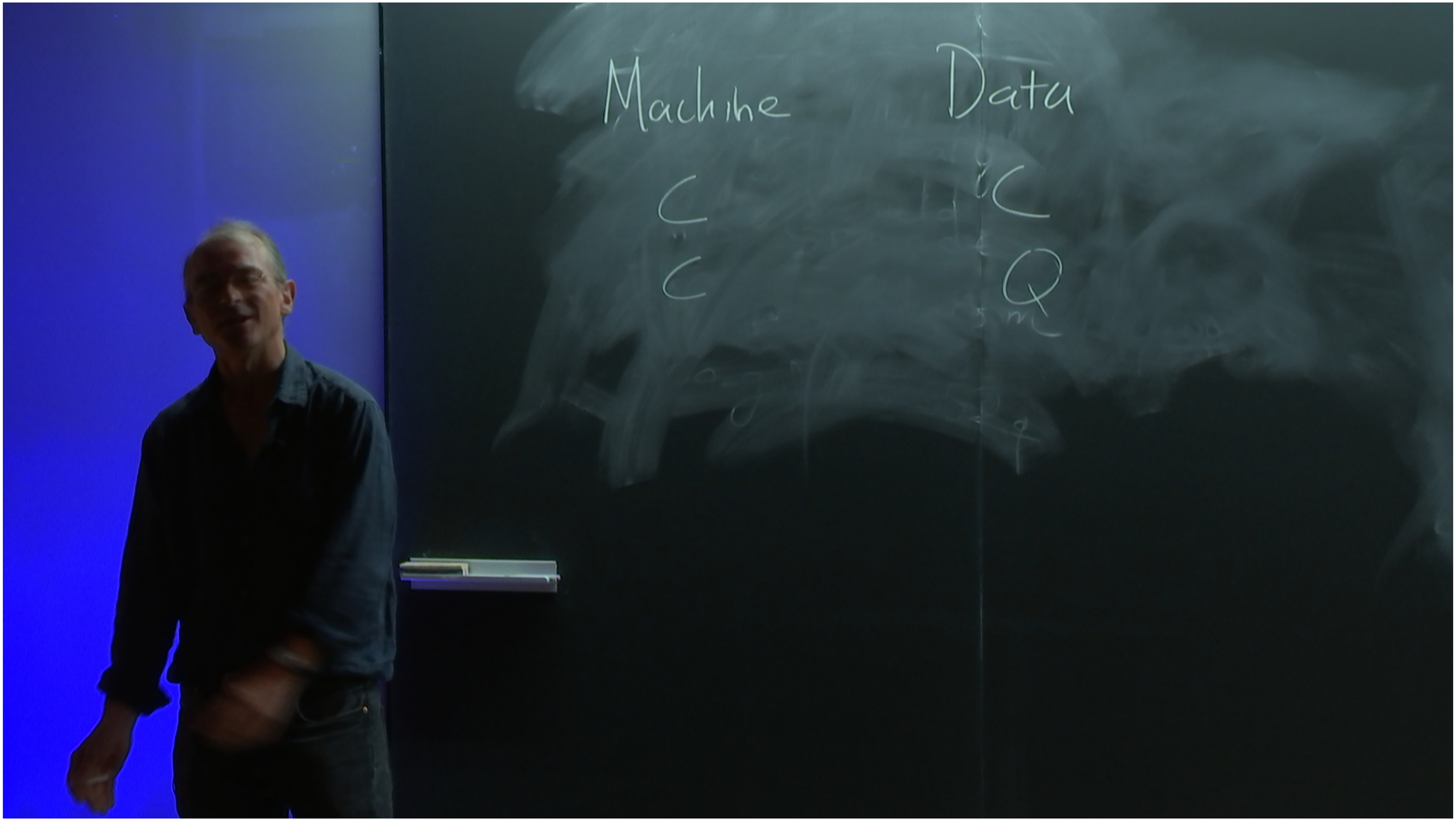
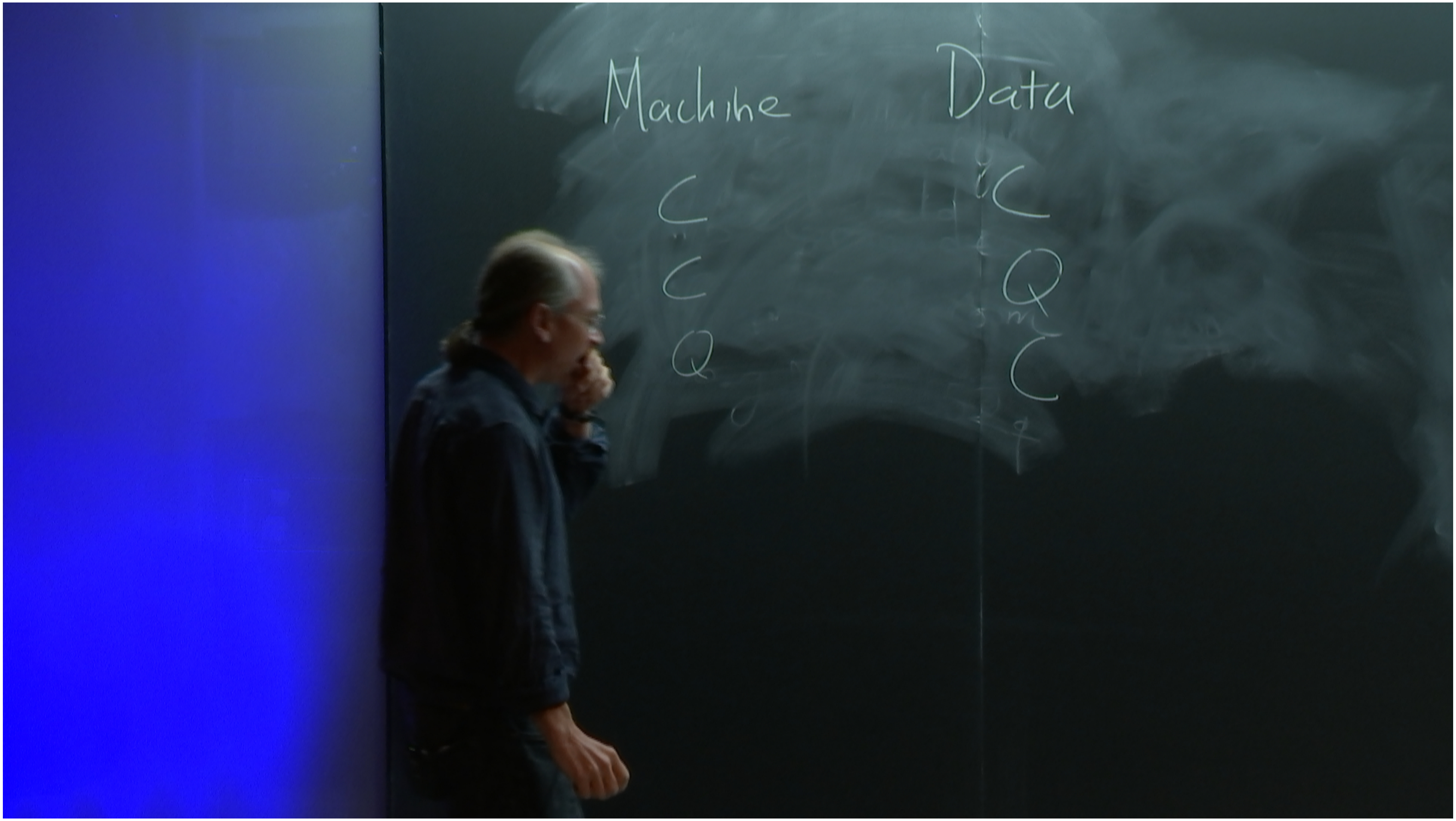
Title: Quantum algorithm for topological analysis of data

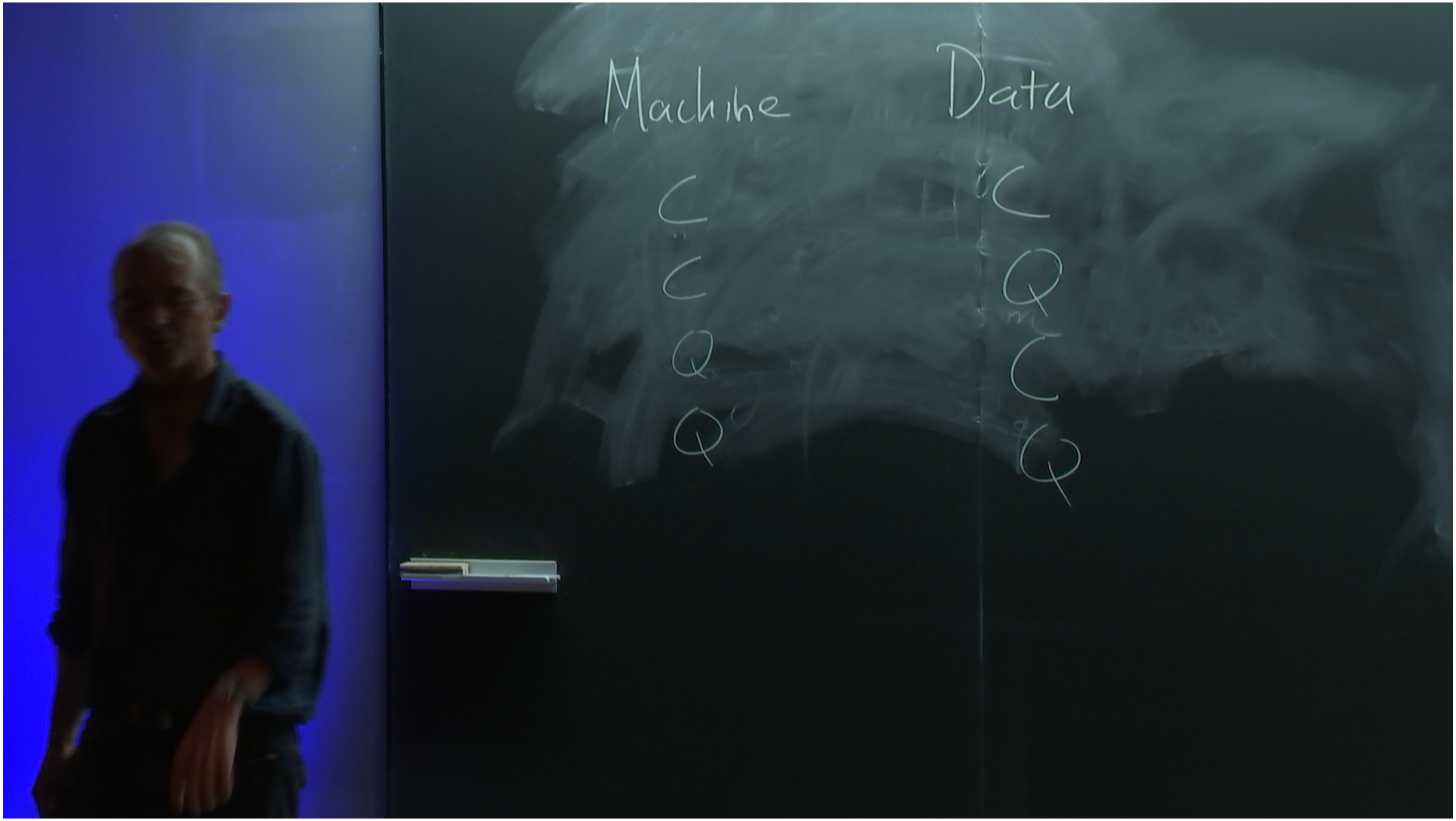
Date: Aug 12, 2016 11:00 AM

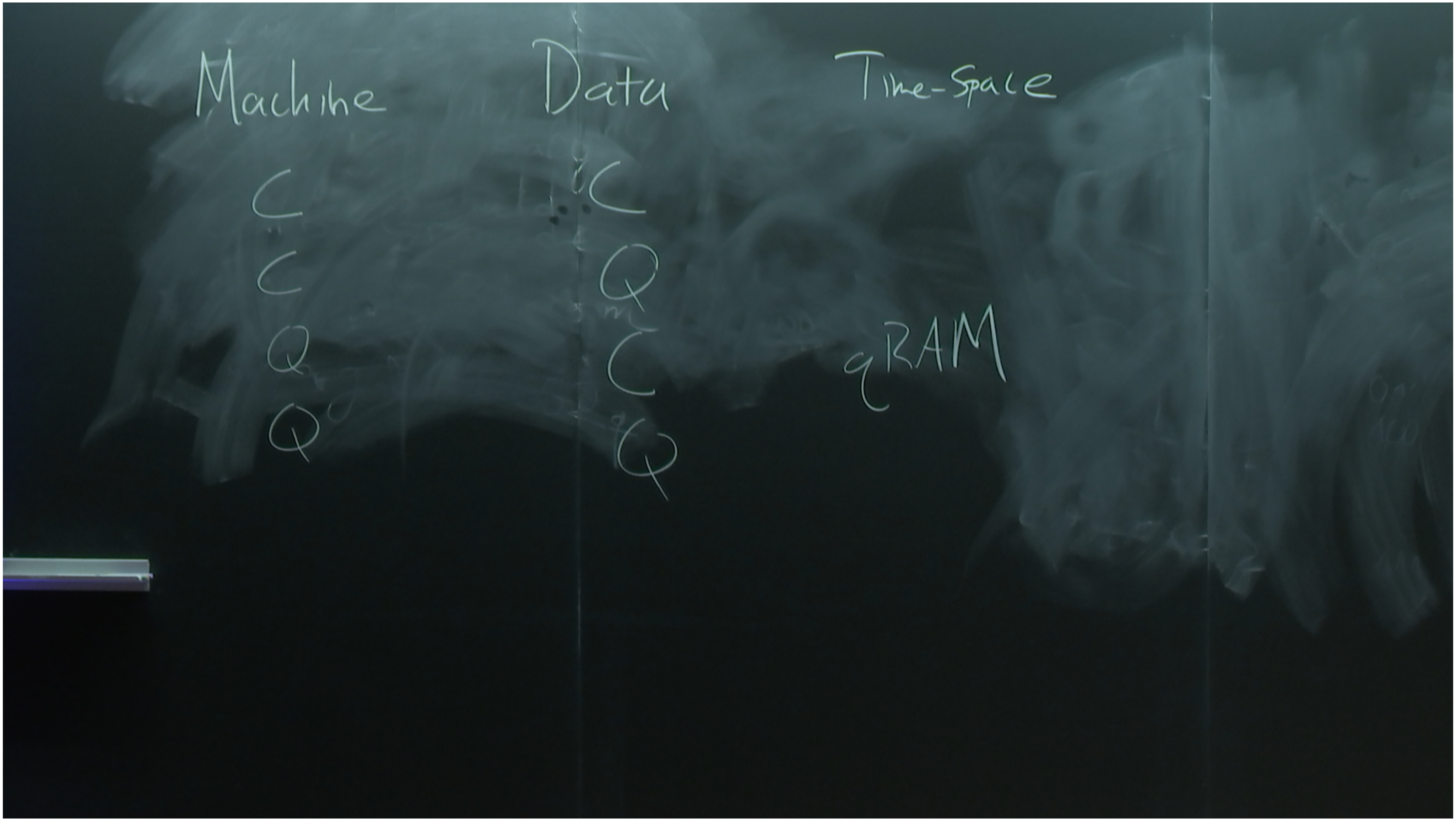
URL: http://pirsa.org/16080020

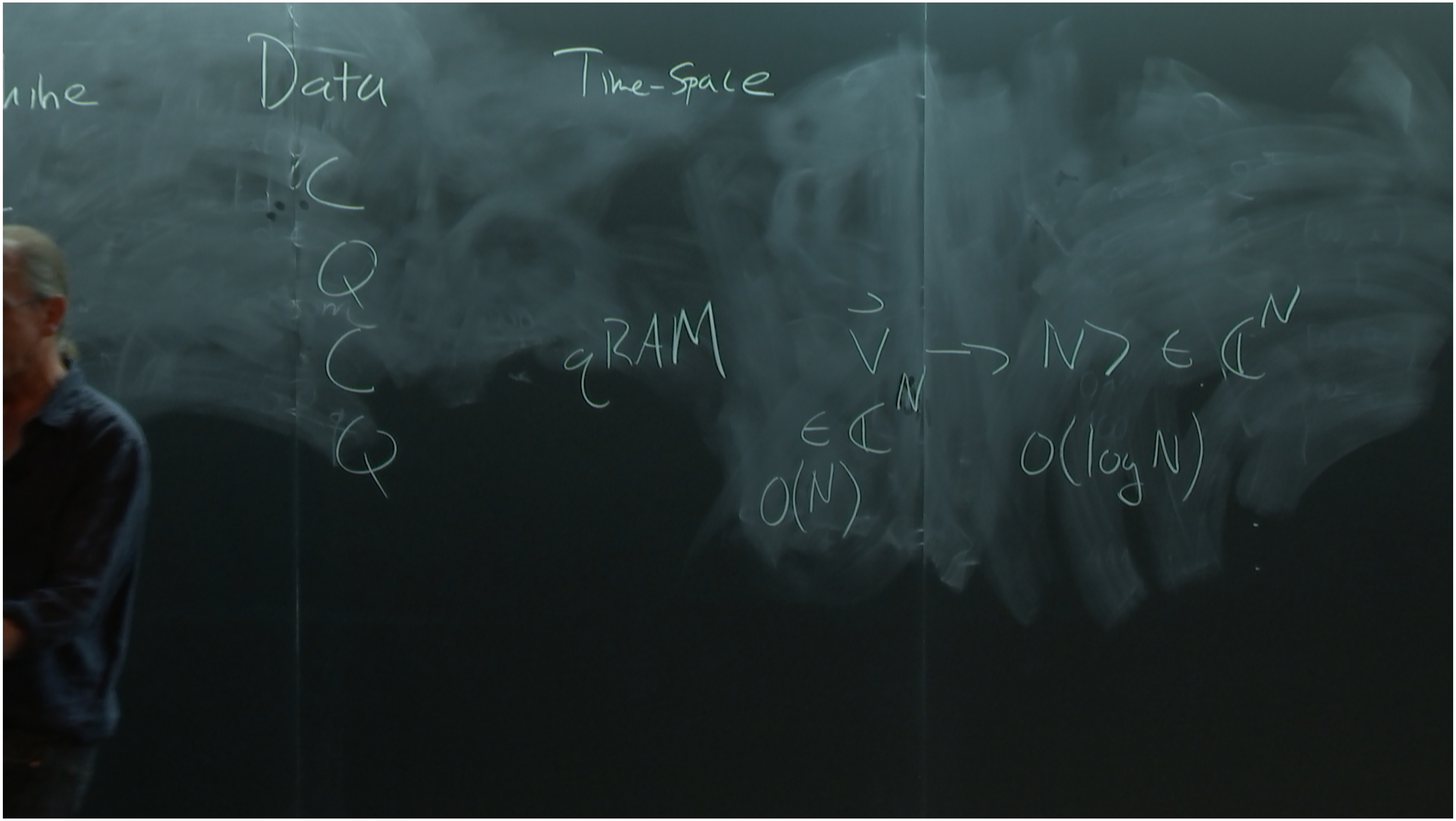
Abstract: This talk presents a quantum algorithm for performing persistent homology, the identification of topological features of data sets such as connected components, holes and voids. Finding the full persistent homology of a data set over n points using classical algorithms takes time O(2^{2n}), while the quantum algorithm takes time O(n^2), an exponential improvement. The quantum algorithm does not require a quantum random access memory and is suitable for implementation on small quantum computers with a few hundred qubits.

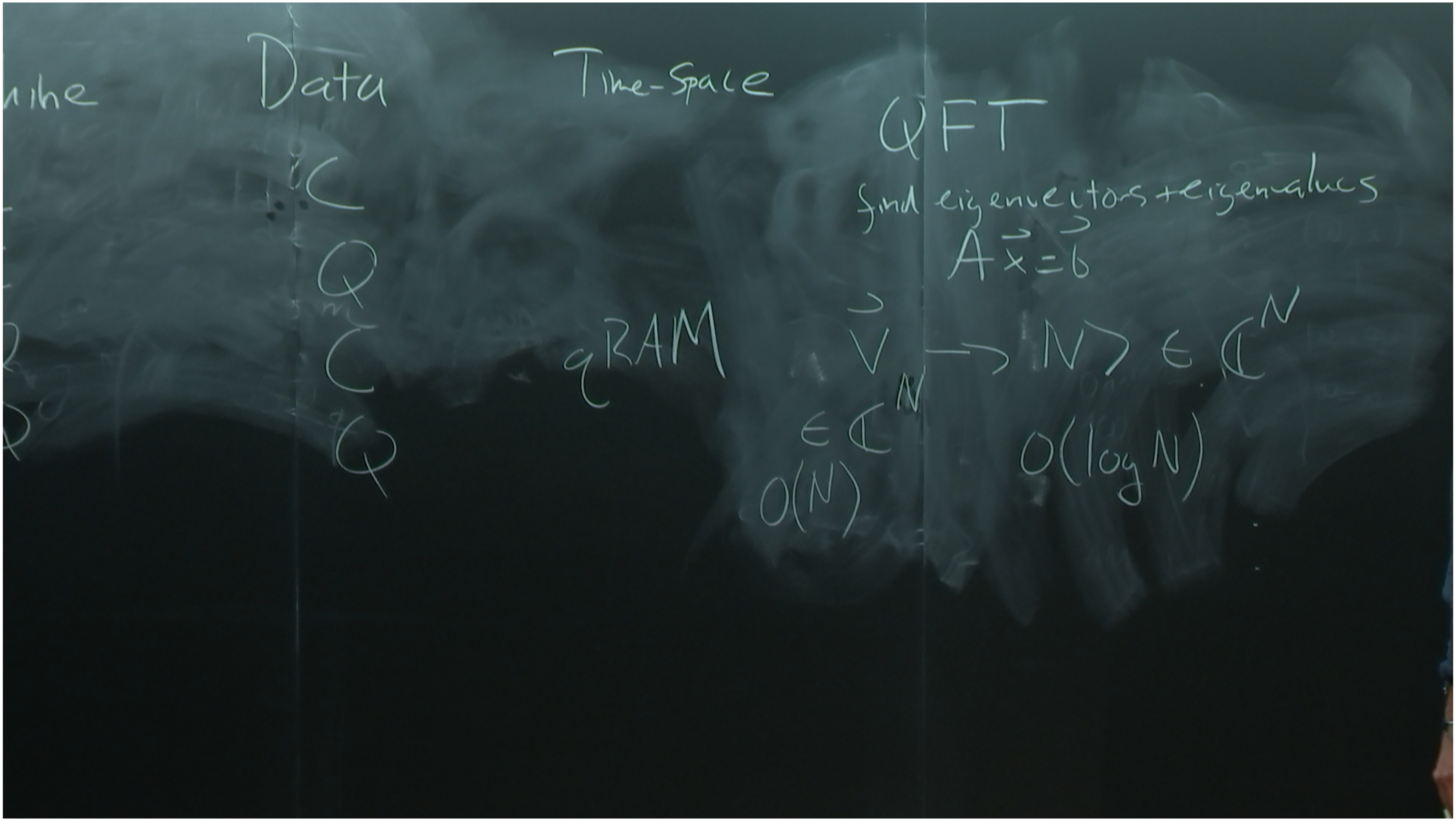

Machine
Data
C
C
C
Q

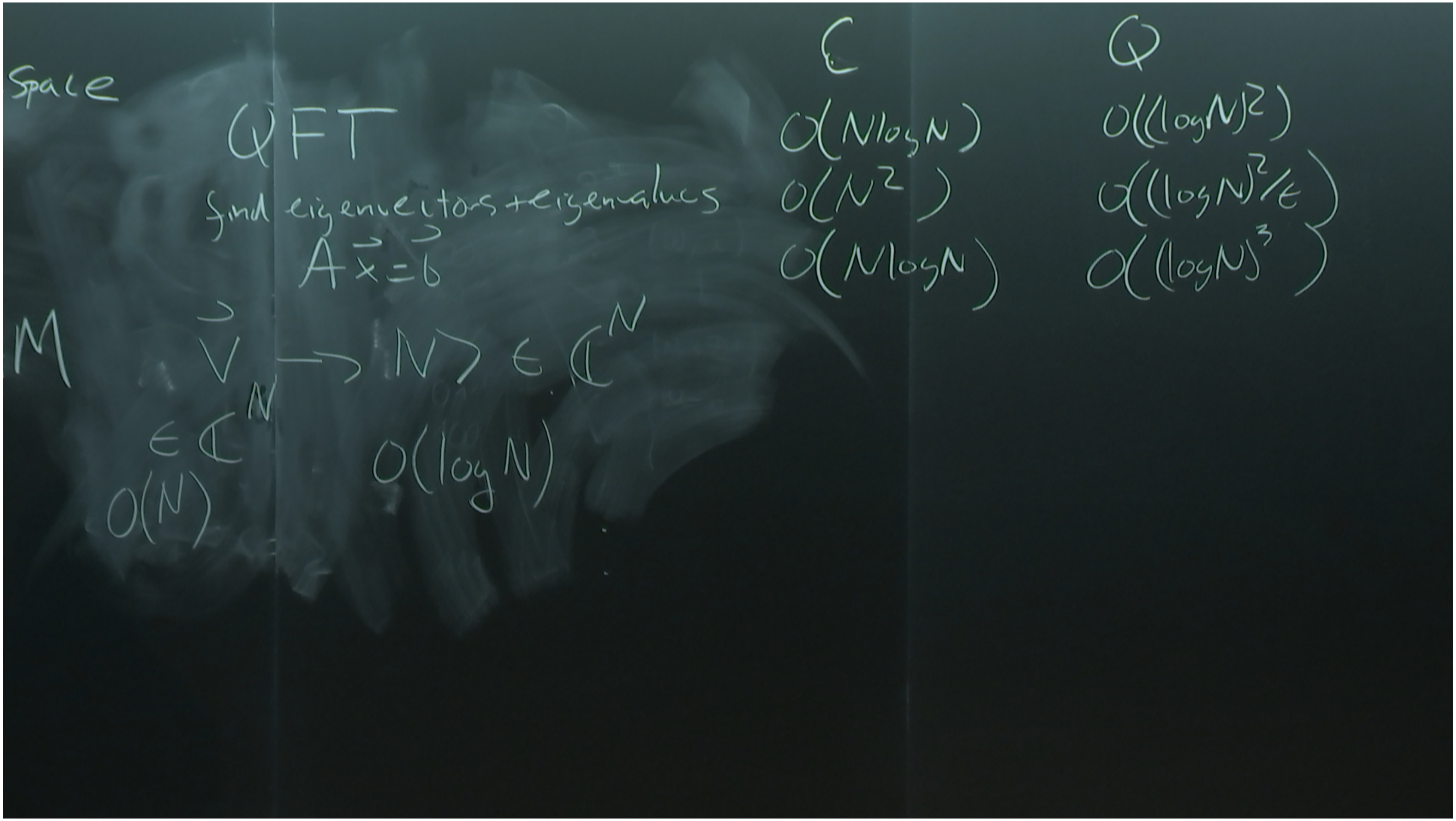

Machine
Data
C
iC
C
Q
Q
C

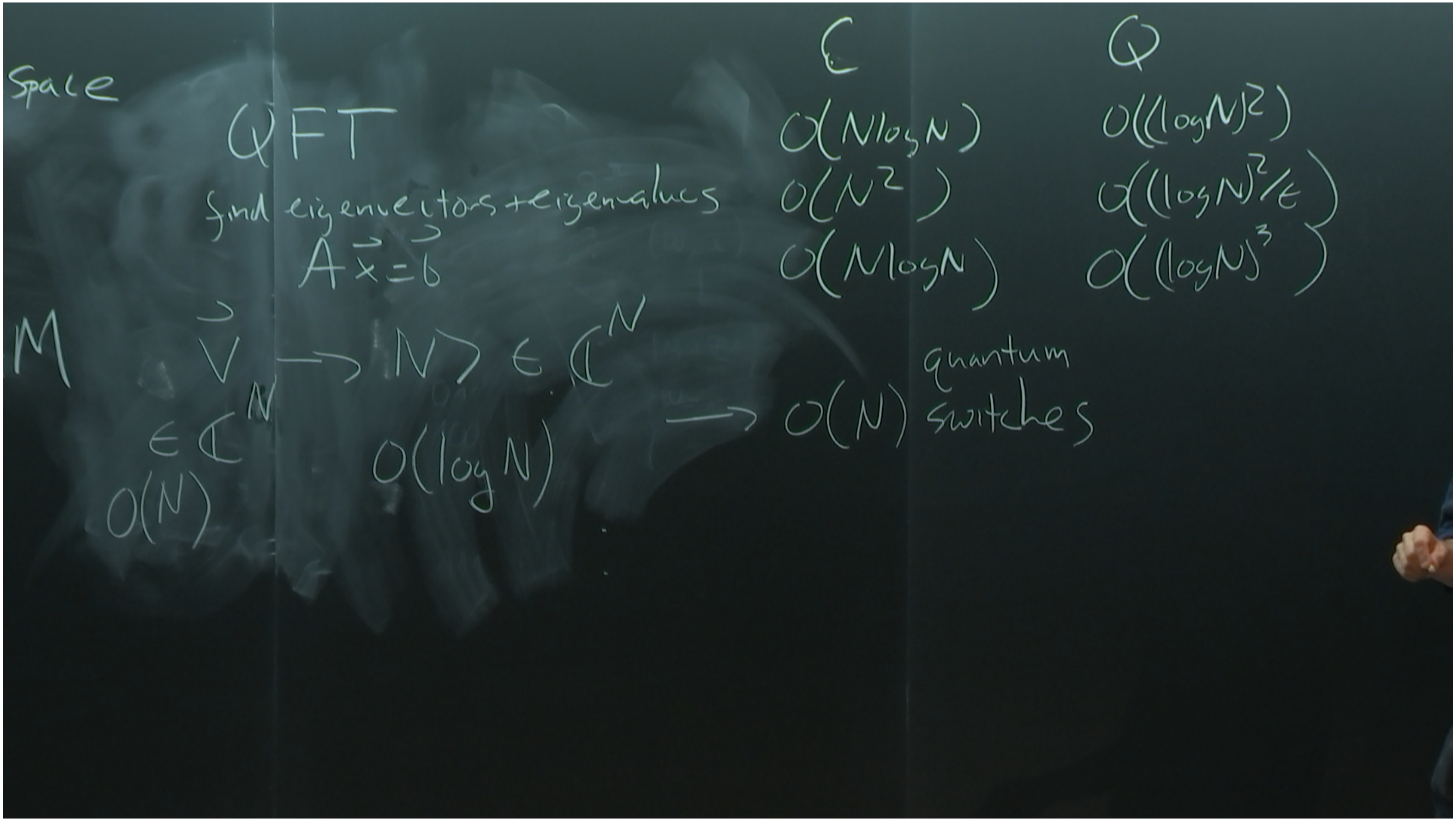

Machine
Data
C
C
C
Q
Q
C
Q
Q

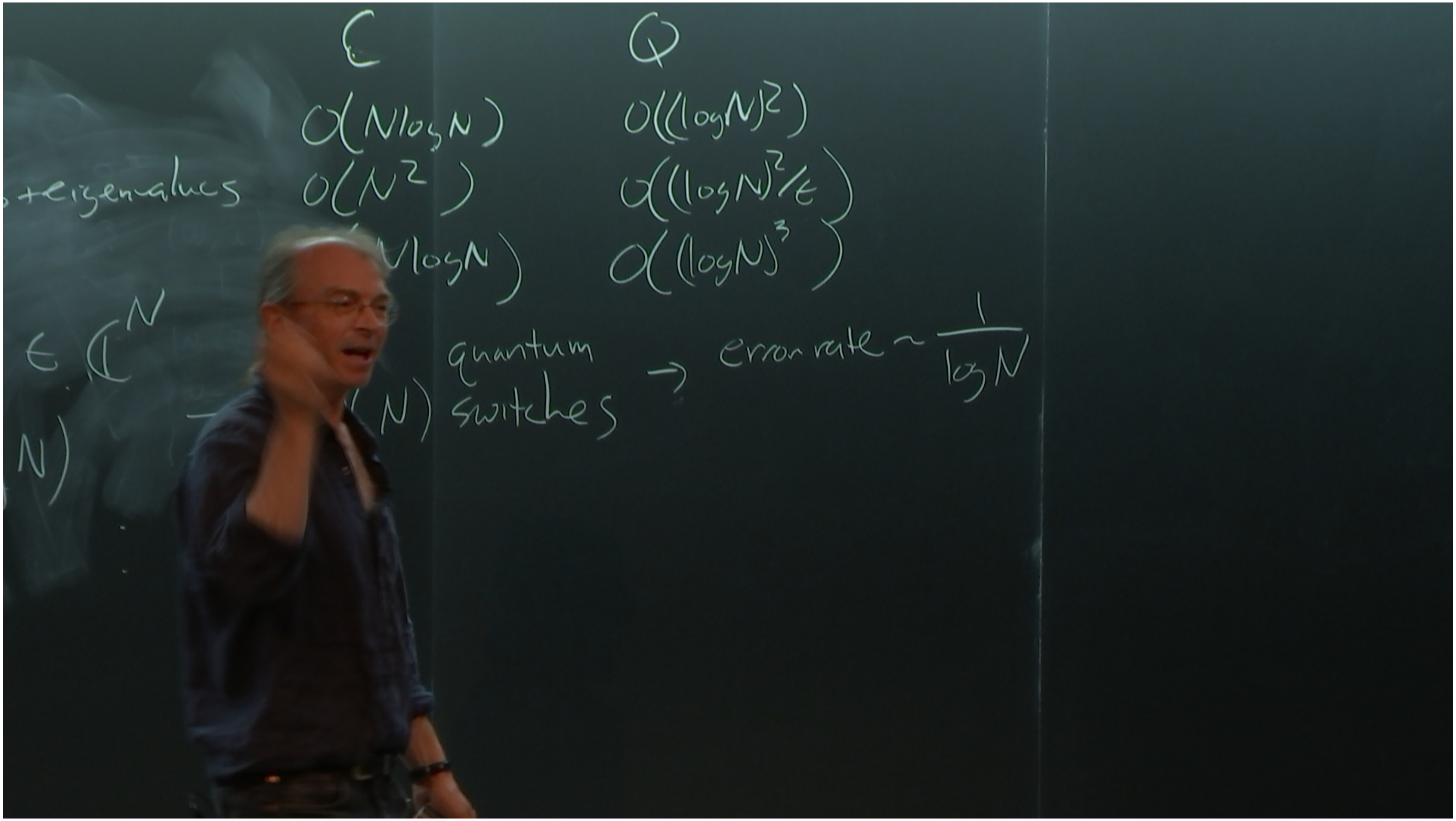

Machine
Data
Time-Space
C
C
Q
Q
C
Q
C
Q
qRAM

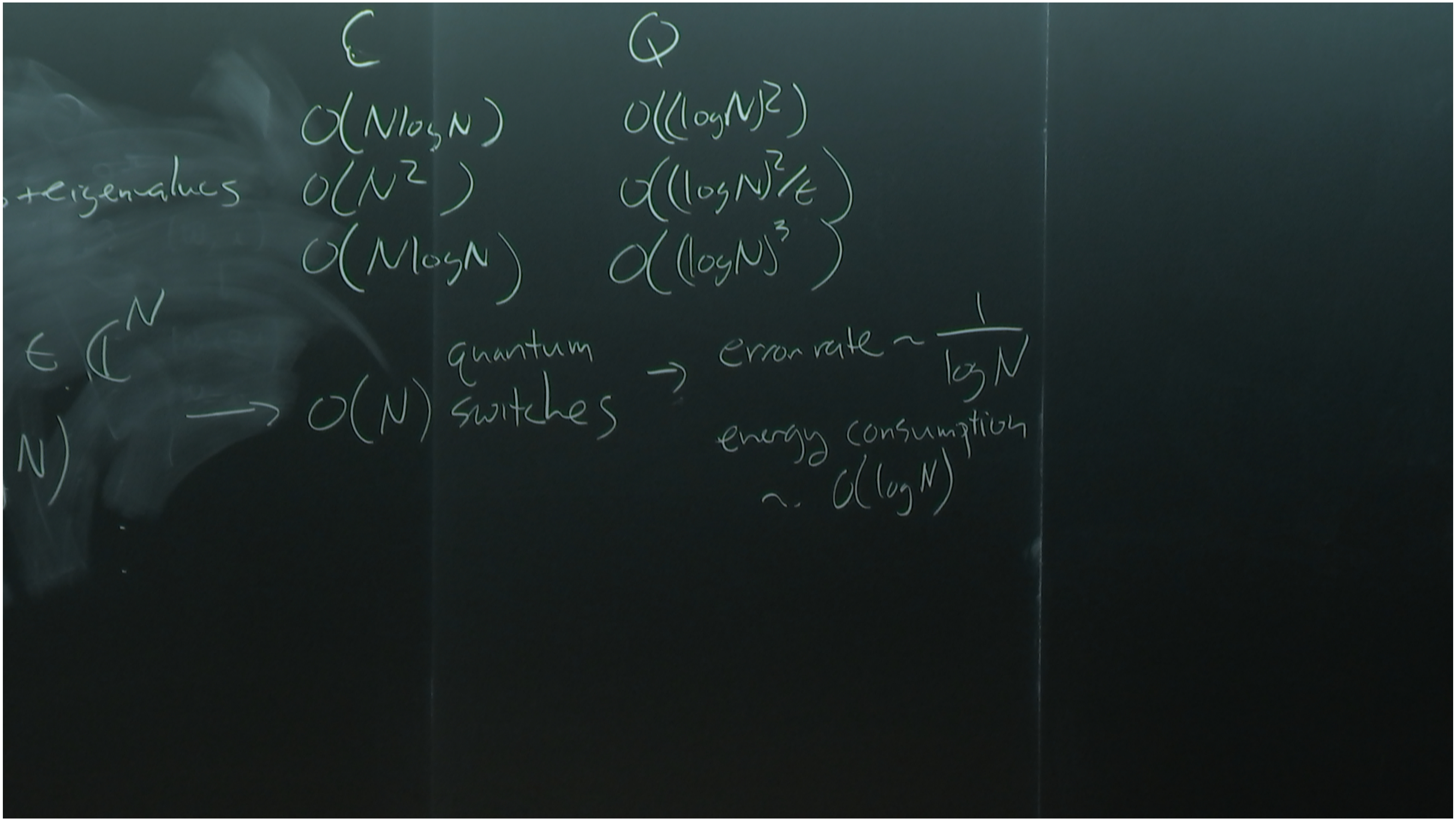

Data
Time-Space
qRAM
O(N)
O(log N)

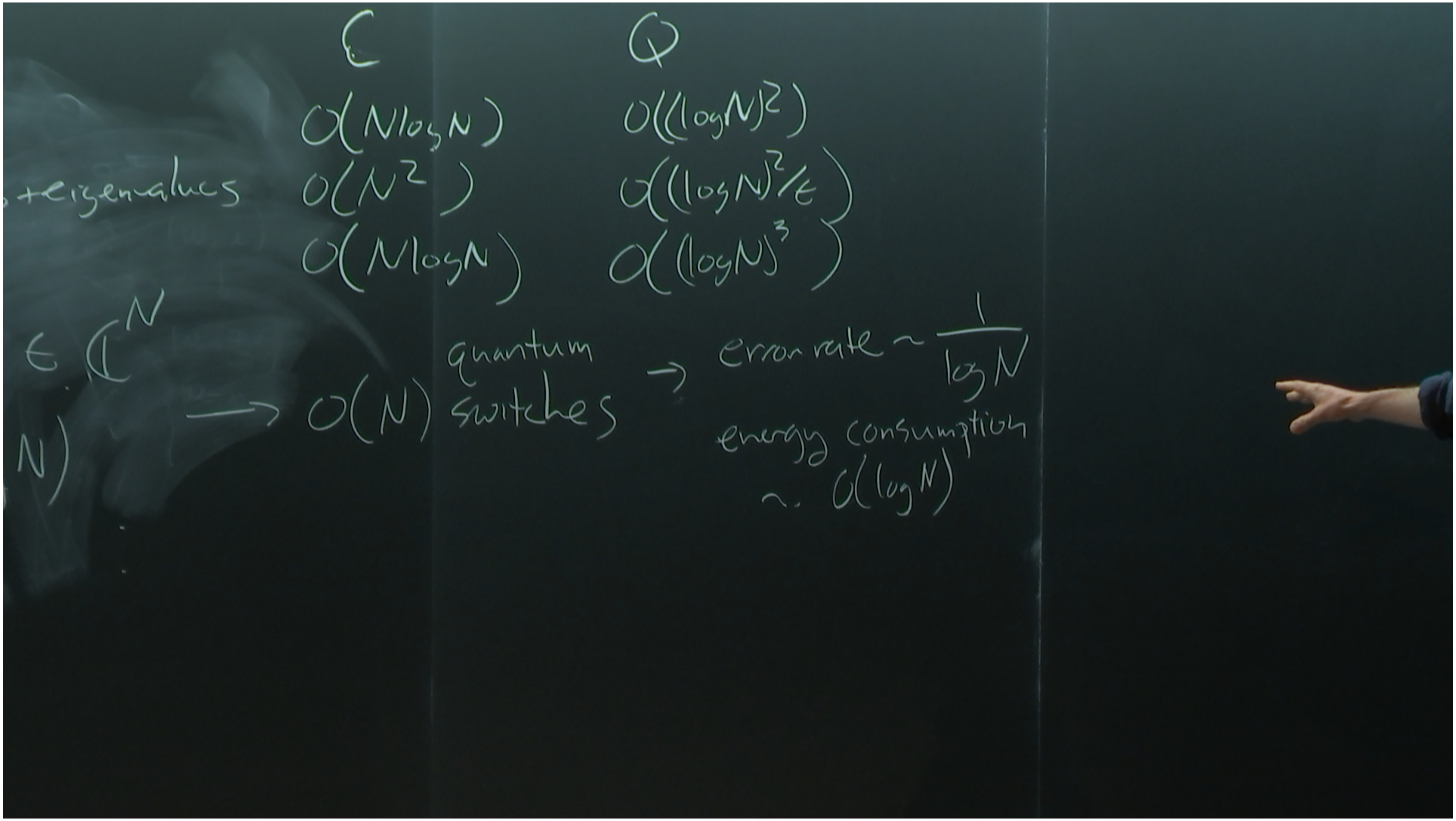

Data
Time-Space
QFT
find eigenvectors + eigenvalues
A x = b
qRAM
O(N)
O(log N)

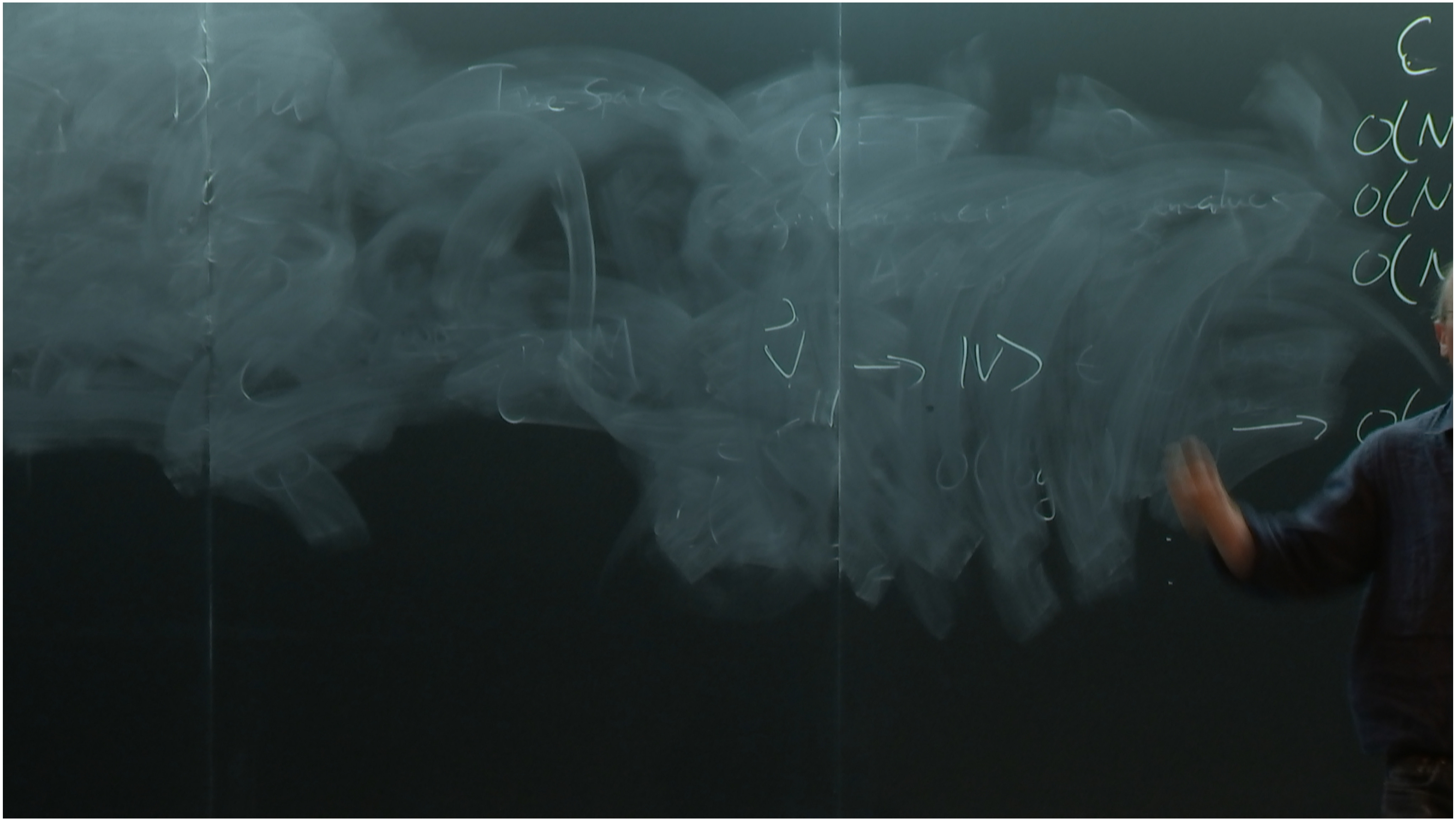

Space
QFT
find eigenvectors + eigenvalues
$A\vec{x} = \vec{b}$
M
$\vec{v} \rightarrow |N\rangle \in \mathbb{C}^N$
$\in \mathbb{C}^N$
$O(N)$
$O(\log N)$
C
Q
$O(N \log N)$
$O((\log N)^2)$
$O(N^2)$
$O((\log N)^2/\epsilon)$
$O(N \log N)$
$O((\log N)^3)$

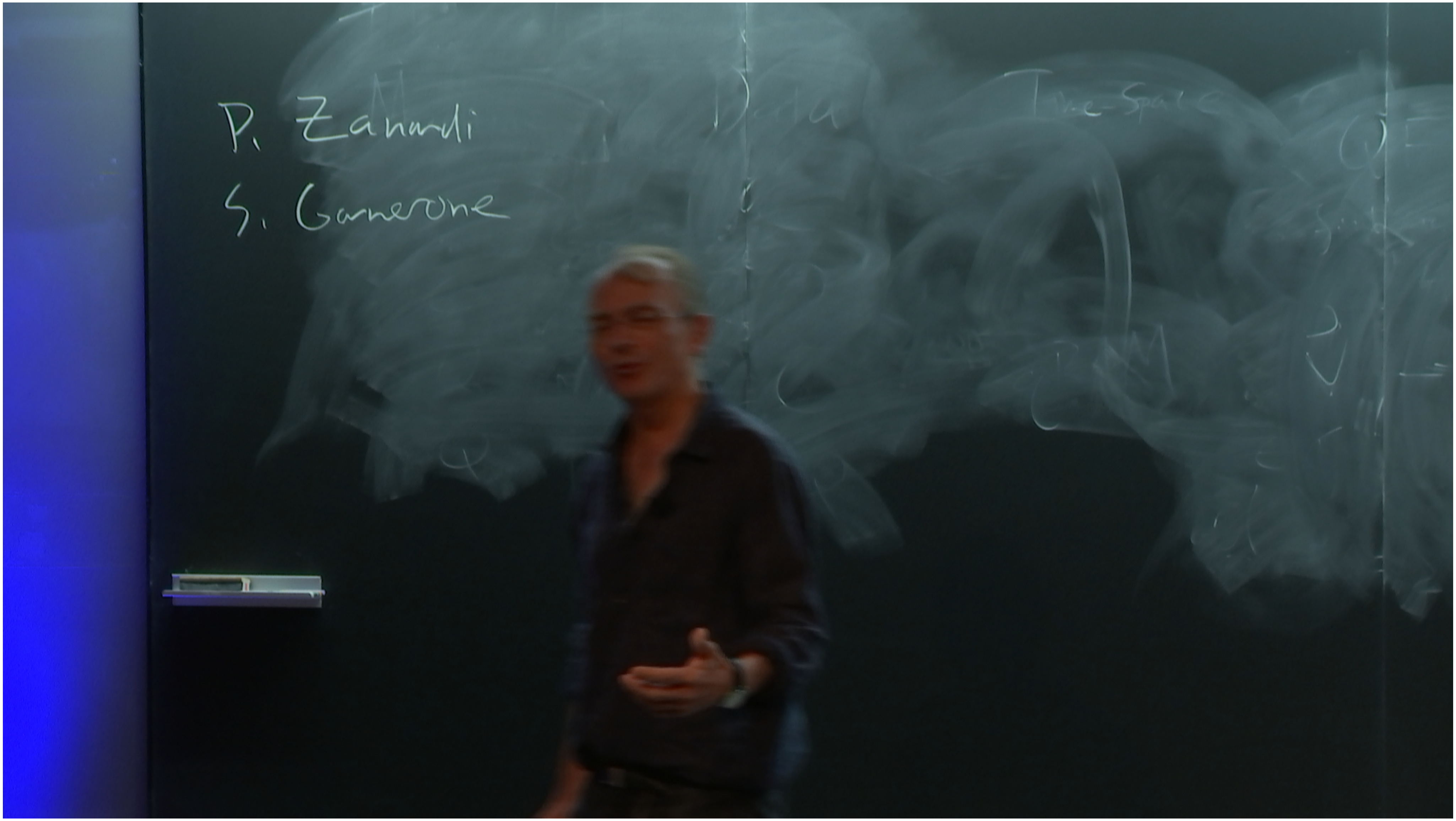

Space
QFT
find eigenvectors + eigenvalues
A x = b
M
V → |N> ∈ C^N
∈ C^N
O(N)
O(log N)
→ O(N) quantum switches
C
Q
O(N log N)
O((log N)^2)
O(N^2)
O((log N)^2/ε)
O(N log N)
O((log N)^3)

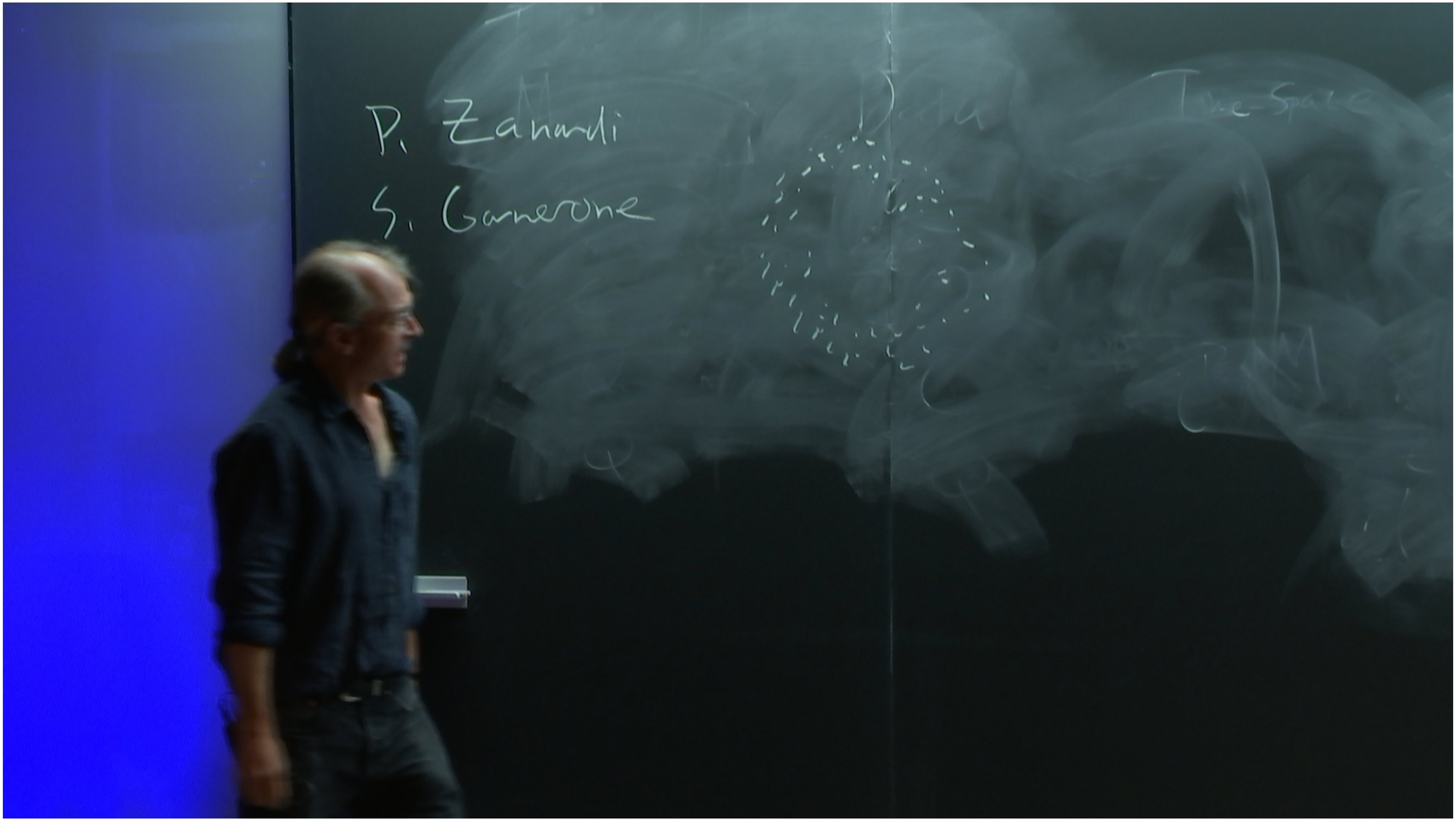

C
Q
O(N log N)
O((log N)^2)
eigenvalues
O(N^2)
O((log N)^2/ε)
O((log N)^3)
quantum
switches
error rate ~ 1/log N

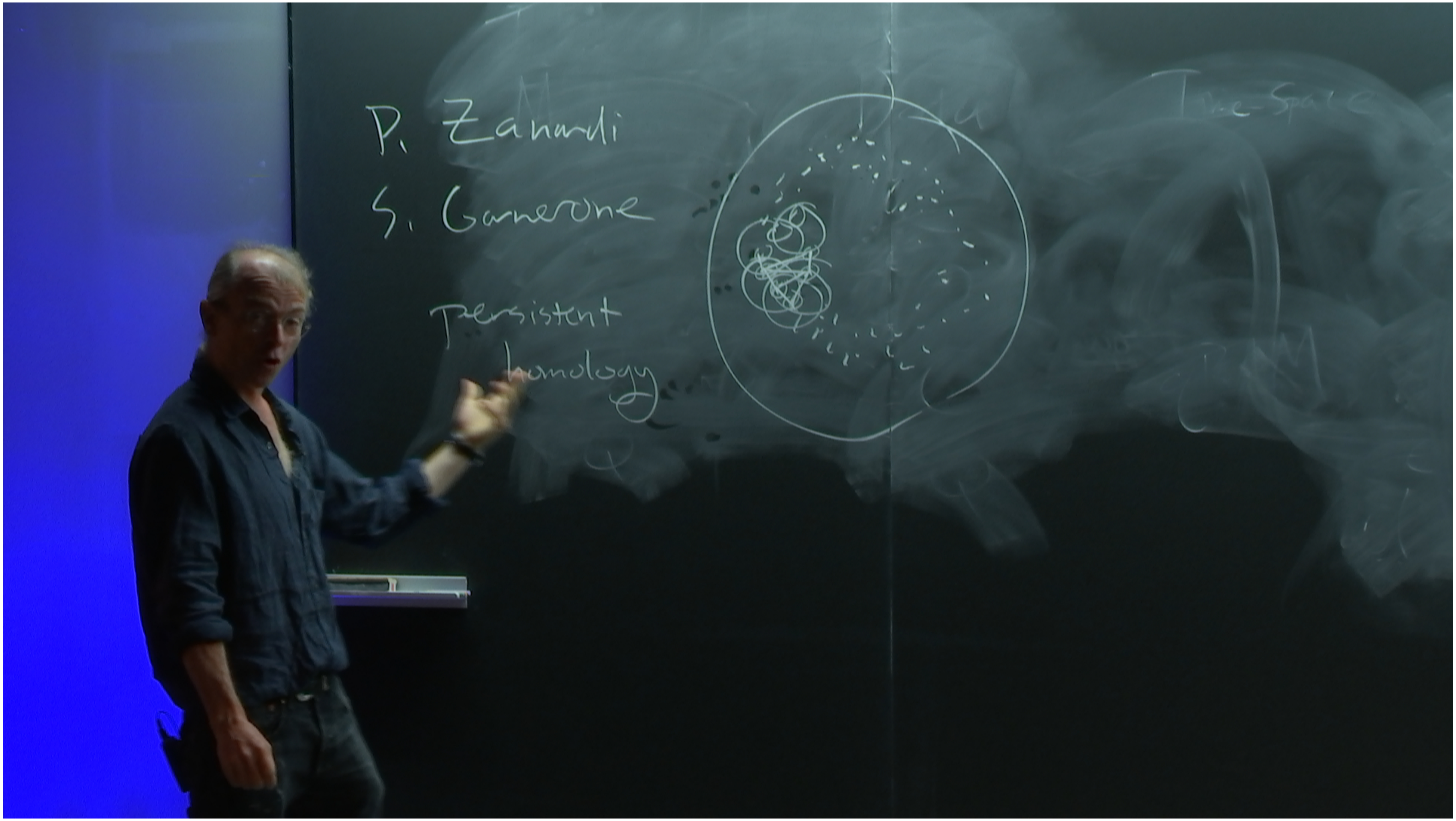

C
Q
O(NlogN)
O((logN)^2)
+eigenvalues
O(N^2)
O((logN)^2/ε)
O(NlogN)
O((logN)^3)
∈ ℂ^N
→ O(N) quantum switches
→ error rate ~ 1/logN
energy consumption
~ O(logN)

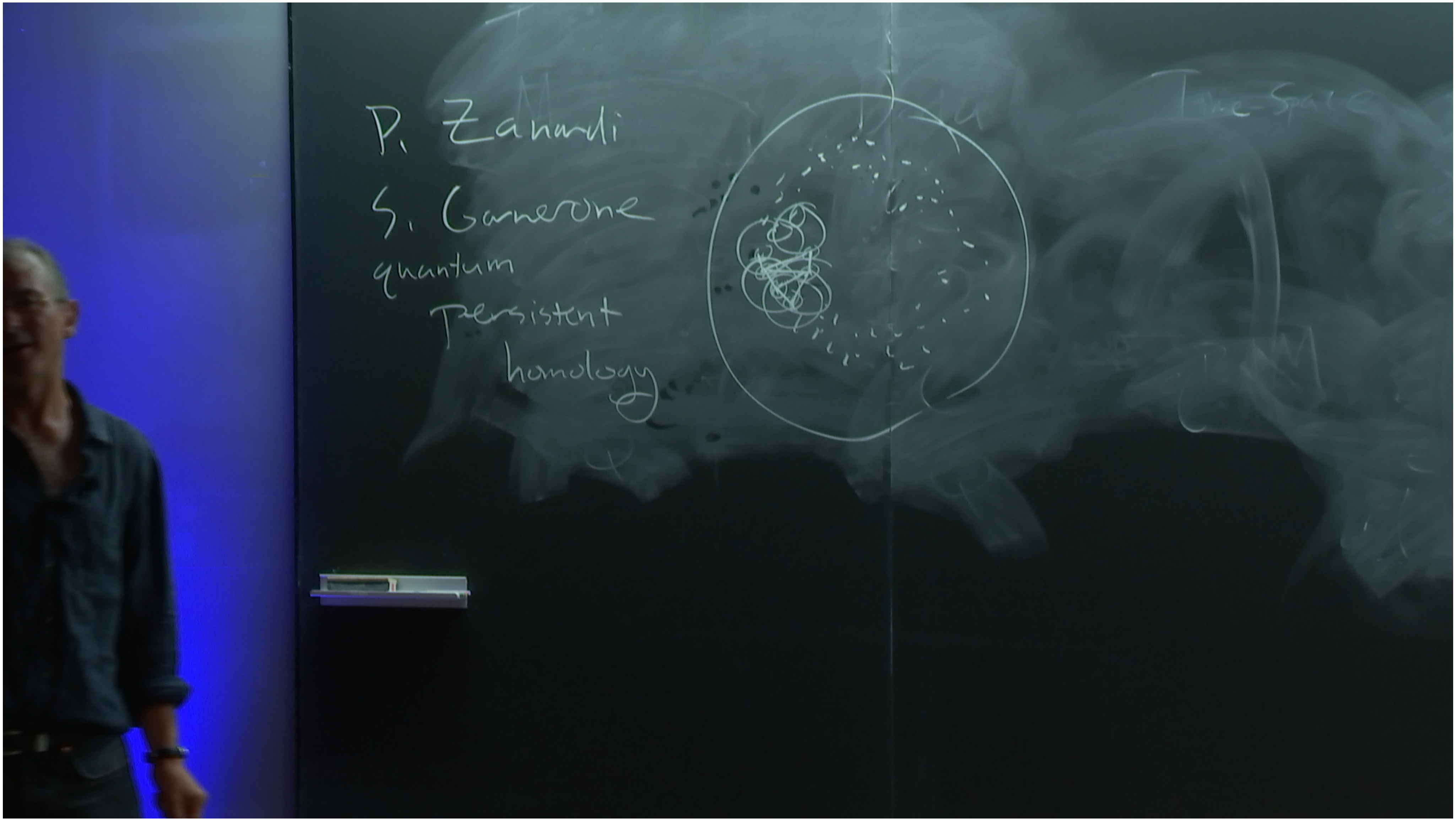

C
Q
O(N log N)
O((log N)^2)
eigenvalues
O(N^2)
O((log N)^2/ε)
O(N log N)
O((log N)^3)
∈ ℂ^N
→ O(N) quantum switches
→ error rate ~ 1/log N
energy consumption
~ O(log N)

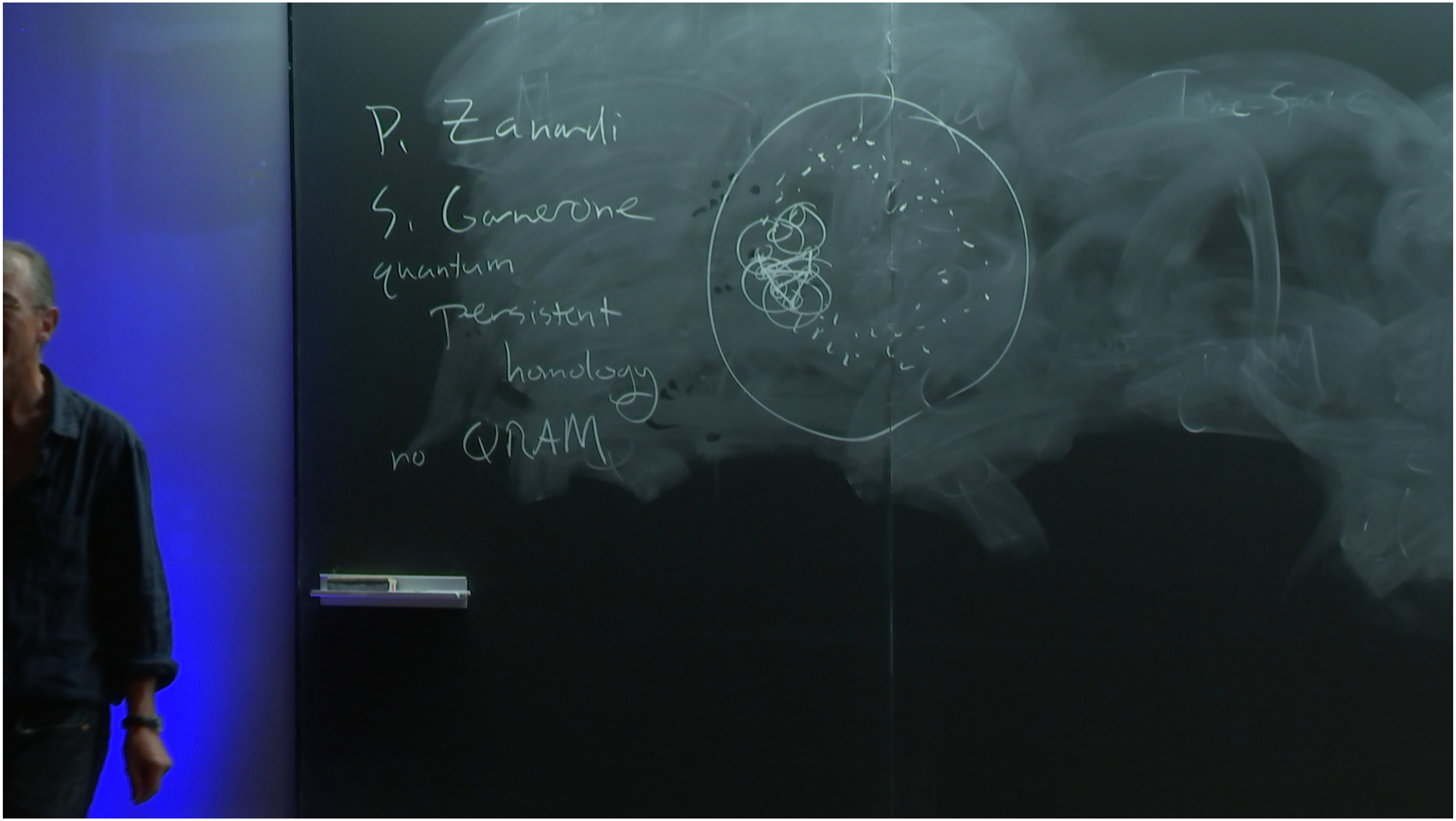


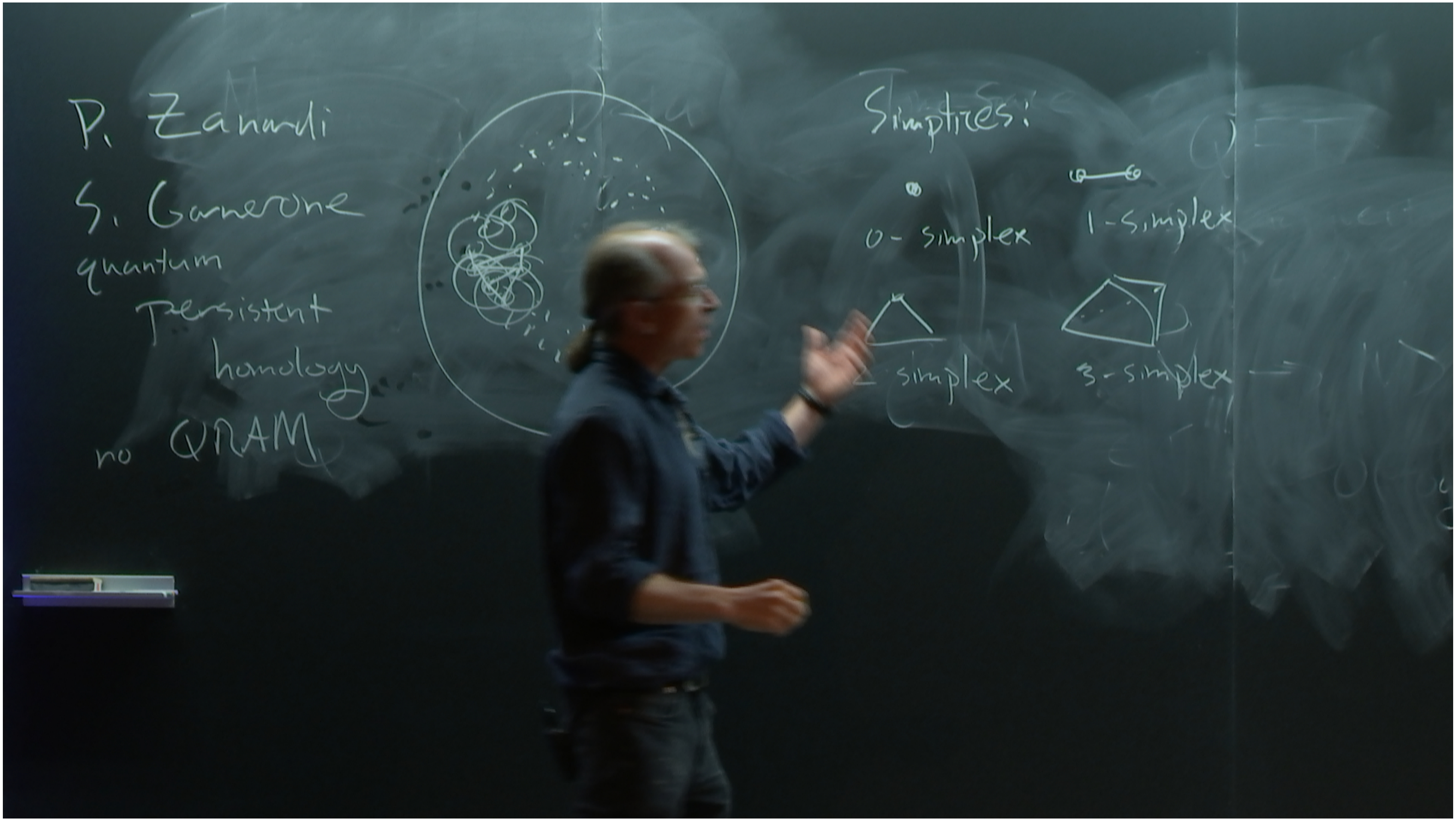

P. Zanardi
S. Garnerone

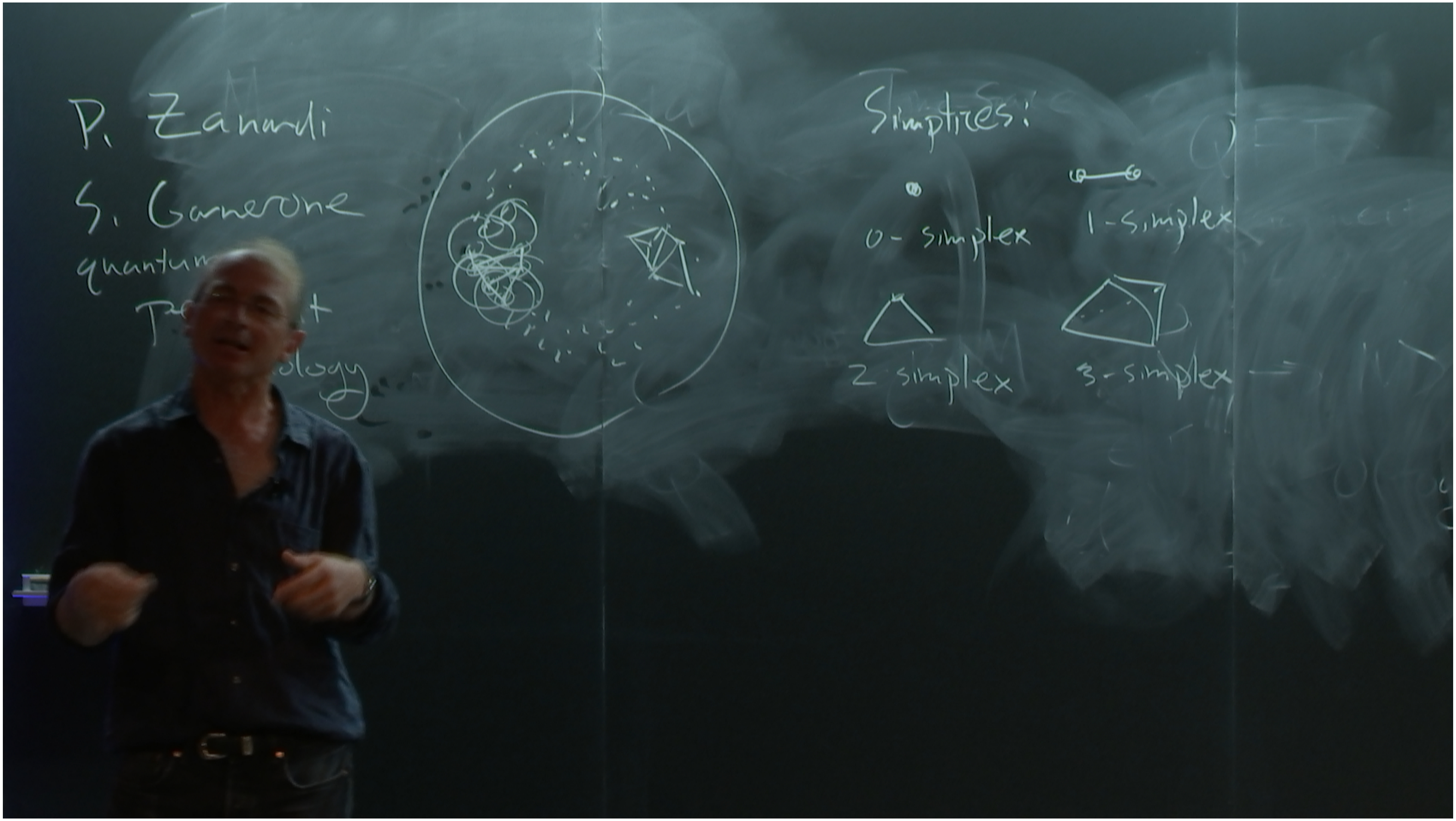

P. Zanardi
S. Garnerone


P. Zanardi
S. Garnerone
persistent
homology

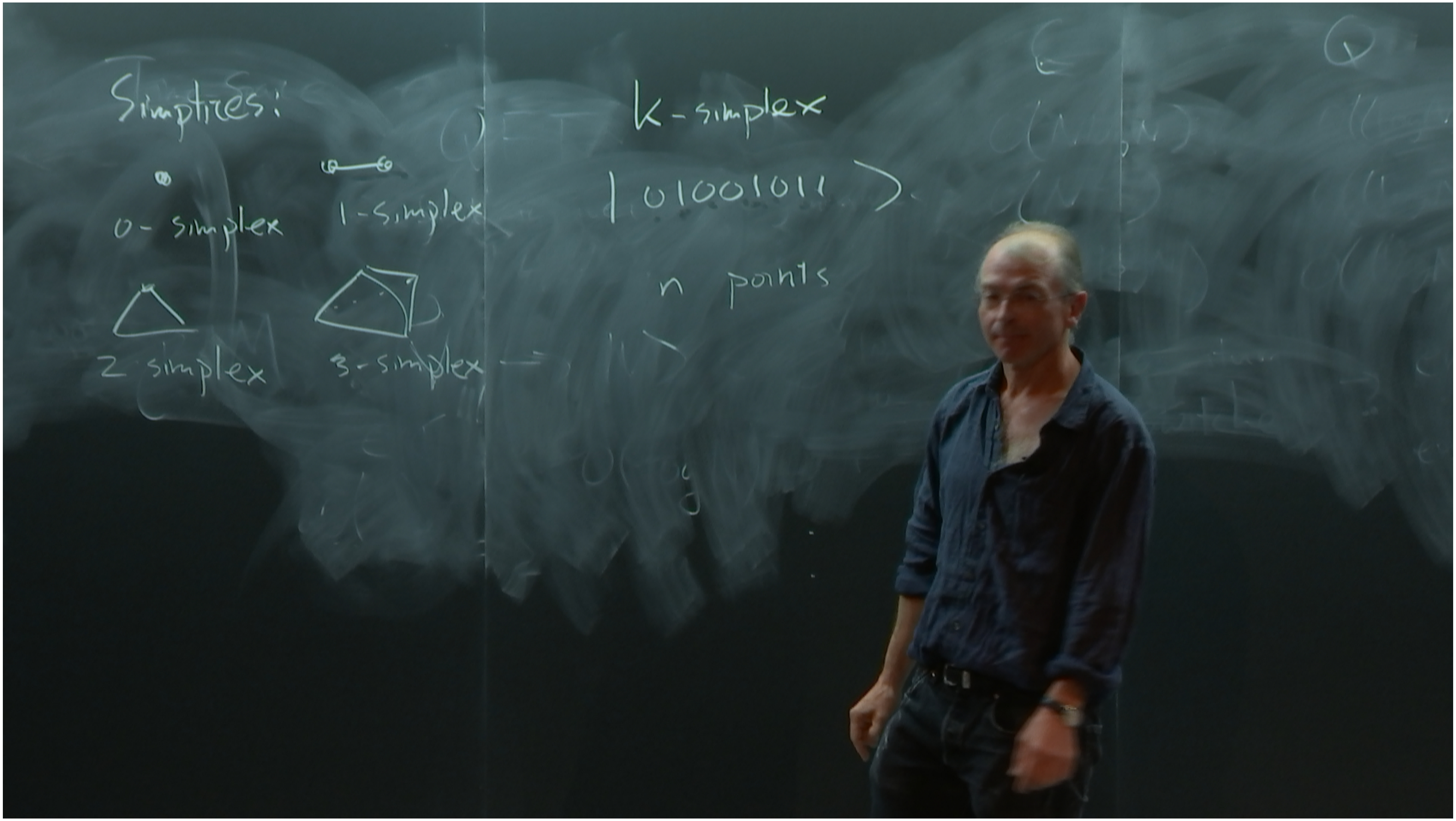

P. Zanardi
S. Garnerone
quantum
persistent
homology

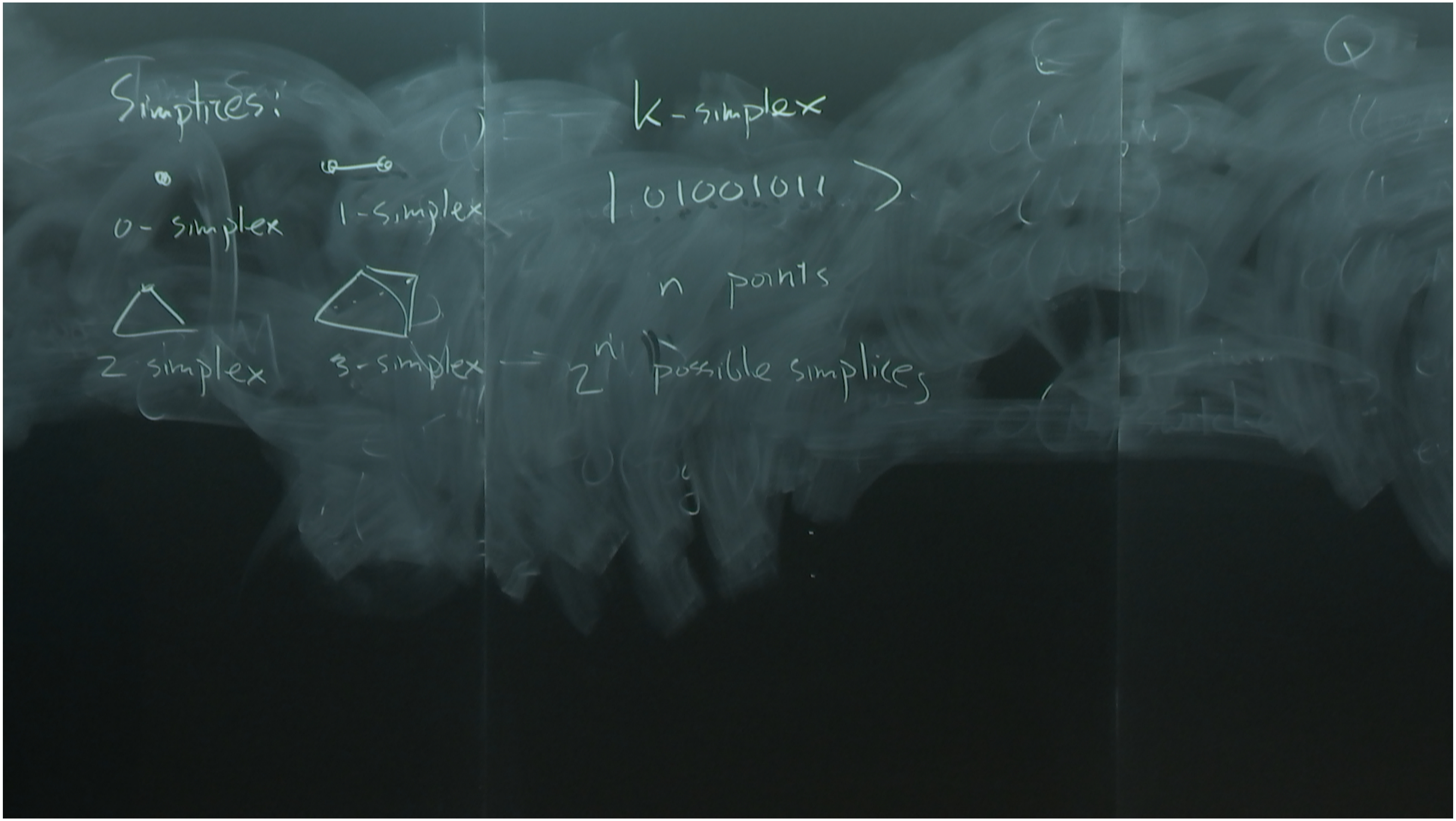

P. Zanardi
S. Garnerone
quantum
persistent
homology
no QRAM

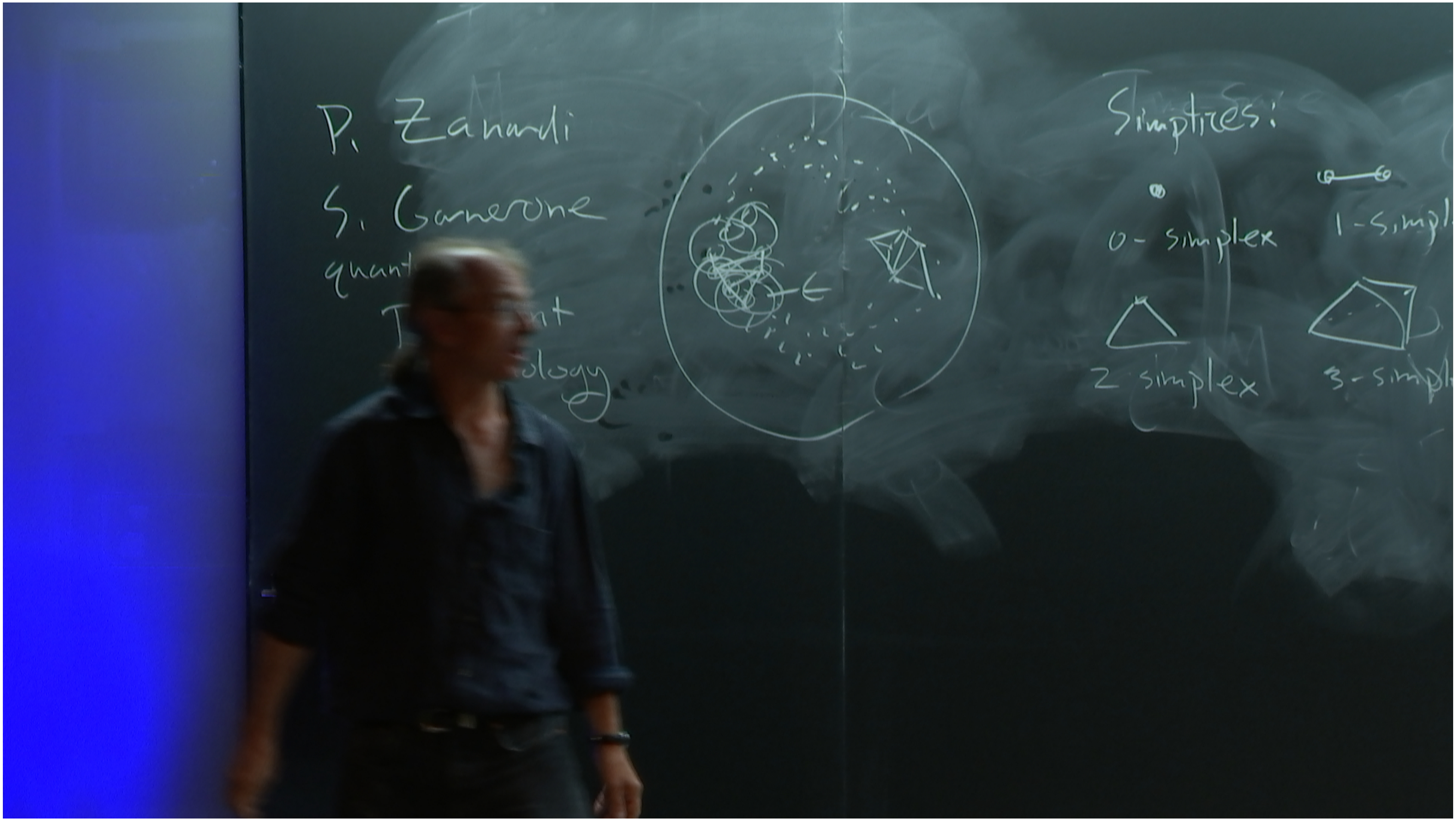

P. Zanardi
S. Garnerone
quantum
persistent
homology
no QRAM
Simplices:
0- simplex
1-simplex
2-simplex
3-simplex

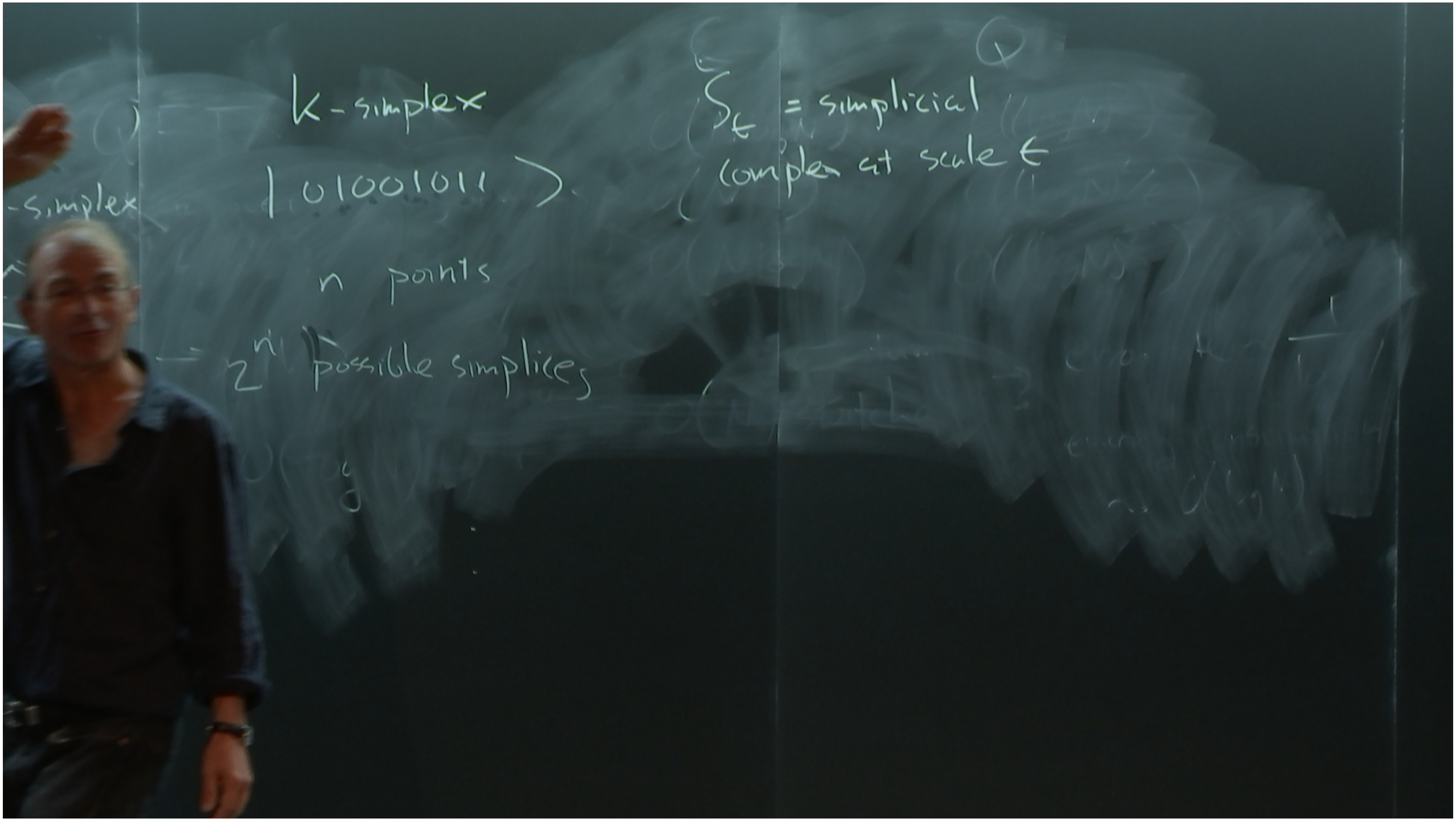


P. Zanardi
S. Garnerone
quantum
Simplices:
0- simplex
1-simplex
2 simplex
3-simplex

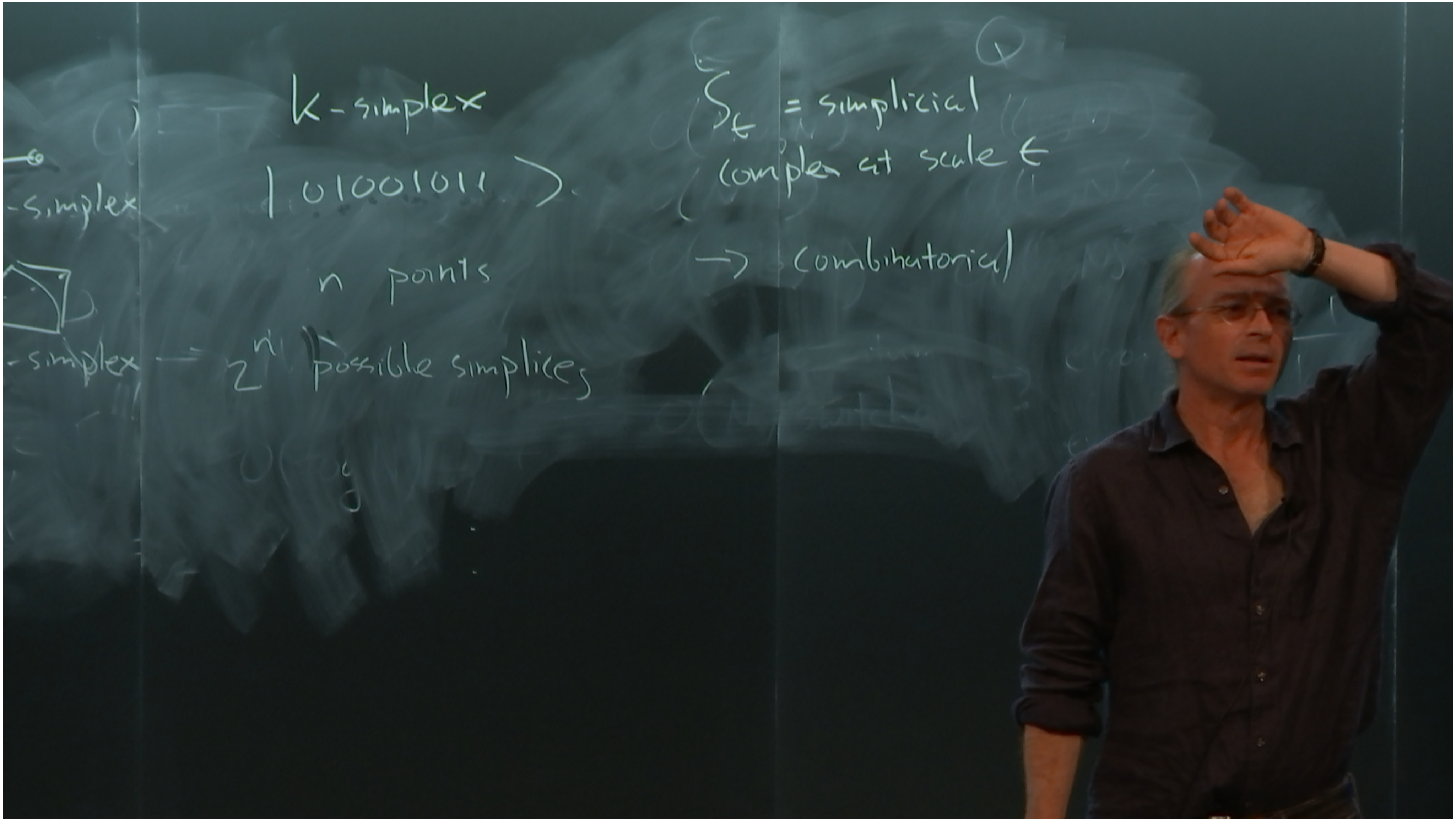

Simplices:
0- simplex
1-simplex
2 simplex
3-simplex
k-simplex
|0100101 1 >
n points

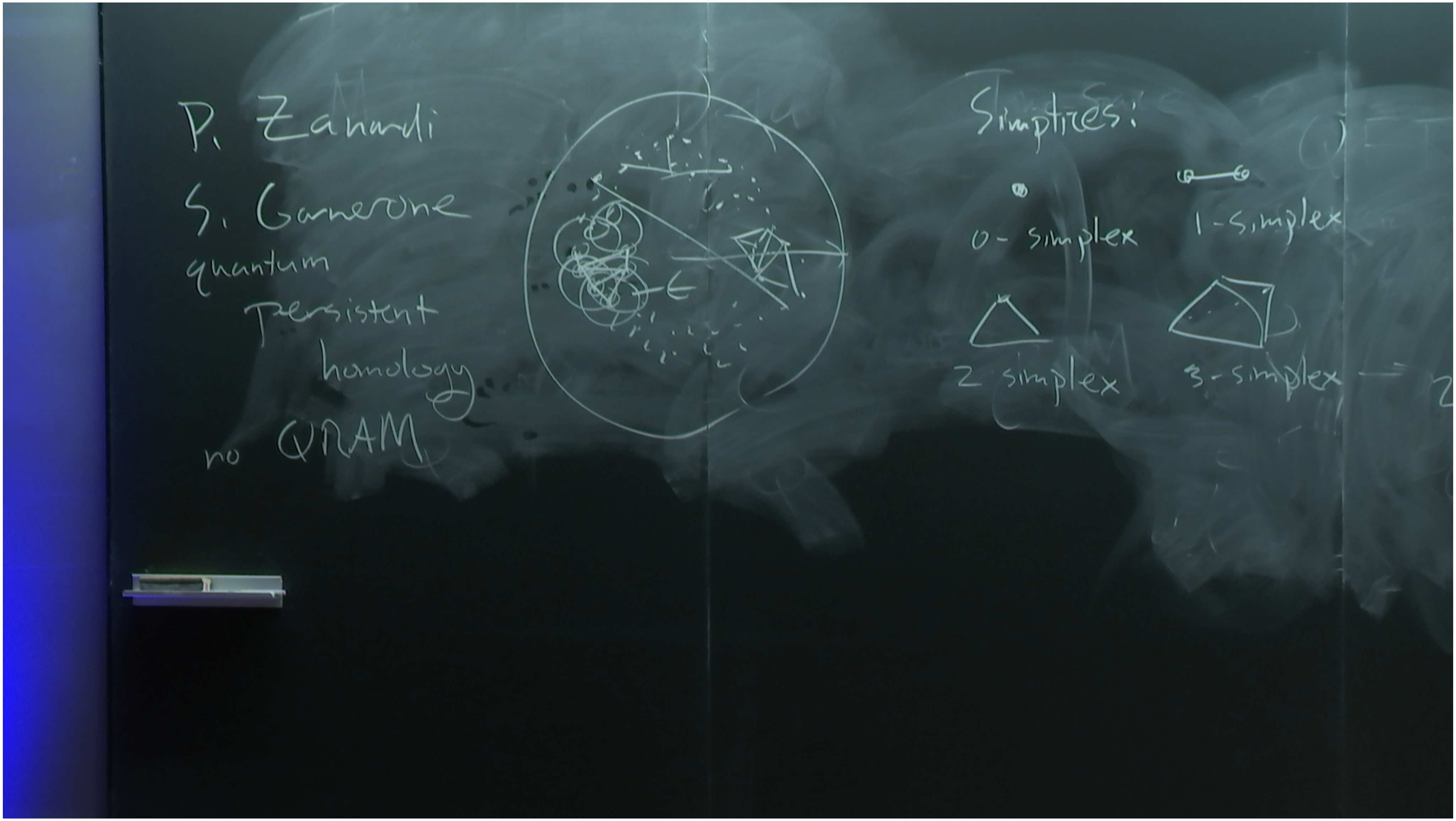

Simplices:
0- simplex
1-simplex
2 simplex
3-simplex
k-simplex
|01001011⟩
n points
2^n possible simplices

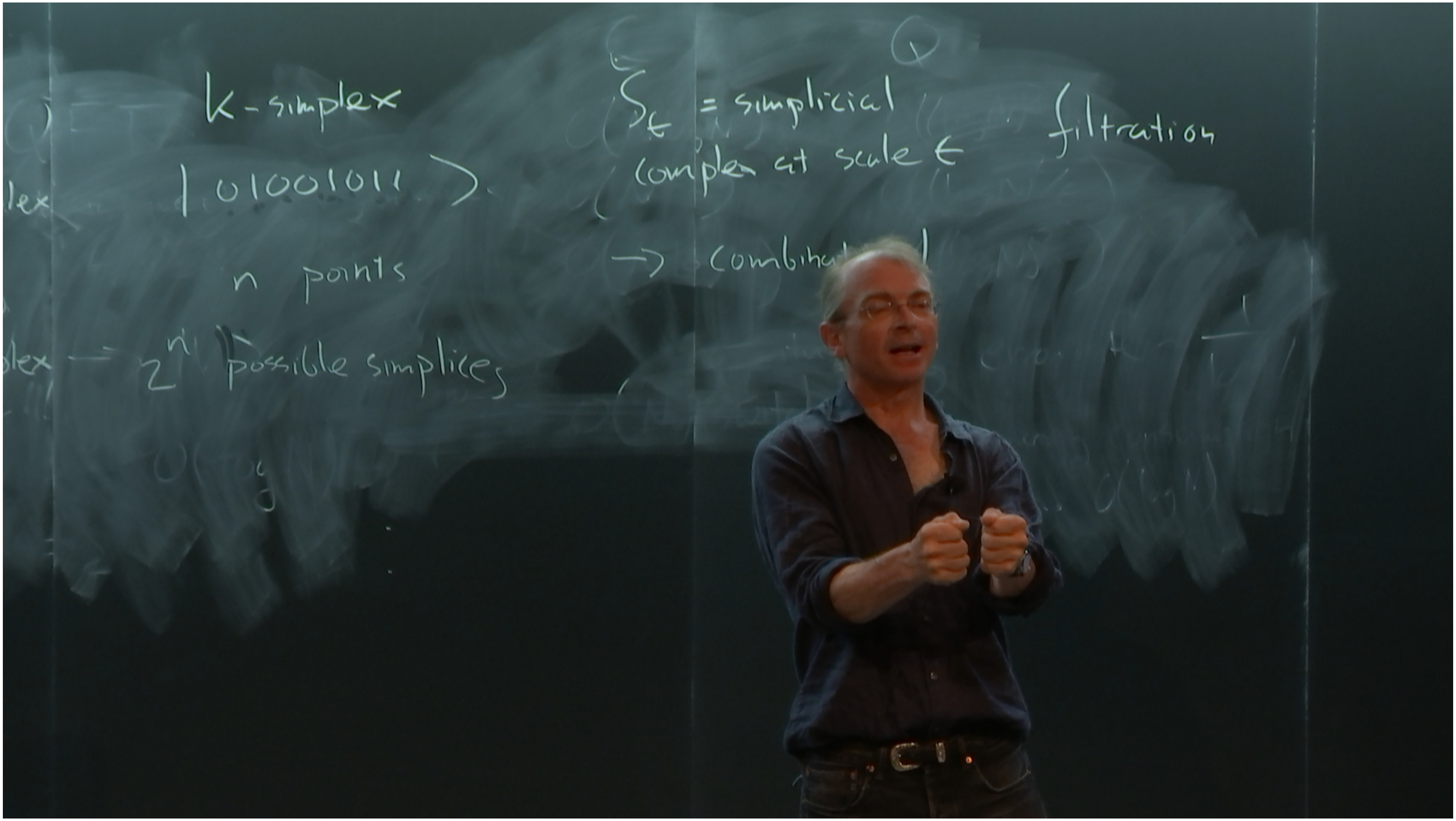

P. Zanardi
S. Garnerone
Simplices:
0- simplex
1-simpl
2 simplex
3-simpl

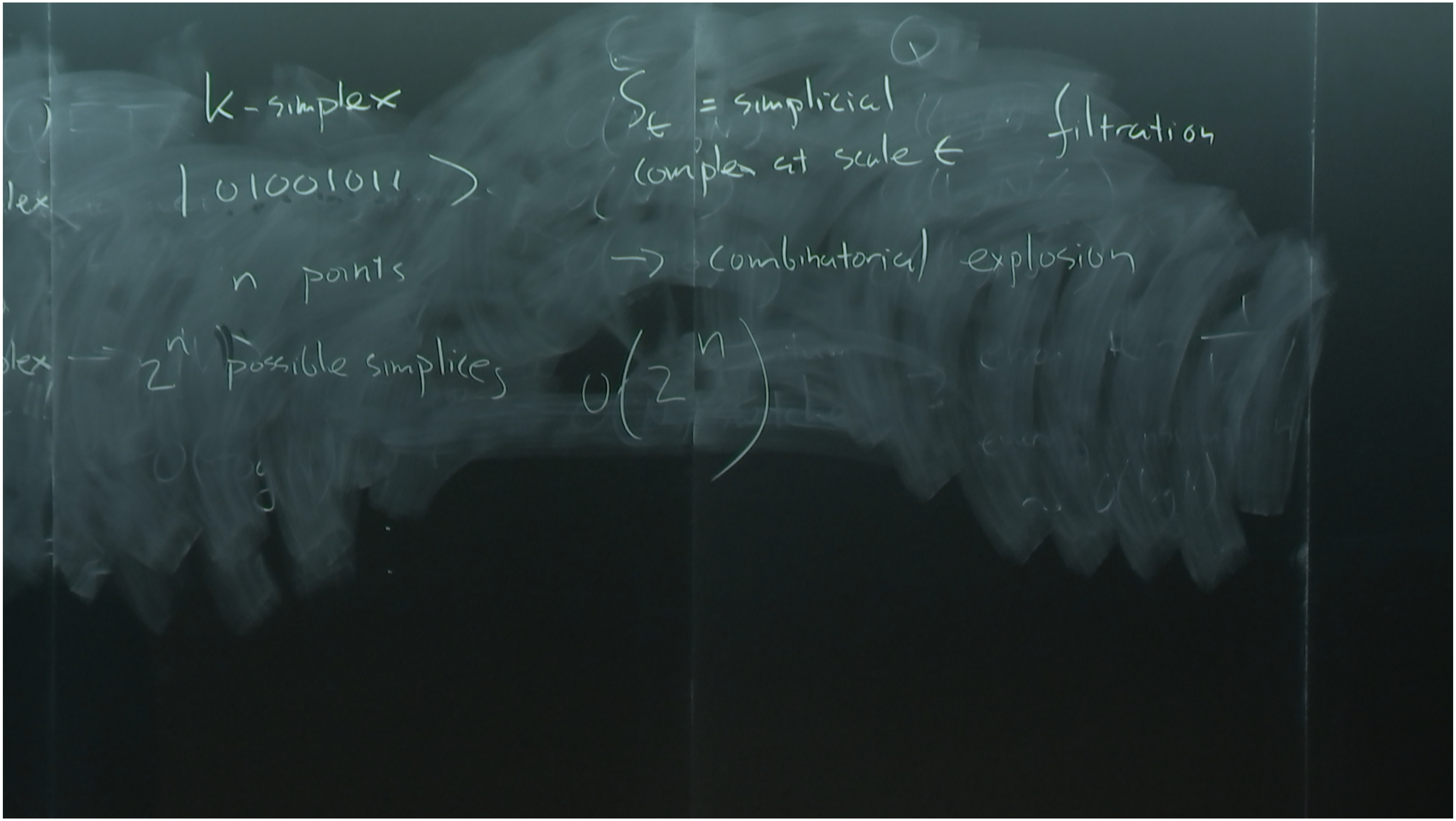


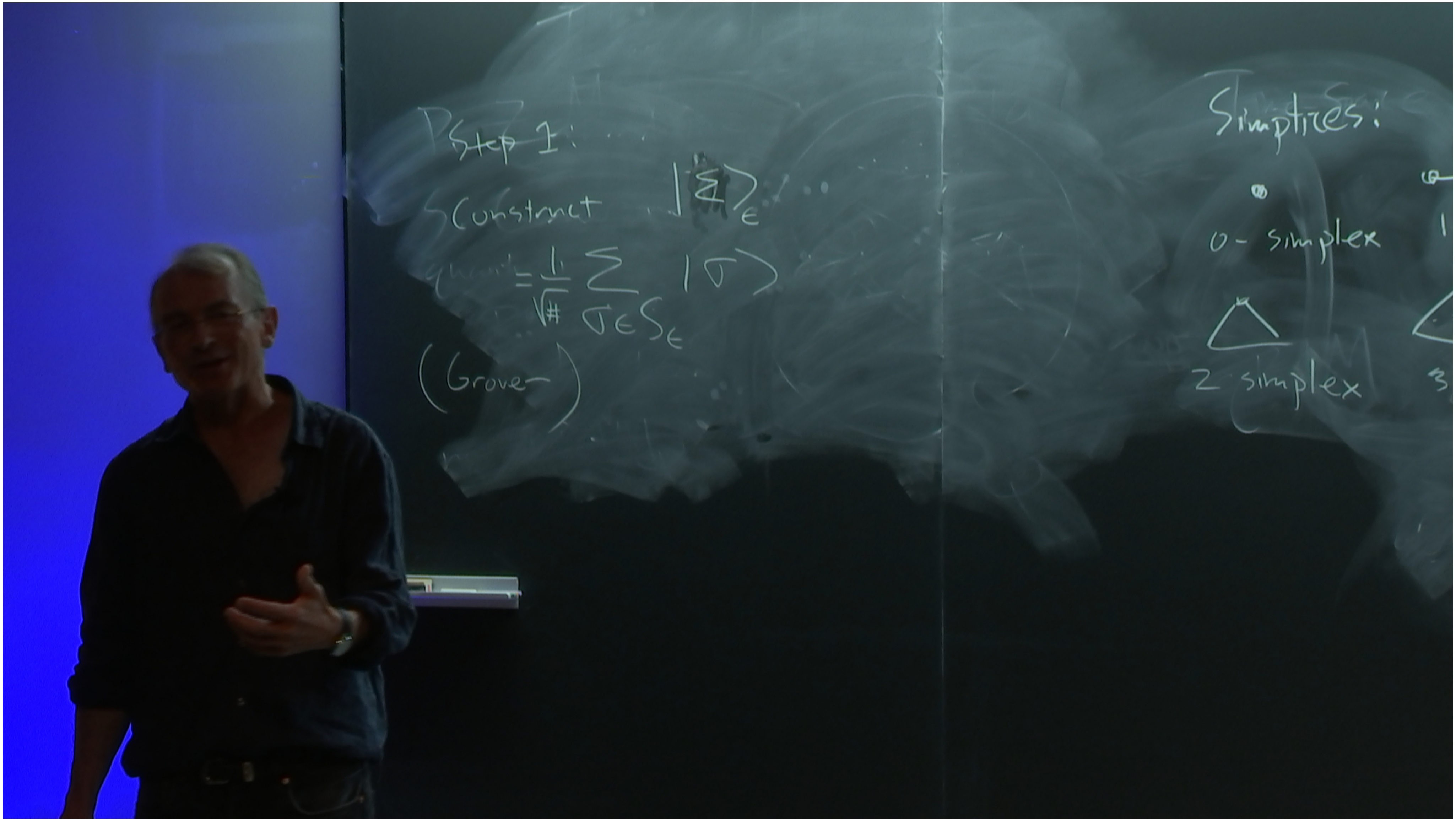
k-simplex
|01001011 >
-simplex
n points
2n possible simplices
S_ε = simplicial
complex at scale ε

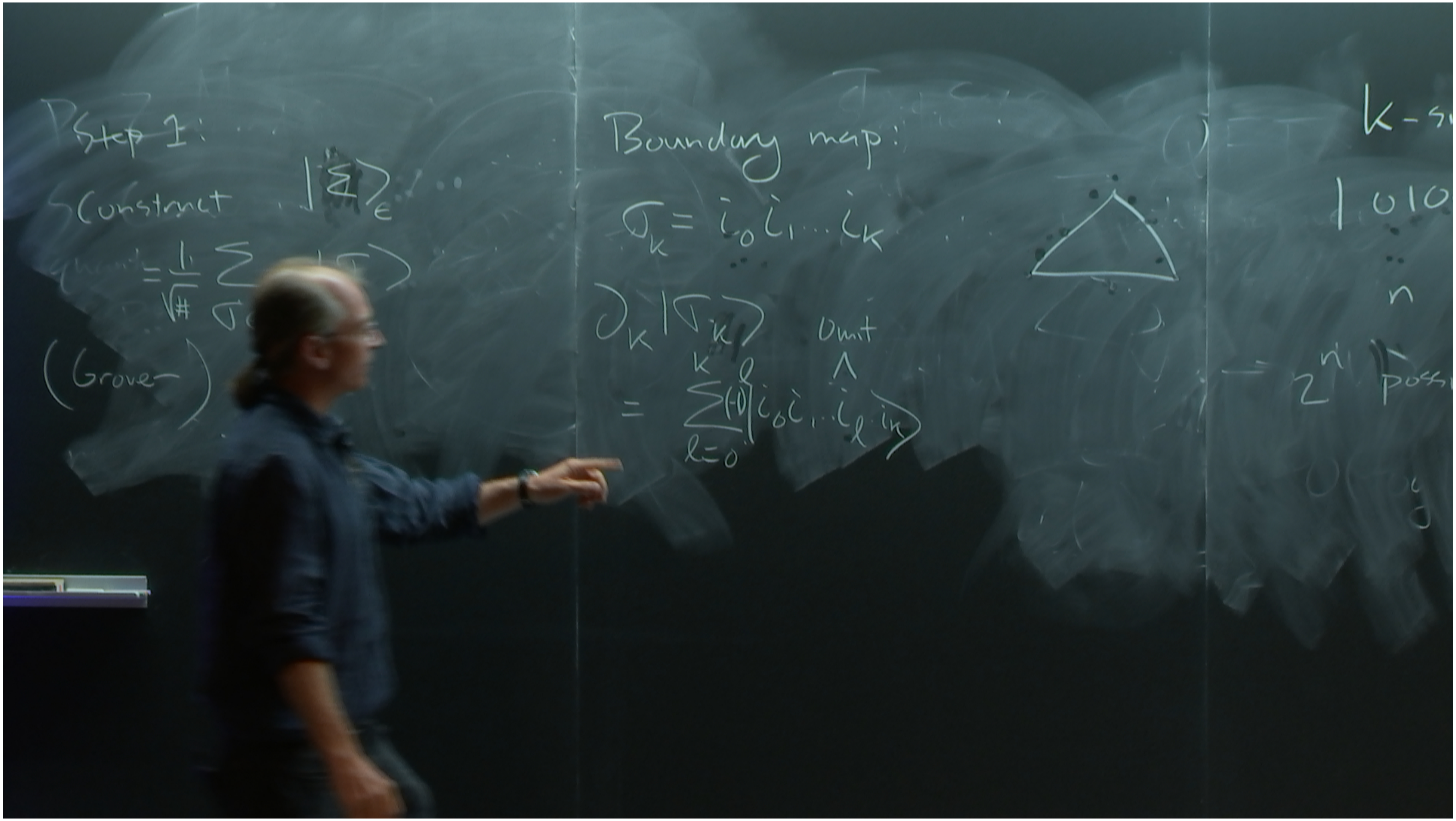

k-simplex
|01001011⟩
n points
2^n possible simplices
-simplex
-simplex
S_ϵ = simplicial complex at scale ϵ
→ (combinatorial)

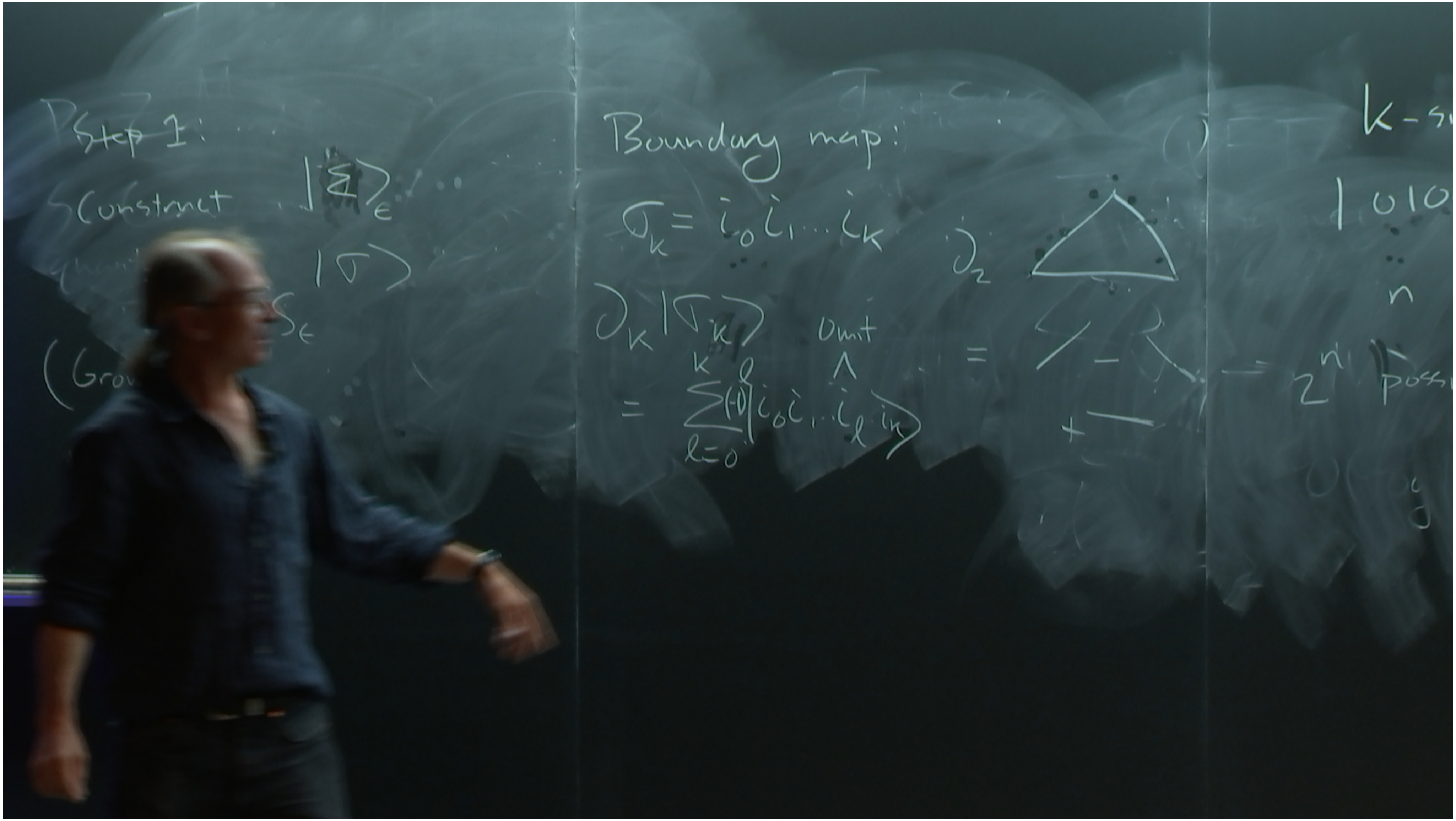

P. Zanardi
S. Garnerone
quantum
persistent
homology
no QRAM
Simplices:
0- simplex
1-simplex
2 simplex
3-simplex

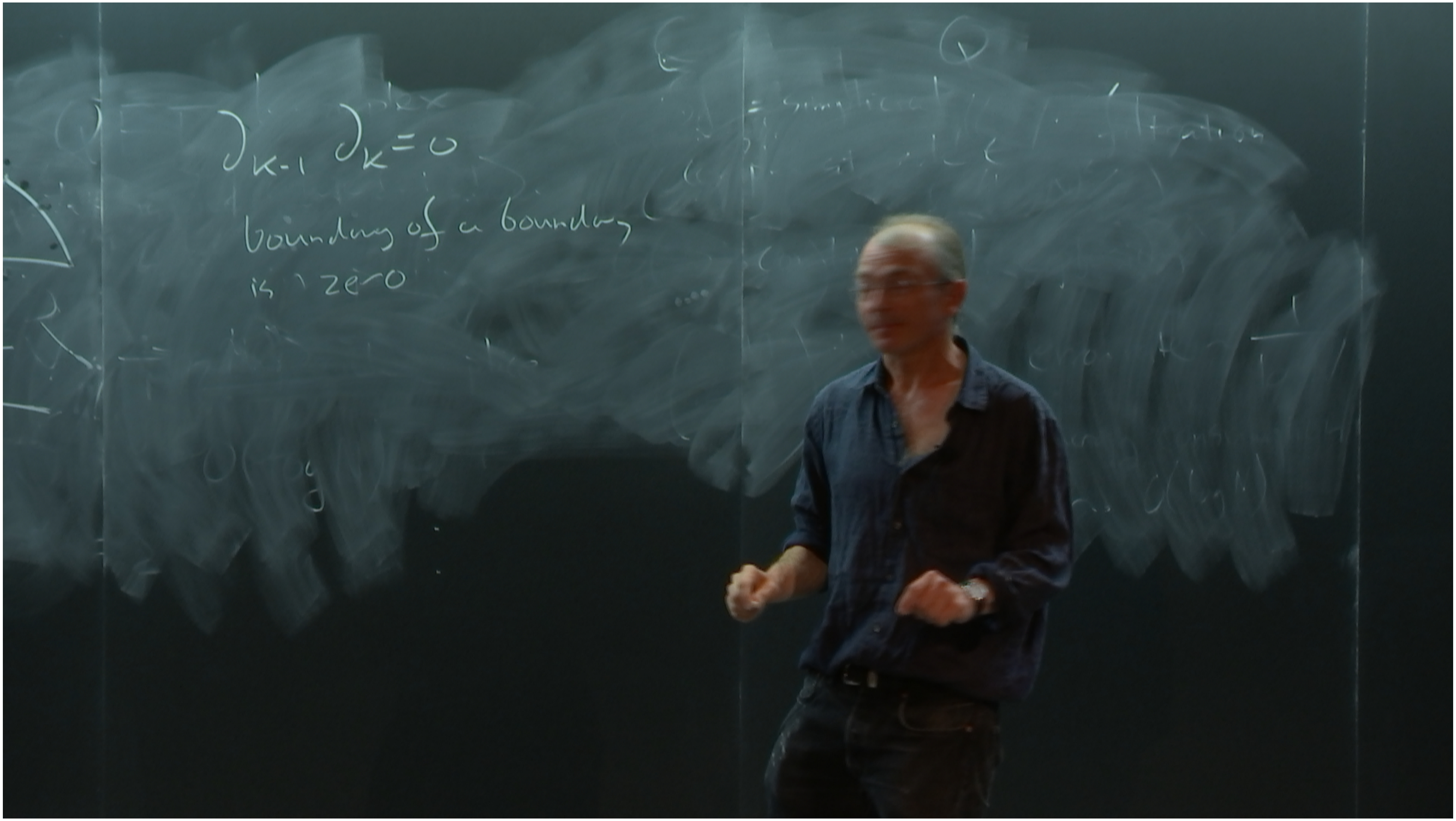

k-simplex
|01001011 >
n points
2^n possible simplices
S_ε = simplicial
complex at scale ε
filtration

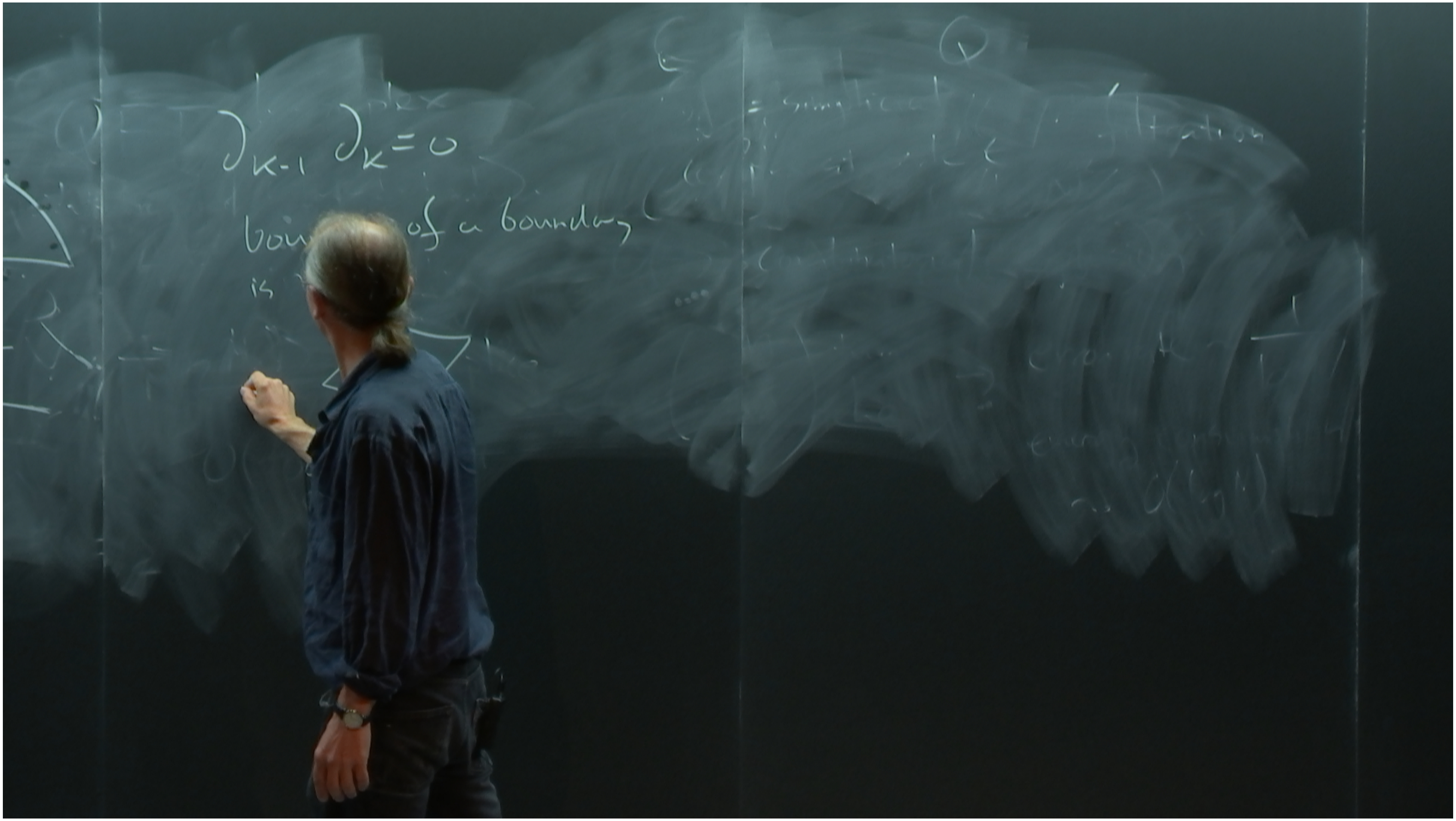

k-simplex
|01001011⟩
n points
2^n possible simplices
S_ε = simplicial complex at scale ε
filtration
→ combinatorial explosion
O(2^n)

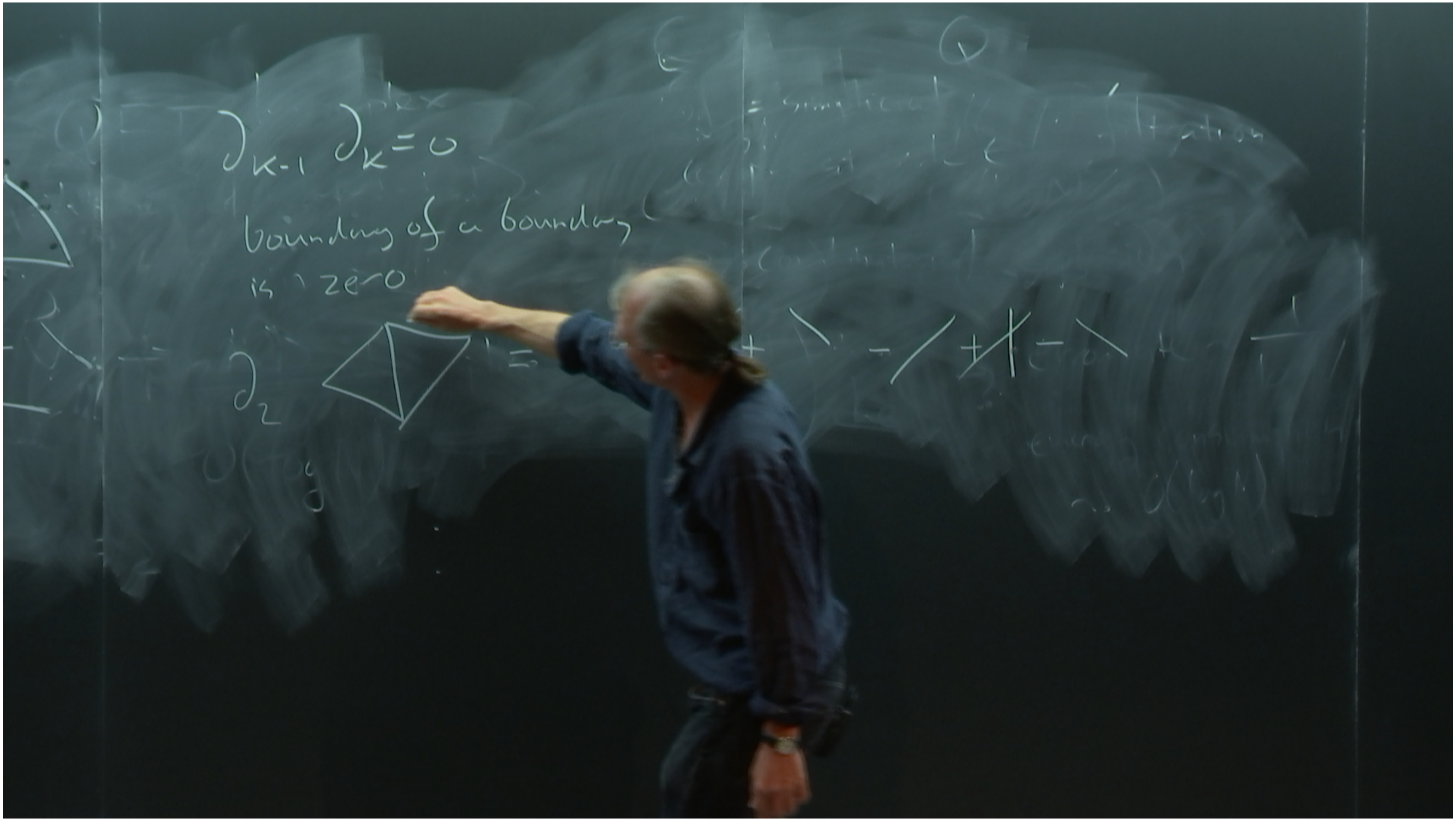

Step 1:
Construct $|\Sigma\rangle_\epsilon$
$= \frac{1}{\sqrt{\#}} \sum_{\sigma \in S_\epsilon} |\sigma\rangle$
(Grover)
Simplices:
0- simplex
2 simplex

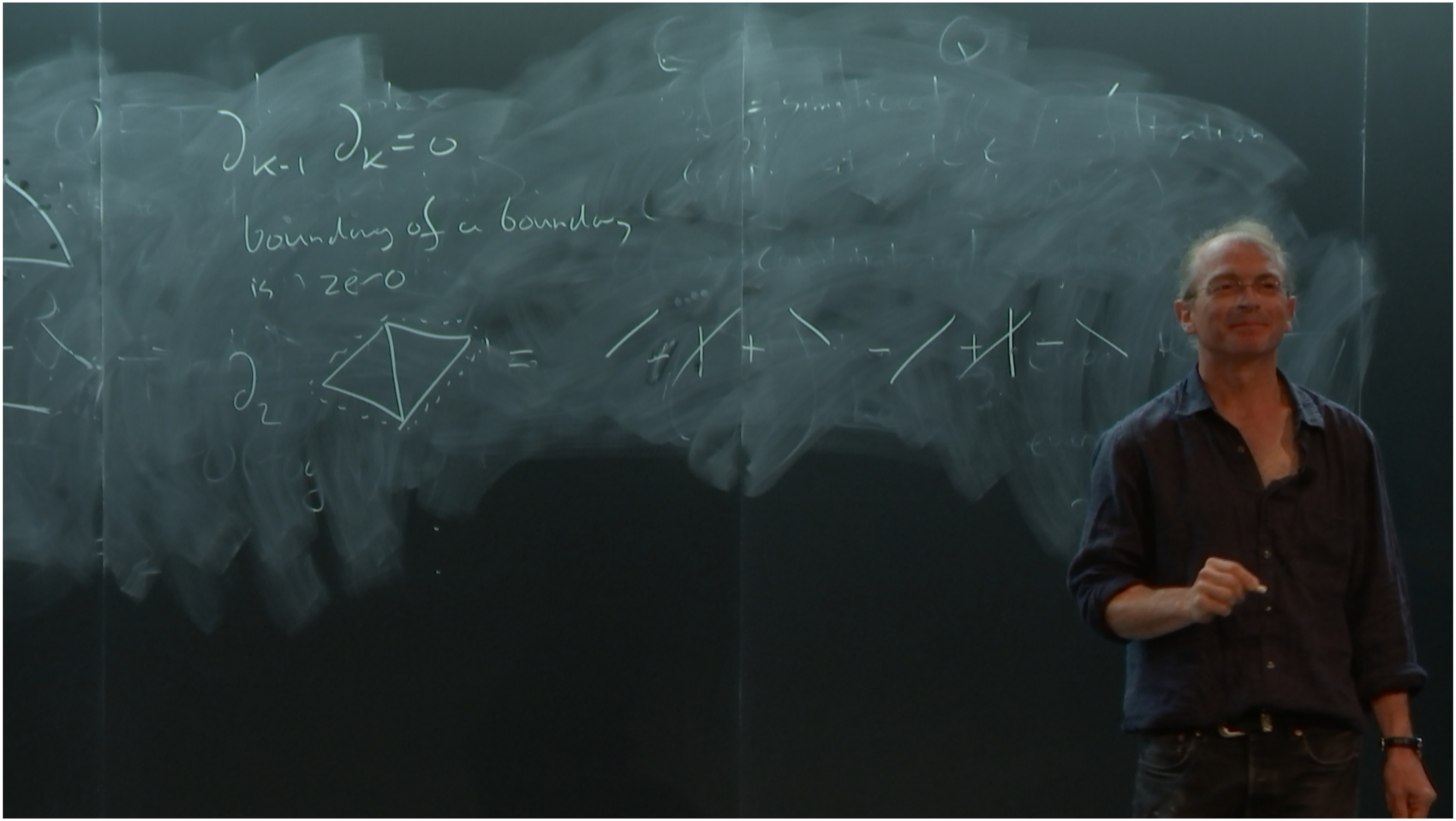

Step 1:
Construct
(Grover)
Boundary map:
Unit
k-s

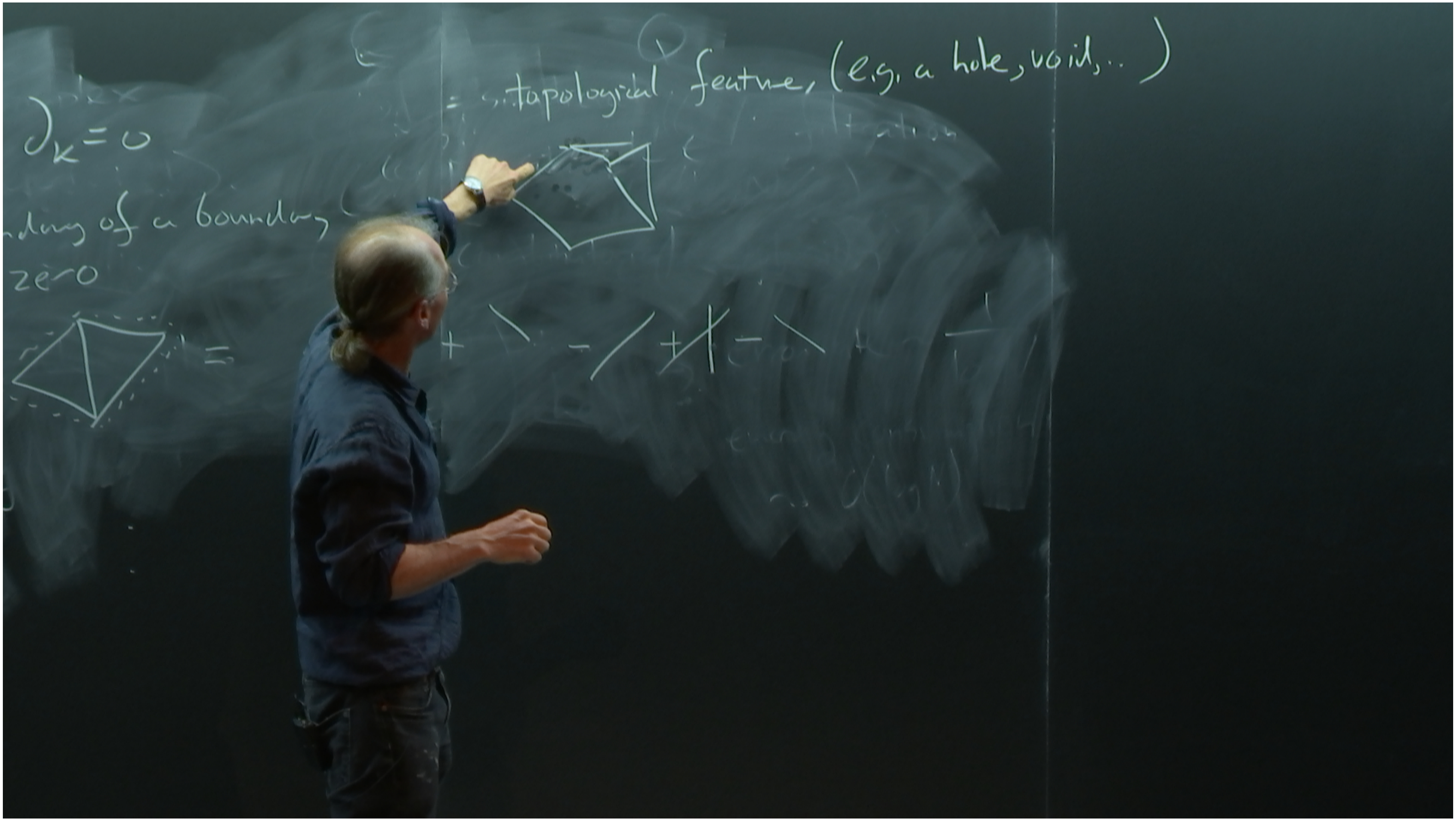

Step 1:
Construct
Boundary map:
omit

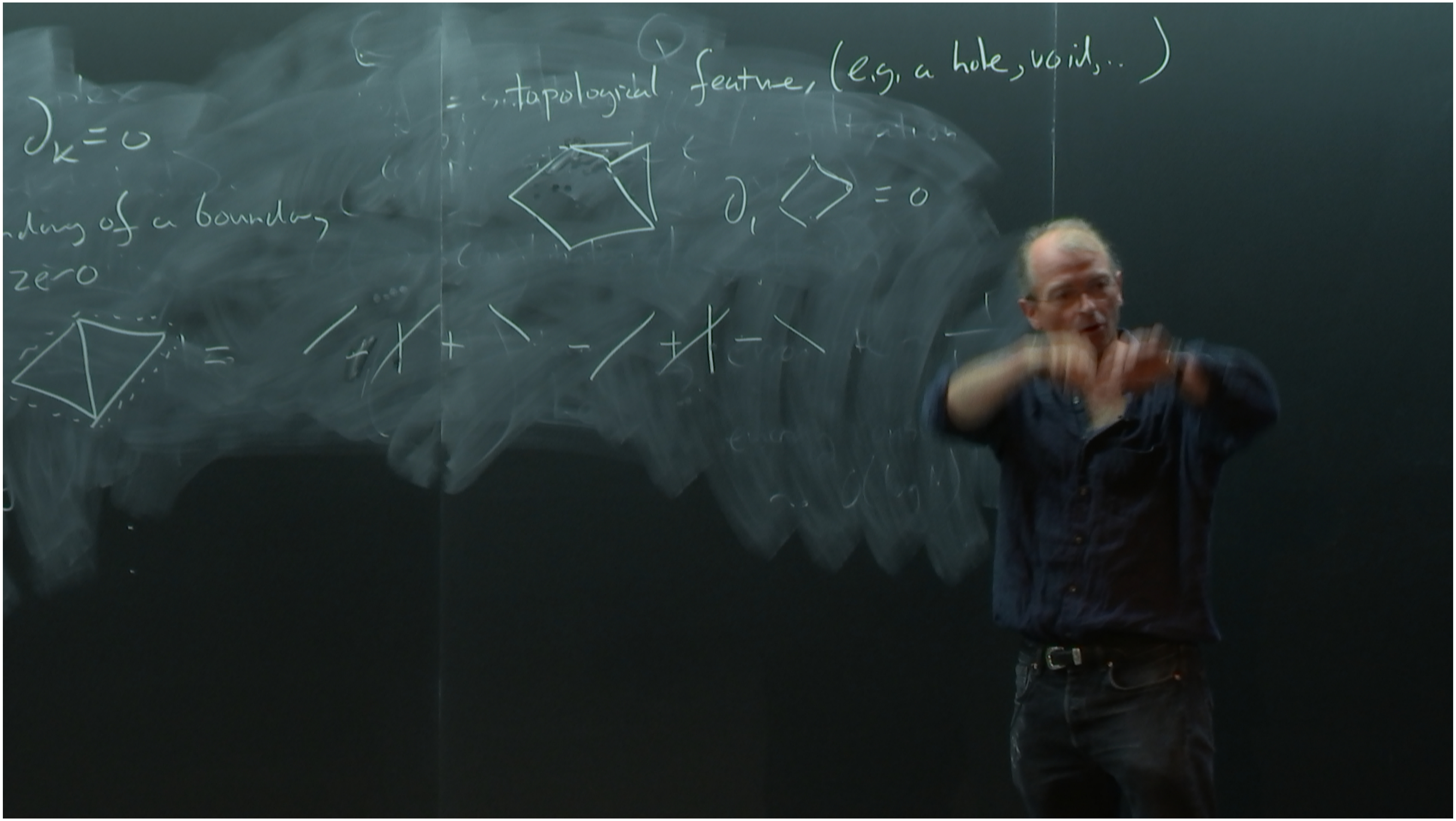

$\partial_{k-1} \partial_k = 0$
bounding of a boundary
is zero

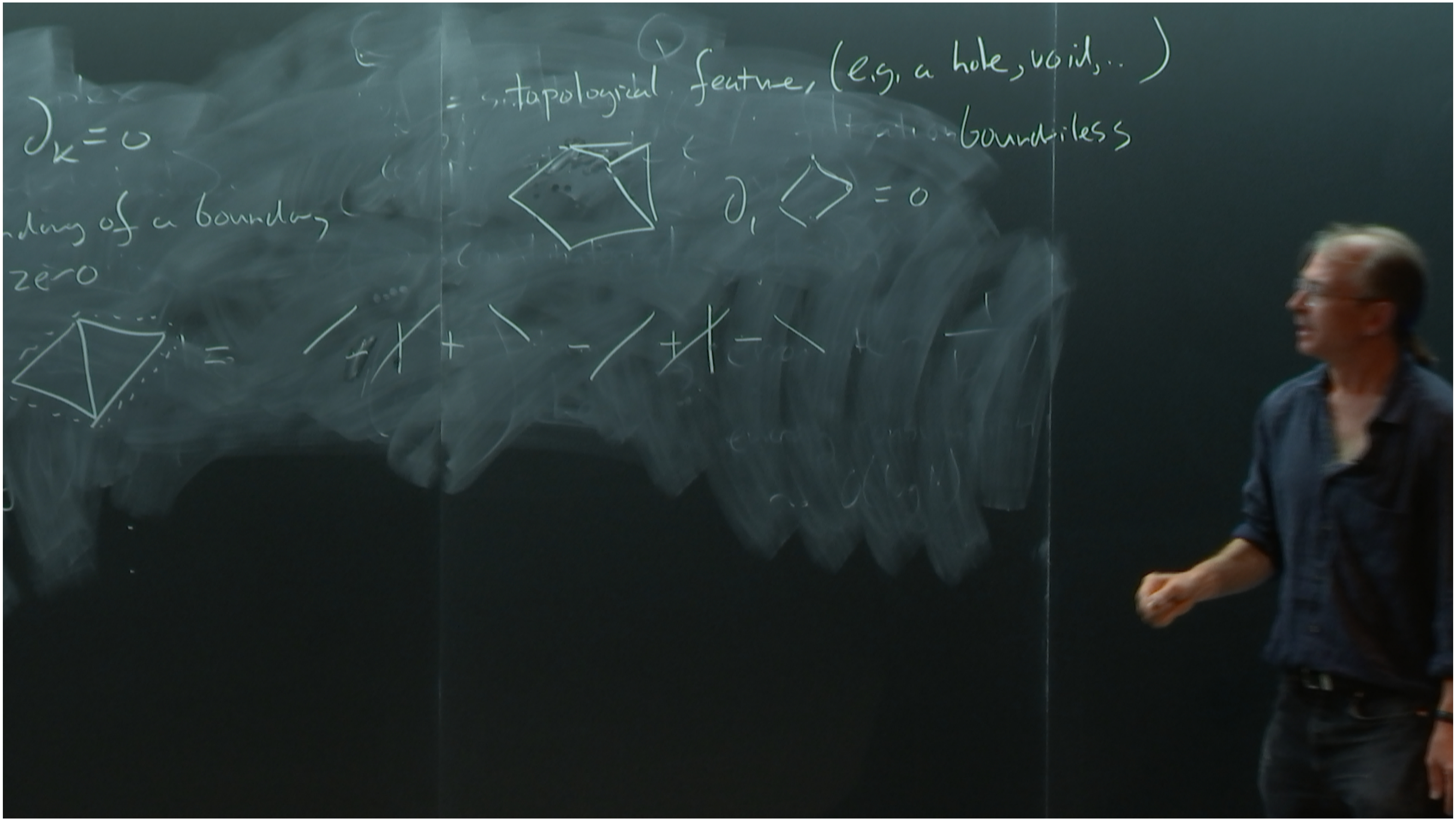

$\partial_{k-1} \partial_k = 0$
of a boundary
is

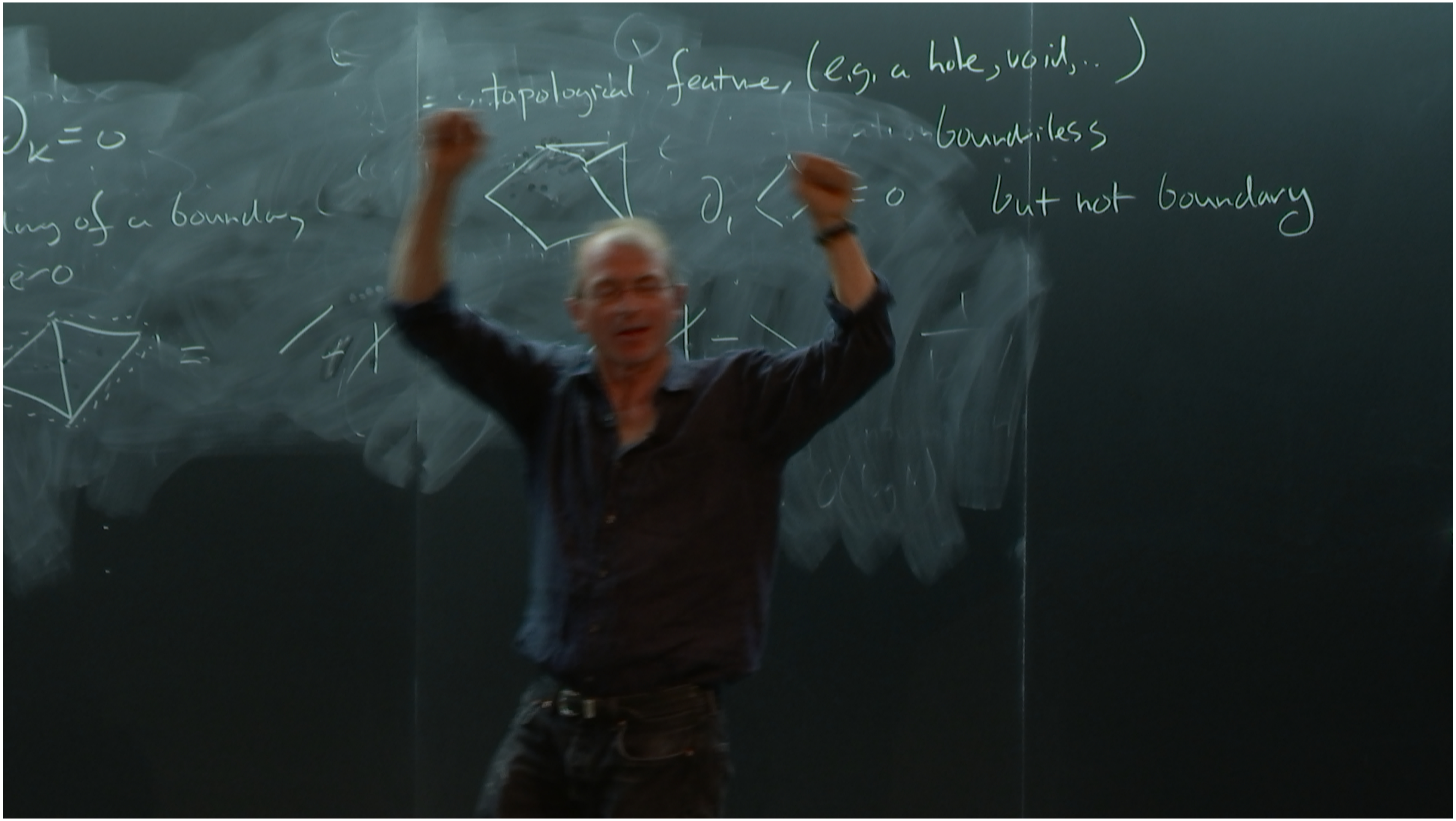

$\partial_{k-1}\partial_k = 0$
boundary of a boundary
is zero
∂_2

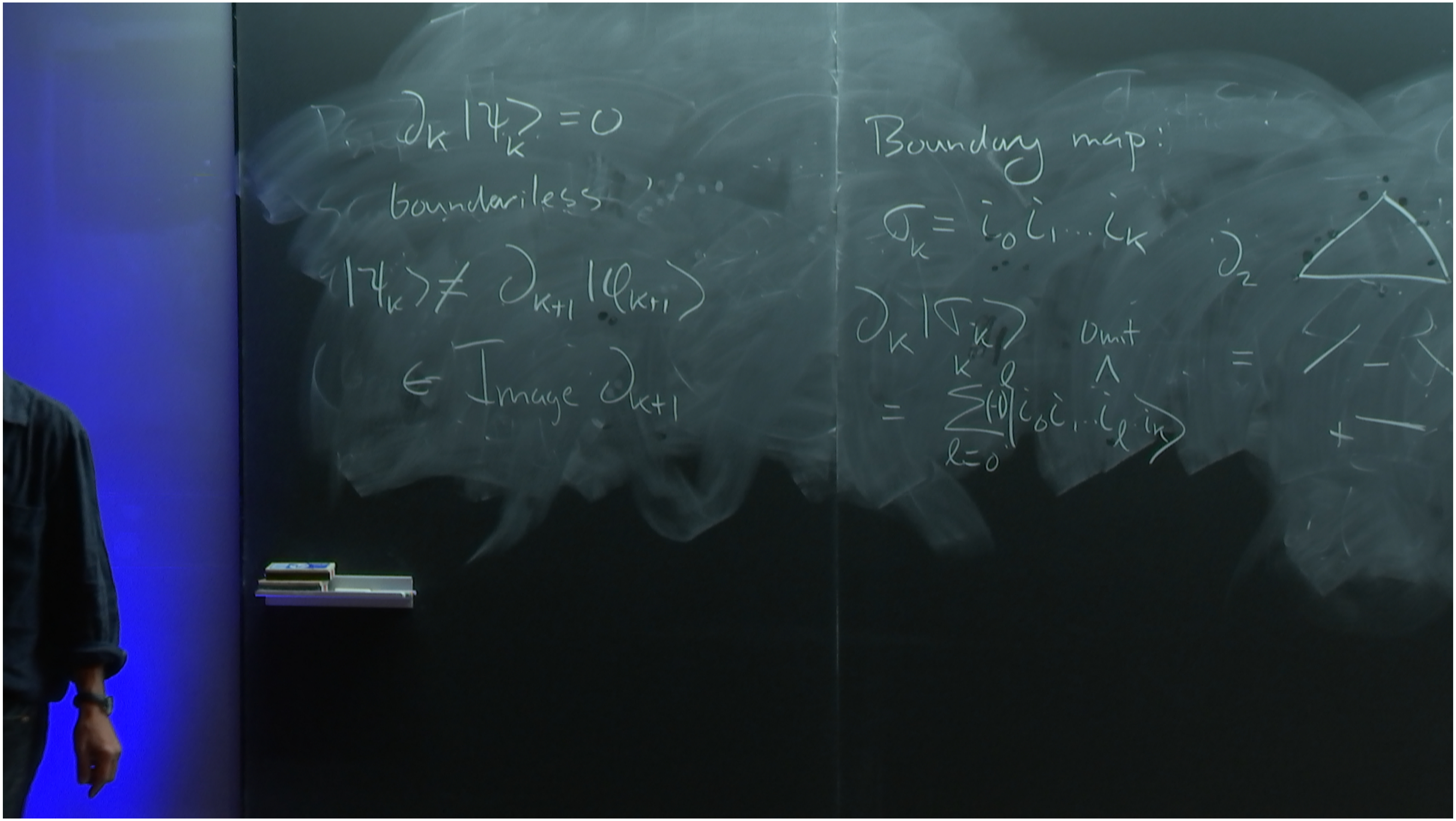

∂k-1 ∂k = 0
bounding of a boundary
is zero
∂2


topological feature, (e.g. a hole, void, ..)
∂k = 0
of a boundary
zero

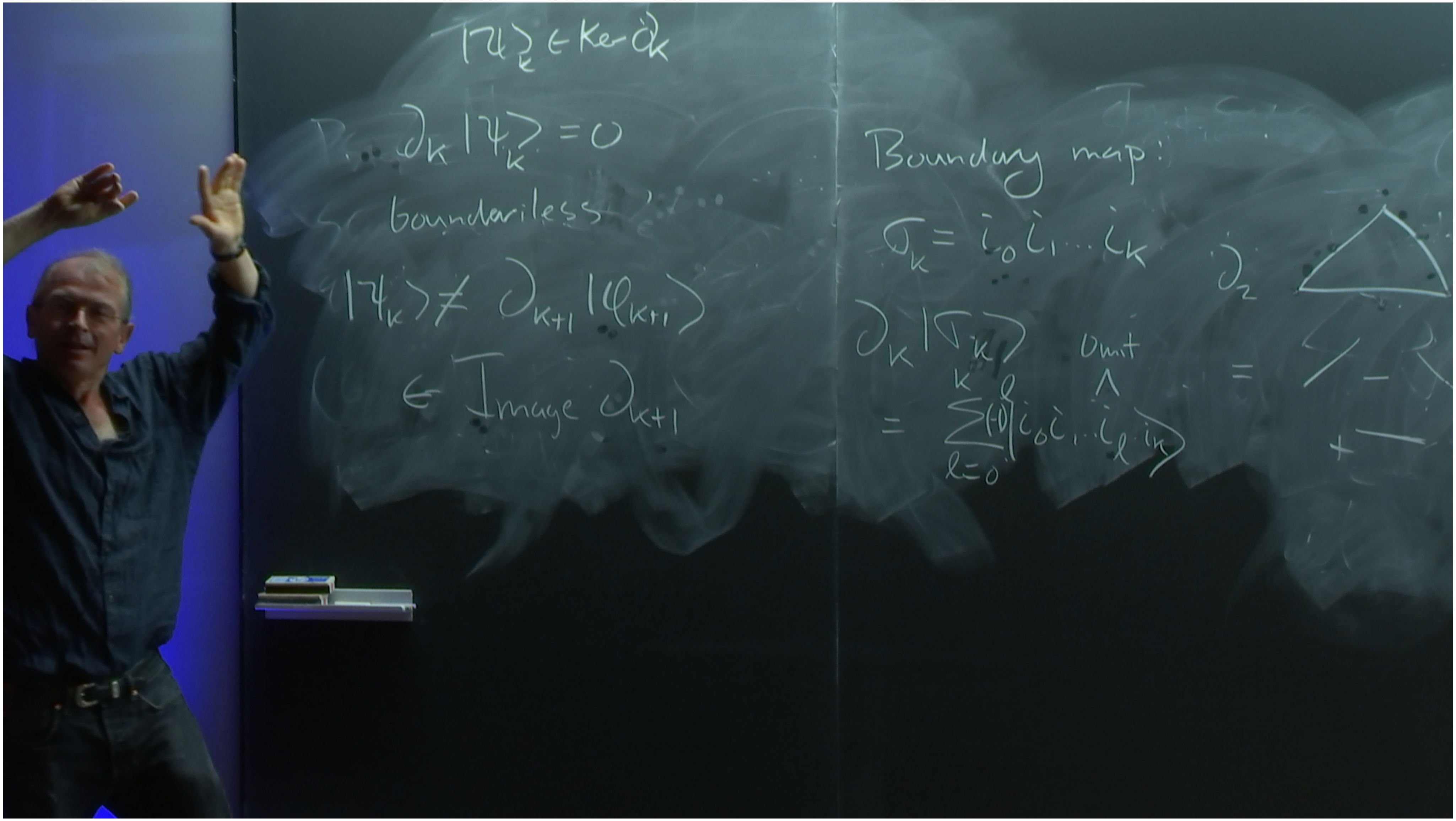

topological feature, (e.g. a hole, void, ..)
∂_k = 0
nding of a boundary
zero
∂_1 = 0

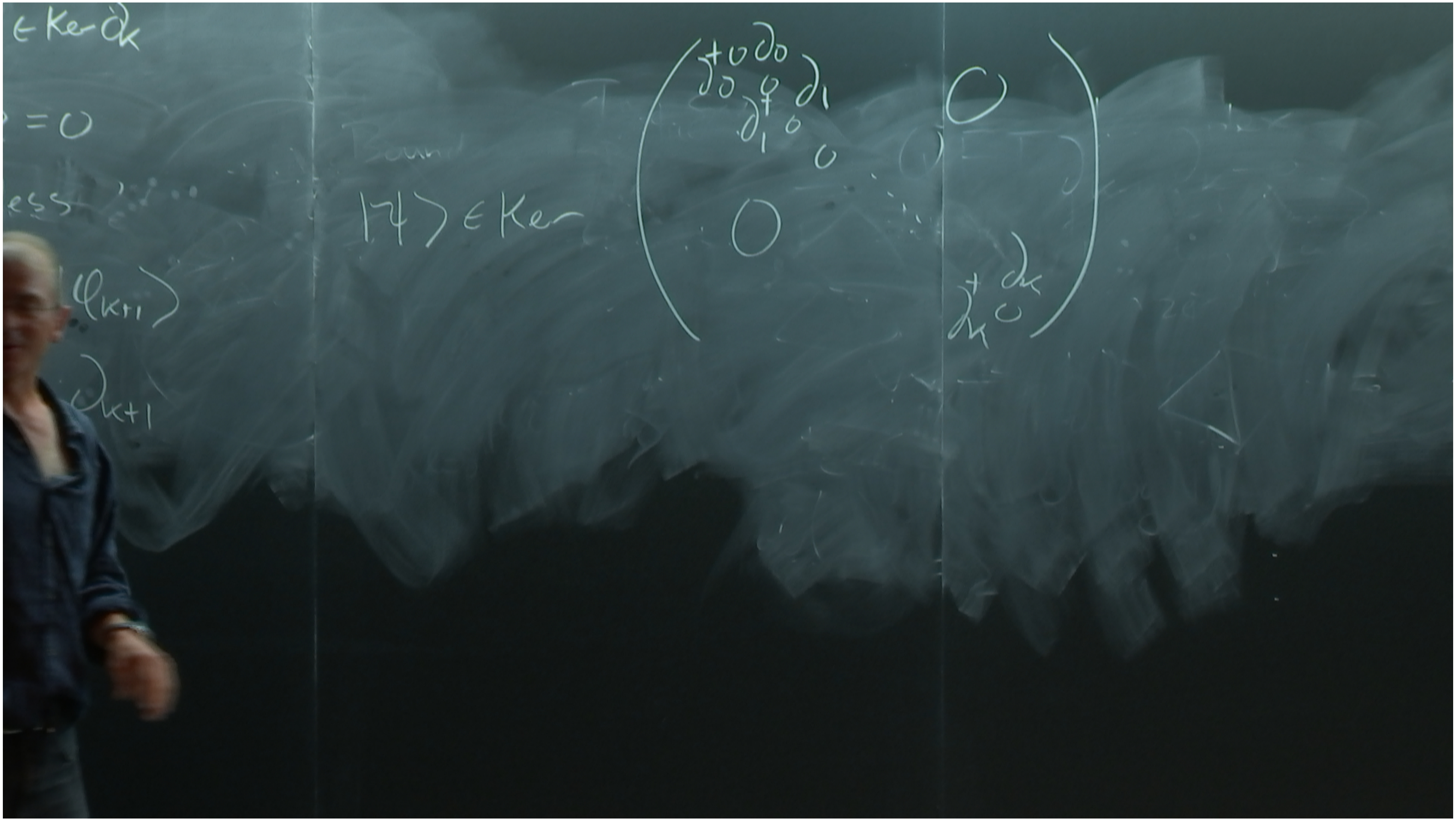

topological feature, (e.g. a hole, void, ..)
boundariless
$\partial_k = 0$
∂_1 = 0
nding of a boundary
zero

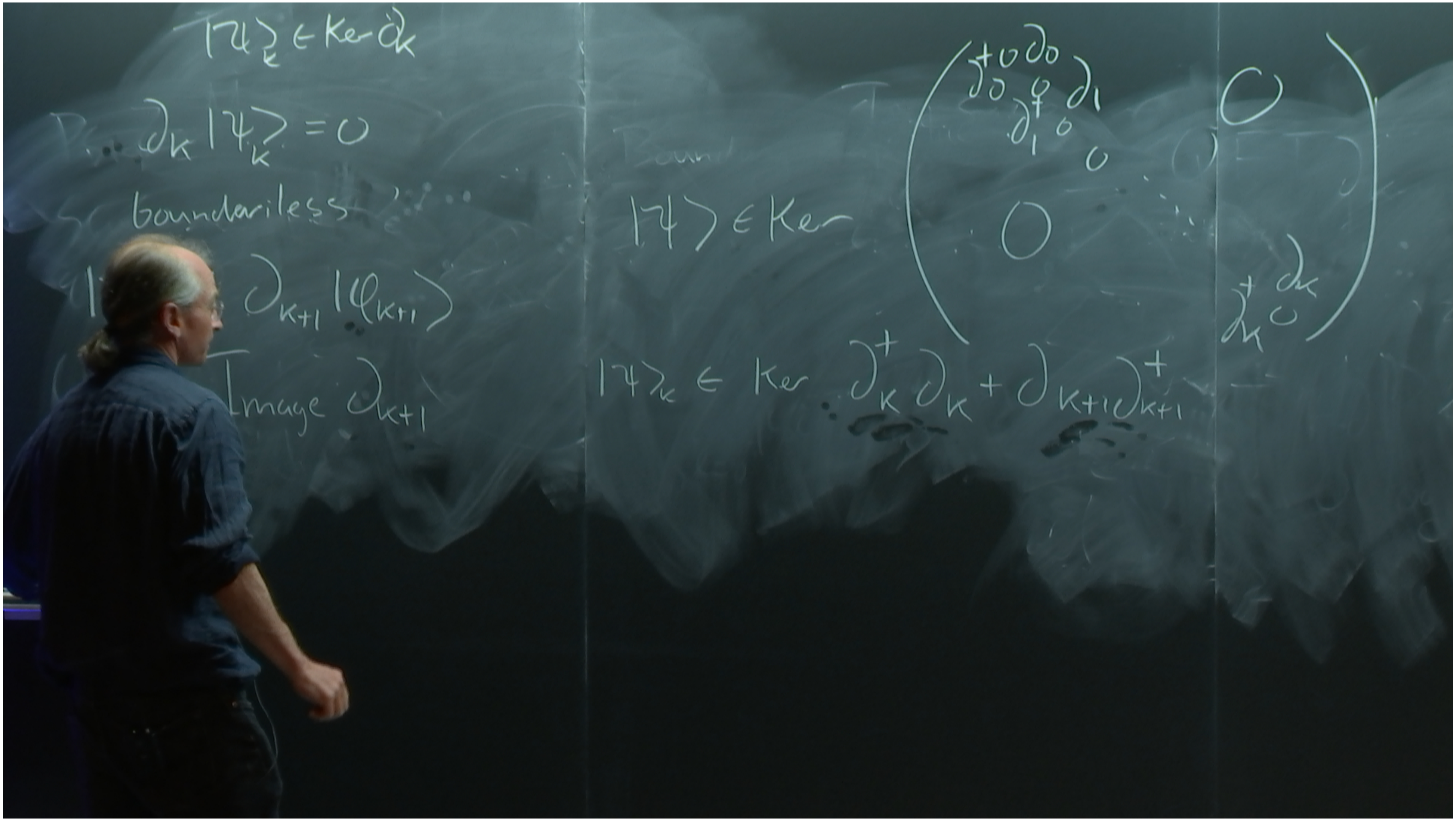

topological feature, (e.g. a hole, void, ..)
boundariless
but not boundary
of a boundary

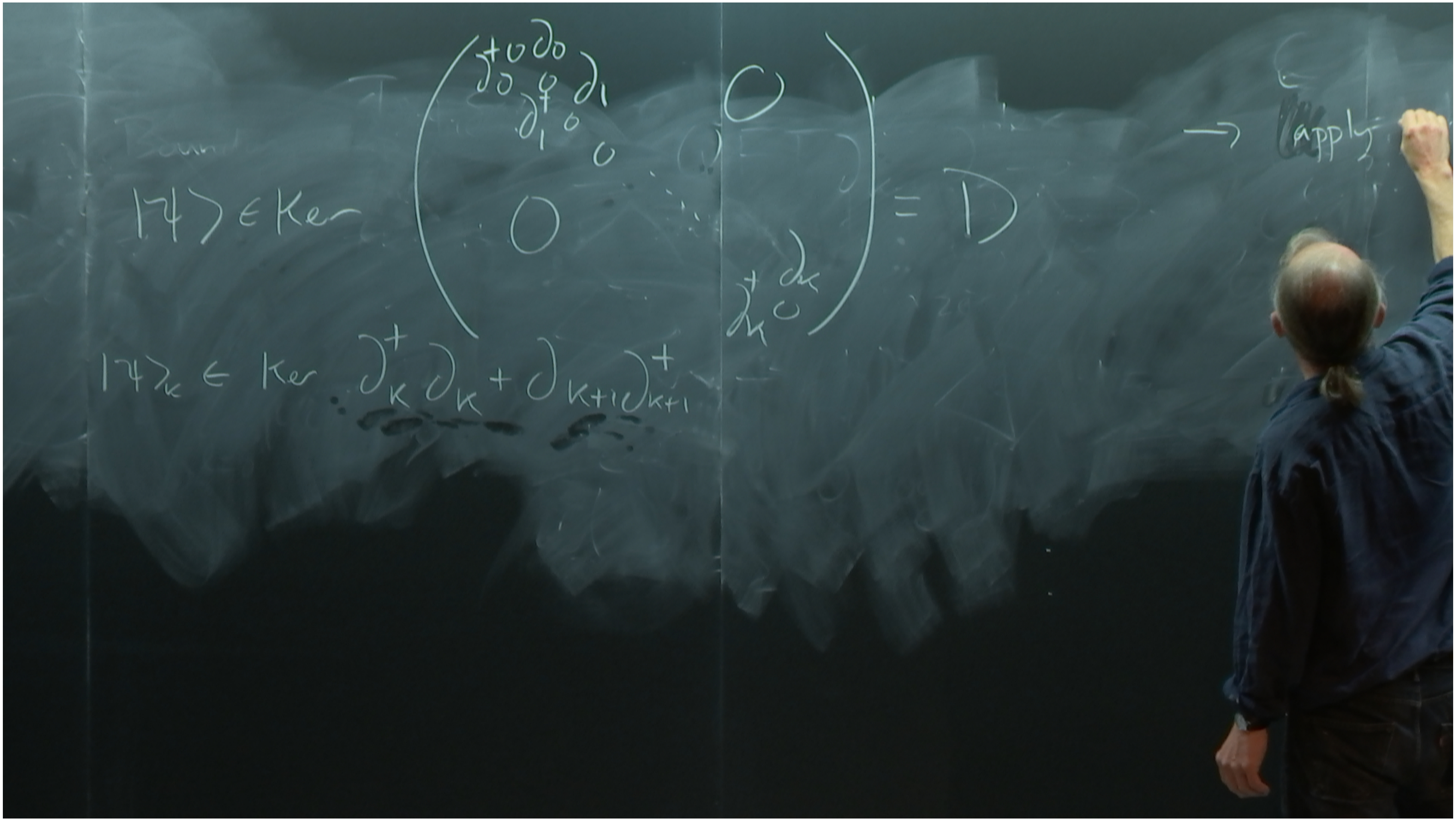


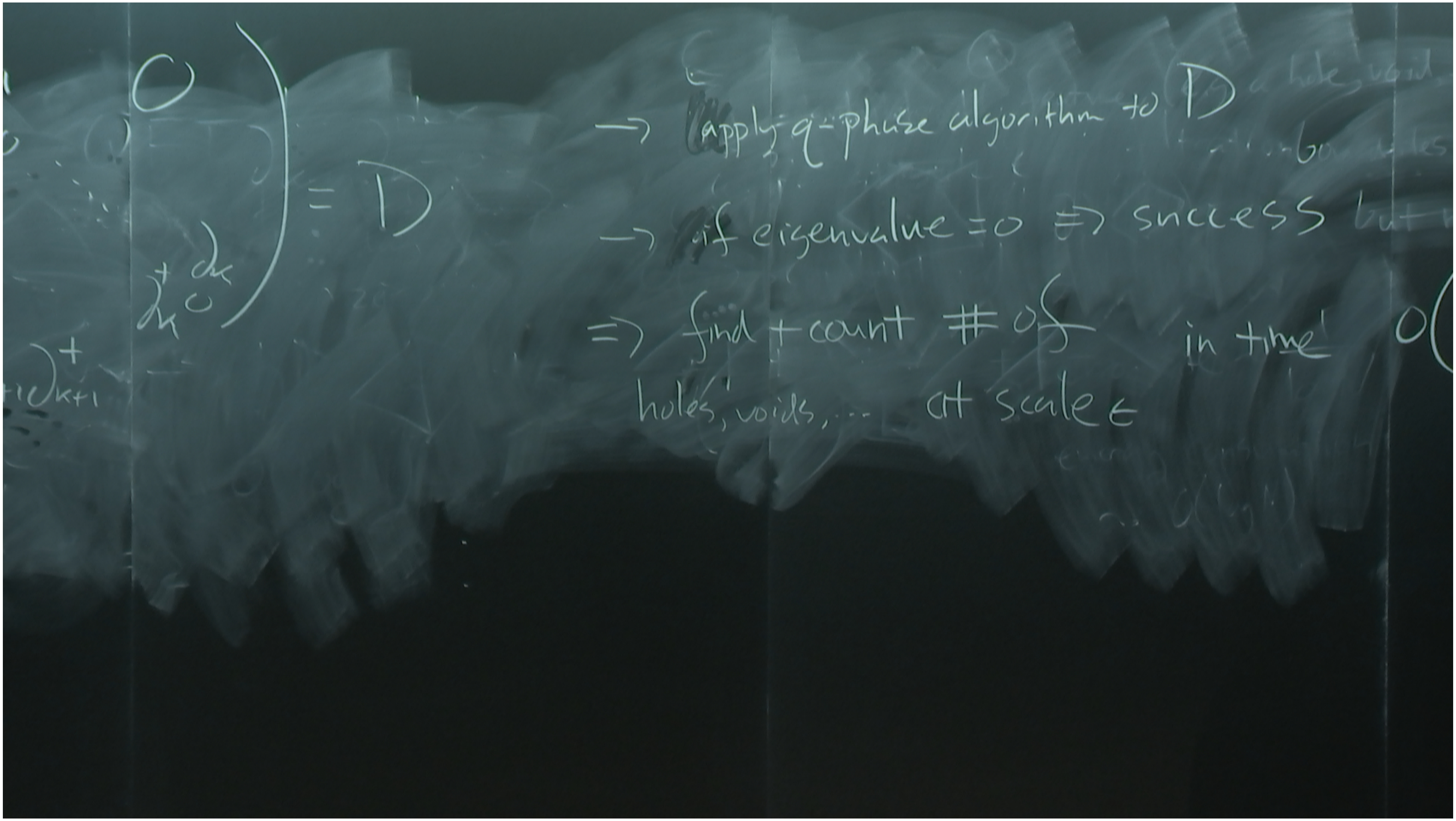
∂_k |ψ_k> = 0
boundariless
|ψ_k> ≠ ∂_{k+1} |φ_{k+1}>
∈ Image ∂_{k+1}
Boundary map:
σ_k = i_0 i_1 ... i_k
∂_k |σ_k>
omit
= Σ_{l=0}^{k} (-1)^l |i_0 i_1 ... î_l ... i_k>
∂_2

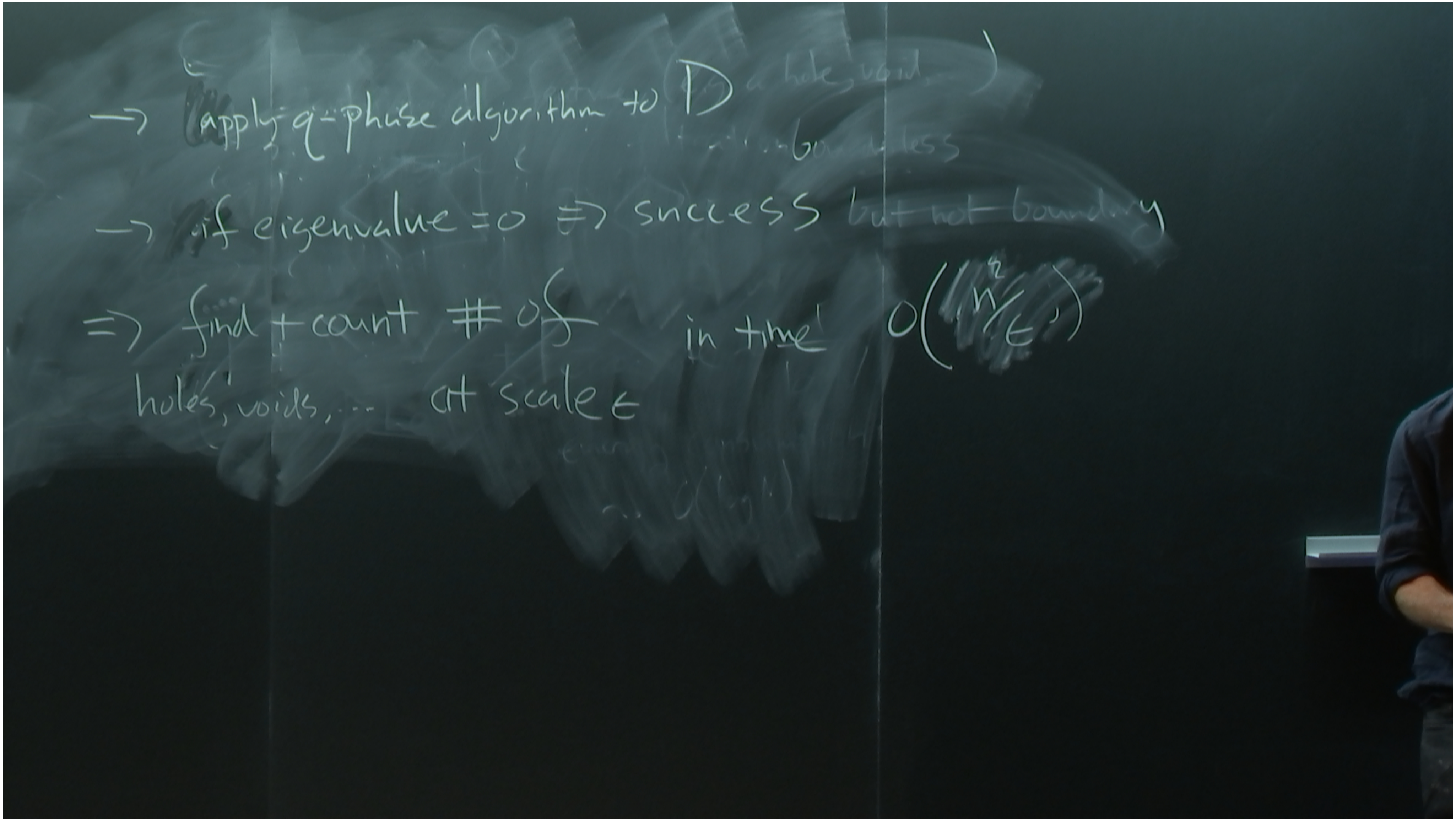

|ψ⟩_k ∈ Ker ∂_k
∂_k |ψ_k⟩ = 0
boundariless
|ψ_k⟩ ≠ ∂_{k+1} |φ_{k+1}⟩
∈ Image ∂_{k+1}
Boundary map:
σ_k = i_0 i_1 … i_k
∂_k |σ_k⟩
omit
= Σ_{l=0}^{k} (-1)^l |i_0 i_1 … î_l … i_k⟩
∂_2
=
+

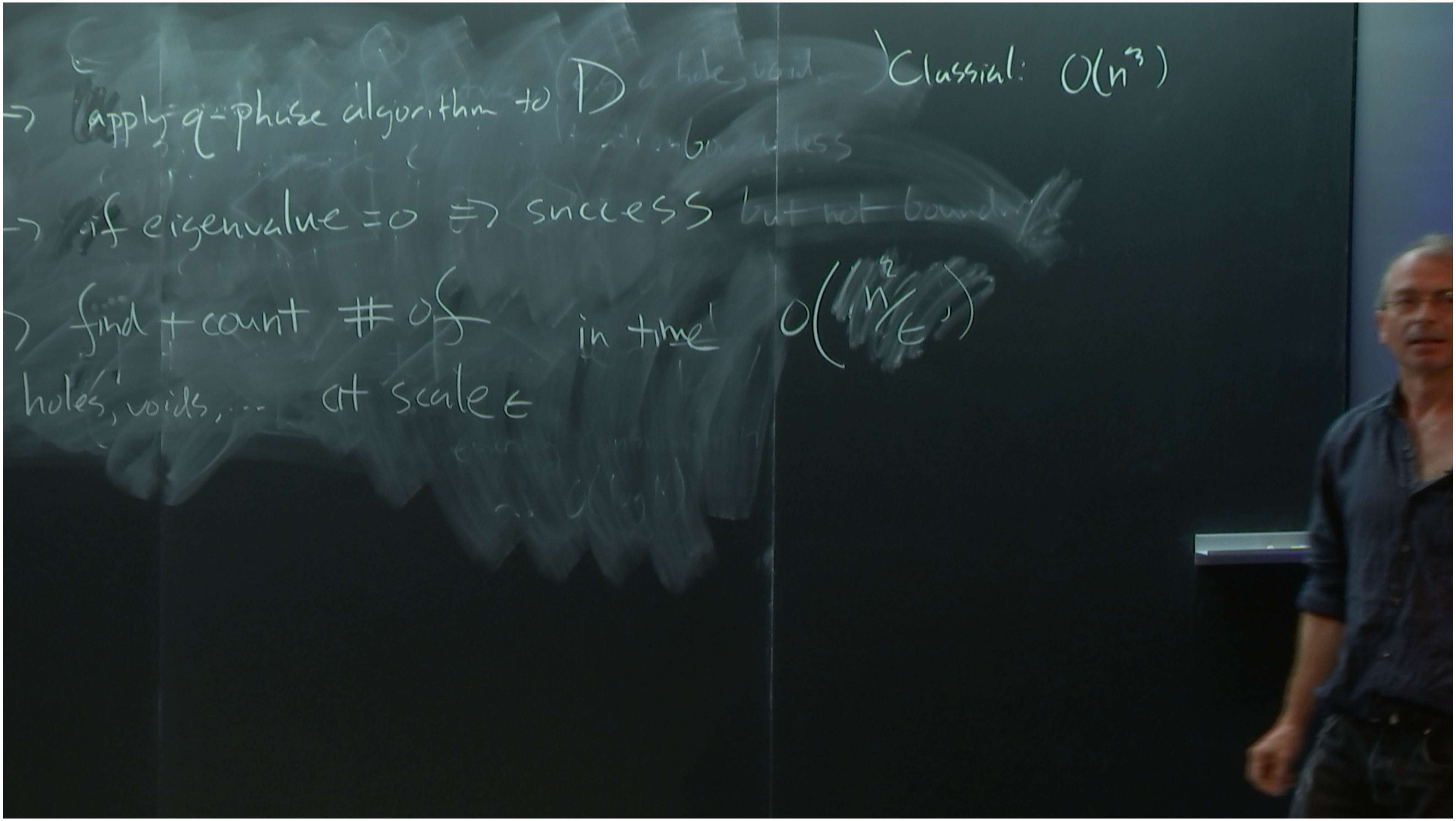

∈ Ker ∂k
= 0
|ψ⟩ ∈ Ker


boundariless
Image

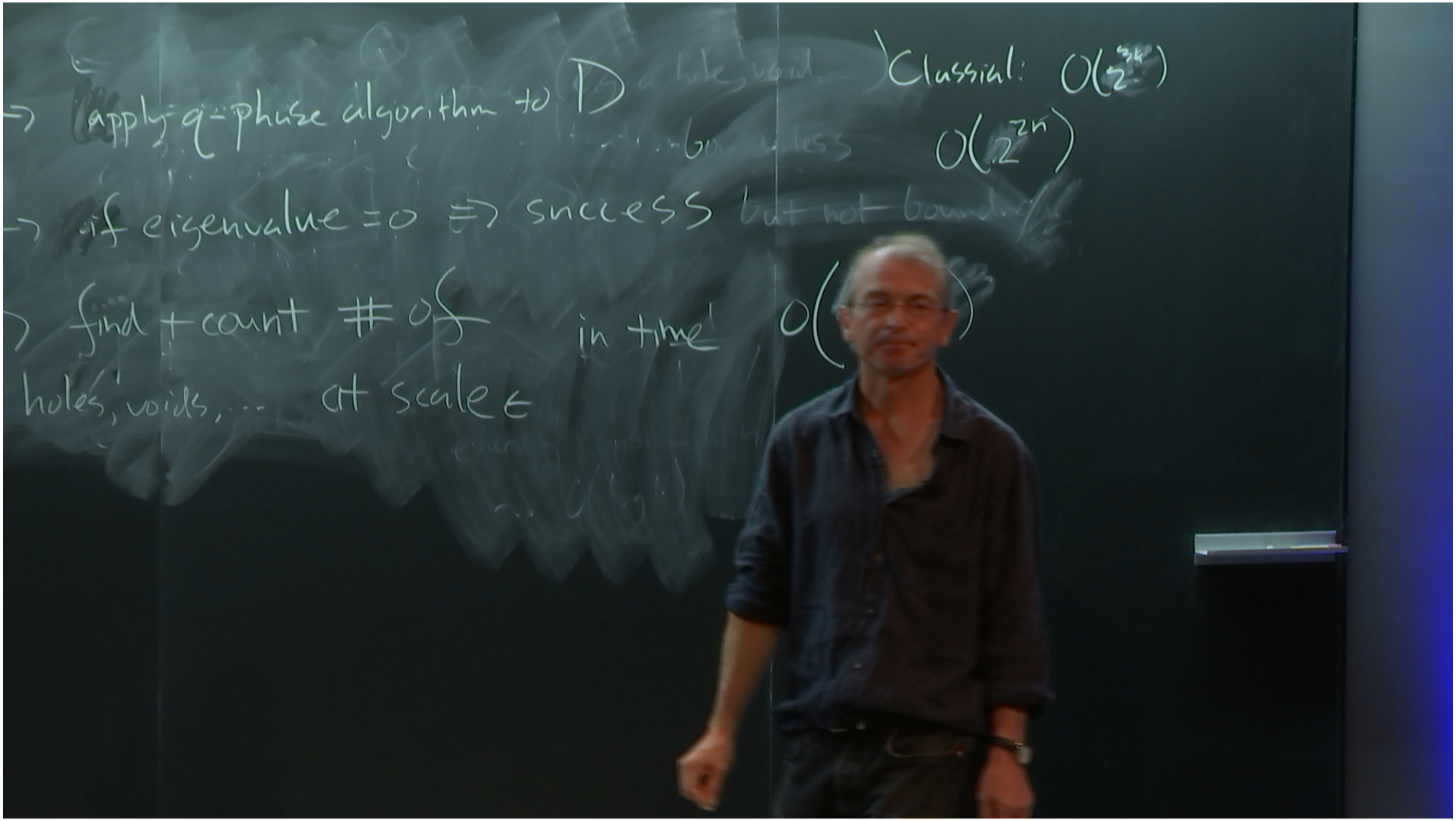

$|\psi\rangle \in \mathrm{Ker}$
$= D$
$|\psi\rangle_k \in \mathrm{Ker}\ \partial_k^\dagger \partial_k + \partial_{k+1}\partial_{k+1}^\dagger$
→ apply

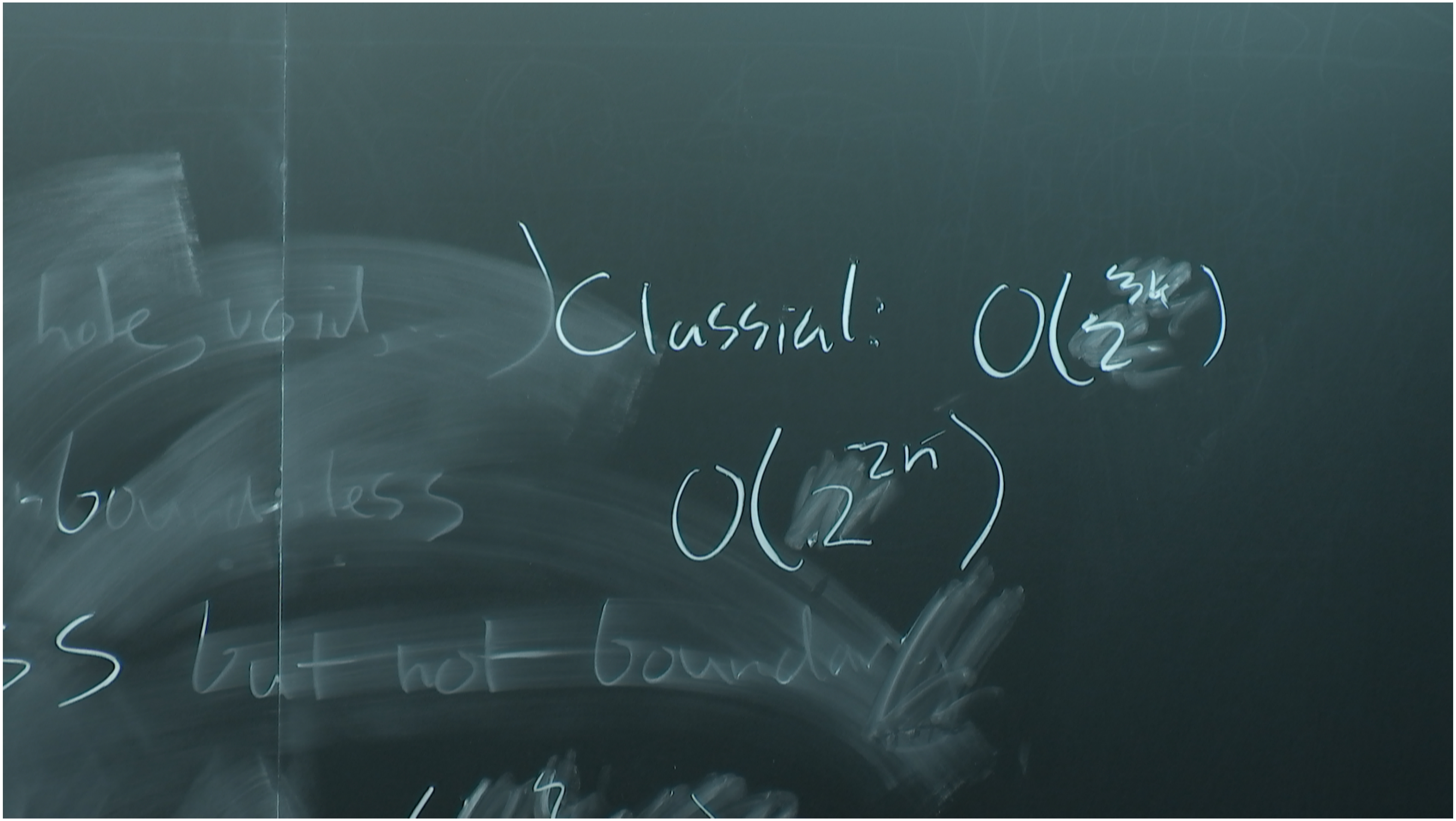

= D
→ apply q-phase algorithm to D
→ if eigenvalue = 0 ⇒ success
⇒ find + count # of
in time
holes, voids, ... at scale ε

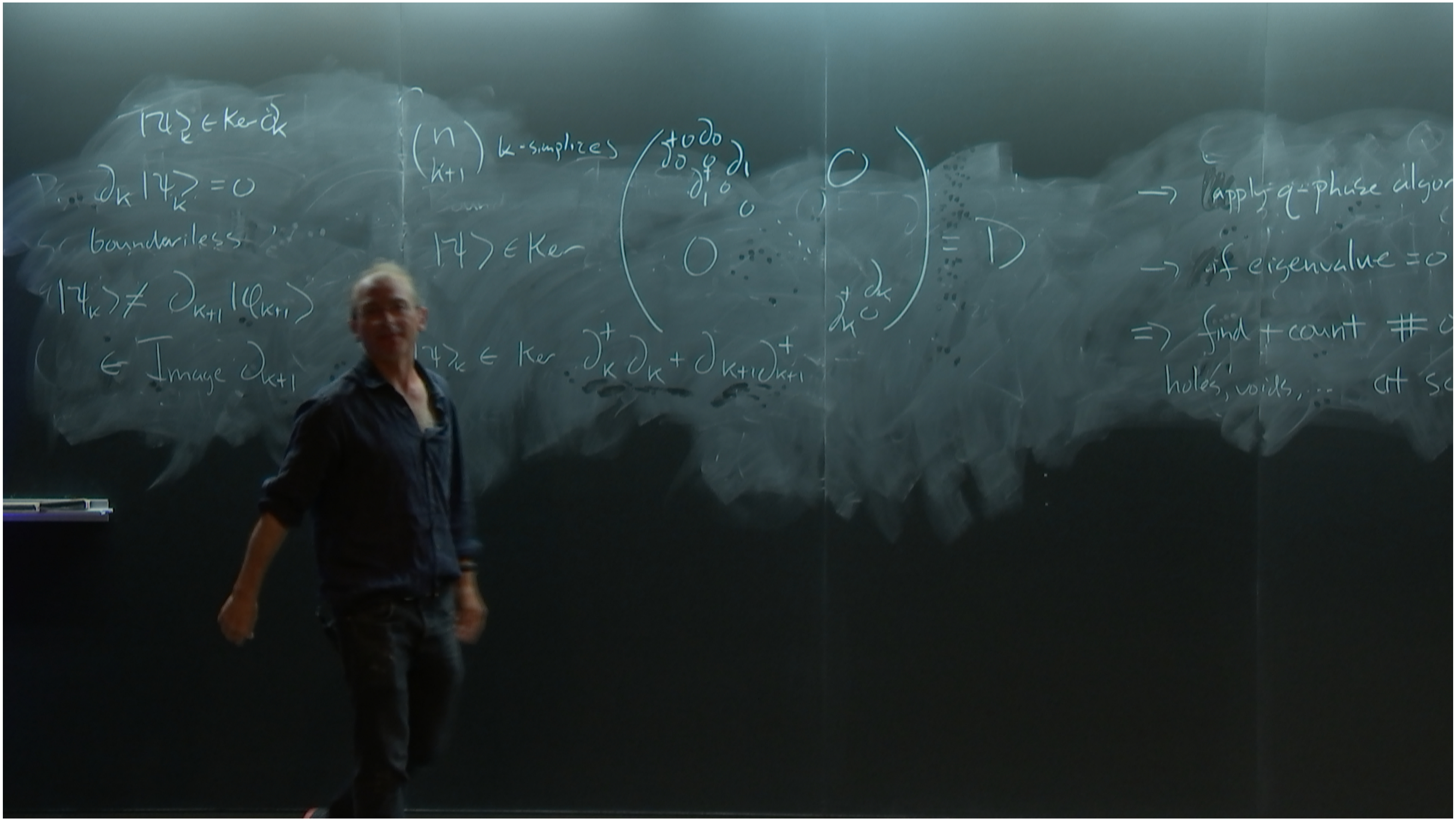

→ apply q-phase algorithm to D
→ if eigenvalue = 0 ⇒ success
⇒ find + count # of
holes, voids, ... at scale ε
in time! O(



→ apply q-phase algorithm to D
Classical: O(n³)
→ if eigenvalue = 0 ⇒ success
→ find + count # of
in time
holes, voids, ... at scale ε


→ apply q-phase algorithm to D
→ if eigenvalue = 0 ⇒ success
→ find + count # of holes, voids, ... at scale ε in time O(
Classical: O(2^3k)
O(2^2n)


Classial:


k-simplizes
boundariless
Image
apply q-phase algor
if eigenvalue = 0
find + count # o
holes, voids, ...
at sc