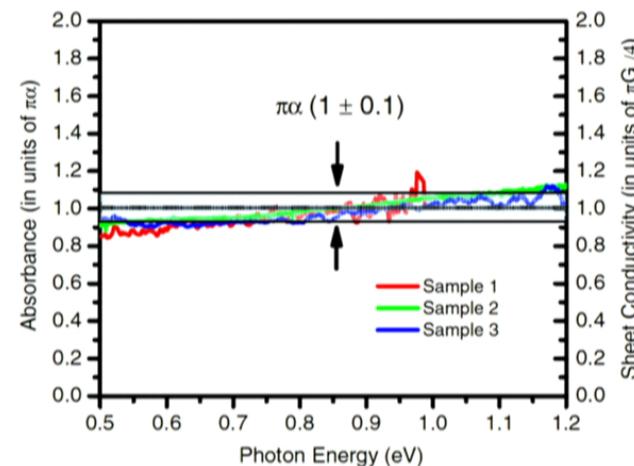
(Received 28 June 2008; published 7 November 2008)

Optical reflectivity and transmission measurements over photon energies between 0.2 and 1.2 eV were performed on single-crystal graphene samples on a SiO₂ substrate. For photon energies above 0.5 eV, graphene yielded a spectrally flat optical absorbance of $(2.3 \pm 0.2)\%$. This result is in agreement with a constant absorbance of $\pi\alpha$, or a sheet conductivity of $\pi e^2/2h$, predicted within a model of noninteracting massless Dirac fermions.

$$\sigma_\infty = 1/4 = 4 \times 1/16$$

❖ Graphene = free
Dirac CFT

$$\frac{\hbar\omega}{k_B T} \sim \frac{1 \text{ eV}}{300 \text{ K}} \sim 39 = \infty$$



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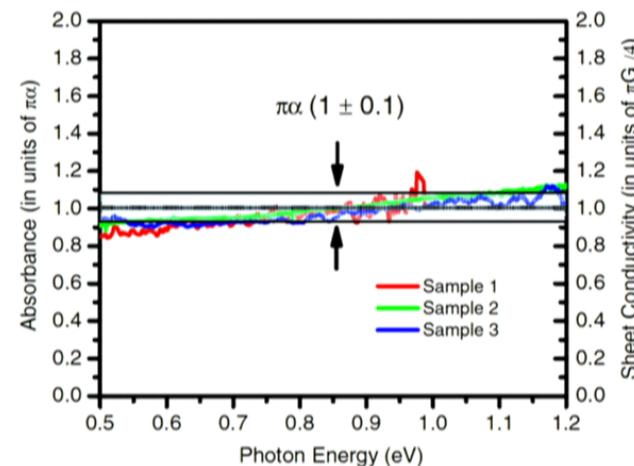
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Kubo for CFTs

- ❖ Ground-state JJ correlation function, constrained by conformal symmetry (Osborn & Petkou, Ann. Phys. '94)

$$\langle J_\mu(x) J_\nu(0) \rangle = C_J \frac{I_{\mu\nu}(x)}{|x|^4}$$
$$\sigma_\infty = \frac{\pi^2}{2} C_J$$

- ❖ Can C_J be computed at our SUSY QCP?

N=2 SCFTs in (2+1)D

- ❖ U(1) current and stress tensor are related by SUSY

$$\mathcal{J}_\mu = J_\mu - (\theta \gamma^\nu \bar{\theta}) 2T_{\nu\mu} + \dots$$

- ❖ $\langle JJ \rangle$ and $\langle TT \rangle$ are related by SUSY

$$\langle J_\mu(x) J_\nu(0) \rangle = C_J \frac{I_{\mu\nu}(x)}{|x|^4}$$

$$\langle T_{\mu\nu}(x) T_{\rho\sigma}(0) \rangle = C_T \frac{I_{\mu\nu,\rho\sigma}(x)}{|x|^6}$$

$$\boxed{\frac{C_J}{C_T} = \frac{5}{3}}$$

N=2 SCFTs in (2+1)D

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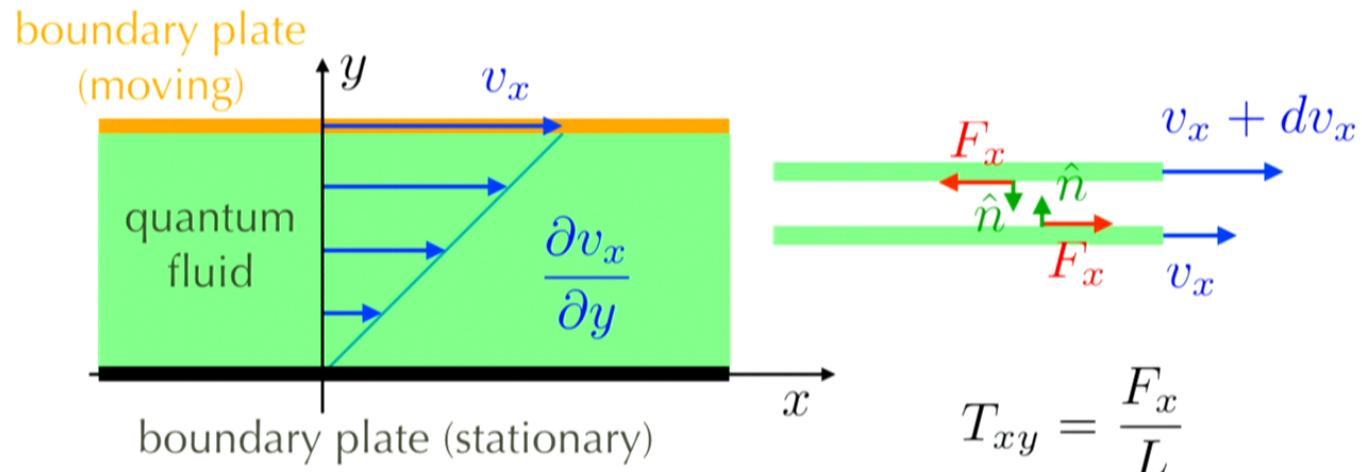
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$$\boxed{\frac{C_J}{C_T} = \frac{5}{3}}$$

Shear viscosity



$$T_{xy} = \frac{F_x}{L}$$

shear stress

$$T_{xy} = \eta \frac{\partial v_x}{\partial y} = \eta \delta \dot{g}_{xy}$$

Conductivity vs viscosity

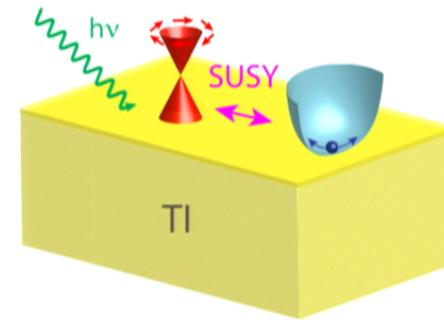
$$\frac{\sigma_\infty}{\eta_\infty} = 40$$

- ❖ Exact universal ratio at the QCP: consequence of SUSY
- ❖ C_T can be calculated exactly for the N=2 WZ model by localization: partition function on squashed S^3 (Closset et al., JHEP '13; Nishioka & Yonekura, JHEP '13)

Exact universal conductivity

$$\sigma(\omega, 0) = \frac{5(16\pi - 9\sqrt{3})}{243\pi} \frac{e^2}{\hbar} \approx 0.2271 \frac{e^2}{\hbar}$$

- ❖ Exact result for T=0 conductivity (and shear viscosity) of "realistic" strongly coupled quantum fluid in (2+1)D



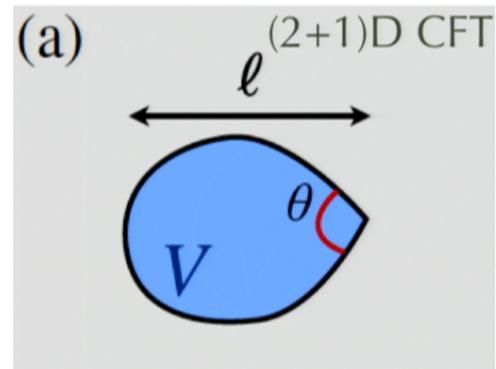
Witczak-Krempa and JM, PRL '16

Exact universal conductivity

	Dirac SM-SC	Gaussian	SC-Ins. (Cooper pairs)
σ_∞	$\frac{5(16\pi - 9\sqrt{3})}{243\pi} \approx 0.227$	$\frac{5}{16} = 0.3125$	0.226

- ❖ Reduced conductivity = increase scattering due to interactions

Corner entanglement entropy



$$S = B\ell/\delta - a(\theta) \ln(\ell/\delta) + \dots$$

$$a(\theta) \simeq \lambda(\pi - \theta)^2$$

(Casini, Huerta, Leitao, NPB '09)

$$\lambda = \pi^2 C_T / 24$$

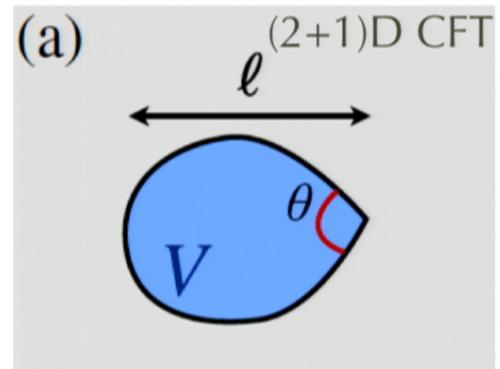
(Bueno, Myers, Witczak-Krempa, PRL '15;
Bueno & Myers, JHEP '15;
Faulkner, Leigh, Parrikar, JHEP '16)

- ❖ Exact result for SM-SC QCP:

$$\lambda = \frac{16\pi - 9\sqrt{3}}{972\pi} \simeq 0.011356$$

Witczak-Krempa and JM, PRL '16

Corner entanglement entropy



$$S = B\ell/\delta - a(\theta) \ln(\ell/\delta) + \dots$$

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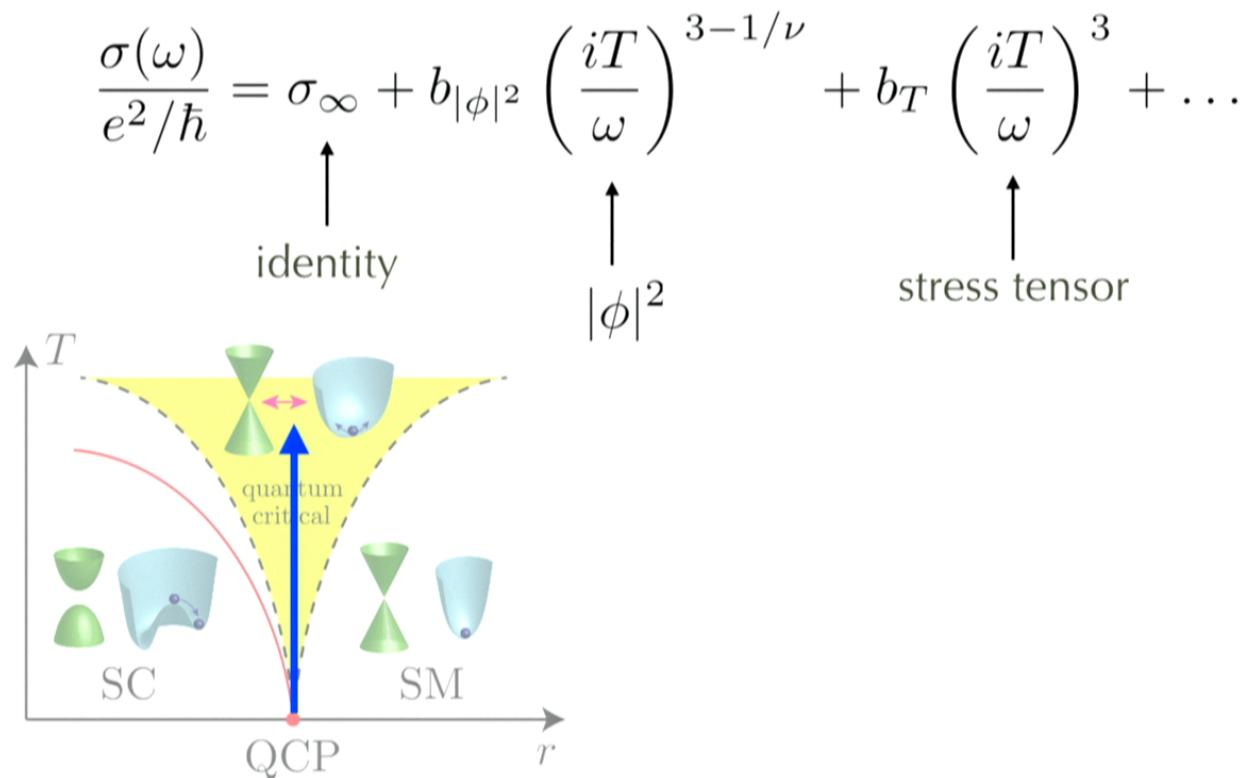
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Witczak-Krempa and JM, PRL '16

Finite temperature?



Katz et al., PRB '14

OPE and high-frequency conductivity

$$J_\mu(x) J_\nu(0) \sim \sum_a \frac{C_{\mu\nu a} \mathcal{O}_a(0)}{|x|^{4-\Delta_a}}$$



$$\frac{\sigma(i\omega_n)}{e^2/\hbar} \sim \frac{\langle J_x(\omega_n) J_x(-\omega_n) \rangle_T}{\omega_n} \sim \sum_a C_{xxa} \frac{\langle \mathcal{O}_a \rangle_T}{|\omega_n|^{\Delta_a}}$$
$$\omega_n \gg T$$

OPE and high-frequency conductivity

$$J_\mu(x) J_\nu(0) \sim \sum_a \frac{C_{\mu\nu a} \mathcal{O}_a(0)}{|x|^{4-\Delta_a}}$$



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OPE and high-frequency conductivity

$$\frac{\sigma(i\omega_n)}{e^2/\hbar} \sim \sum_a C_{xxa} \frac{\langle \mathcal{O}_a \rangle_T}{|\omega_n|^{\Delta_a}}$$

$$\langle \mathcal{O}_a \rangle_T = c_a T^{\Delta_a}$$

$$\frac{\sigma(\omega)}{e^2/\hbar} = \sum_a b_a \left(\frac{iT}{\omega} \right)^{\Delta_a}$$

Finite temperature?

$$\frac{\sigma(\omega)}{e^2/\hbar} = \sigma_\infty + b_{|\phi|^2} \left(\frac{iT}{\omega} \right)^{3-1/\nu} + b_T \left(\frac{iT}{\omega} \right)^3 + \dots$$

- ❖ Can't say much about $b_{|\phi|^2}$: probably nonzero

Finite temperature?

$$\frac{\sigma(\omega)}{e^2/\hbar} = \sigma_\infty + b_{|\phi|^2} \left(\frac{iT}{\omega} \right)^{3-1/\nu} + b_T \left(\frac{iT}{\omega} \right)^3 + \dots$$

- ❖ b_T : related to $\langle \text{JJT} \rangle$ correlation function
- ❖ Combine conformal invariance + Ward identities (Osborn & Petkou, Ann. Phys. '94), and SUSY (Buchbinder, Kuzenko, Samsonov, JHEP '15):

$$b_T = 0$$

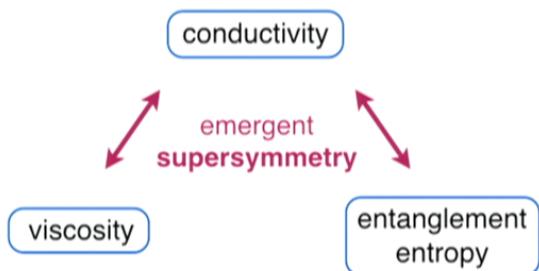
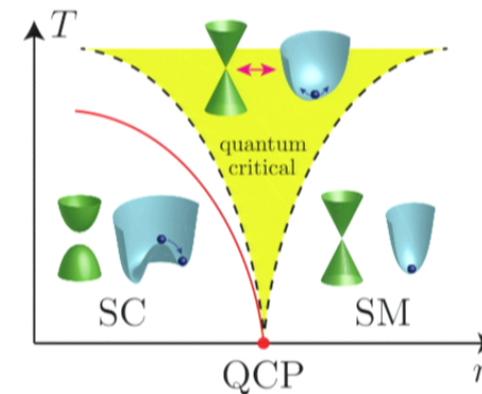
(Witczak-Krempa and JM,
PRL '16)

for all (2+1)D QCPs with N=2 SUSY!

- ❖ Exact result for finite-T, dynamical response of strongly coupled quantum fluid in (2+1)D

Summary

- ❖ Emergent N=2 SUSY at SM-SC QCP of 2D Dirac fermions holds at 3-loop level



- ❖ SUSY allows us to calculate exactly response & entanglement properties, at zero and finite T