Title: Universal Diffusion and the Butterfly Effect

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Abstract: In 2014 Hartnoll proposed that the diffusion constants of incoherent metals should be bounded as $D \geq \hbar v^2/ (k_B T)$, where v is a characteristic velocity. In this talk I will describe a large class of holographic theories that saturate such a bound, with v being the velocity of the butterfly effect. Our results suggest a novel connection between transport at strong coupling and the field of quantum chaos.


Universal Diffusion and the
Butterfly Effect
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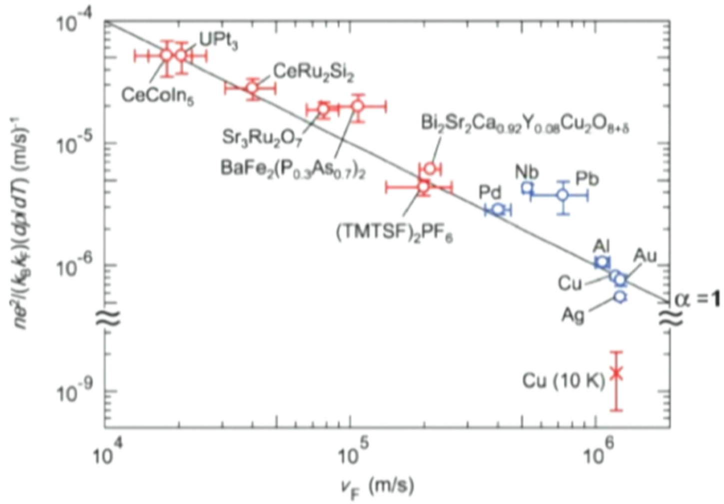
Universal timescale

- Universal timescale governs transport in many strongly correlated materials

$$\tau \sim \frac{\hbar}{k_B T}$$

- E.g. in many materials with a linear resistivity



Mackenzie et al

$$\left[\sigma = \frac{ne^2}{m}\tau\right]$$

Planckian dissipation

- τ represents a `shortest possible' relaxation timescale.

Sachdev, Zaanen

- Saturated at strong coupling. For relativistic theories

$$\frac{\eta}{s} \sim \tau T \qquad \Longrightarrow \qquad \frac{\eta}{s} \sim \frac{\hbar}{k_B}$$

Kovtun, Son & Starinets

- c.f. weakly coupled gauge theory

$$\frac{\eta}{s} \sim \frac{\hbar}{k_B} \frac{1}{\lambda^2 \log \lambda}$$

- Viscosity is equivalent to momentum diffusion

$$D_p = \frac{\eta c^2}{sT} \sim \frac{\hbar c^2}{k_B T}$$

- Hartnoll proposed analogous behaviour could govern charge and energy diffusion

$$D \sim \frac{\hbar v^2}{k_B T}$$

- These are related to conductivities σ, κ via Einstein relations.

Holography

- Holographic theories provide an opportunity to test the existence of such a bound.

Strongly coupled large N gauge theory ⟷ Classical gravity in asymp-AdS spacetime.

- For a holographic CFT dual to an Einstein-Maxwell theory

$$D_c = \frac{\hbar c^2}{4\pi k_B T}\frac{d+1}{d-1}$$

Kovtun

The Butterfly velocity

- Outside of a relativistic setting it's not clear how to define a characteristic speed.
- One natural candidate is provided by the butterfly effect

$$\langle [\hat{W}_x(t_w), \hat{V}_y(0)]^2 \rangle_\beta \sim f_1 e^{\lambda_L (t_w - t_* - |x-y|/v_B)}$$

- v_B describes speed at which chaos/quantum information propagates.
- Can also be defined in non-holographic theories.

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- Quantum chaos has recently been extensively studied in holographic theories. Stanford and Shenker etc
- Both λ_L and v_B can be calculated from near-horizon geometry of black hole.
- Similarly `membrane paradigm' ties DC transport coefficients to the black hole horizon.
- Therefore natural to propose

$$D \sim \frac{\hbar v_B^2}{k_B T}$$

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This talk: Test whether this relationship holds in simple holographic models with a particle-hole symmetry.

- Charge Diffusion
- Energy Diffusion

arXiv: 1603.08510 MAB (to appear in PRL)
arXiv: 1604.01754 MAB

Charge diffusion

- At zero net charge density, the electrical current decouples from momentum.
- Charge diffusion constant is finite even in translationally invariant theories.
- We can provide an initial test of our proposal by calculating D_c for a general family of 'hyperscaling-violating' geometries.

- These are a class of holographic metrics that capture many interesting aspects of strongly correlated materials

$$ds^2_{d+2} = -U(r)dt^2 + \frac{dr^2}{U(r)} + V(r)d\vec{x_d}^2$$

$$U(r) = L_t^{-2} r^{u_1}\left(1 - \frac{r_0^\delta}{r^\delta}\right) \quad V(r) = L_x^{-2} r^{2v_1}$$

$$u_1 = \frac{2z - 2\theta/d}{z - 2\theta/d} \quad 2v_1 = \frac{2 - 2\theta/d}{z - 2\theta/d}$$

- Can arise as infra-red solutions in Einstein-Maxwell-Dilaton gravity.

Gouteraux et al

- Dual to theories with non-relativistic scaling properties

$$[x] = -1,\ [T] = -[t] = z \quad [f] = z + d - \theta = z + d_\theta$$

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- Previous studies of chaos focused on CFTs in which v_B is just a constant.

Roberts, Stanford and Susskind

- Generalising these shock-wave calculations to our more general metrics gives

$$\lambda_L = 2\pi T \qquad v_B^2 = \frac{4\pi T}{dV'(r_0)}$$

$$\hbar = k_B = 1$$

- λ_L is universal, but v_B is model-dependent.

- For hyperscaling-violating geometries this characteristic velocity scales as

$$v_B^2 \sim T^{2-2/z}$$

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Roberts & Swingle

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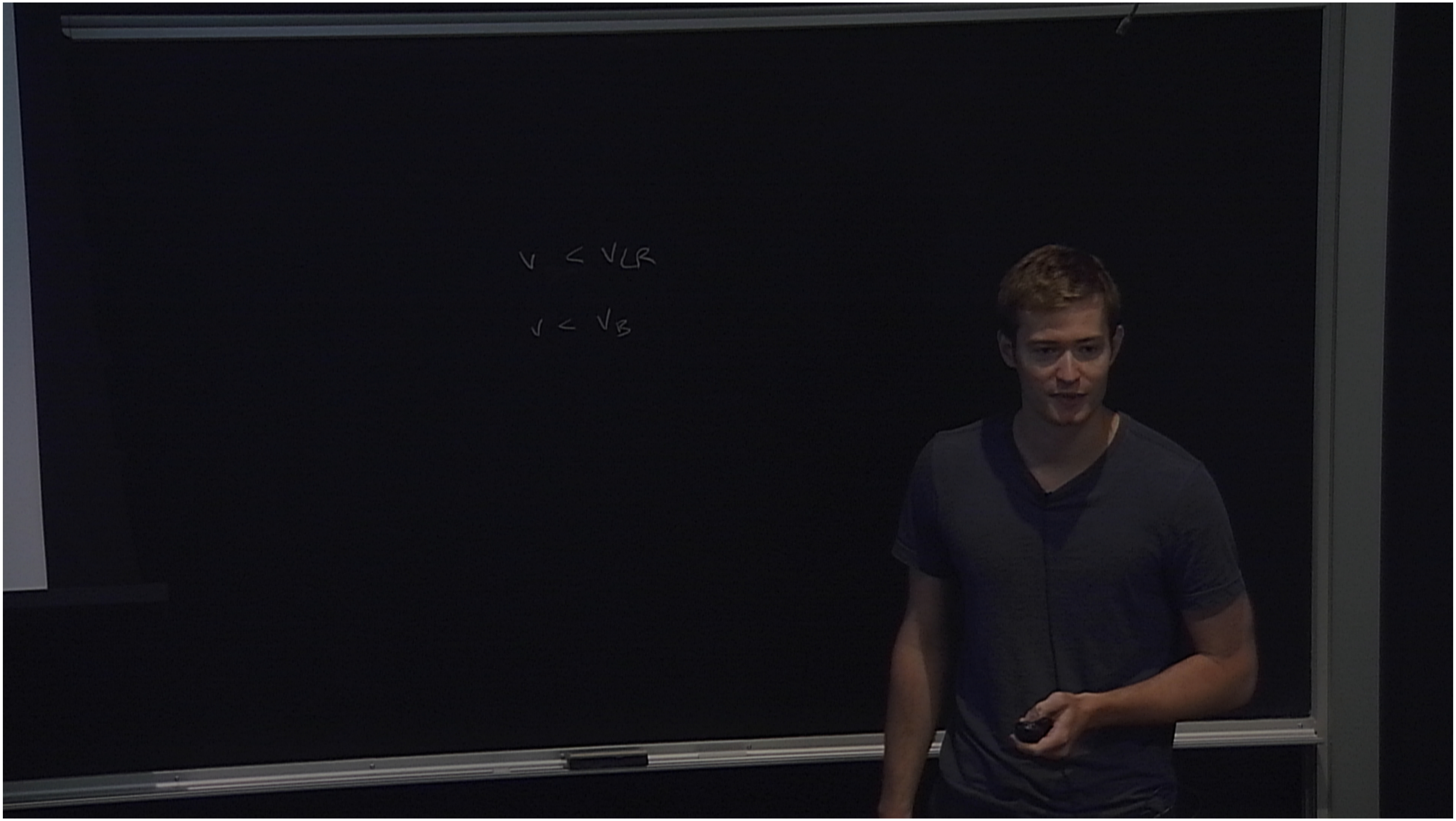
v < vLR
v < vB

- To calculate the diffusion constant, need to consider a gauge field propagating on this background metric

$$S = \int \mathrm{d}^{d+2}x\sqrt{-g}\Big[-\frac{1}{4}Z(r)F^{\mu\nu}F_{\mu\nu}\Big] \qquad Z(r) = Z_0 r^{\gamma}$$

- Extract diffusion constant from Einstein relation $D_c = \sigma/\chi$
- Find two regimes depending on scaling dimension of susceptibility Δ_χ/z

- If $\Delta_\chi/z < 0$ then diffusion constant is sensitive to UV physics

$$D_c \sim \left(\frac{\Lambda_{UV}}{T}\right)^{-\Delta_\chi/z} T^{1-2/z}$$

- This is parametrically larger than v_B^2/T
- If $\Delta_\chi/z > 0$ then diffusion constant is solely tied to IR (horizon data) .
- Find a universal result

$$D_c = \frac{d_\theta}{\Delta_\chi} \frac{v_B^2}{2\pi T}$$

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Energy diffusion

- D_e is infinite in a translationally invariant theory. For weak momentum relaxation

$$D_e \propto \Gamma^{-1}$$

- Momentum relaxation rate Γ is sensitive to details of how translational symmetry is broken.

- Hartnoll proposed that universality could emerge at strong momentum relaxation

$$D_e \sim \frac{v^2}{T}$$

Axion model

$$S = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\left[R + \frac{6}{L^2} - \frac{1}{4e^2}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}g^{\mu\nu}\partial_\mu\chi_{\mathcal{A}}\partial_\nu\chi_{\mathcal{A}}\right]$$

$$\mathcal{A} = 1, 2$$

- Simplest holographic theory with broken translational symmetry.

Andrade & Withers

- Exact black hole solution

$$\chi_{\mathcal{A}} = kx_{\mathcal{A}}$$

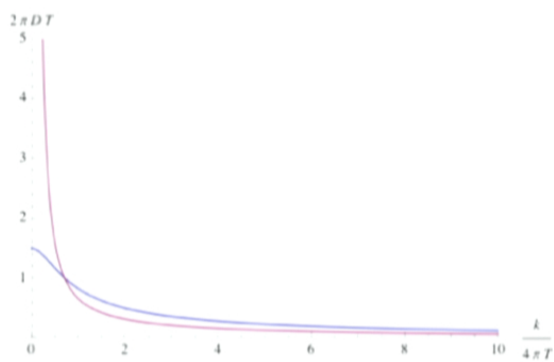
$$U(r) = L^{-2}r^2\left[1 - \frac{k^2L^4}{2r^2} - \left(1 - \frac{k^2L^4}{2r_0^2}\right)\frac{r_0^3}{r^3}\right] \quad V(r) = L^{-2}r^2$$

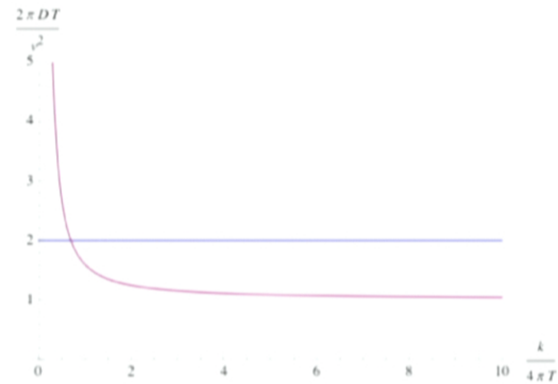
- k/T characterises strength of momentum relaxation.

- For this metric the velocity of the butterfly effect is

$$v_B^2 = \frac{\pi T L^2}{r_0}$$

- Straightforward to extract both D_c and D_e using Einstein relations





- In the \`incoherent' regime $k/T \gg 1$

$$D_c = \frac{L^2}{r_0} = \frac{v_B^2}{\pi T} \qquad D_e \approx \frac{L^2}{2r_0} = \frac{v_B^2}{2\pi T}$$

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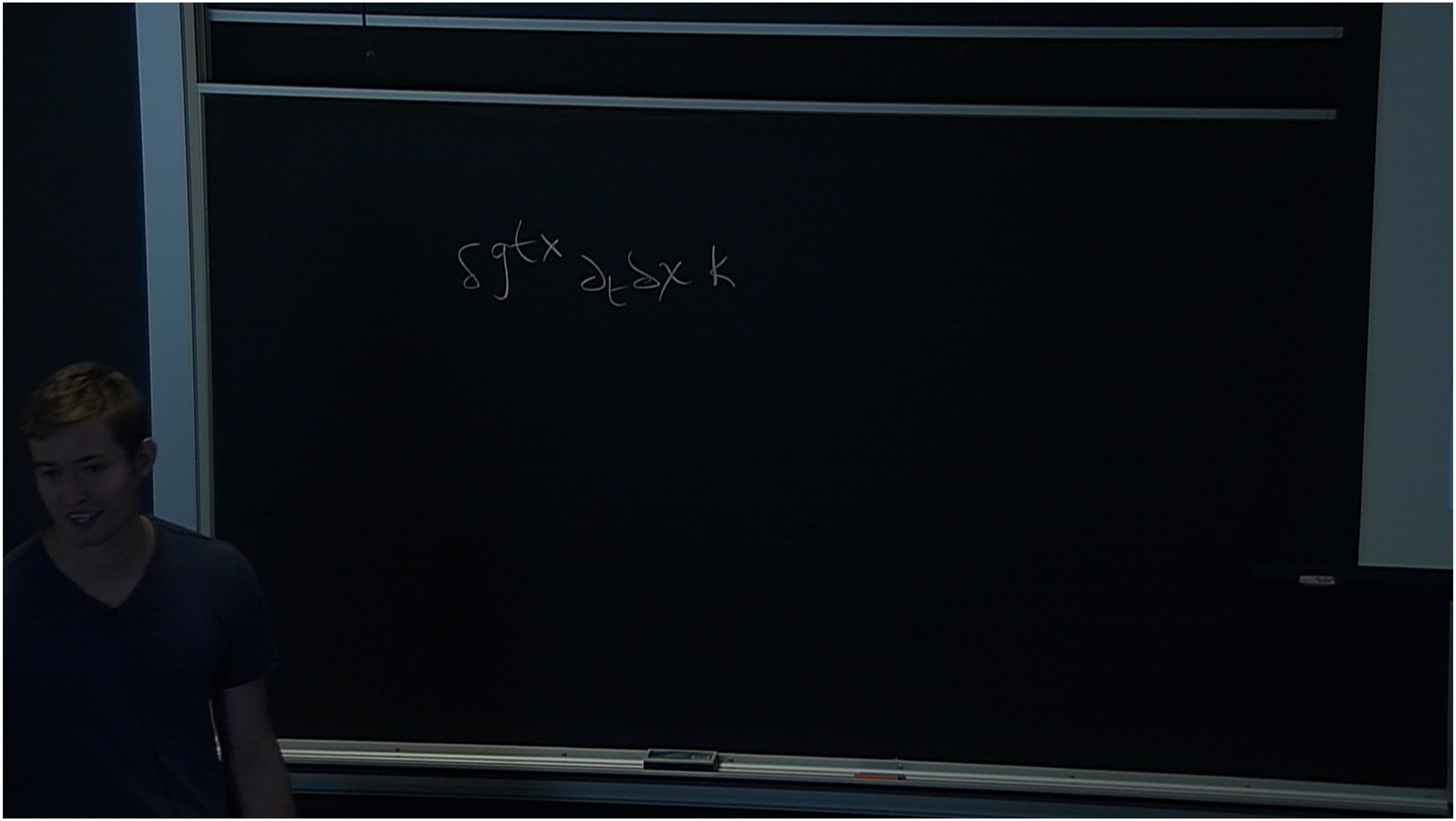
δg^tx ∂t∂x k

Q-lattices

- More general family of incoherent black holes in which a large lattice sources a hyperscaling-violating geometry.

Donos & Gauntlett, Gouteraux

- Charge diffusion is just as we discussed in our earlier examples.
- After using Einstein's equations one can also show

$$D_e = \frac{z}{2z-2}\frac{v_B^2}{2\pi T}$$

- The geometry supported by the lattice is precisely such that the diffusion constant is related to v_B.

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Conclusions

- In simple holographic theories can use v_B to formulate Hartnoll's diffusion bound.

$$D \sim \frac{\hbar v_B^2}{k_B T}$$

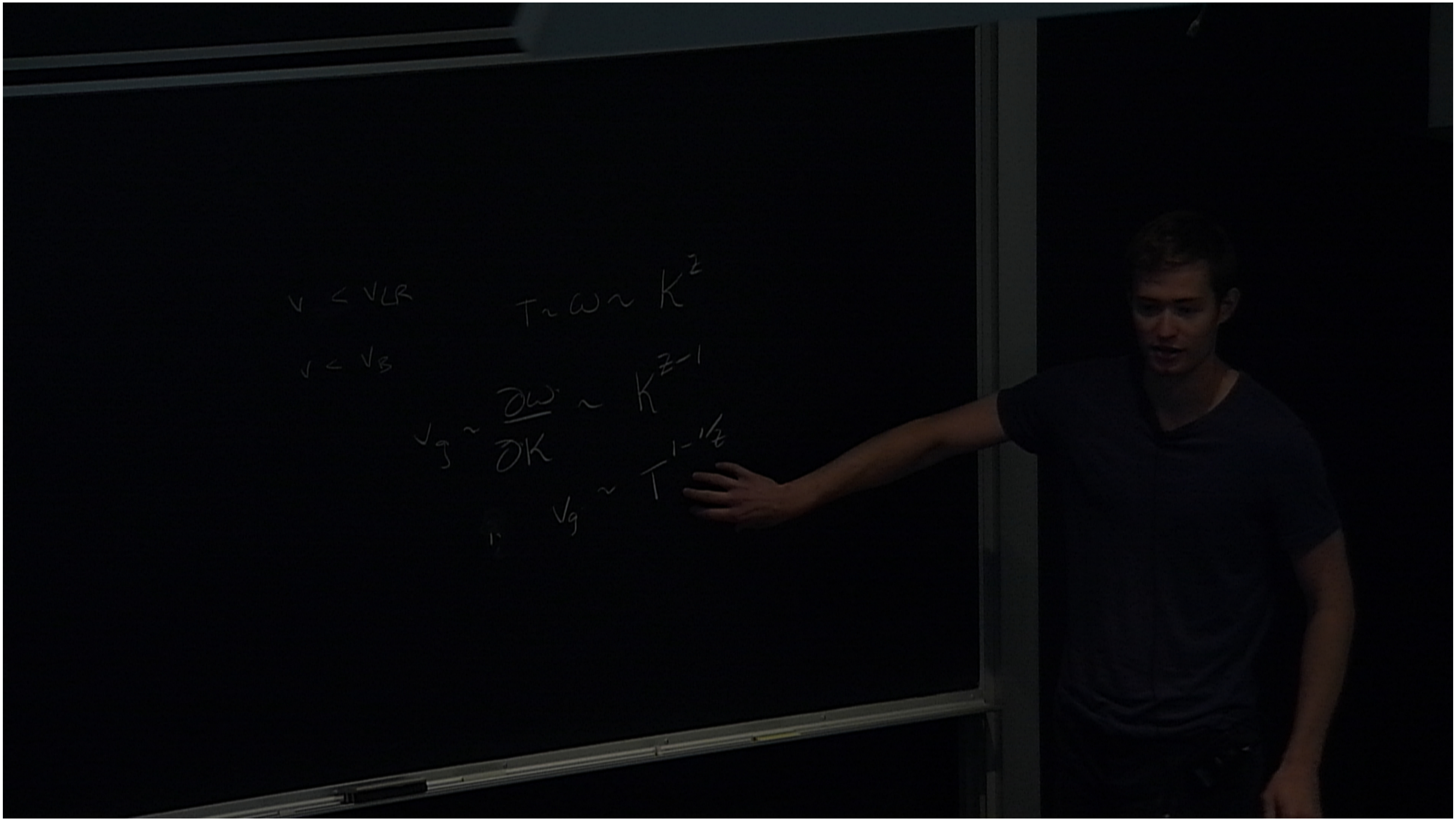
- Generalisation to charged geometries? Inhomogeneous models? Non-holographic theories?

Lucas & Steinberg

- Other connections between the butterfly effect and transport? c.f. chaos bound

$$\lambda_L \leq 2\pi k_B T/\hbar$$

Maldacena, Shenker & Stanford


v < v_LR
v < v_B
T ~ ω ~ K^z
v_g ~ ∂ω/∂K ~ K^(z-1)
v_g ~ T^(1-1/z)