Title: Does relativistic causality constrain interference phenomena?

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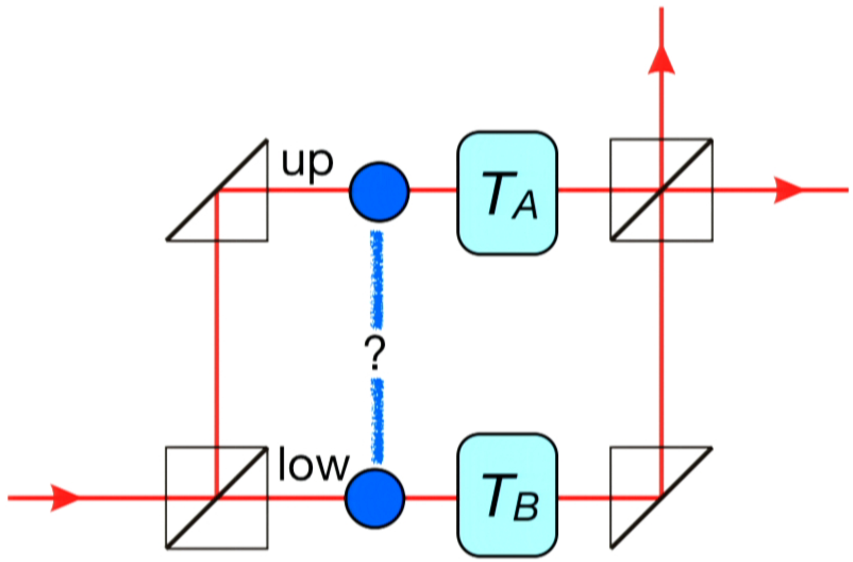
Abstract:

Does relativistic causality constrain interference phenomena?

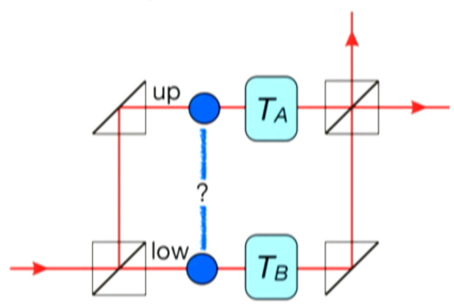
Markus P. Müller
Departments of Applied Mathematics and Philosophy, UWO
Perimeter Institute for Theoretical Physics, Waterloo



Outline

1. General probabilistic theories

2. Relativity of simultaneity on an interferometer



How to describe a "general probabilistic theory"

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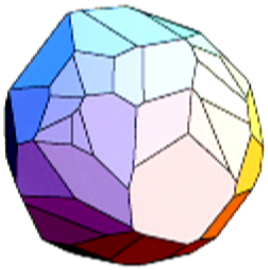
Essentially by an arbitrary convex state space.
And here's why & how.



1. General probabilistic theories

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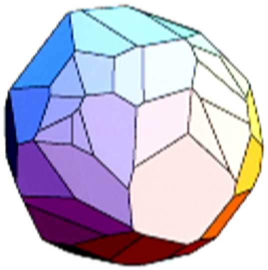
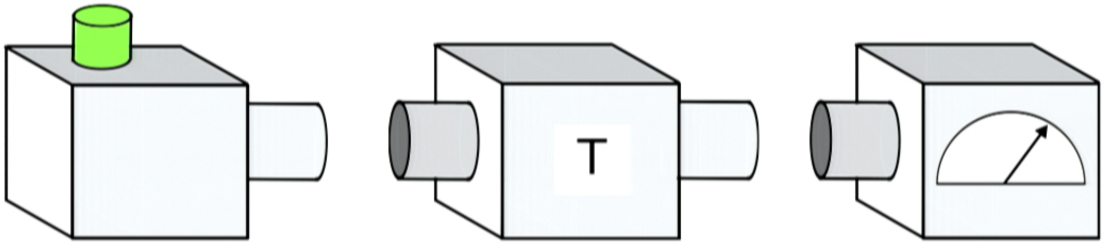




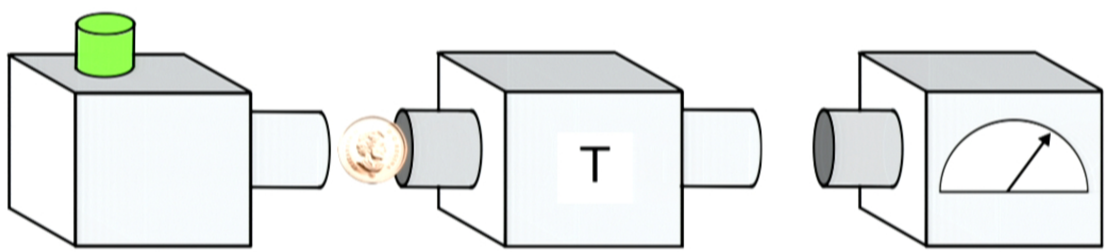
Preparation, transformation, measurement.

1. General probabilistic theories

How to describe a "general probabilistic theory"

Example: classical coin toss.

- On every push of button, the preparation device performs a biased coin toss.



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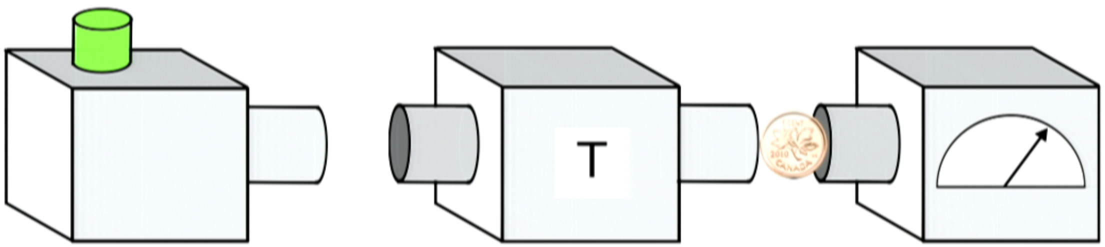
- The transformation device, for example, inverts the coin (if heads then tails, and vice versa).

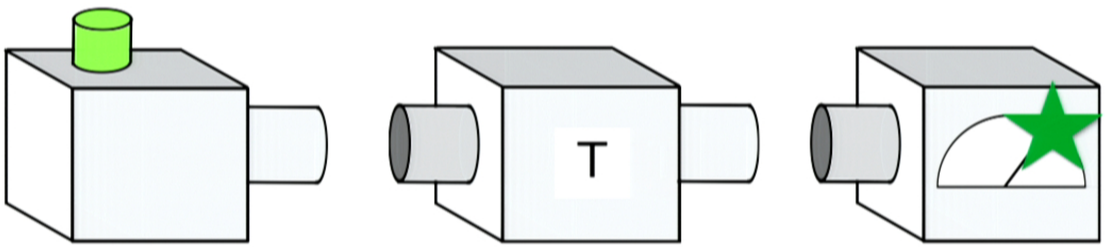


Preparation, transformation, measurement.

How to describe a "general probabilistic theory"

Example: classical coin toss.

- On every push of button, the preparation device produces a biased coin toss.
- The transformation device, for example, inverts the coin (if heads then tails, and vice versa).
- The measurement outcome is "heads" or "tails".



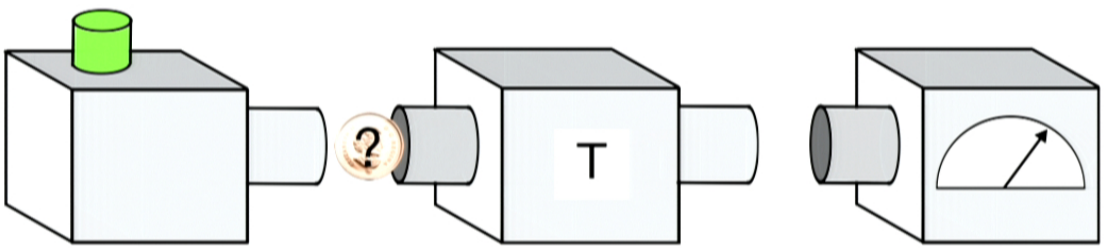
Preparation, transformation, measurement.

How to describe a "general probabilistic theory"

Example: classical coin toss.

- The preparation device prepares a physical system in a state ω. Here

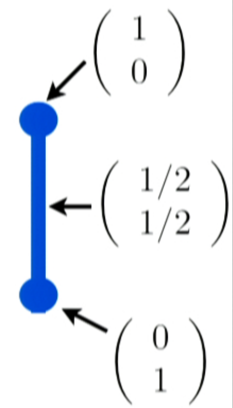
$$\omega = \begin{pmatrix} \mathrm{Prob(heads)} \\ \mathrm{Prob(tails)} \end{pmatrix} = \begin{pmatrix} p \\ 1-p \end{pmatrix}.$$

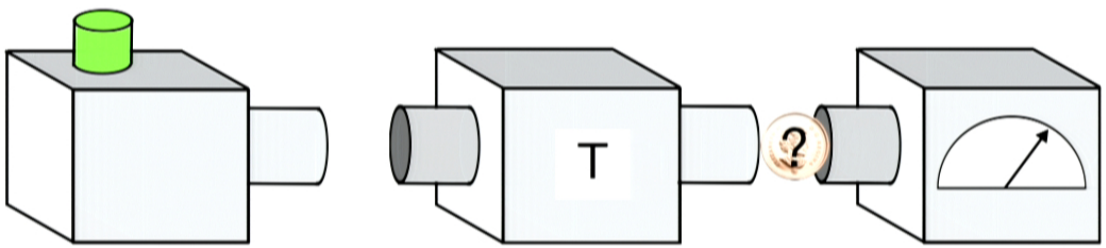


How to describe a "general probabilistic theory"

Example: classical coin toss.

- The preparation device prepares a physical system in a state ω.
- Transformation: $T\begin{pmatrix} p \\ 1-p \end{pmatrix} = \begin{pmatrix} 1-p \\ p \end{pmatrix}$





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Example: classical coin toss.

- The preparation device prepares a physical system in a state ω.
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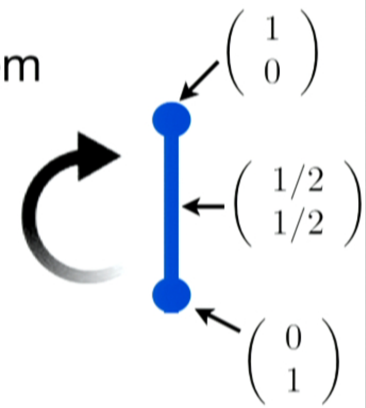
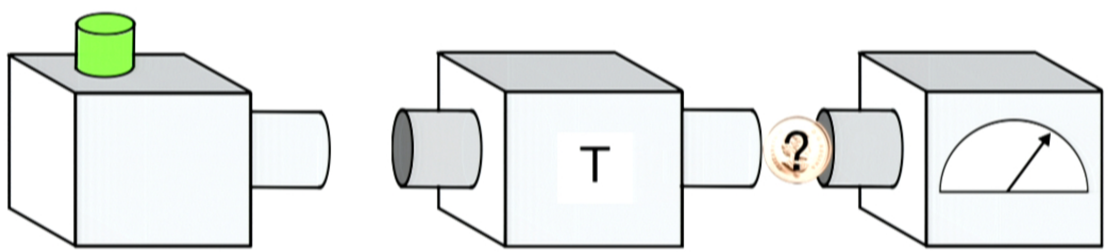
  Maps states to states and is linear.

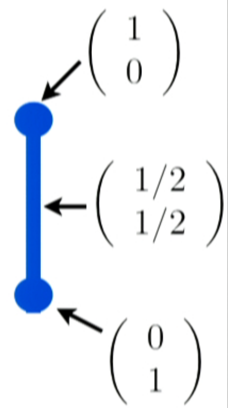


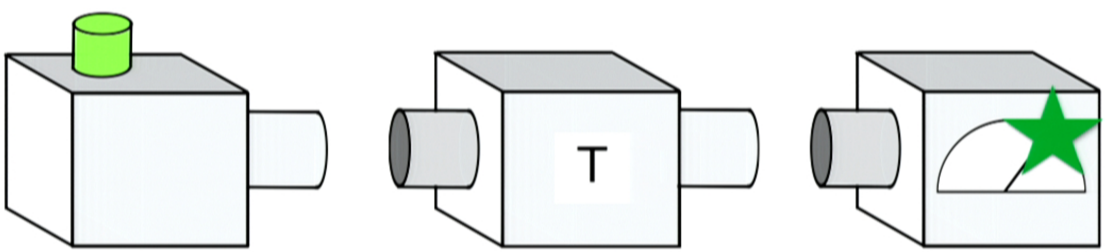


How to describe a "general probabilistic theory"

Example: classical coin toss.

- Every measurement outcome has a probability, depending linearly on the state:



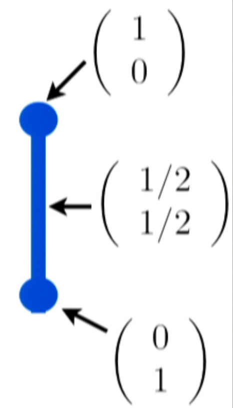


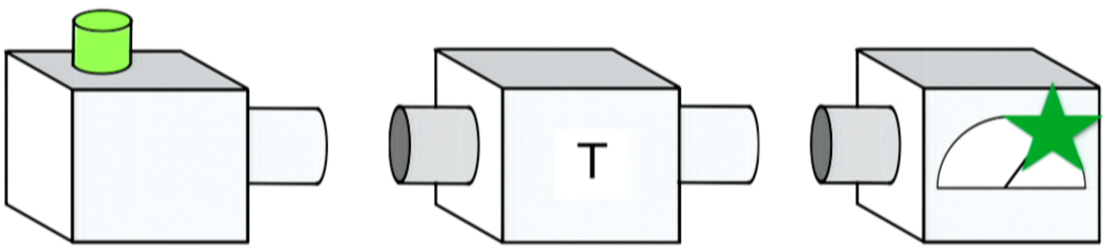
How to describe a "general probabilistic theory"

Example: classical coin toss.

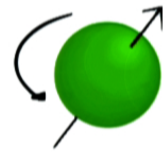
- Every measurement outcome has a probability, depending linearly on the state:

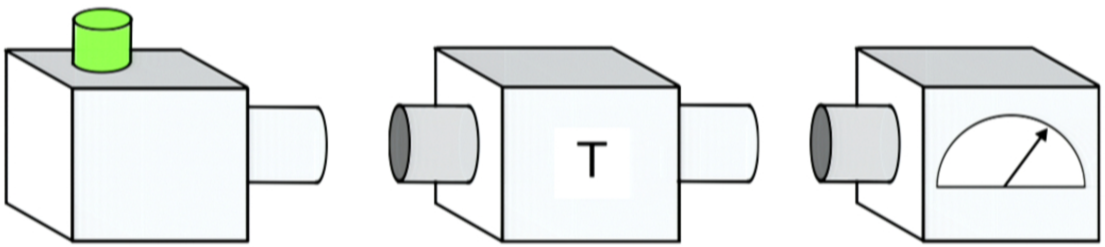
$$\mathrm{Prob}(\mathrm{heads}|\omega) = p = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} p \\ 1-p \end{pmatrix} = e \cdot \omega.$$





How to describe a "general probabilistic theory"

Example: quantum spin-1/2 particle. 



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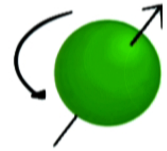
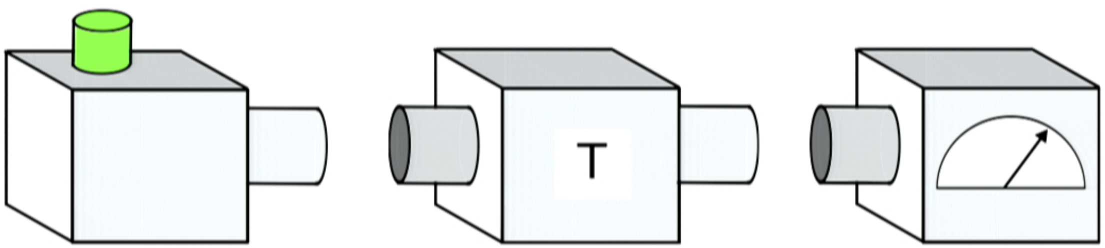


- The preparation device prepares a spin-1/2 particle in quantum state ω.

$$\alpha|\uparrow\rangle + \beta|\downarrow\rangle$$

More generally: ω is 2x2 density matrix.



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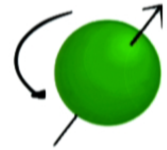

- The preparation device prepares a spin-1/2 particle in quantum state ω.

$$\cos\frac{\theta}{2}|\uparrow\rangle + e^{i\phi}\sin\frac{\theta}{2}|\downarrow\rangle$$

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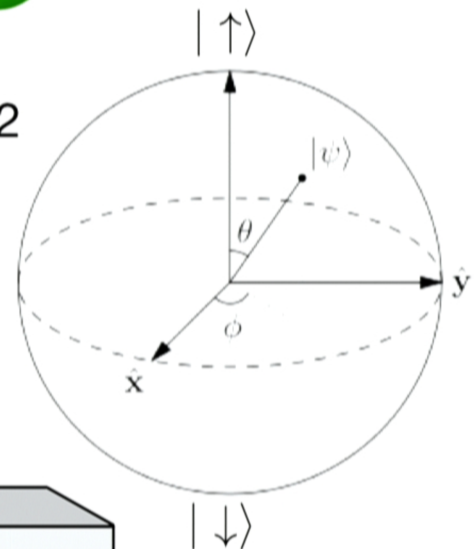

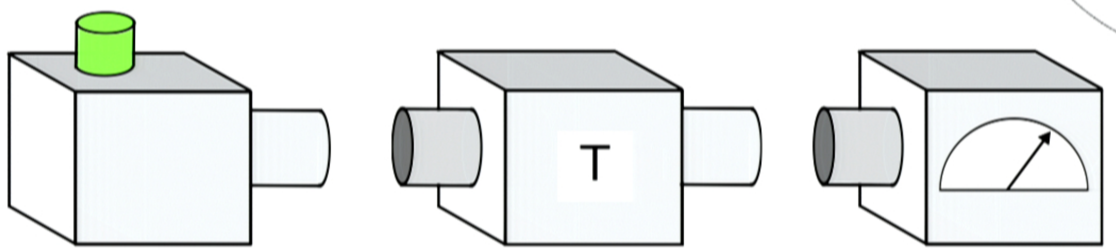

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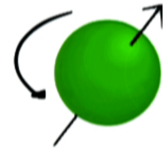
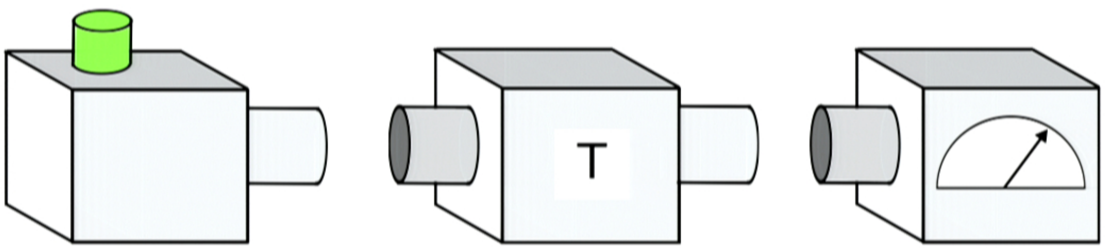


- Unitary transformation of the density matrix:

$$\omega \mapsto U\omega U^\dagger.$$



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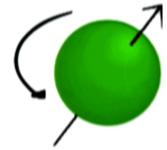
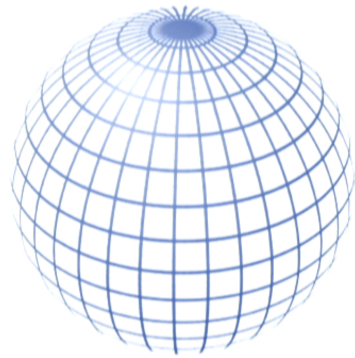


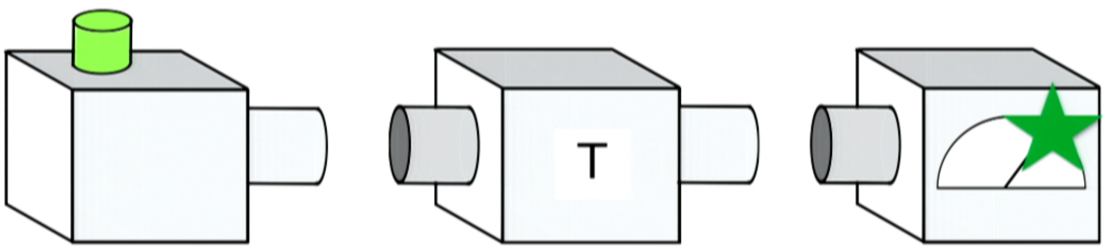


- Unitary transformation of the density matrix:

$$\omega \mapsto U\omega U^\dagger.$$

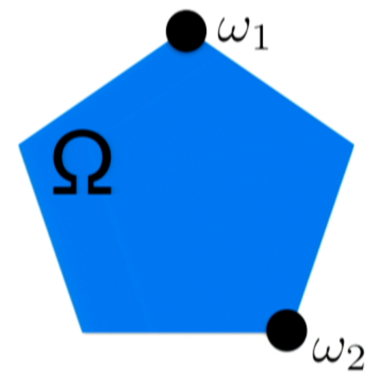
- Measurement in arbitrary spin direction *d*:

$$\mathrm{Prob}(\uparrow\ |\omega) = \mathrm{Tr}(P_d\,\omega)$$



How to describe a "general probabilistic theory"

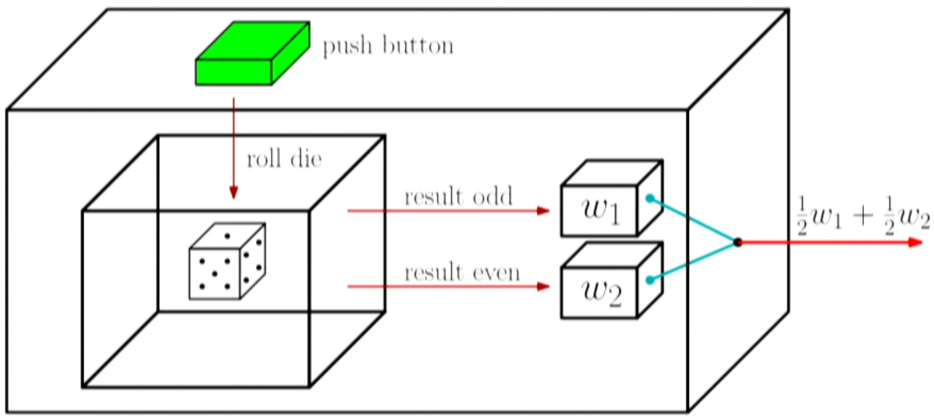
The set of all possible states of a given physical system is called the state space Ω.

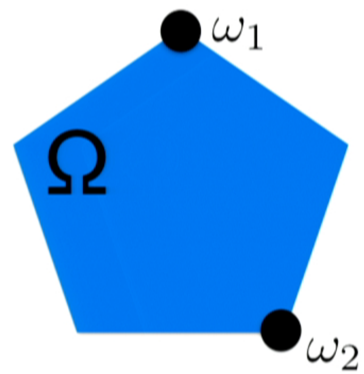


How to describe a "general probabilistic theory"

The set of all possible states of a given physical system is called the state space Ω.

Preparation of statistical mixtures: $\omega = \lambda\omega_1 + (1-\lambda)\omega_2$





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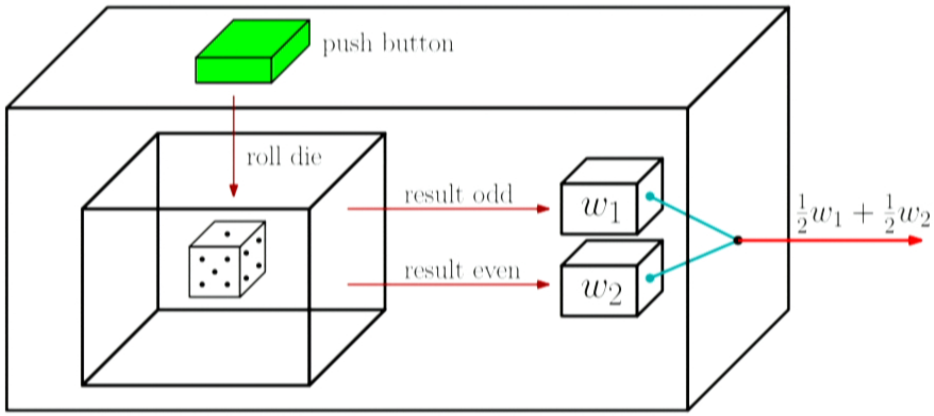


Thus Ω is a convex set.

How to describe a "general probabilistic theory"

The set of all possible states of a given physical system is called the state space Ω.

Preparation of statistical mixtures: $\omega = \lambda\omega_1 + (1-\lambda)\omega_2$

QT: Ω_N = set of N×N density matrices
CPT: Ω_N = set of prob. distributions $(p_1, \ldots, p_N)$.

Thus Ω is a convex set.

How to describe a "general probabilistic theory"

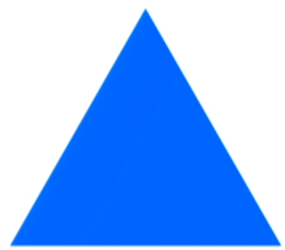
(Almost) everything can be inferred from shape of state space.

classical
bit

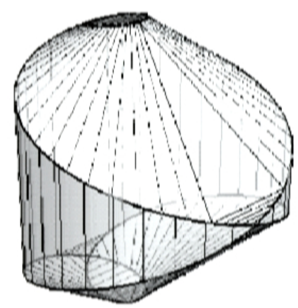
quantum
bit

"gbit"

Arbitrary convex
state space



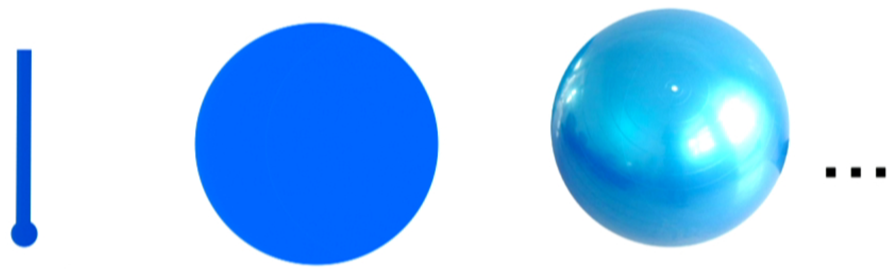
Classical trit
(3-level-system)



Quantum trit:
8D "orbitope"

1. General probabilistic theories

d-dimensional Bloch ball state spaces



$d = 1$ classical bit

$d = 2$ quantum bit over $\mathbb{R}$

$d = 3$ quantum bit over $\mathbb{C}$

$d = 5:$ quantum bit over $\mathbb{H}$

$d = 9:$ quantum bit over $\mathbb{O}$

other *d*: state spaces of Euclidean Jordan algebras

Relativity of simultaneity on an interferometer

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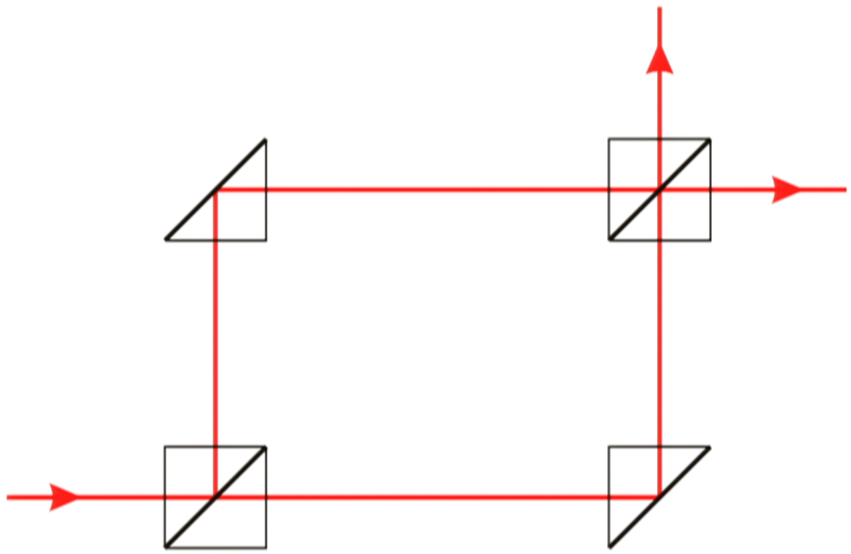
A. Garner, **MM**, O. Dahlsten, arXiv:1412.7112



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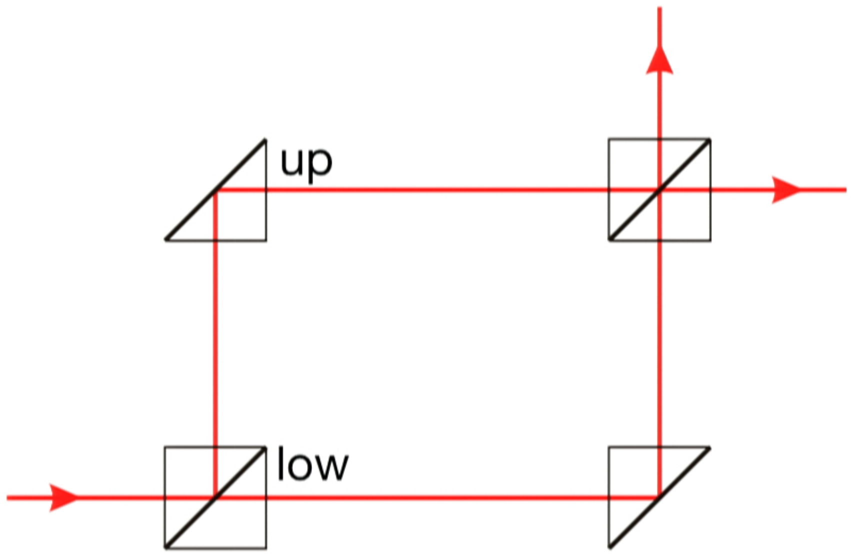

2. Relativity of simultaneity on an interferometer

Does relativistic causality constrain interference phenomena?

Markus P. Müller

Western

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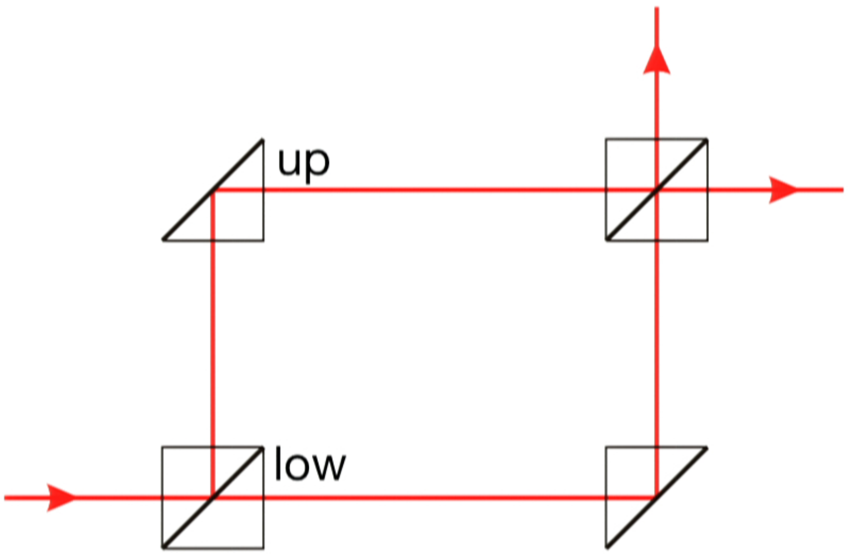

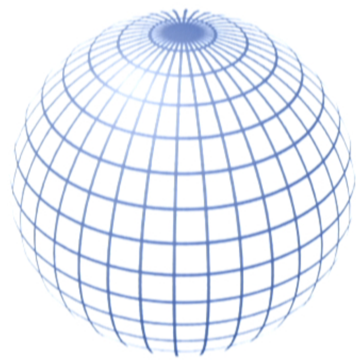

d-dim. "Bloch sphere"

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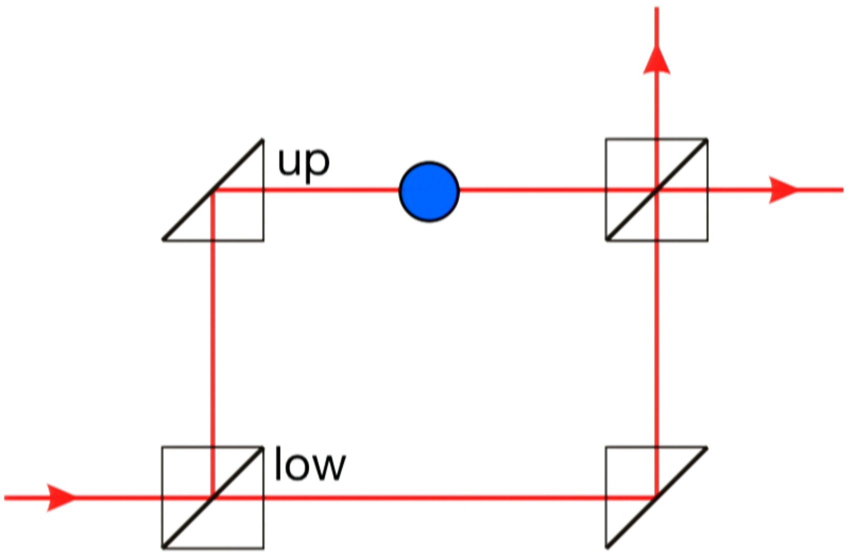

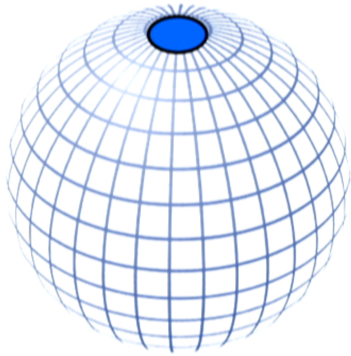

d-dim. "Bloch sphere"

North-pole state: particle definitely in upper branch.

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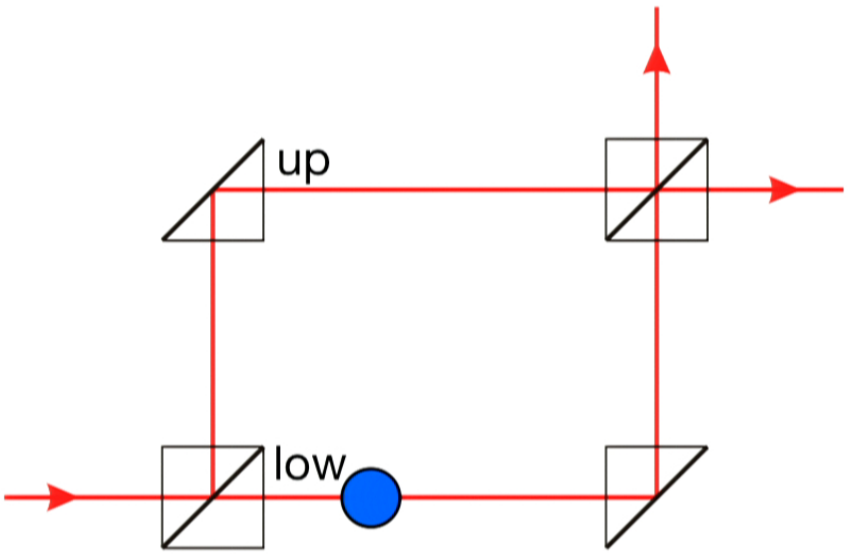

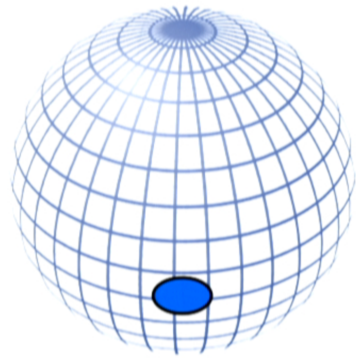

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South-pole state: particle definitely in lower branch.

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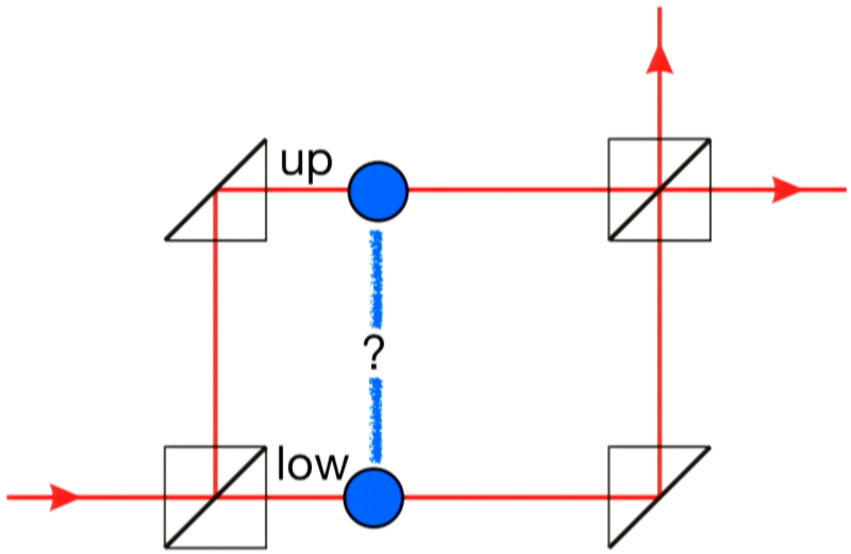

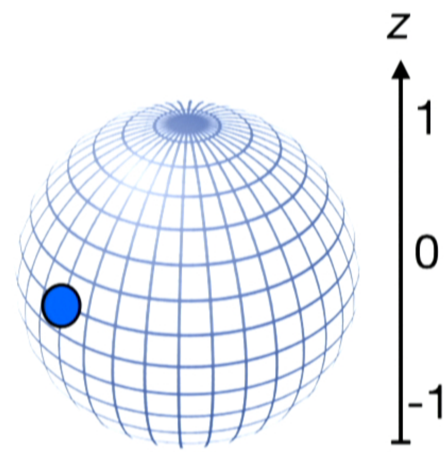

d-dim. "Bloch sphere"

State on equator $z=0$: probability 1/2 for each.

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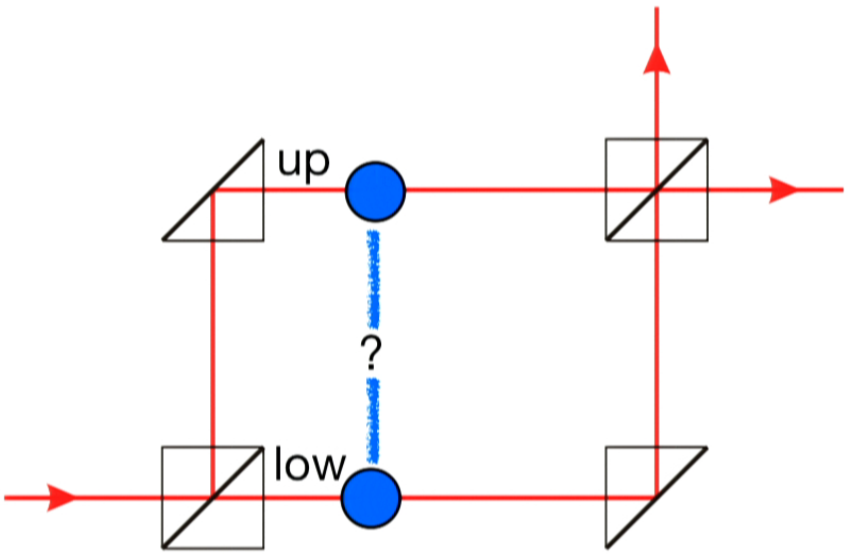

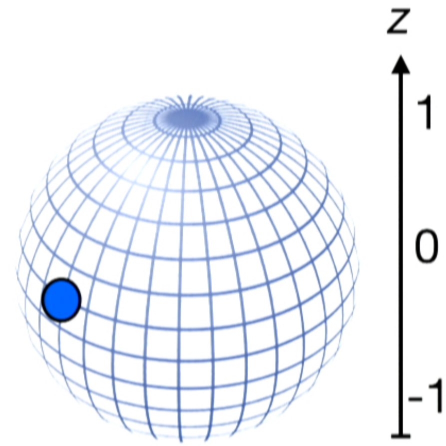

d-dim. "Bloch sphere"

State on equator *z=0*: probability 1/2 for each.

$$p(\text{up}) = \frac{1}{2}(z+1)$$

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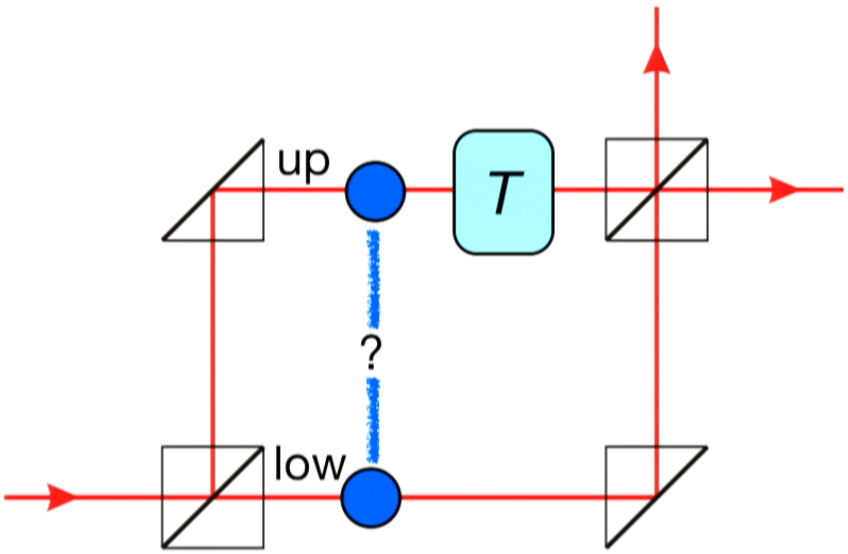

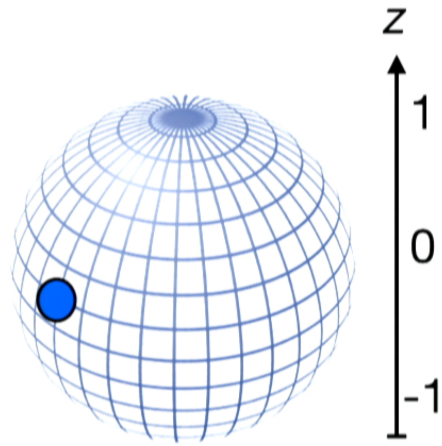

d-dim. "Bloch sphere"

What transformations *T* can we perform locally in one arm...
... without any information loss?

Relativity of simultaneity on an interferometer

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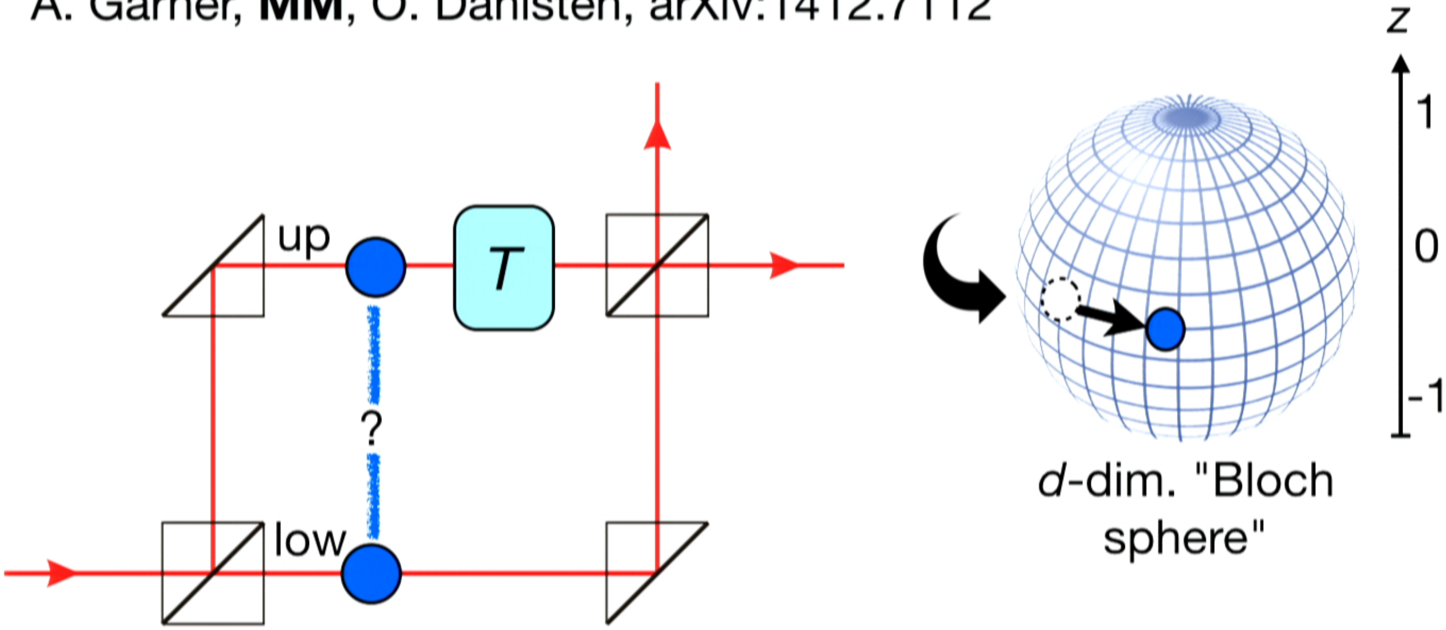

T must be a rotation of the Bloch ball (reversible+linear)...
... and must preserve *p*(up), i.e. preserve the *z*-axis.

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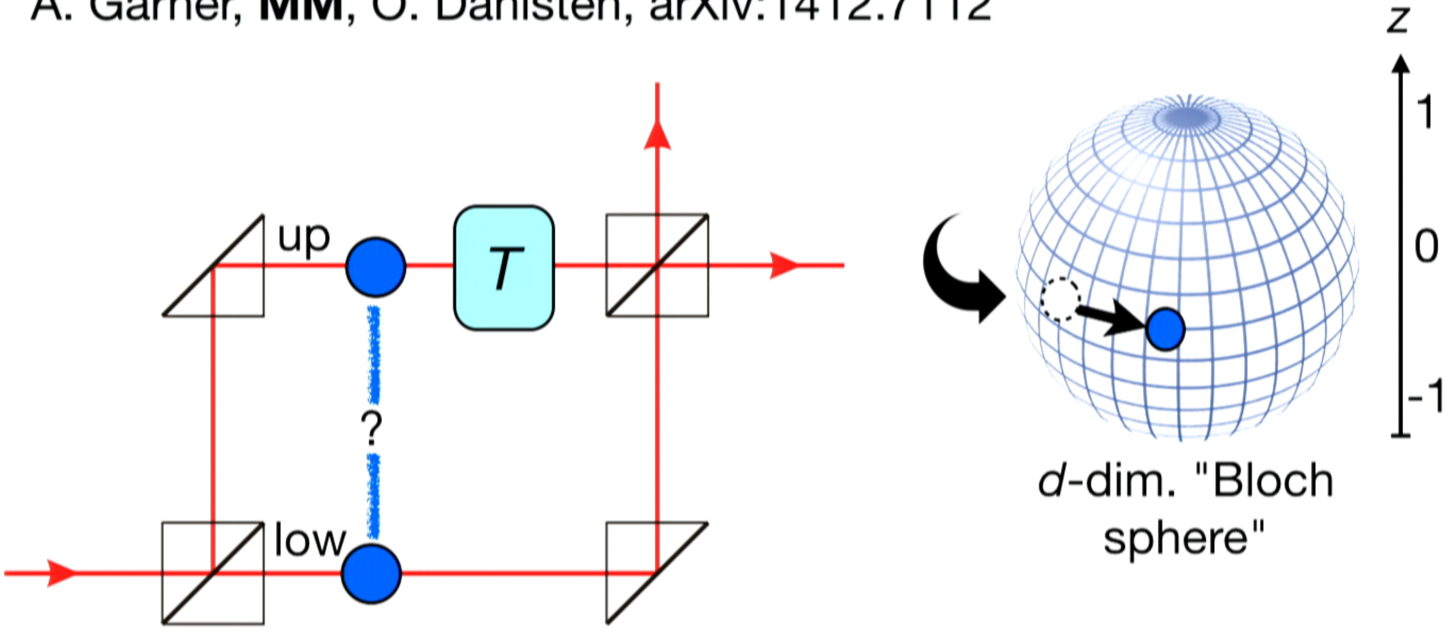

T must be a rotation of the Bloch ball (reversible+linear)...
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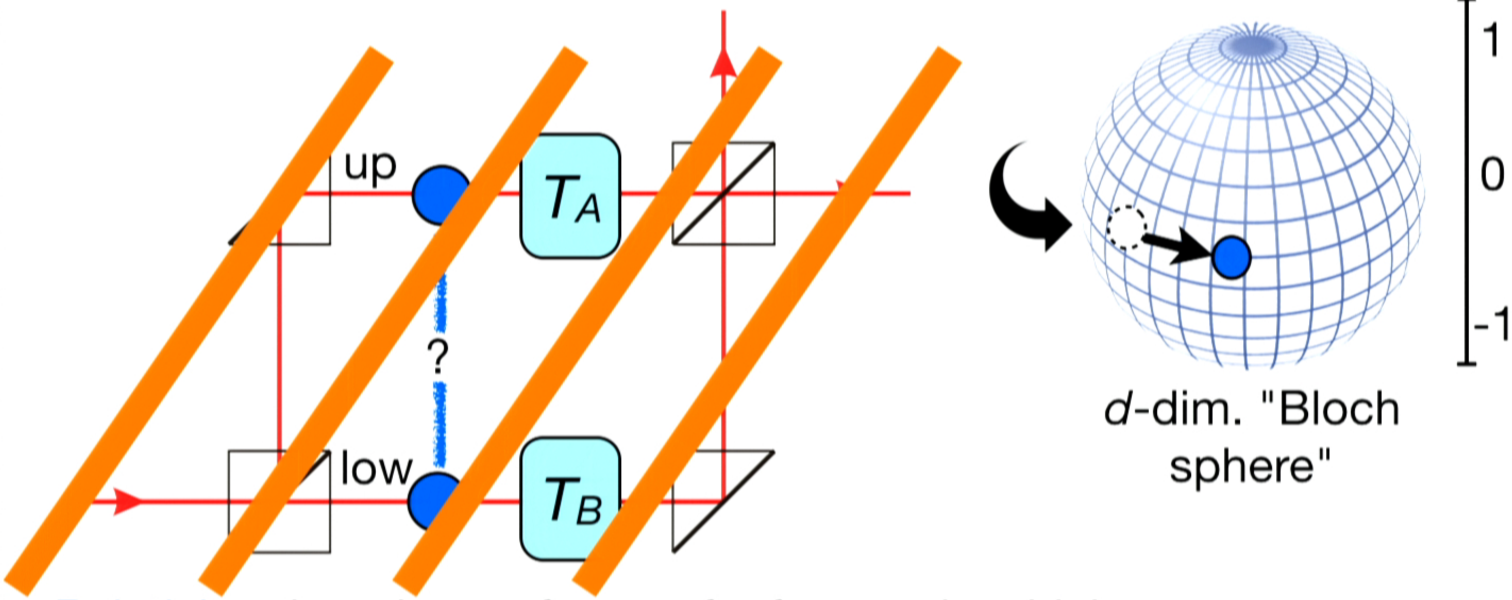
Relativity of simultaneity on an interferometer

Assumption: $\mathcal{G}_A = \mathcal{G}_B \simeq \mathrm{SO}(d-1)$.



Relativity: there is one frame of reference in which T_A happens first, and then T_B...

Relativity of simultaneity on an interferometer

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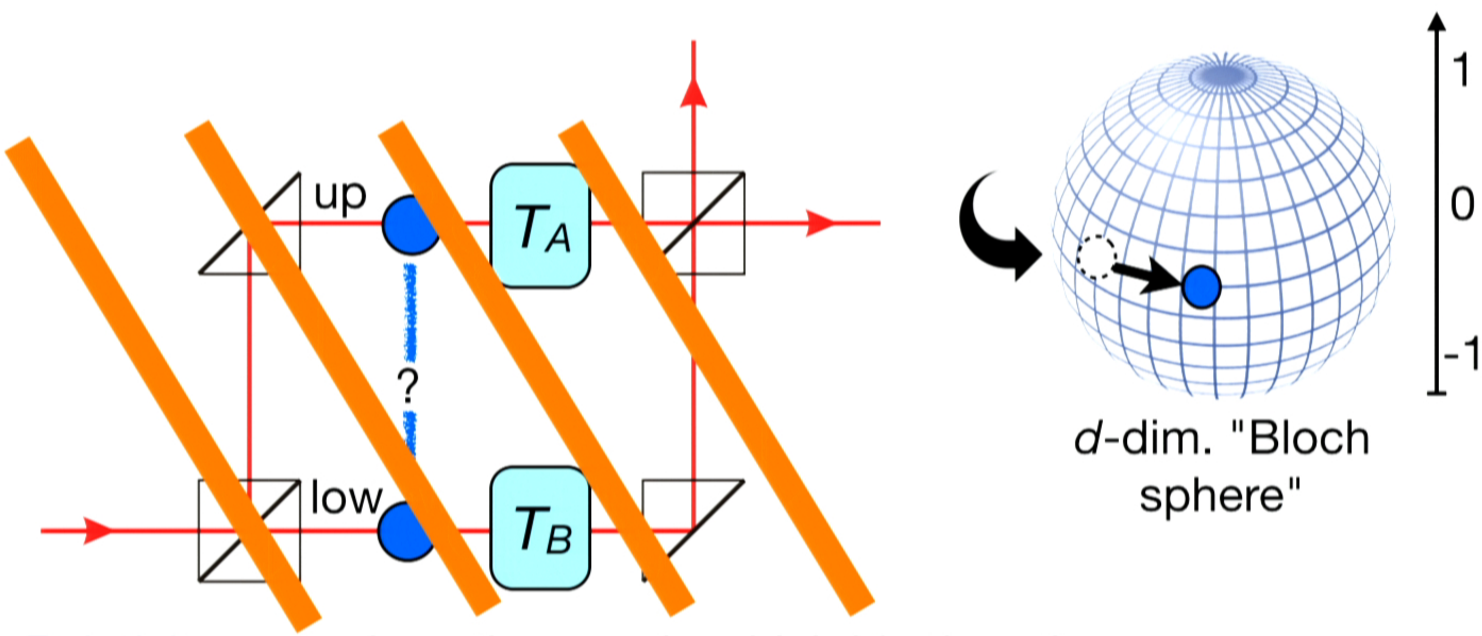
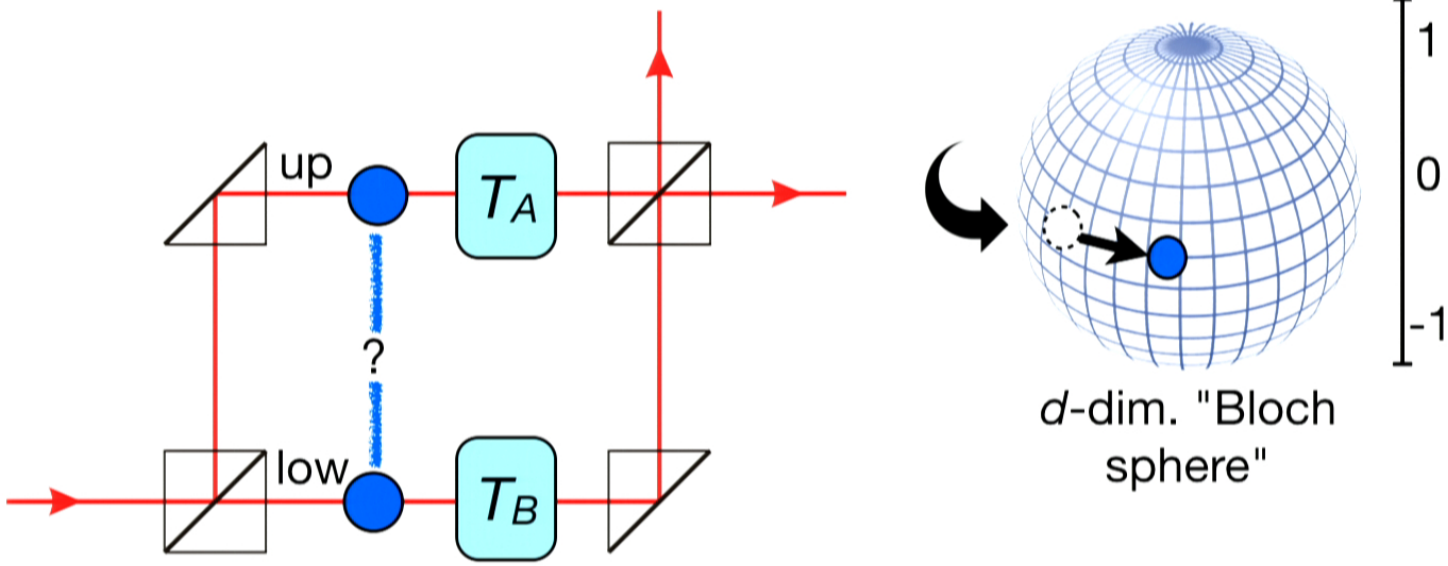


Relativity: ... and another one in which it's the other way around.

Relativity of simultaneity on an interferometer

Assumption: $\mathcal{G}_A = \mathcal{G}_B \simeq \mathrm{SO}(d-1)$.

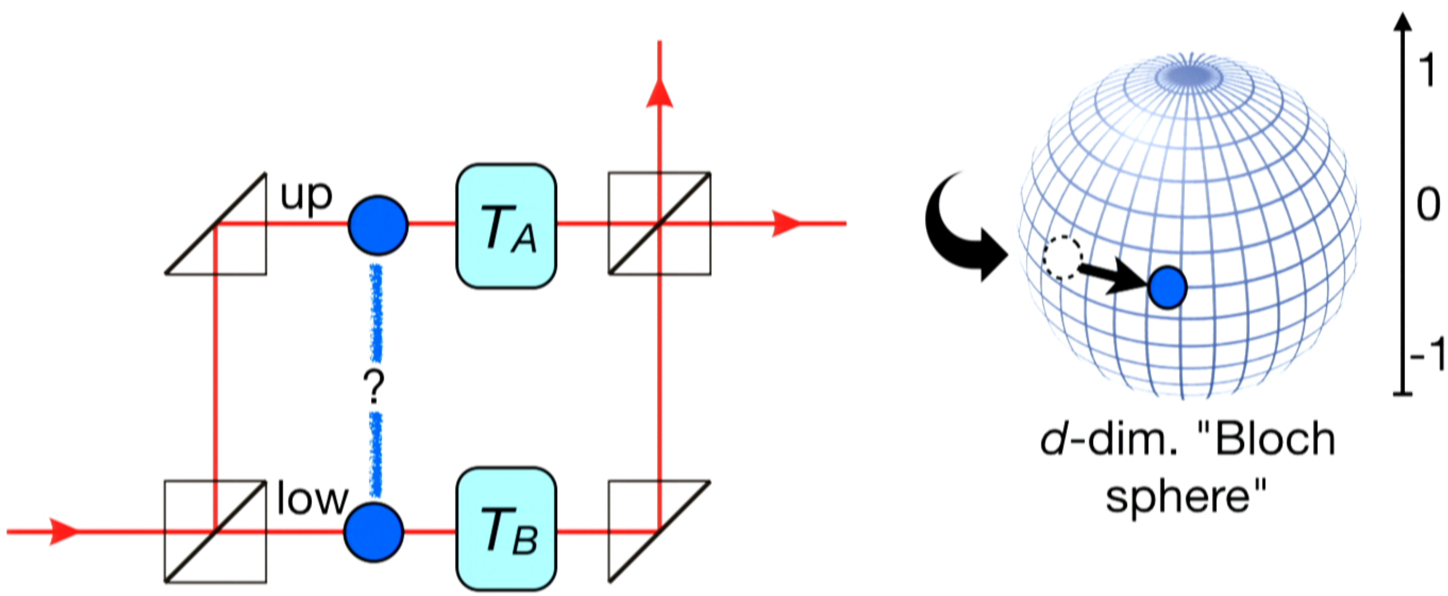


Detector click statistics is Lorentz-invariant

$\Rightarrow T_A T_B = T_B T_A$ for all $T_A, T_B \in \mathrm{SO}(d-1)$.

Relativity of simultaneity on an interferometer

Assumption: $\mathcal{G}_A = \mathcal{G}_B \simeq \mathrm{SO}(d-1)$.



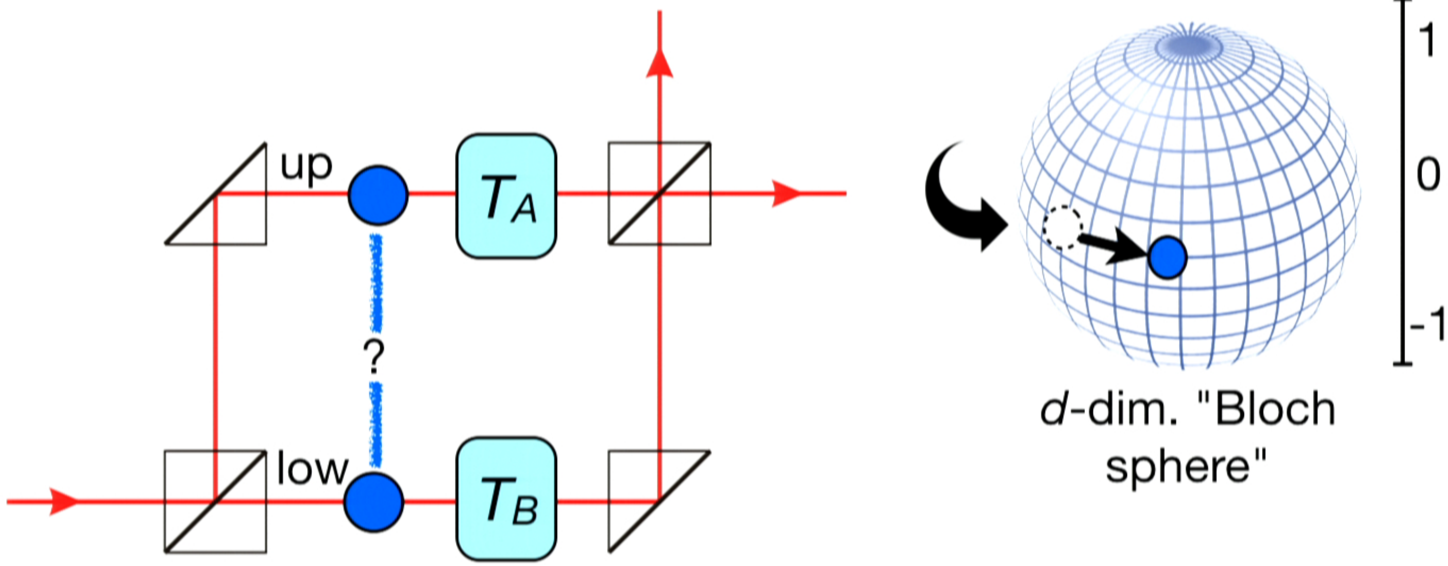
d-dim. "Bloch sphere"

Detector click statistics is Lorentz-invariant

$\Rightarrow T_A T_B = T_B T_A$ for all $T_A, T_B \in \mathrm{SO}(d-1)$.

Relativity of simultaneity on an interferometer

$\Rightarrow \quad d \leq 3$ (In fact, *d*=*3*, otherwise these transformations are all trivial.)



Detector click statistics is Lorentz-invariant

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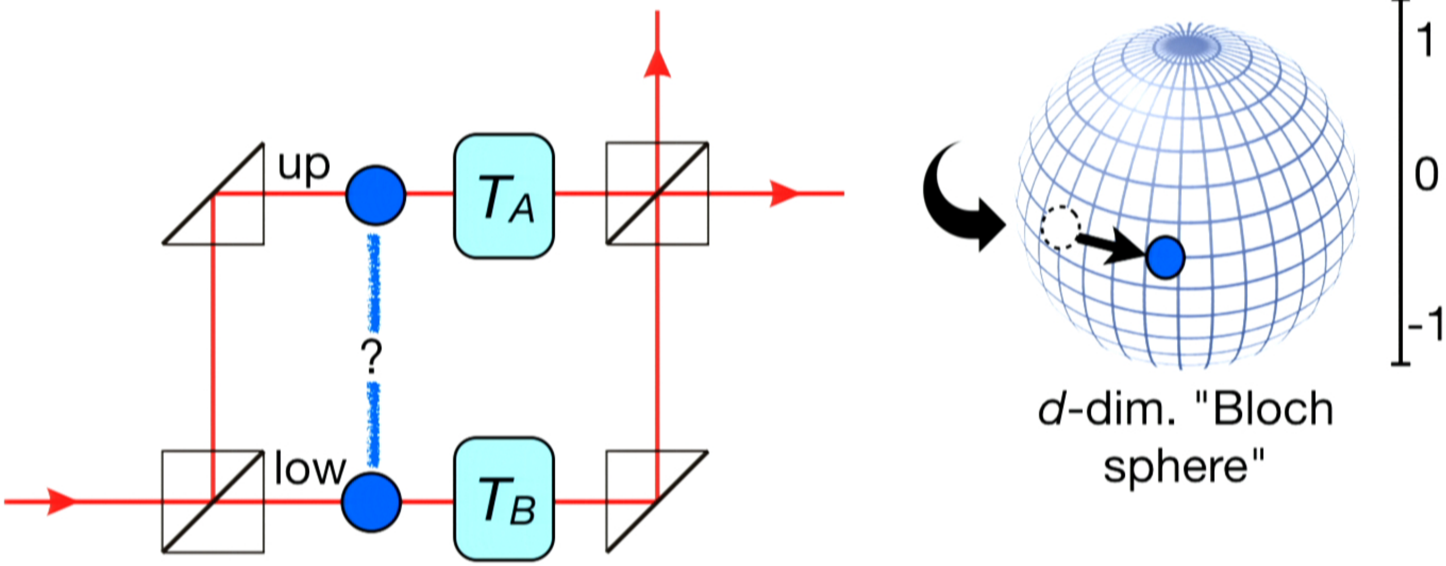
Relativity of simultaneity on an interferometer

Weaker assumption: $\mathcal{G}_A$ and $\mathcal{G}_B$ isomorphic



$\Rightarrow \quad d \leq 5.$ Quaternionic QM survives.

Classification of possibilities

A. Garner, **MM**, O. Dahlsten, arXiv:1412.7112

Theorem 2. *Suppose that (i) $\mathcal{G}_A$ and $\mathcal{G}_B$ are isomorphic; (ii) they generate the full phase group; (iii) every pure state can be mapped to every other by a reversible transformation. Then relativity of simultaneity allows for the following possibilities and no more:*

- $d = 2$ *(the quantum bit over the real numbers), with* $\mathcal{G} = \mathrm{O}(2)$ *and* $\mathcal{G}_A = \mathcal{G}_B = \mathbb{Z}_2$;
- $d = 3$ *(the standard complex quantum bit), with* $\mathcal{G} = \mathrm{SO}(3)$ *and* $\mathcal{G}_A = \mathcal{G}_B = \mathrm{SO}(2) = \mathrm{U}(1)$;
- $d = 4$, *with* $\mathcal{G} \simeq \mathrm{U}(2)$ *and* $\mathcal{G}_A = \mathcal{G}_B = \mathrm{SO}(2) = \mathrm{U}(1)$,
- $d = 5$ *(the quaternionic quantum bit), with* $\mathcal{G} = \mathrm{SO}(5)$, $\mathcal{G}_A$ *the left- and* $\mathcal{G}_B$ *the right-isoclinic rotations in* $\mathrm{SO}(4)$ *(or vice versa), such that both are isomorphic to* $\mathrm{SU}(2)$ *and* $\mathcal{G}_A \cap \mathcal{G}_B = \{+\mathbb{1}, -\mathbb{1}\}$.

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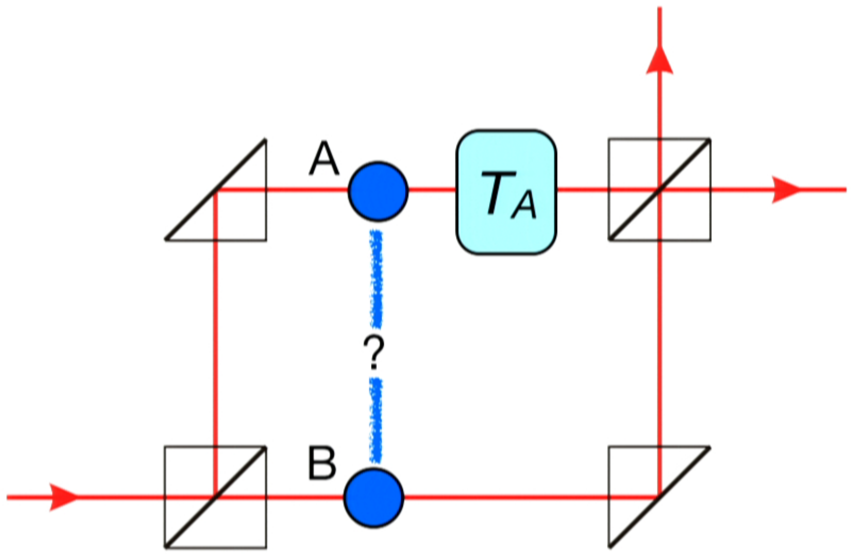
$\mathcal{G}_A = \mathcal{G}_B$

$\mathcal{G}_A \simeq \mathcal{G}_B$

2. Relativity of simultaneity on an interferometer

Classification of possibilities

A. Garner, **MM**, O. Dahlsten, arXiv:1412.7112



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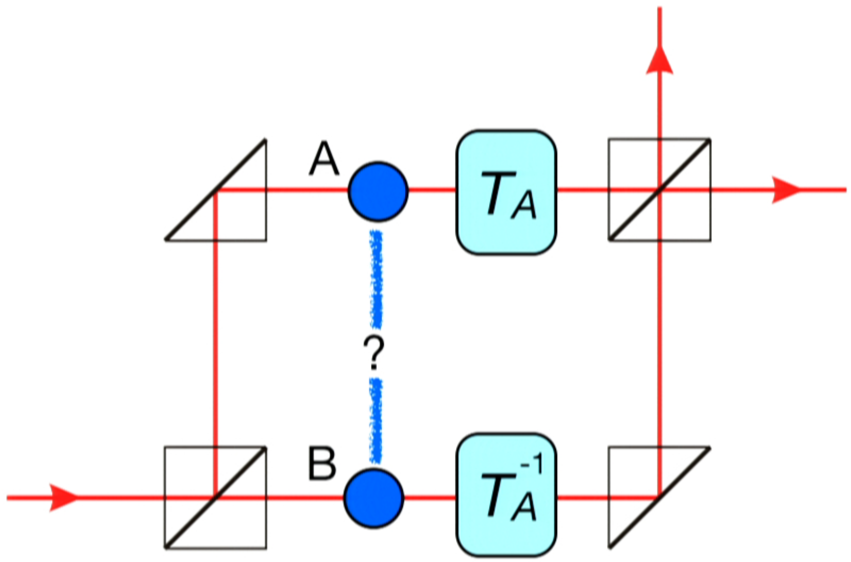

complex QT

$\mathcal{G}_A = \mathcal{G}_B$

$\mathcal{G}_A \simeq \mathcal{G}_B$

Classification of possibilities

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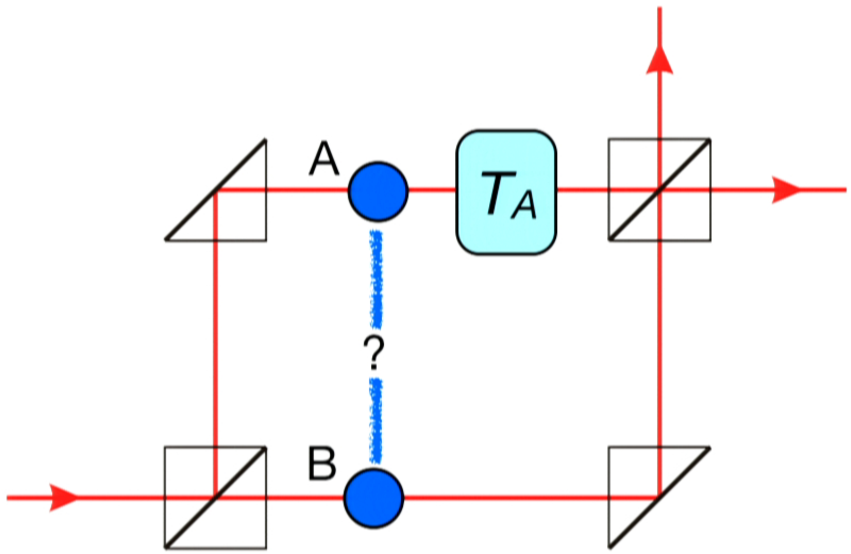

$\mathcal{G}_A = \mathcal{G}_B$

quaternionic QT

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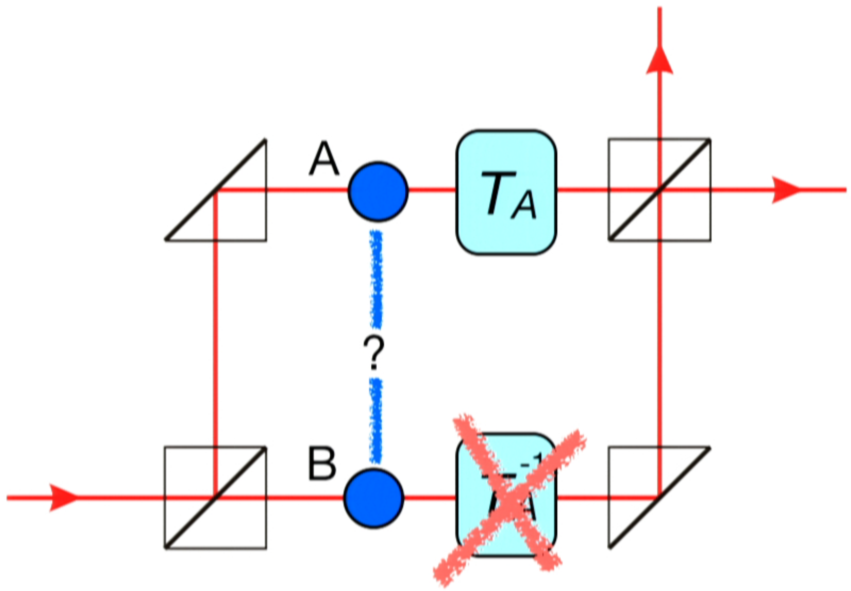

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Relativistic constraints on the state space

Consequences for actual interference experiments:

PHYSICAL REVIEW LETTERS

VOLUME 42 | 12 MARCH 1979 | NUMBER 11

Proposed Test for Complex versus Quaternion Quantum Theory

Asher Peres
Department of Physics, Technion–Israel Institute of Technology, Haifa, Israel
(Received 7 December 1978)

If scattering amplitudes are ordinary complex numbers (not quaternions) then there is a universal algebraic relationship between the six coherent cross sections of any three scatterers (taken singly and pairwise). A violation of this relationship would indicate either that scattering amplitudes are quaternions, or that the superposition principle fails. Some experimental tests are proposed, involving neutron diffraction by crystals made of three different isotopes, neutron interferometry, and K_S-meson regeneration.

2. Relativity of simultaneity on an interferometer

Relativistic constraints on the state space

Consequences for actual interference experiments:

- Generalized Peres Test
  - Quaternion quantum mechanics?
  - Octonion quantum mechanics?

PHYSICAL REVIEW LETTERS

VOLUME 42

Proposed Test fo

Department of Physic

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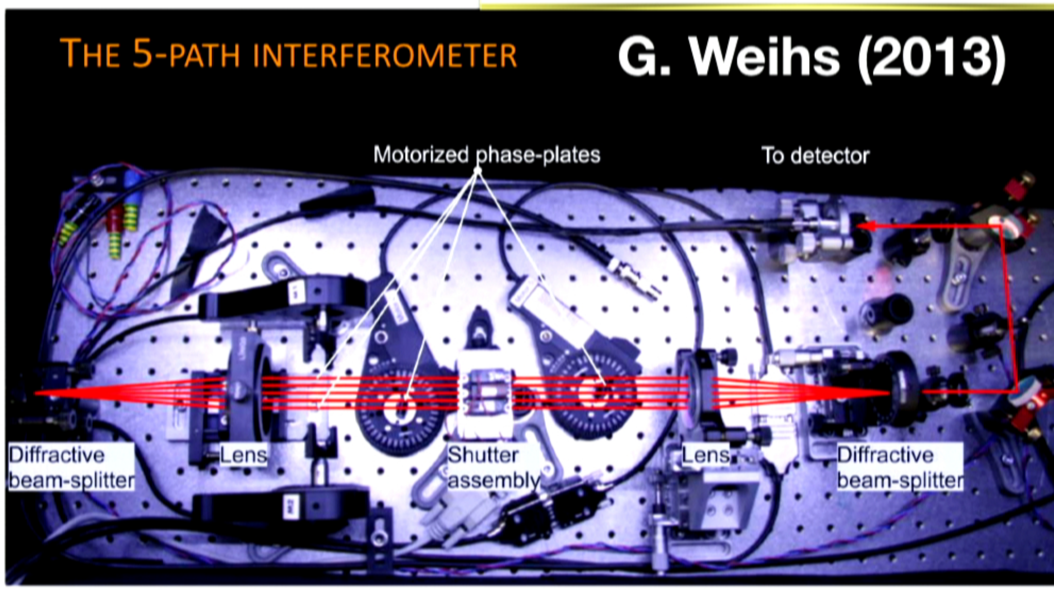


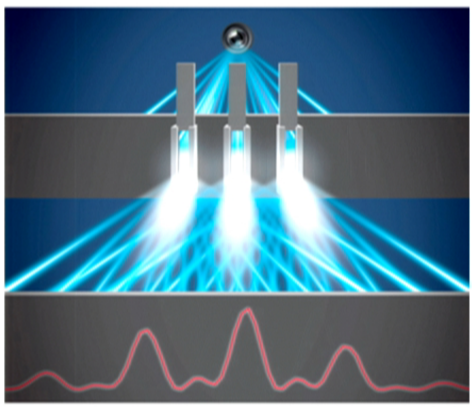
2. Relativity of simultaneity on an interferometer

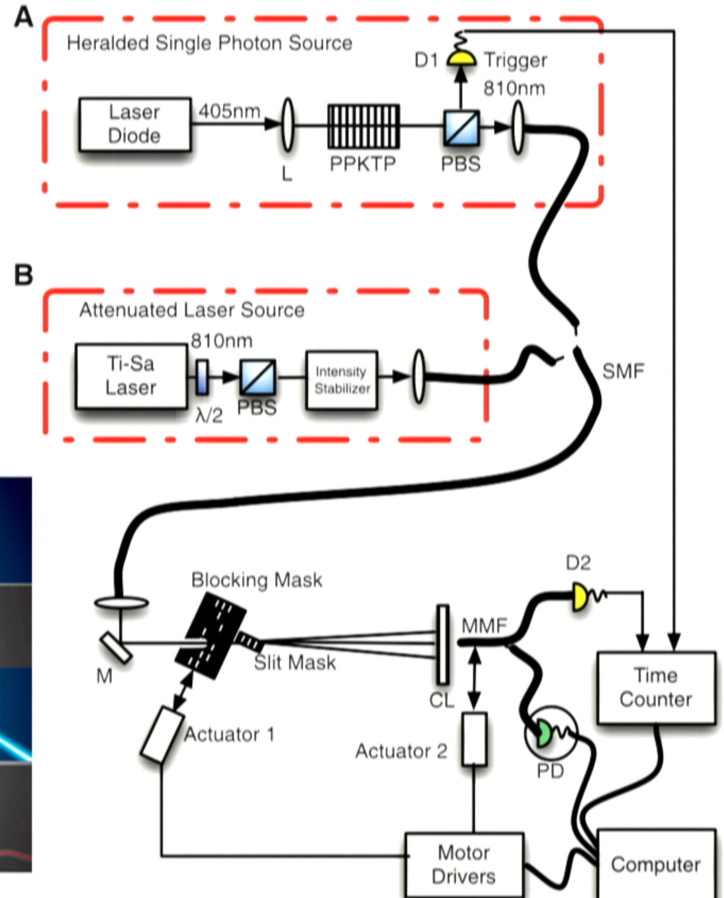
Does relativistic causality constrain interference phenomena? Markus P. Müller Western

Relativistic constraints on the state space

Science
AAAS

U. Sinha, C. Couteau, T. Jennewein, R. Laflamme, G. Weihs, *Ruling Out Multi-Order Interference in Quantum Mechanics*, Science **329**, 418 (2010).





2. Relativity of simultaneity on an interferometer

Western

A well-informed speculation

2. Relativity of simultaneity on an interferometer

Does relativistic causality constrain interference phenomena?

Markus P. Müller

Western

A well-informed speculation

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  Maybe **down to 2nd-order** interference?

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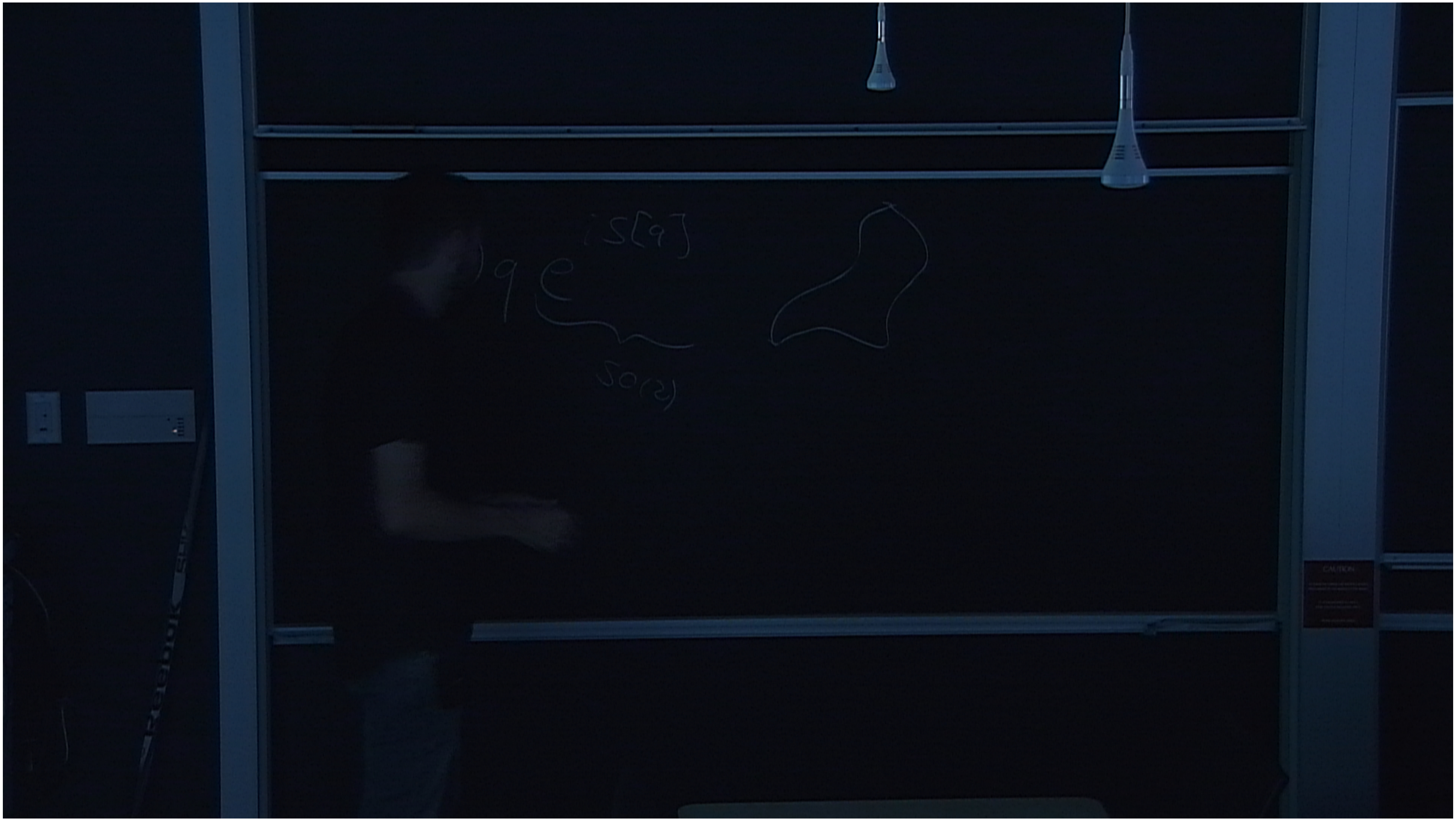
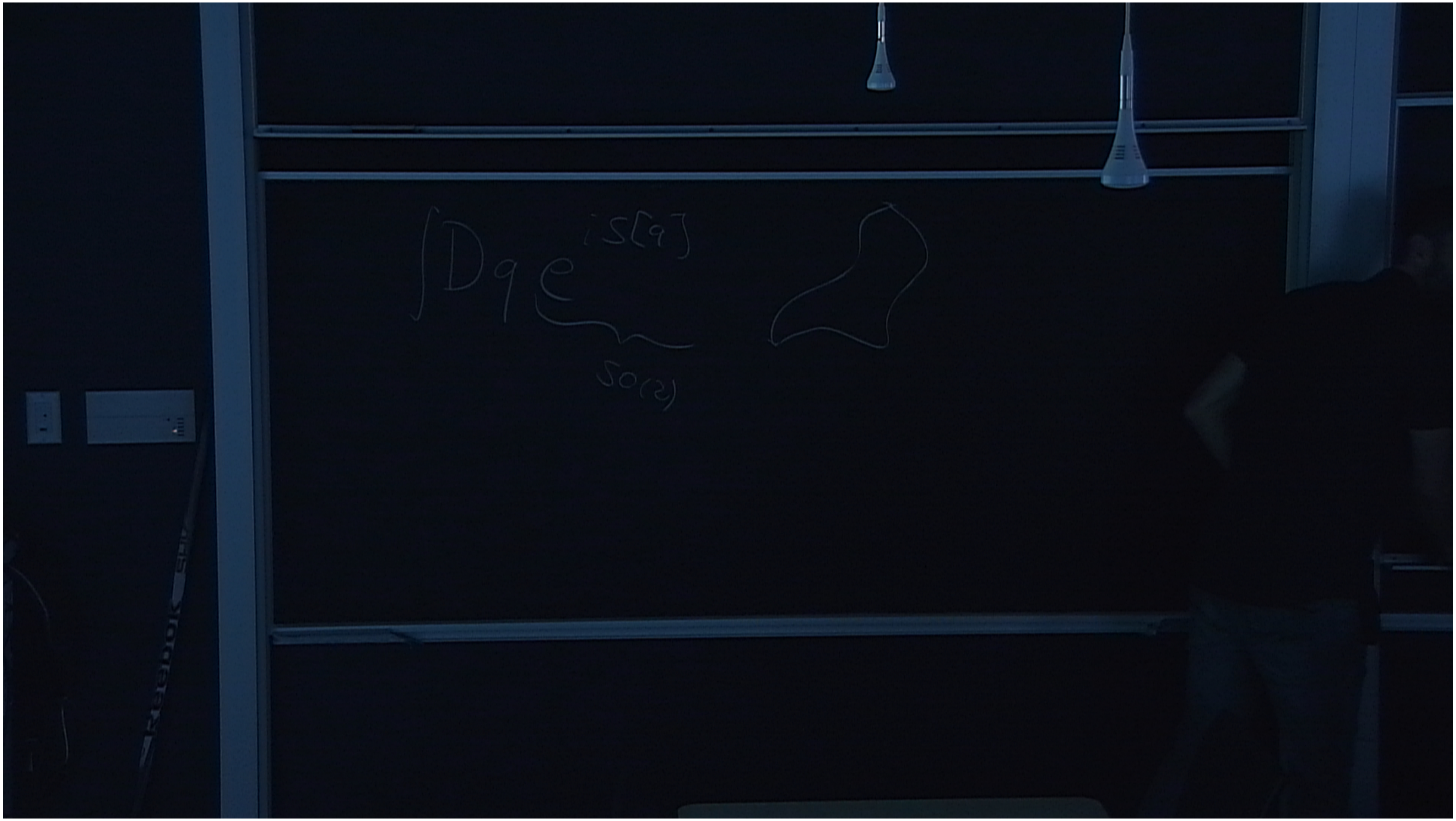
- Or, on the contrary, can relativistic causality be used to **certify "projectiveness"?**

$$[P_i, P_j] = 0$$




∫Dq e^iS[q]
So(2)

A well-informed speculation

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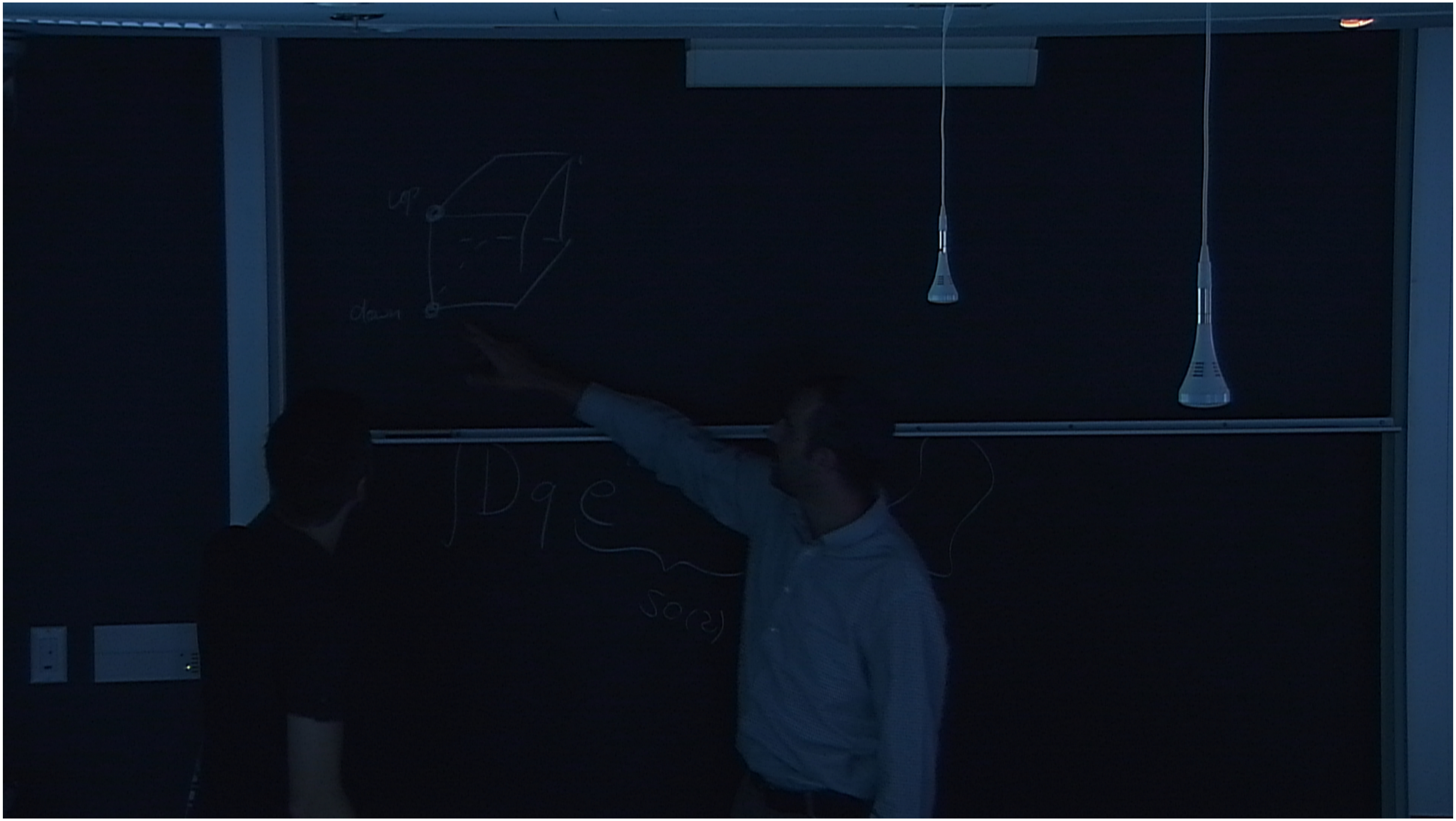
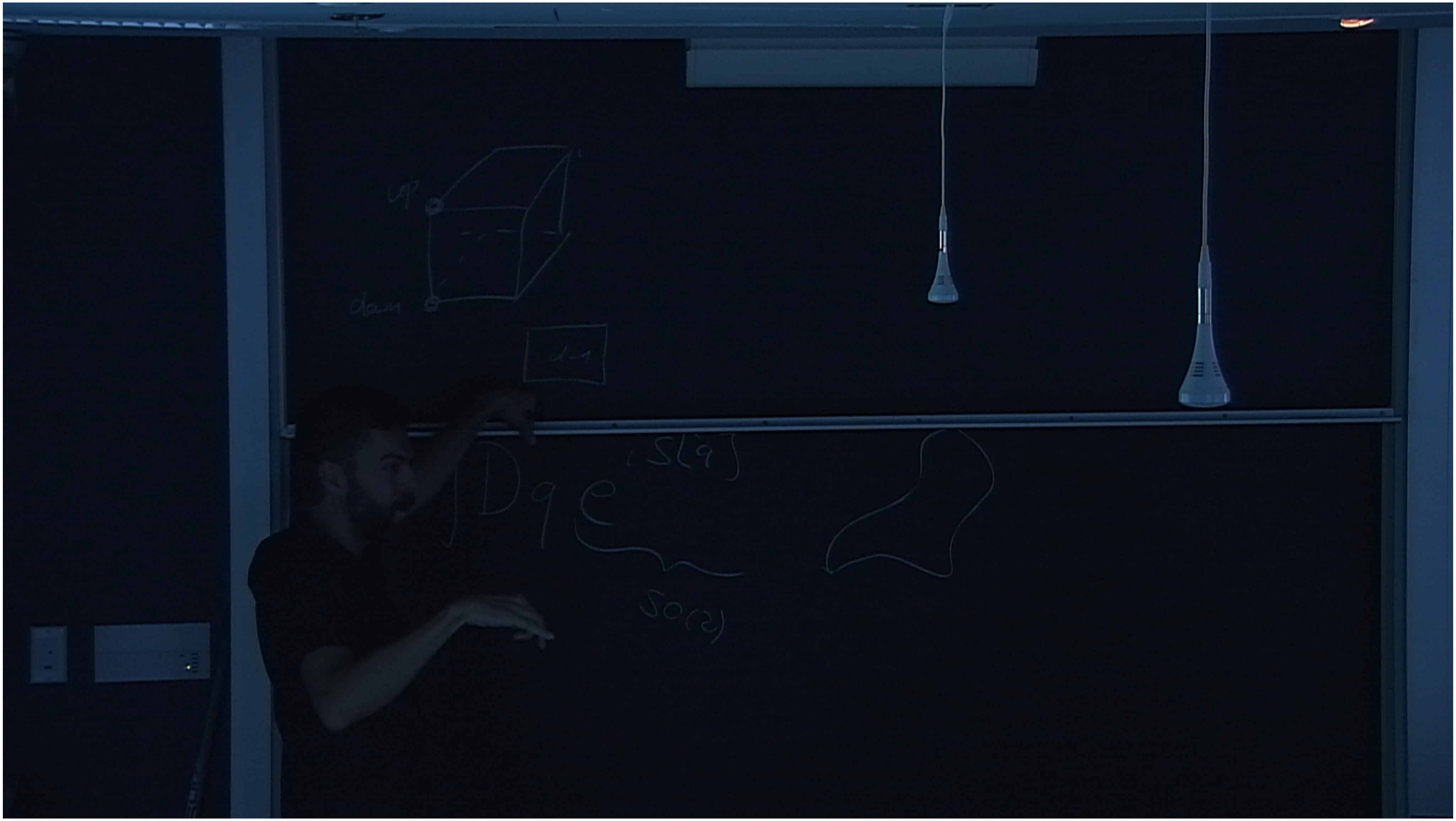
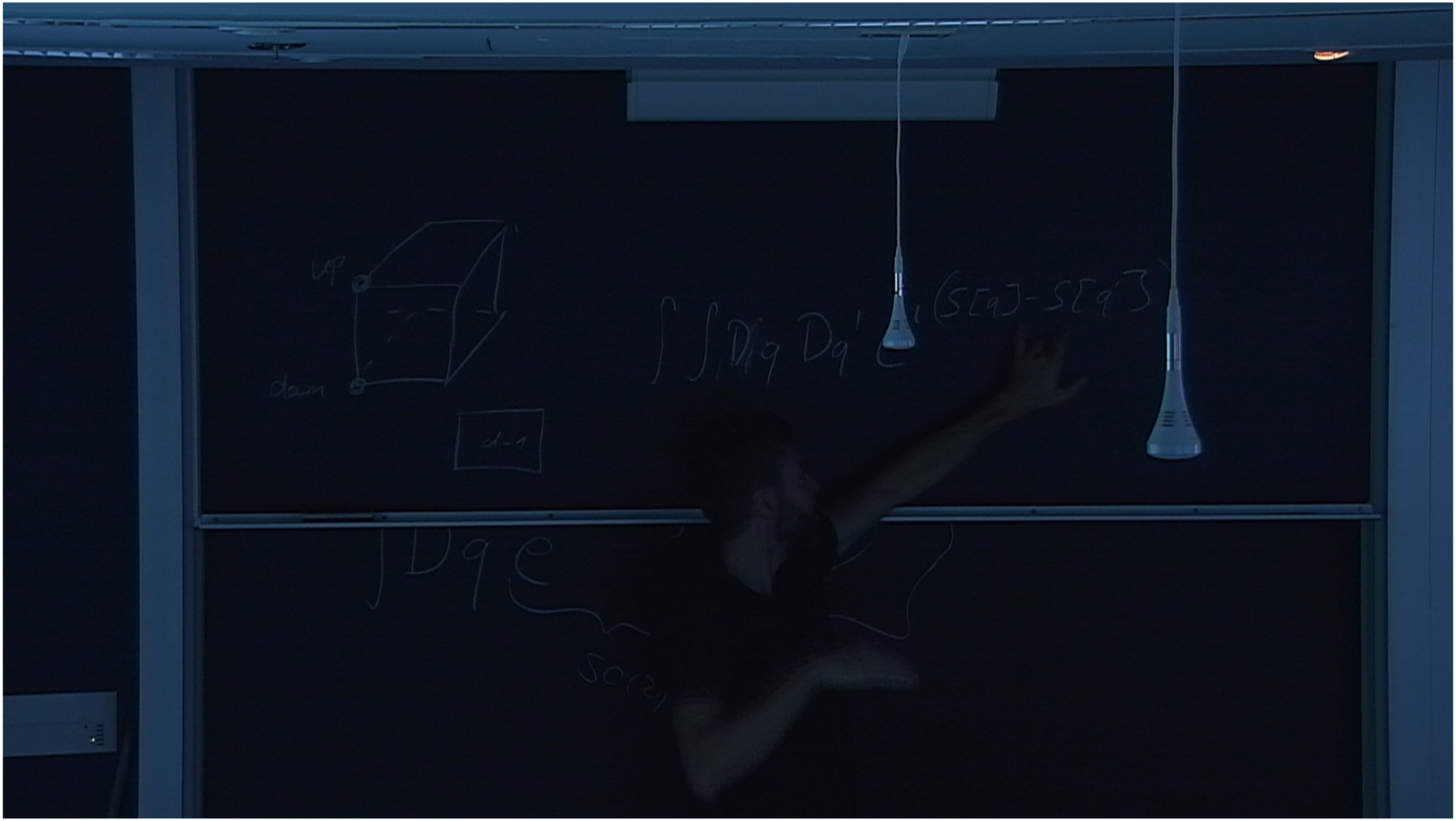

up
down
SO(2)




up
down
d-1
SO(2)