Title: Topological phases and their transitions in 1D

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Abstract: <p>One dimensional symmetry protected topological (SPT) phases are gapped phases of matter whose edges are degenerate if the Hamiltonian respects a particular symmetry. With their interacting classification having been understood since 2010, we would like to further our understanding by addressing the following two questions: (1) Is there a unified way of understanding some of the exactly soluble models for 1D SPTs? And (2) if we are given two arbitrary SPTs, can we predict the structure of the phase transition between them? The answers turn out to be surprisingly simple. The first is given by relating various models to the Kitaev chain, uncovering new facts about some well-known SPTs. As for the transition between two arbitrary SPTs, it is generically described by a free conformal field theory which is determined by the topological class of the gapped phases.</p>

Topological Phases & Their Transitions in 1D

with
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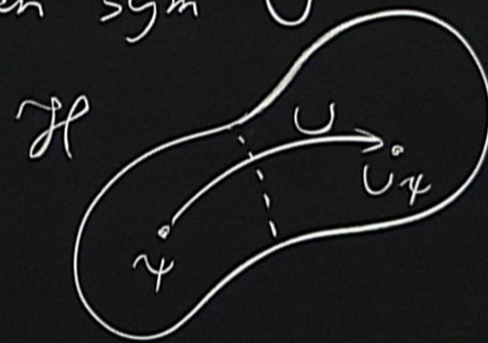
Classification
- → Fidkowski & Kitaev
- → Turner, Pollmann, Berg (2010)
- → Chen, Gu, Wen

Aim
1. relations btwn known models ⇒ new insights
2. „If I have 2 topological phases, can I predict their transitions?"

Recap of „SPT" (sym. prot. top.)

Take a chain w/ unbroken sym U

$U = U_L \otimes \mathbb{1} \otimes U_R$



$\boxed{U = U_L U_R}$ „sym. fract."

Conseq: • $L \cap R = \emptyset \implies U_L, U_R$ are sym

• $U = U_L U_R$ e.g. $UV = VU \implies U_L V_L = \pm V_L U_L$

$V = V_L V_R$ $U^2 = 1$

$U, V, \ldots$ rep of G

$U_L, V_L, \ldots$ proj rep of G

If $\xi < \infty$, any sym. induces a proj rep on the edge

If non-triv, we say the sym „protects" the phase (SPT)

<u>Ex</u>: Cluster model, $H = -\sum X_{n-1} Z_n X_{n+1}$

(2000, Briegel, Raussendorf)

(2011, Son, Hamma, et al)

$$\underbrace{\uparrow\ \uparrow\ \uparrow}_{\sigma^x \otimes \sigma^z \otimes \sigma^x}\ \uparrow\ \uparrow\ \uparrow$$

$$P_1 = Z_1 \quad Z_3 \quad Z_5$$

$$P_2 = \quad Z_2 \quad Z_4 \quad Z_6$$

$$\cap$$

$$\mathbb{Z}_2 \times \mathbb{Z}_2$$

<u>Claim</u>: $P_1^c P_2^c = -P_2^c P_1^c$

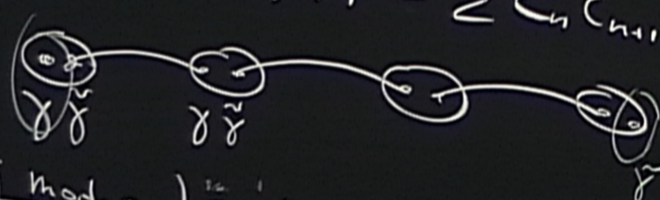
Proof: in gs subspace: $X_{n-1} Z_n X_{n+1} = 1$

$$1 = (X_1 Z_2 \cancel{X_3})(\cancel{X_3} Z_4 \cancel{X_5})(\cancel{\ } \dots) = X_1 \underbrace{Z_2 Z_4 Z_6 \cdots Z_{N-2}}_{P_2 Z_N} X_{N-1}$$

$$\Rightarrow \boxed{P_2 = \underbrace{X_1}_{P_2^L} \underbrace{X_{N-1} Z_N}_{P_2^R}} \qquad P_1^L = Z_1 X_2$$

∎

Ex. Kitaev chain, $H = - \sum c_n^\dagger c_{n+1} + c_n^\dagger c_{n+1}^\dagger + h.c.$



Sym. A, B, $AB = -BA$

$\Rightarrow$ deg

$A|\psi\rangle = a|\psi\rangle$

$B|\psi\rangle = b|\psi\rangle$

Maj modes: $\begin{cases} c + c^\dagger = \gamma \\ \frac{c - c^\dagger}{i} = \tilde{\gamma} \end{cases} \Rightarrow H = i \sum \tilde{\gamma}_n \gamma_{n+1}$

$P = \prod_i (1 - 2n_i) = P_L P_R$, $P_L = i\gamma_1$ $P_R = \tilde{\gamma}_N$ $\boxed{P_L P_R = - P_R P_L}$

Topic 1. Relations btwn SPTs

Claim ①: Kitaev chain is J.W. dual to the Q. Ising chain

$$\begin{cases} \gamma_n = Z_1 Z_2 \cdots Z_{n-1} X_n \\ \tilde{\gamma}_n = Z_1 Z_2 \cdots Z_{n-1} Y_n \end{cases}$$

$$H = i \sum \tilde{\gamma}_n \gamma_{n+1} = i \sum Y_n (Z_n X_{n+1}) = - \sum X_n X_{n+1}$$

Intermezzo: $\{ |\text{triv}\rangle, |\text{SPT } 1\rangle, |\text{SPT } 2\rangle, \ldots \}_G$ is itself a group!

e.g. {SPTs, with P, T}

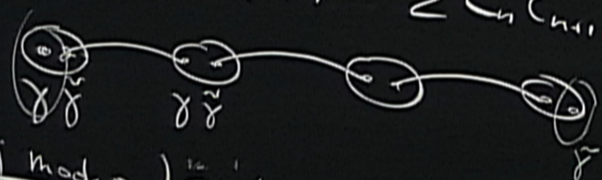
Proof: in gs subspace: $X_{n-1} Z_n X_{n+1} = 1$

$$1 = (X_1 Z_2 \cancel{X_3})(\cancel{X_3} Z_4 \cancel{X_5})(\cancel{\cdots}\ ) = X_1 \underbrace{Z_2 Z_4 Z_6 \cdots Z_{N-2}}_{P_2 Z_N} X_{N-1}$$

$$\Rightarrow \boxed{P_2 = \underbrace{X_1}_{P_2^L} \underbrace{X_{N-1} Z_N}_{P_2^R}} \qquad P_1^L = Z_1 X_2$$

∎

Ex. Kitaev chain, $H = -\sum c_n^\dagger c_{n+1} + c_n^\dagger c_{n+1}^\dagger + h.c.$



Maj modes: $\begin{cases} c + c^\dagger = \gamma \\ \dfrac{c - c^\dagger}{i} = \tilde\gamma \end{cases} \Rightarrow H = i \sum \tilde\gamma_n \gamma_{n+1}$

$P = \prod (1 - 2n_i) = P_L P_R$, $P_L = i\gamma_1$, $P_R = \tilde\gamma_N$, $\boxed{P_L P_R = -P_R P_L}$

Sym A, B, $AB = -BA$

$\Rightarrow$ deg

$A|\psi\rangle = a|\psi\rangle$

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Note: $T = K$, $[H, T] = 0$

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ansitions?"

Topic 1. Relations btwn SPTs

Claim 0: Kitaev chain is J.W. dual to the Q. Ising chain

$$\begin{cases} \gamma_n = Z_1 Z_2 \cdots Z_{n-1} X_n \\ \tilde{\gamma}_n = Z_1 Z_2 \cdots Z_{n-1} Y_n \end{cases}$$

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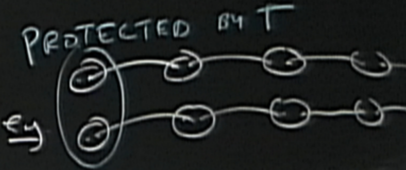
Intermezzo: $\{ |\mathrm{triv}\rangle, |\mathrm{SPT}\,1\rangle, |\mathrm{SPT}\,2\rangle, \dots \}_G$ is itself a group!



e.g. $\{\text{SPTs with } P, T\} = \mathbb{Z}_8$

Claim 1: 2x Kitaev is JW dual to the cluster model

$$H_2 = i \sum \tilde{\gamma}_n \gamma_{n+2} = - \sum X_n Z_{n+1} X_{n+2}$$

T

$P_1, P_2 \in \mathbb{Z}_2 \times \mathbb{Z}_2$

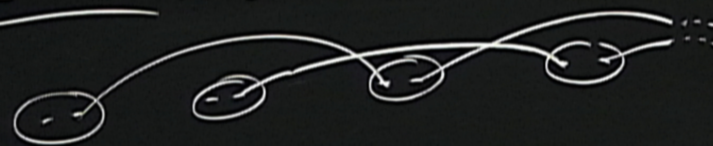
$PT = (\prod Z) K$ new!

Intermezzo: Haldane Phase, spin chain protected by $\begin{cases} R^x_\pi, R^z_\pi \in \mathbb{Z}_2 \times \mathbb{Z}_2 \\ T_{spin} = e^{i\pi S^y} K \end{cases}$

AKLT state

e.g.: {SPTs, with P, T} = $\mathbb{Z}_8$ e.g.

Claim 1: 2× Kitaev is JW dual to the cluster model



$$H_2 = i \sum \tilde{\gamma}_n \gamma_{n+2} = - \sum X_n Z_{n+1} X_{n+2}$$

T $\qquad$ $P_1, P_2 \in \mathbb{Z}_2 \times \mathbb{Z}_2$

$PT = (\prod Z) K$ new!

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$$H = \sum \vec{S}\cdot\vec{S} + \frac{1}{3}(\vec{S}\cdot\vec{S})^2 \quad \text{AKLT}$$

Claim 2: Cluster state is (the fixed pt limit) of the AKLT state

$\boxed{\uparrow\ \uparrow}\ \boxed{\uparrow\ \uparrow}\ \boxed{\uparrow\ \uparrow}$ $\quad \exists$ 2-site trans $\Rightarrow$ $\quad$ $S=\frac{1}{2}$ $\quad S=0$ $\quad S=0$ $\quad S=\frac{1}{2}$ $\quad \boxed{\uparrow\ \uparrow}\ \boxed{\uparrow\ \uparrow}\ \boxed{\uparrow\ \uparrow}$ $\quad \underbrace{\quad}_{XX+ZZ}\ \underbrace{\quad}_{XX+ZZ}$

$P_1 = Z_1 \quad Z_3 \quad Z_5 \; \cdots \; \dashrightarrow R^x_\pi$

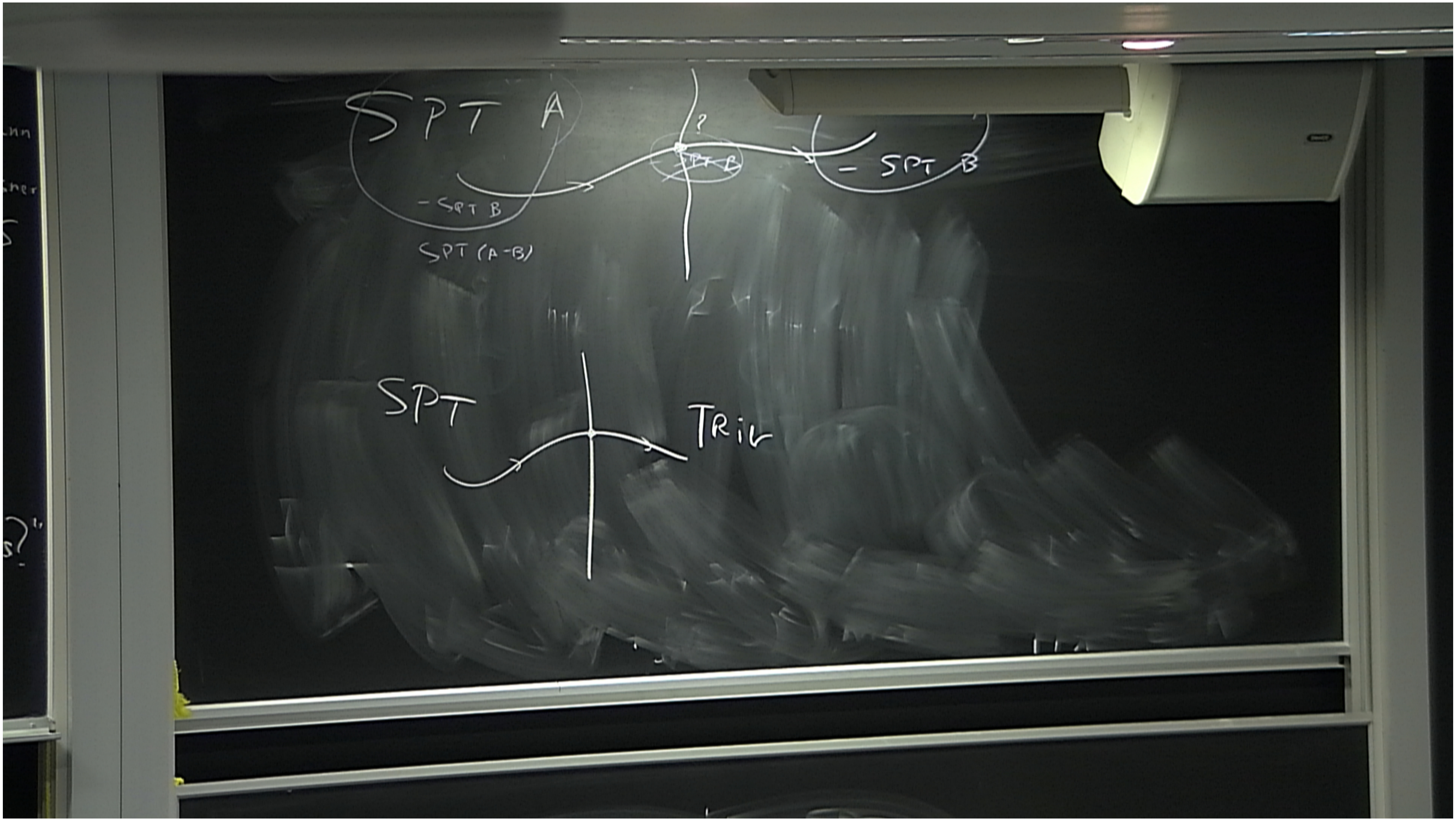
$P_2 = \quad Z_2 \quad Z_4 \; \cdots \; \dashrightarrow R^y_\pi$

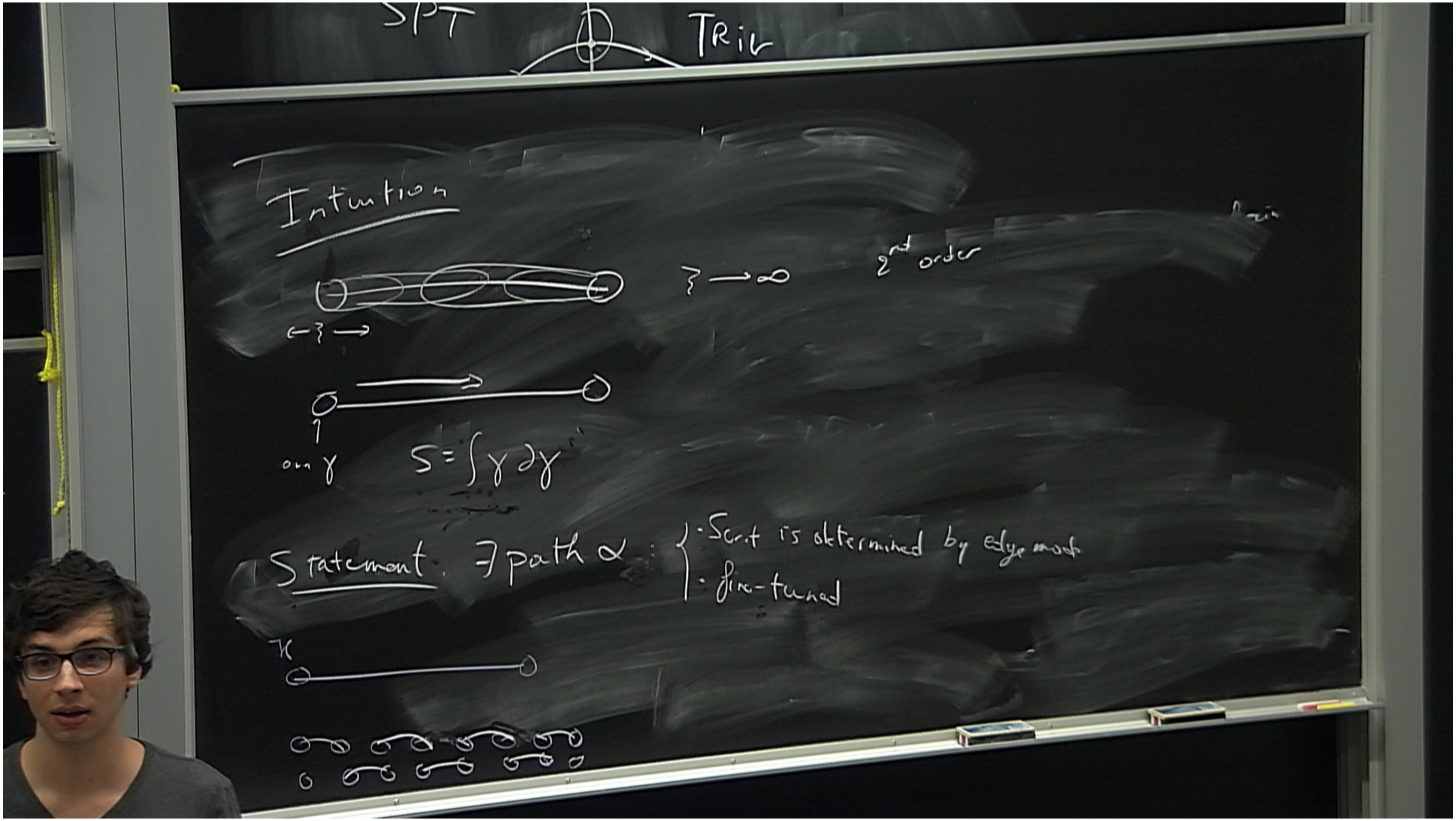
$PT = Z_1 Z_2 Z_3 \cdots K \; \dashrightarrow T_{spin}$

$H = -\sum XZX \qquad H = \sum_{bonds} XX + ZZ \cancel{(+YY)}$

Claim 3: 2x Kitaev = SSH model

Claim 4: 4x Kitaev = Hubbard ladder $\xrightarrow{\text{Mott limit}}$ AKLT state


SPT A
- SPT B
SPT (A-B)
?
SPT B
- SPT B
SPT
Triv


SPT
TRiv
Intuition
ξ → ∞
2nd order
S = ∫ γ ∂γ
Statement: ∃ path α :
· fine-tuned