Title: Towards Quantum Supremacy with Near-Term Devices

Date: Aug 10, 2016 09:30 AM

URL: http://pirsa.org/16080008

Abstract: Can quantum computers outperform classical computers on any computational problem in the near future? We study the problem of sampling from the output distribution of random quantum circuits.

Sampling from this distribution requires an exponential amount of classical computational resources. We argue that quantum supremacy can be achieved in the near future with approximately fifty superconducting qubits and without error correction despite the fact that quantum random circuits are extremely sensitive to errors.

Quantum supremacy

J. Preskill, 2012

- With a quantum device
  - Solve a well-defined computational problem
  - Beyond the capabilities of state-of-the-art classical supercomputers
  - In the near future
    - without error correction.

- Not necessarily a practical problem.

Approaches to quantum supremacy

- Optimization of classical functions
  - quantum annealing.
  - quantum approximate optimization algorithm (E. Farhi et. al).
- Hamiltonian evolution that cannot be simulated on classical computers.
- Variational quantum eigensolver.
  - Ground state energy of a Hamiltonian.
- Approximate sampling from a well defined distribution.
  - Commuting quantum circuits (M. Bremner et. al).
  - Boson sampling (Aaronson and Arkhipov).
- Sampling from the output of universal random quantum circuits.

Requisites for quantum supremacy in the near-term

- Not many qubits.
- No error correction in quantum devices.
- Shallow quantum circuits.
- Cost exponential in the number of qubits on classical computers.
- Specific figure of merit for the computational task.
  - Unfortunately, we lack witness.
  - Measure the figure of merit up to quantum supremacy frontier.
  - Well understood extrapolation of the figure of merit beyond the quantum supremacy frontier where it can not be measured
  - Naturally related to fidelity.
- Predictions from theory for the figure of merit.

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Random quantum circuits

- Links correspond to Controlled Z (CZ) gates.
- Experimental constraints: two CZ gates cannot be share a bond
- Layer 0
  - Hadamard gates.
- Layer i from 1 to d
  - select (i mod 8)th CZ configuration.
  - on unoccupied sites, place at random single (subject to some constraints, arXiv:1608.00263) qubit gates from the set $X^{1/2}$, $Y^{1/2}$, T
- Good set in terms of the required circuit depth.

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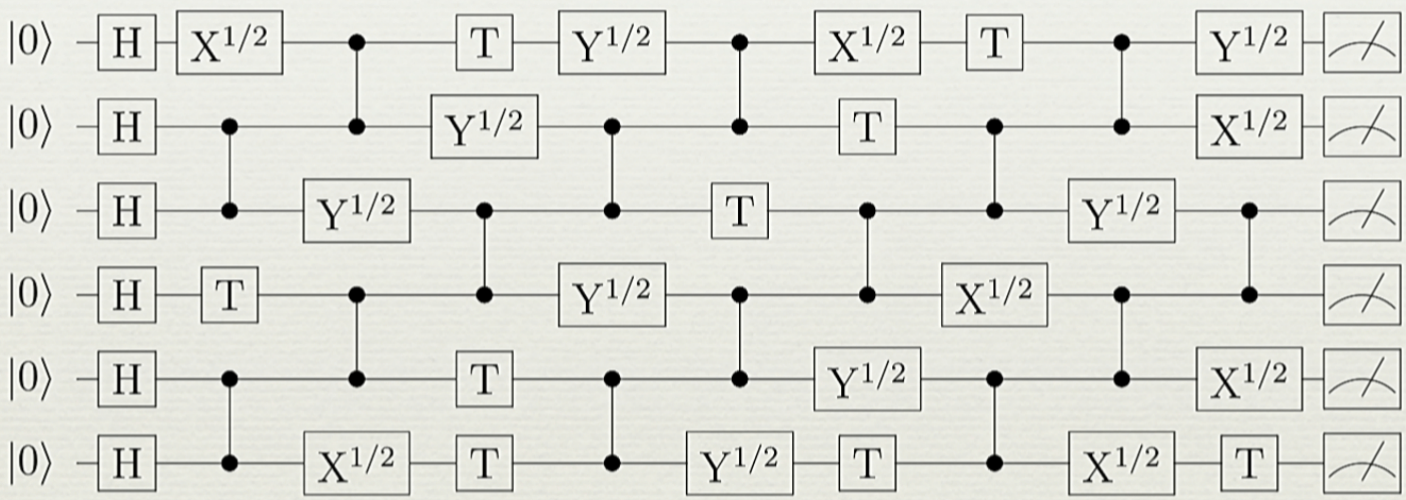

Porter-Thomas distribution

- Random circuit U

$$|\Psi\rangle = U|0\rangle = \sum_{i=1}^{N} c_i |x_i\rangle$$

- Sample from the output distribution.
- Outcome probabilities $p_i = |c_i|^2 = |\langle x_i|U|0\rangle|^2$

- Real and imaginary parts of c_i are uniformly distributed on a 2N-dimensional sphere if the circuit is of sufficient depth.

- Porter-Thomas distribution $f_{\mathrm{PT}}(p) = N \exp(-Np)$

- Circuit should have sufficient depth to reach the Porter-Thomas regime.

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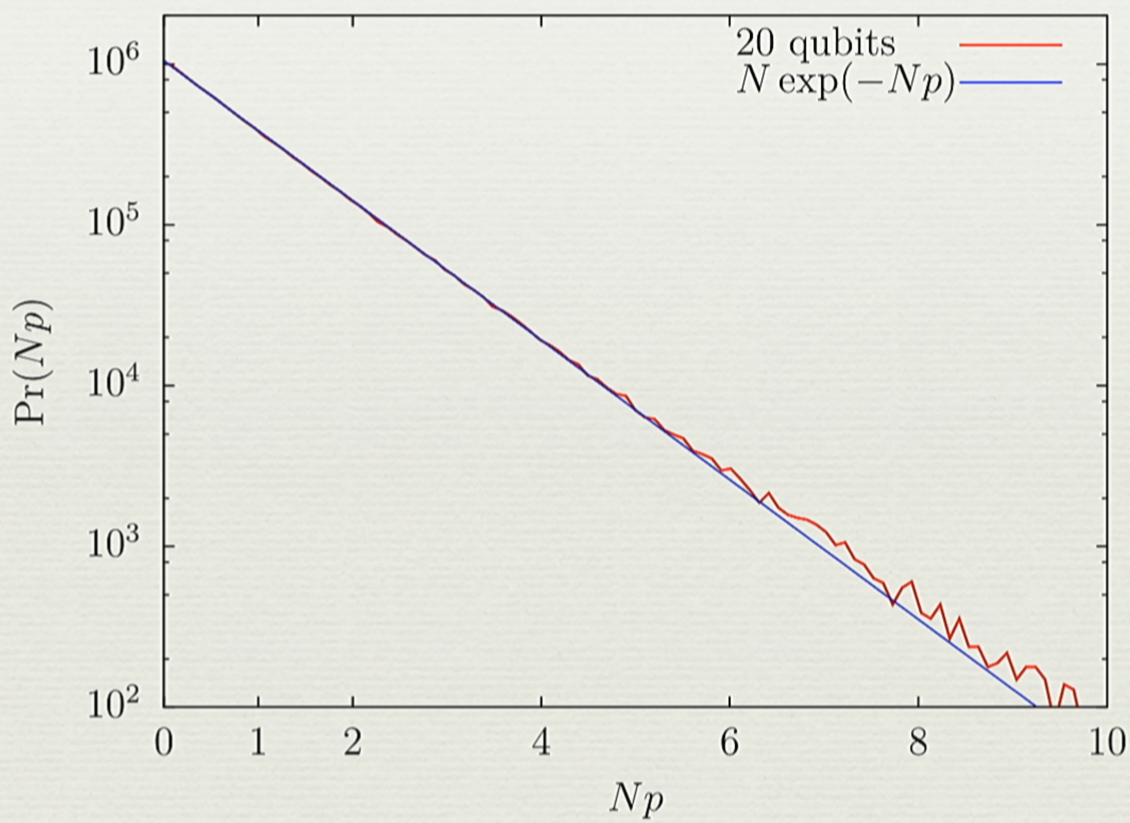


Entropy

- Entropy

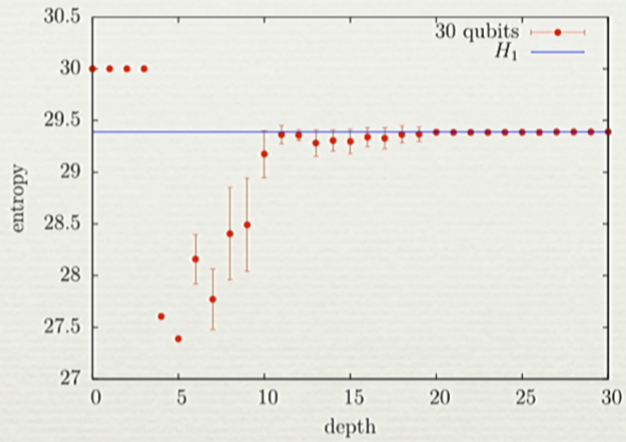
$$H = -\sum_{i=1}^{N} p_i \ln p_i$$

- For Porter-Thomas distribution $f_{\rm PT}(p) = N\exp(-Np)$

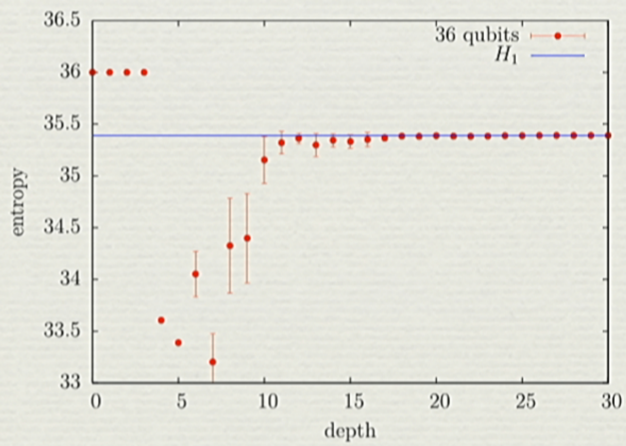
$$H_1 = -\int_0^\infty N e^{-Np} p \ln p \, N dp = \ln N - 1 + \gamma$$

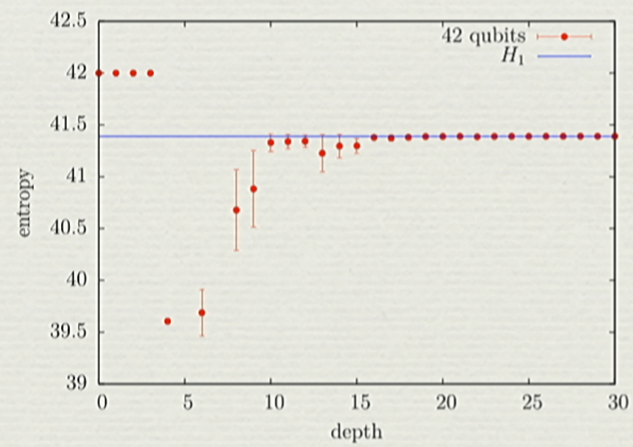
$\gamma \approx 0.577$ Euler's constant

Convergence to Porter-Thomas



$$H_1 = \log_2 N - \frac{1+\gamma}{\log_2}$$

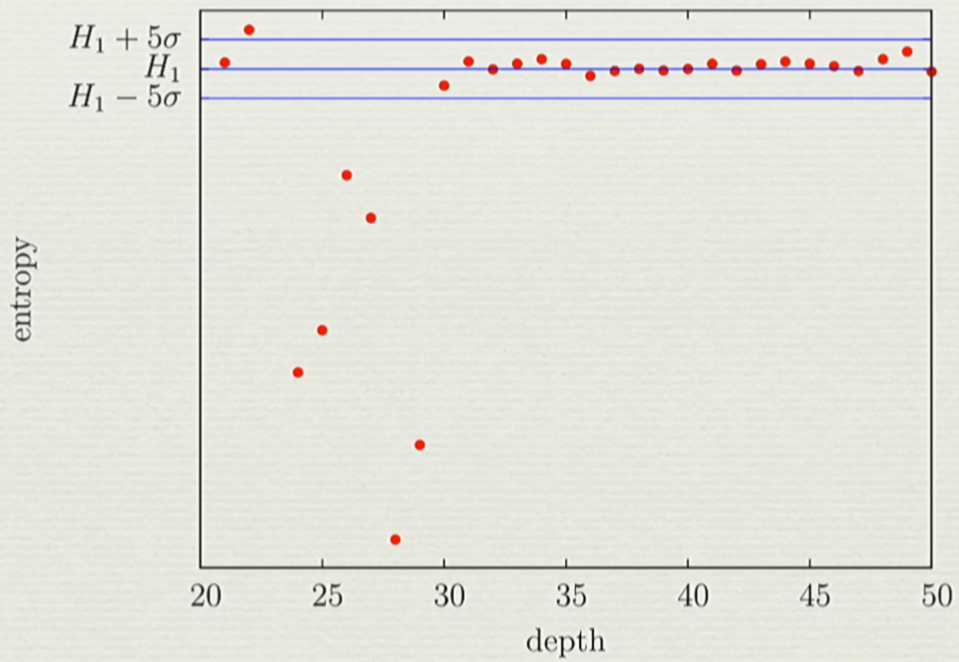




- Simulations for 36 and 42 qubits by Misha Smelyanskiy.

Convergence to Porter-Thomas

- Entropy is a Gaussian variable
- Standard deviation $\sigma \approx \frac{3}{4}\frac{1}{N^{1/2}}$



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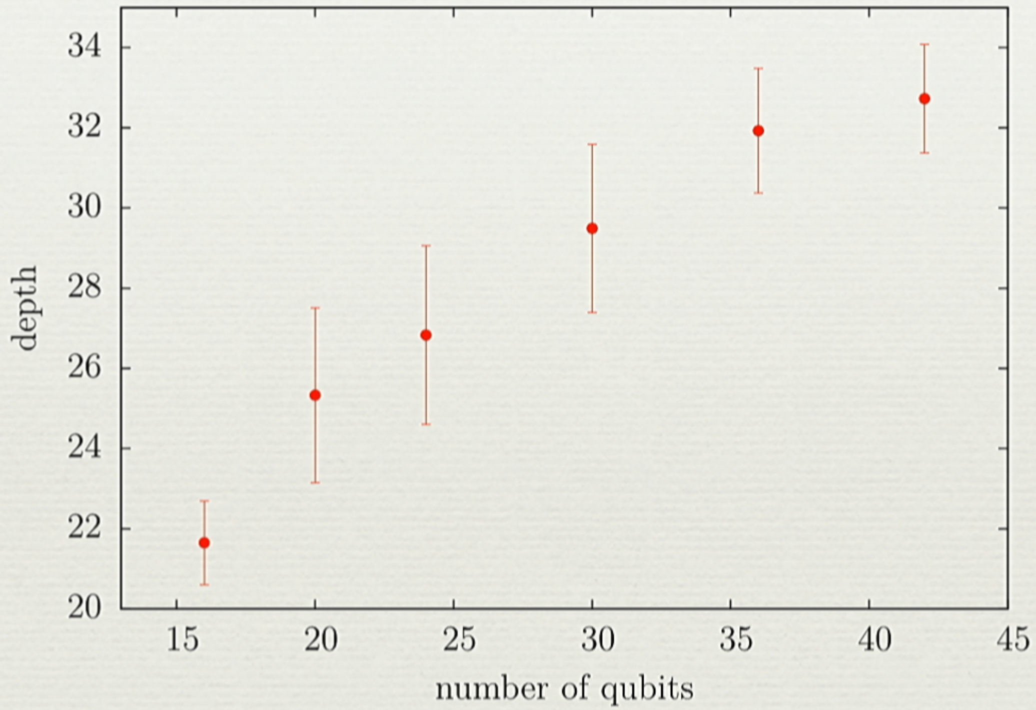
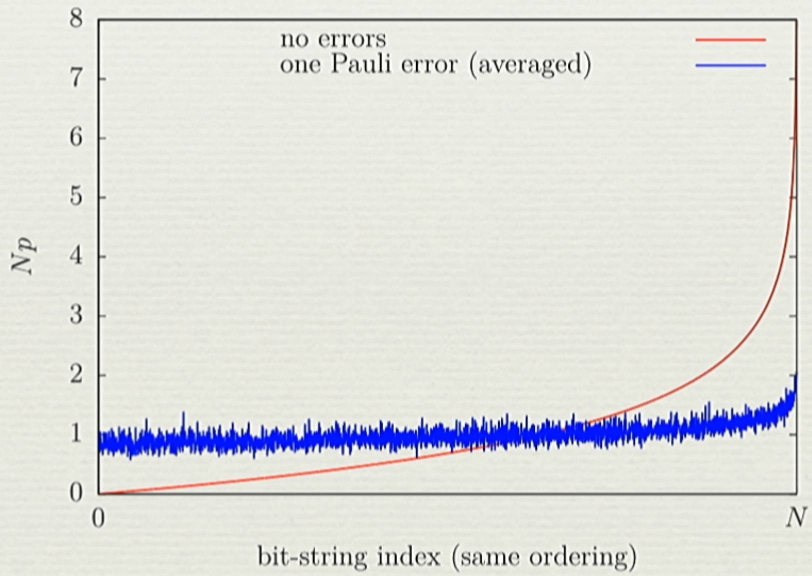


- Sublinear
- Should be proportional to $\sqrt{n}$ in 2D (Beals et al, 2013).

Relation chaos

- Random quantum circuits are examples of chaotic systems
- Very sensitive to errors
  - single Pauli error completely destroys the output distribution



- Requires very high fidelity classical simulations

Sampling on classical computers

- Evolving the full wave function.
  - Memory requirement grows exponentially with the number of qubits.
  - Requires at least 2.5 Petabytes for 48 qubits — the limit of what can be done on todays supercomputers.
- Calculating $\langle x_i|U|0\rangle$ using tensor transactions
  - Exponential in the treewidth of the quantum circuit.
  - Up to depth 25 for 48 qubits.
- Using the stabilizer mechanism (Bravyi and Gosset, 2016).
  - Exponential in the number of T gates.
- Mapping to an Ising model with imaginary temperature

$$p_i = \lambda \left| \sum_s e^{i\theta H_i(s)} \right|^2 \qquad H_i(s) = h_i \cdot s + s \cdot \hat{J} \cdot s$$

  - No structure.
  - Strong sign problem.

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Relation to computational complexity

$$Z = \sum_s e^{i\theta H_i(s)}$$

- Strong computational complexity conjecture: Z can not be probabilistically approximated asymptotically with an NP-oracle (Fujii and Morimae 2013, Bremner et. al. 2015).
- Theorem: if p_i can be classically sampled then Z can be approximated with an NP-oracle (Bremner et. al. 2015).
- Contradiction.

Sampling on classical computers

- Classical simulations fail for random quantum circuits with more than approximately $n = 48$ and depth 25.
- The best what classical computers can do in polynomial time is to sample from the uniform distribution.

Figure of merit

- Sample on a classical computer
  - basically from the uniform distribution.
- Sample on a quantum device
  - from $p_i = |\langle x_i|U|0\rangle|^2$ or approximation.
- Need a figure of merit to distinguish the distributions, to measure the quality of quantum sampling and to test quantum supremacy.

Cross entropy

- Cross entropy $H(p_1,p_2) = -\sum_{i=1}^{N} p_{1i} \ln p_{2i}$

$$p_{0i} = \frac{1}{N}$$

$$H_0 \equiv H(p_0, p_U) = -\sum_{i=1}^{N} \frac{1}{N} \ln p_U(x_i) = -\int_0^\infty N e^{-Np} \ln p \, dp = \ln N + \gamma$$

$$H_1 = -\sum_{i=1}^{N} p_U(x_i) \ln p_U(x_i) = \ln N - 1 + \gamma$$

$$\Delta H = H_0 - H_1 = 1$$

- Hints to a test of quantum supremacy.

Figure of merit (cross entropy difference)

- Algorithm A that approximates circuit U.

$$\alpha \equiv H_0 - H(p_A, p_U) = H_0 + \sum_i p_A(x_i|U) \ln p_U(x_i)$$

- Shows the performance of algorithm A.
- Sampling from the uniform distribution $\alpha = 0$
- Quantum device without errors $\alpha = 1$

- Quantum supremacy is achieved if $1 \geq \alpha_{\mathrm{mea}} > C$
  - C is given by the performance of the best classical algorithm.

- How can we measure alpha?

Figure or merit

- Take sufficiently large sample $\{x_1^{\text{exp}}, x_2^{\text{exp}}, \ldots, x_m^{\text{exp}}\}$ of bit strings using (noisy) experimental quantum implementation.

- For small circuits, we can calculate $p_U(x_i^{\text{exp}})$

- Approximation for alpha

$$\alpha \approx H_0 + \frac{1}{m} \sum_{i=1}^{m} \ln p_U(x_i^{\text{exp}})$$

- For large enough circuits, $p_U(x_i^{\text{exp}})$ cannot be obtained numerically.

Circuit fidelity

- Alpha is related to circuit fidelity.

- Output of the experimental realization K of a random circuit U

$$\rho_K = \tilde{\alpha} U|0\rangle\langle 0|U^\dagger + (1-\tilde{\alpha})\sigma_K$$

- $\tilde{\alpha}$ is the circuit fidelity, σ_K represents the effect or errors

$$p_K(x_i) = \tilde{\alpha} p_U(x_i) + (1-\tilde{\alpha})\langle x_i|\sigma_K|x_i\rangle$$

$$H_0 - H(p_K, p_U) = H_0 - \tilde{\alpha} H_1 + (1-\tilde{\alpha}) \sum_i \langle x_i|\sigma_K|x_i\rangle \ln p_U(x_i)$$

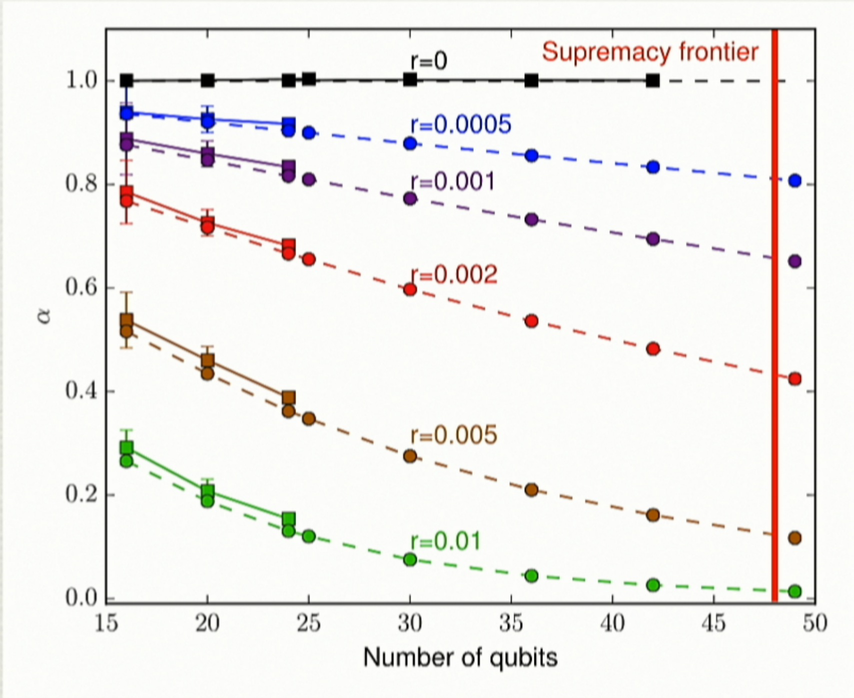
$$\alpha = \mathbb{E}_U[H_0 - H(p_K, p_U)] = H_0 - \tilde{\alpha} H_1 + (1-\tilde{\alpha}) H_0 = \tilde{\alpha}$$

- Depolarizing channel $\rho_K = \tilde{\alpha} U|0\rangle\langle 0|U^\dagger + (1-\tilde{\alpha})\frac{\mathbb{I}}{N}$ $\alpha = \tilde{\alpha}$

Test of quantum supremacy

- Digital error model

$$\tilde{\alpha} \approx \exp(-r_1 g_1 - r_2 g_2 - r_{\text{init}} n - r_{\text{mes}} n)$$



$$r = 10 r_1 = r_2 = r_{\text{init}} = r_{\text{mes}}$$

Test of quantum supremacy

- Cross entropy difference can be measured up to the quantum supremacy frontier with the help of supercomputers.
- Cross entropy difference can be extrapolated by varying the number of qubits, the number of non Clifford gates and the circuit depth.
- Fit to theory: $\tilde{\alpha} \approx \exp(-r_1 g_1 - r_2 g_2 - r_{\mathrm{init}} n - r_{\mathrm{mes}} n)$
- Better error model.

- Observation of a close correspondence between experiment, numerics and theory.

Summary

- Expect to be able to approximately sample the output distribution of shallow random circuits of 7 x 7 qubits with significant fidelity in the near term.
- It is impossible to approximately sample the output distribution of shallow random quantum circuits of about 48 qubits with state-of-the-art supercomputers (depth 25 or larger).
- Quantum supremacy.
- Relation to quantum chaos.
- Relation to computational complexity.
- New method to benchmark complex quantum circuits.
- The cross entropy method applies to other sampling problems.

Quantum machine learning

- Probability distributions produced by random quantum circuits are likely to be useless for machine learning.

- What about probability distributions produced by some other class of shallow circuits? Can be useful for machine learning?