Title: Particle-Vortex duality and Topological Quantum Matter

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Abstract:

Some literature

- Based on:

  J.M, Horatiu Nastase, Jonathan Shock, Nitin Raghoonauth, 1404.5926

  J.M, Horatiu Nastase, 1506.04090, 1512.08926

  J.M, Horatiu Nastase, 1606.01912

- Related:

  Metlitski & Vishwanath, 1505.05142

  Seiberg & Witten, 1602.04251

  Seiberg, Senthil, Wang & Witten, 1606.01989

  Karch & Tong, 1606.01893

  Hsin & Seiberg, 1607.07457

  Radičević, Tong & Turner, 1608.04732

- Recommended:

  McGreevy, 1607.01878

Jeff Murugan (UCT)

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A summary...by analogy

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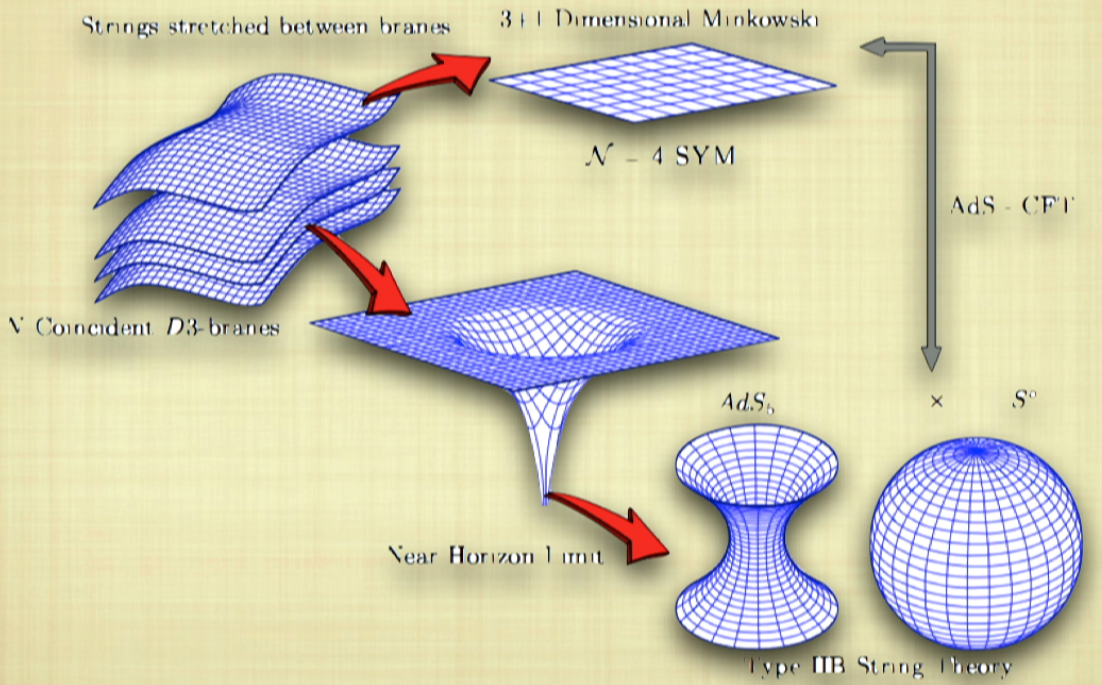
Duality I: Gauge/Gravity Duality



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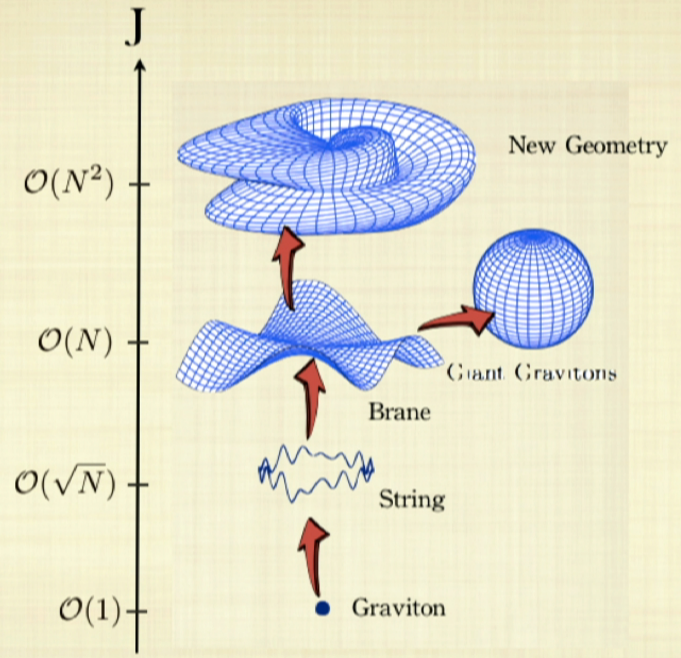

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Duality IV: Massive Thirring model

- (1+1)-dimensional **massive Thirring model**:

$$S_{mT} = \int d^2x \left[i\bar{\psi}\partial\!\!\!/\psi - m_F\bar{\psi}\psi - \frac{g}{2}(\bar{\psi}\gamma^\mu\psi)^2 \right]$$

- Properties:
  - The massless 4-Fermi theory is exactly solvable for coupling $g > \pi$
  - The massive theory is not exactly solvable but has a well-defined perturbative expansion.
  - No solitons in its spectrum.
  - Spectrum contains perturbative fermions and $\psi - \bar{\psi}$ bound states.

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Duality V: bosonization...

- Classically, the two are related by **bosonization**

$$\psi_\pm = \exp\left[-i\frac{2\pi}{\beta}\int_{-\infty}^{x}\frac{\partial\phi(x')}{\partial t}\,dx' \mp i\frac{\beta}{2}\phi(x)\right]$$

- This is a strong/weak duality since the coupling constants are related through

$$\frac{\beta}{4\pi^2} = \frac{1}{1+g\pi}$$

- At the quantum level:

$$\psi_\pm = C_\pm : \exp\left[-i\frac{2\pi}{\beta}\int_{-\infty}^{x}\frac{\partial\phi(x')}{\partial t} \mp i\frac{\beta}{2}\phi(x)\right]:$$

- Under the duality:
  - Perturbative Greens functions of sG mesons map to fermion bound states.
  - Topological solitons of the sG model map to Thirring fermions.

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$$(\partial\phi)^2 + (\cos\phi - 1) \leftrightarrow i\bar{\psi}\partial\!\!\!/\psi - m\bar{\psi}\psi - (\bar{\psi}\gamma^\mu\psi)$$

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Particle-Vortex Duality I: Heuristics

[Lee & Fisher, Peskin, Dasgupta & Halperin, Burgess & Dolan.]

- The planar abelian-Higgs model for a complex scalar ϕ coupled to a U(1) gauge field A_μ contains localized zeros of ϕ with non-trivial winding number called **vortices**.
- With $\phi = ve^{i\theta}$ the Lagrangian density

$$\mathcal{L}_{AH} = \frac{v^2}{2}\left(\partial_\mu\theta - qA_\mu\right)^2 = -\frac{1}{2v^2}\xi_\mu^2 + \xi^\mu\left(\partial_\mu\theta - qA_\mu\right)$$

- Winding around a (anti)vortex changes the phase θ by $(-)2\pi$ with the interaction energy between a vortex and anti-vortex separated by a distance R $\sim \int_a^R r(\nabla\theta)^2 \sim \ln(R/a)$
- Since $\int \frac{e^{i\vec{k}\cdot\vec{x}}}{k^2}\, d^2k \sim \ln(|\vec{x}|/a)$ vortices and anti-vortices interact like “point charges” with Coulomb interaction.

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Particle-Vortex Duality II: formalism

- The scalar phase can be split into a "smooth" part and a "vortex" part

$$\mathcal{L}_{AH} = -\frac{1}{2v^2}\xi_\mu^2 + \xi^\mu(\partial_\mu\theta_{\rm smooth} + \partial_\mu\theta_{\rm vortex} - qA_\mu)$$

- Integrating out the smooth part $\Rightarrow \xi^\mu = \epsilon^{\mu\nu\lambda}\partial_\nu a_\lambda$

$$\Rightarrow \mathcal{L} = -\frac{1}{4v^2}f_{\mu\nu}^2 + \epsilon^{\mu\nu\lambda}\partial_\nu a_\lambda\,(\partial_\mu\theta_{\rm vortex} - qA_\mu)$$

$$= -\frac{1}{4v^2}f_{\mu\nu}^2 + 2\pi a_\mu j^\mu_{\rm vortex} + A_\mu J^\mu$$

Electromagnetic current

$$\text{``}=\text{''} -\frac{1}{4v^2}f_{\mu\nu}^2 + \frac{1}{2}\,|(\partial_\mu - 2\pi i a_\mu)\Phi|^2 - W(\Phi^\dagger\Phi) - A_\mu J^\mu$$

Vortex potential

Vortex field

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Particle-Vortex Duality III: path integration

- Start from the partition function for a complex scalar $\Phi = \Phi_0 e^{i\theta}$ coupled to a U(1) gauge field

$$Z = \int \mathcal{D}a_\mu \mathcal{D}\Phi_0 \mathcal{D}\theta \exp\left\{-\frac{i}{2}\int d^3x\ |(\partial_\mu - iea_\mu)\Phi|^2\right\}$$

- Incorporating $\lambda_\mu = \partial_\mu\theta$ as a new independent variable constrained to $\epsilon^{\mu\nu\rho}\partial_\nu\lambda_\rho = 0$ leads to a "master" action

Split according to smooth fluctuations and vortex ones

$$Z = \int \mathcal{D}a_\mu \mathcal{D}b_\mu \mathcal{D}\Phi_0 \mathcal{D}\lambda_\mu \exp\left\{-\frac{i}{2}\int d^3x \left[(\partial_\mu\Phi_0)^2 + (\lambda^\mu_{\rm smooth} + \lambda^\mu_{\rm vortex} + ea^\mu)^2\Phi_0^2 + \frac{1}{e}\epsilon^{\mu\nu\rho}b_\mu\partial_\nu\lambda_\rho\right]\right\}$$

Lagrange multiplier

- Note that:
  - Integrating out b_μ reproduces the original action.
  - Integrating out λ_μ on the other hand, results in the equation of motion

$$(\lambda^\mu_{\rm smooth} + \lambda^\mu_{\rm vortex} + ea^\mu)e\Phi_0^2 = -\epsilon^{\mu\nu\rho}\partial_\nu\lambda_\rho$$

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Particle-Vortex Duality IV: results

- Substituting back into the master action then yields the **dual action**

$$Z=\int \mathcal{D}\Phi_0\mathcal{D}a_\mu\mathcal{D}b_\mu\exp\left\{-i\int d^3x\left[\frac{1}{4e^2\Phi_0^2}f^{(b)}_{\mu\nu}f^{(b)\mu\nu}+\epsilon^{\mu\nu\rho}b_\mu\partial_\nu a_\rho-\frac{2\pi}{e}j^\mu_{\rm vortex}b_\mu+\frac{1}{2}(\partial_\mu\Phi_0)^2\right]\right\}$$

- Note that:
  - The vortex current $j^\mu_{\rm vortex}=\frac{1}{2\pi}\epsilon^{\mu\nu\lambda}\partial_\nu\partial_\lambda\theta$ appears as an explicit source term in the dual action.
  - The duality relates it to the scalar current through $j^\mu_{\rm vortex}=\frac{1}{2\pi e\Phi_0^2}\epsilon^{\mu\nu\lambda}\partial_\nu j_\lambda$
  - The Lagrange multiplier b_μ of the original theory is promoted to a fully dynamical Maxwell gauge field dual to the phase θ
  - The field modulus Φ_0 of the original theory plays the role of the coupling constant for b_μ

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$$|D_A\phi|^2 - |\phi|^4 \leftrightarrow |D_a\varphi|^2 - |\varphi|^4 - \frac{1}{2\pi}A\,da$$

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The ABJM model

[Aharony et.al.'08]

- The (2+1)-dimensional ABJM model:
  - is an $\mathcal{N}=6$ super-Chern-Simons field theory with gauge group $U(N)_k \times U(N)_{-k}$
  - arises as the worldvolume theory for N coincident M2-branes living in $\mathbb{R}^{2,1}\times\mathbb{C}^4/\mathbb{Z}_k$
  - contains Chern-Simons gauge fields A_μ and $\hat{A}_\mu$, bifundamental scalars C^I and fermions ψ^I .
  - is the CFT dual to the type IIA superstring on $AdS_4\times\mathbb{CP}^3$, in the limit when $k \gg N^{1/5}$
  - is defined through the (not particularly enlightening) action

$$\begin{aligned} S &= \int d^3x \Bigg(\frac{k}{4\pi}\epsilon^{\mu\nu\lambda}\left(A_\mu\partial_\nu A_\lambda + \frac{2i}{3}A_\mu A_\nu A_\lambda - \hat{A}_\mu\partial_\nu\hat{A}_\lambda - \frac{2i}{3}\hat{A}_\mu\hat{A}_\nu\hat{A}_\lambda\right) \\ &- D_\mu C_I^\dagger D^\mu C^I - i\psi^{I\dagger}\gamma^\mu D_\mu\psi_I + \frac{4\pi^2}{3k^2}\Big(C^I C_I^\dagger C^J C_J^\dagger C^K C_K^\dagger \\ &+ C_I^\dagger C^I C_J^\dagger C^J C_K^\dagger C^K + 4C^I C_J^\dagger C^K C_I^\dagger C^J C_K^\dagger - 6C^I C_J^\dagger C^J C_I^\dagger C^K C_K^\dagger\Big) \\ &+ \frac{2\pi i}{k}\Big(C_I^\dagger C^I \psi^{J\dagger}\psi_J - \psi^{J\dagger}C^I C_I^\dagger\psi_J - 2C_I^\dagger C^J\psi^{I\dagger}\psi_J + 2\psi^{J\dagger}C^I C_J^\dagger\psi_J \\ &+ \epsilon^{IJKL}C_I^\dagger\psi_J C_K^\dagger\psi_L - \epsilon_{IJKL}C^I\psi^{J\dagger}C^K\psi^{L\dagger}\Big)\Bigg) \end{aligned}$$

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Massive ABJM and abelian projections

[Gomis et.al.'08, Terashima'08,Mohammed, JM & Nastase'12]

- ABJM allows for a maximally supersymmetric one-parameter mass deformation that breaks the R-symmetry to $SU(2) \times SU(2) \times U(1)_A \times U(1)_B \times \mathbb{Z}_2$
- The massive ABJM theory supports a rich solitonic spectrum including vortices, Q-balls and domain walls and fuzzy sphere ground states represented by matrices

Bifundamental under the gauge group →

$$G^\alpha = G^\alpha G^\dagger_\beta G^\beta - G^\beta G^\dagger_\beta G^\alpha$$

Ajoint combinations

- The ansatz $A_\mu = a_\mu^{(2)} G^1 G_1^\dagger + a_\mu^{(1)} G^2 G_2^\dagger,\ \hat{A}_\mu = a_\mu^{(2)} G_1^\dagger G^1 + a_\mu^{(1)} G_2^\dagger G^2,\ Q^\alpha = \phi_\alpha G^\alpha,\ R^\alpha = \chi_\alpha G^\alpha$ reduces the full ABJM action to an abelian effective action

$$S = -\frac{N(N-1)}{2}\int d^3x \left[\frac{k}{4\pi}\epsilon^{\mu\nu\lambda}\left(a_\mu^{(2)} f_{\nu\lambda}^{(1)} + a_\mu^{(1)} f_{\nu\lambda}^{(2)}\right) + |D_\mu \phi_i|^2 + |D_\mu \chi_i|^2 + U(|\phi_i|, |\chi_i|)\right]$$

- This system also possesses vortex like solutions which can be understood as D-brane configurations in the dual theory on $AdS_4 \times \mathbb{CP}^3$

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Aside: The Mukhi-Papageorgakis mechanism

[Mukhi-Papageorgakis'08,Chu et.al.'10]

- Higgs-Englert-Brout-Guralnik-Hagen-Kibble mechanism: a massless gauge field "eats" a scalar and becomes massive.
- Is there an analogue for **Chern-Simons theories?**
- Consider a complex scalar coupled to a topological (CS) field in (2+1)-dimensions

$$S = -\int d^3x \left[\frac{k}{2\pi}\epsilon^{\mu\nu\rho}a_\mu\partial_\nu a_\rho + \frac{1}{2}|(\partial_\mu - iea_\mu)\Psi|^2 + V(|\Psi|^2)\right]$$

Chern-Simons level #

- Implement a Higgs mechanism by
  - Expanding scalar degrees of freedom around the vacuum as $\Psi = (b+\delta\psi)e^{i\delta\theta}$
  - Shifting the Chern-Simons gauge field $a_\mu \to a'_\mu = a_\mu + \frac{1}{e}\partial_\mu\theta$
  - Integrating out the phase field to get

The gauge field is now dynamical

Nonlinear terms in the fluctuations

$$S = \int d^3x \left[-\frac{k^2}{16\pi^2 b^2}(\tilde{f}_{\mu\nu})^2 - \frac{1}{2}(\partial_\mu\delta\psi)^2 + \dots\right]$$

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Son's conjecture and particle-hole symmetry

[Son '15, Metlitski & Vishwanath '16]

A niggling problem in flat-land: A system of nonrelativistic particles with a 2-body interaction is P and T symmetric. In a uniform B-field, the system is PT-symmetric BUT the physics of the lowest Landau level also exhibits another discrete **particle-hole symmetry**!

$$i\bar{\Psi}\not{D}_A\Psi - \frac{1}{8\pi}A\,dA \leftrightarrow i\bar{\psi}\not{D}_{-a}\psi + \frac{1}{4\pi}A\,da - \frac{1}{8\pi}A\,dA$$

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Particle-Hole symmetry is **emergent** on projecting onto the LLL and not realised as a local operation acting on fields.

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The composite fermion is a **massless Dirac fermion** which, under particle-hole conjugation transforms as $\psi \to i\sigma_2\psi$

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The composite fermion should be interpreted as a kind of fermionic vortex that arises from a **fermionic particle-vortex duality**

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A web of (abelian) 3-D dualities

[Aharony '16, JM & Nastase, Seiberg et.al., Karch & Tong, Kachru et.al.]

$$i\bar{\Psi}\not{D}_A\Psi \leftrightarrow |D_b\phi|^2 - |\phi|^2 + \frac{1}{4\pi} b\, db + \frac{1}{2\pi} b\, dA$$

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Particle-Vortex duality and it's fermionic counterpart are part of a larger, richer set of 3-dimensional dualities. Key to this web is **3-D bosonization**

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For $N = k = N_f = 1$ the dualities have the **structure**

A fermion coupled to $U(1)_{-1/2}$	$\leftrightarrow$	A scalar
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These dualities can be "derived" by starting from $\mathcal{N} = 4$ **mirror symmetry** in 3-dimensions and breaking the SUSY in a controlled way by adding a background D-term.

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Can the duality web be generalised to the **nonabelian** case?

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Nonabelian T-duality

[de la Ossa & Quevedo, Roček & Verlinde]

$$S = \int d^2\sigma \Big[Q_{\mu\nu}\partial_+ X^\mu \partial_- X^\nu + Q_{\mu i}\partial_+ X^\mu L^i_- + Q_{i\mu} L^i_+ \partial_- X^\nu + E_{ij} L^i_+ L^j_- \Big]$$

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Abelian T-duality is a symmetry of the perturbative string path integral and can be implemented as a duality transformation on the worldsheet acting on commuting abelian isometries. **How does it generalise to nonabelian isometries**?

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Step 1: **Gauge** global $SU(2)$ symmetry with $\partial_\pm g \to D_\pm g = (\partial_\pm - A_\pm)g$

Step 2: Constrain the gauge field to be trivial by imposing $F_\pm = 0$ through a **Lagrange multiplier** term $-i\mathrm{Tr}[vF_{+-}]$

Step 3: Integrate out the Lagrange multiplier to return to the original action or **integrate out the gauge field** to get the T-dual theory

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$$S_{\mathrm{dual}} = \int d^2\sigma \Big[Q_{\mu\nu}\partial_+ X^\mu \partial_- X^\nu + (\partial_+ v_i + Q_{\mu i}\partial_+ X^\mu) M^{-1}_{ij} (\partial_- v_j - Q_{j\mu}\partial_- X^\mu)\Big]$$

$$\Phi(x,v) = \Phi(x) - \frac{1}{2}\ln(\det M)$$

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Abelian T-duality is a symmetry of the perturbative string path integral and can be implemented as a duality transformation on the worldsheet acting on commuting abelian isometries. **How does it generalise to nonabelian isometries?**

Step 1: **Gauge** global $SU(2)$ symmetry with $\partial_\pm g \to D_\pm g = (\partial_\pm - A_\pm)g$

Step 2: Constrain the gauge field to be trivial by imposing $F_\pm = 0$ through a **Lagrange multiplier** term $-i\mathrm{Tr}[vF_{+-}]$

Step 3: Integrate out the Lagrange multiplier to return to the original action or **integrate out the gauge field** to get the T-dual theory

$$S = \int d^2\sigma \Big[Q_{\mu\nu}\partial_+ X^\mu \partial_- X^\nu + Q_{\mu i}\partial_+ X^\mu L^i_- + Q_{i\mu} L^i_+ \partial_- X^\nu + E_{ij} L^i_+ L^j_-\Big]$$

Implementing the nonabelian T-duality is **non-trivial**, even in (1+1)-dimensions, with well-known **global issues** and unresolved questions about the coordinate ranges

$$S_{\rm dual} = \int d^2\sigma \Big[Q_{\mu\nu}\partial_+ X^\mu \partial_- X^\nu + (\partial_+ v_i + Q_{\mu i}\partial_+ X^\mu) M^{-1}_{ij} (\partial_- v_j - Q_{j\mu}\partial_- X^\mu)\Big]$$

$$\Phi(x,v) = \Phi(x) - \frac{1}{2}\ln(\det M)$$

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Nonabelian particle-vortex duality

[JM & Nastase, Radičević, Tong & Turner]

$$S_{\rm master} \quad = \quad \int d^3x \Big[-\frac{1}{2}(\partial_\mu \Phi_0^k)^2 - \frac{1}{2}(\Phi_0^k)^2 g^{\mu\nu} \tilde{L}^i_\mu \tilde{L}^j_\nu E_{ij} + \epsilon^{\mu\nu\rho} v^i_\mu F^i_{\nu\rho} \Big]$$

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Nonabelian particle-vortex duality

[JM & Nastase, Radičević, Tong & Turner]

To nonabelianize the standard particle-vortex duality, we **lift the 1+1-dimensional nonabelian T-duality.** Again, integrating out the Lagrange multipliers v^i_μ yeilds a flat connection and allows us to set $A_\mu = 0$

$$S_{\rm original} = \int d^3x \left[-\frac{1}{2}(\partial_\mu \Phi^k_0)^2 - \frac{1}{2}(\Phi^k_0)^2 g^{\mu\nu} L^i_\mu L^j_\nu E_{ij} \right]$$

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To get the **dual theory**, we gauge fix $g = 1$ in , $\tilde{L}^i_\mu = -i[t^i g^{-1} D_\mu g]$ then integrate out the gauge field A_μ to get the action

$$S_{\rm dual} = -\frac{1}{2} \int d^3x \left[(\partial_\mu \Phi_0^k)^2 + V_i^\mu M_{ij}^{-1} V_j^\rho \right]$$

$$S_{\rm master} \quad = \quad \int d^3x \left[-\frac{1}{2}(\partial_\mu \Phi_0^k)^2 - \frac{1}{2}(\Phi_0^k)^2 g^{\mu\nu} \tilde{L}^i_\mu \tilde{L}^j_\nu E_{ij} + \epsilon^{\mu\nu\rho} v^i_\mu F^i_{\nu\rho} \right]$$

To identify this transformation with **particle-vortex duality** in 2+1-dimensions, we need to be able to:

- derive $S_{\rm original}$ from a standard vortex-admitting action
- couple the theory to a nontrivial external gauge field and
- add a vortex current term to the action

We are only able to implement the duality on a **specific ansatz**

This procedure furnishes a **duality transformation** at the level of the path integral in 2+1-dimensional theories of the general form above. But is it particle-vortex?

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Example: semi-local vortices

$$\mathcal{L} = -\frac{1}{2}|D_\mu \Phi|^2 - \frac{\lambda}{4}\left(\Phi^\dagger \Phi - v^2\right)^2 - \frac{1}{4} f_{\mu\nu} f^{\mu\nu}$$

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Example: semi-local vortices

Semi-local vortices are topological defects in 3+1-dimensions with **local and global symmetries** present. Unlike cosmic strings, they can exist even if the vacuum manifold is simply connected.

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In our example, we have an $SU(2)_G \times U(1)_L$ symmetry with the scalar transforming in the fundamental of the flavor group. At critical coupling, this model admits **Bogomolnyi vortices**

$$a_\theta = \frac{v}{\sqrt{2}}\frac{n}{r}a(r); \quad a_r = 0; \quad \Phi^a = v\varphi^a(r)e^{in\alpha_a}$$

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This form is compatible with our ansatz $\Phi^a = \Phi^a_0 \exp\left(i\int dx^\mu L^i_\mu F_i\right)$ and gives the master action

$$\begin{aligned} S_{\rm master} &= \int d^3x\Big[-\frac{1}{2}(\partial_\mu\Phi^a_0)^2 - \frac{1}{2}(\Phi^a_0)^2 g^{\mu\nu}\sum_{i,j=1}^{4}\tilde{\tilde{L}}^i_\mu \tilde{\tilde{L}}^j_\nu E_{ij} \\ &- \frac{1}{4}f^2_{\mu\nu} - V(\Phi) + \epsilon^{\mu\nu\rho}\sum_{i=1,2,3} v^i_\mu F^i_{\nu\rho}\Big] \end{aligned}$$

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The dual action is obtained by integrating out the gauge field with the **duality between particle and vortex currents** expressed through

$$j^\mu_{\rm vortex} = \epsilon^{\mu\nu\rho}\partial_\nu\left(\frac{j_\rho}{2(\Phi_0^a)^2}\right)$$

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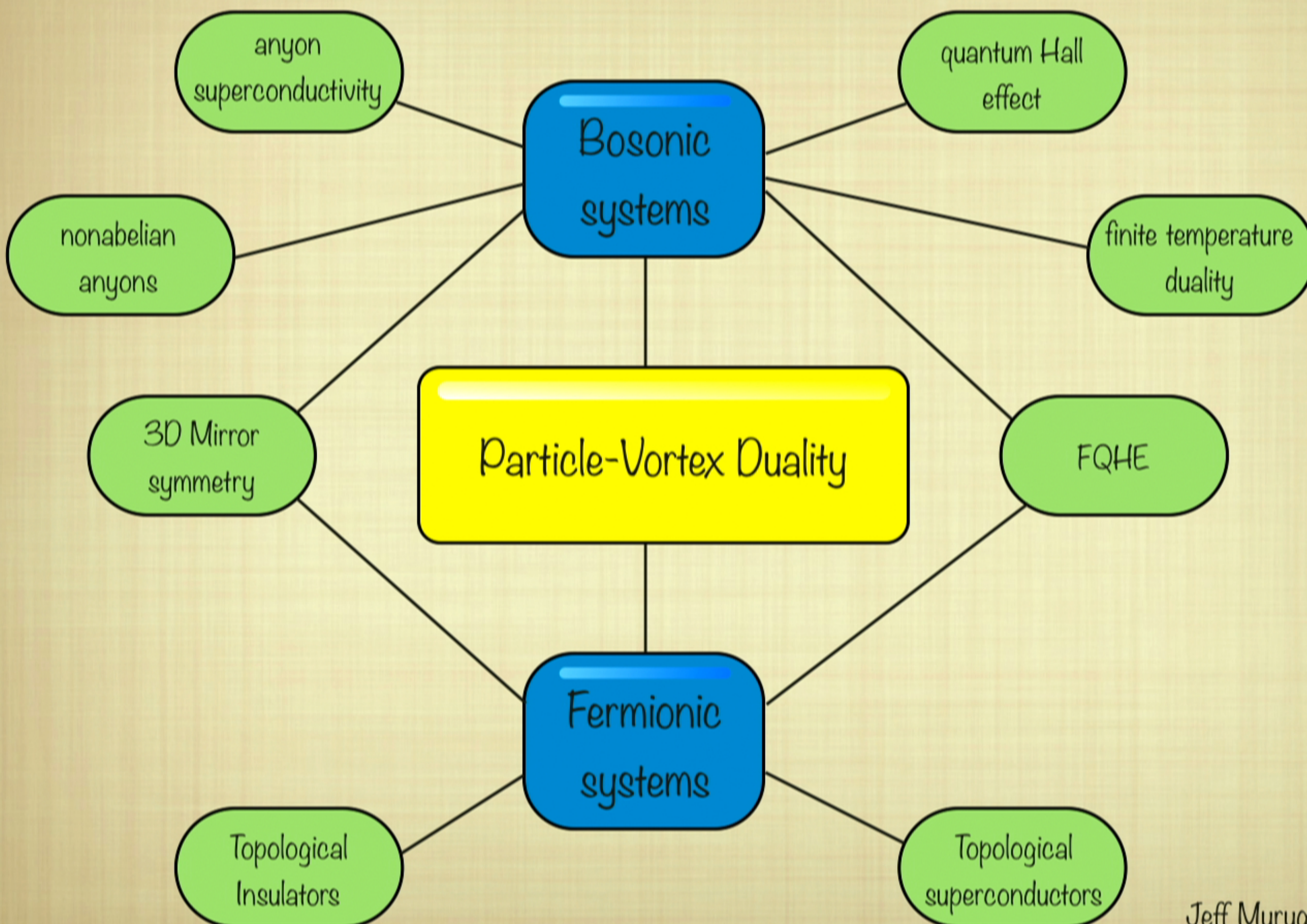
Conclusions and connections



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մերսի
Hristo! 감사합니다 спасибі! תודה
நன்றி Ndiyabulela! Ke a leboha!
σας ευχαριστώ! Gracies! Ngeyabonga! Baie Dankie!
Ukhani! Thank You! Merci! Asante
Obrigado! Grazias!
Ihe edn! Inkomu! Siyabonga! Danke! Ďakujem
धन्यवाद ¡Gracias धन्यवाद Grazie! ありがとう
Eskerrik asko!
Suksema! Juspajaraña شكرا Təşəkkür edirəm!
Dzięki Obrigadu! Дзякуй
Jeff Murugan (UCT)