Title: Hierarchical growth of entangled states

Date: Aug 22, 2016 02:15 PM

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Abstract: This talk, based on work with Brian Swingle, will describe the s-sourcery program. Its goal is to extend the lessons of the renormalization group to quantum many body states.

Hierarchical growth of entangled states

or

'*s*-sourcery'

John McGreevy (UCSD)
based on work
(arXiv:1407.8203, 1505.07106, 1602.02805, 1607.05753, in progress)
with
Brian Swingle (Stanford)
and
Shenglong Xu (UCSD)

PLAN:

- Gapped groundstates
- Gapless groundstates
- Mixed states

Big goal:

Understand the structure of entanglement in physical states of quantum field theories

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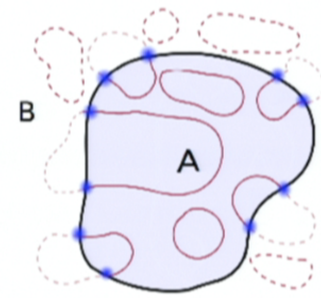
useful as a diagnostic

- (how to distinguish different phases with the same symmetries?)

[Fig: T. Grover]



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Understand the structure of entanglement in physical states of quantum field theories

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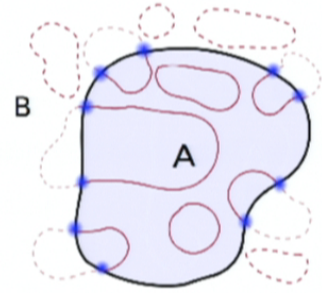
  (how much resources are required? where in Hilbert space to look?)

- useful as a diagnostic
  (how to distinguish different phases with the same symmetries?) [Fig: T. Grover]



- a crucial point of contact with holographic duality
  (entanglement entropy $\simeq$ area)

Context

- $\mathcal{H} = \otimes_x \mathcal{H}_x$
- $H = \sum_x H_x$ hamiltonian 'motif'

  (rules out many horrible pathologies). support of H_x is localized.
- families of systems labelled by (linear) system size L:
  H_L with groundstate(s) $\{|\psi_L\rangle\}$

Coarsely-stated, impossible desideratum: low-depth unitary **U** which constructs the groundstate *from smaller unentangled subsystems* :

$$|\psi_L\rangle \stackrel{??}{=} \mathbf{U}|0\rangle^{\otimes L} \qquad \mathbf{U} \stackrel{??}{=}$$ 

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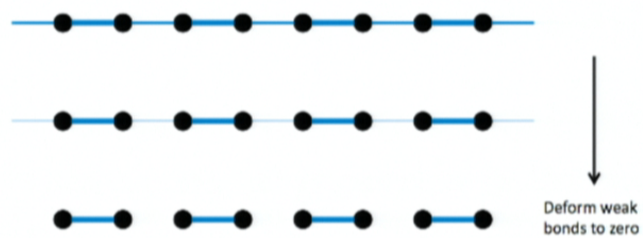
Warmup example ($d = 1, s = 0$):

$H(\eta) = \sum_n (1 + (-1)^n \eta) \mathbf{c}_n^\dagger \mathbf{c}_{n+1} + hc$

adiabatically deform 1d band insulator to product state



Given:

$$H(0) \xrightarrow{H(\eta)} H(1)$$

gapped

product ground state $|\psi(0)\rangle$ — $|\psi(\eta)\rangle$ — ground state of interest $|\psi(1)\rangle$

Construct: $\mathbf{U} \overset{?}{=} \mathrm{P}e^{\mathbf{i}\int_0^1 d\eta \mathbf{H}(\eta)}$

There are two problems with this plan, in general

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Solution [Hastings, Wen]:

Find quasilocal $\mathbf{K}$ such that

$\mathbf{i}\partial_\eta|\psi(\eta)\rangle = \mathbf{K}(\eta)|\psi(\eta)\rangle$

$\rightsquigarrow$ Produce quasi-local $\mathbf{U} = e^{\mathbf{i}\int_0^1 d\eta \mathbf{K}(\eta)}$.

$$K = -\mathbf{i}\int_{-\infty}^{\infty} dt F(t) e^{\mathbf{i}H(\eta)t}\partial_\eta H(\eta) e^{-\mathbf{i}H(\eta)t}$$

$F(t)$ odd, rapidly decaying, $\tilde{F}(0) = 0$,

$\tilde{F}(\omega) = -\frac{1}{\omega}, |\omega| \geq \Delta$.

Quasilocal means:

$U = e^{\mathbf{i}K}, \quad K = \sum_x K_x, \quad K_x = \sum_r K_{x,r}$

$K_{x,r}$ supported on disk of radius r, $\| K_{x,r} \| \leq e^{-r^{1-d}}$

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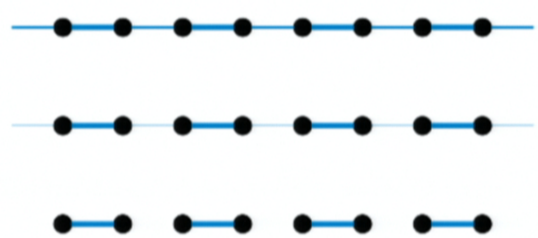

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Expanding universe strategy

[Swingle, JM, 1407.8203, PRB]

Instead, we are going to *grow* the system $|\psi_L\rangle \to |\psi_{2L}\rangle$ with local unitaries.

$$\mathbf{U} \sim \cdots \circ U_{4L_0 \leftarrow 2L_0} \circ U_{2L_0 \leftarrow L_0}$$

$\mathbf{U}$ will in general not have finite depth.
but $\mathbf{U}$ will have an RG structure.

Assumptions:

- Raw material: a bath of 'ancillas' $\otimes|0\rangle^M$ is freely available.
- For rigorous results, energy gap Δ for all excitations.
- There may be groundstate degeneracy $G(H_L)$ but the groundstates are *locally indistinguishable* (a necessary condition for the state to be stable)

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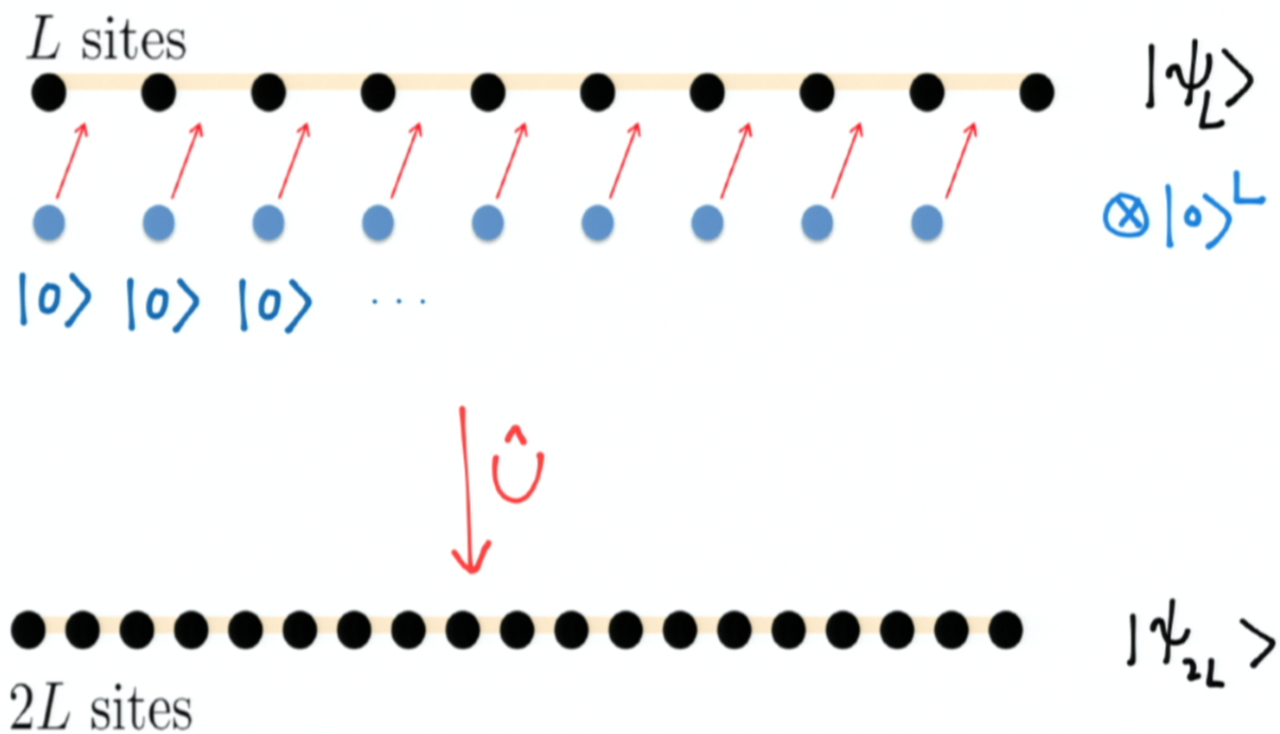
$d = 1, s = 1$ example:

(Not like crystal growth!)



$$|\psi_{2L}\rangle = U\left(|\psi_L\rangle \otimes |0\rangle^L\right).$$

An s-source RG fixed point

(in d dimensions) is a system whose groundstate on $(2L)^d$ sites can be made from the groundstate on L^d sites (plus unentangled ancillas) using a quasilocal unitary.

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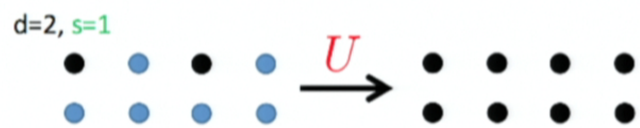

How to construct **U**

Construct **U** by quasiadiabatic evolution :

(For $s = 1$ we must start with $s = 1$ copy at size L.)

Given:

$H(\eta)$ gapped

$H(0) \longrightarrow H(1)$ $\rightsquigarrow$ Produce quasi-local **U** .

ground state on size L (and product states) $|\psi(0)\rangle$

$|\psi(\eta)\rangle$

ground state on size 2L $|\psi(1)\rangle$

[Hastings, Wen]

Reminder: quasilocal means:

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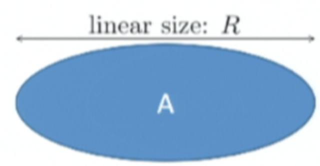


Basic property: Recursive entropy bounds:

(Uses Small Incremental Entangling result of

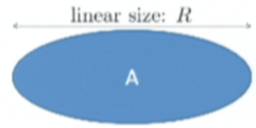
[Kitaev, Bravyi, van Acoleyn-Marien-Verstraete 2014].)

$$S(2R) \leq sS(R) + kR^{d-1}$$
$$S(2R) \geq sS(R) - k'R^{d-1}$$

Why is s-source RG fixed point a useful notion?

1. Such a circuit controls the growth of entanglement with system size: Area law theorem: any $s \leq 1$ fixed point in $d > 1$ enjoys an area law for EE of subregions.

$$S(A) \equiv -\mathrm{tr}\rho_A \log \rho_A \leq k|\partial A| = kR^{d-1}.$$



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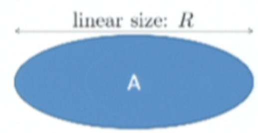
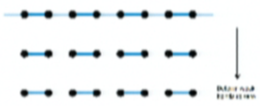
   

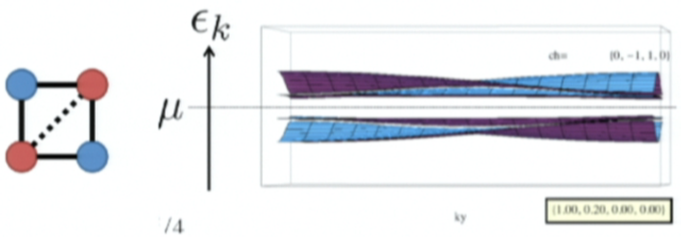
   $s \geq 2^{d-1}$ is required to violate the area law.
2. The groundstate degeneracy satisfies: $G(2L) = G(L)^s$
3. s (smallest possible) is a property of the phase (since by definition an adiabatic path connects any two representatives) $\Longrightarrow$ classification axis.
4. The circuit implies a MERA representation of the groundstate.

Many interesting states are s-source fixed points

- Mean field symmetry-breaking states $(s = 0)$



- Chern insulators, IQH $(s = 1)$



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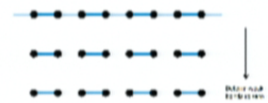

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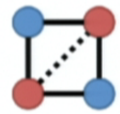
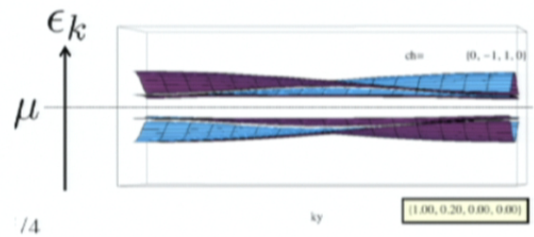




- Topological states (discrete gauge theory, fractional QH), including chiral ones $(s = 1)$

- Any *topological quantum liquid*

  $\equiv$ insensitive to smooth deformations of space $\simeq$ gapped QFT

  has $s = 1$.

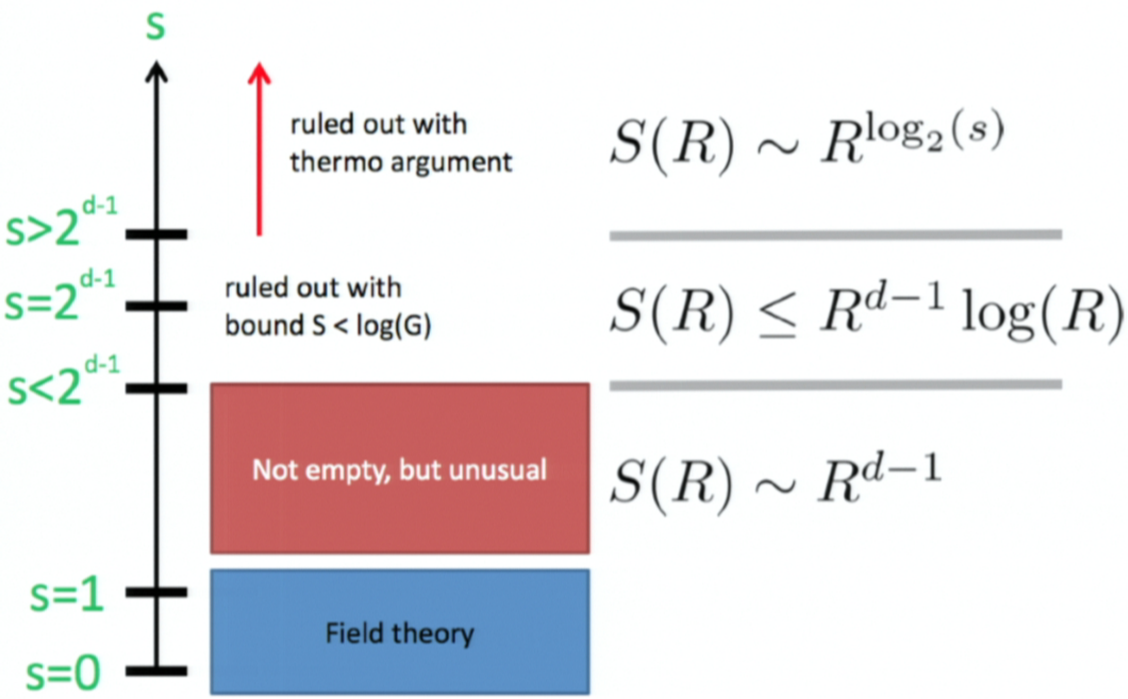
Why: place it in an expanding universe $ds^2 = -d\eta^2 + a(\eta)^2 d\vec{x}^2$

Experimental example: QCD

- Our universe is expanding, $t_{\text{doubling}} \sim 10^{10}$years.
- The QCD gap stays open $(m_\pi, m_p > 0)$.
- This is a gapped path from $|\psi_L\rangle$ to $|\psi_{2L}\rangle$.
- $\implies$ $\exists$ a quasilocal unitary which constructs the QCD groundstate from a small cluster plus ancillas.
(*i.e.* QCD has $s = 1$).

This suggests a new approach to simulating its groundstate which is in principle very efficient.

Reason to care #3: Classification of gapped states by *s*



← *e.g.*:
Layers of FQHE
or
Haah's cubic code

$d = 3, s = 2$

$\implies$ no continuum description!

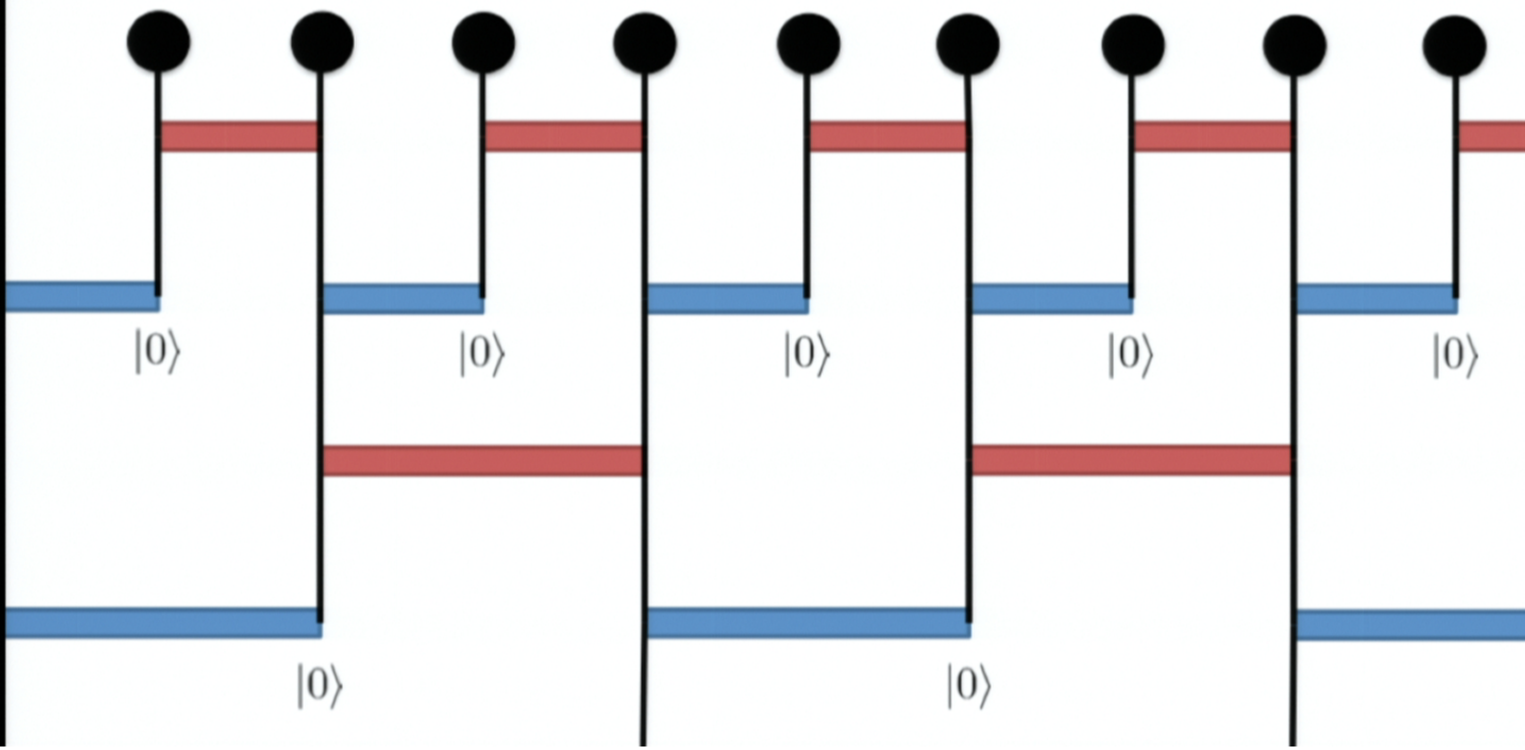
extensive GSD!

Reason to care #4: **U** $\rightsquigarrow$ MERA

A MERA is a representation of the groundstate which: [Vidal]

- allows efficient computation of observables (few contractions)
- organizes the information by scale (like Wilson and AdS/CFT taught us to do)
- geometrizes the entanglement structure [Swingle]

(Best representation of 1d critical states, very hard to find in $d > 1$.)



MERA representations of $s = 1$ fixed points

Quasilocal $\mathbf{U} \overset{\text{Trotter}}{\rightarrow}$ low-depth circuit:

$$|\psi_L\rangle \simeq \mathbf{U}_{\text{circuit}} |\psi_{L/2}\rangle |0\rangle^{L/2}$$

finite overlap requires $\hat{\ell} \sim \log^{1+\delta}(L)$

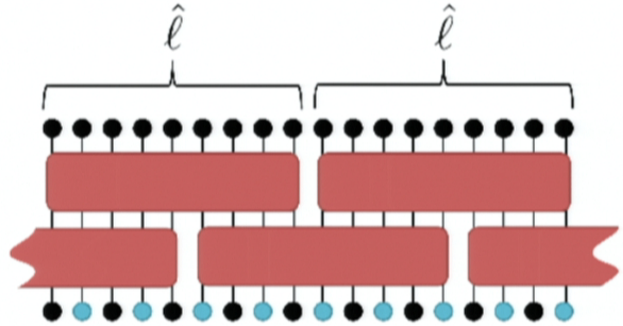
$$\Longrightarrow$$

bond dimension $\sim e^{\hat{\ell}^d} \sim e^{c \log^{d(1+\delta)}(L)}$

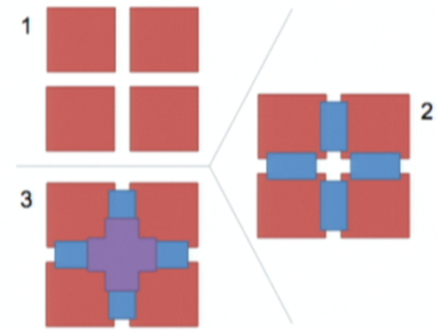
Crucial point: This construction of $\mathbf{U}_{\text{circuit}}$ requires no variational sweeps on large system.

Numerical implementation...?

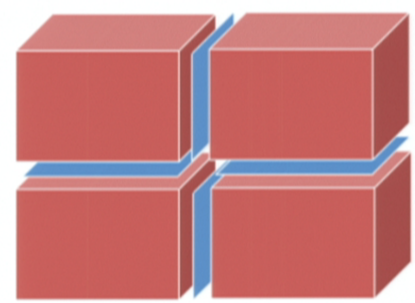
[getting started with Snir Gazit]



$d = 2$:



$d = 3$:



Further payoff: Invertible states

- A robust notion of 'short-range-entangled' Related ideas: [Kitaev, Freed]
  'Invertible states,' $|\psi\rangle$ means $\exists|\psi^{-1}\rangle, \mathbf{U}$ s.t.

$$|\psi\rangle \otimes |\psi^{-1}\rangle = \mathbf{U}|0\rangle^{\otimes 2L^d} \text{ has } s = 0.$$

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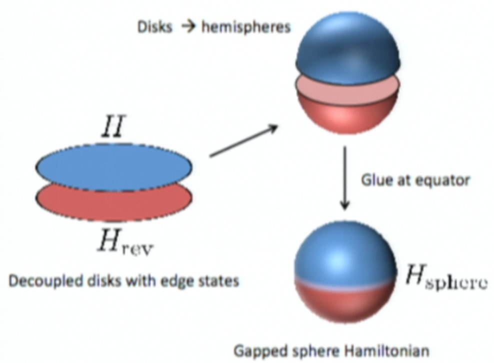
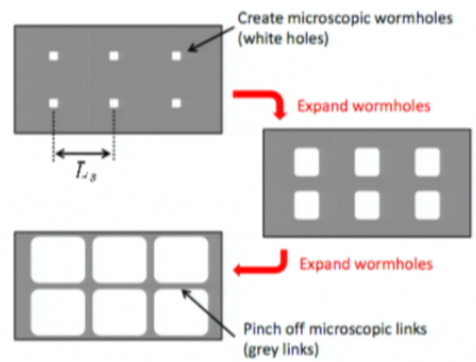
Graphical proof of weak area law:

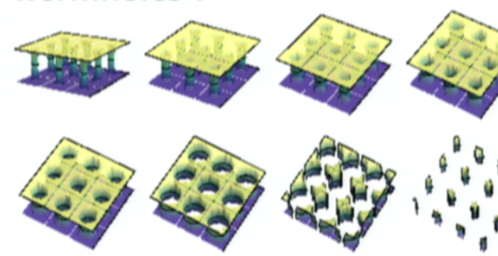


step 1: 'edge inverse' kills edge states



step 2: make adiabatic path to $|0\rangle^{\otimes}$ on T^d

side view of $H + H_{rev}$ + wormholes :



Gapless states and s-sourcery

- 'Entanglement Thermodynamics' constrains area law violation by gapless states
- and gives a relation between s and scaling exponents ($s = 2^{\theta}$).
- Examples of RG circuits for nontrivial critical points.

Entanglement bounds for gapless states

The area law is violated in groundstates of metals: $S \sim R^{d-1} \log k_F R$.

This violation is a symptom of many low-energy *extended* modes.

$\Longrightarrow$ can be seen in thermodynamics.

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Result: [Swingle-JM, 1505.07106, PRB]

If: thermal entropy of a scale-invariant state is $s(T) \sim T^{\frac{d-\theta}{z}}$

$z \equiv$ dynamical exponent

$\theta \equiv$ hyperscaling violation exponent

(anomalous dimension of T_{tt})

Then: the groundstate EE obeys the area law when $\theta < d - 1$ and $0 < z < \infty$.

(Recall: a Fermi surface has $\theta = d - 1$.)

Entanglement thermodynamics

Idea: Recast EE as local thermodynamics problem ($T = T_x$)

Find $\quad \boldsymbol{\sigma}_A \simeq Z^{-1} e^{-\sum_x \frac{1}{T_x} \mathbf{H}_x} \quad$ ($\mathbf{H} \equiv \sum_x \mathbf{H}_x$. local Gibbs state)

such that $\quad S(\boldsymbol{\sigma}_A) \geq S(\boldsymbol{\rho}_A)$.

Entanglement thermodynamics

Crucial Fact (local thermodynamics): For scaling purposes,

$$\begin{aligned} \mathrm{tr}\mathbf{H}_A \boldsymbol{\sigma}_A &\simeq E_{g,A} + \int_A d^d x \; e(T_x) \\ -\mathrm{tr}\boldsymbol{\sigma}_A \log \boldsymbol{\sigma}_A &\simeq \int_A d^d x \; s(T_x) \end{aligned}$$

$e(T_x) = Ts(T_x)$, bulk thermodynamic densities at temp T_x.

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Why: True if $1 \gg \frac{\nabla T_x}{T_x} \cdot \xi_x$ (for all x) ($\xi_x \equiv$ local correlation length).

But: let $\boldsymbol{\sigma}_A(\tau) \equiv Z(\tau)^{-1} e^{-\frac{1}{\tau}\sum_x \tilde{\mathbf{H}}_x / T_x} \overset{\tau \to 1}{\to} \boldsymbol{\sigma}_A$.

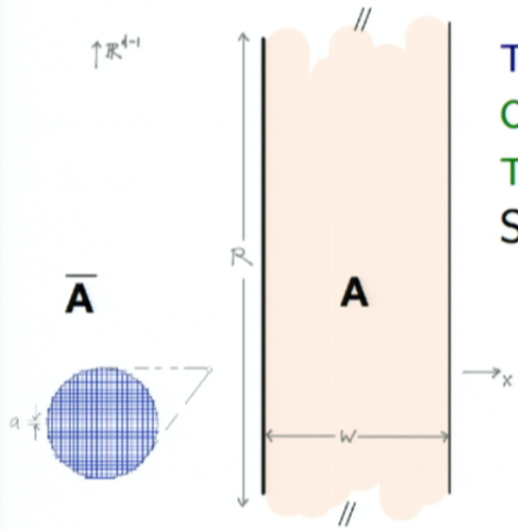
This state has temperature $T_x(\tau) = \tau T_x$, $\implies \xi_x(\tau) \sim T_x(\tau)^{-1/z} \propto \tau^{1/z}$

So (unless $z = \infty$!) the figure of merit for local thermo in state $\boldsymbol{\sigma}_A(\tau)$ is

$$1 \gg \underbrace{\frac{\nabla T_x(\tau)}{T_x(\tau)}}_{\sim \tau^0} \cdot \underbrace{\xi_x(\tau)}_{\sim \tau^{-1/z}} \overset{\tau \to \infty}{\to} 0.$$

$S(\boldsymbol{\sigma}_A(\tau)) = \tau^{\frac{d-\theta}{z}} S(\boldsymbol{\sigma}_A) \implies$ scales the same way with region size.

Scaling in strip geometry



To use local thermo, we need T_x.

Our question is local. Choose convenient geometry.

Translation invariant in $d-1$ dims (PBC). $R \gg w \gg a$.

Scale invariance $\Longrightarrow$

$$T_x \sim \begin{cases} x^{-z} & \\ \infty & \text{(no)} \\ 0 & \text{(sometimes: frustration free } \mathbf{H}\text{)} \end{cases}$$

$$\Longrightarrow \quad e(T_x) \sim x^{-z+\theta-d}, \quad s(T_x) \sim x^{\theta-d}$$

$$S_A \leq -\mathrm{tr}\boldsymbol{\sigma}_A \ln \boldsymbol{\sigma}_A \quad \sim R^{d-1} \int_a^w dx\, x^{-d+\theta}$$

$$\sim R^{d-1} \quad \left(a^{-d+\theta+1} - w^{-d+\theta+1}\right) \overset{w\to\infty}{\to} \infty \text{ only if } d < 1+\theta$$

Hence: scale invariant states with $\theta < d-1$ obey the area law.

Connection to s-sourcery

[Swingle-JM, 1505.07106]

If our scaling theory is an s-source RG fixed point

$$S(2R) \leq sS(R) + kR^{d-1} \; .$$

Assume saturated (if not, can use smaller s) $\Longrightarrow$

$$\begin{aligned} S_A &= k\left(\frac{R}{a}\right)^{d-1} \sum_{n=0}^{\log_2(w/a)} \left(\frac{s}{2^{d-1}}\right)^n \\ &\overset{R\gg w\gg a}{\simeq} k\left(\frac{R}{a}\right)^{d-1} \left(1 - \left(\frac{a}{w}\right)^{d-1-\log_2 s} + \cdots\right) \end{aligned}$$

Compare subleading terms in EE of strip:

$$\boxed{s = 2^\theta}$$

(Fermi surface has $\theta = d-1$, hence $s = 2^{d-1}$, marginally violates area law. ✓)

Gapless states with explicit $s = 1$ RG circuits

Expectation: CFTs are $s = 1$ fixed points.

∞ many examples of $d = 2$ quantum critical points

which are exact $s = 1$ fixed points: **'Square-root states'** [Kimball 1979]

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• Classical stat mech model in d space dimensions	$\longrightarrow$	• Quantum system in d space dimensions
• configurations s	$\longrightarrow$	• states $\|s\rangle$ (orthonormal)
• Boltzmann weight $e^{-\beta h(s)}$ $\mathcal{Z} \equiv \sum_s e^{-\beta h(s)}$	$\longrightarrow$	• g.s. wavefunction $\|h, \beta\rangle = \mathcal{Z}^{-1/2} \sum_s e^{-\beta h(s)/2} \|s\rangle$
• coolness $\beta = 1/T$	$\longrightarrow$	• coupling

e.g. near-neighbor Ising model: $h(s) = \sum_{\langle ij \rangle} s_i s_j$ $\longrightarrow$ $\mathbf{Z}_i|s\rangle = s_i|s\rangle$. Parent Hamiltonian: $\mathbf{H} = \sum_i \left(-\mathbf{X}_i + e^{-\beta \mathbf{Z}_i \sum_{\langle i|j \rangle} \mathbf{Z}_j} \right)$

correlations $\langle Z_r Z_{r'} \rangle$ $=$ correlations of diagonal operators $\langle \text{gs}|\mathbf{Z}_r \mathbf{Z}_{r'}|\text{gs}\rangle$

classical critical point $\longrightarrow$ quantum critical point

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classical critical point	$\longrightarrow$	quantum critical point
• real-space RG scheme	$\longrightarrow$	• quantum RG circuit with $s = 1$

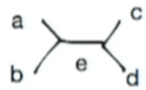
RG circuits for square root states

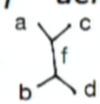
2d classical Ising TRG scheme: $\mathcal{Z} = \sum_{abcd\ldots} T_{abc} T_{ade} \cdots$ 

Two parts of classical RG step

[Levin-Nave]:

$1: \sum_e T_{abe} T_{cde} = \sum_f S_{acf} S_{bdf}$

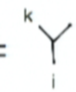
$\sum_e$ 

$= \sum_f$ 

[Different use of related machinery: Evenbly-Vidal, TNR]

$2: \sum_{abc} S_{akc} S_{cjb} S_{bia} = T'_{ijk}$

$\sum_{abc}$ 

$=$ 

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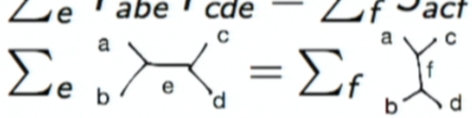

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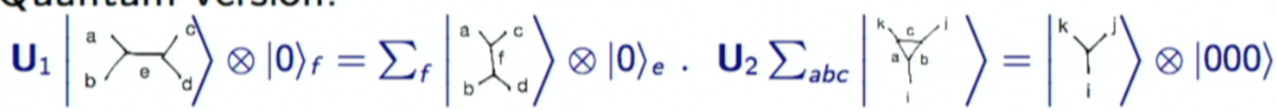
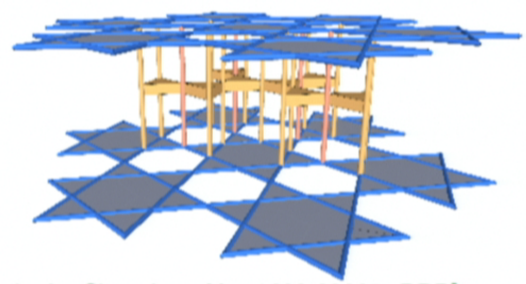
Quantum version:



$\mathbf{U} = \prod \mathbf{U}_2 \prod \mathbf{U}_1$

Fixed point of classical TRG

$\Longrightarrow$ $s = 1$ fixed point.



[JM, B Swingle, Shenglong Xu, 1602.02805, PRB]

Mixed s-sourcery

The extension of tensor network ideas to open quantum systems will be useful.

Even for thermal equilibrium, given $\boldsymbol{\rho} = Z^{-1}e^{-\beta \mathbf{H}}$, expectations are not, in general, computable.

Mixed s-sourcery

[Swingle-JM, 1607.05753]

What should replace the unitaries in the s-source RG circuit?

A sequence of states $\{\rho_L\}$ form a **purified s source fixed point** if there exists a sequence of purifications $\{|\sqrt{\rho_L}\rangle_{12}\}$ with $\mathrm{tr}_2(|\sqrt{\rho_L}\rangle\langle\sqrt{\rho_L}|_{12}) = \rho_L$ and

$$|\sqrt{\rho_{2L}}\rangle = \tilde{V}\left(\underbrace{|\sqrt{\rho_L}\rangle \otimes \ldots \otimes |\sqrt{\rho_L}\rangle}_{s \text{ times}} \otimes |0\ldots0\rangle\right)$$

where $|0\ldots0\rangle$ is a product state of the appropriate size and $\tilde{V}$ is a quasi-local unitary on A^sE. *i.e.*: $\exists$ a quasilocal channel $\rho_{2L} = \mathcal{E}\left(\rho_L^{\otimes s} \otimes |0\ldots0\rangle\langle 0\ldots0|\right)$

- The entropy can be volume law, but the mutual info is still area law:

$$I(A_{2R}, A_{2R}^c) \leq sI(A_R, A_R^c) + kR^{d-1}.$$

- Local channel preserves locality of operators $\implies$ efficiently contractible.

Local free fermions are mixed $s = 0$

[Swingle-JM, 1607.05753]

$$H = \sum_{xy} c_x^\dagger h_{xy} c_y + h.c., \qquad \text{with } h_{xy} \to 0 \text{ for } |x-y| \gg a$$

thermal eqbm: $\boldsymbol{\rho}_T = e^{-\mathbf{H}/T}/Z = \mathrm{tr}_2 \underbrace{\sum_E \sqrt{\frac{e^{-\beta E}}{Z}} |E\rangle_1 |E\rangle_2}_{\equiv |T\rangle}$ is $s = 0$.

$|T\rangle$ is the groundstate of $(f_k = \frac{1}{e^{\epsilon_k}+1})$

$$H_T \equiv \sum_k \left(-d_k^\dagger d_k + \tilde{d}_k^\dagger \tilde{d}_k \right), \qquad \left(\begin{smallmatrix} d_k \equiv \sqrt{f_k} c_k + \sqrt{1-f_k} \tilde{c}_k, \\ \tilde{d}_k \equiv -\sqrt{f_k} c_k + \sqrt{f_k} \tilde{c}_k \end{smallmatrix} \right)$$

which is gapped, local and adiabatically connected to

$$H_\infty = -\sum_x \left(c_x^\dagger c_x + \tilde{c}_x^\dagger \tilde{c}_x \right), \quad |\mathrm{gs}_\infty\rangle = \prod_x \frac{c_x + \tilde{c}_x}{\sqrt{2}} |0\rangle \qquad \text{(ultralocal)}.$$

So the resulting a quasiadiabatic **U** gives a quasilocal channel:
$\boldsymbol{\rho}_T \to \mathrm{tr}_2 \mathbf{U} |T\rangle\langle T| \mathbf{U}^\dagger =$ product state.

A sufficient condition for mixed $s = 0$

$$S(A) = c_1 \text{vol}(A) + \int_{\partial A} \left(c_2 + \sum_{i>2} c_i f_i(K, R) \right) + \mathcal{O}(\ell^d e^{-\ell/\xi}) \quad (\star)$$

$\ell \equiv$ linear size of A .

$$\implies I(A:C|B) \approx 0 \text{ if } {}^{AB+BC-B-ABC=0}_{\text{and } \partial B + \partial(AC) = \emptyset} .$$

[Fawzi-Renner 15]: approximate quantum Markov chains can be reconstructed from marginals via a channel on the buffer.

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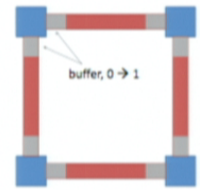

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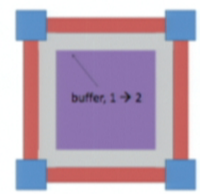
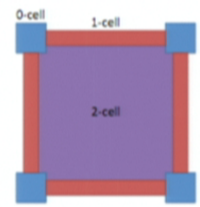


Make a cellular decomposition of space (e.g. $d = 2$) $\longrightarrow$
(all regions $> \xi$)



$$I(p\text{-cells} : (p-1)\text{-cells}|\text{buffer}) \approx \mathcal{O}(N_{\text{cells}} e^{-\ell/\xi}).$$

If so, then here is the state:

$$\rho = \rho_{\text{2-cells}\cup\text{1-cells}\cup\text{0-cells}} \approx \mathcal{N}_{1\to 2}(\mathcal{N}_{0\to 1}(\mathcal{N}_{\emptyset\to 0}(\cdot)))$$

3d:

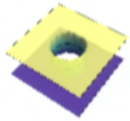
When is cellular reconstruction possible?

$(\star)$ is true for:

- invertible states.
- CFT at finite temperature.
- states with classical gravity duals.
- states which are not finite-T quantum memories [Hastings def of TO] :

adiabatically connected to $T = \infty \implies$ quasilocal channel to product.

Run the construction backwards: an array of bubbles-of-Nothing.

bubble of Nothing:  [Witten 1985]

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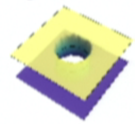
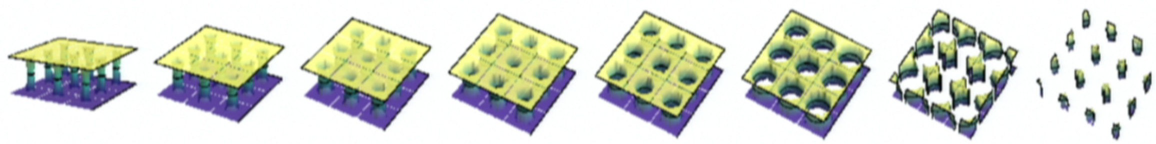
bubble of Nothing:  [Witten 1985]



Two possible obstructions: edge modes and TEE [Preskill-Kitaev].

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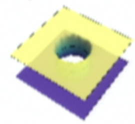
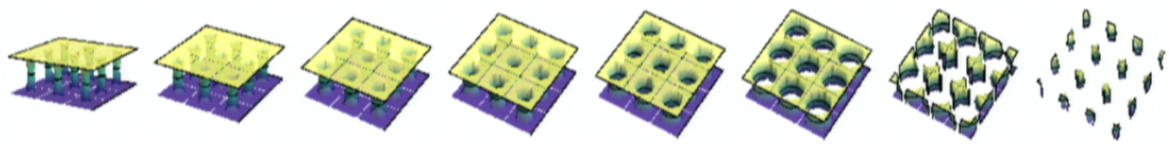
bubble of Nothing:  [Witten 1985]



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This construction was used in [Mahajan et al, 1608.05074] to make efficient representations of non-eqbm steady states associated with dissipative transport.

The idea: despite extensive von Neumann entropy, such states have low entanglement, hence tensor network representations.

Questions

Q: Is the thermal double $\sum_n \sqrt{\frac{e^{-\beta \mathbf{H}}}{Z}} |n\rangle |n\rangle$ always the groundstate of a local, gapped $\mathbf{H}$?

We showed 'yes' for free fermions and for sqrt states.

'Yes' lets us use groundstate s-sourcery.

Q: Can we improve the structure of the channel? The range of the resulting circuits is the thermal correlation length ($\to \infty$ as $T \to 0$).

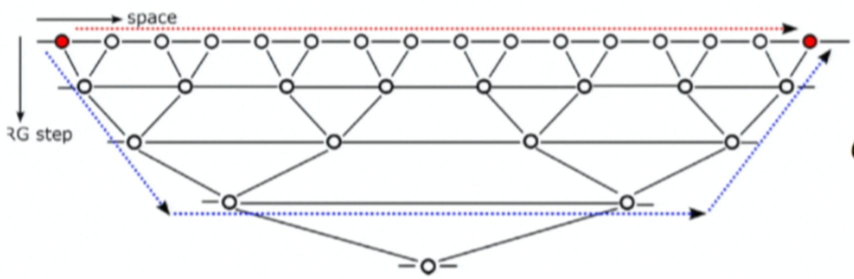
Fawzi-Renner result doesn't take advantage of locality within the buffer B.

U will be more local if we incorporate the $s = 1$ groundstate circuit near the IR.

Geometry is made of entanglement

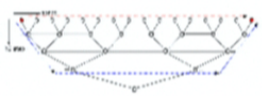
This is a step in a larger program to understand the emergence of space in gauge/gravity duality:

entanglement determines (much of)* bulk geometry [Swingle, van Raamsdonk, ...]



$$ds^2 = dS^2$$

Entanglement of a subregion bounded by the minimum number of bonds which must be cut to remove it from the graph.

$\mathcal{H}, \mathbf{H} \overset{\text{RG circuits}}{\longrightarrow}$  $\overset{\text{[Swingle-van R, Faulkner et al]}}{\longrightarrow} G_{\mu\nu} = T_{\mu\nu}$

* Interesting exception: behind horizons, where time is emergent, extra data about the *complexity* of the state is required. [Stanford group]

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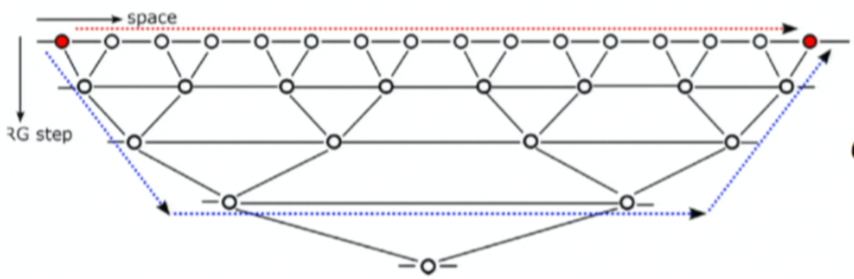

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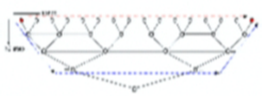
$\mathcal{H}, \mathbf{H} \overset{\text{RG circuits}}{\longrightarrow}$  $\overset{\text{[Swingle-van R, Faulkner et al]}}{\longrightarrow}$ $G_{\mu\nu} = T_{\mu\nu}$

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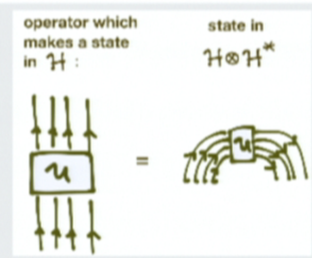
A unification of these quantities is in order!



The end.

Thank you for listening.

State of matter	z	s	θ	EE
Insulators, etc.	Gap	0	n/a	Area
SSB, discrete	Gap	0	n/a	Area
IQHE (invertible)	Gap	1	n/a	Area
FQHE	Gap	1	n/a	Area
Topological states	Gap	1	n/a	Area
Haah's cubic code ($d = 3$)	Gap	2	n/a	Area
SSB, continuous ($d > 1$)	1	1	0	Area
QCP (conformal), $d = 1$	1	1	0	Area*Log
QCP (conformal), $d > 1$	1	1	0	Area
Quadratic band touching	2	≤ 1	0	Area
Fermi liquids	1	2^{d-1}	$d - 1$	Area*Log
Spinon Fermi surface	3/2?	2^{d-1}	$d - 1$	Area*Log
Diffusive metal, $d = 3$	2	2^{d-2}	$d - 2$	Area
QED	1	1	0	Area
QCD	Gap	1^*	n/a	Area